國 立 交 通 大 學

電信工程學系

碩 士 論 文

多重輸入多重輸出正交分頻多工空間分割存取下鏈系統在

通道估計錯誤下之共同波束設計

Joint Tx/Rx MMSE Beamforming Design for Multi-user MIMO-OFDM SDMA Downlink System under Channel Estimation Error

研究生：林偉凱

指導教授：紀翔峰

中 華 民 國 九十五年八月

多重輸入多重輸出正交分頻多工空間分割存取下鏈系統在通道估計錯誤下之共同波束設計

研究生：林偉凱　　　　　　指導教授：紀翔峰 博士

國立交通大學

電信工程學系碩士班

摘要

近年來，多重輸入多重輸出正交分頻多工空間分割多重存取技術已成為現今寬頻無線網路系統的主要方法之一，在此論文中，我們將著重此技術在多用戶下鏈系統中的波束設計及應用。

過去，已有不少的文獻研究收發機共同波束設計在多重輸入多重輸出系統的應用，然而在這些研究都考慮單一用戶系統，此外，他們也都假設在有完美的通道估計來設計。然而，這個假設在實際通訊系統中是不存在的，故在此論文中，我們將針對多用戶系統，在通道估計有誤差的情況下，提供一個收發機共同波束設計方法能有效抵抗通道錯誤所造成的系統效能損失。

收發器共同波束技術能將原本空間上互相干擾的多重輸入多重輸出系統轉

換成一個子通道間互相獨立不干擾的等效系統。而在我們所提出的多重輸入多重輸出正交分頻多工空間分割多重存取下鏈系統中，藉由在傳送端利用零空間技巧，我們能將多用戶系統轉換為一個用戶間互相獨立的系統，如此便可將文獻中的單一用戶收發機共同波束技術應用在每個獨立用戶上。此外，在有多路徑的頻率選擇通道下，我們將採用正交分頻多工技術應用在上述系統中。

進一步的，先前所提到的設計方法皆假設在有完美的通道估計條件下，然而在實際系統上通道估計通常會有誤差，此誤差將造成顯著的系統效能損失，此損失可歸咎於因為不完美零空間矩陣所造成多用戶間互相的干擾，以及因為不完美收發機共同波束設計所造成每個用戶本身資料串之間的干擾。在此論文中，我們將考慮多用戶多重輸入多重輸出正交分頻多工空間分割多重存取下鏈系統在通道估計有誤差的情況下，提供一個較能抵抗因為通道錯誤所造成系統效能損失的收發機共同波束設計。最後，我們利用電腦模擬來評估比較系統效能。

Joint Tx/Rx MMSE Beamforming Design for Multi-user MIMO-OFDM SDMA Downlink System under Channel Estimation Error

Student：Wei-Kai Lin Advisor：Dr. Hsiang-Feng Chi

Department of Communication Engineering

National Chiao Tung University

Hsinchu, Taiwan

Abstract

The techniques of MIMO-OFDM SDMA have recently considered one of the candidates for the broad-band multiple-access wireless systems. In this thesis, we focus on the research of beamforming in the applications of multi-user MIMO-OFDM SDMA downlink wireless system.

Many researchers have used the joint Tx/Rx beamforming approach in the MIMO systems. However, they only considered the single user system. In addition, their methods are proposed under the perfect channel estimation assumption, which is no true in the real world. In this thesis, we will aim at multi-user MIMO SDMA systems to propose a MMSE beamforming algorithm, which has better immunity to

the channel estimation errors than the previously published methods.

The joint Tx/Rx beamforming technique can transform a mutually cross-coupled MIMO transmission system into a set of independent eigen subchannels so that subchannel signal can be decoded without the interference from other antennas. In our proposed MIMO-OFDM SDMA downlink system, by using null-space technique in the base station (Tx side), the multi-user MIMO downlink system can be simplified into several independent single user MIMO subsystems. The MMSE beamforming algorithm is further applied to each single user MIMO subsystem achieving the spatial diversity gain. In addition, to combat the frequency-selective channel caused by multi-paths, we focus the aforementioned problems in the MIMO-OFDM systems.

However, all the previously proposed methods are derived under the assumption of perfect channel estimation. But in real wireless systems, the channel estimate always contains estimation errors which may cause significant performance degradation when these methods are used. The performance loss can be imputed to the multi-user interference induced by the imperfect null-space matrix and the inter-stream interference caused by imperfect Tx/Rx beamforming. In this thesis, without the assumption of perfect channel estimation, we will consider the joint Tx/Rx beamforming design for multi-user MIMO-OFDM SDMA downlink system which is more robust to channel estimation error than the previous methods. Finally, the performance of the proposed schemes will be evaluated by computer simulations.

誌謝

光陰似箭，短短兩年碩士班的研究生涯，以此論文之告成，即將劃上句號。在這兩年裡，首當感謝指導教授 紀翔峰老師於研究以及論文上給予充分且詳實的指導與教誨鼓勵，令學生受益匪淺。

其次感謝交通大學電信系教授 吳文榕老師以及教授 黃家齊老師擔任口試委員的指導與建議，在此也表現由衷的感謝。

衷心的感謝實驗室的學長、同學及學弟們給予研究上的指導幫助才能順利完成碩士生涯，也共同度過許多快樂的時光，更有難忘的回憶，謝謝你們。

最後，要特別感謝我的父母以及家人還有女友及其家人，特別是父母的辛苦栽培以及默默付出，還有女友平時的照顧，使得我得以全心投入學業。再次衷心的謝謝以上諸位，由於你們的幫忙，此論文得以完成，衷心感謝。

僅以此篇論文獻給對我付出關心的你們。

民國九十五年八月

研究生林偉凱謹識於交通大學

Contents

List of Figures

Chapter 1

Introduction

In the recent years, due to the rapid increase in broadband access demand, the wireless system with high data rates and high link quality services has become one of the hot research topics. The MIMO-OFDM system is considered as one of the most promising technologies to provide these services. In this chapter, we will begin with the brief overview of MIMO systems, OFDM technology, and the MIMO-OFDM system. Then, we describe the motivation of the research. Finally, the organization of this thesis will be presented in the end of this chapter.

1.1 MIMO-OFDM Based System

To meet the requirement of high data rates and good link quality, the multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) is considered as one of the promising technologies for broadband wireless communication systems, such as 802.11n and 802.16e. In the following sections, we will briefly introduction the basics of MIMO [1] [2] [3] and OFDM [4] technologies and describe how they can provide high data rate and link quality services. These two technologies are further combined [5] to solve the multi-path channel problem in broadband wireless systems. Each subcarrier of OFDM system can be treated as a simple flat-fading MIMO channel.

1.1.1 Overview of MIMO System

1.1.1.1 MIMO Channel Models

The basic signal model for a single-carrier flat-fading MIMO channel with M transmit and N receive antennas is illustrated in Figure 1-1 where $\mathbf{H}=\{h_{nm}\}$ is the channel matrix with dimension $N \times M$, $\mathbf{s}(t)=\{s_m(t)\}$ is input vector and $\mathbf{y}(t)=\{y_n(t)\}$ is output vector. The discrete-time model with matrix form can be wrote as

$$\mathbf{y}(k)=\mathbf{H}\mathbf{s}(k)+\mathbf{n}(k) \qquad (1.1.1.1\text{-}1)$$

where $\mathbf{y}(k)=[y_1(k), y_2(k), \cdots, y_N(k)]^T$ is the receive signal vector, $\mathbf{x}(k)=[x_1(k), x_2(k), \cdots, x_M(k)]^T$ is the transmit signal vector, $\mathbf{H}=\{h_{nm}\}$ is channel matrix with dimension $N \times M$ and $\mathbf{n}(k)$ is noise vector.

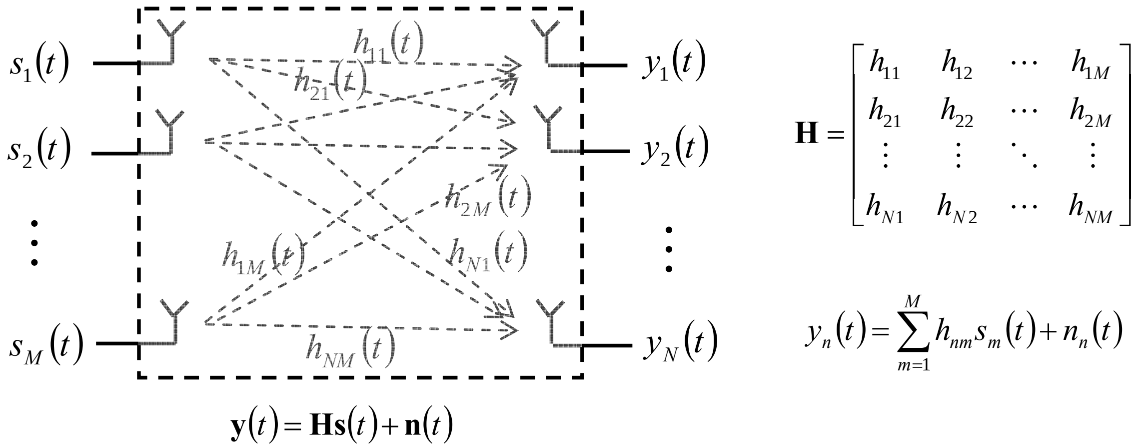


Figure 1-1 A MIMO system with M transmit and N receive antennas under flat-fading channel

We now consider the frequency-selective fading channel. The signal model for such environment is shown as following Figure 1-2

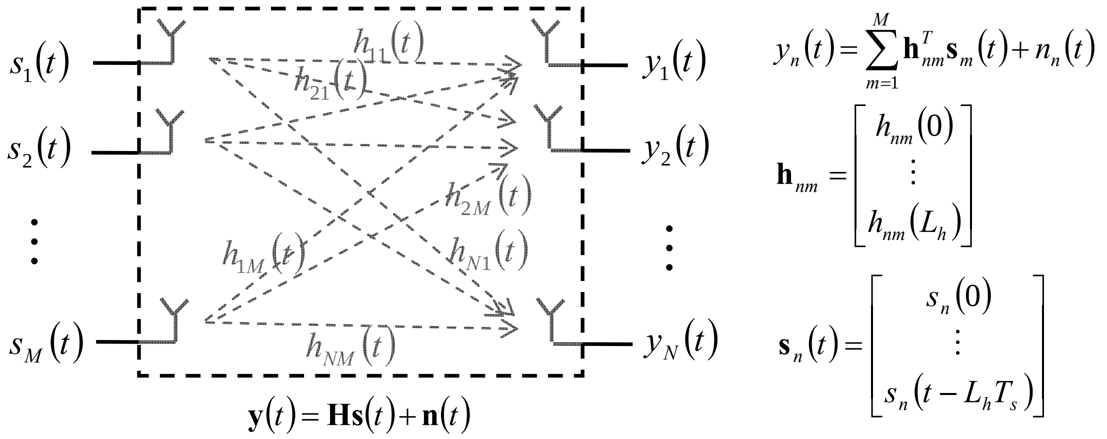


Figure 1-2 A MIMO system with M transmit and N receive antennas under frequency selective channel

where $\mathbf{H} = \{\mathbf{h}_{nm}^T\}$ is the MIMO channel matrix with dimensions $N \times M(L_h + 1)$, and L_h is maximum channel order. The MIMO system can be described by the discrete-time model as

$$\mathbf{y}_k = \sum_{l=0}^{L} \mathbf{H}_l \mathbf{s}_{k-l} + \mathbf{n}_k \qquad (1.1.1.1\text{-}2)$$

$$\begin{bmatrix} y_{1,k} \\ y_{2,k} \\ \vdots \\ y_{N,k} \end{bmatrix} = \sum_{l=0}^{L} \begin{bmatrix} h_{11,l} & h_{12,l} & \cdots & h_{1M,l} \\ h_{21,l} & h_{22,l} & \cdots & h_{2M,l} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1,l} & h_{N2,l} & \cdots & h_{NM,l} \end{bmatrix} \begin{bmatrix} s_{1,k} \\ s_{2,k} \\ \vdots \\ s_{M,k} \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n_{2,k} \\ \vdots \\ n_{N,k} \end{bmatrix} \qquad (1.1.1.1\text{-}3)$$

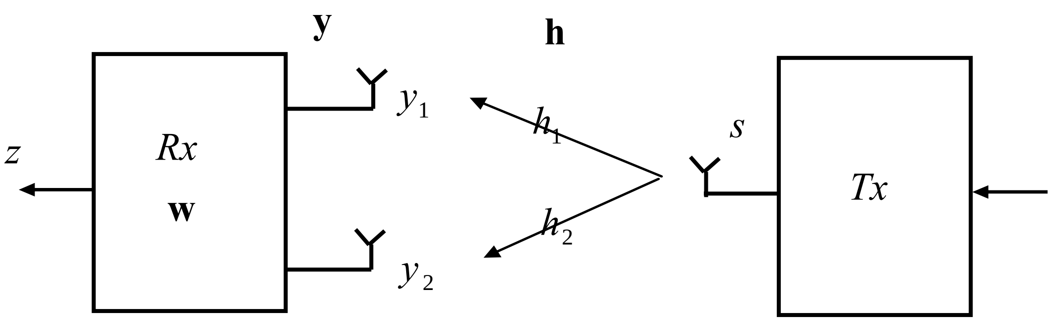
Note that when $M = 1$, the MIMO channel reduces to a single-input multiple output (SIMO) channel. Similarly, when $N = 1$, the MIMO channel reduces to a multiple-input single-output (MISO) channel. When both $M = 1$ and $N = 1$, the MIMO channel simplifies to a simple traditional SISO channel. Following, we will discuss some properties of MIMO channels.

1.1.1.2 Properties of The MIMO Channel

MIMO channels have a number of advantages over traditional SISO channels such as the beamforming (or array) gain, the diversity gain, the interference suppression gain and the multiplexing gain [2] [6]. The beamforming and the diversity gains exist not only in MIMO channels but also in MISO or SIMO channels. However, the multiplexing gain is a unique feature that exists only in MIMO channels. Some gains can be simultaneously achieved while others compete and become a tradeoff [2] [6]. We briefly introduce these gains as follows [2] [6]:

- Beamforming (or array) Gain: Multiple antennas (or array antenna) at transmit or receive terminal coherently combine the signal energy improving the signal-to-noise ratio (SNR) and suppress interference. Therefore, if the bit-error-ratio (BER) of a communication system is plotted with respect to the transmitted or received power per antenna using a logarithmic scale, the beamforming or array gain is easily characterized as a shift of the performance curve due to the gain in SNR.

  Besides, beamforming is a term traditionally associated with array processing or smart antennas in wireless communication systems where an array antenna can be arranged either at transmitter or at receiver. For illustration purposes, we consider a SIMO system shown in Figure 1-3 with the received signal given by $\mathbf{y} = \mathbf{h}s + \mathbf{n}$ where $\mathbf{h}$ is the channel of the desired signal and $\mathbf{n}$ is white noise with $E\left[\mathbf{n}\mathbf{n}^H\right] = \sigma^2\mathbf{I}$.



$SNR = SNR_1 + SNR_2$

Figure 1-3 Beamforming gain obtained from a SIMO system

The receiver uses a beamforming vector $\mathbf{w}$ to combine all the elements of $\mathbf{y}$ in a coherent way as $z = \mathbf{w}^H \mathbf{y}$. If the beamvector matches the channel, that is if $\mathbf{w} = \mathbf{h}$, the SNR is maximized and given by

$$\text{SNR} = \|\mathbf{h}\|^2 / \sigma^2 \qquad (1.1.1.2\text{-}1)$$

which clearly shows the increase of SNR with respect to using a single receive dimension

$$\text{SNR} = |h|^2 / \sigma^2 \qquad (1.1.1.2\text{-}2)$$

- Diversity Gain: Diversity gain obtained from multiple antennas helps to combat channel fading and enhance the link reliability. The receiver receives replicas of the information signal through independently fading links, branches, or dimensions occurred by multiple antennas equipped at the transmit or the receive terminal. This type of gain is clearly related to the random nature of the channel and is closely connected to the statistics of channels. If the BER of a communication system is plotted with respect to the transmitted power or the received power per antenna using a logarithmic scale, the diversity gain is easily characterized as the increase of the slope of the performance curve in the low BER region [2]. The basic idea is that with high probability, at least one or more of these links will not be in a fade at any given instant. In other words, the use of multiple dimensions reduces the fluctuations of the received signal and eliminates the deep fades.

Clearly, this concept is suitable for wireless communications where fading exists due to multipath effects and it may not be useful for wireline communications where the fading effect does not exist.

There are three main forms of traditional diversity used in wireless communication systems, temporal diversity, frequency diversity, and spatial diversity. The spatial diversity gain can be obtained from receive and transmit antenna array.

- Interference suppression gain: Interference suppression gain is obtained from multiple antennas equipped at receiver where adaptively combines to selectively cancel or avoid interference and pass the desired signal.
- Multiplexing Gain: Spatial multiplexing gain uses multiple antennas at both ends to create multiple channels and increase of rate, at no additional power consumption. While the beamforming and the diversity gains can be obtained when multiple antennas are equipped at either transmit or the receive side, multiplexing gain requires multiple dimensions at both ends of the link. The basic idea is to exploit the multiple dimensions to create several parallel subchannels within the MIMO channel, which lead to a linear increase in capacity. The multiplexing property allows the

transmission of several symbols simultaneously. Figure 1-4 shows the spatial multiplexing gain obtain from a MIMO system.

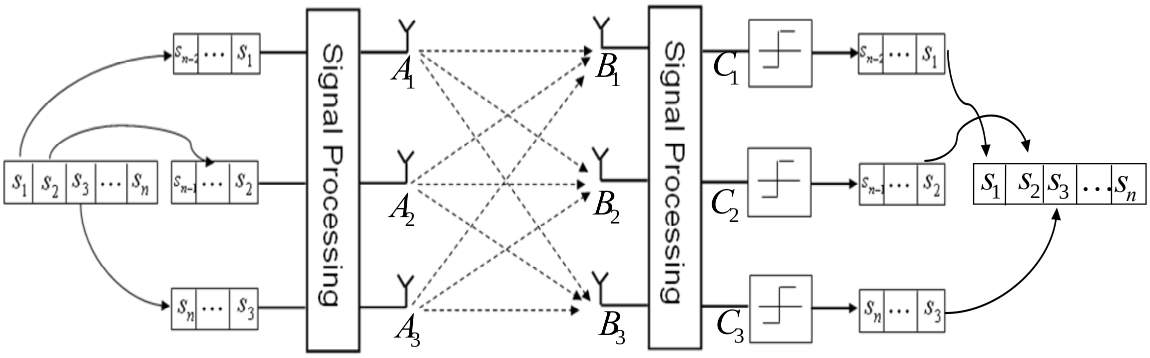


Figure 1-4 Multiplexing gain obtained from a MIMO system

These types of smart antenna can be used to improve coverage, link quality, data rates and system capacity. We summarize the above descriptions in Figure 1-5.

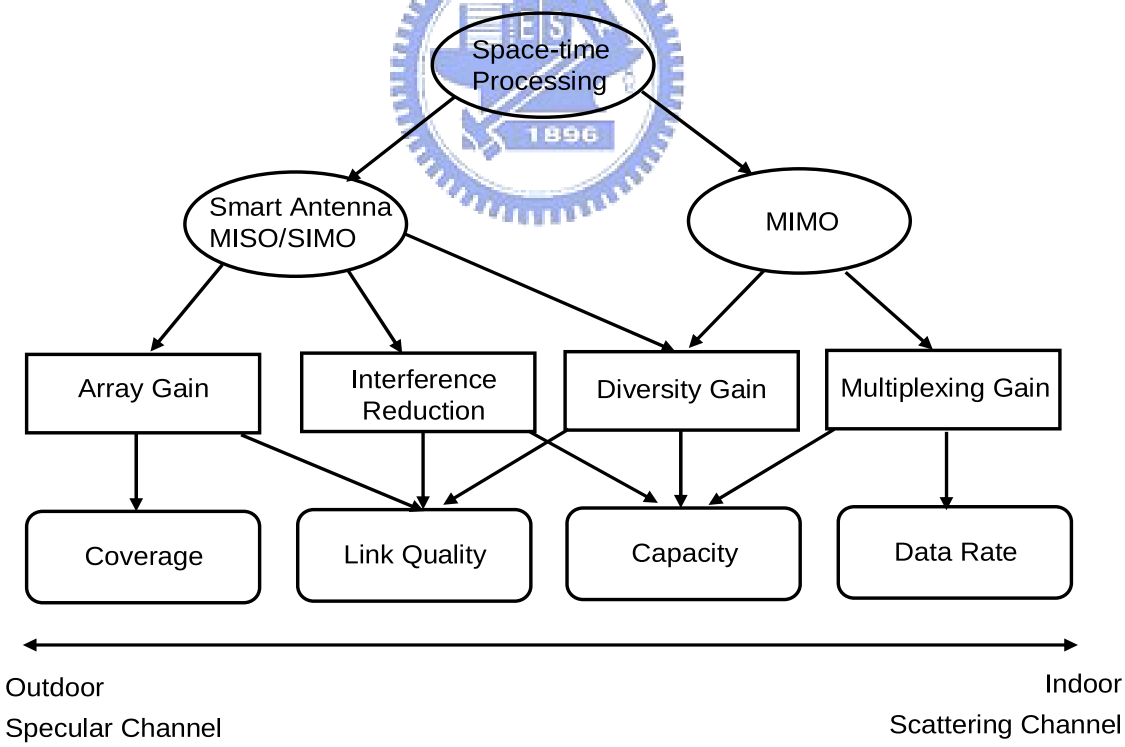


Figure 1-5 Smart antennas overview

1.1.1.3 Tradeoffs Between Gains

- Beamforming and Diversity Gains

Beamforming gain is a concept that refers to combine multiple copies of the same signal for a specific channel realization not for the statistics of a channel. However, diversity gain is directly related to the statistical behavior of a channel. With multiple antennas at receive terminal, both array and diversity gains can be achieved at the same time by a coherent combination of the received signals and there is no tradeoff between them. With multiple antennas at transmit terminal, beamforming gain requires CSI (Channel State Information) at the transmitter while diversity gain can be achieved even when the CSI is unknown.

