Receiver window designs for radio frequency interference suppression in DMT systems

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Abstract: Windowing is often applied at the DMT (discrete multitone) receiver to suppress radio frequency interference (RFI). The spectral roll-off of the window determines how neighbouring tones are affected by RFI. However, the roll-off of the window is not of equal importance in all frequency range. In particular, the roll-off in high frequency will be inconsequential when the side lobes are so small that RFI is not the dominating noise. The window is designed here to minimise total interference. The frequency response of the proposed window achieves a good trade-off in spectral roll-off between high and low frequencies. As a result, fewer tones will be dominated by RFI than in the cases of commonly used Hanning and Blackman windows, in which the stopband in high frequency are often over designed. We have considered the case for informed receiver (RFI information available to the receiver) and uninformed receiver (RFI information unavailable to the receiver). In either case, the proposed window is channel-independent and can be obtained in a closed form.

1   Introduction

The very high speed digital subscriber line (VDSL) transmission system shares its spectrum with different types of radio transmission, for example, amplitude-modulation stations and amateur radio [1, 2]. These radio signals can be coupled into telephone wires and interfere with the VDSL signal at the receiving side. This type of noise in a VDSL transmission system is known as radio frequency interference (RFI) ingress [3]. The VDSL signal can also interfere with radio transmission. In the VDSL technology [1, 2], windowing is applied at the transmitting side on extended DMT symbols (cyclic prefix and cyclic suffix) to reduce out-of-band emission. The large side lobes of the rectangular window in conventional DMT systems lead to spectral leakage. As a result, many neighbouring tones can be affected. The signal-to-interference-noise ratio (SINR) of these tones are reduced and the total transmission rate is decreased. To improve RFI suppression, several methods of receiver windowing have been proposed [4–8]. The window helps to shape the frequency response of the rectangular window and significantly enhance the roll-off in high frequency. Commonly used windows include Hanning and Blackman windows [9]. One method that minimises the total error, which may include RFI and channel noise

at the DFT output is given in [4]. A frequency-domain or time-domain windowing followed by decision feedback equaliser for RFI suppression is given in [5]. A combination of raised-cosine window and per tone equaliser are proposed to suppress RFI interference in [6]. However, the channel information is required in these designs. In [7], channel-independent windows are designed by minimising the side lobe energy. In this case, ISI (inter symbol interference) is introduced and post-processing is required to cancel ISI. Using statistics of channel noise and RFI, a joint design of the time-domain equaliser and the receiving window for maximising bit rates is given in [8].

The spectral roll-off of the window determines how the tones are affected by RFI interference. A faster roll off means that the effect of RFI diminishes faster and fewer neighbouring tones are affected. However, the roll-off in high frequency becomes inconsequential when the side lobes are so small that RFI is not the dominating noise. There is a trade-off between high and low frequency roll-offs. In this paper, we design the window by minimizing total interference at the receiver outputs. The frequency responses of the proposed windows achieve a good trade-off in spectral roll-off between high and low frequencies. As a result, fewer tones will be dominated by RFI

interference than in the cases of commonly used Hanning and Blackman windows, in which the stopband in high frequency are often over designed. We will consider both the cases when the statistics of the interference is available to the receiver (informed receiver) and the case when it is not (uninformed receiver). The proposed windows in both cases are channel-independent and can be obtained in a closed form solution. In the simulations, we will see that the performances of the proposed informed and uninformed windows are better than that of the rectangular window, Hanning window, Blackman window, Kaiser window and the window design method in [4].

This paper is organised as follows. In Section 2, we will give the equivalent filterbank representation for the convenience of analysis. In Section 3, we will design the window for the informed receiver. In Section 4, we will design the window for the uninformed receiver. In Section 5, we will evaluate the performance of the proposed windows by simulations.

