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零相關區域序列之研究

On Zero-Correlation Zone Sequences

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中文摘要

在許多的通訊與雷達系統裡常常需要用到有著良好自相關性(Autocorrelation)跟良好的交互相關性(Crosscorrelation)之序列。在有延遲的情況下，非零的自相關性會導致符元間干擾(Intersymbol Interference)，而在許多使用者的情況下，非零的交互相關性會導致多重路徑干擾(Multiple-access Interference)。在一個多重路徑衰弱(Multi-path Fading)的環境下，如果已知延遲的邊界，那序列必須在延遲的時間裡面，滿足良好的相關性。這一有良好關係的延遲範圍稱做零相關區域(Zero-Correlation Zone)。

在本論文中，我們提出四種有系統的架構法，去架構出零相關區域序列。這些方法更有彈性的去選擇序列的長度和零相關區域的長度，也可以產生出有限訊號群集點(Finite Constellation Points)之序列。我們舉了許多用數值表示的零相關區域序列並且展示了一些應用例子去證實我們的方法是有效的。

On Zero-Correlation Zone Sequences

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Abstract

Sequences with desired autocorrelation (AC) and cross-correlation (CC) properties are often needed in many communication and radar systems applications. Nonzero AC values at nonzero lags result in intersymbol or self interference (ISI) while nonzero CC values give rise to multiuser or multiple-access interference (MAI). For use in a multipath fading enviroment with known a delay spread bound, a family of sequences needs to meet these desired correlation requirements for only those correlation lags that lie within a range called zero-correlation zone (ZCZ) or interference free window (IFW). This thesis presents four systematic methods for constructing families of ZCZ sequences. The proposed methods unify various existing ZCZ sequence-generating algorithms. They provides more flexibility in choosing the sequence length, the ZCZ size, and the signal constellation. We give various numerical sequences and show several application examples to demonstrate the usefulness of our approaches.

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Chapter 1

Introduction

Many communication and radar applications necessitate the use of sets of sequences with good correlation properties. For use either as the training signal in the preamble or as the signature codes of a spread spectrum multiple access network, one would prefer to have a family of sequences whose autocorrelation (AC) function has a single peak at the zero delay ($\tau = 0$) and whose cross-correlation (CC) values are identically zero. Such sequences can be used to avoid or minimize (i) the interference from other users or other antennas if multiple transmit antennas were in place and (ii) self-interference (e.g., inter-symbol interference, ISI) due to multiple propagation paths. Practical considerations also require that the sequence length be arbitrary and the family size be as large as possible while maintaining the desired AC and CC properties. Similar requirements are called for in designing pulse compressed radar signal or two-dimensional array wave that has a time-frequency ambiguity function which achieves the minimum resolution.

For periodic sequences correlation peaks at τ equals multiple of a period are inevitable. Besides these periodic peaks, it is impossible to have zero periodic CC and AC at all other lags. In fact the bounds on CC and AC of sequences derived in [1] and [2] indicate that there is a tradeoff between AC and CC when designing sequences. In a multipath fading environment, however, the ideal correlation properties are not required to suppress the interference belongs to categories (i) and (ii). In fact, if the channel's maximum delay spread T_m and the maximum co-channel users' (distance) separation D_m are known, then one only needs to require that the correlation is low enough within a period of $T_m + D_m/c$ seconds, where c is the speed of light, to minimize the interference. The period is called zero-correlation zone (ZCZ), interference-free window (IFW) or low-correlation zone (LCZ) [3] [4], depending on if the zero-correlation or low-correlation requirement is imposed. Sequences that meet these requirements are referred to as ZCZ or LCZ sequences. For these sequences, correlation values outside the ZCZ (LCZ) are of no concern because they have little or no impact on the system performance. In this thesis, we focus on the design of ZCZ sequences. Note that the zero-correlation requirement imposes a severe design constraint, hence a ZCZ sequence set usually do not have a family size as large as that of a LCZ family with the same sequence length and signal constellation.

Several ZCZ families have been proposed. The PS sequences [8] have zero AC values at some $\tau \neq 0$ and zero CC for all τ. Binary, ternary, [9], quadriphase, and polyphase sequence sets [10] have been constructed. In [11] a family of polyphase sequences was generated based on generalized Chirp-like sequences. Unfortunately, all these sequences are generated in an heuristic manner and there is no theorizing as to why they were so constructed. An exception is the transform domain approach suggested in [13] where a systematic method to generate families of sequences that have zero CC and periodic impulse-like AC was presented.

In this thesis, we present four systematic approaches for generating families of sequences whose periodic AC and CC functions satisfy a variety of ZCZ requirements. Our approach for constructing ZCZ sequences is elementary and simple. Based upon a basic binary sequence that satisfies the ZCZ requirement for AC, the first approach generates a ZCZ family via some unitary matrices. The second approach involves the notion of complementary sets [14],[15]. It uses a basic binary sequence which satisfies the ZCZ requirement for AC and a class of mutually orthogonal complementary sets to generate ZCZ families. We use the fact that a basic binary sequence can also be decomposed into a set of binary ZCZ sequences. The third approach is a combination of both the first and second methods. The ZCZ property is invariant under modulation if the modulating sequence is one with perfect AC. One can easily deduce the ZCZ properties and determine the family size. The last approach is a transform domain method.

The rest of this thesis is organized as follows. In Chapter 2, we give basic definitions of perfect AC sequences and complementary sequences, summarize transform domain characterization of AC and CC, and describe some existing ZCZ sets. Chapter 4 contains our main results, presenting four systematic procedures for constructing families of ZCZ sequences. The following chapter provides several numerical examples, showing that most existing ZCZ sequences can be produced by our approaches. More importantly we show that many new ZCZ families with better properties can be generated by judicious choices of the design parameters.

Chapter 2

Definitions and Some Basic Properties

2.1 The Welch-Sarwate bound

Sets of periodic sequences with good correlation properties are desired in many communication applications. Oftentimes we hope to have a set of sequences whose AC function has a single peak at the zero delay and whose CC values are identically zero. Such sequences can be used to avoid or minimize the interference from other antennas (or other users) and eliminate the ISI due to a multi-path channel. However, it is observed that a set of sequences having good AC properties, e.g., PN sequences and Gold sequences, does not have good CC properties. On the other hand, the ideal AC requirement can not be met if the set has good CC properties. Walsh-Hadamard code is a typical example. For convenience of reference we begin our discourse with

Definition 1 *Let X denote a set of K complex-valued sequences of period N, i.e., for every sequence $u \in X$, $u \in X$, $|i|_N = i \ (mod\ N)$, for all $i \in \mathbf{Z}$, $\mathbb{Z}$ being the set of integers. The periodic CC function $\theta_{uv}(\cdot)$ for sequences u, $v \in X$ is defined by*

$$\theta_{uv}(\tau) = \sum_{i=0}^{N-1} u(i)v(|i-\tau|_N)^*, \quad \tau \in \mathbb{Z}, \tag{2.1}$$

where and a^* denotes the complex conjugate of a. The periodic AC function $\theta_u(\tau)$ for the sequence u is simply $\theta_{uu}(\tau)$.

We assume that $\theta_{uu}(0) = N$ for all $u \in X$ then it is obvious that $|\theta_u(\tau)| \leq N$ and $|\theta_{uv}(\tau)| \leq N$ for all $u, v \in X$.

For a set of sequences X, the maximum periodic CC magnitude $\widetilde{\theta}_c$, and the maximum out-of-phase periodic AC magnitude $\widetilde{\theta}_a$ defined by

$$\begin{aligned} \widetilde{\theta}_c &= \max\{|\theta_{uv}(\tau)| : u, v \in X, u \neq v, 0 \leq l \leq N-1\} \\ \widetilde{\theta}_a &= \max\{|\theta_u(\tau)| : u \in X, 0 < l \leq N-1\} \end{aligned}$$

must satisfy an inequality known as the Welch-Sarwate bound [1], i.e.,

Theorem 1 *For any set X of K sequences of period N satisfying $\theta_u(0) = N$ for all $u \in X$,*

$$\left(\frac{\widetilde{\theta}_c^{\,2}}{N}\right) + \frac{N-1}{N(K-1)}\left(\frac{\widetilde{\theta}_a^{\,2}}{N}\right) \geq 1. \tag{2.2}$$

The above theorem implies that $\widetilde{\theta}_a \geq N\sqrt{\frac{K-1}{N-1}}$ and for the special case $K = 2$, we have $\widetilde{\theta}_a \geq \frac{N}{\sqrt{N-1}} \geq \sqrt{N}$. Thus even for a set of only two sequences of length N with perfect CC properties, i.e., $\theta_c(l) = 0$ for all l, it is impossible for them to yield the ideal AC function $\theta_a(n) = 0$. Since one can not have both ideal AC and CC properties, the next best thing one can expect is to have the ideal properties within a limited range.

Definition 2 *A set of K sequences $\boldsymbol{C} = \{C_0, C_1, \cdots, C_{K-1}\}$ of period N is called a ZCZ family (or sequence set) if the periodic AC and CC functions of all its member sequences satisfy the requirements of an ideal set for $|\tau| < T, T < N$. In other words, $\theta_{c_ic_j}(\tau) = 0, \theta_{c_ic_i}(\tau) = \theta_{c_i}(0)\delta(\tau)$, for $c_i \neq c_j, |\tau| \leq T$.*

Because of the AC and CC properties, ZCZ sequences can be used as the signature sequences for a cellular CDMA system that operates in an environment whose delay spread is less than the period of the sequences. Without the bandlimiting effect, such a system is free from ISI and multiple access interference (MAI) that limit the system capacity.

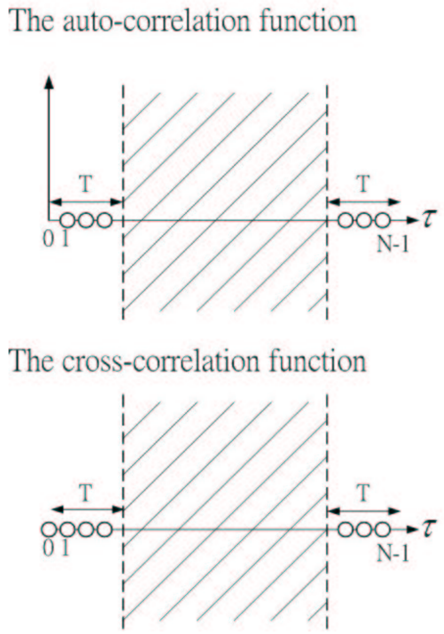


Figure 2.1: The correlation properties of a ZCZ family.

ZCZ sequences can also be used as training sequences for a MIMO-OFDM receiver to estalish a link within the preamble period. The link setup process includes at least package detection, frame and frequency/carrier synchronization and channel estimation. Such a synchronization procedure involves the detection and estimation of some signal and channel parameters in a multiple antenna scenario. Conventional maximum likelihood (ML) paradigm solves this data-aided estimation and detection problem by an estimator-correlator type receiver structure and necessitates the ideal AC and CC properties on the part of the training sequences. Invoking *Theorem 1*, we assign properly-selected sequences with period $N(N \geq 2)$ to different transmit antennas. For a MIMO receive it is necessary to separate signals emitting from different transmitting antennas so as to resolve and estimate the impulse response of each sub-channels between any pair of transmit-receive antennas. One way to achieve near-optimal channel estimation is to use pilot sequences that have perfect CC properties, i.e., $\widetilde{\theta_c} = 0$. If there are K transmit antennas, we need at least K different preamble sequences.

Although there are many proposals for generating ZCZ sequences, there still lacks a systematic theory behind the existing construction methods. Our intention is to present a simple and systematic theory in constructing families of ZCZ sequences. It will be shown that the new derivations contain as special cases many if not all of the existing constructions.

2.2 Perfect AC sequences

In this section, we present some perfect AC sequences that have a Dirac-like periodic AC functions whose values are zeros for all non-zero lags. Notations and definitions are given first and then the so-called FZC sequences is introduce. Last we introduce a way In [8] and [13] of generating perfect AC sequences, which have lengths of square integers and polyphase components in both time and frequency domain.

2.2.1 Notations and definitions

Definition 3 *Let us define the* $N \times N$ *DFT matrix with index* m *as*

$$F^{(N,m)}(k,l) = [W_N^{-klm}] = (W_N^m)^{-kl}, \tag{2.3}$$

where m *is an integer,* $k, l = 0, 1, \ldots, N-1$, $W_N = e^{j2\pi/N}$ *and* $j = \sqrt{-1}$.

Definition 4 *The diagonalized matrix* $D(\{x_l\})$ *associated with the sequence* $\{x_l\}$ *is defined as*

$$D(\{x_l\}) = diag(\{x_l\}). \tag{2.4}$$

Lemma 1 *The periodic autocorrelation function of* $x(n)$, $\theta_{xx}(n)$, *is equivalent to the circular convolution function between* $x(n)$ *and* $x^*(-n)$.

Proof:

The periodic autocorrelation function of a sequence of length N, $\{x(n)\}$, is defined as

$\theta_{xx}(n) \triangleq \sum_{\tau=0}^{N-1} x(\tau)x^*(\tau-n)$. The circular convolution function, which is denoted as $\otimes$, between $x(n)$ and $x^*(-n)$ is

$$\begin{aligned} x(n) \otimes x^*(-n) &= \sum_{\tau=0}^{N-1} x(\tau)x^*(\tau-n) \\ &= \theta_{xx}(n) \end{aligned} \quad (2.5)$$

Using the same argument, we conclude that the cross-correlation function $\theta_{xy}(n)$ is equivalent to $x(n) \otimes y^*(-n)$.

