# Chapter 3

# Iterative Decoding of BICM Systems

In the receiver with BICM, we separate the receiver into two parts: the inner detector and the outer decoding. With iterations between the MIMO inner detector and the outer decoder, the soft information of MAP decoder can be passed to the inner detector to improve the signal detection, and vice versa. The bit metrics (soft information) of the inner detector is computed according to [4] and the BCJR algorithm [16] is applied to calculate the a posteriori log likelihood ratios (LLRs) of the coded bits. In this chapter, we will describe the methods of calculating the bit metrics of the inner detector and the BCJR algorithm.

## 3.1 APP detector

The APP detector computes the a posteriori probability (APP) for coded bit $c_i$, i=0,…, $m$ , where the constellation size |A| $= 2^m$, given the channel received signal $y$ ( see equation (2.1) , $y = x + n$ ), as follows,

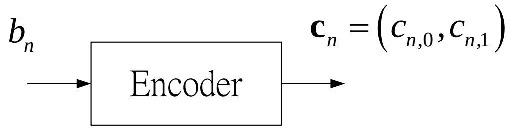
$$
\begin{aligned}
L_{D,M}(c_i) &= ln\frac{p(c_i=1|y)}{p(c_i=0|y)} \\
&= ln\frac{p(c_i=1)}{p(c_i=0)} + ln\frac{p(y|c_i=1)}{p(y|c_i=0)} \\
&= L_{A,M}(c_i) + ln\frac{\sum_{x\in\chi_{i,1}} P(y|x)\prod_{j\neq i} P(c_j)}{\sum_{x\in\chi_{i,0}} P(y|x)\prod_{j\neq i} P(c_j)} \\
&= L_{A,M}(c_i) + ln\frac{\sum_{x\in\chi_{i,1}} P(y|x)\exp\left(\sum_{j\in J_{i,x}} L_{A,M}(c_j)\right)}{\sum_{x\in\chi_{i,0}} P(y|x)\exp\left(\sum_{j\in J_{i,x}} \mathbf{L}_{A,M}(c_j)\right)} \\
&= L_{A,M}(c_i) + L_{E,M}(c_i) \qquad (3.1)
\end{aligned}
$$

where $L_{A,M}(c_i) = ln\frac{p(c_i=1)}{p(c_i=0)}$ is the a priori information of the detector, $\chi_{i,b}$ is the set of symbols with $c_i = b$, $b\in\{1,0\}$, $J_{i,x}$ is the set of indices j with $J_{i,x} = \{ j \mid j = 0,\ldots,m-1,\ j\neq i,\ c_j = 1\}$.

## 3.2 MAP decoder

In the BICM-ID system, soft information is passed from the detector to the decoder and vice versa at bit level. Therefore, we need to calculate the bit metrics before entering the bit-level de-interleaver. The BCJR algorithm is utilized to calculate the bit metrics we need.

Assume the code rate of the convolutional code is $R_c = 1/2$, $b_n$ is the $n^{\text{th}}$ information bit at the input of the decoder and $\mathbf{c}_n = (c_{n,0}, c_{n,1})$ is the output bits of the decoder.




The a posteriori LLR of $c_{i,j}$ for the MAP decoder is defined as

$$\begin{aligned} L_{D,Dec}\left(c_{n,j}\right) &\triangleq ln \frac{p\left[c_{n,j}=1 \mid \left\{L_{A,Dec}\left(\mathbf{c}_i\right)\right\}_{i=0}^{N_b-1} \text{ ; decoding}\right]}{p\left[c_{n,j}=0 \mid \left\{L_{A,Dec}\left(\mathbf{c}_i\right)\right\}_{i=0}^{N_b-1} \text{ ; decoding}\right]} \\ &= ln \frac{\sum_{\Omega_j^{(1)}} \sigma_n\left(m', m\right)}{\sum_{\Omega_j^{(0)}} \sigma_n\left(m', m\right)} \end{aligned} \tag{3.2}$$

and

$$\sigma_n\left(m', m\right) = \frac{p\left[S_{n-1}=m';\ S_n=m;\ \left\{L_{A,Dec}\left(\mathbf{c}_i\right)\right\}_{i=0}^{L_b-1}\right]}{p\left[\left\{L_{A,Dec}\left(\mathbf{c}_i\right)\right\}_{i=0}^{L_b-1}\right]} \tag{3.3}$$

where

$L_{A,Dec}\left(\mathbf{c}_n\right) = \left\{L_{A,Dec}\left(c_{n,0}\right), L_{A,Dec}\left(c_{n,1}\right)\right\}$