2 Filterbank representation

In this section, we derive the filterbank representation of the receiver with windowing. The representation will be useful in formulating the interference of individual tones. Fig. 1 shows a typical M-subchannel DMT receiver. After the removal of cyclic extension, M-pt DFT W is applied, where $[W]_{mn} = 1/\sqrt{M}e^{-j(2\pi mn/M)}$, $0 \leq m, n \leq M-1$. The scalar multipliers λ_i at the DFT outputs are known as frequency domain equaliser. Assume that the cyclic prefix (CP) is of length P. The transmitted block size is $N = P + M$. Suppose the channel order is l_c with $l_c < P$ and we have $L = P - l_c$ samples of cyclic prefix not affected by the channel. Therefore there are $M + L$ samples free from interblock interference for each block. To apply windows, the receiver takes these $M + L$ samples, multiplies the first L samples by the coefficients w_n, $n = 0, 1, \ldots, L-1$, and multiplies the last L samples by $1 - w_n$, where w_n are free parameters [4]. In other words, the $M + L$ samples are applied by a window of the following form.

$$g = \begin{bmatrix} w \\ \mathbf{1}_{M-L} \\ \mathbf{1}_L - w \end{bmatrix} \quad (1)$$

where $w = [w_0 \cdots w_{L-1}]^T$, and the notation $\mathbf{1}_n$ denotes an $n \times 1$ column vector whose elements are equal to 1. After
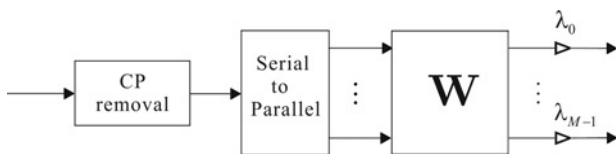
applying the window g, the receiver folds the first L samples and adds to the last L samples. Fig. 2 shows the DMT receiver with windowing.

The windowing operation in Fig. 2 can be represented by an $M \times N$ matrix B. The matrix B is given by

$$B = \begin{bmatrix} \mathbf{0} \\ I_L \quad I_M \end{bmatrix} \text{diag}(g) \begin{bmatrix} \mathbf{0} & I_{M+L} \end{bmatrix} \quad (2)$$

where $\text{diag}(g)$ is a diagonal matrix with the elements of g on its diagonal. To analyse the RFI in each subchannel, we can use the equivalent filterbank representation [10] as shown in Fig. 3.

The M receiving filters $H_i(z)$ for $i = 0, 1, \cdots, M-1$ are related to B and W by

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = WB \begin{bmatrix} 1 \\ z \\ \vdots \\ z^{N-1} \end{bmatrix} \quad (3)$$

Using the expression of B in (1), we can verify that the coefficients of the first receiving filter $h_0(n)$ are given by

$$h_0(n) = \frac{1}{\sqrt{M}}$$

$$\begin{cases} w_{-n-P+L}, & -(P-1) \leq n \leq -(P-L) \\ 1, & -(N-L-1) \leq n \leq -P \\ 1 - w_{-n-N+L}, & -(N-1) \leq n \leq -(N-L) \\ 0, & \text{otherwise} \end{cases} \quad (4)$$
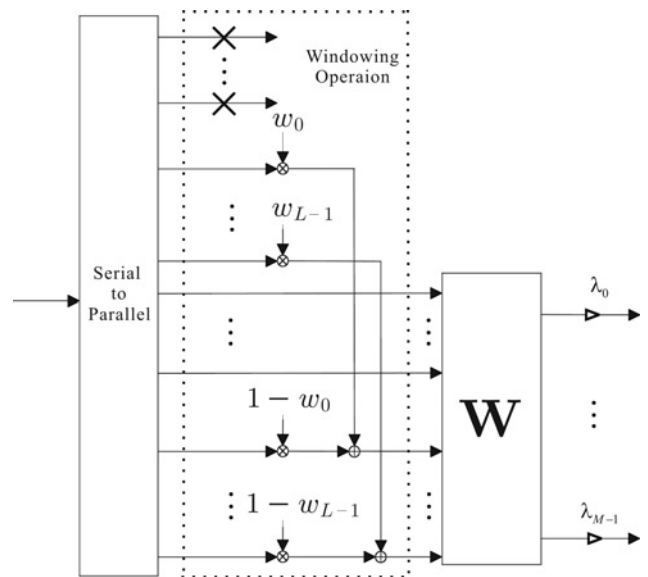