Lemma 2 *The DFT of the periodic CC function $\theta_{xy}(\tau)$ of two period-N sequences, $\{x(n)\}$ and $\{y(n)\}$, is equal to X(k)Y*(k), where $X(k)$ and $Y(k)$ are the DFTs of $\{x(n)\}$ and $\{y(n)\}$, respectively.*

Corollary 1 *The AC function $\theta_{x,x}(n)$ is equivalent to $x(n) \otimes x^*(-n)$ and $\Theta_{xx}(k) = DFT[\theta_{xx}(n)] = |X(k)|^2$, where $\otimes$ denotes circular convolution. Hence a sequence $\{x(n)\}$ has an impulse-like AC, i.e., $\theta_{xx}(n) = N_c\delta(n)$, iff $|X(k)|^2$ is a constant for all k.*

Corollary 2 *Up-sampling of $\{x(n)\}$ is equivalent to a repetition of $X(k)$. Hence a perfect AC sequence can be generated by up-sampling a shorter perfect AC sequence.*

2.2.2 Frank-Zadoff-Chu (FZC) sequences

The well-known complex sequences, Frank-Zadoff-Chu (FZC) sequences [7], [6] render a Dirac-like periodic AC functions whose values are zeros for all non-zero lags. More specifically,

Definition 5 *A FZC sequence $\{a_k\}$ of length N has entries of unity-modulus complex numbers, i.e., $a_k = e^{j\alpha_k}, k = 0, \ldots, N-1$. When N is even, they are given by*

$$a_k = \exp\left(j\frac{M\pi k^2}{N}\right), \quad (2.6)$$

where M is an integer prime to N, while if N is odd,

$$a_k = \exp\left(j\frac{M\pi k(k+1)}{N}\right), \quad (2.7)$$

where M is also an integer prime to N.

It is proved that $\theta_a(n) = N\delta(n)$, for $n = 0, 1, \ldots, N-1$. The single maximum of magnitude N occurs at $n = 0$.

2.2.3 PS sequences: square-length perfect AC sequences

Several definitions are needed for specifying a class of perfect AC sequences.

Definition 6 *A set of constant module numbers,* $\mathbf{B} = \{b_i : ||b_i|| = d > 0, i = 0, \ldots, N_b - 1\}$*, where b_i are not necessarily distinct, is called a set of basic symbols.*

Definition 7 *The quotient and residual functions Q and R corresponding to the devisee and divisor (α, β) are defined as*

$$Q(\alpha, \beta) = q, \quad R(\alpha, \beta) = r, \tag{2.8}$$

where α and q are integers, β is a natural number, and $\alpha = q\beta + r$ with $r = 0, 1, \ldots, \beta - 1$.

Definition 8 *A basic orthogonal sequence matrix G of size $N_b \times N_b$ associated with the set of basic symbols $\mathbf{B} = \{b_i = W_{N_b}^i\}$ and $1 \leq m \leq N_b - 1$ are the matrix*

$$G = F^{(N_b, -m)} D(\{b_i\}) = [g_{i,j}]. \tag{2.9}$$

Definition 9 *The sequence $\{g_p\}$ of length N_b^2 obtained by selecting elements from the matrix G through [5]*

$$g_p = g_{Q(p,N_b),R(p,N_b)} \tag{2.10}$$

is called a basic orthogonal sequence. Regarding the periodic sequence $\{g(p)\}$ as a vector, a basic orthogonal sequence can also be obtained by

$$\vec{g} = [g(0), g(1), \ldots, g(N_b^2 - 1)]^T = vec(G^T), \tag{2.11}$$

where $vec(\cdot)$ denotes the stacking operator,

It can be proved a basic orthogonal sequence has a perfect AC function.

2.3 Complementary sequences

Definition 10 *The aperiodic CC function of two length-ρ sequences $u \equiv \{u(n)\}, v \equiv \{v(n)\}$, $0 \leq n < \rho$ is defined as*

$$\psi_{uv}(\tau) = \sum_{n=0}^{\rho-\tau} u(n)v^*(n-\tau) \tag{2.12}$$

The aperiodic AC function for the sequence $u(n)$ is simply $\psi_{uu}(\tau)$.

Definition 11 *A set of Q sequences $\boldsymbol{E} = \{E_0, E_1, \ldots, E_{Q-1}\}$ forms a complementary set if and only if*

$$\sum_{i=0}^{Q} \psi_{E_iE_i}(\tau) = 0, \ \forall\, \tau \neq 0 \tag{2.13}$$

For the special case of binary sequences, a set is said to be complementary if the total number of pairs of like elements with a given separation is equal to the total number of pairs of unlike elements with the same separation in these sequences.

Definition 12 *A set of sequences $\boldsymbol{F} = \{F_0, F_1, \ldots, F_{Q-1}\}$ is said to be a* **mate** *of the set of sequences $\mathbf{E} = \{E_0, E_1, \ldots, E_{Q-1}\}$ if*

1. *the length of E_i is equal to the length of F_i, for $0 \leq i < Q$,*
2. *the set $\mathbf{F}$ is a complementary set,*
3. *$\sum_{i=0}^{Q-1} \psi_{E_iF_i}(\tau) = 0, \ \forall\, \tau$.*

Definition 13 *A collection of complementary sets of sequences $\{\mathbf{E^0}, \mathbf{E^1}, \ldots, \mathbf{E^n}\}$, where each set contains Q sequences, is said to be* **mutually orthogonal** *if every two complementary sets in the collection are mates of each other.*

It has been shown in [16] that the number of mutually orthogonal sets cannot exceed the number of sequences in the set.

2.4 Some Known ZCZ sequences

2.4.1 Upper bounds

In this section, we give the definition of ZCZ sequences and show a set of ZCZ sequences can be maintained ZCZ correlations via some special modulation.

Definition 14 *An (N, K, T) ZCZ family is a set of K length-(period-)N ZCZ sequences $\boldsymbol{C}$ whose zero-correlation zone has a width of T.*

Lemma 3 *The AC and CC functions of a set of sequences are invariant up to a scale factor under the modulating operation by a perfect AC sequence A. The modulating operation, denoted by $\ominus$, is defined by*

$$\widetilde{U}(\tau) = U \ominus A = \theta_{UA}(\tau) = \sum_{i=0}^{N-1} U(i)A(|i-\tau|_N)^*, \quad \tau = 0, 1, \ldots, N-1, \tag{2.14}$$

where U is one sequence of a set of sequences.

Proof:

Given two sequences U, V and a prefect AC sequence A, all having the same period N. Denote by

$$\overline{A} = (\overline{a}_0, \overline{a}_1, \cdots, \overline{a}_{N-1})$$

$$\overline{U} = (\overline{u}_0, \overline{u}_1, \cdots, \overline{u}_{N-1})$$

$$\overline{V} = (\overline{v}_0, \overline{v}_1, \cdots, \overline{v}_{N-1})$$

the DFTs of A, U, and V, respectively, and by Θ_{UV} the DFT of θ_{UV}. Consider the modulated sequences

$$\widetilde{U} = U \ominus A, \quad \widetilde{V} = V \ominus A$$

Corollary 1 and the normalization $\{|\overline{a}_k|^2 = 1, k = 0, 1, \cdots, N-1\}$ implies that $\Theta_{UV} = \Theta_{\widetilde{U}\widetilde{V}}$ since the kth entry $\Theta_{\widetilde{U}\widetilde{V}}(k)$ is equal to the kth entry $\Theta_{UV}(k)$:

$$\Theta_{\widetilde{U}\widetilde{V}}(k) = \overline{u}_k\overline{a}_k^*\overline{v}_k^*\overline{a}_k = |\overline{a}_k|^2\overline{u}_k\overline{v}_k^* = \overline{u}_k\overline{v}_k = \Theta_{UV}(k) \tag{2.15}$$

In [4], the bounds on the correlation of the ZCZ sequences are established as follow

Corollary 3 *For a set of K ZCZ sequences of period N, the length T of zero-correlation zone is upper-bounded by*

$$K(T+1) \leq N \tag{2.16}$$

2.4.2 PS ZCZ sequences

Using the basic orthogonal sequence $\{g_p\}$ generated in Section 2.2.3, we form the $N_s \times K$ matrix H

$$H = [h_{i,k}] \tag{2.17}$$

where $N_s = KN_b^2$, K is a natural number,

$$h_{i,k} = \sum_{p=0}^{N_b^2-1} g_p \delta(i-k-pK) \tag{2.18}$$

and $\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$.

The *PS sequence matrix* C of size $N_s \times K$ (or $KN_b^2 \times K$) is defined as

$$C = [c_{l,k}] = \frac{1}{N_b} F^{(N_s,-1)} H. \tag{2.19}$$

Each column vector of C, $\{c_{l,k}, l = 0, 1, \cdots, N_s - 1\}$, is a period of a sequence called a *PS sequence.*

2.4.2.1 Properties of the PS sequences

(PS.1) Autocorrelation function:

The AC function of the PS sequence is given by

$$\begin{aligned} \theta(c)(\tau) &= \sum_{l=0}^{N_s-\tau-1} c_{l+\tau,k} c_{l,k}^* + \sum_{l=N_s-\tau}^{N_s-1} c_{l+\tau-N_s,k} c_{l,k}^* \\ &= N_s W_{N_s}^{\tau k} \delta(R(\tau, N_b^2)). \end{aligned} \tag{2.20}$$

The AC function has a nonzero value only when $R(\tau, N_b^2) = 0$; i.e., $\tau = IN_b^2$, where I is an integer. We can control the interval or period by properly choosing the value of N_b^2. On the contrary, the PN sequence has nonzero values of the AC function at all intervals. The PS sequence has better CC properties than the PN sequence. Fig. 2.2 is a typical plot for the AC function of the PS sequences.

(PS.2) Cross-correlation function:

Let us denote two PS sequences as $\{c_{l,k^I}\}$ and $\{c_{l,k^{II}}\}$. The CC function of the two sequences is 0 if $k^I \neq k^{II}$.

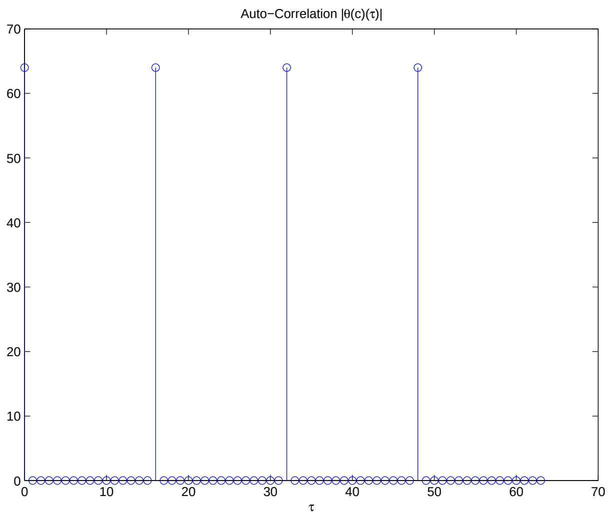


Figure 2.2: The autocorrelation function of the PS sequence.($K = 4, N_b^2 = 16$, and $N_s = KN_b^2 = 64$)

2.4.3 Ternary ZCZ sequences

The method of constructing ternary ZCZ sequences described in [9] starts with M ternary subsets of sequences. Each subset is created from a different binary seed set, where M is the number of seed sets. The seed sets are of size $L_0 \times L_0$ and although sequences within each seed are orthogonal, no orthogonality between subsets is required. Equal length zero padding vectors are inserted between elements of each sequence of every seed set, assuming the length of zero padding is Z_0. Each seed set is transformed into a subset of sequences of length $N = L_0(Z_0 + 1)$ with $T = Z_0$. The ZCZ properties between subsets are provided by chip shifting each subset a different number of chips, τ_m for $m = 1, 2, \ldots, M$. Each sequence created from the same seed set is shifted the same number of chips. s_m^l represents the l-th sequence, where $l = 1, 2, \ldots, L_0$, created from zero padding the m-th seed set of ML_0 sequences is given by P_M and their overall ZCZ is given in

$$p_M = \begin{bmatrix} S_1(\tau_1) \\ S_2(\tau_2) \\ \vdots \\ S_M(\tau_M) \end{bmatrix}. \tag{2.21}$$

$$T = Z_M \tag{2.22}$$

where Z_M is the minimum number of zeros between elements of all sequences.

When a specific ZCZ is required, Z_M can be used to calculate the chip-shit applied to each set, $\tau_m = (Z_M + 1)(m - 1)$ and to ensure the complete set of sequences have ZCZ properties. An example is illustrated in Fig. 2.3 for $M = 2$,$L_0 = 4$, $Z_0 = 3$, $\tau_1 = 0$ and $\tau_2 = 1$

Figure 2.3: Example of Ternary ZCZ sequences for for $M = 2$,$L_0 = 4$, $Z_0 = 3$,$\tau_1 = 0$ and $\tau_2 = 1$

2.4.4 ZCZ Sets Derived From Perfect Sequences and Unitary Matrices

In [10], two algorithms are proposed to derive polyphase ZCZ sets from perfect sequences and unitary matrices.

(Poly.1) Let $A_0 = (a_0^0, a_1^0, \ldots, a_{N'}^0)$ be a perfect AC sequence of period N'. Select two integers N'' and N_r that are related by

$$N' = N'' \cdot N_r, \; 1 \leq N'' < N', \; 1 < N_r \leq N' \tag{2.23}$$

Using these integers, we obtain N_r perfect sequences $A^i (0 \leq i \leq N_r - 1)$ via

$$\begin{aligned} A^i &= (a_0^i, a_1^i, \ldots, a_{N'-1}^i) \\ &= (a_{iN''}^0, a_{iN''+1}^0, \ldots, a_{N'-1}^0, a_0^0, \ldots, a_{iN'-1}^0) \end{aligned} \tag{2.24}$$

That is, A^i is a perfect AC sequence derived from shifting A^0 cyclically to the left by $i \cdot N''$ positions.