$L_{A,Dec}\left(c_{n,j}\right)$: a priori LLR of coded bit $c_{n,j}$ at the input of the decoder at time $n$

$S_n$: the state of information bit at time $n$

$\Omega_j^{(k)}$: the set of state transition from $m'$ to $m$ and the $j^{th}$ bit of output $\mathbf{c}_n$ is $k \in (0,1)$

Define the forward metrics $\alpha_n(m)$ as

$$\alpha_n(m) \triangleq p\left[S_n = m;\ \left\{L_{A,Dec}\left(\mathbf{c}_i\right)\right\}_{i=0}^{n}\right] \tag{3.4}$$

Define the backward metrics $\beta_n(m)$ as

$$\beta_n(m) \triangleq p\left[\left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=n+1}^{L_b-1} \mid S_n = m\right] \tag{3.5}$$

And define the transition metrics $\gamma_n(m',m)$ as

$$\begin{aligned}
\gamma_n(m',m) &\triangleq p\left[S_n = m; \left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=0}^{L_b\text{-}1} \mid S_{n-1} = m'\right] \\
&= p\left[\left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=0}^{L_b\text{-}1} \mid S_{n-1} = m', S_n = m\right] \cdot p\left[S_n = m \mid S_{n-1} = m'\right] \\
&= p\left[S_n = m \mid S_{n-1} = m'\right] \cdot \prod_{j=0}^{1} p\left[c_{n,j} \mid S_{n-1} = m', S_n = m\right]
\end{aligned} \tag{3.6}$$

where

$$p\left[c_{n,j} = 1 \mid S_{n-1} = m', S_n = m\right] = \frac{\exp\left(L_{A,Dec}(c_{n,j})\right)}{1+\exp\left(L_{A,Dec}(c_{n,j})\right)}$$

$$p\left[c_{n,j} = 0 \mid S_{n-1} = m', S_n = m\right] = \frac{\exp\left(-L_{A,Dec}(c_{n,j})\right)}{1+\exp\left(L_{A,Dec}(c_{n,j})\right)}$$

According to [16] ,

$$\begin{aligned}
\alpha_n(m) &= \sum_{m'=0}^{M-1} p\left[S_{n-1} = m';\ S_n = m; \left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=0}^{n}\right] \\
&= \sum_{m'} p\left[S_{n-1} = m';\ \left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=0}^{n}\right] \cdot p\left[S_n = m; \left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=0}^{n} \mid S_{n-1} = m'\right] \\
&= \sum_{m'} \alpha_{n-1}(m') \cdot \gamma_n(m',m)
\end{aligned} \tag{3.7}$$

and

$$\begin{aligned}
\beta_n(m) &= \sum_{m'=0}^{M-1} p\left[S_{n+1} = m'; \left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=n+1}^{L_b-1} \mid\ S_n = m\right] \\
&= \sum_{m'} p\left[S_{n+1} = m';\ L_{A,Dec}(\mathbf{c}_{n+1}) \mid\ S_n = m\right] \cdot p\left[\left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=n+2}^{L_b-1} \mid S_{n+1} = m'\right] \\
&= \sum_{m'} \beta_{n+1}(m') \cdot \gamma_{n+1}(m,m')
\end{aligned} \tag{3.8}$$

Therefore

$$\begin{aligned}\sigma_n(m',m) &= p\left[S_n=m;\ \left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=0}^{n}\right]\cdot p\left[S_n=m;\left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=0}^{L_b\text{-}1} \mid S_{n-1}=m'\right]\\ &\quad\cdot p\left[\left\{L_{A,Dec}(\mathbf{c}_i)\right\}_{i=n+1}^{L_b-1} \mid S_n=m\right]\\ &=\alpha_{n-1}(m')\cdot\gamma_n(m',m)\cdot\beta_n(m)\end{aligned} \tag{3.9}$$