- Beamforming and Multiplexing Gains

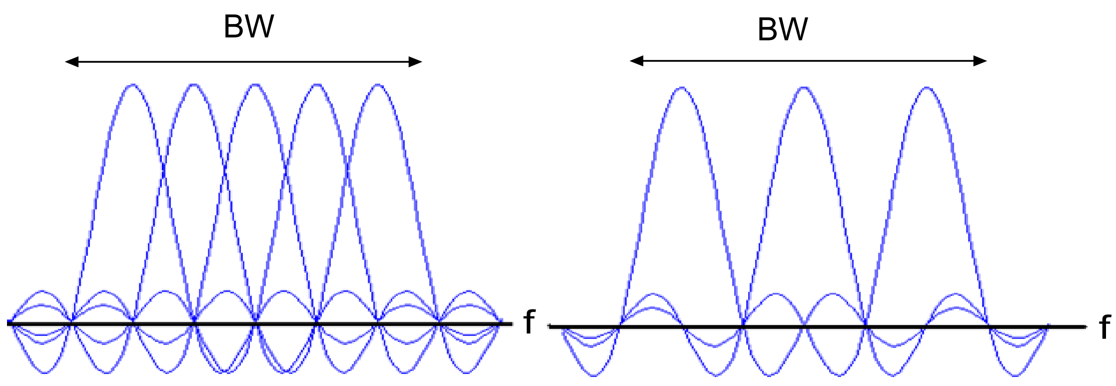
Maximum beamforming gain on a MIMO channel implies that only the maximum singular value of the channel should be used. However, for multiplexing gain, the optimum approach is to use a set of the channel singular values according to a water-filling strategy. In other words, maximum beamforming gain requires establishing only a single substream for communication while maximum multiplexing gain requires establishing several substreams at the same time.

- Diversity and Multiplexing Gains

Traditionally, the design of systems has been focused on either extracting maximum diversity gain or maximum multiplexing gain. However, both of these gains can be simultaneously obtained, but there is a fundamental tradeoff between them. Since the diversity gain is related to the BER and the multiplexing gain is related to the data rate, the tradeoff between them is essentially the fundamental tradeoff between the error probability and the data rate of a system.

1.1.2 Introduction of OFDM Technology

OFDM is a promising technique to achieve high data rate and combat multipath fading effect in wireless communications. OFDM can be thought of as a hybrid of multi-carrier modulation (MCM) and frequency shift keying (FSK) modulation [4]. MCM is the principle of transmitting data by dividing the stream into several parallel sub-streams and modulating each of these data sub-streams onto individual carriers; FSK modulation is a technique which data is transmitted on one carrier from a set of orthogonal carriers in each symbol duration. Orthogonality among these carriers is achieved by separating these carriers by an integer multiple of the inverse of symbol duration of the parallel bit streams. Using OFDM technique, all the orthogonal carriers are transmitted simultaneously. In other words, the entire allocated channel is occupied through the aggregated sum of the narrow orthogonal subbands. Figure 1-6 illustrates the spectral efficiency of OFDM compared to conventional MCM.



(a) (b)

Figure 1-6 (a) OFDM (b) Conventional MCM

We now briefly describe some fundamental principles of FSK modulation as they pertain to OFDM modulation. The input sequence determines which of the carriers is transmitted by its symbol duration, that is,

$$\mathbf{s}_i(t) = A\exp(j2\pi f_i t)\Pi(t/T) \tag{1.1.2-1}$$

where

$$f_i = f_c + i/T \ , \quad i = 0,1\ldots, L-1 \tag{1.1.2-2}$$

$$\Pi(t/T) = \begin{cases} 1 \ , & \text{for } -T/2 \le t \le T/2 \\ 0 \ , & \text{otherwise} \end{cases}$$

L is the total number of carriers and T is the symbol duration. In order to avoid that the carriers interfere with each other during detection, the spectral peak of each carrier must coincide with the zero crossing of all the other carriers as depicted in Figure 1-6. Thus, the difference between the center lobe and the first zero crossing represents the minimum required spacing and is equal to $1/T$. An OFDM signal is constructed by assigning parallel bit streams to these subcarriers with minimum required spacing, normalizing the signal energy, and extending the bit duration, i.e.,

$$\mathbf{s}(n) = \frac{A}{N}\mathbf{x}_i(n)\exp(j2\pi f_i t), \ \text{ for } 0 \le n \le L, 0 \le i \le N \tag{1.1.2-3}$$

where $x_i(n)$ is the n-th bit of the i-th data stream. Recall that from the Discrete Fourier Transform (DFT) pairs, the above equation is just the inverse DFT (IDFT) of $x_i(n)$ scaled by A. The output sequence $s(n)$ is transmitted one symbol at a time across the channel.

Before transmission, a cyclic prefix (CP) is a copy of the last part of the OFDM symbol. This makes a portion of the transmitted signal $\tilde{s}$ periodic with period L, i.e.:

$$\tilde{s}(n-m)=\tilde{s}(L+n-m), \quad \text{for } n\text{-}m \le L_{CP} \tag{1.1.2-4}$$

where L_{CP} is length of CP. Hence the received signal using vector notation is given by

$$\tilde{\mathbf{y}}(n)=\tilde{\mathbf{s}}(n)*\mathbf{h}(n)+\mathbf{n}(n) \tag{1.1.2-5}$$

where $*$ denotes linear convolution, $\mathbf{h}$ is the channel impulse response vector, and $\mathbf{n}$ is the additive noise vector. Now if length of CP is longer than the delay spread of the channel, received useful part $\mathbf{y}(n)$ becomes circular convolution one

$$\mathbf{y}(n)=\mathbf{s}(n)\otimes_L \mathbf{h}(n)+\mathbf{n}(n) \tag{1.1.2-6}$$

where $\otimes_L$ denotes L-point circular convolution. Note that the DFT transform for the convolution theory is based on circular convolution. Hence, the received signal at the k-th subcarrier after DFT results a simple scalar multiplication and can be written as

$$\mathbf{Y}_k=\mathbf{S}_k\cdot\mathbf{H}_k+\mathbf{N}_k \tag{1.1.2-7}$$

where $\mathbf{Y}_k$, $\mathbf{S}_k$ and $\mathbf{H}_k$ are the k-th subcarrier of DFT of $\mathbf{y}$, $\mathbf{s}$ and $\mathbf{h}$ respectively. Figure 1-7 illustrates the classical scheme of an OFDM communication system. Because of the advantages of OFDM, it becomes a popular transmission scheme for wireless communication systems. For examples, OFDM has been adopted in many standards such as digital audio and video broadcasting (DAB and DVB) and wireless local area networks (WLAN) standards etc.

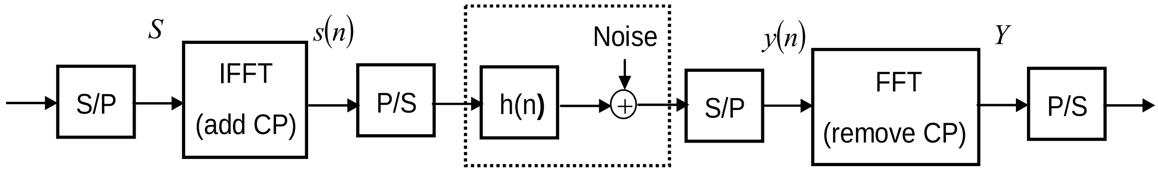


Figure 1-7 Classical scheme of an OFDM communication system

1.1.3 MIMO-OFDM System

After describing the features of MIMO and OFDM techniques, we now combine these two techniques to form MIMO-OFDM systems [5]. Figure 1-8 and 1-9 illustrate the basic structure of transmitter and receiver of MIMO-OFDM system respectively.

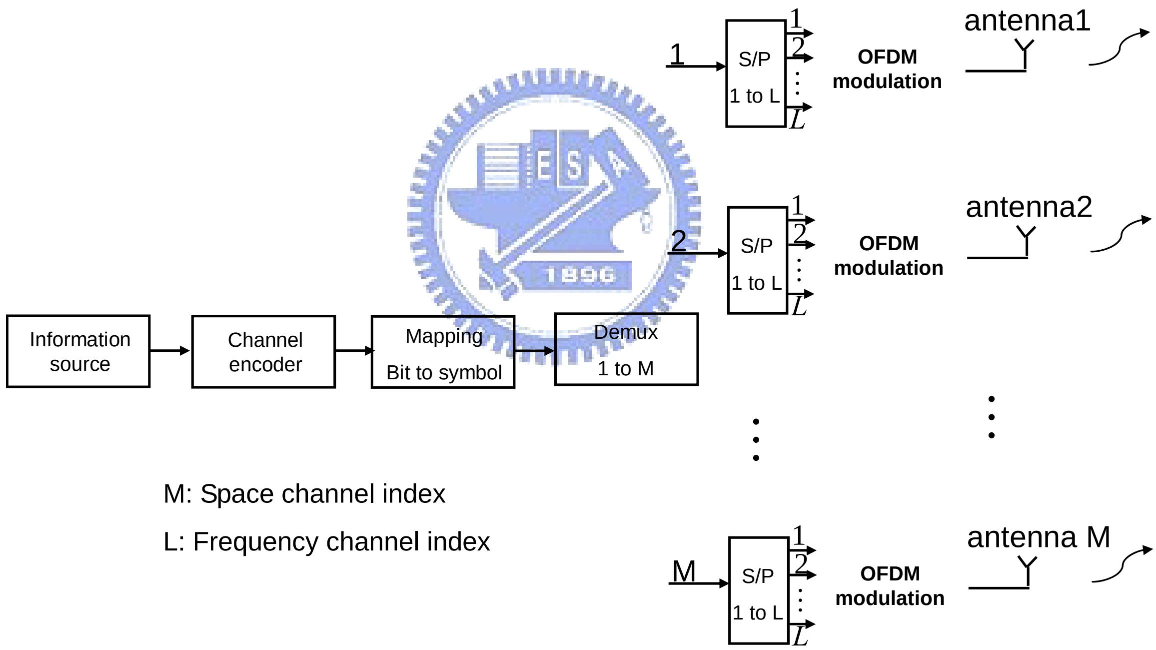


Figure 1-8 MIMO-OFDM transmit side

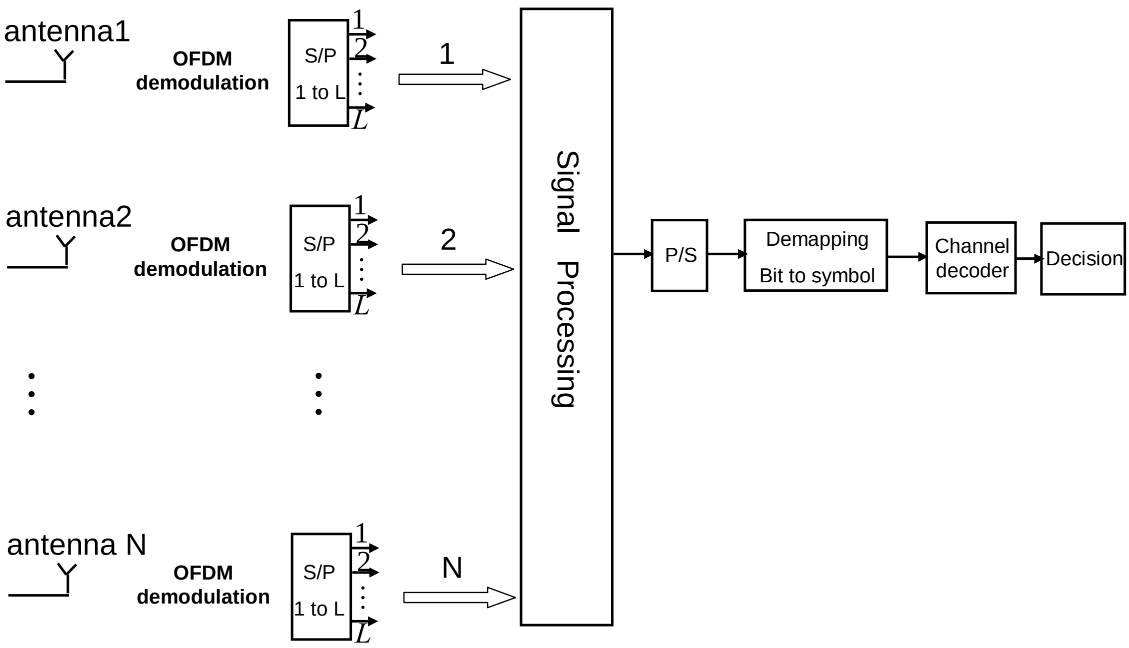


Figure 1-9 MIMO-OFDM receive side

At the transmitter, the input data symbols are de-multiplexed to M branches, M means the number of transmit antennas. Thus for each branch, we perform the OFDM modulation and transmitted from each antenna. At the receiver, we perform the OFDM demodulation. After removing CP and passing the signal to DFT, we obtain the major feature, that is, for each subcarrier of OFDM demodulation, it can be treated as a simple flat-fading MIMO channel. Following equation shows the receiver signal property after OFDM demodulator at k-th subcarrier:

$$\begin{bmatrix} y_{1,k} \\ y_{2,k} \\ \vdots \\ y_{N,k} \end{bmatrix} = \begin{bmatrix} H_{11,k} & H_{12,k} & \cdots & H_{1M,k} \\ H_{21,k} & H_{22,k} & \cdots & H_{2M,k} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1,k} & H_{N2,k} & \cdots & H_{NM,k} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \vdots \\ x_{M,k} \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n_{2,k} \\ \vdots \\ n_{N,k} \end{bmatrix} \quad (1.1.3\text{-}1)$$

where $H_{ij,k}$ represents the channel frequency response at k-th subcarrier from i-th antenna to j-th antenna. We have explored the MIMO-OFDM based system and obtained the flat-fading feature at each subcarrier of OFDM. We can begin to study the joint Tx-Rx beamforming design under flat-fading MIMO channel and easily extend to MIMO-OFDM based system which obtains spectral efficiency and combats the frequency selective channel environment. It will be analyzed in chapter 2.

1.2 Motivation

MIMO wireless communication systems have attracted a lot of interest in the recent years. Since they offer multiplicity of spatial channels, they provide significant capacity and performance increase, compared to conventional SISO communication systems. Space-time coding [7] [8] and spatial multiplexing [9] are two primary techniques of achieving high data rate over MIMO channels. Spatial multiplexing involves transmitting independent data streams across multiple antennas, whereas space-time coding appropriately maps data symbol streams across space and time for transmit diversity and coding gain at a given data rate. Both of these schemes do not require channel knowledge at the transmitter. In a number of applications, the CSI may be available at the transmitter by sending back the CSI from the receiver. When the CSI is known at both transmitter and receiver, the best performance can be obtained by the use of EVD (Eigen Value Decomposition) weighting combined with water-pouring strategy [10]. However, this approach is feasible by adaptively controlling the number of data streams and the modulation/coding schemes in each stream. The high complexity makes this approach impractical in the real wireless systems. A sub-optimal approach which uses a fixed number of data streams and the predefined identical modulation/coding schemes is proposed in [11]. The design is based on the joint Tx/Rx minimal mean square error (MMSE) beamformings (commonly termed as the pre-filter and the post-filter). Other joint Tx/Rx designs with different criterions and constraints have also been presented in [6], [12], [13], and [14]. These joint designs commonly have the solutions which are scalable with respect to the number of antennas, the size of the coding block, and the power constraints. The solutions are shown to be able to convert the mutually cross-coupled MIMO transmission system into a system with a set of parallel eigen subchannels (also termed channel eigenmodes).

These solutions cannot be directly applicable to Multi-user MIMO space division multiple access (SDMA) systems [15-17], where a multi-antenna base station (BS) communicates at the same time with several multi-antenna terminals. The joint Tx-Rx design problem in multi-user MIMO-SDMA system can be decoupled into several single user design problems by using a null-space constraint [18], where each joint design problem only depends on his single user MIMO channel. That means the null-space constraint block-diagonalizes the whole MIMO channel into several single user MIMO channels and results zero multi-user interference (MUI) between each user.

In addition, in terms of spectral efficiency, a frequency-selective MIMO channel can be handled by using the multi-carrier approach, such as the OFDM technology, which treats each sub-carrier as a flat-fading MIMO channel. In this thesis, we will apply the joint beamforming design to the multi-user MIMO-OFDM SDMA system. Furthermore, we will consider the system in which the channel estimation contains errors. These channel estimation errors cause significant performance degradation in the Tx/Rx beamforming. To cope with the channel information error in multi-user joint design problem, we combine two methods published in [6] and [18] to improve the performance. We also apply the moving average approach in the design for slow-fading wireless channel environments.

1.3 Thesis Organization

The focus of this thesis is on the joint beamforming design of the transmitter and receiver for multi-user MIMO-OFDM SDMA under channel estimation error. In order to clearly and completely describe the whole system, we separate the system into three topics and then analyze these topics in following chapters step by step.

The thesis is organized as follows. In chapter 2, we analyze the joint Tx/Rx MMSE beamforming design for single user MIMO channel case when the CSI is perfect known at both terminals and extend it to MIMO-OFDM system according to principles of chapter 1.1. In chapter 3, we investigate the joint beamforming design problem in multi-user MIMO-OFDM SDMA system with perfect CSI. Chapter 4 considers the case that the channel estimation contains errors. We apply some robust methods to improve the system performance. Furthermore, we use the moving average approach to enhance the system performance under slow time-variant channel environment. In last chapter, we conclude the thesis with some respectives.

Chapter 2

Joint Tx/Rx MMSE Beamforming Design for Single User MIMO-OFDM Downlink System with Perfect CSI

In chapter 1, we have introduced the basics of OFDM system, channel models and properties of MIMO systems, and the MIMO-OFDM based system. We now concentrate on the joint Tx/Rx beamforming design for single user MIMO downlink system under the flat-fading channel and perfect CSI conditions. According the concepts of MIMO-OFDM system described in section 1.1.3, we then extend the joint beamforming design problem to MIMO-OFDM based system which obtains spectral-efficiency and combats the frequency selective channel environment.

2.1 Introduction

For MIMO system, space-time coding and spatial multiplexing are promising techniques for achieving high data rates requirement. Neither of the two techniques require CSI at transmit side. However, in some situations, channel information could be available at the transmitter because of the feedback information from the receiver. If the CSI is known at both of transmit and receive terminals, the optimal solution is provided by the eigen-value decomposition (EVD) weighting scheme combined with a water-pouring strategy [10]. However, instead of using the optimal solution, a sub-optimal solutions using a fixed number of data streams and a fixed identical modulation/coding scheme have been proposed [11]. Other joint designs subject to different constraints and criteria also have been presented [6] [12-14]. The joint Tx/Rx design diagonalizes the MIMO channel into eigen subchannels; achieve the symbol by symbol detection and the system structure can be scalable with respect to the number of antennas, size of the coding block, and transmit power. In this chapter, we focus on the joint MMSE beamforming design which minimizes the accumulative mean square error subject to the total transmitted power as the constraint.

2.2 Joint Tx/Rx Beamforming MIMO System Models

2.2.1 Single Carrier Flat-fading Case

The model of joint Tx/Rx beamforming design for single user MIMO downlink system in flat-fading channel environment is illustrated in Figure 2-1. We consider a wireless communication system with M transmit and N receive antennas under the flat-fading environment. Thus, the flat-fading MIMO channel $\mathbf{H}$ can be represented by a channel matrix with dimension $N \times M$. The input symbol streams are passed through the transmit beamforming $\mathbf{F}$ (pre-filter) which is optimized for a known channel and then the pre-filter output is transmitted into the flat-fading MIMO channel. The received signal is processed by the receive beamforming **G** (post-filter). In this thesis, we will not consider coding and modulation design and only concentrate on transmit and receive beamforming design.