Let $\mathbf{H_{N_r}^n}$ be an $N_r \times N_r$ unitary matrix defined by

$$\mathbf{H_{N_r}^n} = \frac{1}{\sqrt{N_r}} \begin{bmatrix} h_{0,0}^n & h_{0,1}^n & \cdots & h_{0,N_r-1}^n \\ h_{1,0}^n & h_{1,1}^n & \cdots & h_{1,N_r-1}^n \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r-1,0}^n & h_{N_r-1,N_r-1}^n & \cdots & h_{N_r-1,N_r-1}^n \end{bmatrix} \tag{2.25}$$

Let $\mathbf{C^0}$ be a set of N_r perfect sequences of period N' given by

$$\begin{aligned} \mathbf{C^0} &= \{C_0^0, C_1^0, \ldots, C_{N_r-1}^0\} = \{A^0, A^1, \ldots, A^{N_r-1}\} \\ C_i^0 &= (c_0^{0,i}, c_1^{0,i}, \ldots, c_{N'-1}^{0,i}) = (a_0^i, a_1^i, \ldots, a_{N'-1}^i), \; 0 \leq i \leq N_r - 1 \end{aligned} \tag{2.26}$$

By using $\mathbf{H^n}$ and $\mathbf{C^0}$, we obtain the sequence set

$$\mathbf{C^n} = \{C_i^n, \; 0 \leq i \leq N_r - 1\} \tag{2.27}$$

where

$$C_i^n = \left(c_0^{n,i}, c_1^{n,i}, \ldots, c_j^{n,i}, \ldots, c_{N'N_r^n-1}^{n,i}\right), \; 0 \leq j \leq N'N_r^n - 1 \tag{2.28}$$

and $c_j^{n,i}$ is derived from the following recursive procedure:

$$c_j^{n,i} = h_{i,|j|_{N_r}}^n \cdot c_{\lceil j/N_r \rceil}^{n-1,|j|_{N_r}} \tag{2.29}$$

with $\lceil j/N_r \rceil \overset{def}{=}$ the largest integer $\leq j/N_r$, and $|x|_N \overset{def}{=} x \pmod N$.

Theorem 2 *The sequence set* $\mathbf{C^n}$ *derived from* (2.23) − (2.29) *is a ZCZ sequence set with* $(N, K, T) = (N'N_r^n, N_r, (N' - 2)N_r^{n-1})$

(Poly.2) Let $N'' = 1$, $N_r = N'$ and define N_r' and N''' via

$$N_r' = N''' \cdot N', \; N''' > N' \tag{2.30}$$

Let $\mathbf{D^n}$ be an $N_r' \times N_r'$ unitary matrix whose (i,j)th entry is $d_{i,j}^n/\sqrt{N_r'}$ and $\mathbf{E^0}$ be a sequence set composed of N_r' perfect sequences of period N' defined by

$$\begin{aligned} \mathbf{E^0} &= \{E_0^0, E_1^0, \ldots, E_i^0, \ldots, E_{N_r'-1}^0\} \\ &= \{A^0, A^1, \ldots, A^{|i|_{N'}}, \ldots, A^{N'-1}\}, \; 0 \leq i \leq N_r' - 1 \\ C_i^0 &= (e_0^{0,i}, e_1^{0,i}, \ldots, e_j^{0,i}, \ldots, e_{N'-1}^{0,i}) \\ &= (a_0^{|i|_{N'}}, a_1^{|i|_{N'}}, \ldots, a_j^{|i|_{N'}}, \ldots, a_{N'-1}^{|i|_{N'}}), \; 0 \leq j \leq N' - 1 \end{aligned} \tag{2.31}$$

Based on $\mathbf{D^n}$ and $\mathbf{E^0}$, we construct the sequence set $\mathbf{E^n}$ as follows.

$$\begin{aligned} \mathbf{E^n} &= \{E_0^n, E_1^n, \ldots, E_i^n, \ldots E_{N_r'-1}^n\} \\ E_i^n &= (e_0^{n,i}, e_1^{n,i}, \ldots, e_j^{n,i}, \ldots, e_{N'N_r'^n-1}^{n,i}) \end{aligned} \tag{2.32}$$

where

$$e_j^{n,i} = d_{i,|j|_{N_r'}}^n \cdot c_{\lceil j/N_r' \rceil}^{n-1,|j|_{N_r'}}, \; 0 \leq i \leq N_r' - 1, \; 0 \leq j \leq N'N_r'^n - 1 \tag{2.33}$$

Theorem 3 *The sequence set* $\mathbf{E^n}$ *defined by* (2.23), (2.24) *and* (2.30) − (2.33) *is a ZCZ sequence set with* $(N, K, T) = (N'N_r'^n, N_r', (N' - 2)N_r'^{n-1})$

2.4.5 LA codes

Large Area (LA) codes belong to a family of ternary codes having the elements of ± 1 or 0. Their maximum correlation magnitude is unity and they also exhibit an zero-correlation zone. Denote the family of the K number of orthogonal ternary codes employing K number of ± 1 pulses by LA(N, K, T), which exhibit a minimum spacing of T-chip duration between non-zero pulses, while having a code length of N. All the codes in an LA code family share the same legitimate pulse positions.

Example 1 *The construction of LA*$(847, 16, 38)$ *code was described in [17] and [18], where the 16 pulse positions,* p_k*,* $k = 0, 1, \ldots, 15$*, at*

$$\{p_k\} = \{0, 38, 78, 120, 164, 210, 258, 308, 360, 414, 470, 530, 660, 732, 808\} \tag{2.34}$$

The autocorrelation function of LA$(847, 16, 38)$ *is plotted in Fig. 2.4*

In [17], the author proposed a scheme for determining the LA positions. Define the pulse spacing d_k, which is related to the difference of the pulse positions, by

$$d_k \triangleq \begin{cases} p_{k+1} - p_k & \text{for } 0 \leq k < K-1 \\ N - p_{K-1} & \text{for } k = K-1 \end{cases} \tag{2.35}$$

The constraints imposed on the pulse spacing d_k of the LA code are [17]

(LA.1) d_k should be even except for d_{K-1}

(LA.2) $d_k \neq d_{k'}$ for $0 \leq k \neq k' < K$

(LA.3) $\sum_{k \in \mathbf{S}} \neq \sum_{k' \in \mathbf{S}'}$ for any $\mathbf{S}$, $\mathbf{S}' \subset \{k | 0 \leq k < K\}$

These three constraints form a sufficient condition, guaranteeing that the number of pulses satisfying $p_k + n = p_{k'}$ is at most one for $0 < n < N$ and $0 \leq k, k' < K$. It then follows that the maximum correlation magnitude is simply one.

The proposal suggested in [20] imposes the same constraints except that d_k needs not have to be even. This modification helps reducing the required sequence length under the same family size and same ZCZ width constraints as those of [17].

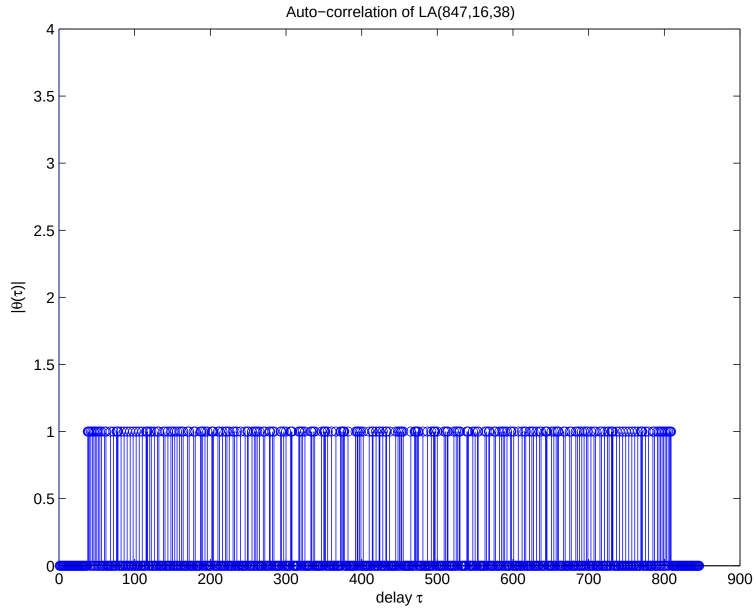


Figure 2.4: The autocorrelation function of LA$(847, 16, 38)$

2.4.6 LS codes

Although the correlations of the LS codes are aperiodic as defined in Definition 10, we can consider aperiodic correlations as periodic correlations in a window of length W_0 for padding a string of W_0 zeros in the end of codes. In this subsection, we construct the LS codes in the way of aperiodic correlations, and show the LS codes have a aperiodic zero-correlation zone. If padding enough zeros in the end of the LS codes, the periodic correlations of the the LS codes are the same as aperiodic correlations in the window of zero-correlation zone.

Assume there is a complementary set which consists of two complementary sequences of length ρ, we express the two complementary sequences in z-transform form, which are denoted as $C_0(z)$ and $S_0(z)$. This complementary's mate is consists of two complementary sequences, $C_1(z)$ and $S_1(z)$, which are given by

$$C_1(z) = z^{\rho-1} S_0(z^{-1}),\ S_1(z) = -z^{\rho-1} C_0(z^{-1}) \tag{2.36}$$

We use the sequences $C_0(z)$, $C_1(z)$, $S_0(z)$, and $S_1(z)$ described above, each of length ρ, to obtain a set of $K = 2^n$ sequences of length $N = K\rho + d$, where n is a natural number and $d = rho - 1$. We need an arbitrary $ptimesp$ Hadamard matrix $\mathbf{H}$, where $p = K/2$. We use the vector $\pi = [\pi_1, \pi_2, \ldots, \pi_p]$, $\pi_k \in \{0, 1\}$ to denote a binary expansion of an arbitrary integer, $0 \le n < 2^p$ so that $n = \sum_i \pi_{i=1}^p 2^i$.

Theorem 4 *Suppose that* $\mathbf{H} = [\mathbf{h_{i,j}}]$ *is a* $p \times p$ *Hadamard matrix, and* $(C_1(z), S_1(z)$ *is a mate of* $(C_0(z), S_0(z)$*, where the sequences have the length* ρ*. We define the sequence* $G_k(z)$*,* $1 \le k \le p$ *of length* $N = K\rho + d$ *as*

$$G_k(z) = \sum_{i=1}^{p} h_{k,i}[C_{\pi_i}(z) + z^{p\rho+d} S_{\pi_i}(z)] z^{(i-1)\rho} \tag{2.37}$$

where $K = 2p$ *and* $d = \rho - 1$*. Further, the sequences* $G_{p+1}, G_{p+2}, \ldots, G_K$ *are obtained when the binary expansion vector* π *in the formula above is replaced by its complementary* $\pi^* = [\pi_1^*, \ldots, \pi_p^*]$ *with* $\pi_k^* = |\pi_k + 1|_2$ *for each* $1 \le k \le p$*. Then,*

$$\psi_{g_k, g_l}(\tau) = 0, \ \forall_{|\tau| \le d} \tag{2.38}$$

for every $1 \le k, l \le K$ *and* $\tau \ne 0$ *if* $k = l$

The sequence set $\mathbf{G} = \{G_1(z), G_2(z), \ldots, G_{2p-1}(z)\}$ is called the LS codes. It has an aperiodic zero-correlation zone with length d, which is proved in [22]. If we pad d zeros in the end of each sequence of the sequence set $\mathbf{G}$, the zero-padding set has a periodic zero-correlation zone with length d and is a $(N, 2p, d)$ ZCZ set. Note that $d \le \rho - 1$. If $d > \rho - 1$, the length of ZCZ is still $\rho - 1$.

2.4.7 LAS spreading codes

LAS spreading codes are based on LA codes and LS codes. Each LS code is pulse position modulated (ppm) by an LA code (Fig. 2.5). The set of LA codes can be regarded as an (N_1, K_1, T_1) ZCZ family and that of LS codes forms an (N_2, K_2, T_2) ZCZ family. For $N_2 < T_1$, LAS spreading codes constitute an (N_1, K_1K_2, T_2) ZCZ family.

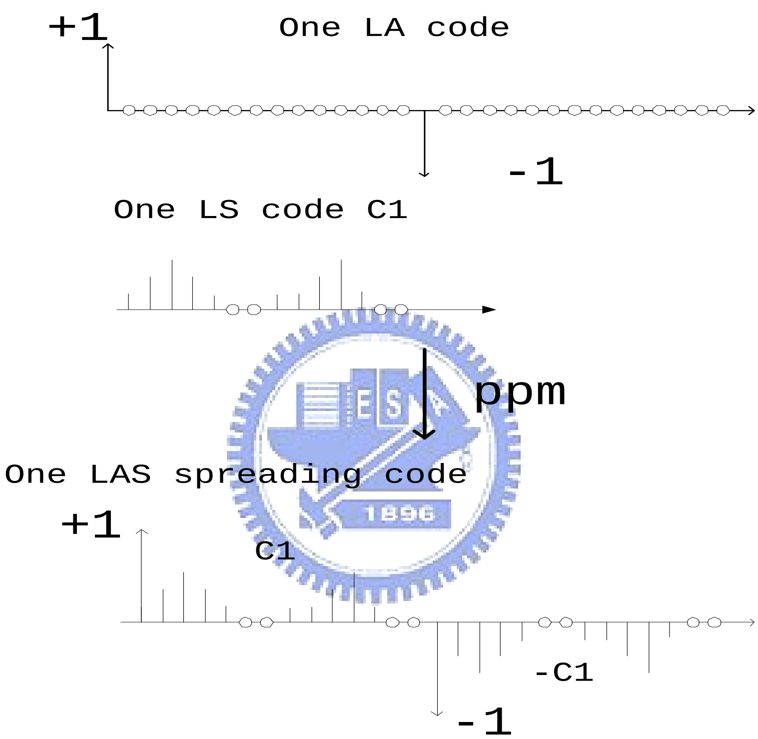


Figure 2.5: One LS code is pulse position modulated by one LA code(ppm)

Chapter 3

Methods of Generating ZCZ Families

3.1 Definitions and Fundamental Results

Definition 15 *A binary sequence of period N satisfies the ZCZ width constraint T on AC is called a basic (N,T) ZCZ sequence.*

A basic sequence can be obtained by the simple rule given in

Lemma 4 *A binary sequence $B=(b_0,b_1,\cdots,b_{N-1}), b_i\in\{0,1\}$ is a basic (N,T) ZCZ sequence if the minimum runlength of 0's between two consecutive 1's is T, where a run refers to a string of the same symbols. T is also called the minimum spacing of the sequence B.*

We start with a simple decomposition of a basic ZCZ sequence.

