國 立 交 通 大 學

電信工程學系

碩 士 論 文

正交分頻多工系統之
通道估計與 I/Q 失衡補償

Joint Maximum Likelihood Estimation of Channel and I/Q Imbalance for OFDM Systems

研 究 生：許宸睿

指導教授：沈文和 博士

中 華 民 國 九 十 五 年 七 月

正交分頻多工系統之

通道估計與 I/Q 失衡補償

Joint Maximum Likelihood Estimation of

Channel and I/Q Imbalance for OFDM Systems

研 究 生：許宸睿　　Student：Chen-Jui Hsu

指導教授：沈文和 博士　　Advisor：Dr. Wern-Ho Sheen

國 立 交 通 大 學

電 信 工 程 學 系 碩 士 班

碩 士 論 文

A Thesis

Submitted to Institute of Communication Engineering

College of Electrical Engineering and Computer Science

National Chiao Tung University

in Partial Fulfillment of the Requirements

for the Degree of

Master of Science

in

Communication Engineering

July 2006

Hsinchu, Taiwan, Republic of China.

中 華 民 國 九 十 五 年 七 月

正交分頻多工系統之

通道估計與 I/Q 失衡補償

研究生：許宸睿　　　　　　　　　　　指導教授：沈文和 博士

國立交通大學

電信工程學系碩士班

摘要

現今許多的應用需要具備高傳輸速率的無線通訊系統，而正交分頻多工(OFDM)系統正是滿足此特性的主流技術之一。在接收機架構方面，無論是在業界或是學術界，直接轉換接收機(Direct Conversion Receiver)皆被廣泛地採用，因為其具備有小體積，低成本和高整合度等優勢。然而，此架構因為射頻元件不完美的緣故，無可避免地會產生一些射頻損傷，像是直流偏移(DC Offset)、頻率偏移(Frequency Offset)和 I/Q 失衡(I/Q Imbalance)等。此論文中將對正交分頻多工系統同時考慮通道估測以及 I/Q 失衡估測補償兩項議題，並在假設頻率偏移可被預先被估測出的情況下結合頻率偏移補償。I/Q 失衡有兩種類型，第一種為不隨頻率變動，第二種為隨頻率變動，兩者皆在本研究範圍之內。不同於傳統方法， 我們的可能性函數(Likelihood Function)是由頻率域上所載的接收資料所組成，利用最大可能性(Maximum Likelihood)來估測 I/Q 失衡及通道，這個方法能使我們利用到正交分頻多工系統之特性及不同載波間的高相關性優點。論文內容也包含了最大可能性估計的效能分析與電腦模擬的結果，以驗證演算法的正確性和可行性，在訊號雜訊能量比(SNR)高到一定程度下其效能可達到 Cramer-Rao 下限(Lower Bounds)。

Joint Maximum Likelihood Estimation of Channel and I/Q Imbalance for OFDM Systems

Student: Chen-Jui Hsu Advisor: Dr. Wern-Ho Sheen

Department of Communication Engineering

National Chiao Tung University

Abstract

High data rate wireless communication is required by a variety of applications, and OFDM is one of the promising technologies too meet such a demand. On the other hand, direct-conversion RF receiver (DCR) architecture has become a trend in industry and academic world nowadays because it is small, cheap and low power-consuming. This kind of architecture, however, accompanies with some radio frequency (RF) imperfections such as the direct current (DC) offset, frequency offset, in-phase/quadrature (I/Q) imbalance etc. In this thesis, for OFDM system, we propose an algorithm for joint estimation of delay-spreading channel and IQ imbalance effect combined with frequency offset compensation assuming that frequency offset has been prior estimated. Both of frequency-independent and frequency-dependent IQ imbalance are covered. Different from traditional approach, our likelihood function is constructed by using frequency-domain data to estimate time-domain channel response and IQ imbalance based on ML criterion. This approach enables us to take advantage of characteristics of OFDM and smoothing property. Simulation results show that the estimated values approach their CRLBs once SNR is above certain level