Then

$$\begin{aligned}L_{D,Dec}(c_{n,j}) &= ln\frac{\sum_{\Omega_j^{(1)}}\alpha_{n-1}(m')\cdot\gamma_n(m',m)\cdot\beta_n(m)}{\sum_{\Omega_j^{(0)}}\alpha_{n-1}(m')\cdot\gamma_n(m',m)\cdot\beta_n(m)}\\ &= ln\frac{\sum_{\Omega_j^{(1)}}\alpha_{n-1}(m')\cdot\beta_n(m)\cdot p\left[S_n=m\mid S_{n-1}=m'\right]\cdot\prod_{j=0}^{1}p\left[c_{n,j}\mid S_{n-1}=m',S_n=m\right]}{\sum_{\Omega_j^{(0)}}\alpha_{n-1}(m')\cdot\beta_n(m)\cdot p\left[S_n=m\mid S_{n-1}=m'\right]\cdot\prod_{j=0}^{1}p\left[c_{n,j}\mid S_{n-1}=m',S_n=m\right]}\\ &= ln\frac{\sum_{\Omega_j^{(1)}}\alpha_{n-1}(m')\cdot\beta_n(m)\cdot p\left[S_n=m\mid S_{n-1}=m'\right]\cdot\prod_{j\neq j}p\left[c_{n,j}\mid S_{n-1}=m',S_n=m\right]}{\sum_{\Omega_j^{(0)}}\alpha_{n-1}(m')\cdot\beta_n(m)\cdot p\left[S_n=m\mid S_{n-1}=m'\right]\cdot\prod_{j\neq j}p\left[c_{n,j}\mid S_{n-1}=m',S_n=m\right]}\\ &\quad+ln\frac{p\left[c_{n,j}=1\mid S_{n-1}=m',S_n=m\right]}{p\left[c_{n,j}=0\mid S_{n-1}=m',S_n=m\right]}\\ &=L_{E,Dec}(c_{n,j})+L_{A,Dec}(c_{n,j})\end{aligned}$$

(3.10)

where

$$L_{E,Dec}(c_{n,j}) = ln\frac{\sum_{\Omega_j^{(1)}}\alpha_{n-1}(m')\cdot\beta_n(m)\cdot p\left[S_n=m\mid S_{n-1}=m'\right]\cdot\prod_{j\neq j}p\left[c_{n,j}\mid S_{n-1}=m',S_n=m\right]}{\sum_{\Omega_j^{(0)}}\alpha_{n-1}(m')\cdot\beta_n(m)\cdot p\left[S_n=m\mid S_{n-1}=m'\right]\cdot\prod_{j\neq j}p\left[c_{n,j}\mid S_{n-1}=m',S_n=m\right]}$$

is the extrinsic information and (3.11)

$$L_{A,Dec}(c_{n,j}) = ln\frac{p\left[c_{n,j}=1\mid S_{n-1}=m',S_n=m\right]}{p\left[c_{n,j}=0\mid S_{n-1}=m',S_n=m\right]} \tag{3.12}$$

is the intrinsic (a priori) information

The a posteriori LLR of information bit $b_n$ is

$$L_D\left(b_n\right)=ln\frac{\sum_{\mathrm{B}_j^{(1)}}\alpha_{n-1}\left(m'\right)\cdot\gamma_n\left(m',m\right)\cdot\beta_n\left(m\right)}{\sum_{\mathrm{B}_j^{(0)}}\alpha_{n-1}\left(m'\right)\cdot\gamma_n\left(m',m\right)\cdot\beta_n\left(m\right)} \tag{3.13}$$

where

$\mathrm{B}_j^{(k)}$: the set of state transition from $m'$ to $m$ and information bit $b_n$ is $k\in\left(0,1\right)$

To estimate information bit $\hat{b}_n$,

$$\begin{cases}\hat{b}_n=1, & \text{if } L_D\left(b_n\right)\geq 0\\ \hat{b}_n=0, & \text{if } L_D\left(b_n\right)<0\end{cases}$$