Corollary 4 *If a basic (N,T) ZCZ sequence B, regarding as a real vector of dimension N, can be expressed as the sum of K orthogonal N-dimensional binary vectors $B^i, K\leq w_H(B), \sum_i w_H(B^i)=w_H(B)$, then the set $\{B^i\}$ is a binary (N,K,T) ZCZ family. The decomposition of the binary vector B into the sum of B^i is called an orthogonal tone decomposition.*

Definition 16 *Let $\boldsymbol{V}=[v_{n_3n_2n_1}]$ be a $N_3\times N_2\times N_1$ array with Hamming weight $w_H(\boldsymbol{V})=k$ and $\mathbf{H_k}=[h_{m,n}]$ be an arbitrary $k\times k$ matrix. vec$(\mathbf{V})$ is the row vec-*

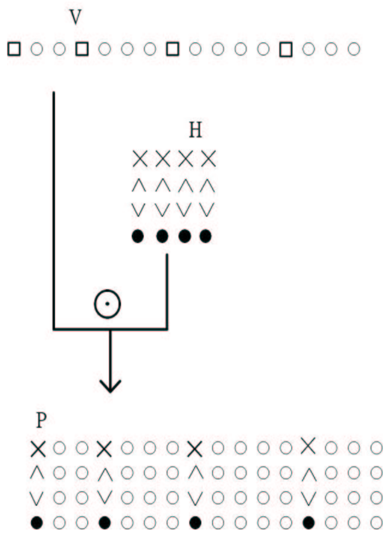


Figure 3.1: The operating concept of Definition 16

tor obtained by stacking up elements of $\boldsymbol{V}$*, along each dimension as follow*

$$vec(\mathbf{V}) \stackrel{def}{=} \Big[v_{000}, v_{001}, \cdots, v_{00(n_1-1)}, v_{010}, v_{011}, \cdots, v_{01(n_1-1)},$$
$$\cdots v_{(n_3-1)00}, v_{(n_3-1)01}, \cdots, v_{(n_3-1)0(n_1-1)}, \cdots$$
$$v_{(n_3-1)(n_2-1)0}, v_{(n_3-1)(n_2-1)1}, \cdots, v_{(n_3-1)(n_2-1)(n_1-1)}\Big] \quad (3.1)$$

Define the $k \times N_3N_2N_1$ *"product matrix"* $\mathbf{P}$ *via the operation* $\mathbf{P} = \mathbf{H_k} \odot \boldsymbol{V} = [p_{ij}]$ *where*

$$p_{m,v(n)} = \begin{cases} h_{mn}, & v(n) = \text{the coordinate of the } n\text{th nonzero entry of } vec(\mathbf{V}) \\ 0, & \text{otherwise} \end{cases} \quad (3.2)$$

Lemma 5 *Rows of the product matrix* $\boldsymbol{P} = \boldsymbol{H} \odot V$ *form a* $(N, w_H(V), T)$ *ZCZ family if* V *is a basic* (N,T) *ZCZ sequence and* $\mathbf{H}$ *is a* $k \times k$ *unitary matrix.*

Proof (i) Let $\mathbf{H} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \cdots, \mathbf{h}_k^T]$ and define $|\mathbf{h}_i|$ as the column vector whose coordinates are the absolute values of those of $\mathbf{h}_i$ then the N-dimensional column vector $\sum_{i=1}^k |\mathbf{h}_i|$ has a minimum spacing of T. (ii) The CC at $\tau = 0$ is zero because $\mathbf{H}$ is an unitary matrix. It follows that both AC and CC functions have the same ZCZ width T. We immediately have

Corollary 5 *Let B be a basic (N,T) sequence and $\{B^i, i = 0,1,\cdots,M-1\}$ be an orthogonal tone decomposition of B. Then the set of all rows of the M matrices $\{\mathbf{P^i} = \mathbf{H^i_{m_i}} \odot B^i, 0 \le i < M\} = \{P^i_n,\ 0 \le n < w_H(B^i)-1, 0 \le i < M\}$ forms an $(N, w_H(\boldsymbol{B}), T)$ ZCZ family, where $m_i = w_H(B^i)$ and $\mathbf{H^i_{m_i}}$ are (not necessarily distinct) the $m_i \times m_i$ unitary matrix for B^i.*

3.2 Direct Methods

The above theorem suggests that an (N,K,T) ZCZ family can be generated by the following three steps.

(A.1) Let B be a basic (N,T) sequence with $w_H(B) = K$ and $\mathbf{B} \overset{def}{=} \{B^i = (b^i_0, b^i_1, \cdots, b^i_{N-1}),\ 0 \le i < M \le K\}$ be an orthogonal tone decomposition of B.

(A.2) Compute the M product matrices $\mathbf{P^i} = \mathbf{H^i_{m_i}} \odot B^i$, where $m_i = w_H(B^i)$ and $\mathbf{H^i_{m_i}}$ are unitary matrices (not necessarily distinct).

(A.3) Let $A = (a_0, a_1, \ldots, a_{N-1})$ be a perfect sequence with period N. Modulating each row of $\mathbf{P^i}$, where $0 \le i < M$, with A through modulating operation, we obtain a set of modulated sequences $\mathbf{C} = \{C_0, C_1, \ldots, C_{K-1}\}$

Theorem 5 *The sequence sets obtained at step (A.1) are (N,M,T) ZCZ families, and those obtained at step (A.2) and (A.3) are (N,K,T) ZCZ families.*

The set obtained in (A.1) is obviously an (N,M,T) ZCZ family. A larger ZCZ family with size $K > M$ is derived from this (N,M,T) family in (A.2). A perfect AC sequence A is used to modulate the ZCZ sequences into sequences of finite constellation signals in (A.3). The above procedure can be generalized by replacing the $m_i \times m_i$ matrix $\mathbf{H^i_{m_i}}$ in (A.2) by a ZCZ matrix defined by

Definition 17 *An (k,n,t) ZCZ matrix is a matrix whose rows are members of an (n,k,t) ZCZ family.*

Corollary 6 *If one replaces the matrix* $\mathbf{H^i_{m_i}}$ *in (A.2) by an* (K', K, T') *ZCZ matrix then one obtains an* $(N, K', T + T')$ *ZCZ family.*

We can also use complementary sets to construct ZCZ sequences.

3.3 Complementary Methods

Definition 18 *Let* U *be a* $1 \times N$ *vector with Hamming weight* $W_H(U) = k$*. A collection* $\mathcal{E}$ *of complementary sets of sequences* $\{\mathbf{E^0}, \mathbf{E^1}, \ldots, \mathbf{E^{n-1}}\}$*, where each set* $\mathbf{E^i} = \{E^i_0, E^i_1, \ldots, E^i_{k-1}\}$ *contains* k *sequences with* $E^i_j = (e^{ij}_0, e^{ij}_1, \ldots, e^{ij}_{\rho-1})$*,* $0 \le i < n$*,* $0 \le j < k$*,* $n \le k$*. The* $n \times (N + k(\rho - 1))$ **concatenated-product matrix** $\mathbf{\Delta}$ *is obtained by* $\mathbf{\Delta} = \mathcal{E} \oslash V = [\Delta_{p,q}]$ *where*

$$\Delta_{i,j(\rho-1)+v(j)+m} = \begin{cases} e^{ij}_m, & v(j) = \text{the coordinate of the } j\text{th nonzero entry of } V \\ & 0 \le m < \rho - 1,\ 0 \le i < n, \text{ and } 0 \le j < k \\ 0, & \text{otherwise} \end{cases} \tag{3.3}$$

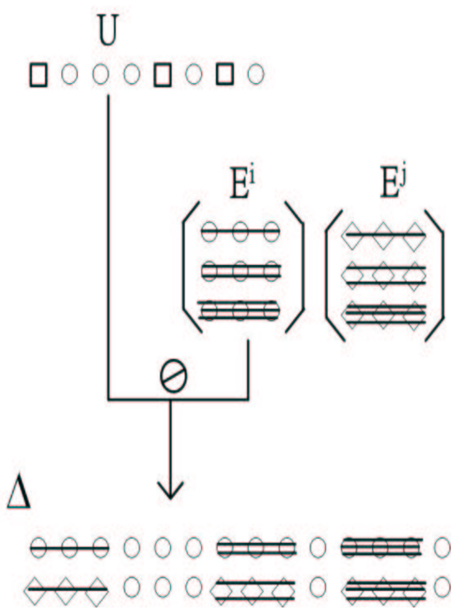


Figure 3.2: The operating concept of Definition 18

Assume a collection $\mathcal{E}$ of K complementary sets of sequences $\{\mathbf{E^0}, \mathbf{E^1}, \ldots, \mathbf{E^{K-1}}\}$ is mutually orthogonal. $\mathbf{E}^j = \{E^j_0, E^j_1, \ldots, E^j_{Q-1}\}$ and the length of E^j_i is ρ, where $0 \le$

$j < K$ and $0 \leq i < Q$. An $(N + Q(\rho - 1), K, T)$ ZCZ family can be generated by the following three steps.

(B.1) Let B be a basic (N, T) sequence with $w_H(B) = Q$.

(B.2) Compute the matrix $\Delta = \mathcal{E} \oslash B$.

(B.3) Let $A = (a_0, a_1, \ldots, a_{N+Q(\rho-1)-1})$ be a perfect sequence with period $N + Q(\rho - 1)$. Modulating each row of Δ by A through the "modulating" operation. Denote rows of Δ by the set of modulated sequences $\mathbf{C} = \{C_0, C_1, \ldots, C_{K-1}\}$.

In other words,

Theorem 6 *The sequence sets obtained in* $(B.2)$ *and* $(B.3)$ *are* $(N + Q(\rho - 1), K, T)$ *ZCZ families.*

Because the correlation properties of complementary sets, we can consider each complementary sequence on one complementary ZCZ sequence as an element. The length of ZCZ is just the minimum run-length of zeros between two nonzero elements in a sequence. In $(B.2)$, an $(N + Q(\rho - 1), K, T)$ ZCZ family is derived from a basic (N, T) sequence. A perfect sequence A is used to **modulate** the complementary ZCZ sequences into sequences of finite constellation signals in $(B.3)$. The cardinality of a complementary ZCZ family depends on the size of the mutually orthogonal complementary set we use.

3.4 Hybrid Methods

One can also combine ingredients of the direct methods and the complementary methods to generate other families of ZCZ sequences.

(C.1) By using $(A.2)$ of the Direct Methods, we can obtain an (N, K, T) ZCZ family.

(C.2) Assume a collection $\mathcal{E}$ of K' complementary sets of sequences $\{\mathbf{E^0}, \mathbf{E^1}, \ldots, \mathbf{E^{K'-1}}\}$ are mutually orthogonal. $\mathbf{E}^j = \{E_0^j, E_1^j, \ldots, E_{Q-1}^j\}$ and the length of E_i^j is ρ, where $0 \leq j < K$ and $0 \leq i < Q$. Let B' be a basic (N', T') sequences with $w_H(B') = Q$. By applying the complementary methods, we obtain a set of $(N'+Q(\rho-1), K', T')$ ZCZ sequences.

(C.3) Let $N' + Q(\rho - 1) \leq T$, and $(N' + Q(\rho - 1), K', T')$ ZCZ sequences are pulse position modulated (ppm) by each sequence of the (N, K, T) ZCZ family to get a (N, KK', T') ZCZ family. Through the modulating operation, elements of each sequences in the (N, KK', T') ZCZ family can be modulated into non-zero elements.

We now present an alternate transform domain approach for generating ZCZ sequences.

3.5 Transform Domain Methods

Definition 19 *The matrices*

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{3.4}$$

and

$$\mathbf{H}_{2^n} = \begin{bmatrix} \mathbf{H}_{2^{n-1}} & \mathbf{H}_{2^{n-1}} \\ \mathbf{H}_{2^{n-1}} & -\mathbf{H}_{2^{n-1}} \end{bmatrix}, \; n = 2, 3 \cdots \tag{3.5}$$

are called standard Hadamard matrices.

Theorem 7 *Let $\mathbf{H} = [\mathbf{h}_0, \mathbf{h}_1, \cdots, \mathbf{h}_{N-1}]$ be a standard Hadamard matrix of order $N = 2^n$, where $\mathbf{h}_i$ is the ith column of $\mathbf{H}$. Partition $\mathbf{H}$ into $N/K = m, m = 2^p, N \times K$ submatrices, $\mathbf{A}_0, \mathbf{A}_1, \cdots, \mathbf{A}_{m-1}$, where each submatrix is formed by K consecutive columns of $\mathbf{H}$, i.e., $\mathbf{A}_i = [\mathbf{h}_{iK}, \cdots, \mathbf{h}_{(i+1)K-1}] \stackrel{def}{=} [\overline{C}_{i0}^T, \overline{C}_{i1}^T, \cdots, \overline{C}_{i(K-1)}^T]$[1]. Denote the N-point IDFT of $\overline{C}_{ij}$ by C_{ij}. Then the set $\boldsymbol{C} \stackrel{def}{=} \{C_{i0}, \cdots, C_{i(K-1)}\}$ of K period-N sequences is an $(2^n, K, m-1)$ ZCZ family.*

[1]For convenience, such a partition is referred to as a **regular partition** of order p or the regular pth-order partition.

Proof

We first note that the AC of any sequence is obviously perfect because the components of the vectors $\overline{C}_{ij}$ all have unit magnitudes. Let $\overline{C}_i$ and $\overline{C}_j$ $(i \neq j)$ be any two rows in a submatrix and denote the IDFTs of these two vectors by C_i and C_j. Taking IDFT on the Hadamard product $\Theta_{C_iC_j} = \overline{C}_i \odot \overline{C}_j$ of $\overline{C}_i$ and $\overline{C}_j$, we obtain the CC function $\theta_{C_iC_j}(\tau)$ between C_i and C_j. Because rows in a Hadamard matrix are orthogonal, the numbers of +1 and -1 are the same, $\theta_{C_iC_j}(0) = 0$.