Figure 1 Traditional DMT receiver

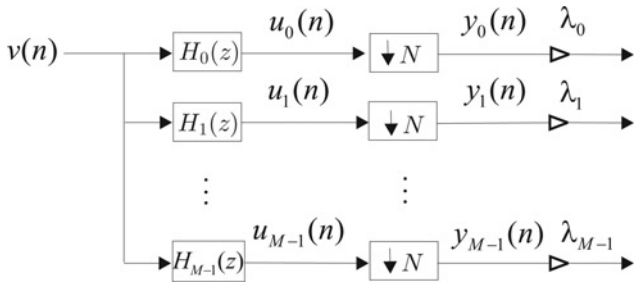
Figure 2 DMT receiver with windowing

Figure 3 *Filterbank representation of a DMT receiver with windowing*

We can further verify that all the receiving filters are shifted versions of the first receiving filter except for some scalars

$$H_i(z) = W^{-iP} H_0(zW^i) \tag{5}$$

where $W = e^{-j(2\pi/M)}$.

3 Informed window

In this section, we assume that the statistics of RFI are available to the receiver (informed receiver). We will design the informed window by minimising the interference of the tones that are used for transmission. The RFI is known to be a narrow band noise. For the duration of one VDSL symbol, it can be considered as sinusoids. We assume that RFI interference occurs at frequency ω_l with amplitude α_l and phase θ_l, $l = 0, \cdots, R-1$. Thus, we can model the interference as

$$v(n) = \sum_{l=0}^{R-1} \alpha_l \cos(\omega_l n + \theta_l) \tag{6}$$

To analyse the effect of $v(n)$, we apply the interference-only signal $v(n)$ to the receiver (Fig. 3). The output of the ith receiving filter is

$$u_i(n) = \frac{1}{2} \sum_{l=0}^{R-1} \alpha_l [c_{l,i} e^{j(\omega_l n + \theta_l)} + c'_{l,i} e^{-j(\omega_l n + \theta_l)}] \tag{7}$$

where $c_{l,i} = H_i(e^{j\omega_l})$ and $c'_{l,i} = H_i(e^{-j\omega_l})$. The interference at the ith receiver output is $y_i(n) = u_i(Nn)$, which has the same amplitude as $u_i(n)$.

The RFI interference of the ith individual tone is given by

$$J_i = \sum_{l=0}^{R-1} \alpha_l^2 [|c_{l,i}|^2 + |c'_{l,i}|^2] \tag{8}$$

where

$$c_{l,i} = W^{-iP} H_0(e^{j(\omega_l - 2\pi i/M)}) \quad \text{and}$$
$$c'_{l,i} = W^{-iP} H_0(e^{-j(\omega_l + 2\pi i/M)}) \tag{9}$$

The total RFI interference is given by

$$J = \sum_{i \in U} J_i$$
$$= \sum_{i \in U} \sum_{l=0}^{R-1} \alpha_l^2 [|H_0(e^{j(\omega_l - 2\pi i/M)})|^2$$
$$+ |H_0(e^{-j(\omega_l + 2\pi i/M)})|^2] \tag{10}$$

where U is the set of tones that are used for the current transmission. From (4) we can verify that $H_0(e^{j(\omega_l - 2\pi i/M)})$ can be given in terms of $\boldsymbol{w}$ as

$$H_0(e^{j(\omega_l - 2\pi i/M)}) = b_{l,i} + \boldsymbol{a}_{l,i}^\dagger \boldsymbol{w} \tag{11}$$

where the notation '$\dagger$' denotes Hermitian, $b_{l,i}$ is a scalar and $\boldsymbol{a}_{l,i}$ is an $L \times 1$ column vector given, respectively, by

$$b_{l,i} = \sum_{k=P}^{P+M-1} e^{j(\omega_l - 2\pi i/M)k}$$
$$[\boldsymbol{a}_{l,i}]_m = e^{j(\omega_l - 2\pi i/M)(P-L+m)} - e^{j(\omega_l - 2\pi i/M)(N-L+m)} \tag{12}$$