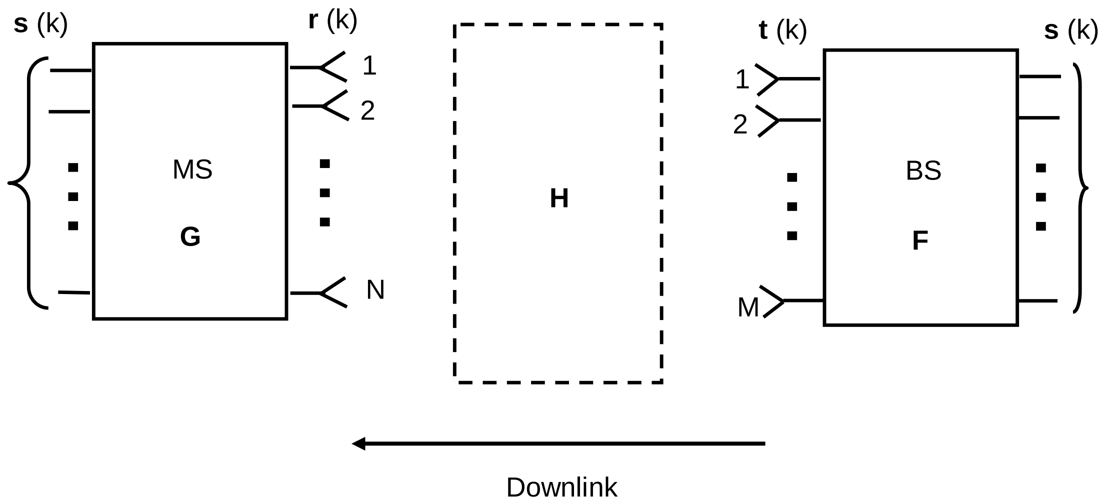


Figure 2-1 Joint Tx/Rx beamforming design for single user MIMO system

For a MIMO channel without any delay-spread, the joint beamforming design system equation can be written as

$$\hat{\mathbf{s}} = \mathbf{GHFs} + \mathbf{Gn} \qquad (2.2.1\text{-}1)$$

where $\mathbf{H}$ is the MIMO channel matrix described as above, $\hat{\mathbf{s}}$ is the $B \times 1$ received vector, and $\mathbf{s}$ is the $B \times 1$ transmitted vector. Note that

$$B = rank(\mathbf{H}) \le \min(M, N) \qquad (2.2.1\text{-}2)$$

is the number of parallel transmitted data streams. $\mathbf{F}$ is the $M \times B$ transmit beamforming matrix, $\mathbf{G}$ is the $B \times N$ receive beamforming matrix and $\mathbf{n}$ is the $N \times 1$ noise vector. The transmit beamforming adds a redundancy of $M - B$ across space, because the number of input symbols is just B but produces M output symbols transmitted simultaneously through M transmit antennas. That results the performance improvement due to the diversity gain. Besides, the receive beamforming removes the redundancy that be introduced by the transmit beamforming and results B output data for detection.

To derive the beamforming, we assume the following properties:

$$E\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}; \quad E\{\mathbf{n}\mathbf{n}^H\} = \mathbf{R}_{\mathbf{nn}}; \quad E\{\mathbf{s}\mathbf{n}^H\} = 0; \qquad (2.2.1\text{-}3)$$

where the superscript H represents the conjugate transpose (Hermitian) operation. For simplicity of analysis, we assume that the transmitted signals are uncorrelated and normalized to unit power. $B = rank(\mathbf{H})$, the full-loaded case, the elements of MIMO channel matrix are uncorrelated with full rank. If $B < rank(\mathbf{H})$, under-loaded case, we can also apply it to our system. However the over-loaded case $B > rank(\mathbf{H})$ is not possible for any practical system that transmits independent data streams more than rank of the MIMO channel $\mathbf{H}$ in order to have an acceptable performance.

2.2.2 OFDM-based Case

The OFDM technique which has attracted a lot of interest in the recent years as it can combat delay spread, easily deal with frequency selective channels and achieve the spectral efficiency described in section 1.2.1. Now, we apply the above MIMO system to the frequency selective channel with OFDM transmission, where the flat-fading conditions prevail on each subcarrier. The MIMO-OFDM system model in each subcarrier is similar to Figure 2-1. The difference is that the pro- and post-filter are processed at each subcarrier of OFDM. The MIMO-OFDM system equation has the similar form as the flat-fading case (2.2.1-1) and is shown as below where subscript notation k of denotes the subcarrier index

$$\hat{\mathbf{s}}_k = \mathbf{G}_k \mathbf{H}_k \mathbf{F}_k \mathbf{s}_k + \mathbf{G}_k \mathbf{n}_k \quad (2.2.2\text{-}1)$$

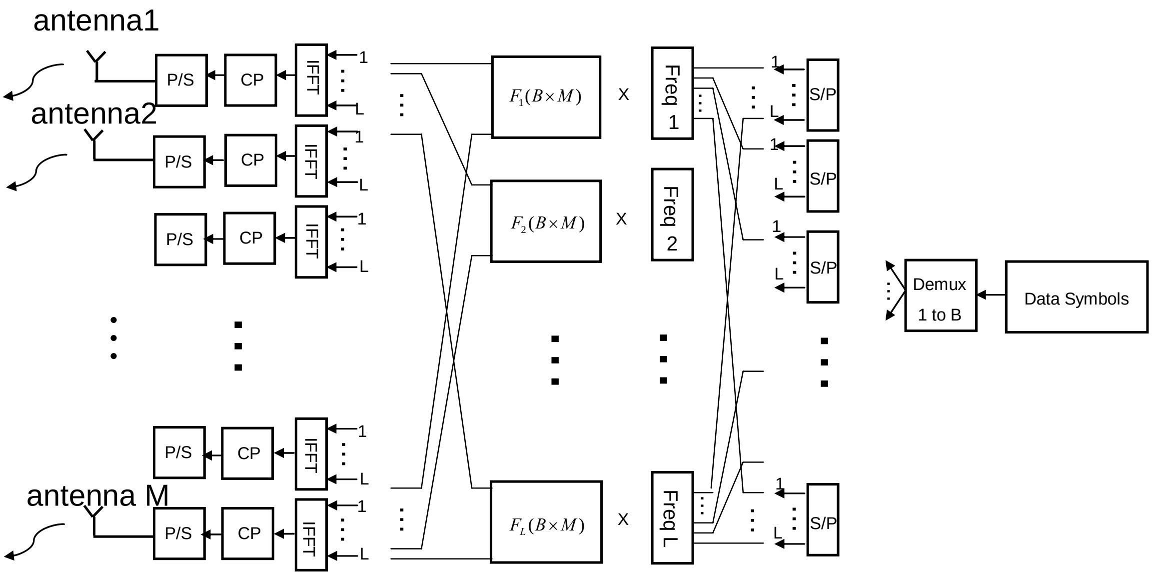


Figure 2-2 Joint Tx/Rx beamforming processing at transmitter of a single user MIMO-OFDM system

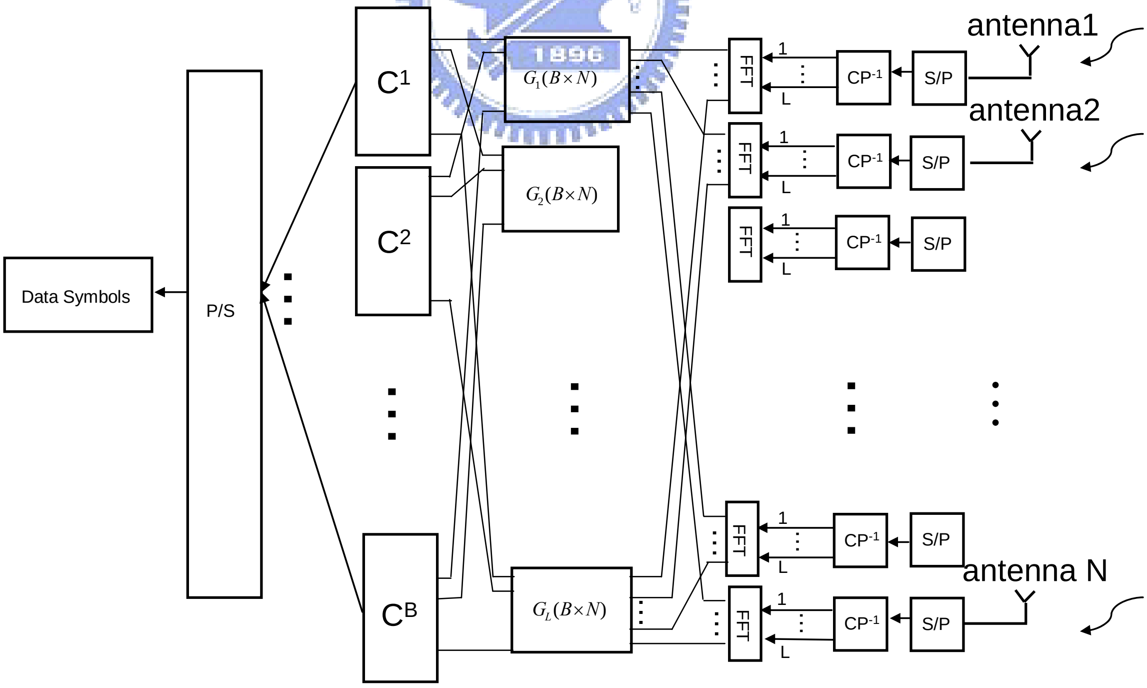


Figure 2-3 Joint Tx/Rx beamforming processing at receiver of a single user MIMO-OFDM system

Note that, in MIMO-OFDM based system, the transmit beamforming is performed in frequency domain, that is, before the OFDM modulation. And the receive beamforming should be also performed in frequency domain, that is, after OFDM demodulation. Figure 2-2 and Figure 2-3 illustrate the detail operations at transmitter (BS) and receiver (MS) respectively.

2.3 Joint Tx/Rx Beamforming Design for Single User Case

2.3.1 Problem Description

Now we will design transmit and receive beamforming matrices $\mathbf{F}$ and $\mathbf{G}$ to minimize the accumulative MSE in the flat-fading MIMO channel environment. However, as explained before, the flat-fading characteristic is preserved at each subcarrier of OFDM. Thus, we modify the above MIMO system and directly design $\mathbf{F}_k$ and $\mathbf{G}_k$ for the MIMO-OFDM system. The MSE matrix at each subcarrier of MIMO-OFDM system is defined as the covariance matrix of the error vector and showed as below:

$$\mathbf{E}_k(\mathbf{F}_k,\mathbf{G}_k) = MSE(\mathbf{F}_k,\mathbf{G}_k) = E\{\mathbf{e}_k\mathbf{e}_k^H\} \qquad (2.3.1\text{-}1)$$

where $\mathbf{e}_k$ is the symbol estimation errors and defined as $\mathbf{e}_k = (\hat{\mathbf{s}}_k - \mathbf{s}_k)$. Replacing the system equation (2.2.2-1) into (2.3.1-1), we obtain the MSE matrix as below:

$$\begin{aligned}
&\mathbf{E}_k(\mathbf{F}_k,\mathbf{G}_k) = E\{\mathbf{e}_k\mathbf{e}_k^H\} = E\{(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H\} \\
&= E\{[(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{G}_k\mathbf{n}_k) - \mathbf{s}_k][(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{G}_k\mathbf{n}_k) - \mathbf{s}_k]^H\} \\
&= \mathrm{E}\{(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{G}_k\mathbf{n}_k)(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{G}_k\mathbf{n}_k)^H\} \\
&- E\{(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{G}_k\mathbf{n}_k)(\mathbf{s}_k)^H\} \\
&- E\{(\mathbf{s}_k)^H(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{G}_k\mathbf{n}_k)\} + E\{(\mathbf{s}_k)(\mathbf{s}_k)^H\}
\end{aligned} \qquad (2.3.1\text{-}2)$$

We now use the assumptions in equation (2.2.1-3) and assume the channel matrix $\mathbf{H}$

is fixed and known at the transmitter and the receiver The MSE matrix in equation (2.3.1-2) can be simplified as:

$$\begin{aligned}\mathbf{E}_k(\mathbf{F}_k,\mathbf{G}_k) &= (\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k)(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k)^H + \mathbf{G}_k\mathbf{R}_{\mathbf{nn},k}\mathbf{G}_k^H + \mathbf{I} \\ &\quad -(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k) - (\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k)^H\end{aligned} \qquad (2.3.1\text{-}3)$$

Thus, the minimize MSE problem can be stated as follows:

$$\begin{aligned}&\min_{\mathbf{F}_k,\mathbf{G}_k} E\left\{\|\mathbf{e}_k\|^2\right\} = trace(\mathbf{E}_k(\mathbf{F}_k,\mathbf{G}_k)) \\ &\text{subject to trace}(\mathbf{F}_k\mathbf{F}_k^H) = p_{T,k}\end{aligned} \qquad (2.3.1\text{-}4)$$

where $p_{T,k}$ is the transmitted power constraint at k-th subcarrier. Note that the above equation (2.3.1-4) is based on the Frobenius norm:

$$E\left\{\|\mathbf{e}_k\|^2\right\} = E\left\{trace(\mathbf{e}_k\mathbf{e}_k^H)\right\} = trace\left(E\left\{(\mathbf{e}_k\mathbf{e}_k^H)\right\}\right) \qquad (2.2.1\text{-}5)$$

After formulating the problem of the joint beamforming design for transmitter and receiver over flat-fading channel and frequency-selective channel (OFDM-based). In the next section, we will continue to derive two other methods which have the same solution as the above transmit and receive beamformer.

2.3.2 Optimum Transmit and Receive Beamformings

2.3.2.1 Lagrange Multiplier Method

First, we use the method of Lagrange duality and Karush-Kuhn-Tucker (KKT) conditions to solve the joint design problem in equation (2.3.1-8). We add the Lagrange multiplier μ_k to form the Lagrangian shown as below:

$$L(\mu_k,\mathbf{F}_k,\mathbf{G}_k) = trace\left(E\left\{\mathbf{e}_k\mathbf{e}_k^H\right\}\right) + \mu_k\left[trace\left(\mathbf{F}_k\mathbf{F}_k^H - P_{T,k}\right)\right] \qquad (2.3.2.1\text{-}1)$$

Replacing the equation (2.3.1-3) into above equation, we obtain

$$L(\mu_k, \mathbf{F}_k, \mathbf{G}_k) = trace\Big[(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k)(\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k)^H + \mathbf{G}_k\mathbf{R}_{\mathbf{nn},k}\mathbf{G}_k^H + \mathbf{I} \\ - (\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k) - (\mathbf{G}_k\mathbf{H}_k\mathbf{F}_k)^H\Big] + \mu_k\Big[trace(\mathbf{F}_k\mathbf{F}_k^H - P_{T,k})\Big] \quad (2.3.2.1\text{-}2)$$

The following KKT conditions are necessary and sufficient to solving the optimal transmit and receive beamforming $\mathbf{F}_k$ and $\mathbf{G}_k$. $\mathbf{F}_k$ and $\mathbf{G}_k$ are optimal solutions if and only if there is a μ_k that together with $\mathbf{F}_k$ and $\mathbf{G}_k$ satisfy the conditions:

$$\nabla_{\mathbf{F}_k} L(\mu_k, \mathbf{F}_k, \mathbf{G}_k) = 0 \quad (2.3.2.1\text{-}3)$$

$$\nabla_{\mathbf{G}_k} L(\mu_k, \mathbf{F}_k, \mathbf{G}_k) = 0 \quad (2.3.2.1\text{-}4)$$

$$\nabla_{\mu_k} L(\mu_k, \mathbf{F}_k, \mathbf{G}_k) = 0 \quad (2.3.2.1\text{-}5)$$

From the equation (2.3.2.1-3) and (2.3.2.1-2), we can obtain

$$\mathbf{H}_k\mathbf{F}_k\mathbf{F}_k^H\mathbf{H}_k^H\mathbf{G}_k^H - \mathbf{H}_k\mathbf{F}_k + \mathbf{R}_{\mathbf{nn},k}\mathbf{G}_k^H = 0 \quad (2.3.2.1\text{-}6)$$

and from the equation (2.3.2.1-4) and (2.3.2.1-2), we can obtain

$$\mathbf{H}_k\mathbf{F}_k\mathbf{F}_k^H\mathbf{H}_k^H\mathbf{G}_k^H\mathbf{G}_k\mathbf{H}_k - \mathbf{G}_k\mathbf{H}_k + \mu_k\mathbf{F}_k^H = 0 \quad (2.3.2.1\text{-}7)$$

To obtain above two equations, we have to use the fact:

$$(\partial\, trace(\mathbf{AXB}))/(\partial\mathbf{X}) = \mathbf{BA} \quad (2.3.2.1\text{-}8)$$

$$(\partial\, trace(\mathbf{AX}^H\mathbf{B}))/(\partial\mathbf{X}) = \mathbf{0} \quad (2.3.2.1\text{-}9)$$

Now we are going to solve the two equations (2.3.2.1-6) and (2.3.2.1-7) to obtain the optimal transmit and receive beamformer. First of all, we define the SVD of following equation:

$$\mathbf{H}_k^H\mathbf{R}_{\mathbf{nn},k}^{-1}\mathbf{H}_k = \begin{pmatrix}\mathbf{U}_k & \tilde{\mathbf{U}}_k\end{pmatrix}\begin{pmatrix}\mathbf{S}_k & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{S}}_k\end{pmatrix}\begin{pmatrix}\mathbf{V}_k & \tilde{\mathbf{V}}_k\end{pmatrix}^H \quad (2.3.2.1\text{-}10)$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are orthogonal matrices which form a basis of $\mathbf{H}_k^H\mathbf{R}_{\mathbf{nn},k}^{-1}\mathbf{H}_k$

and $\mathbf{S}$ is a diagonal matrix with B nonzero singular values with decreasing order. $\tilde{\mathbf{S}}_k$ is a diagonal matrix with zero singular values; $\tilde{\mathbf{U}}_k$ and $\tilde{\mathbf{V}}_k$ are orthogonal matrices with dimensions $M \times (M - B)$ which form a basis of the null space of $\mathbf{H}_k^H \mathbf{R}_{\mathbf{nn},k}^{-1} \mathbf{H}_k$. Note that we have assume that the rank of $\mathbf{H}_k$ is B for simplicity.

Applying the similar approaches used in [7], we can obtain the transmit and receive beamforming matrices with structures as follows:

$$\mathbf{F}_k = \mathbf{V}_k \mathbf{\Phi}_{\mathbf{F}_k} \quad (2.3.2.1\text{-}11)$$

$$\mathbf{G}_k = \mathbf{\Phi}_{\mathbf{G}_k} \mathbf{U}_k^H \mathbf{H}_k^H \mathbf{R}_{\mathbf{nn},k}^{-1} \quad (2.3.2.1\text{-}12)$$

Where $\mathbf{\Phi}_{\mathbf{F}_k}$ and $\mathbf{\Phi}_{\mathbf{G}_k}$ are diagonal matrices with nonnegative values and with dimension $B \times B$. Thus, the transmit and receive beamforming matrices diagonalize the MIMO channel matrix into a set of eigen subchannels. We will explain the results in section 2.3.3. The diagonal matrices $\mathbf{\Phi}_{\mathbf{F}_k}$ and $\mathbf{\Phi}_{\mathbf{G}_k}$ in above two equations are given by:

$$\mathbf{\Phi}_{\mathbf{F}_k} = \left(\mu_k^{-1/2} \mathbf{S}_k^{-1/2} - \mathbf{S}_k^{-1} \right)_+^{1/2} \quad (2.3.2.1\text{-}13)$$

$$\mathbf{\Phi}_{\mathbf{G}_k} = \left(\mu_k^{1/2} \mathbf{S}_k^{-1/2} - \mathbf{S}_k^{-1} \right)_+^{1/2} \mathbf{S}_k^{-1/2} \quad (2.3.2.1\text{-}14)$$

The subscript notation $+$ denoted that the negative elements of the diagonal matrices are replaced by zero and μ_k in the above two equations is chosen to satisfy the transmit power constraint and given by:

$$\mu_k^{1/2} = \frac{trace\left(\mathbf{S}_k^{-1/2} \right)}{P_{T,k} + trace\left(\mathbf{S}_k^{-1} \right)} \quad (2.3.2.1\text{-}15)$$

Up to now, we have showed the Lagrange multiplier approach to derive transmit and receive beamforming matrices. In next section, we show another method to derive these beamforming matrices. These approaches are different, but the results are identity since both of them are looking for the optimal solutions.

2.3.2.2 Two Step Method

Recall that the minimized accumulative MSE problem is stated in (2.3.1-4) and the accumulative MSE matrix is given in equation (2.3.1-3). We now use the two-step derivation approach to design the system. In first step, we derive the optimal receive beamforming matrix $\mathbf{G}_k$ by assuming that the transmit beamforming matrix is fixed and then leave the difficult part which is to derive the transmit beamforming matrix $\mathbf{F}_k$ to next step.

The optimal receive beamforming solution $\mathbf{G}_{k,opt}$ that minimizes the MSE matrix is the same as the Wiener solution which is known to minimize the $trace(MSE(\mathbf{F}_k, \mathbf{G}_k))$ and is given by the following equation

$$\nabla_{\mathbf{G}_k} trace(MSE(\mathbf{G}_k)) = \mathbf{0} \tag{2.3.2.2-1}$$

And then the optimal solution $\mathbf{G}_{k,opt}$ can be obtained as below:

$$\mathbf{G}_{k,opt} = \mathbf{F}_k \mathbf{H}_k \left(\mathbf{H}_k \mathbf{F}_k \mathbf{F}_k^H \mathbf{H}_k^H + \mathbf{R}_{\mathbf{nn},k} \right)^{-1} \tag{2.3.2.2-2}$$

The optimal receive beamforming is exactly the Wiener filter solution. Replacing the optimal receive matrix $\mathbf{G}_{k,opt}$ into the MSE matrix, we obtain the following concentrated error matrix:

$$\mathbf{E}_k\left(\mathbf{F}_k,\mathbf{G}_{k,opt}\right)=\mathbf{I}-\mathbf{F}_k^H\mathbf{H}_k^H\left(\mathbf{H}_k\mathbf{F}_k\mathbf{F}_k^H\mathbf{H}_k^H+\mathbf{R}_{\mathbf{nn},k}\right)^{-1}\mathbf{H}_k\mathbf{F}_k$$
$$=\left(\mathbf{I}+\mathbf{F}_k^H\mathbf{H}_k^H\mathbf{R}_{\mathbf{nn},k}^{-1}\mathbf{H}_k\right)^{-1} \quad (2.3.2.2\text{-}3)$$

Thus, the joint beamforming design problem is simplified to the design of the transmit beamforming with the receive beamforming matrix given by Wiener solution (2.3.2.2-2). Note that, without any constraint, the minimization of (2.3.2.2-3) will lead to the trivial solution of increasing to infinity of the norm of $\mathbf{F}_k$. Thus, the solution of the optimization problem with transmit power as a constraint:

$$\min_{\mathbf{F}_k,\mathbf{G}_k} \ trace\left(\mathbf{E}_k\left(\mathbf{F}_k,\mathbf{G}_{k,opt}\right)\right)$$
$$\text{subject to } trace\left(\mathbf{F}_k\mathbf{F}_k^H\right)=p_{T,k} \quad (2.3.2.2\text{-}4)$$

is given by $\mathbf{F}_k=\mathbf{V}_k\mathbf{\Phi}_{\mathbf{F}_k}$, where $\mathbf{\Phi}_{\mathbf{F}_k}$ is a $B\times B$ diagonal matrix with the following elements

$$\phi_{ii,k}=\left(\frac{P_{T,k}+\sum_{j=1}^{B}\lambda_{jj,k}^{-1}}{\sum_{j=1}^{B}\lambda_{jj,k}^{-1/2}}\lambda_{ii,k}^{-1/2}-\frac{1}{\lambda_{ii,k}}\right)_+^{1/2} \quad (2.3.2.2\text{-}5)$$

where λ_k denotes the singular values of the matrix $\mathbf{S}_k$. The above equation is actually equivalent to the elements of equation (2.3.2.1-13) which is the matrix form. If we replace the transmit beamforming into the optimal receive beamforming matrix derived in first step, we can obtain the same optimal receive solution given in section 2.3.2.1.