Let $\Theta_{C_iC_j} = [\Theta_{C_iC_j}(0), \Theta_{C_iC_j}(1), \cdots, \Theta_{C_iC_j}(N-1)]$. The structure of the standard Hadamard matrices and our partition imply that the sequence $\{\Theta_{C_iC_j}(\lambda)\}$ is periodic with period K, i.e., it consists of m consecutive identical K-tuples.

$$
\begin{aligned}
&\big(\Theta_{C_iC_j}(0), \Theta_{C_iC_j}(1), \cdots, \Theta_{C_iC_j}(K-1)\big) = \big(\Theta_{C_iC_j}(K), \Theta_{C_iC_j}(K+1), \cdots, \Theta_{C_iC_j}(2K-1)\big) \\
= \quad &\cdots \quad = \big(\Theta_{C_iC_j}((m-1)K), \Theta_{C_iC_j}((m-1)K+1), \cdots, \Theta_{C_iC_j}(N-1)\big)
\end{aligned}
$$

Taking IDFT, we obtain

$$
\begin{aligned}
&\theta_{C_iC_j}(\tau) = \sum_{\lambda=0}^{N-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\tau\lambda}{N}} \\
= \quad &\sum_{\lambda=0}^{K-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\tau\lambda}{N}} + \sum_{\lambda=K}^{2K-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\tau\lambda}{N}} + \cdots + \sum_{\lambda=(m-1)K-1}^{N-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\tau\lambda}{N}}
\end{aligned}
$$

Let q be an odd number, $(2^{p-1})q = \beta$ and define $x \pmod N \stackrel{def}{=} |x|_N$. Since $2^{n-p} = N/m = K$, we have

$$
\begin{aligned}
&\theta_{C_iC_j}(|\beta|_N) = \sum_{\lambda=0}^{N-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\tau\lambda}{N}} \\
= \quad &\sum_{\lambda=0}^{K-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\beta\lambda}{2^n}} + \sum_{\lambda=K}^{2K-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\beta\lambda}{2^n}} + \cdots + \sum_{\lambda=(m-1)K-1}^{N-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi\beta\lambda}{2^n}} \\
= \quad &\sum_{\lambda=0}^{K-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi q\lambda}{2K}} + \sum_{\lambda=K}^{2K-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi q\lambda}{2K}} + \cdots + \sum_{\lambda=(m-1)*K-1}^{N-1} \Theta_{C_iC_j}(\lambda) e^{\frac{j2\pi q\lambda}{2K}} \\
= \quad &0 \qquad (3.6)
\end{aligned}
$$

Note that a 2^p-period sequence $\{\Theta_{C_iC_j}(\lambda)\}$ can also be regarded as a 2^t-period sequence for $0 \le t < p$. Therefore $\theta_{C_iC_j}(\tau) = 0$, for $\tau \in \{|2^t q|_N, 0 \le t < p, q \in I_o\} \overset{def}{=} \mathcal{Z}_0$, where I_o denotes the set of odd integers. Obviously, the set $\{\pm 1, \pm 3, \pm 5, \cdots, \pm 2^p - 1\} \subset \mathcal{Z}_0$. Even integers between $-(2^p-1)$ and 2^p-1 are of the form $\pm 2^t q, t = 1, 2, \cdots, p-1$. But $\pm 2^p \notin \mathcal{Z}_0$ for otherwise we have $q2^t = \pm 2^p \pmod N$ for some $1 \le t < 2^p$ and $q \in I_o$, which implies $\pm 2^t(2^{p-t} \mp q) = 0 \pmod N$, a contradiction. Hence $\mathcal{Z}_0$ contains the subset $\{\pm 1, \pm 2, \cdots, \pm 2^p - 1 = \pm(m-1)\}$. ■

A generalization is given by

Theorem 8 *Let $\mathbf{U}_2$ be any 2×2 complex unitary matrix and $\mathbf{U}_{2^n}$ be recursively generated by the standard procedure (3.5). ZCZ families can be obtained by applying the regular mth-order partition described in Theorem 2 on the matrix $\mathbf{U}_{2^n}$.*

Proof

Consider the complex unitary matrix

$$\mathbf{U}_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{3.7}$$

and its generalization

$$\mathbf{U}_{2^n} = \begin{bmatrix} \mathbf{U}_{2^{n-1}} & \mathbf{U}_{2^{n-1}} \\ \mathbf{U}_{2^{n-1}} & -\mathbf{U}_{2^{n-1}} \end{bmatrix} = [\mathbf{u}_0, \mathbf{u}_1, \cdots, \mathbf{u}_{2^n-1}] \tag{3.8}$$

Define the corresponding *signed* matrices $\widetilde{\mathbf{U}}_{2^l}$, $l = 1, 2, \cdots$, as the hard-limited versions of $\mathbf{U}_{2^l}$, i.e., $\widetilde{\mathbf{U}}_{2^l} = [\text{sgn}(u_{ij}) \cdot 1]= [\tilde{\mathbf{u}}_0, \tilde{\mathbf{u}}_1, \cdots, \tilde{\mathbf{u}}_{2^n-1}]$, where u_{ij} is the entry of $\mathbf{U}_{2^n}$ in the ith row and the jth column, and the *sgn* function is defined by $\text{sgn}(x) = 1$, if $x > 0$, and $\text{sgn}(x) = -1$ if $x < 0$.

We first note that the period T of a given Hadamard product vector $\Theta_{\mathbf{u}_i\mathbf{u}_j} = \mathbf{u}_i \odot \mathbf{u}_j^*$ is the least common multiple of the period of the Hadamard product of the associated signed vectors $\Theta_{\tilde{\mathbf{u}}_i\tilde{\mathbf{u}}_j} = \tilde{\mathbf{u}}_i \odot \tilde{\mathbf{u}}_j$ and that of $\left|\Theta_{\mathbf{u}_i\mathbf{u}_j}\right|$. But the way the Hadamard-like matrices (3.8) are constructed implies that the Hadamard products of any pairs of column vectors from their signed counterparts $\widetilde{\mathbf{U}}_{2^n}$ must have a period of 2^l, for some $l \ge 1$.

We also find that the magnitude of the Hadamard product of any two rows of $\mathbf{U}_{2^n}$ yields one of the four row vectors, i.e.,

$$\begin{aligned} \left|\Theta_{\mathbf{u}_i\mathbf{u}_j}\right| &= \left|\mathbf{u}_i \odot \ \mathbf{u}_j^*\right| \\ &= \begin{bmatrix} |a|^2 \\ |c|^2 \\ |a|^2 \\ |c|^2 \\ \vdots \\ |a|^2 \\ |c|^2 \end{bmatrix} \text{ or } \begin{bmatrix} ab^* \\ cd^* \\ ab^* \\ cd^* \\ \vdots \\ ab^* \\ cd^* \end{bmatrix} \text{ or } \begin{bmatrix} |b|^2 \\ |d|^2 \\ |b|^2 \\ |d|^2 \\ \vdots \\ |b|^2 \\ |d|^2 \end{bmatrix} \text{ or } \begin{bmatrix} ba^* \\ dc^* \\ ba^* \\ dc^* \\ \vdots \\ ba^* \\ dc^* \end{bmatrix} \end{aligned} \tag{3.9}$$

The components of each of the four vectors form a periodic sequence of length 2^n and period 2, i.e., each vector consists of 2^{n-1} consecutive identical 2-tuples.

From the above two observations we conclude that T is identical to the period of $\Theta_{\tilde{\mathbf{u}}_i\tilde{\mathbf{u}}_j}$. The corollary will then be proved if we can show that the Hadamard product of any two column vectors of the submatrices resulted from a regular pth-order partition of $\widetilde{\mathbf{U}}_{2^n}, 0 \le p < n$, gives a periodic sequence of length 2^n and period 2^{n-p}. We prove this claim by induction.

1. For the signed matrix

$$\widetilde{\mathbf{U}}_2 = \begin{bmatrix} +1 & +1 \\ +1 & +1 \end{bmatrix} \tag{3.10}$$

The 0th order regular partition results in only $2^0 = 1$ submatrix and all the Hadamard product vectors $\Theta_{\tilde{\mathbf{u}}_i\tilde{\mathbf{u}}_j}$ have $2^0 = 1$ identical tuple only.

2. Suppose the claim is true for $p = m$ and $n = k-1$. That is, the mth-order regular partition of $\widetilde{\mathbf{U}}_{2^{k-1}}$ gives 2^m submatrices, $\breve{\mathbf{U}}^l_{2^{k-1}}$, where $0 \le m < k-1$, $0 \le l < 2^m$, such that the Hadamard product $\Theta_{\mathbf{u}_i\mathbf{u}_j}$ of any two rows within a submatrix yields a vector of period 2^{k-1-m}. For $n = k$, we perform the m'-order partition on the signed matrix $\widetilde{\mathbf{U}}_{2^k}$ to get two submatrices

$$\breve{\mathbf{U}}^l_{2^k} = \begin{bmatrix} \breve{\mathbf{U}}^l_{2^k} \\ \breve{\mathbf{U}}^l_{2^k} \end{bmatrix} \qquad \breve{\mathbf{U}}^{l+2^{n-1}}_{2^k} = \begin{bmatrix} \breve{\mathbf{U}}^l_{2^k} \\ -\breve{\mathbf{U}}^l_{2^k} \end{bmatrix} \tag{3.11}$$

$\Theta_{\mathbf{u}_i^T \mathbf{u}_j^T}$ of each of the two submatrices has $2^{m'}$ identical tuples, where $m' = m + 1$. When $m' = 0$, for $\mathbf{u}_i^T \in \breve{\mathbf{U}}_{2^n}^k$ and $\mathbf{u}_j^T \in \breve{\mathbf{U}}_{2^n}^{k+2^{n-1}}$, $\Theta_{\mathbf{u}_i^T \mathbf{u}_j^T}$ clearly has 2^0 identical tuple. ■

Chapter 4

Applications: Generating ZCZ Sequences

4.1 PS-like sequences

A set of PS-like sequences with size 3 can be generated by one of the *Direct Methods* presented in Section III. More specifically, we let $M = 1, B^0 = (100010001000)$ and use the unitary matrix

$$\mathbf{H_3^0} = \begin{bmatrix} W_3^0 & W_3^0 & W_3^0 \\ W_3^0 & W_3^1 & W_3^2 \\ W_3^0 & W_3^2 & W_3^1 \end{bmatrix}$$

along with the perfect AC sequence $A = (100100100-100)$ to obtain

$$\begin{aligned} P_0^0 &= (W_3^0 000 W_3^0 000 W_3^0 000) \\ P_1^0 &= (W_3^0 000 W_3^1 000 W_3^2 000) \\ P_2^0 &= (W_4^0 000 W_3^2 000 W_3^1 000) \end{aligned} \tag{4.1}$$

and then the family of PS-like sequences

$$\begin{aligned} C_0 &= P_0^0 \ominus A = (W_6^0 W_6^0 W_6^0 W_6^3 W_6^0 W_6^0 W_6^0 W_6^3 W_6^0 W_6^0 W_6^0 W_6^3) \\ C_1 &= P_1^0 \ominus A = (W_6^0 W_6^2 W_6^4 W_6^3 W_6^2 W_6^4 W_6^0 W_6^5 W_6^4 W_6^0 W_6^2 W_6^1) \\ C_2 &= P_2^0 \ominus A = (W_6^0 W_6^4 W_6^2 W_6^3 W_6^4 W_6^2 W_6^0 W_6^1 W_6^2 W_6^0 W_6^4 W_6^5) \end{aligned} \tag{4.2}$$

It is easily verifiable that

$$\theta_{C_i,C_j}(\tau) = 0,\ 0\ \leq \tau < 12\ \ (i \neq j), \quad \theta_{C_i}(\tau) = 12\delta(|\tau|_4) \tag{4.3}$$

and $\mathbf{C} = (C_0, C_1, C_2)$ is a $(12, 3, 3)$ ZCZ family. The member sequences of $\mathbf{C}$ are called *PS-like sequences*. Note that the PS-like family has the same correlation properties as those of PS sequences but only use a constellation whose size is only half of that required by the original PS sequences. The autocorrelation function of C_1 is shown in Fig. 4.1 while the cross-correlation function of C_0 and C_1 is shown in Fig. 4.2.

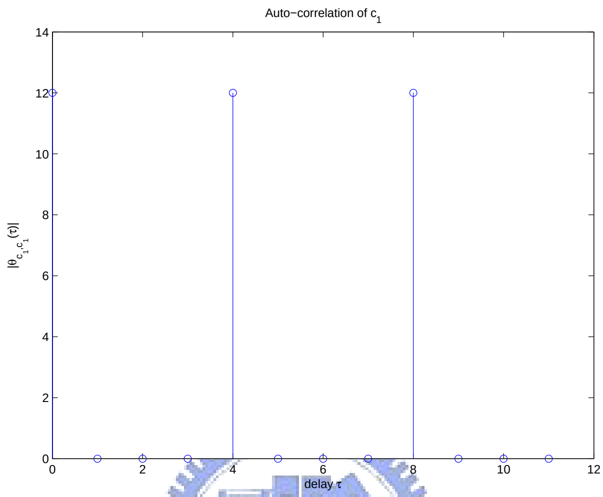


Figure 4.1: The autocorrelation function of C_1 in section 4.1

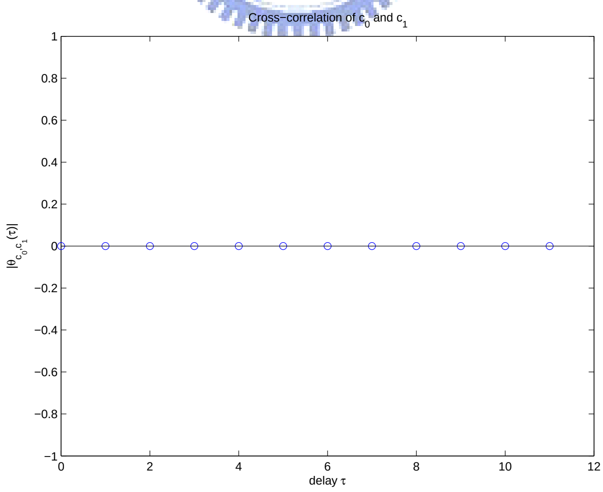


Figure 4.2: The crosscorrelation function of C_0 and C_1 in section 4.1

4.2 Ternary ZCZ sequences

Let $M = 2$ and use the two basic sequences $B^0 = (0001000100010001)$, $B^1 = (0100010001000100)$ and

$$\begin{aligned}
\mathbf{H_4^0} &= \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix} \\
\mathbf{H_4^1} &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \\
A &= (1000000000000000)
\end{aligned} \tag{4.4}$$

we obtain the ternary sequences shown in Fig. 2.3.

$$\begin{aligned}
C_0 = P_0^0 &= (000+000+000+000-) \\
C_1 = P_1^0 &= (000+000+000-000+) \\
C_2 = P_2^0 &= (000+000-000+000+) \\
C_3 = P_3^0 &= (000+000-000-000-) \\
C_4 = P_0^1 &= (0+000+000+000+00) \\
C_5 = P_1^1 &= (0+000-000+000-00) \\
C_6 = P_2^1 &= (0+000+000-000-00) \\
C_7 = P_3^1 &= (0+000-000-000+00)
\end{aligned} \tag{4.5}$$

where " + " and " − " denote +1 and −1, respectively. It can be shown that the set $\mathbf{C} = \{C_1, C_2, \cdots, C_7\}$ is a $(16, 8, 1)$ ZCZ family.