誌　謝

本篇論文得以順利完成，要特別感謝三個人，我的指導教授 沈文和博士、徐進發學長以及程瑞曦博士的不斷指教。本篇論文是徐進發學長先前未完成研究的工作及延伸，在此特別感激他找到這麼好的研究題目並盡其所能的傳承給我，包括了指導我對其中不解的疑惑問題讓我能很快投入其中步上此研究軌道、並把實驗所需之模擬程式一一介紹給我讓我能進入狀況，最後並把此論文需要繼續研究或加以改善的結果指示給我讓我能繼續往此方向鑽研其中並做出不錯的結果。

在兩年研究過程中，指導教授沈老師給予非常多的指導與建議，讓我對通訊系統有更深一層的思考與認識，尤其對研究謹慎小心的態度，讓我獲益匪淺。而在本篇論文研究中，沈老師及程瑞曦教授給了我很大的指引方向，教我如何把一個研究做完善的思考及呈現，鼓勵我不斷突破才得以做出更佳的研究成果，從中也學到了許多自己不足的專業知識觀念。另外，要感謝口試委員 資通所經理賴癸江的指正與建議，使我的碩士論文更加完善。

最後感謝實驗室全體同仁大家互相幫忙照顧，讓我能順利完成碩士研究、以及郭志成學長的不吝指教，很有耐心的為我解答不解的問題。也感謝家人給我的照顧與鼓勵，讓我無後顧之憂的進行碩士班研讀及研究，再次甚重感謝大家。

民國九十五年八月

研究生許宸睿謹識於交通大學

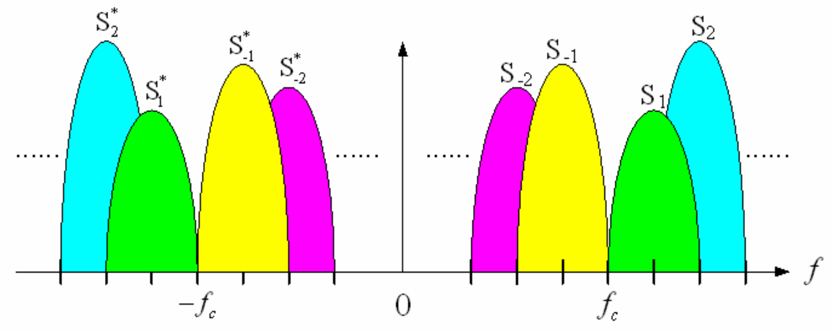
Contents

List of Figures

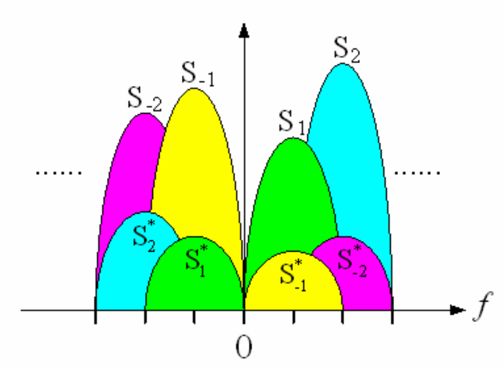
Chapter 1 Introduction

Demands for high data rate wireless communication services and cost-effective devices are getting stronger for past years. In response, OFDM has been adopted in several communication standards due to its higher spectral efficiency and effectiveness in dealing with multi-path delay spreading [1-3]. To combat multi-path delay spreading, a cyclic-prefix is inserted in front of each OFDM symbols so that OFDM symbols can be compensated in frequency domain using one-tap equalizer [4-6]. On the other hand, to reduce device cost, RF transceivers adopt direct-conversion architecture have been in favor due to fewer external components required in this approach [7-8]. However, direct-conversion architecture can easily introduce IQ imbalance effect.