Similarly, we can verify that $H_0(e^{-j(\omega_l + 2\pi i/M)})$ can be expressed by

$$H_0(e^{-j(\omega_l + 2\pi i/M)}) = b'_{l,i} + \boldsymbol{a}'^\dagger_{l,i} \boldsymbol{w} \tag{13}$$

where $b'_{l,i}$ and $\boldsymbol{a}'_{l,i}$ are, respectively,

$$b'_{l,i} = \sum_{k=P}^{P+M-1} e^{-j(\omega_l + 2\pi i/M)k}$$
$$[\boldsymbol{a}'_{l,i}]_m = e^{-j(\omega_l + 2\pi i/M)(P-L+m)}$$
$$- e^{-j(\omega_l + 2\pi i/M)(N-L+m)} \tag{14}$$

Using (11) to (14), the objective function can be written in terms of $\boldsymbol{w}$ as

$$J = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{A} \boldsymbol{w} + \boldsymbol{w}^{\mathrm{T}} \boldsymbol{b} + \boldsymbol{b}^\dagger \boldsymbol{w} + c \tag{15}$$

where $\boldsymbol{A}$ is an $L \times L$ matrix, $\boldsymbol{b}$ is an $L \times 1$ vector, and c is a scalar given, respectively, by

$$\boldsymbol{A} = \sum_{i \in U} \sum_{l=0}^{R-1} \alpha_l^2 [\boldsymbol{a}_{l,i} \boldsymbol{a}_{l,i}^\dagger + \boldsymbol{a}'_{l,i} \boldsymbol{a}'^\dagger_{l,i}]$$
$$\boldsymbol{b} = \sum_{i \in U} \sum_{l=0}^{R-1} \alpha_l^2 [\boldsymbol{a}_{l,i} b_{l,i} + \boldsymbol{a}'_{l,i} b'_{l,i}]$$
$$c = \sum_{i \in U} \sum_{l=0}^{R-1} \alpha_l^2 [|b_{l,i}|^2 + |b'_{l,i}|^2] \tag{16}$$

To minimise the objective function in (15), we can use the method of optimisation in [11] to obtain a closed-form solution. In particular, when the objective function J in

(15) is minimal, the optimal $\boldsymbol{w}$ must satisfy $\partial J/\partial \boldsymbol{w} = 0$. The optimal solution can be written as

$$\boldsymbol{w} = -[\mathfrak{R}(\boldsymbol{A})]^{-\mathrm{T}}\mathfrak{R}(\boldsymbol{b}) \qquad (17)$$

where notation $\mathfrak{R}(X)$ denotes the real part of X. In this solution, no channel information is required; only the statistics of the RFI interference are needed.

4 Uninformed window

We now consider the case when the statistics of RFI interference are not available to the receiver (uninformed receiver). In this case, the frequency and amplitude of RFI are not known. We can minimise the total interference by minimising the stopband energy of $H_0(e^{j\omega})$

$$\Phi = \int_{\omega_s}^{2\pi-\omega_s} |H_0(e^{j\omega})|^2 \frac{\mathrm{d}\omega}{2\pi} \qquad (18)$$

where ω_s is the stopband bandedge. From (4), we can write $H_0(e^{j\omega})$ as

$$H_0(e^{j\omega}) = \boldsymbol{s}^\dagger \boldsymbol{g} \qquad (19)$$

where $\boldsymbol{g}$ is the window vector and $\boldsymbol{s}$ is an $(M+L) \times 1$ column vector given by

$$\boldsymbol{s} = \begin{bmatrix} e^{j\omega(P-L)} \\ e^{j\omega(P-L+1)} \\ \vdots \\ e^{j\omega(P+M-1)} \end{bmatrix} \qquad (20)$$

Then the stopband energy, Φ, can be rewritten as

$$\Phi = \int_{\omega_s}^{2\pi-\omega_s} (\boldsymbol{g}^\dagger \boldsymbol{s}\boldsymbol{s}^\dagger \boldsymbol{g}) \frac{\mathrm{d}\omega}{2\pi} = \boldsymbol{g}^\dagger \boldsymbol{Q}\boldsymbol{g} \qquad (21)$$

where

$$\boldsymbol{Q} = \int_{\omega_s}^{2\pi-\omega_s} \boldsymbol{s}\boldsymbol{s}^\dagger \frac{\mathrm{d}\omega}{2\pi} \qquad (22)$$