4.3 Binary ZCZ sequences

Most of the basic sequences have equally spaced nonzero entries. But one can also build ZCZ sequences based on non-uniformly spaced basic sequence. For example, if we let

$M = 1, B^0 = (1000000100100100)$, invoke the unitary matrix

$$H_4^0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

and the perfect AC sequence $\overline{A} = (1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0,\ 0, -1, 0, 0, 0)$, we obtain

$$\begin{aligned} P_0^0 &= (1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0) \\ P_1^0 &= (1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 1, 0, 0, -1, 0, 0) \\ P_2^0 &= (1, 0, 0, 0, 0, 0, 0, 1, 0, 0, -1, 0, 0, -1, 0, 0) \\ P_3^0 &= (1, 0, 0, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0, 1, 0, 0) \end{aligned}$$

The resulting binary (16,4,2) ZCZ family consists of

$$\begin{aligned} C_0 &= (1, -1, 1, 1, -1, 1, 1, 1, 1, 1, 1, -1, 1, 1, -1, 1) \\ C_1 &= (1, 1, 1, -1, -1, -1, 1, -1, 1, -1, 1, 1, 1, -1, -1, -1) \\ C_2 &= (1, 1, -1, 1, -1, -1, -1, 1, 1, -1, -1, -1, 1, -1, 1, 1) \\ C_3 &= (1, -1, -1, -1, -1, 1, -1, -1, 1, 1, -1, 1, 1, 1, 1, -1) \end{aligned} \tag{4.6}$$

This is one ZCZ polyphase set in theorem 2, given the pefrect AC sequence $\overline{A}$, the unitary matrix H_4^0, $N'' = 1$, $N_r = N' = 4$, and $n = 0$. The autocorrelation function of C_0 is shown in Fig. 4.3 and the crosscorrelation function of C_0 and C_1 is shown in Fig. 4.4.

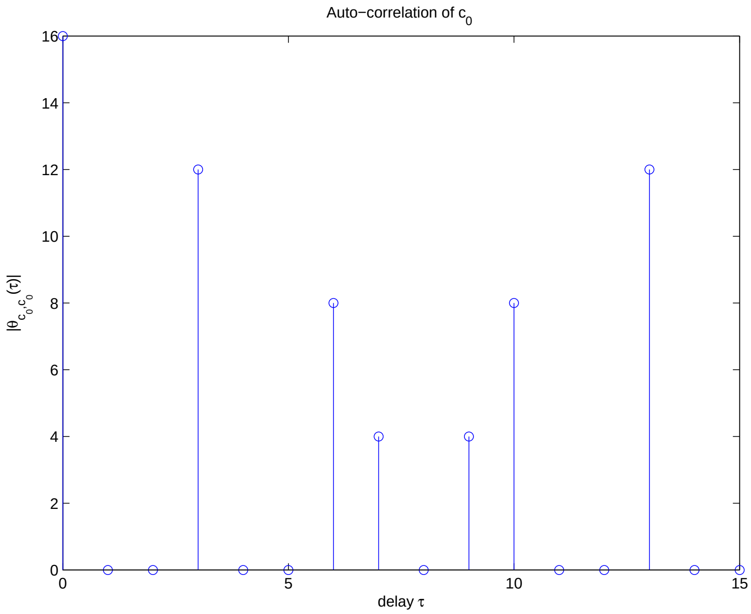


Figure 4.3: The autocorrelation function of C_0 in section 4.3

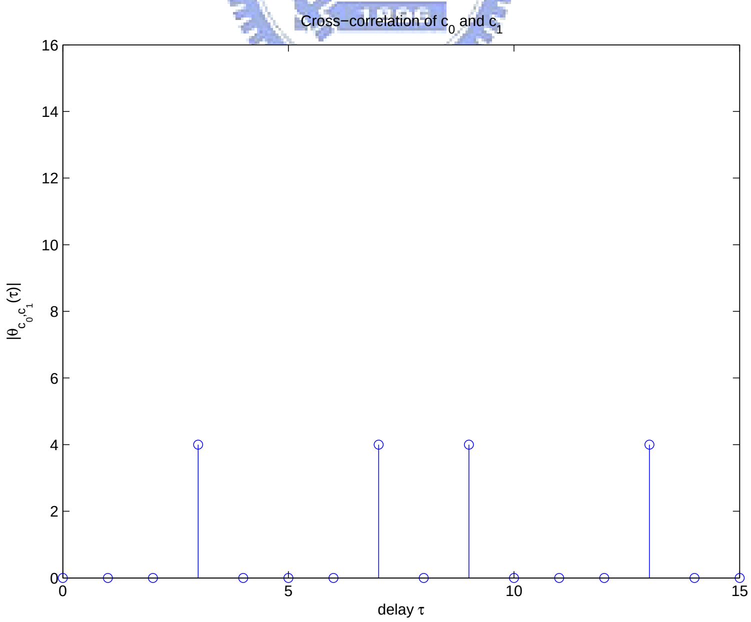


Figure 4.4: The crosscorrelation function of C_0 and C_1 in section 4.3

4.4 Hadamard ZCZ sequences

Let $M = 4, B^0 = (10000000), B^1 = (00001000), B^2 = (01000010)$. Using two trivial unitary matrices $H_1^0 = H_1^1 = 8$,

$$H_2^2 = \begin{bmatrix} 4-4i & 4+4i \\ 4+4i & 4-4i \end{bmatrix} \quad (4.7)$$

and $A = (10000000)$, we obtain

$$\begin{aligned} C_0 &= (8,0,0,0,0,0,0,0) \\ C_1 &= (0,0,0,0,8,0,0,0) \\ C_2 &= (0,4-4i,0,0,0,0,4+4i,0) \\ C_3 &= (0,4+4i,0,0,0,0,4-4i,0) \end{aligned} \quad (4.8)$$

which is a $(8,4,1)$ ZCZ family consisting of four perfect AC sequences. Taking DFT on these sequences, we obtain the first four rows of an 8×8 Hadamard matrix

$$\begin{aligned} \overline{C_1} &= (+1,+1,+1,+1,+1,+1,+1,+1) \\ \overline{C_2} &= (+1,-1,+1,-1,+1,-1,+1,-1) \\ \overline{C_3} &= (+1,+1,-1,-1,+1,+1,-1,-1) \\ \overline{C_4} &= (+1,-1,-1,+1,+1,-1,-1,+1) \end{aligned} \quad (4.9)$$

Because of this special property, we refer to this family as a *Hadamard* ZCZ family. It is clear that this the Hadamard ZCZ sequences can be generated by the method described in *Theorem 2*. Although these Hadamard ZCZ sequences consist of a lot zeros, we can use another perfect AC sequence instead of $A = (10000000)$, which are also binary in frequency domain (i.e. $\overline{A} = (1,-1,1,1,-1,1,1,-1)$), to modulate them into non-zero ZCZ sequences via modulating operation. The new ZCZ sequences are also binary in frequency domain.

4.5 New Polyphase ZCZ Sequences

Based on Theorem 1, we can generate new polyphase sequences given specific basic sequences, unitary matrices, and a specific perfect AC sequence. Assume the lengths of basic sequences are N, the length of a perfect polyphase AC sequence A' is N', which $N = N'N_r$, and A' consists of elements which are signals of $W^l_{N_{A'}}$, $0 \leq l < N_{A'}$, $2 \leq N_{A'} \leq N'$. A up-samples A' by inserting $N_r - 1$ zeros between each entry of A', so the length of A is N. *Corollary 2* implies that A is also a perfect AC sequence. For convenience, assume that there is only one basic binary sequence B with $w_H(B) = N_r$ and length N and define the unitary matrix $H^0_{N_r}$ by

$$H^0_{N_r} = \begin{bmatrix} W^0_{N_r} & W^0_{N_r} & \dots & W^0_{N_r} \\ W^0_{N_r} & W^1_{N_r} & \dots & W^{N_r-1}_{N_r} \\ \vdots & \vdots & \ddots & \vdots \\ W^0_{N_r} & W^{N_r-1}_{N_r} & \dots & W^{(N_r-1)^2}_{N_r} \end{bmatrix} \tag{4.10}$$

Let $\zeta = lcm(N_r, N')$, where $lcm(k, l)$ denotes the least common multiple of k and l.

(D.1) When $\frac{N}{\zeta} > 1$, let $B = (b_0, b_1, \dots, b_{N-1})$ be permuted by

$$b_i = \begin{cases} 1, & i = \alpha N', \dots, \gamma\zeta + (\frac{N}{\zeta} - \gamma) + \alpha N' \\ & \alpha = 0, \dots, \frac{\zeta}{N'} - 1, \gamma = 1, \dots, \frac{N}{\zeta} - 1 \\ 0, & \text{otherwise} \end{cases} \tag{4.11}$$

Using the perfect sequence A and unitary matrix $H^0_{N_r}$ mentioned above, we obtain $(N, N_r, N' - 2)$ polyphase ZCZ families whose sequences consist of elements drawn from the constellation $W^l_{lcm(N_{A'}, N_r)}$, $0 \leq l \leq lcm(N_{A'}, N_r)$. Section 4.3 is a special case of this condition.

(D.2) When $\frac{N}{\zeta} = 1$, then $\frac{\zeta}{N'} = N_r$. Let $B = (b_0, b_1, \dots, b_{N-1})$ be permuted by

$$b_i = \begin{cases} 1, & i = \alpha N', \alpha = 0, 1, \dots, N_r - 1 \\ 0, & \text{otherwise} \end{cases} \tag{4.12}$$

By using the perfect sequence A and unitary matrix $H^0_{N_r}$ mentioned above, we obtain $(N, N_r, N' - 1)$ polyphase ZCZ families with sequence elements belong to the constellation $\{W^l_{lcm(N_{A'},N_r)}\}$, where $0 \leq l \leq lcm(N_{A'}, N_r)$. Section 4.1 is a special case of this condition and the corresponding polyphase ZCZ sequences are PS-like sequences which keep the correlation property of PS sequences but use a smaller signal constellation $\{W^l_{lcm(N_{A'},N_r)}\}$ (PS sequences use the constellation $\{W^l_N\}$)

The basic sequence B with length $N = N'N_r$ can be arranged into an $N' \times N_r$ basic array. The modulating operation is achieved by another way of multiplying the cyclic-shifted perfect sequence A' by the basic array. If there is only one nonzero element in each column of the basic array, the modulated sequence is polyphase for that A' and the nonzero elements in the basic array are polyphase. Fig.4.5 illustrates the procedure. Fig.4.6 shows how we arrange the basic sequence in $(D.1)$ and $(D.2)$ to make each column of the basic array coming from the basic sequence has only one nonzero element. If N_r is a power of 2, we can use a $N_r \times N_r$ Hadamard matrix instead of $H^0_{N_r}$ to reduce $lcm(N_{A'}, N_r)$ to $W^l_{lcm(N_{A'},2)}$, but the ZCZ polyphase sequences produced in (D.2) are not PS-like sequences. The polyphase ZCZ sequences generated in (D.1) are generalizations of some ZCZ sequences in section 2.4.4. In section 2.4.4, The familiy size must be a multiple of a factor of N'. However, we can generate a polyphase ZCZ family by (D.1) with length $N = N_rN'$, where N' is the length of a perfect AC sequence and N_r is a natural number.

[12] suggests a method to generate sequences similar to those of (D.2) under the constraint that $N' + 1$ is a multiple of N_r. In (D.2), the constraint on $lcm(N', N_r) = N$ is more flexible. From (3), under fixed N and K, the ZCZ length of the sequences generated from (D.2) achieve the bound and those generated from (D.1) achieve the bound less that 1. We can also generate a lot of polyphase ZCZ sequences with ZCZ length less than those from (D.1) and (D.2) under fixed N and K by arranging the basic sequence B in another way. Some of these sequences are introduced in [12].