Typically, two types of IQ imbalance exist. For one thing, the imperfect local oscillator (LO) results in frequency-independent I/Q imbalance. The complex carrier generated by imperfect LO is not a true quadrature signal, and the amplitudes of carrier signal for I/Q branches are not equal in practice. For another, I/Q imbalance results from the unavoidable mismatch among all the analog elements on I/Q branches. The signals of I and Q branches are processed by individual analog devices, such as amplifiers, low-pass filter (LPF), A/D converters, and etc. Nevertheless, it is difficult to produce two analog devices with exactly identical responses or properties, especially when the bandwidth (BW) of the system is large, e.g. 40 MHz mode of IEEE 802.11n. This kind of I/Q imbalance is frequency-dependent; in other words, the I/Q imbalance effects tend to vary with frequency. The frequency-independent and frequency-dependent I/Q imbalance have the received signal interfered by its image signal as Fig.1-1 shows. Distorted OFDM symbols can result in higher error rate, if no effective IQ imbalance compensations are applied [9-11].



(a) Received signal in passband



(b) Received signal which is interfered by image signal in baseband

Fig.1-1 Spectra of the received signal in DCR with I/Q imbalance

Both of channel equalization and IQ compensation requires channel and IQ imbalance estimation beforehand. Previous research has offered several methods for these two estimations [12-15]. However, most of them treated these two estimations separately. Consequently, estimators they obtained are sub-optimal only. Recently researchers started to consider joint estimation for these two effects [16-18]. *Gil et al* [16] started to develop joint ML estimation of carrier offset, channel, IQ imbalance and DC offset using time-domain data. Their method did not take characteristics of OFDM into consideration and focus on frequency-independent IQ imbalance only. *Tarighat et al* [17] considered their joint estimation of IQ imbalance and channel specifically for OFDM systems. Their estimation is performed completely in frequency-domain and treats all carriers independently. As a result, it requires several training symbols to converge the estimation in their approach. Also, their proposed method in frequency domain can not deal with frequency offset which may arising inter carrier interferences (ICI). *Xing et al* [18] considered joint effect of frequency offset, IQ imbalance (both types of IQ imbalance) in OFDM system. However, their method requests special design of training symbols to achieve expected performance.

We develop joint channel and IQ imbalance estimation using ML criterion. The IQ imbalance estimation covers both of frequency-independent and dependent cases. The likelihood function is constructed using frequency-domain data to estimate time-domain channel effect and IQ imbalance. This approach enables us to make use of characteristics of OFDM as well as correlation between subcarriers. We also consider carrier frequency offset (CFO) effect and propose a compensation method before our joint estimation of I/Q imbalance and channel, but we assume that CFO has been previously estimated within acceptable level. This assumption can be justified as seeing most communication standards using different types of training symbols for estimation.

This paper is organized as follows. Chapter 2 describes the model used for IQ imbalance and formulates its equivalent effect both in time domain and frequency domain to develop I/Q imbalance compensation scheme. In chapter 3, algorithms using ML estimation are derived, as well as take into account frequency offset compensation before applying the joint estimation. Performance of proposed joint estimators are analyzed in chapter 4, and the Cramer-Rao Lower Bounds (CRLBs) of the estimators are derived in chapter 5. Computer simulation based on IEEE 802.11a as well as results compared with analysis and CRLBs are discussed in chapter 6. Finally, Chapter 7 concludes the thesis.

Chapter 2 System Model

2.1 Time Domain I/Q Imbalance Model

Fig.2-1 shows a typical architecture of a direct-conversion receiver, and its mathematical model with I/Q imbalance effect is represented in Fig.2-2. The I/Q imbalance due to imperfect LO is frequency-independent. After down conversion, the I and Q branch signals are generated by individual analog components, such as local oscillator (LO), amplifiers, low pass filters (LPF) and A/D converters which in general cause the frequency dependent I/Q imbalance. We use $h_I[n]$ and $h_Q[n]$ model the cascade effects of analog branch component, and frequency-dependent IQ imbalance occurs once $h_I[n]$ and $h_Q[n]$ show different responses.