2.3.3 Equivalent Decomposition of MIMO-OFDM system

The major characteristic of the joint beamforming design is to convert the mutually cross-coupled MIMO transmission system into a set of parallel eigen subchannels (also termed channel eigenmodes) system. From the results of previous section, the matrix model of MIMO-OFDM system at k-th subcarrier similar to flat-fading case can be illustrated in Figure 2-4.

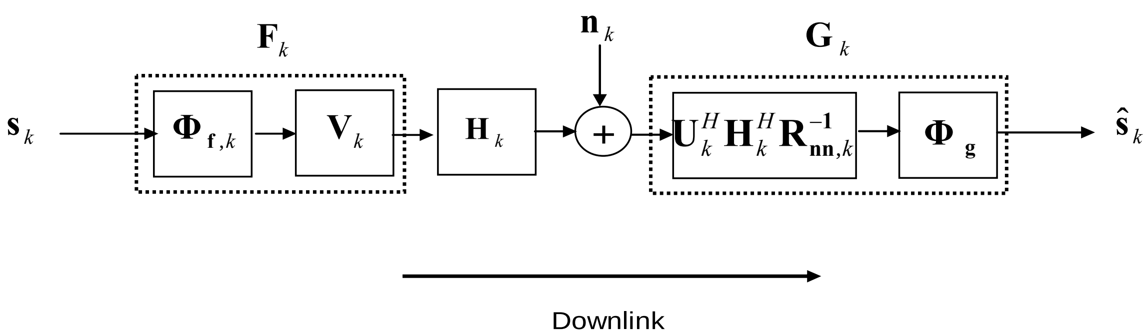


Figure 2-4 The matrix model of a single user MIMO-OFDM system at k-th subcarrier

Mathematically, the product of optimal beamforming matrices $\mathbf{F}_{k,opt}$ and $\mathbf{G}_{k,opt}$ in equation (2.3.2.1-11) and (2.3.2.1-12) and the channel matrix $\mathbf{H}_k$ becomes:

$$\mathbf{G}_{k,opt}\mathbf{H}_k\mathbf{F}_{k,opt} = \mathbf{\Phi}_{\mathbf{G}_k}\mathbf{U}_k^H\mathbf{H}_k^H\mathbf{R}_{\mathbf{nn},k}^{-1}\mathbf{H}_k\mathbf{V}_k\mathbf{\Phi}_{\mathbf{F}_k} \quad (2.3.3\text{-}1)$$

Using the SVD in equation (2.3.2.1-10), it can be rewritten as:

$$\mathbf{G}_{k,opt}\mathbf{H}_k\mathbf{F}_{k,opt} = \mathbf{\Phi}_{\mathbf{G}_k}\mathbf{U}_k^H\mathbf{U}_k\mathbf{S}_k\mathbf{V}_k^H\mathbf{V}_k\mathbf{\Phi}_{\mathbf{F}_k} = \mathbf{\Phi}_{\mathbf{G}_k}\mathbf{S}_k\mathbf{\Phi}_{\mathbf{F}_k} \quad (2.3.3\text{-}2)$$

Since $\mathbf{\Phi}_{\mathbf{G}_k}$, $\mathbf{S}_k$ and $\mathbf{\Phi}_{\mathbf{F}_k}$ are all diagonal matrices, the MIMO channel are decoupled into parallel eigen subcarriers. Figure 2-5 illustrates the equivalent MIMO-OFDM system at k-th subcarrier.

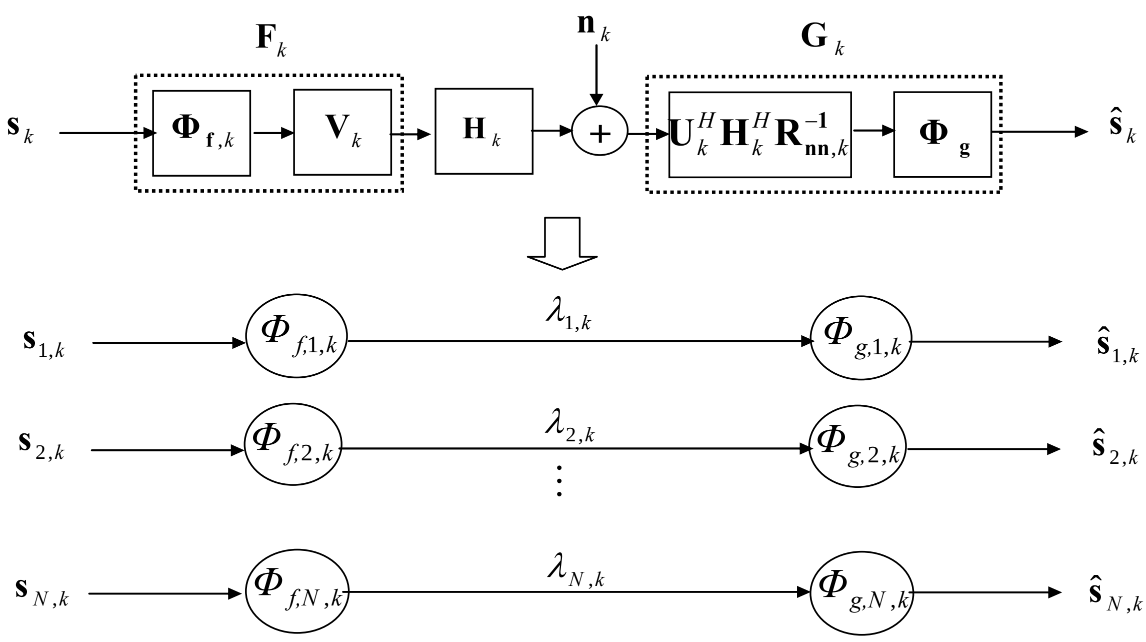


Figure 2-5 Equivalent decomposition of MIMO-OFDM system at k-th subcarrier

The major interest of the diagonalized structure is that all the matrix equations can be substituted with scalar ones with the consequent great simplification and only the symbol by symbol detection need to be performed.

2.4 Conclusions

In this chapter, we have shown the system model for flat-fading channel environment and extend to OFDM-based system which can combat the frequency selective channel and achieve the spectral efficiency. Each subcarrier of MIMO-OFDM based system can be treated as flat-fading case and solved by the same ways. We have also developed two forms of joint MMSE transmitter-receiver beamforming: the Lagrangian and the two-step methods. Both approaches result in the same close-form solution.

The joint design problem shows that the optimal beamforming matrices $\mathbf{F}_{k,opt}$

and $\mathbf{G}_{k,opt}$ in equation (2.3.2.1-11) and (2.3.2.1-12) cascaded with the channel matrix $\mathbf{H}_k$ in between will result in a diagonal matrix. That is, the original mutually cross-coupled MIMO transmission system is decoupled into a set of parallel eigen subchannels system. It means that the matrix equations can be simplified to scalar ones so that we can perform symbol by symbol detection similar to a set of parallel SISO systems. This joint design approach makes the number of antennas, the size of the coding block, and the transmit power become scalable.

Chapter 3

Joint Tx/Rx MMSE Beamforming Design for Multi-user MIMO-OFDM SDMA Downlink System with Perfect CSI

In this chapter, we extend the single user joint Tx/Rx beamforming design to the multi-user case so that we can achieve the multiple access via the space domain the so called spatial-division multiple access (SDMA) [15] [18]. Compare to the conventional frequency-division multiple access (FDMA) and time-division multiple access (TDMA), the SDMA can conduct multi-user transmission at the same time and frequency. That is, we can reuse the frequency bandwidth. However, the multi-user interference (MUI) can potentially cause performance degradation. The technique of null-spacing [18] is proposed to handle the MUI problem.

3.1 Introduction

Different from most of the SDMA systems which assume a single antenna equipped at mobiles, the MIMO system we consider is the system with multiple antennas used at both transmitter and receiver. We will further extend the MIMO SDMA system to MIMO-OFDM SDMA system by the concepts mentioned in chapter 2. In the multiple access system, the MUI is a major problem which causes significant performance loss. A way of solving this problem is to use the null-space constraint to decouple the multi-user MIMO SDMA joint design problem into several single user problems which have been described in previous section, where each problem only depends on each single user MIMO channel. In other words, the product of the MIMO channel and the null-space matrix at transmit side results in a block-diagonal matrix, which means the MUI between each user is completely removed. Thus, each user terminal only has to deal with its own inter-stream interference.

In this chapter, we first model the single carrier flat-fading MIMO SDMA system which combines the joint Tx/Rx beamforming design with the null-space technique. Then we extend such system to MIMO-OFDM SDMA case which preserves the flat-fading property at each subcarrier. Thus, the beamforming and null-space matrices have to be designed based at each subcarrier. That is, we have to perform the pre-filter with the null-space constraint before OFDM modulation at the transmitter and at the receiver. The post-filter is also performed after OFDM demodulation. The null-space matrix design technique will be introduced in section 3.3.1. Thereafter, we will introduce the combination of the joint Tx/Rx beamforming and the null-space constraint to deal with the multi-user MIMO-OFDM SDMA downlink system.

3.2 Joint Tx/Rx Beamforming MIMO SDMA System Models

3.2.1 MIMO SDMA under Single Carrier Flat-fading Channel

Figure 3-1 illustrates a multi-user MIMO SDMA downlink system under single carrier flat-fading channel. We consider the transmit side (BS) equipped with M antennas simultaneously communicates with U user terminals (mobile station or MS). Each user terminal has N^u receiver antennas. The BS transmits several data symbol streams towards the U user terminals simultaneously. C^1 data streams are transmitted towards user terminal 1, C^2 data streams are transmitted towards user terminal 2, and so on.

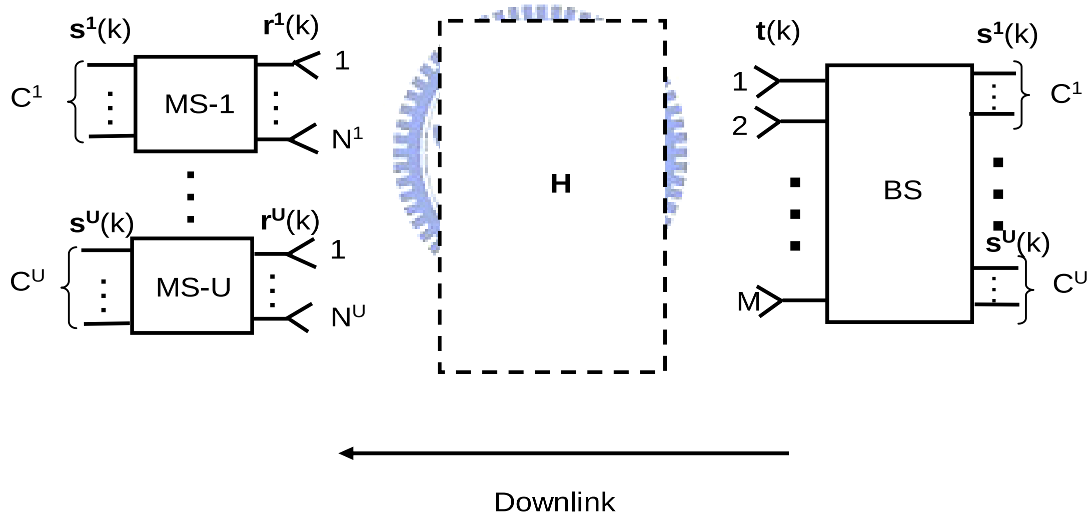


Figure 3-1 Joint Tx/Rx beamforming design for a multi-user MIMO SDMA downlink system

If we do nothing at BS, each user terminal will receive the mixture of all data streams and needs to recover its own streams. Note that the receiver antennas N^u of each user terminal is greater or equal to the number of data streams C^u in order to make sure an acceptable performance.

The joint Tx/Rx beamforming design combined with null-space matrix can be depicted in Figure 3-2.

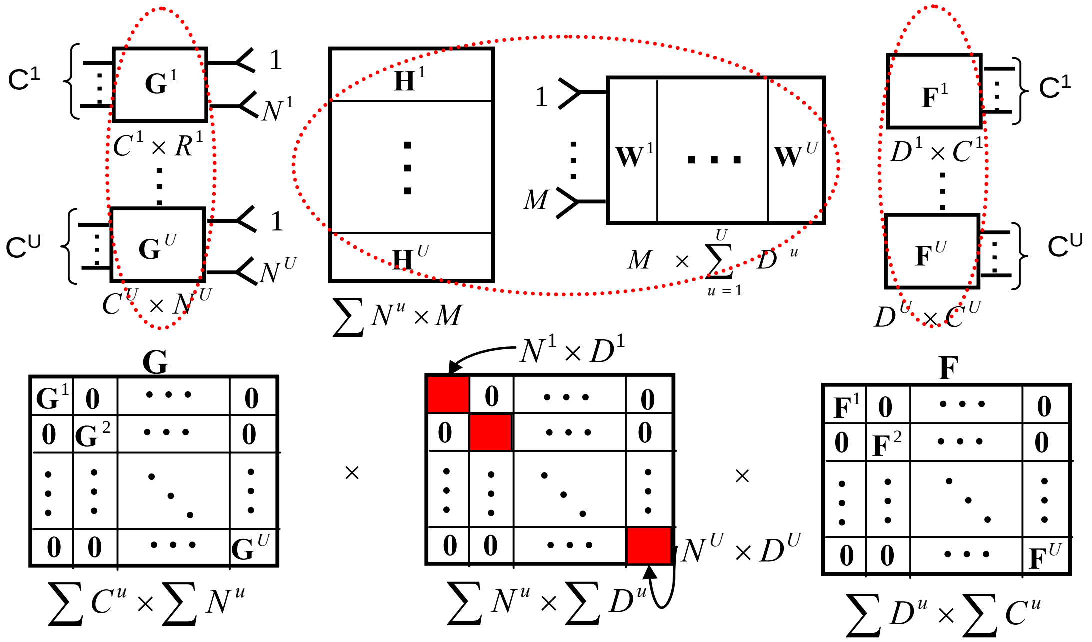


Figure 3-2 Joint Tx/Rx beamforming design for a multi-user MIMO downlink system with null-space matrix

3.2.2 MIMO-OFDM SDMA

By using the OFDM technology, the transmit beamforming and null-space matrix is designed based on each subcarrier and performed before OFDM modulation at the transmit side. At each user terminal, the receive beamforming is also designed based on each subcarrier and performed after OFDM demodulation. And at each subcarrier, the flat-fading conditions prevail and can be treated as above single carrier flat-fading MIMO system shown in Figure 3-2.

3.3 Joint Tx/Rx Beamforming Design for Multi-user Case

3.3.1 Null-space Constraint Design

Now we introduce the design of the null-space matrix which block-diagonalizes the MIMO channel. The following design is based on MIMO-OFDM system where the subscript notation k denotes the subcarrier index. In order to remove the MUI between each user, a null-space matrix denoted by $\mathbf{W}_k$ is designed that the product of the MIMO channel matrix and the null-space matrix $\mathbf{H}_k\mathbf{W}_k$ at k-th subcarrier results a block-diagonal matrix with u-th block in the diagonal which is u-th user's data streams. That is the MUI is completely eliminated and leaves only each user's inter-stream interference which can be deal with by each user's processing.

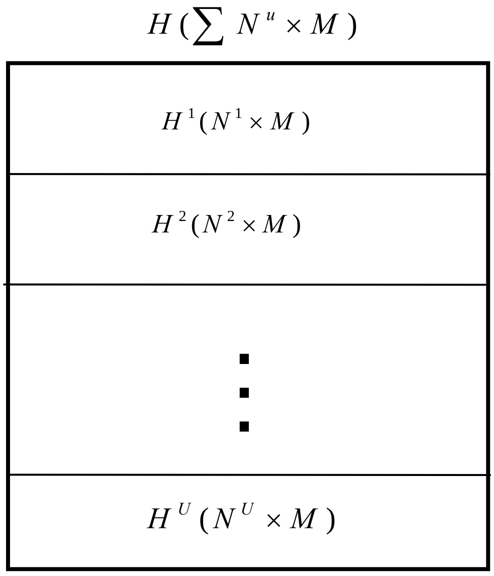


Figure 3-3 The vertical concatenation representation of a multi-user MIMO channel matrix

First of all, the multi-user MIMO channel matrix $\mathbf{H}_k$ at k-th subcarrier can be viewed as a vertical concatenation of U MIMO subchannels matrix $\mathbf{H}_k^u$ which means the BS to u-th user's MIMO subchannel at k-th subcarrier and with dimension $N^u \times M$. We illustrate the whole multi-user MIMO channel by Figure 3-3.

In order to block-diagonalize the whole MIMO channel matrix $\mathbf{H}_k$, we have to design the null-space matrix $\mathbf{W}_k$ with horizontal concatenation of U sub-matrices

$\mathbf{W}_k^u$ depicted in Figure 3-4. The MIMO channel matrix and null-space matrix at k-th subcarrier can be represented in mathematic form as follows (where the superscript T denotes the transport operation):

$$\mathbf{H}_k = \left[\mathbf{H}_k^{1\,T} \quad \mathbf{H}_k^{2\,T} \quad \cdots \quad \mathbf{H}_k^{U\,T} \right]^T \tag{3.3.1-1}$$

$$\mathbf{W}_k = \left[\mathbf{W}_k^1 \quad \mathbf{W}_k^2 \quad \cdots \quad \mathbf{W}_k^{U1} \right] \tag{3.3.1-2}$$

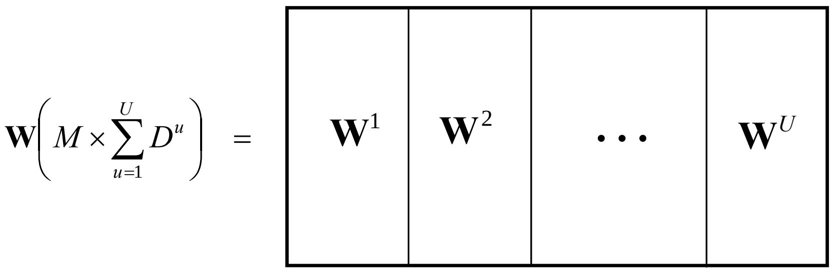


Figure 3-4 The horizontal concatenation representation of a multi-user null-space matrix

We can see that the block-diagonal condition is fulfilled if each column of $\mathbf{W}_k^u$ lies in the null-space of $\mathbf{H}_{k,C}^u$ where $\mathbf{H}_{k,C}^u$ is obtained by removing N^u rows from $\mathbf{H}_k$ and has dimension $\sum_{i=1,i\neq u}^{U} N^u \times M$. We can represent it by the following mathematic equation:

$$\mathbf{W}_k^u \in null\left\{ \mathbf{H}_{k,C}^u \right\} \Leftrightarrow \mathbf{H}_{k,C}^u \cdot \mathbf{W}_k^u = \mathbf{0} \tag{3.3.1-3}$$

For example, $\mathbf{W}_k^1$, the first columns of $\mathbf{W}_k$, is a set of orthogonal basis of the null space of $\mathbf{H}_{k,C}^1$; $\mathbf{W}_k^2$ is built by the orthogonal basis of the null space of $\mathbf{H}_{k,C}^2$, and so on. It is easy to see that each $\mathbf{W}_k^u$ has D_k^u columns where D_k^u is given by the dimension theorem and is given by:

$$D_k^u = M - \sum_{i=1,i\neq u}^{U} N^u \tag{3.3.1-4}$$

The null-space matrix $\mathbf{W}_k^u$ which block-diagonalizes the channel matrix $\mathbf{H}_k$ can be illustrated by Figure 3-5. The block-diagonalized matrix means that the MUI is completely eliminated and each user terminal receives only its own data streams.