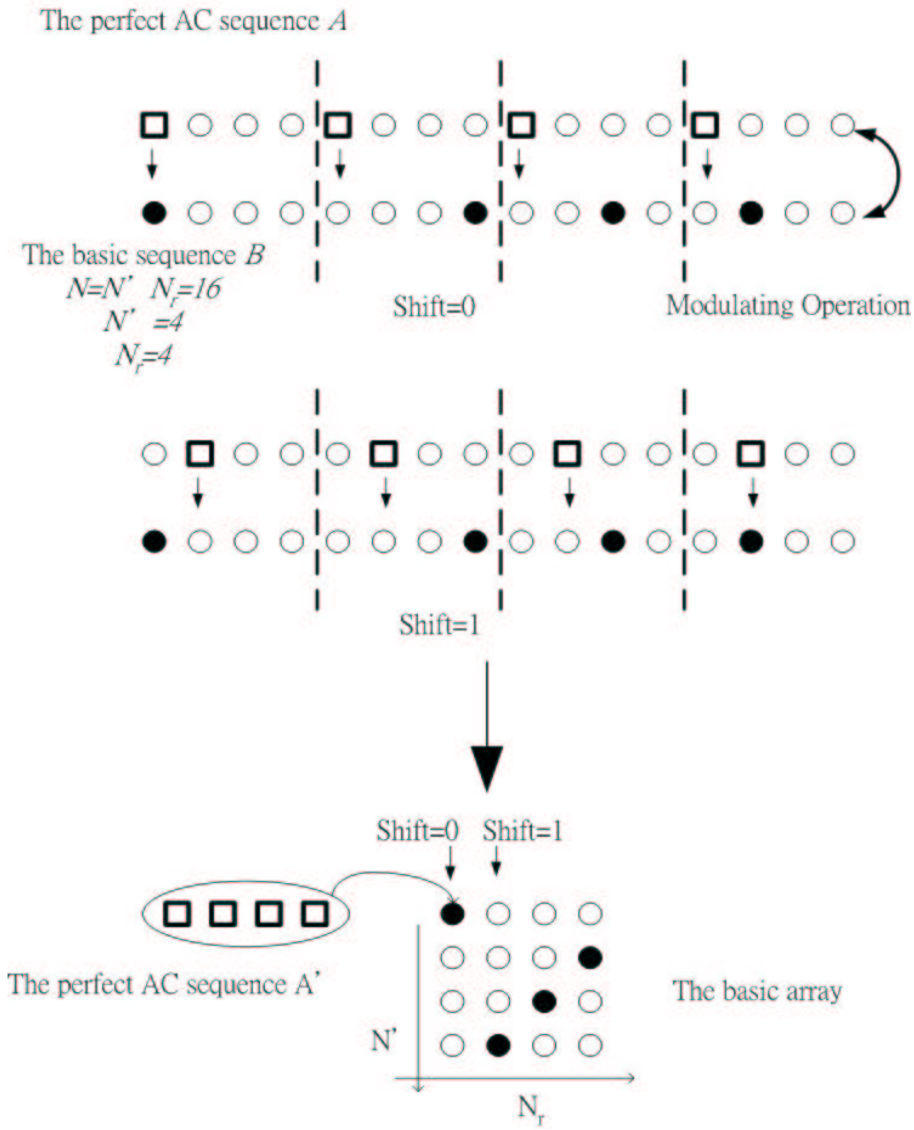
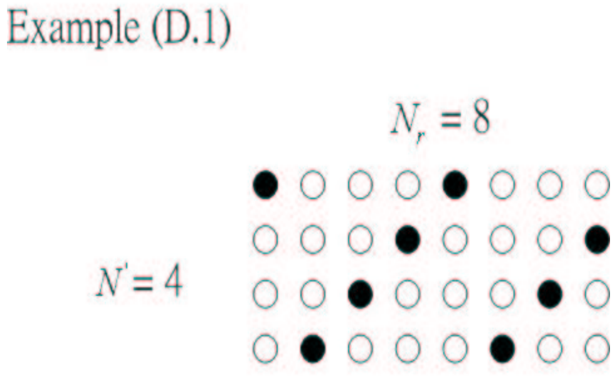


Figure 4.5: The modulating operation is achieved by another way of multiplying the cyclic-shifted perfect sequence A' by the basic array.



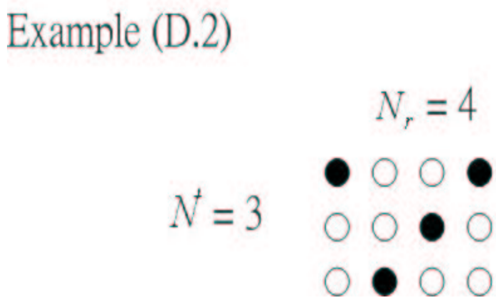


Figure 4.6: The basic array coming from the basic sequence in (4.11) and (4.12) has only one nonzero element. In the example of $(D,1)$, $l.c.m(N', N_r) = \zeta = 8 < N = 32$. In the example of $(D,2)$, $l.c.m(N', N_r) = \zeta = N = 12$.

4.6 New Polyphase ZCZ Sequences based on Mutually Orthogonal Complementary Sets

Theorem 2 says that one can derive polyphase ZCZ sequence families from mutually orthogonal complementary sets by using a perfect AC sequence and a basic sequence.

Let $\mathcal{Z}^\rho$ be the $1 \times \rho$ all-zero vector and B be a basic sequence B of length N with $w_H(B) = Q$. Denote by A' a perfect polyphase AC sequence of length N', $N'|N$, whose elements are drawn from the set $\{W^l_{N_{A'}}, 0 \le l < N_{A'}, 2 \le N_{A'} \le N'\}$. Up-sampling A' by ρN_r-fold, we obtain a length$-N_r N'\rho$ sequence A with perfect AC (see *Corollary 2*).

Theorem 9 *Let $\mathcal{E}$ be a mutually orthogonal collection of K polyphase complementary sets of sequences $\mathbf{E} = \{\mathbf{E^0}, \mathbf{E^1}, \ldots, \mathbf{E^{K-1}}\}$, where $\mathbf{E^j} = \{E^j_0, E^j_1, \ldots, E^j_{N_r-1}\}$, E^j_i is a length-ρ polyphase sequence with elements from the constellation $\{W^l_{N_c}\}$, $N_c < \rho$, $0 \le l < N_c$, $K < N_r$, $0 \le j < K$ and $0 \le i < N_r$.*

$(E.1)$ *Let $\zeta = lcm(N_r, N')$. If $\frac{N}{\zeta} > 1$, permute B according to (4.11), and if $\frac{N}{\zeta} = 1$, permute B by (4.12).*

$(E.2)$ *Compute $\mathbf{\Delta} = \mathcal{E} \oslash B$.*

$(E.3)$ *Extend the $K \times (N + (\rho - 1)N_r)$ matrix $\mathbf{\Delta}$ into a $K \times \rho N$ matrix $\mathbf{\Delta'}$ by replacing each zero in $\mathbf{\Delta}$ with the zero vector $\mathcal{Z}^\rho$.*

$(E.4)$ *Modulate each row of $\mathbf{\Delta'}$ by A and denote the set of rows by $\mathbf{C} = \{C_0, C_1, \ldots, C_{K-1}\}$.*

Then

(i) $\mathbf{C}$ *is a $(\rho N, K, \rho(N' - 2))$ polyphase ZCZ family if $\frac{N}{\zeta} > 1$.*

(ii) $\mathbf{C}$ *is a $(\rho N, K, \rho(N' - 1))$ polyphase ZCZ family if $\frac{N}{\zeta} = 1$.*

Fig. 4.7 shows how the extended sequences through $(E.1) - (E.3)$ are arranged to form an extended array so that each column of the extended basic array has only one nonzero element. The polyphase ZCZ sequences based on mutually orthogonal complementary sets consists of the polyphase elements of the form $W^{l}_{lcm(N_{A'},N_c)}$. This is a generalization of some ZCZ sequences given in Section 2.4.4. Sequences presented in Section 2.4.4 require that the family size be a multiple of a factor of N' but there is no such constraint in our approach. Although using a proper $\mathcal{E}$ can increase the duration of ZCZ, our method guarantees the minimum ZCZ length for all $\mathcal{E}$.

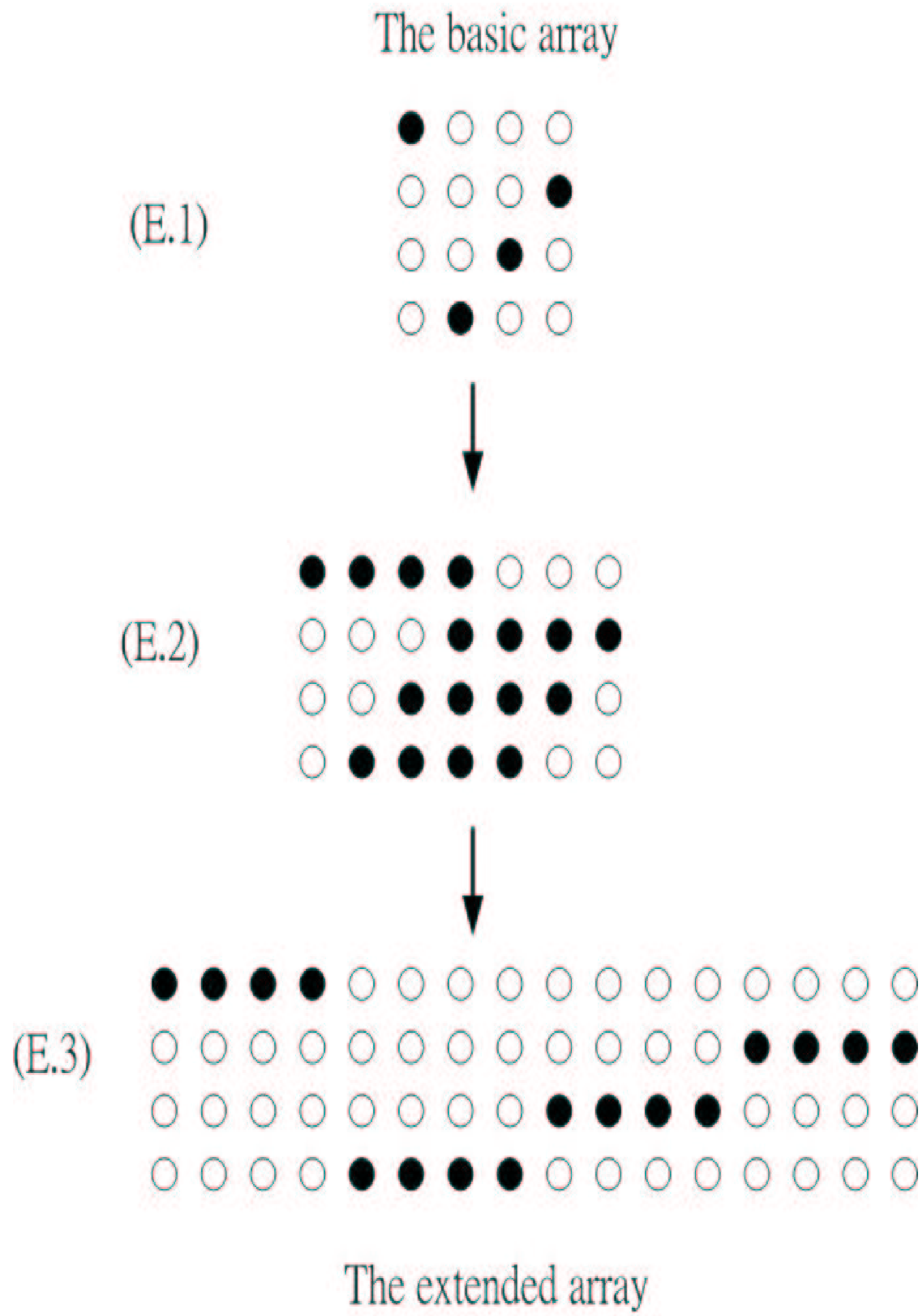


Figure 4.7: The extended sequence in (E.1)-(E.3) is arranged to form an extended array

Example 2 *Let* $N = 16$*,* $N' = 4$*,* $N_r = 4$*,* $A' = (W_4^0 W_4^1 W_4^2 W_4^1)$ *and use the permuted basic sequence* $B = (1000000100100100)$*. The mutually orthogonal collection* E *of complementary sets of sequences are given by [14]*

$$\mathbf{E^0} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\mathbf{E^1} = \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\mathbf{E^2} = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{E^3} = \begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{bmatrix} \qquad (4.13)$$

Using Steps (E.2)-(E.4) mentioned above, we obtain a set of $(64, 4, 8)$ *quadriphase ZCZ sequences. Because the length of the sequences is very long, we only give* C_0 *and* C_1 *below.*

$$\begin{aligned} C_0 = (&W_4^0W_4^0W_4^0W_4^0W_4^3W_4^1W_4^1W_4^3W_4^0W_4^2W_4^0W_4^2W_4^1W_4^1W_4^3W_4^3W_4^3W_4^3W_4^3W_4^3 \\ &W_4^2W_4^0W_4^0W_4^2W_4^1W_4^3W_4^1W_4^3W_4^2W_4^2W_4^0W_4^0W_4^2W_4^2W_4^2W_4^2W_4^3W_4^1W_4^1W_4^3 \\ &W_4^2W_4^0W_4^2W_4^0W_4^1W_4^1W_4^3W_4^3W_4^3W_4^3W_4^3W_4^3W_4^0W_4^2W_4^2W_4^0W_4^1W_4^3W_4^1W_4^3 \\ &W_4^0W_4^0W_4^2W_4^2) \\ C_1 = (&W_4^0W_4^0W_4^2W_4^2W_4^3W_4^1W_4^3W_4^1W_4^0W_4^2W_4^2W_4^0W_4^1W_4^1W_4^1W_4^1W_4^3W_4^3W_4^1W_4^1 \\ &W_4^2W_4^0W_4^2W_4^0W_4^1W_4^3W_4^3W_4^1W_4^2W_4^2W_4^2W_4^2W_4^2W_4^2W_4^0W_4^0W_4^3W_4^1W_4^3W_4^1 \\ &W_4^2W_4^0W_4^0W_4^2W_4^1W_4^1W_4^1W_4^1W_4^3W_4^3W_4^1W_4^1W_4^0W_4^2W_4^0W_4^2W_4^1W_4^3W_4^3W_4^1 \\ &W_4^0W_4^0W_4^0W_4^0) \end{aligned} \qquad (4.14)$$

The AC of C_0 is given by

$$\begin{aligned}|\theta_{C_0C_0}(\tau)| = (&64,0,0,0,0,0,0,0,0,4,8,4,0,4,8,4,0,0,0,0,0,8,\\ &0,8,0,4,8,12,0,4,8,4,0,4,8,4,0,12,8,4,0,8,\\ &0,8,0,0,0,0,0,4,8,4,0,4,8,4,0,0,0,0,0,0,0,0)\end{aligned} \tag{4.15}$$

The CC between C_0 and C_1 is

$$\begin{aligned}|\theta_{C_0C_1}(\tau)| = (&0,0,0,0,0,0,0,0,0,4,8,4,16,4,8,4,0,0,0,0,0,8,0,8,\\ &0,12,8,12,16,4,8,4,0,4,8,12,16,4,8,12,0,8,0,\\ &8,0,0,0,0,0,4,8,12,16,12,8,4,0,0,0,0,0,0,0,0)\end{aligned} \tag{4.16}$$

The autocorrelation function of C_0 is shown in Fig. 4.8 and the crosscorrelation function of C_0 and C_1 is shown in Fig. 4.9.