The elements of Q are given by

$$[\boldsymbol{Q}]_{mn} = \begin{cases} -\frac{\sin{(m-n)\omega_s}}{\pi(m-n)} & m \neq n \\ 1 - \frac{\omega_s}{\pi} & m = n \end{cases} \qquad (23)$$

The window vector $\boldsymbol{g}$ can be written as

$$\boldsymbol{g} = \boldsymbol{d} + \boldsymbol{E}\boldsymbol{w} \qquad (24)$$

where $\boldsymbol{d}^{\mathrm{T}} = [\,\boldsymbol{0} \quad \boldsymbol{1}_M^{\mathrm{T}}\,]$, and $\boldsymbol{E}^{\mathrm{T}} = [\,\boldsymbol{I}_L \quad \boldsymbol{0} \quad -\boldsymbol{I}_L\,]$.

As a result, the objection function can be given in terms of $\boldsymbol{w}$

$$\Phi = (\boldsymbol{d} + \boldsymbol{E}\boldsymbol{w})^{\mathrm{T}}\boldsymbol{Q}(\boldsymbol{d} + \boldsymbol{E}\boldsymbol{w}) \qquad (25)$$

Similar to the informed window, using the method of optimisation in [11], we can obtain the following optimal uninformed solution $\boldsymbol{w}$ that minimises the stopband energy

$$\boldsymbol{w} = -(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{E})^{-\mathrm{T}}(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{d}) \qquad (26)$$

In this case, neither the channel nor the RFI information is needed for obtaining the window.

5 Simulations

In this section, we will evaluate the proposed window design technique. The channels used for our evaluations are seven VDSL loops [1]. The DFT size is $M = 1024$, cyclic prefix $P = 80$ and window length $L = 10$. The channel noise consists of AWGN of -140 dBm, FEXT and NEXT cross-talk as described in [1]. The time-domain equaliser of length 20 is used to shorten the channel to length less than 70 [12]. The RFI interference is of differential mode with strength -55 dBm [1]. Three RFI sources with frequencies at 1.44, 1.9 and 2.0 MHz are considered. In this simulation, the RFI signal is generated as in [1]. We will first use VDSL loop 1 of length 4500 ft as an example to examine the frequency response of the proposed window and demonstrate the effect on subchannel interference and SINR.

Frequency response: Suppose the statistics of RFI is available to the receiver. We compute $\boldsymbol{w}$ using (17) and obtain the informed window form (1). Fig. 4 shows the frequency response of the informed window. For comparison, we have also shown the frequency responses of the Hanning, Blackman, and Kaiser windows with shape parameter $\beta = 5$ [9]. We can see that the informed window has a faster roll-off in low frequency whereas the other three windows have much smaller side lobes in high frequency. However, the roll-off in high frequency will not be
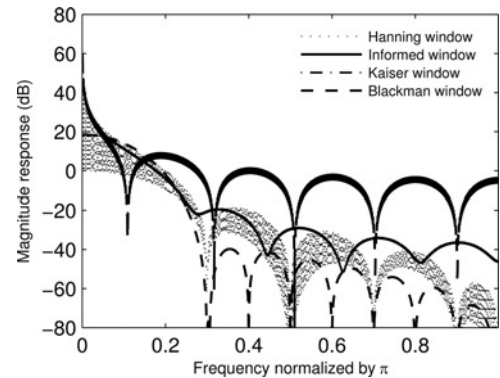

**Figure 4** *Frequency response of receiving windows*

important when the side lobes are so small that RFI is not the dominating noise. As the proposed window has the characteristics of fast roll-off in low frequency, fewer tones will be dominated by RFI as we will see next.