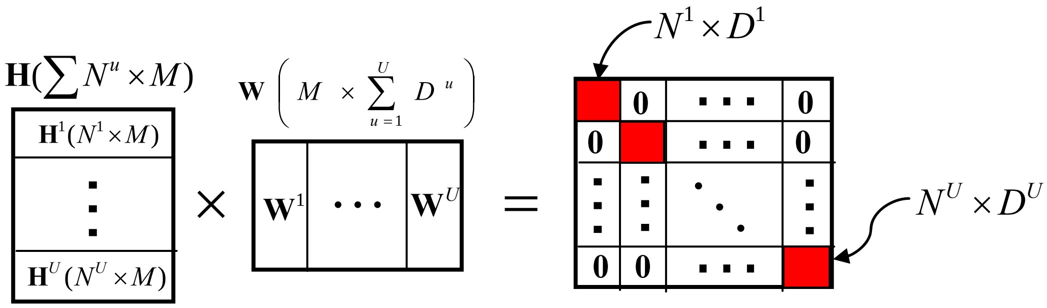


Figure 3-5 The product of the MIMO channel and the null-space matrix

3.3.2 Joint Tx/Rx Design with Null-space Constraint

We have introduced how to design the null-space matrix in previous section. Now we combine the joint Tx/Rx beamforming design with null-space constraint to deal with multi-user MIMO-OFDM SDMA system. The multi-user MIMO-OFDM SDMA system on k-th subcarrier can be modeled as shown in Figure 3-6 which is similar to the flat-fading case as shown in Figure 3-2. The difference between the two systems is that we process these operations in time or frequency domain. For MIMO-OFDM based system, these operations are performed in frequency domain while for MIMO system they are processed in time domain. Thanks to the null-space constraint that the transmit beamforming matrix can be calculated independently for each user and each user terminal only needs to know its part of the multi-user MIMO channel to calculate the receive beamforming matrix. It is reasonable for practical systems. Figure 3-7 and Figure 3-8 illustrate the details of joint Tx/Rx beamforming design for multi-user MIMO-OFDM SDMA downlink system at transmitter and receiver respectively.

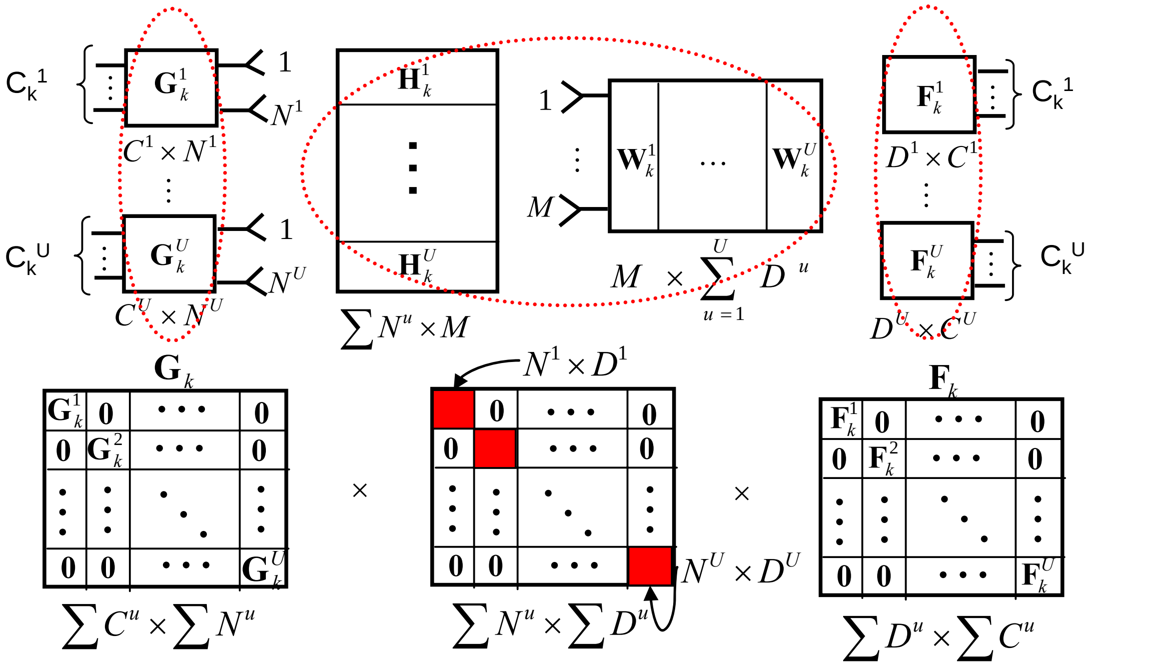


Figure 3-6 Joint Tx/Rx beamforming for multi-user MIMO-OFDM downlink system with null-space matrix

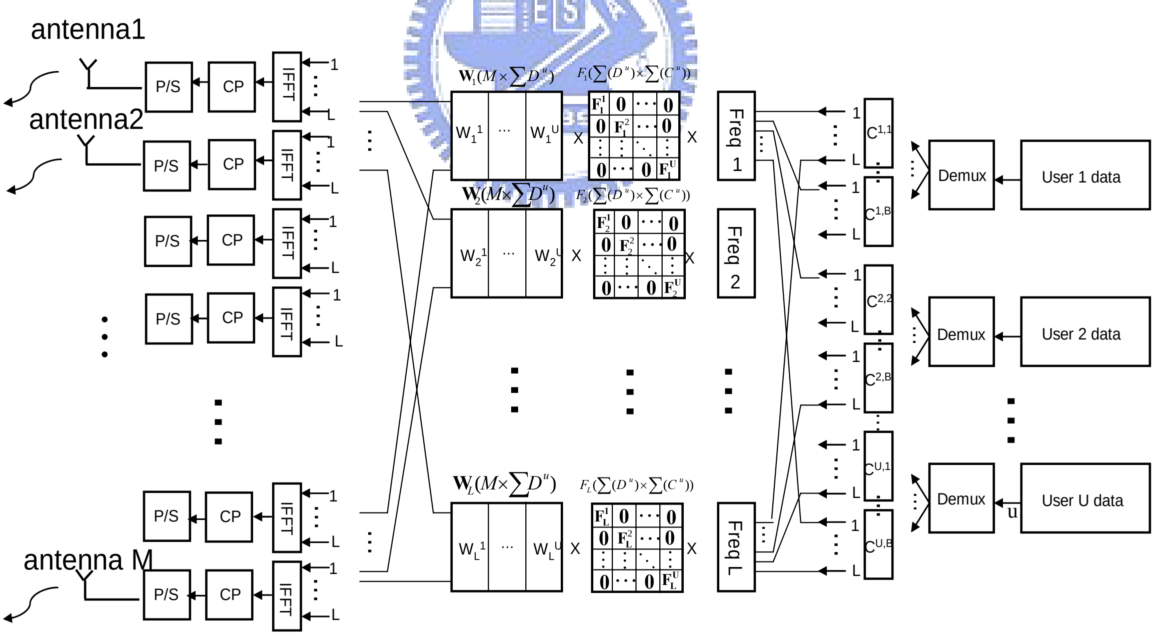


Figure 3-7 Joint Tx/Rx beamforming for multi-user MIMO-OFDM downlink system at transmitter

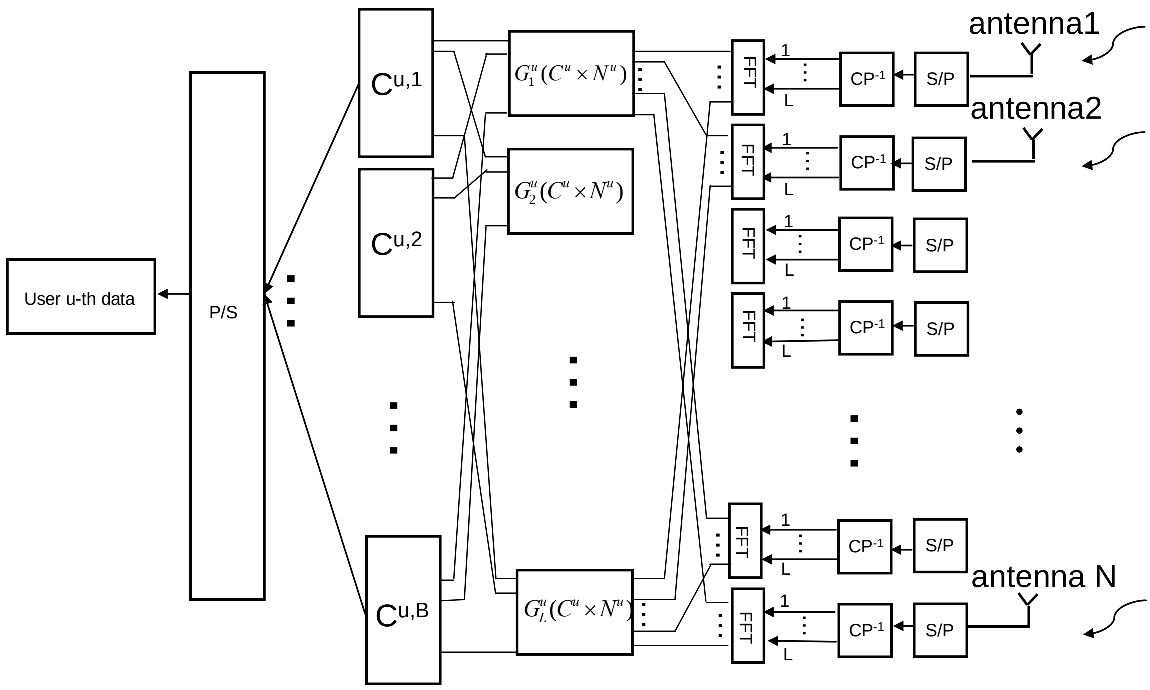


Figure 3-8 Joint Tx/Rx beamforming for multi-user MIMO-OFDM downlink system at u-th user receiver

3.3.3 Derivation of The Joint Tx/Rx Design with Null-space Constraint

We now describe the idea by using mathematic equations and explain the joint Tx/Rx MMSE beamforming design with the null-space constraint for multi-user MIMO-OFDM SDMA downlink system as follows. The superscript notation u denotes user index, and for each user the joint beamforming design is performed over the equivalent channel $\mathbf{H}_k^u \mathbf{W}_k^u$ which means we have taken the block-diagonalization constraint into account and results MUI free.

The system equation for user u and subcarrier k is

$$\hat{\mathbf{s}}_k^u = \mathbf{G}_k^u \mathbf{H}_k^u \mathbf{W}_k^u \mathbf{F}_k^u \mathbf{s}_k^u + \mathbf{G}_k^u \mathbf{n}_k^u \qquad (3.3.3\text{-}1)$$

And the problem is formulated as

$$\begin{aligned} &\min_{\mathbf{F}_k,\mathbf{G}_k} \; E\left\{\left\|\mathbf{e}_k^u\right\|^2\right\} \\ &\text{subject to } trace\left(\mathbf{F}_k^{u^H} \mathbf{W}_k^{u^H} \mathbf{W}_k^u \mathbf{F}_k^u\right) = p_{T,k}^u \end{aligned} \qquad (3.3.3\text{-}2)$$

where $\mathbf{e}_k^u$ is error vector and equal to $\mathbf{s}_k^u - \hat{\mathbf{s}}_k^u$ and $p_{T,k}^u$ denotes the transmit power constraint of user u at subcarrier k. Combining the system equation (3.3.3-1) and above equations, we have

$$\begin{aligned} &\min_{\mathbf{F}_k,\mathbf{G}_k} E\left\{\left\|\mathbf{s}_k^u - \mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\mathbf{s}_k^u + \mathbf{G}_k^u\mathbf{n}_k^u\right\|^2\right\} \\ &\text{subject to } trace\left(\mathbf{F}_k^{u\,H}\mathbf{W}_k^{u\,H}\mathbf{W}_k^u\mathbf{F}_k^u\right) = p_{T,k}^u \end{aligned} \tag{3.3.3-3}$$

In the same way, we use the Frobenius norm and then the minimization problem can be rewritten as minimizing accumulative MSE matrix

$$\begin{aligned} &\min_{\mathbf{F}_k,\mathbf{G}_k} trace\left\{MSE\left(\mathbf{F}_k^u, \mathbf{G}_k^u\right)\right\} \\ &\text{subject to } trace\left(\mathbf{F}_k^{u\,H}\mathbf{W}_k^{u\,H}\mathbf{W}_k^u\mathbf{F}_k^u\right) = p_{T,k}^u \end{aligned} \tag{3.3.3-4}$$

The MSE matrix $MSE\left(\mathbf{F}_k^u, \mathbf{G}_k^u\right)$ can be derived as below

$$\begin{aligned} &MSE\left(\mathbf{F}_k^u, \mathbf{G}_k^u\right) \\ &= E\left[\left(\mathbf{s}_k^u - \mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\mathbf{s}_k^u + \mathbf{G}_k^u\mathbf{n}_k^u\right)\left(\mathbf{s}_k^u - \mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\mathbf{s}_k^u + \mathbf{G}_k^u\mathbf{n}_k^u\right)^H\right] \\ &= \mathrm{E}\left\{\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\mathbf{s}_k^u + \mathbf{G}_k^u\mathbf{n}_k^u\right)\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\mathbf{s}_k^u + \mathbf{G}_k^u\mathbf{n}_k^u\right)^H\right\} \\ &- E\left\{\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\mathbf{s}_k^u + \mathbf{G}_k^u\mathbf{n}_k^u\right)\left(\mathbf{s}_k^u\right)^H\right\} \\ &- E\left\{\left(\mathbf{s}_k^u\right)^H\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\mathbf{s}_k^u + \mathbf{G}_k^u\mathbf{n}_k^u\right)\right\} + E\left\{\left(\mathbf{s}_k^u\right)\left(\mathbf{s}_k^u\right)^H\right\} \end{aligned} \tag{3.3.3-5}$$

And we have the same assumptions similar to (2.2.1-3), that is

$$E\left\{\mathbf{s}_k^u\mathbf{s}_k^{u\,H}\right\} = \mathbf{I};\quad E\left\{\mathbf{n}_k^u\mathbf{n}_k^{u\,H}\right\} = \mathbf{R}_{\mathbf{nn},k}^u;\quad E\left\{\mathbf{s}_k^u\mathbf{n}_k^{u\,H}\right\} = \mathbf{0}; \tag{3.3.3-6}$$

So the MSE matrix $MSE\left(\mathbf{F}_k^u, \mathbf{G}_k^u\right)$ can be simplified as

$$\begin{aligned} MSE\left(\mathbf{F}_k^u, \mathbf{G}_k^u\right) = \left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right)\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right)^H + \mathbf{G}_k^u\mathbf{R}_{\mathbf{nn},k}\mathbf{G}_k^{u\,H} \\ + \mathbf{I} - \left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right) - \left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right)^H \end{aligned} \tag{3.3.3-7}$$

Furthermore, we can solve the optimization problem by Lagrangian or two-step approaches. Following show the Lagrange duality method which transforms the constrained optimization problem into an unconstrained one.

$$
\begin{aligned}
L\left(\mu_k^u, \mathbf{F}_k^u, \mathbf{G}_k^u\right) = trace\Big\{ & \left(\mathbf{G}_k^u \mathbf{H}_k^u \mathbf{W}_k^u \mathbf{F}_k^u\right)\left(\mathbf{G}_k^u \mathbf{H}_k^u \mathbf{W}_k^u \mathbf{F}_k^u\right)^H + \mathbf{G}_k^u \mathbf{R}_{\mathbf{nn},k} \mathbf{G}_k^{u^H} + \mathbf{I} \\
& - \left(\mathbf{G}_k^u \mathbf{H}_k^u \mathbf{W}_k^u \mathbf{F}_k^u\right) - \left(\mathbf{G}_k^u \mathbf{H}_k^u \mathbf{W}_k^u \mathbf{F}_k^u\right)^H \Big\} \\
& + \mu_k^u \left[trace\left(\mathbf{F}_k^{u^H} \mathbf{W}_k^{u^H} \mathbf{W}_k^u \mathbf{F}_k^u\right) - p_{T,k}^u \right]
\end{aligned}
\tag{3.3.3-8}
$$

where μ_k^u is the Lagrange multiplier and the KKT conditions are

$$\nabla_{\mathbf{F}_k} L\left(\mu_k^u, \mathbf{F}_k^u, \mathbf{G}_k^u\right) = 0 \tag{3.3.3-9}$$

$$\nabla_{\mathbf{G}_k^u} L\left(\mu_k^u, \mathbf{F}_k^u, \mathbf{G}_k^u\right) = 0 \tag{3.3.3-10}$$

$$\nabla_{\mu_k^u} L\left(\mu_k^u, \mathbf{F}_k^u, \mathbf{G}_k^u\right) = 0 \tag{3.3.3-11}$$

We obtain the similar results in chapter 2.

$$\mathbf{F}_k^u = \mathbf{V}_k^u \mathbf{\Phi}_{\mathbf{F}_k^u} \tag{3.3.3-12}$$

$$\mathbf{G}_k^u = \mathbf{\Phi}_{\mathbf{G}_k^u} \mathbf{U}_k^{u^H} \mathbf{H}_k^{u^H} \mathbf{W}_k^{u^H} \mathbf{R}_{\mathbf{nn},k}^{u\ -1} \tag{3.3.3-13}$$

$$\mathbf{\Phi}_{\mathbf{F}_k^u} = \left(\mu_k^{u^{-1/2}} \mathbf{S}_k^{u^{-1/2}} - \mathbf{S}_k^{u^{-1}} \right)_+^{1/2} \tag{3.3.3-14}$$

$$\mathbf{\Phi}_{\mathbf{G}_k^u} = \left(\mu_k^{u^{1/2}} \mathbf{S}_k^{u^{-1/2}} - \mathbf{S}_k^{u^{-1}} \right)_+^{1/2} \mathbf{S}_k^{u^{-1/2}} \tag{3.3.3-15}$$

where $\mathbf{V}_k^u$, $\mathbf{U}_k^u$ and $\mathbf{S}_k^u$ are obtained from the SVD of

$$\mathbf{H}_k^{u^H} \mathbf{W}_k^{u^H} \mathbf{R}_{\mathbf{nn},k}^{-1} \mathbf{W}_k^u \mathbf{H}_k^u = \left(\mathbf{U}_k^u \quad \tilde{\mathbf{U}}_k^u\right) \begin{pmatrix} \mathbf{S}_k^u & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{S}}_k^u \end{pmatrix} \left(\mathbf{V}_k^u \quad \tilde{\mathbf{V}}_k^u\right)^H \tag{3.3.3-16}$$

The meanings of above parameters are equivalent to the description in chapter 2. Particularly, μ_k^u is chosen to satisfy the transmit power constraint and given in following equation:

$$\mu_k^{u\frac{1}{2}} = \frac{trace\left(\mathbf{S}_k^{u^{-\frac{1}{2}}}\right)}{P_{T,k}^u + trace\left(\mathbf{S}_k^{u^{-1}}\right)} \tag{3.3.3-17}$$

We can expect that the same results can be obtained from the two-step approach where we use the equivalent MIMO channel $\mathbf{H}_k^u \mathbf{W}_k^u$ instead of $\mathbf{H}_k^u$.

3.4 Simulation Results and Comments

We will show the performance of above joint Tx/Rx beamforming design for multi-user MIMO-OFDM SDMA downlink system by computer simulations. In this set-up, we first assume that the channel estimations are perfect known at both transmit and receive terminals; the elements of the MIMO channel are independent-identically-distribution (i.i.d) complex Gaussian distribution with zero mean and variance 1 and the channel length is L_C.

At BS which is equipped with M transmit antennas, each user's data are QPSK modulated and de-multiplexed into B parallel paths and each user is equipped with N receive antennas. Each path is processed by OFDM modulation. Before passing to OFDM, we have to perform the transmit beamforming for each subcarrier of each user and then process the null-space matrix for each subcarrier of all user. We can see procedures in Figure 3-7. In OFDM, we assume the length of FFT is L and the length of CP is L_{CP} which is larger or equal than the channel length L_C in order to keep the orthogonality between each subcarrier. And then launch the OFDM packets via M transmit antennas. At each receive terminal, thanks to the null-space matrix, each user only receives it own data. After performing OFDM demodulation, the output signal is processing to the receive beamforming for each subcarrier and then passes to an appropriate interleaver to obtain correct data streams.

Figure 3-9 shows the bit error rate (BER) curves of typical multi-user MIMO-OFDM SDMA systems where the BS equipped with 6 to 8 transmit antennas communicates simultaneously with 3 users. Each user is equipped with 2 receive antennas. The FFT length L is 64. Each OFDM packet contains 640 data symbols and 100 MIMO channel realizations described above are simulated and generated independently for each packet. The total transmit power per symbol period across all antennas is normalized to 1. The SNR is defined as the total transmitted power normalized with the noise variance at each subcarrier.