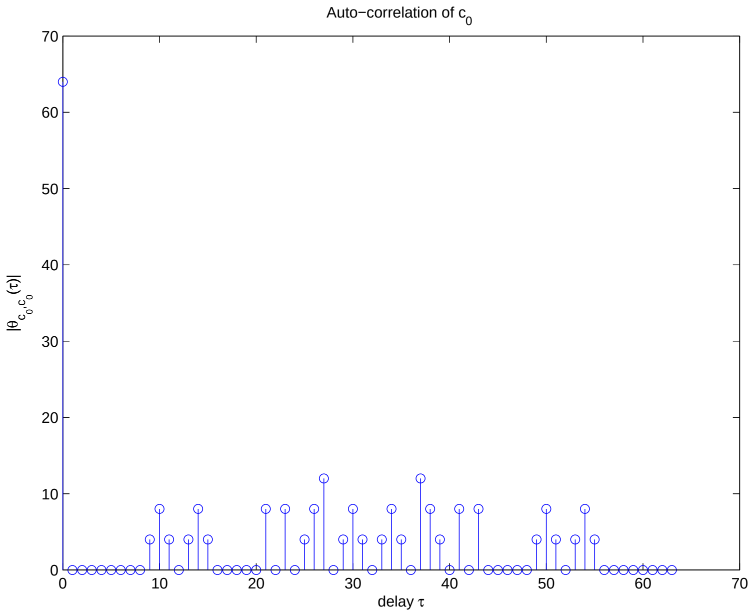


Figure 4.8: The autocorrelation function of C_0 in section 4.6

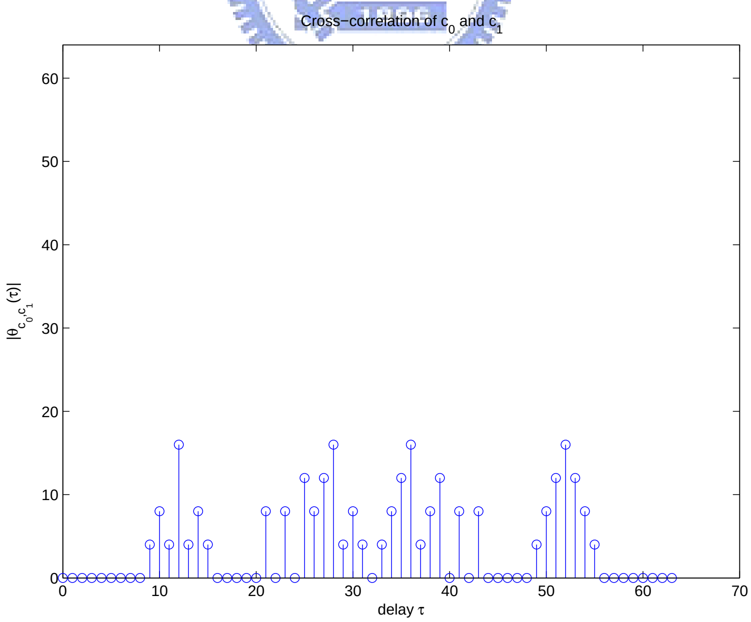


Figure 4.9: The crosscorrelation function of C_0 and C_1 in section 4.6

4.7 LAS-like ZCZ sequences

Large Area (LA) codes [20] [21] are ternary ($\pm 1, 0$)codes with a ZCZ and unity maximum correlation magnitude. Let us use the code which has the element 1 and 0 in a family of LA codes as the basic sequence (N, T) B with $w_H(B) = K$. Choosing suitable complementary sets, we can obtain a set of LAS-like ZCZ sequences and the length of ZCZ depends on the minimum size of zero strings in B' of the Hybrid Method.

The construction of a LAS spreading code is similar to the class of Hybrid Methods except that it uses Loosely Synchronous (LS) codes [19] instead of the collection $\mathcal{E}$ of complementary sets in step (C.2) of the Hybrid Methods. LS codes are constructed from two mates of complementary sets. The basic idea of the LS codes is the insertion of zeros between the complementary sequences in the same set to avoid overlaps between them. The LS codes can have a large family size by using one of the methods suggested in [19] with the constraint that the ZCZ length is fixed for given mates of complementary sets while increasing the sequence length. The advantage of LS codes is that it can be extended to have large family size without increasing the number of zeros, which can improve the duty ratio. The purpose of using LS codes in LAS codes is to achieve the highest possible duty ratio. However the ZCZ length of a LAS spreading code is constrained by the length of complementary sequences, which are used to construct LS codes.

In step (C.2) of the class of hybrid methods, the ZCZ length of LAS-like ZCZ sequences can be extended by increasing the minimum run-length of zero strings in B' using the same length complementary sequences of LS codes. The duty ratio reduction of LAS-like ZCZ sequences because of longer run-lengths can be compensated for by applying the modulating operation.

4.8 Summary and Comparisons

Table 4.1 compares the properties between some existing ZCZ sequences and new sequences generated by our methods. We also indicate in the table that if these ZCZ sequence sets achieve the bound of Corollary 3. The mark $\surd$ is used to indicate which of the four methods can be used to generate the corresponding ZCZ sequence sets. Whether a family achieves the theoretical bound is indicated by either $\circ$ (no) or $\bullet$ (yes).

Table 4.1: Comparison of ZCZ sequence sets

	PS ZCZ sequences in subsection 2.4.2	Ternary ZCZ sequences in subsection 2.4.3
Direct Methods	$\surd$	$\surd$
Complementary Methods		
Hybrid Methods		
Transform Domain Methods		
The bound in Corollary 3	$\bullet$	$\bullet$
	ZCZ sets in subsection 2.4.4	Hadamard ZCZ sequences in section 4.4
Direct Methods	$\surd$	$\surd$
Complementary Methods	$\surd$	
Hybrid Methods		
Transform Domain Methods		$\surd$
The bound in Corollary 3	$\circ$	$\bullet$
	New polyphase sequences in (D.1) in section 4.5	New polyphase sequences in (D.2) in section 4.5
Direct Methods	$\surd$	$\surd$
Complementary Methods		
Hybrid Methods		
Transform Domain Methods		
The bound in Corollary 3	$\circ$	$\bullet$
	New polyphase sequences based on mutually orthogonal complementary set in (E.4-i) in section 4.6	New polyphase sequences based on mutually orthogonal complementary set in (E.4-ii) in section 4.6
Direct Methods		
Complementary Methods	$\surd$	$\surd$
Hybrid Methods		
Transform Domain Methods		
The bound in Corollary 3	$\circ$	$\bullet$
	LAS-like ZCZ sequences in section 4.7	
Direct Methods		
Complementary Methods		
Hybrid Methods	$\surd$	
Transform Domain Methods		
The bound in Corollary 3	$\circ$	

Chapter 5

Multi-Dimensional Arrays

Like the one-dimensional (1D) case, two-dimensional (2D) arrays that possess some desired AC or CC properties are useful in some applications. In this section, we extend the class of *Direct Methods* to two and higher-dimension arrays. (It can be decomposed into a binary set as described in *Direct Methods*).

5.1 Preliminary

Definition 20 *Let a 2D array sequence $C = \{c_{i,j}\}$ be denoted by*

$$C = \begin{bmatrix} c_{0,0} & c_{0,1} & \cdots & c_{0,N1-1} \\ c_{1,0} & c_{1,1} & \cdots & c_{1,N1-1} \\ c_{2,0} & c_{2,1} & \cdots & c_{2,N1-1} \\ \cdots & \cdots & \cdots & \cdots \\ c_{N_2-1,0} & c_{N_2-1,1} & \cdots & c_{N_2-1,N1-1} \end{bmatrix}. \quad (5.1)$$

The 2D periodic AC function between two array sequences C and D having the same dimensions is defined by

$$\theta_{C,D}(\phi,\omega) = C \ominus_{2D} D = \sum_{p=0}^{N_2-1}\sum_{q=0}^{N_1-1} c_{p,q} d^*_{|p+\phi|_{N_2},|q+\omega|_{N_1}} \quad (5.2)$$

where $\ominus_{2D}$ is called 2D modulating operation

Definition 21 *An array is called a perfect array if its periodic AC function satisfies*

$$\theta_{C,C}(\phi,\omega) = \theta_C(\phi,\omega) = \begin{cases} E, & (\phi,\omega) = 0 \\ 0, & (\phi,\omega) \neq 0 \end{cases} \quad (5.3)$$

where $E = \sum_{p=0}^{N_2-1} \sum_{q=0}^{N_1-1} |c_{p,q}|^2$

Definition 22 *A set of* K *arrays* $\boldsymbol{C} = \{C_0, C_1, \ldots, C_{K-1}\}$ *of period* $N_2 \times N_1$ *is called a ZCZ family (or array set) if the periodic AC and CC functions of all its member arrays satisfy the requirement of an ideal set for* $|\tau_2| \leq T_2$*,* $|\tau_1| \leq T_1$*, and* $T_2 < N_2$*,* $T_1 < N_1$*. In other words,* $\theta_{C_i,C_j}(\tau_2, \tau_1) = 0$*,* $\theta_{C_i,C_i}(\tau_2, \tau_1) = \theta_{C_i}(0,0)\delta(\tau_2, \tau_1)$ *for* $C_i \neq C_j$*,* $\tau_2 \leq T_2$*, and* $\tau_1 \leq T_1$*.*
As a ZCZ array set $\boldsymbol{C}$ *is characterized by the parameters* (N_2, N_1, K, T_2, T_1)*, where* N_2 *is a vertical period of the arrays,* N_1 *is a horizontal period of the arrays,* K *is the family size(i.e., the number of arrays),* T_2 *is the length of vertical zero-correlation zone, and* T_1 *is the length of horizontal zero-correlation zone, we call such a array set a* (N_2, N_1, K, T_2, T_1) *2D ZCZ family.*

5.2 Generating of 2-D ZCZ Arrays

The procedure for generating a family of 2D arrays consists of four steps.

(F.1) Let B be a basic (N_2, N_1, T_2, T_1) array with $w_H(B) = K$ and $\mathbf{B} \stackrel{def}{=} \{B^r = \{b_{i,j\}^r}, 0 \leq r < M < K\}$ be an orthogonal tone decomposition of B.

(F.2) Compute the M product matrices $\mathbf{P^r} = \mathbf{H^r_{m_r}} \odot B^r$ where $m_r = w_H(B^r)$ and $H^r_{m_r}$ are unitary matrices(not necessarily distinct).

(F.3) With the definition of vectorization in Definition 16 , permute each row of $\mathbf{P^r}$ into a $N_2 \times N_1$ array, and denote these arrays as a array set $\mathbf{G^r}$

(F.4) Let $A = \{a_{i,j}\}$ be a perfect array with period $N_2 \times N_1$. Modulating each $N_2 \times N_1$ array of $\mathbf{G^r}$, with A through 2D modulating operation, where $0 \leq r < M$, we get a set of modulated array set $\mathbf{C} = \{C_0, C_1, \ldots, C_{K-1}\}$.

Theorem 10 *The array sets obtained in (F.3) and (F.4) are* (N_2, N_1, K, T_2, T_1) *2D ZCZ families. In (F.1), an* (N_2, N_1, M, T_2, T_1) *2D ZCZ family is obtained. A larger* *family with size $K \geq M$ is derived from this (N_2, N_1, M, T_2, T_1) family in (F.2) and (F.3). A perfect 2D AC array* **A** *is used to modulate the ZCZ arrays into arrays of finite constellation signals in (F.4).*

5.3 3-D and Multidimensional ZCZ Arrays

Definition 23 *A ZCZ set of K 3-dimensional (3D) arrays $\boldsymbol{C}=\{C_0, C_1, \ldots, C_{K-1}\}$ of period $N_1 \times N_2 \times N_3$ is characterized by the parameters $(N_1, N_2, N_3, K, T_1, T_2, T_3)$, where T_i is the length of zCZ on N_i-axis ($i = 1, 2, 3$).*

Recall that one of the key steps in generating the ZCZ families presented in the previous subsection is to find a $2D$ perfect array to modulate ZCZ array sequences into ones with elements from a desired constellation. Similarly, to construct a family of $3D$ or multi-dimensional ZCZ arrays, one needs to have a perfect $3D$ or multi-dimensional array (i.e., one whose $3D$ or multi-dimensional AC function is nonzero only at the origin) to begin with. Some works on the syntheses of perfect multidimensional arrays can be found in [13]. For $3D$ ZCZ arrays, one can prove

Corollary 7 *A family of 3D ZCZ arrays can be constructed by the following procedure.*

(G.1) *Let B be a basic $(N_1, N_2, N_3, T_1, T_2, T_3)$ 3D array with $w_H(B) = K$ and $\boldsymbol{B} \stackrel{def}{=} \{B^r = b_\{i, j, q\}^r, 0 \leq r < M < K\}$ be an orthogonal tone decomposition of B.*

(G.2) *Compute the M product matrices $\mathbf{P^r} = \mathbf{H^r_{m_r}} \odot B^r$ where $m_r = w_H(B^r)$ and $H^r_{m_r}$ are unitary matrices (not necessarily distinct).*

(G.3) *Permute each row of $\mathbf{P^r}$ into a $N_3 \times N_2 \times N_1$ 3D array, and denote these 3D arrays as $\mathbf{G^r}$*

(G.4) *Let $A = \{a_{i,j,q}\}$ be a perfect 3D array with period $N_3 \times N_2 \times N_1$. Modulating each $N_3 \times N_2 \times N_1$ 3D array of $\mathbf{G^r}$, with A through 2D modulating operation, where $0 \leq r < M$, we get a set of modulated 3-D array set $\boldsymbol{C} = \{C_0, C_1, \ldots, C_{K-1}\}$.*

Generalization of the above procedure for constructing higher dimensional ZCZ arrays is straightforward and shall be omitted in our discourse.

Chapter 6

Conclusion

In this thesis, we present several systematic approaches for constructing ZCZ sequences. The fact that the AC and CC functions are closely related to the DFTs of the desired sequences enable us to render a simple interpretation. Our approaches yield simple and straightforward generations of many existing ZCZ sequences and are capable of producing new ones with the desired parameters (N, K, T). We show how the parameter values in the classes of *direct methods* and *complementary methods* should be selected to generate polyphase ZCZ families. They are generalizations of [10] but render more flexible choices of parameter values for generating the desired ZCZ families. A new ZCZ family, called Hadamard ZCZ sequences, is particularly worth mentioning, for each of the member sequences has the perfect AC property in addition to the required CC property. We also address the issue of generating multi-dimensional arrays that possess similar desired correlation properties and present a systematic construction method.

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