Subchannel interference: We compute the interference power at the receiver outputs for the receiving windows. Fig. 5 shows the RFI interference power of individual tones for the informed window, uninformed window, window in [4], Hanning window, Blackman window and Kaiser window with shape parameter $\beta = 5$. In Fig. 5a, we compare the window in [4] with the Hanning window. In Fig. 5b, we compare the Blackman and Kaiser windows. We can see that the informed and uninformed windows have lower RFI power than the other four windows near the RFI source frequencies. Also shown in Fig. 5a and b are the combined effects of channel noise (AWGN, FEXT and NEXT) and

the residual ISI for the informed window, uninformed window, window in [4], Hanning window, Blackman window and Kaiser window, which are labelled as 'other noise (informed)', 'other noise (uninformed)', 'other noise (window [4])', 'other noise (Hanning)', 'other noise (Blackman)', and 'other noise (Kaiser)'. In both Fig. 5a and b, the curves of 'other noise' overlap with each other and are indistinguishable in the figure. From Fig. 5, we can see that RFI is dominating in the tones around the RFI frequencies. For the tones away from the interference sources, other noise is dominating. As a result, higher attenuation of the window in high frequency is of little significance. In this case, the commonly used Hanning and Blackman windows are over designed in the high frequency region. The proposed windows, because of their faster roll-off in low frequency, has fewer RFI dominating tones.
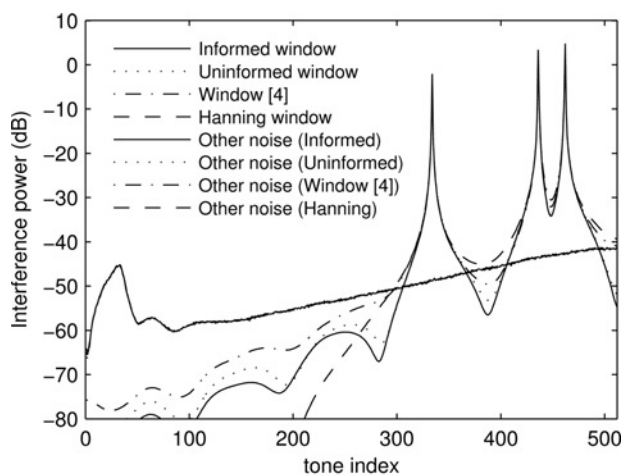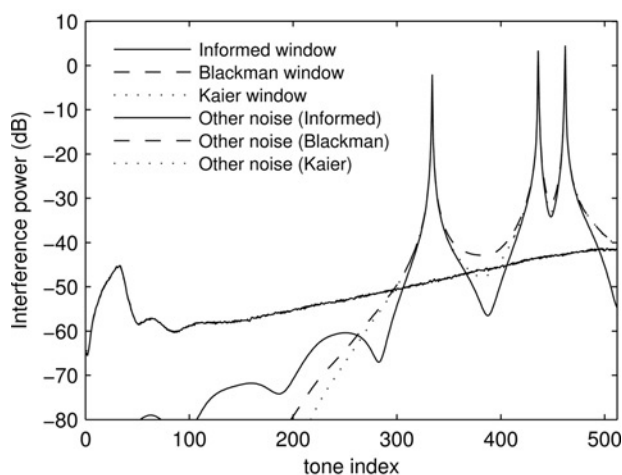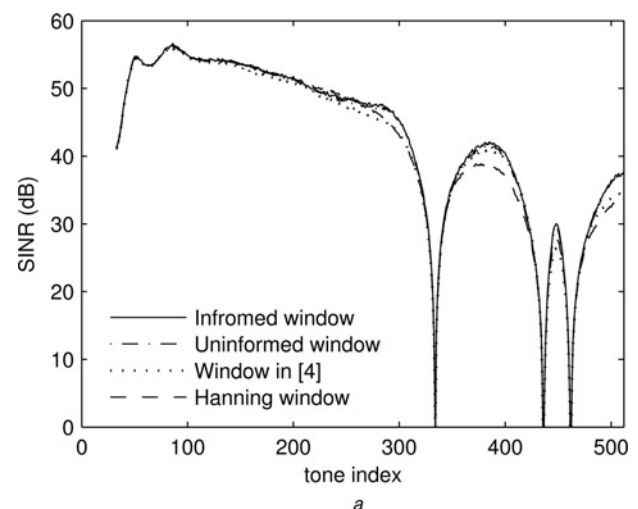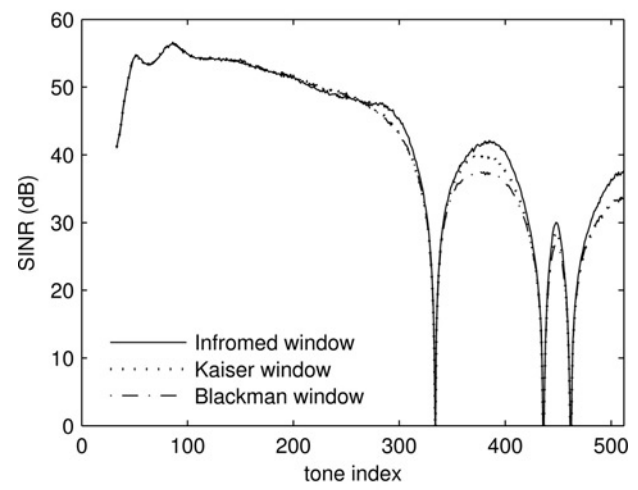