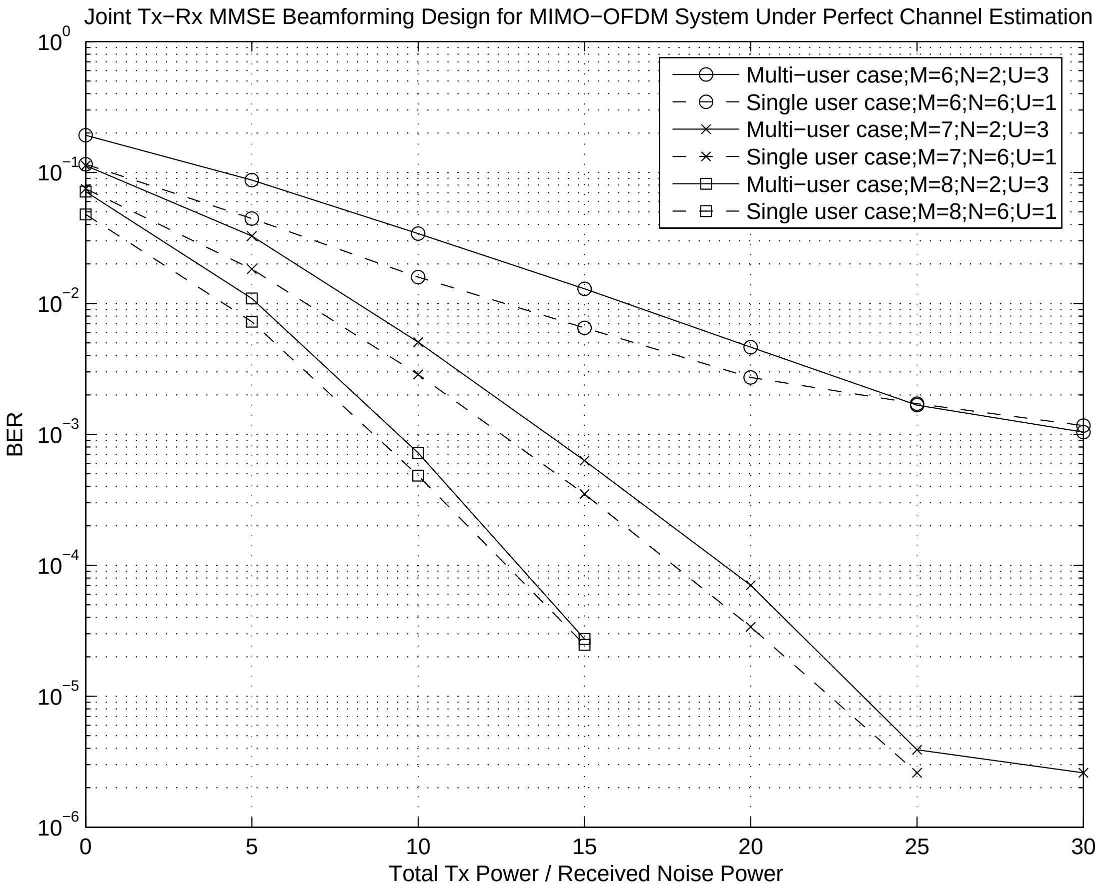


Figure 3-9 Joint Tx/Rx MMSE beamforming design for MIMO-OFDM system under perfect CSI

We call the MIMO system “fully loaded” when the number of parallel streams is equal to the number of BS antennas. When the number of BS antennas is greater than the number of parallel streams, the system is called ”under-loaded”. In this case, the diversity gain can be expected. However, it is impossible to simultaneously transmit more parallel streams (over-loaded case) without inducing irreducible MUI.

In Figure3-9, we also show the single user case which has 6 receive antennas. From the simulation results, we can summarize the following observations.

- Since the single user case with the same number of receive antennas has more degrees of freedom for spatial processing at the receiver, it has a better performance than the multi-user case.
- When the number of BS antennas increases, we can have of course a better performance because of the diversity gain obtained from the transmit beamforming.
- In multi-user case, adding one antenna at the BS provides a diversity gain of 1 to all users.
- The performance difference between the single user and the multi-user case becomes negligible when the number of BS antennas increases.

3.5 Conclusions

In this chapter, we consider the joint design Tx/Rx beamforming for multi-user MIMO-OFDM downlink communications using SDMA under perfect channel estimation. In multi-user system, the MUI is the major problem which needs to be handled. We use a null-space technique to decouple the multi-user MIMO SDMA joint design problem into several single user problems, In other words, the product of the MIMO channel and the null-space matrix results a block-diagonal matrix which means the MUI between each user is completely removed. Thus, each user terminal only has to handle its own inter-stream interference. Furthermore, we extend this flat-fading channel case to frequency selective channel environment by using the MIMO-OFDM based system. Using the null-space constraint, the several decoupled single user designs also have the properties that the system structure can be scalable with respect to the number of antennas, size of the coding block, and transmit power.

Chapter 4

Robust Design of Joint Tx/Rx MMSE Beamforming with Excellent Channel Estimation Error immunity for Multi-user MIMO-OFDM SDMA Downlink System

---

We have discussed the joint Tx/Rx MMSE beamforming design for the multi-user MIMO-OFDM SDMA system in chapter 3. The channel information is assumed to be perfectly known at both transmit and receive terminals. However, the channel estimation always contains errors in real communication systems and only imperfect CSI can be obtained. In a practical wireless environment, channel information has to be estimated periodically because of the time-varying characteristic of channel, especially in the mobile environment. The imperfect CSI has a significant impact on the performance. In this chapter, we will focus the robust beamforming design to enhance the system performance.

4.1 Introduction

If the channel estimation is perfect, we can design the optimal solutions of transmit and receive beamforming matrices in single user MIMO-OFDM case. Furthermore, for a multi-user MIMO-OFDM SDMA system, a perfect null-space matrix that removes the MUI between each user also can be obtained. However, if the channel estimation contains errors, the null-space and Tx/Rx beamforming matrices will be designed imperfectly in multi-user joint design system (see Figure 4-1).

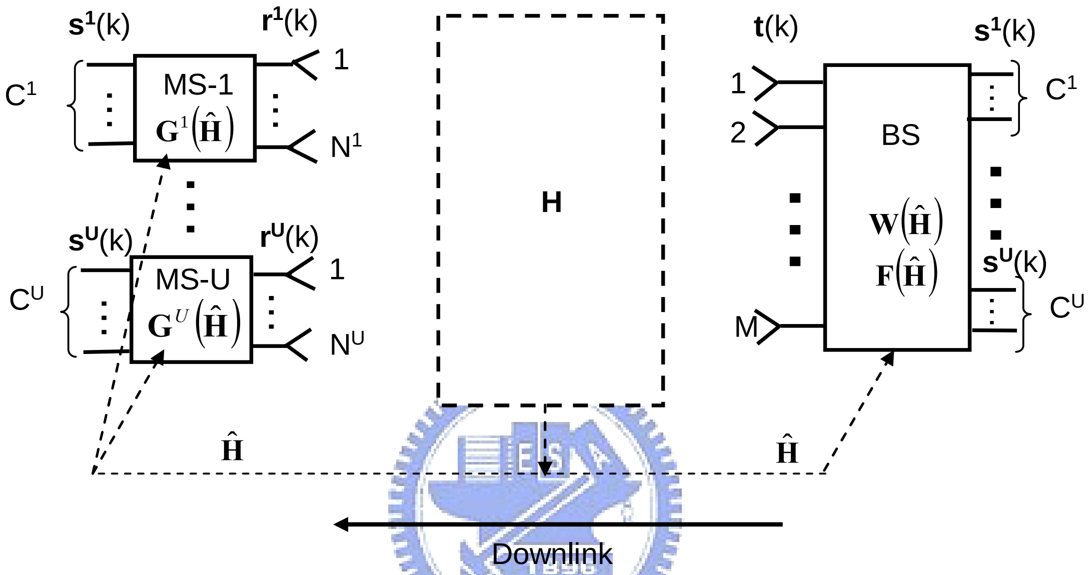


Figure 4-1 Joint Tx/Rx beamforming design for multi-user MIMO downlink system under imperfect CSI

The imperfect null-space design will induce MUI between each user and the imperfect transmit and receive beamforming designs will cause the inter-stream interference at each user terminal. That is why the imperfect CSI will cause significant performance degradation. This is an unavoidable problem in realistic wireless communication systems that the channel estimation always contains errors and only the imperfect CSI can be obtained.

The CSI at receiver can be obtained via the training sequence or pilot symbols that allows to estimate the channel. The CSI at transmitter can be obtained by using feedback channels from the receiver to the transmitter. In many cases, a sufficiently accurate channel information at receiver can be assumed, however, the CSI at transmitter is always far from sufficient accuracy. Hence one can assume that the receiver has perfect CSI to design receive beamforming and the transmitter has imperfect CSI to design transmit beamforming and null-space matrices. But in this chapter, we will consider more general case that both CSI at transmit and receive sides are imperfect to design null-space and Tx/Rx beamforming matrices in multi-user joint design MIMO-OFDM SDMA system.

We are going to apply two robust approaches to our multi-user joint design problem to against the performance degradation caused by MUI and inter-stream interference [6] [18]. Both of these robust methods have the similar performance over fast time-variant and slow time-variant environments. Therefore, we apply the moving average approach that has better performance over slow time-variant environment. Note that all the robust methods need the statistic properties of the estimation errors. In the end, we will show the simulation results and give some comments.

4.2 Robust Design of Joint Tx/Rx MMSE Beamforming

4.2.1 Problem Description

We now consider interferences induced by the imperfect null-space matrix and Tx/Rx beamforming due to the channel information error. Recall that for each subcarrier of OFDM system the product of MIMO channel and null-space matrix results a block-diagonalized matrix which means MUI is perfect removed. That is

$$\mathbf{H}_k^i \mathbf{W}_k^u = \mathbf{0}; \quad \text{if } i \neq u \qquad (4.2.1\text{-}1)$$

where the subscript notation denotes subcarrier index and the superscript notation represents the user terminal index. And each user's Tx/Rx beamforming is also designed due to the exact channel information $\mathbf{H}_k^i$. Figure 4-2 illustrates again the major block-diagonalized operation at each subcarrier.

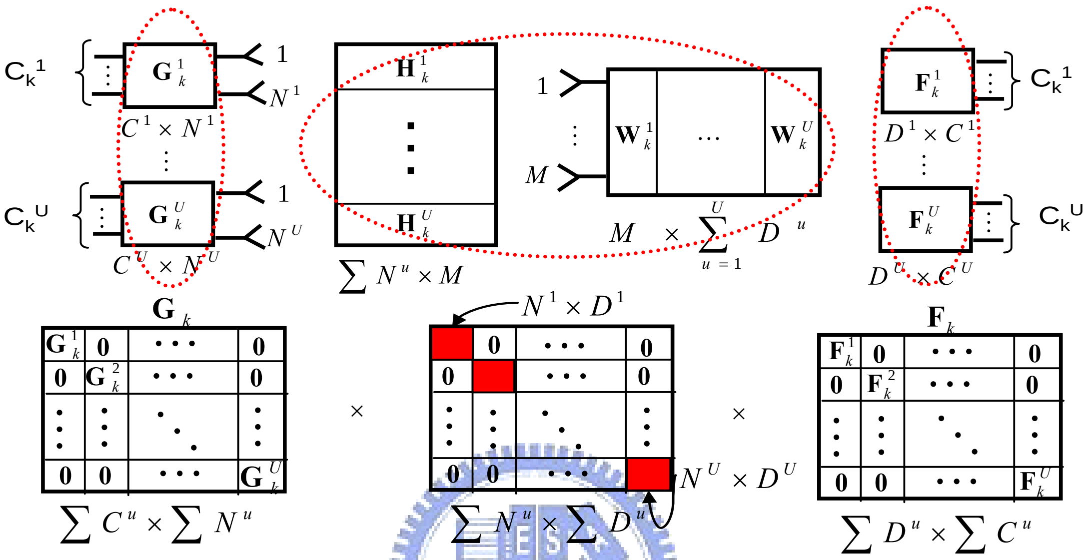


Figure 4-2 Joint Tx/Rx beamforming for multi-user MIMO-OFDM downlink system with null-space matrix

Moreover, we explain the operation by another point of view in order to derive the robust null-space matrix against MUI. Assume that t_k is the total transmitted signal at subcarrier k and it can be formulated as

$$\mathrm{t}_k = \sum_{i=1}^{U} \mathbf{W}_k^i \mathbf{F}_k^i \mathbf{s}_k^i \tag{4.2.1-2}$$

Thus the received signal of user u at subcarrier k for a multi-user joint beamforming design MIMO-OFDM SDMA system becomes

$$\mathbf{y}_k^u = \mathbf{H}_k^u \mathrm{t}_k + \mathbf{n}_k^u = \mathbf{H}_k^u \left(\sum_{i=1}^{U} \mathbf{W}_k^i \mathbf{F}_k^i \mathbf{s}_k^i \right) + \mathbf{n}_k^u \tag{4.2.1-3}$$

By the equation (4.2.1-1), the above equation can be rewritten as

$$\mathbf{y}_k^u = \mathbf{H}_k^u \left(\sum_{i=1}^{U} \mathbf{W}_k^i \mathbf{F}_k^i \mathbf{s}_k^i \right) + \mathbf{n}_k^u = \mathbf{H}_k^u \mathbf{W}_k^u \mathbf{F}_k^u \mathbf{s}_k^u + \mathbf{n}_k^u \tag{4.2.1-4}$$

In (4.2.1-4), it is easy to see that the mutual interferences between co-channel users are removed and leaves only each user's joint design problem to cope with its own inter-stream interference.

Now we assume that the MIMO channel information contains errors and can be represented as

$$\hat{\mathbf{H}}_k = \mathbf{H}_k + \Delta\mathbf{H}_k \tag{4.2.1-5}$$

where $\Delta\mathbf{H}_k$ is channel information error and its elements are independent random variables with zero mean and variance $\sigma^2_{\Delta\mathbf{H}_k}$. Since the BS has the imperfect CSI $\hat{\mathbf{H}}_k$, not the exact CSI $\mathbf{H}_k$, the null-space matrix is obtained under the condition

$$\hat{\mathbf{H}}_k^i \hat{\mathbf{W}}_k^u = \mathbf{0}; \text{ if } i \neq u \tag{4.2.1-6}$$

And we also obtain the imperfect transmit beamforming $\hat{\mathbf{F}}_k^u$ that is designed from the equivalent channel $\hat{\mathbf{H}}_k^i \hat{\mathbf{W}}_k^u$. Thus, the total transmitted signal at each subcarrier becomes

$$\hat{\mathrm{t}}_k = \sum_{i=1}^{U} \hat{\mathbf{W}}_k^i \hat{\mathbf{F}}_k^i \mathbf{s}_k^i \tag{4.2.1-7}$$

And the received signal of user u becomes

$$\mathbf{y}_k^u = \mathbf{H}_k^u \hat{\mathbf{t}}_k + \mathbf{n}_k^u = \mathbf{H}_k^u \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^i + \left(\sum_{i=1, i\neq u}^{U} \mathbf{H}_k^u \hat{\mathbf{W}}_k^i \hat{\mathbf{F}}_k^i \mathbf{s}_k^i \right) + \mathbf{n}_k^u \tag{4.2.1-8}$$

Since the second term $\mathbf{H}_k^u \hat{\mathbf{W}}_k^i$ in (4.2.1-8) is no longer zeros for $u \neq i$, user u will be affected by MUI. Furthermore, at user u's terminal, since the first term of

(4.2.1-8), both of transmit and receive beamforming will be designed imperfectly which leads to the inter-stream interference. In the following section, we will first consider the MUI caused by imperfect null-space matrix (the second term effect), and then consider the inter-stream interference caused by imperfectly designed beamformings (the first term effect).

4.2.2 Robust Designs

4.2.2.1 Robust Design Against MUI

In this section, we derive the robust null-space matrix which minimizes the expected power of MUI by using the statistics of the channel estimation error. Before deriving it, we first consider the statistical dependence among the exact channel $\mathbf{H}_k$, the estimated channel $\hat{\mathbf{H}}_k$ and the estimated channel error $\Delta\mathbf{H}_k$. In general, $\Delta\mathbf{H}_k$ is independent of $\mathbf{H}_k$ but dependent on $\hat{\mathbf{H}}_k$. However, if $\|\Delta\mathbf{H}_k\|$ is much smaller than $\|\mathbf{H}_k\|$, it can be assumed that $\Delta\mathbf{H}_k$ is approximately independent of $\hat{\mathbf{H}}_k$. Thus, in order to derive the robust null-space matrix, we will assume that

$$\hat{\mathbf{H}}_k \perp \mathbf{H}_k;\quad \hat{\mathbf{H}}_k \perp \Delta\mathbf{H}_k \tag{4.2.2.1-1}$$

We now investigate the effect that when user u's signal is transmitted, how it will induce interference to affect other users. The received signal at subcarrier k and all user terminals when user u's signal is transmitted is

$$\mathbf{y}_k^{all} = \mathbf{H}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u = \mathbf{H}_k \mathbf{t}_k^u \tag{4.2.2.1-2}$$

Obviously, if the null-space is perfect designed, that is, if we know the exact CSI, there is no MUI occurred induced by user u's signal and all elements of $\mathbf{y}_k^{all}$ will be zeros except the elements at user u's terminal. We now assume that the transmitted signal of user u at subcarrier k which minimizes the expected power of MUI is

$$\mathbf{t}_k^u = \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u + \mathbf{a}_k^u \tag{4.2.2.1-3}$$

And then (4.2.2.1-2) becomes

$$\begin{aligned} \mathbf{y}_k^{all} &= \mathbf{H}_k\left(\hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u + \mathbf{a}_k^u\right) = \left(\hat{\mathbf{H}}_k - \Delta\mathbf{H}_k\right)\left(\hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u + \mathbf{a}_k^u\right) \\ &= \hat{\mathbf{H}}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u + \hat{\mathbf{H}}_k \mathbf{a}_k^u - \Delta\mathbf{H}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u - \Delta\mathbf{H}_k \mathbf{a}_k^u \end{aligned} \tag{4.2.2.1-4}$$

In above equation, the first term $\hat{\mathbf{H}}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u$ has non-zero only in user u's received signal, and other elements are all zero values, therefore it does not induce any interference to other users. But the other terms in (4.2.2.1-4) can be consider as the MUI induced by user u's signal. In order to minimize the MUI, we would like to find $\mathbf{a}_k^u$ to minimize the expected power of these terms $\left(\hat{\mathbf{H}}_k \mathbf{a}_k^u - \Delta\mathbf{H}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u - \Delta\mathbf{H}_k \mathbf{a}_k^u\right)$ in (4.2.1.1-8). In other words, we are going to find an $\mathbf{a}_k^u$ that satisfies

$$\min_{\mathbf{a}_k^u} \mathrm{E}\left\|\hat{\mathbf{H}}_k \mathbf{a}_k^u - \Delta\mathbf{H}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u - \Delta\mathbf{H}_k \mathbf{a}_k^u\right\|^2 \tag{4.2.2.1-5}$$

Define that

$$\begin{aligned} q^2 &= \left\|\hat{\mathbf{H}}_k \mathbf{a}_k^u - \Delta\mathbf{H}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u - \Delta\mathbf{H}_k \mathbf{a}_k^u\right\|^2 \\ &= \mathbf{a}_k^{u\,H} \hat{\mathbf{H}}_k^{\,H} \hat{\mathbf{H}}_k \mathbf{a}_k^u + \mathbf{s}_k^{u\,H} \hat{\mathbf{F}}_k^{u\,H} \hat{\mathbf{W}}_k^{u\,H} \Delta\mathbf{H}_k^{\,H} \Delta\mathbf{H}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u \\ &\quad + \mathbf{a}_k^{u\,H} \Delta\mathbf{H}_k^{\,H} \Delta\mathbf{H}_k \mathbf{a}_k^u - 2\,\mathrm{Re}\left\{\mathbf{a}_k^{u\,H} \hat{\mathbf{H}}_k^{\,H} \Delta\mathbf{H}_k \hat{\mathbf{W}}_k^u \hat{\mathbf{F}}_k^u \mathbf{s}_k^u \right. \\ &\quad \left. + \mathbf{a}_k^{u\,H} \hat{\mathbf{H}}_k^{\,H} \Delta\mathbf{H}_k \mathbf{a}_k^u - \mathbf{s}_k^{u\,H} \hat{\mathbf{F}}_k^{u\,H} \hat{\mathbf{W}}_k^{u\,H} \Delta\mathbf{H}_k^{\,H} \Delta\mathbf{H}_k \mathbf{a}_k^u \right\} \end{aligned} \tag{4.2.2.1-6}$$

Under the assumptions described in (4.2.2.1-1) and the statistic properties of $\Delta\mathbf{H}_k$, that is, its elements are independent random variables with zero mean and variance $\sigma^2_{\Delta\mathbf{H}_k}$. Thus the expectation of equation (4.2.2.1-6) can be rewritten as

$$\begin{aligned} E\left\{q^2\right\} = \mathbf{a}_k^{u\,H}\hat{\mathbf{H}}_k^{\;H}\hat{\mathbf{H}}_k\mathbf{a}_k^u + \sigma^2_{\Delta\mathbf{H}_k}\mathbf{s}_k^{u\,H}\hat{\mathbf{F}}_k^{u\,H}\hat{\mathbf{W}}_k^{u\,H}\hat{\mathbf{W}}_k^u\hat{\mathbf{F}}_k^u\mathbf{s}_k^u \\ + \sigma^2_{\Delta\mathbf{H}_k}\mathbf{a}_k^{u\,H}\mathbf{a}_k^u + 2\,\mathrm{Re}\left\{\sigma^2_{\Delta\mathbf{H}_k}\mathbf{s}_k^{u\,H}\hat{\mathbf{F}}_k^{u\,H}\hat{\mathbf{W}}_k^{u\,H}\mathbf{a}_k^u\right\} \end{aligned} \tag{4.2.2.1-7}$$

To find an $\mathbf{a}_k^u$ to satisfy (4.2.2.1-5), we take derivative of (4.2.2.1-7) with respect to $\mathbf{a}_k^u$ and set the result to zero. We can obtain

$$\nabla_{\mathbf{a}_k^u} E\left\{q^2\right\} = \hat{\mathbf{H}}_k^{\;H}\hat{\mathbf{H}}_k\mathbf{a}_k^u + \sigma^2_{\Delta\mathbf{H}_k}\mathbf{a}_k^u + \sigma^2_{\Delta\mathbf{H}_k}\hat{\mathbf{W}}_k^u\hat{\mathbf{F}}_k^u\mathbf{s}_k^u \equiv \mathbf{0} \tag{4.2.2.1-8}$$