Figure 5 *Subchannel interference power of the DMT system with windowing*

a Informed window, uninformed window, window in [4] and Hanning window
b Informed, Blackman and Kaiser windows with shape parameter $\beta = 5$

Figure 6 *Subchannel SINRs of the DMT system with windowing*

a Informed window, uninformed window, window in [4] and Hanning window
b Informed, Blackman and Kaiser windows with shape parameter $\beta = 5$

Table 1 Bit rate (Mbits/s) on VDSL loops

Loop	1	2	3	4	5	6	7
Informed	20.74	20.42	18.94	11.25	26.60	22.75	17.97
Uninformed	20.60	20.40	18.90	11.22	26.58	22.70	17.94
Rectangular	19.72	19.49	17.95	10.40	26.38	21.92	16.94
Hanning	20.23	19.96	18.59	10.90	26.48	22.33	17.42
Blackman	20.14	19.86	18.48	10.80	26.39	22.32	17.46
Kaiser $\beta = 5$	20.38	20.02	18.78	10.97	26.46	22.33	17.68
Window [4]	20.24	20.23	18.82	11.06	26.52	22.60	17.79
No RFI	23.34	22.78	21.49	13.45	27.59	22.39	20.57

Subchannel SINRs: Fig. 6 shows the SINRs of the individual tones for both informed and uninformed windows. For comparison, in Fig. 6a, b, we have also shown the SINRs of the window in [4], Hanning, Blackman and Kaiser windows with shape parameter $\beta = 5$. From Fig. 6a, b, we see that the SINRs of the informed and uninformed windows are higher than those of the other windows near the RFI source frequency, that is, in the tones where RFI interference is dominating. This is because of the fact that the proposed windows achieve a better trade-off in low and high frequencies. Therefore we can transmit more bits in the neighbouring tones by using the proposed windows. The two curves corresponding to the two proposed windows almost overlap with each other. This shows that the use of uninformed window leads to only a minor performance degradation.

Table 1 shows the bit rates for seven VDSL loops [1] with window length $L = 10$, where VDSL loops 1 to 4 are of length 4500 ft. For comparison purpose, we have also included the bit rates of the rectangular, Hanning, Blackman, Kaiser windows and the window in [4]. In addition, the bit rates for the case when there is no RFI interference are also shown in the table. From the table, we can see that the proposed windows have better performance for all the test loops.

6 Conclusion

We have proposed a window design method for RFI suppression in DMT systems. The proposed windows strike a balance between low and high frequency response. Thus, fewer tones are dominated by RFI and better bit rates are achieved. We consider both the cases when the receiver knows the statistics of the interference (informed receiver) and the case when the statistics are not available to the receiver (uninformed receiver). In both cases, the windows are channel-independent and can be obtained in a closed form. Windows designed for uninformed receiver (interference-independent window) has the advantage that the window coefficients need not be updated when the statistics of the RFI interference changes. It can also be seen that not knowing the statistics of the RFI source leads to only a minor performance degradation.

8 References

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