From above equation, we get the solution as

$$\mathbf{a}_k^u = -\left(\frac{1}{\sigma^2_{\Delta\mathbf{H}_k}}\hat{\mathbf{H}}_k^{\;H}\hat{\mathbf{H}}_k + \mathbf{I}_{n_T}\right)^{-1}\hat{\mathbf{W}}_k^u\hat{\mathbf{F}}_k^u\mathbf{s}_k^u \tag{4.2.2.1-9}$$

And the transmitted signal of user u in (4.2.2.1-3) becomes

$$\mathbf{t}_k^u = \left\{\mathbf{I}_{n_T} - \left(\frac{1}{\sigma^2_{\Delta\mathbf{H}_k}}\hat{\mathbf{H}}_k^{\;H}\hat{\mathbf{H}}_k + \mathbf{I}_{n_T}\right)^{-1}\right\}\hat{\mathbf{W}}_k^u\hat{\mathbf{F}}_k^u\mathbf{s}_k^u \tag{4.2.2.1-10}$$

Thus, we can obtain the robust null-space matrix $\tilde{\mathbf{W}}_k^u$ which minimizes the expected power of MUI between all users from the equation (4.2.2.1-10)

$$\tilde{\mathbf{W}}_k^u = \left\{\mathbf{I}_{n_T} - \left(\frac{1}{\sigma^2_{\Delta\mathbf{H}_k}}\hat{\mathbf{H}}_k^{\;H}\hat{\mathbf{H}}_k + \mathbf{I}_{n_T}\right)^{-1}\right\}\hat{\mathbf{W}}_k^u \tag{4.2.2.1-11}$$

4.2.2.2 Robust Design Against Imperfect Beamforming

After considering the MUI caused by imperfect null-space matrix, we now consider the inter-stream interference induced from imperfect Tx/Rx beamforming at each user terminal. Recall that for user u at subcarrier k the transmit and receive beamforming are designed from minimizing the instantaneous MSE matrix

$$\begin{aligned}\mathbf{E}_k^u\left(\mathbf{F}_k^u,\mathbf{G}_k^u\right)=&\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right)\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right)^H+\mathbf{G}_k^u\mathbf{R}_{\mathbf{nn},k}^u\mathbf{G}_k^{u\,H}\\&+\mathbf{I}-\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right)-\left(\mathbf{G}_k^u\mathbf{H}_k^u\mathbf{W}_k^u\mathbf{F}_k^u\right)^H\end{aligned}\qquad(4.2.2.2\text{-}1)$$

By the robust null-space matrix, we decouple the multi-user joint design problem with worse MUI into a set of parallel single user joint design with minimized MUI. The per-user joint beamforming design with robust null-space matrix can be represented as

$$\begin{aligned}\mathbf{E}_k^u\left(\mathbf{F}_k^u,\mathbf{G}_k^u\right)=&\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)^H+\mathbf{G}_k^u\mathbf{R}_{\mathbf{nn},k}^u\mathbf{G}_k^{u\,H}\\&+\mathbf{I}-\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)-\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)^H\end{aligned}\qquad(4.2.2.2\text{-}2)$$

In order to enhance the performance, we now consider the minimization of the averaged MSE matrix.

$$\begin{aligned}E\left\{\mathbf{E}_k^u\left(\mathbf{F}_k^u,\mathbf{G}_k^u\right)\right\}=E\Big\{&\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)^H+\mathbf{G}_k^u\mathbf{R}_{\mathbf{nn},k}^u\mathbf{G}_k^{u\,H}\\&+\mathbf{I}-\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)-\left(\mathbf{G}_k^u\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)^H\Big\}\end{aligned}\qquad(4.2.2.2\text{-}3)$$

In the same way, using equations (4.2.1-5), (4.2.2.1-1) and the statistics of $\Delta\mathbf{H}_k^u$, we can have following equation

$$\begin{aligned}&E\left\{\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\mathbf{H}_k^{u\,H}\right\}\\&=E\left\{\left(\hat{\mathbf{H}}_k^u-\Delta\mathbf{H}_k^u\right)\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\left(\hat{\mathbf{H}}_k^u-\Delta\mathbf{H}_k^u\right)^H\right\}\\&=E\left\{\hat{\mathbf{H}}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\hat{\mathbf{H}}_k^{u\,H}\right\}+E\left\{\Delta\mathbf{H}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\Delta\mathbf{H}_k^{u\,H}\right\}\\&=\hat{\mathbf{H}}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\hat{\mathbf{H}}_k^{u\,H}+\sigma_{\Delta\mathbf{H}_k^u}^2\cdot trace\left(\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\right)\cdot\mathbf{I}\end{aligned}\qquad(4.2.2.2\text{-}4)$$

$$E\left\{\mathbf{H}_k^u\right\}=E\left\{\left(\hat{\mathbf{H}}_k^u-\Delta\mathbf{H}_k^u\right)\right\}=\hat{\mathbf{H}}_k^u\qquad(4.2.2.2\text{-}5)$$

Using properties (4.2.2.2-4) and (4.2.2.2-4), the equation (4.2.2.2-3) becomes

$$
\begin{aligned}
&E\left\{\mathbf{E}_k^u\left(\mathbf{F}_k^u,\mathbf{G}_k^u\right)\right\} \\
&=\left\{\mathbf{G}_k^u\left(\hat{\mathbf{H}}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\hat{\mathbf{H}}_k^{u\,H}+\sigma_{\Delta\mathbf{H}_k^u}^2 trace\left(\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\right)\cdot\mathbf{I}\right)\mathbf{G}_k^{u\,H}\right. \\
&\left.+\mathbf{G}_k^u\mathbf{R}_{\mathbf{nn},k}^u\mathbf{G}_k^{u\,H}+\mathbf{I}-\left(\mathbf{G}_k^u\hat{\mathbf{H}}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)-\left(\mathbf{G}_k^u\hat{\mathbf{H}}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\right)^H\right\}
\end{aligned}
\quad (4.2.2.2\text{-}6)
$$

where $trace\left(\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\right)$ is equal to transmit power constraint $P_{T,k}^u$.

Therefore, using the two-step approach, the optimal receive beamforming that minimizes the averaged MSE matrix can be obtained as the Wiener solution

$$
\mathbf{G}_{k,opt}^u=\left(\hat{\mathbf{H}}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u\mathbf{F}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\hat{\mathbf{H}}_k^{u\,H}+\mathbf{R}_{\mathbf{nn},k}^u+\sigma_{\Delta\mathbf{H}_k^u}^2P_{T,k}^u\mathbf{I}\right)^{-1}\hat{\mathbf{H}}_k^u\tilde{\mathbf{W}}_k^u\mathbf{F}_k^u \quad (4.2.2.2\text{-}7)
$$

And we can obtain the transmit beamforming which has the similar form

$$
\mathbf{F}_k^u=\mathbf{V}_k^u\mathbf{\Phi}_{\mathbf{F}_k^u};\mathbf{\Phi}_{\mathbf{F}_k^u}=\left(\mu_k^{u\,-1/2}\mathbf{S}_k^{u\,-1/2}-\mathbf{S}_k^{u\,-1}\right)_+^{1/2} \quad (4.2.2.2\text{-}8)
$$

But we should note that the $\mathbf{V}_k^u$, and $\mathbf{S}_k^u$ in (4.2.2.2-8) are obtained from the SVD of

$$
\hat{\mathbf{H}}_k^{u\,H}\tilde{\mathbf{W}}_k^{u\,H}\left(\mathbf{R}_{\mathbf{nn},k}^u+\sigma_{\Delta\mathbf{H}_k^u}^2P_{T,k}^u\right)\tilde{\mathbf{W}}_k^u\hat{\mathbf{H}}_k^u \quad (4.2.2.2\text{-}9)
$$

Combining these two robust methods, we can obtain a better performance than the naive design which designs the null-space matrix and Tx/Rx beamforming by the imperfect channel information $\hat{\mathbf{H}}_k^u$. By the way, for single user joint design problem under channel estimation error, the null-space matrix becomes an identity matrix and we just need to apply the robust Tx/Rx beamforming to improve the performance.

4.2.2.3 Robust Design Using Moving Average Method

We have introduced the approach that combines robust null-space matrix which minimizes the MUI between each user and robust Tx/Rx beamforming which copes with the inter-stream interference of each user terminal. However, the combination of these two robust methods uses the instantaneous estimated channel information $\hat{\mathbf{H}}_k^u$ to calculus the robust null-space matrix and Tx/Rx beamforming. In other words, once we receive an OFDM packet and estimate the channel by the long preamble of the packet, we use the instantaneous estimated channel information to apply to our combined robust approach to improve the performance. Thus, we can expect that the similar performance can be obtained under fast- and slow-fading channel caused by the Doppler effect of moving user terminals.

However, under the slow time-variant environment we can apply the moving average approach (the same throughput) to improve the channel estimation error instead of using the instantaneous estimated channel information. Using the moving average approach in such multi-user joint design system, we calculus the null-space matrix and Tx/Rx beamforming by the same approach described in chapter 2 and chapter 3. The only difference is that we replace the instantaneous estimated channel information by the moving average of the estimated channel. These robust approaches to improve the system performance are simulated by computer and shown as next section. We also give some comments to these simulation results.

4.3 Simulation Results and Comments

we consider a multi-user MIMO-OFDM SDMA where the BS equipped with 6 to 7 transmit antennas communicates simultaneously with 3 users. Each user is equipped with 2 receive antennas. The other parameters are set up in the same way as that in section 3-4.

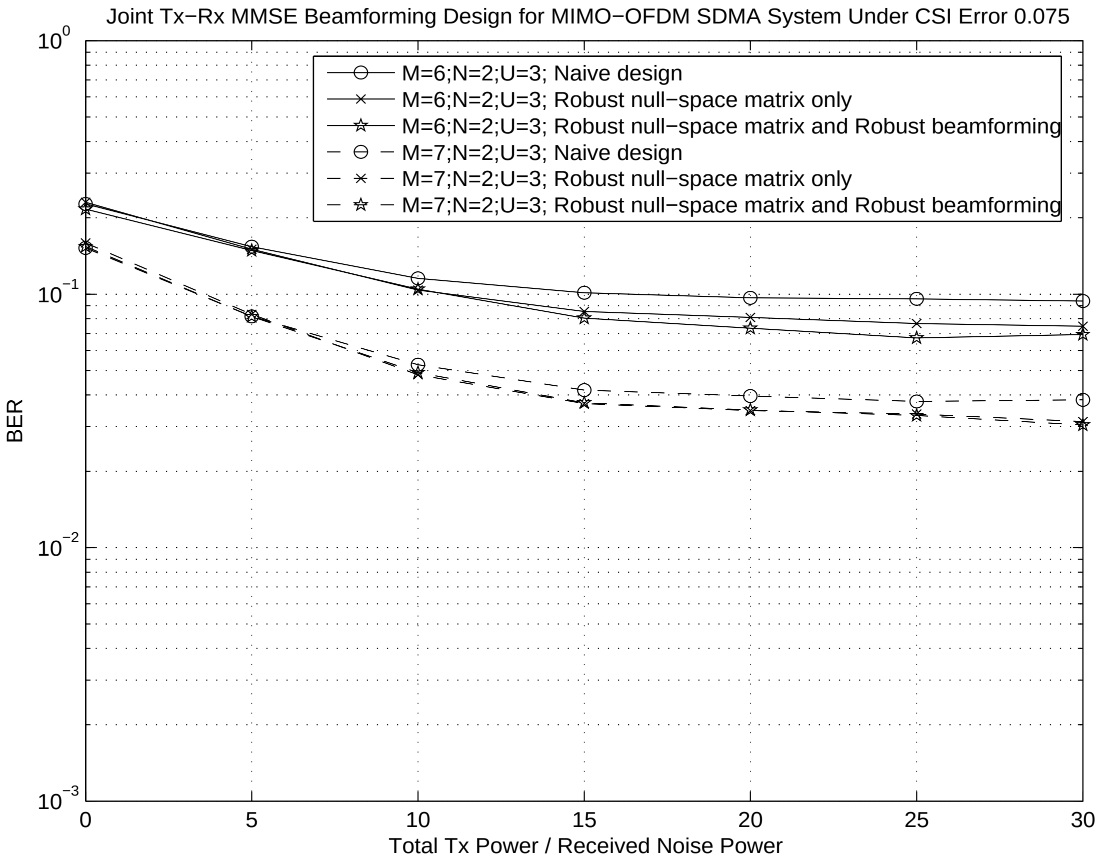


Figure 4-3 Joint Tx/Rx MMSE beamforming design for MIMO-OFDM SDMA system under CSI error 0.075

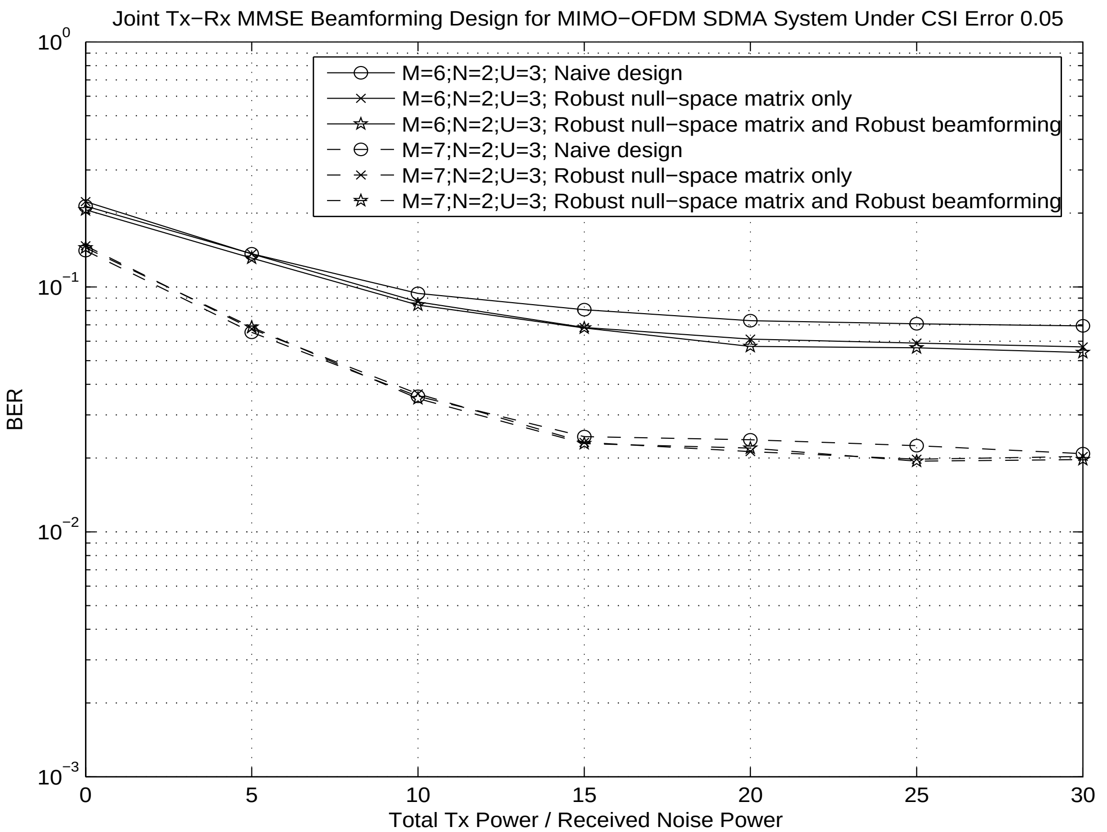


Figure 4-4 Joint Tx/Rx MMSE beamforming design for MIMO-OFDM SDMA system under CSI error 0.05

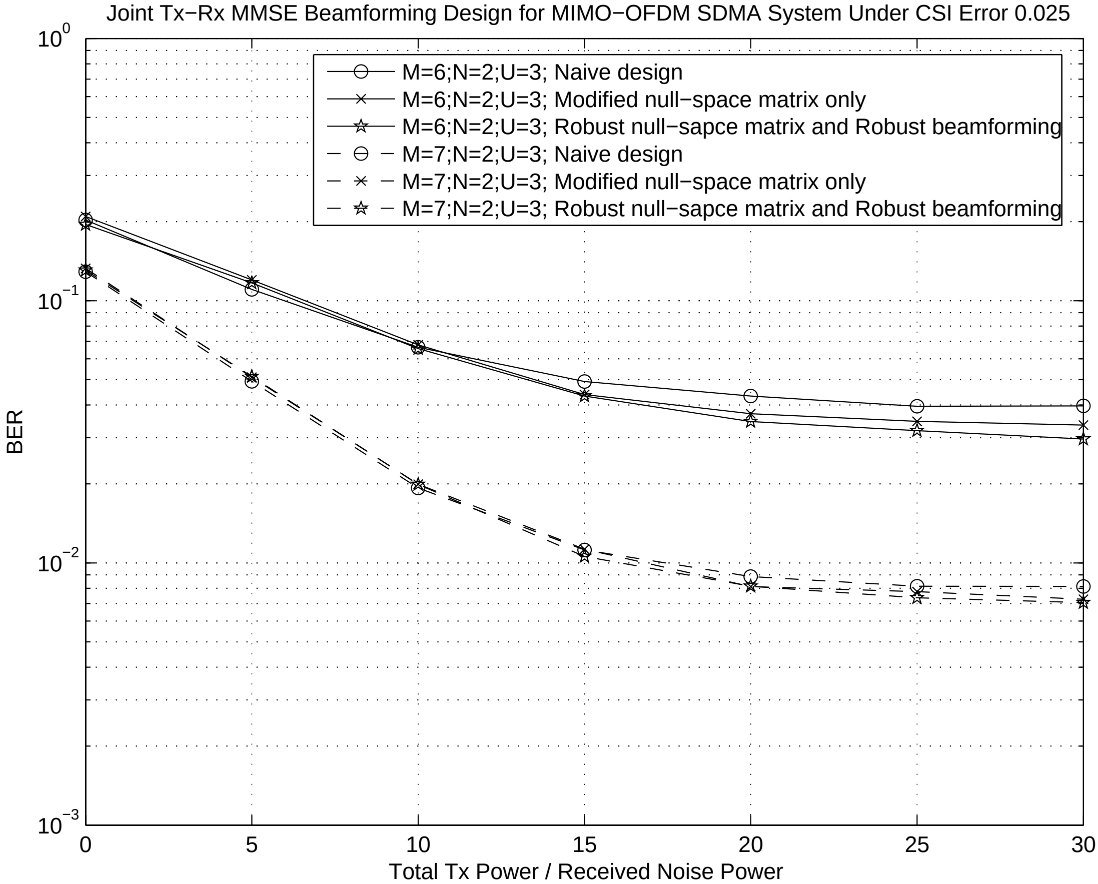


Figure 4-5 Joint Tx/Rx MMSE beamforming design for MIMO-OFDM SDMA system under CSI error 0.025

Figure 4-3, Figure 4-4 and Figure 4-5 are the simulation results of joint Tx/Rx beamforming design for multi-user MIMO-ODFM SDMA downlink system when the variances of channel estimation error are 0.075, 0.05 and 0.025 respectively. Comparing these simulation results, we give the following comments:

- The naive design of course has worst performance due to the imperfect design of null-space matrix and beamforming which induces the MUI and inter-stream interference respectively. However, using the robust null-space matrix to resist the MUI, we can obtain the performance improvement.
- Furthermore, combining the robust null-space matrix with the robust Tx/Rx beamforming to simultaneously resist the MUI and inter-stream interference,

we certainly obtain a better performance than above two designs.

- When the variance of channel estimation error increases, the performance becomes worse. The improvement of the combined robust approach is observable.
- The effect of improvement of the robust null-space matrix is larger than the robust beamforming design since the MUI will cause the more performance loss than the inter-stream interference.
- For the under-loaded case, we can see the performance improvement caused by the increase of diversity gain.

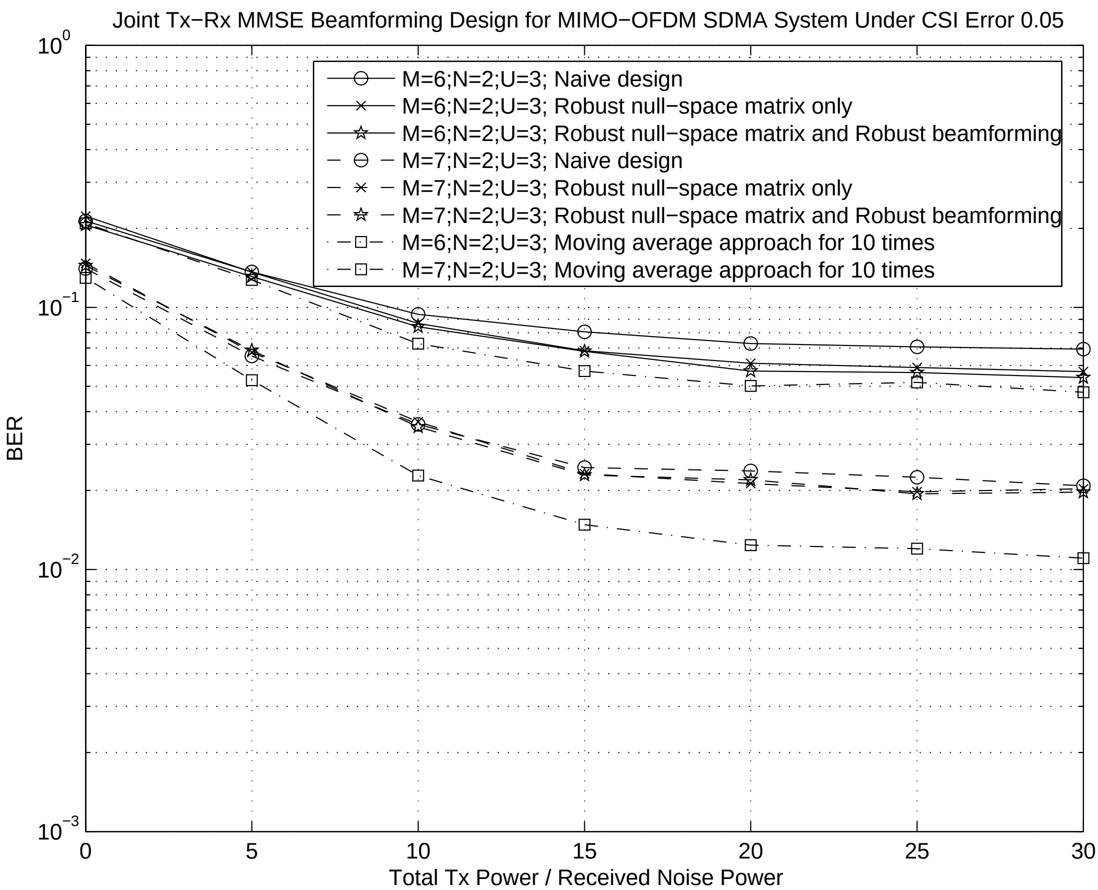


Figure 4-6 Joint Tx/Rx MMSE beamforming design for MIMO-OFDM SDMA system under CSI error 0.05

In the sequel, we are going to apply the moving average approach to the slow-fading environment. Total 100 MIMO channel realizations are simulated and, we apply the moving average approach 10 times to each channel realization. We can see that the moving average approach is superior to the original robust methods in Figure 4-6. But this approach is only suitable for the slow-fading channel. That means only the slight Doppler Effect caused by slow-moving user terminals exists.

We also consider the single user case (Figure 4.7, 4.8 and 4.9), where the null-space matrix is an identity matrix and only the robust beamforming matrix is used.

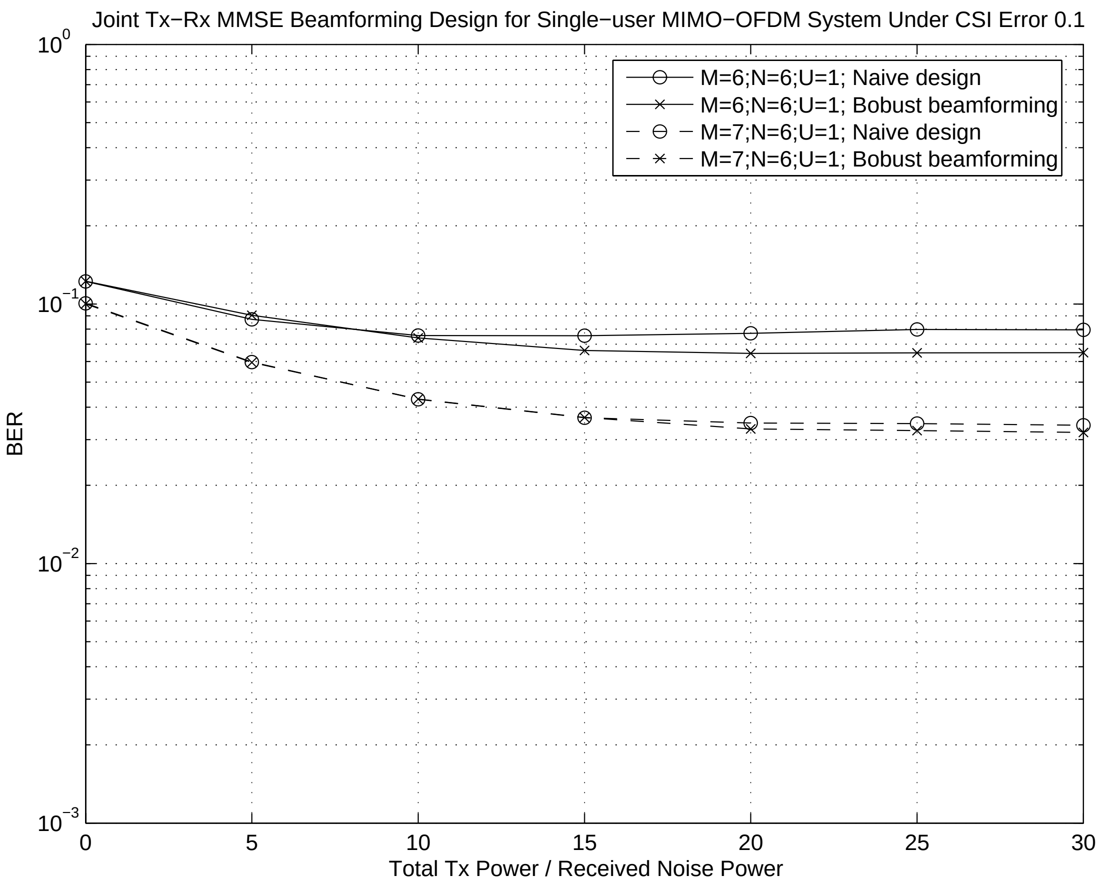


Figure 4-7 Joint Tx/Rx MMSE beamforming design for single user MIMO-OFDM system under CSI error 0.1

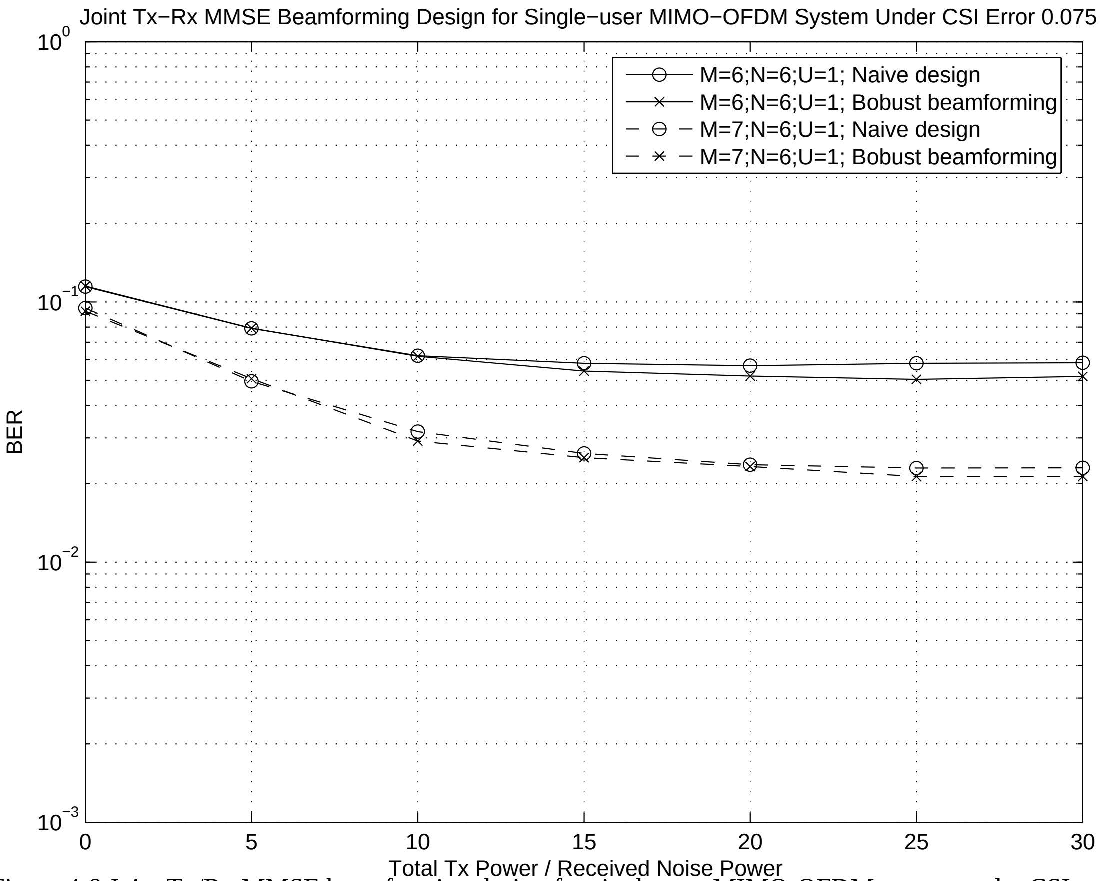


Figure 4-8 Joint Tx/Rx MMSE beamforming design for single user MIMO-OFDM system under CSI error 0.075

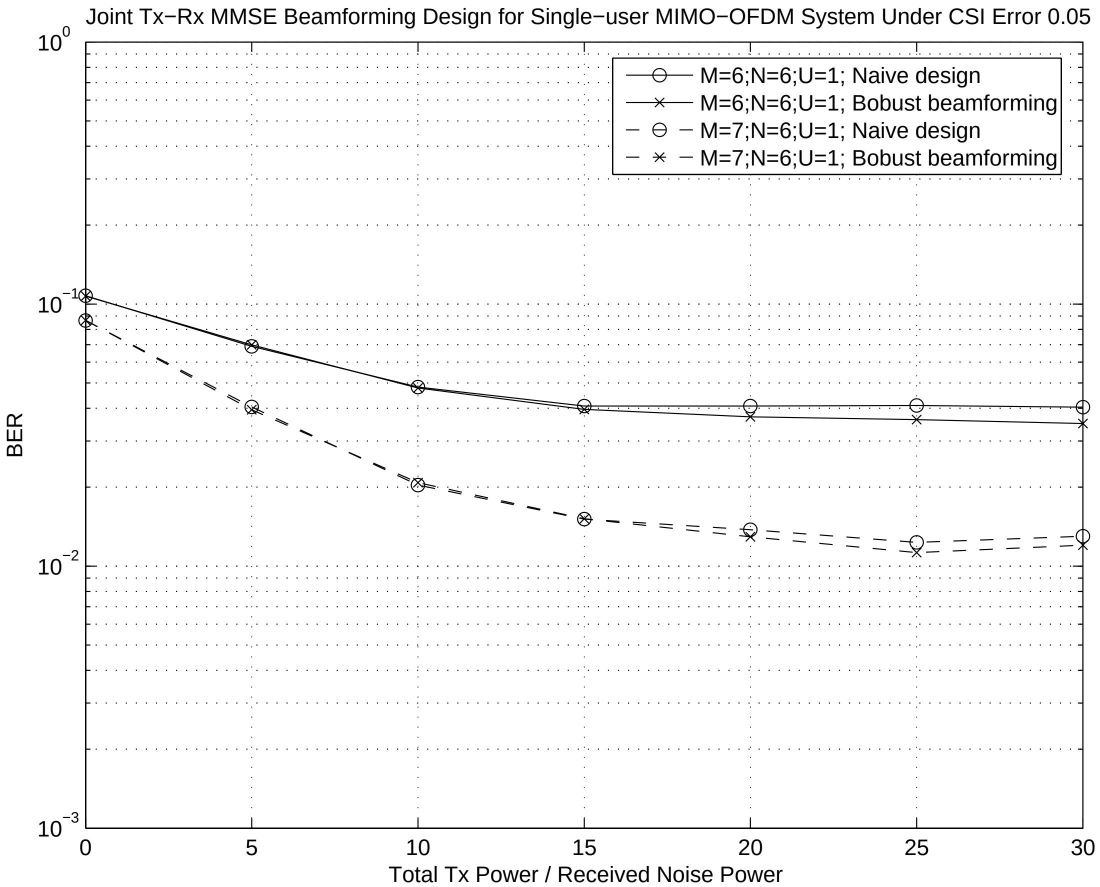


Figure 4-9 Joint Tx/Rx MMSE beamforming design for single user MIMO-OFDM system under CSI error 0.05

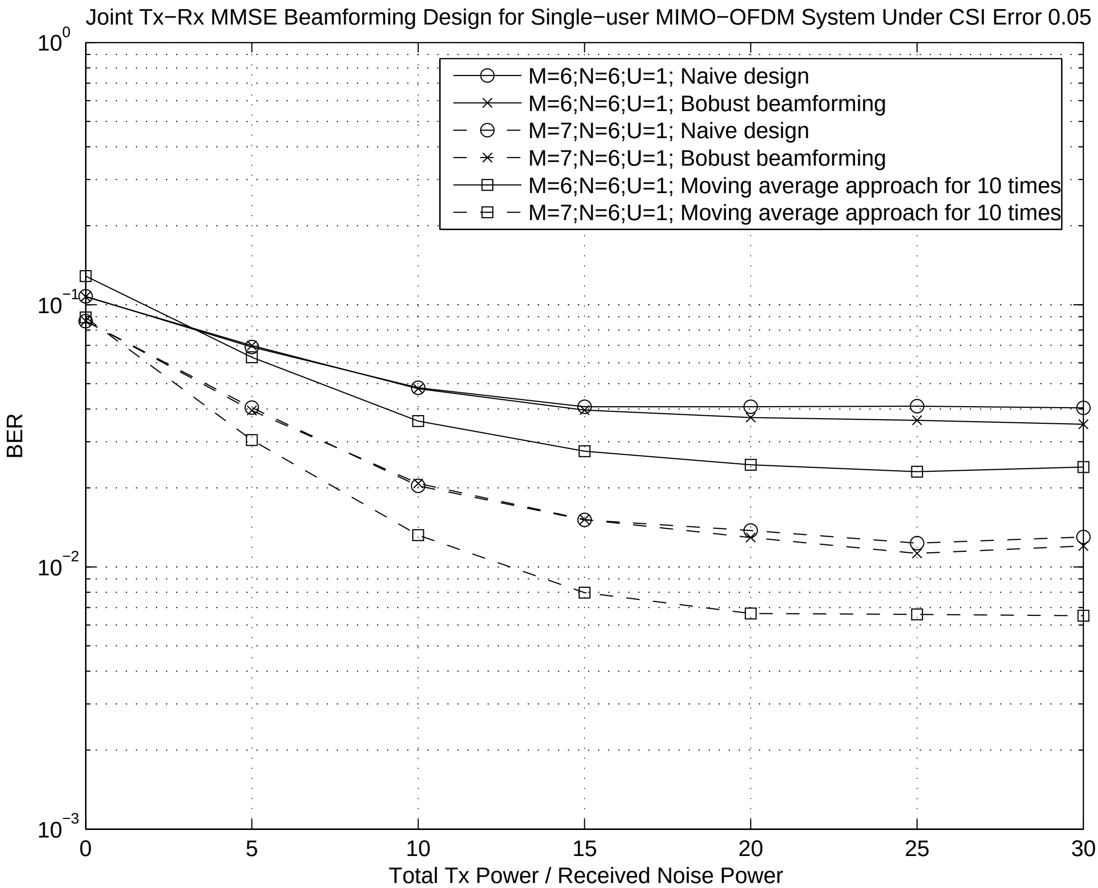


Figure 4-10 Joint Tx/Rx MMSE beamforming design for single user MIMO-OFDM system under CSI error 0.05

Figure 4-10 shows that the moving average approach enhances the performance when the channel error variance is 0.05. We can see that the moving average approach is also superior to the original robust methods. However this approach is only suitable for the slow-fading channel.

4.4 Conclusions

In this chapter, we address the problem of joint Tx/Rx beamforming design for multi-user MIMO-OFDM SDMA downlink system with channel estimation errors. Imperfect CSI will cause significant performance degradation. The performance degradation is caused by the erroneous null-space matrix and Tx/Rx beamforming which will induce the MUI among users and the inter-stream interference of each user respectively. To handle these two kinds of interference, we combine two robust approaches to improve the performance. The combination of these two robust methods utilizes the instantaneous estimated channel information. The similar performance can be obtained under both the fast time-variant and the slow time-variant environments.

Under the slow time-variant environment, we also apply the moving average method to calculus the null-space matrix and transmit and receive beamforming. Using the average approach, we obtain a better performance than the method using the instantaneous channel estimate under slow time-variant environment. But the performance improvement is not obvious for the fast time-variant environment.

Chapter 5

Conclusions and Perspective

In this thesis, we have considered the MIMO wireless communication systems which adopt multiple antennas at both the transmit and the receive sides. Such MIMO systems are thought as the promising structure to provide a significant capacity. Furthermore, the OFDM is a popular technique for achieving high data rate, spectral efficiency, and combating multi-path fading effects in wireless communications. Thus, the MIMO-OFDM based systems become a trend for future wireless communications. All joint design problems discussed in this thesis were based on the MIMO-OFDM system.

5.1 Conclusions

We have clearly introduced the conventional joint Tx/Rx MMSE beamforming design for single user MIMO-OFDM system in chapter 2 and then extend it to multi-user MIMO-OFDM SDMA system in chapter 3. Both of them assume that exact channel information is known at transmitter and receiver. In this thesis, we furthermore consider the practical case in which the channel estimate contains errors may cause significant performance loss.

For a multi-user joint beamforming design system, the performance loss is caused by the multi-user interference induced by the erroneous null-space matrix and the inter-stream interference caused by imperfect CSI. Consequently, we address the problem of designing joint Tx/Rx beamforming for multi-user MIMO-OFDM SDMA downlink system with imperfect channel information at both terminals.

We then combined two robust approaches to suppress these two interferences and obtained performance improvement. The original robust design utilizes the instantaneous estimated channel. It can be applied to fast time-variant and slow time-variant environments and has similar performances. Furthermore, we apply the moving average approach to the slow fading channel environment and obtain performance improvement.

5.2 Perspective

Future wireless applications are requested to provide higher data rate and good link quality wireless access. Owing to the advantages of MIMO and OFDM techniques, some standards such as 802.11n and 802.16e adopt MIMO-OFDM based system to increase capacity, reliability and range etc. Space-time coding and spatial multiplexing are promising techniques for achieving high data rates over MIMO systems. However, the joint Tx/Rx beamforming design scheme can also applied to some applications where the CSI can be available at both transmitter and receiver. This scheme makes that the number of antennas, size of the coding block, and transmit power can be scalable and the solutions are shown to convert the mutually cross-coupled MIMO transmission system into a set of parallel eigen subchannels system. Thus, this scheme may also become a popular solution for future wireless communications.

References

[1] H. Bolcskei and A. J. Paulraj, "Multiple-input multiple-output (MIMO) wireless systems," in The Communications Handbook, J. Gibson, Ed., pp. 90.1–90.14. CRC Press, 2nd edition, 2002.

[2] A. J. Paulraj, D. A. Gore, R. U. Nabar and H. Bölcskei, "An overview of MIMO communications– A key to Gigabit wireless," IEEE Proc., vol. 92, no. 2, pp. 198- 218, Feb. 2004.

[3] S. D. Shan, "Wireless communication using dual antenna arrays," Kluwer Academic, 1999.

[4] J. Terry and J. Heiskala, OFDM wireless LANs: A theoretical and practical guide, Indiana: SAMS, 2001.

[5] L. Giangaspero, L. Agarossi, G. Paltenghi, S. Okamura, M. Okada, S. Komaki, "Co-channel Interference Cancellation Based on MIMO OFDM Systems"

[6] D. P. Palomar, M. A. Lagunas, "A Unified Framework for Communications through MIMO Channels"

[7] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE JSAC, vol. 16, no. 8, pp. 1451- 1458, Oct. 1998.

[8] V. Tarokh, N. Seshdri and A. R. Calderbank, “Space- time codes for high data rate wireless communication: Performance analysis and code construction,” IEEE Trans. Inform. Theory, vol. 44, no. 2, pp. 744- 765, Mar. 1998.

[9] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multiple antennas,” Bell Labs Syst. Tech. J., vol. 1, pp. 41-59, Autumn 1996.

[10] G. G. Raleigh and J.M. Cioffi, “Spatial-temporal coding for wireless communications”, IEEE Transactions on Communications, Vol. 46, No. 3, March 98

[11] H. Sampath and A. Paulraj, ” Joint TX & RX Optimization for High Data Rate Wireless Communication Using Multiple Antennas,” Proceedings of Asilomar 1999, Monterey, California.

[12] A. Scaglione, P. Stoica, S.Barbarossa, G. B. Giannakis, H. Sampath “Optimal Designs for Space-Time Linear Precoders and Decoders”

[13] H. Sampath, P. Stoica and A. Paulraj, “Generalized Linear Precoder and Decoder Design for MIMO Channels Using the Weighted MMSE Criterion,” IEEE Transactions on Communications, Vol. 49, No. 12, December 2001

[14] D. P. Palomar, J. M. Cioffi, M. A. Lagunas “Joint Tx-Rx Beamforming Design for a Unified Framework for Convex Optimization”

[15] P. Vandenameele, L. V. D. Perre, M. G. E. Engels, B. Gyselinckx, H. J. D. Man, "A Combined OFDM/SDMA Approach"

[16] M. J. Syed, V. Meghdadi, G. Ferre, J. P. Cances, J. M. Dumas, "Multi-user Detection in OFDM Space Time Block Code for High Rate Uplink Application"

[17] M. Rim "Multi-user Downlink Beamforming with Multiple Transmit and Receive Antennas"

[18] A. Bourdoux, N. Khaled, "Joint Tx-Rx Optimization for MIMO-SDMA Based on a Null-space Constraint"

[19] K. Lee, J. Chun, "On the Beamforming Weight of the MIMO/SDMA System under Channel Uncertainty"