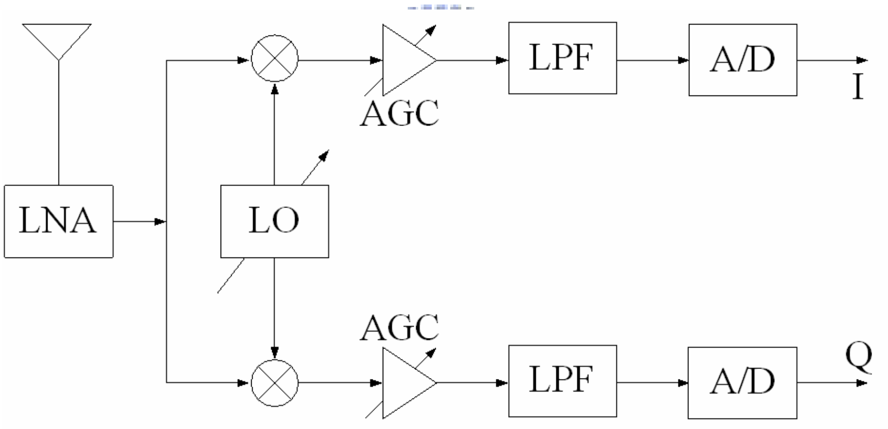


Fig.2-1 General architecture of a direct-conversion receiver.

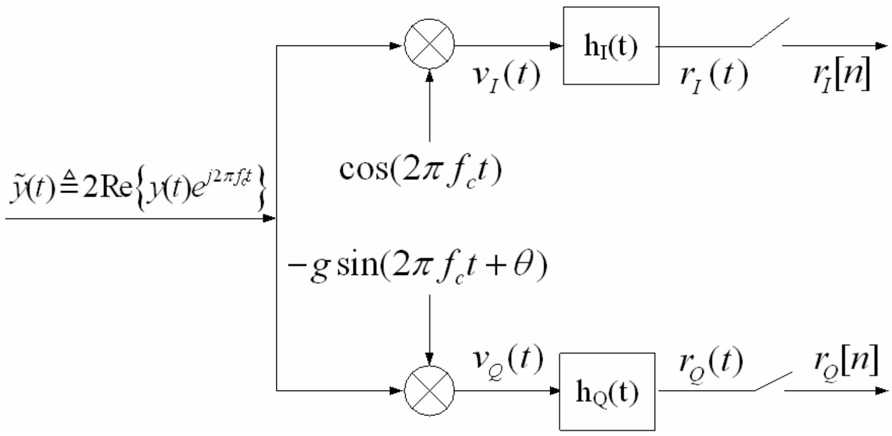


Fig.2-2 Mathematical model of a direct-conversion receiver with I/Q imbalance.

The frequency-independent I/Q imbalance caused by imperfect LO is expressed as $C_{LO}(t)=\cos(2\pi f_c t)-jg\sin(2\pi f_c t+\theta)$, where g and θ denote amplitude and phase imbalance. After some arrangement, we could represent it in a complex form. If the LO is perfect, g , θ and φ_0 should be zero, and γ_0 should be equal to one.

$$C_{LO}(t)=\cos(2\pi f_c t)-jg\sin(2\pi f_c t+\theta) \tag{2.1}$$

$$=\gamma_0 e^{-j2\pi f_c t}+\varphi_0 e^{j2\pi f_c t} \tag{2.2}$$

where $\gamma_0 \triangleq \frac{1}{2}\left(1+ge^{-j\theta}\right)$ *and* $\varphi_0 \triangleq \frac{1}{2}\left(1-ge^{j\theta}\right)$ $\left(\; P.S. \quad |\gamma_0|\approx 1 \text{ and } |\varphi_0|\approx 0 \; \right)$

$$\left(\begin{array}{l} proof: \\ \cos(2\pi f_c t)-jg\sin(2\pi f_c t+\theta)=\frac{1}{2}\left(e^{-j2\pi f_c t}+e^{j2\pi f_c t}\right)+\frac{1}{2}g\left(e^{-j(2\pi f_c t+\theta)}-e^{j(2\pi f_c t+\theta)}\right) \\ \qquad\qquad =\frac{1}{2}\left(1+ge^{-j\theta}\right)e^{-j2\pi f_c t}+\frac{1}{2}\left(1-ge^{j\theta}\right)e^{j2\pi f_c t} \\ \qquad\qquad =\gamma_0 e^{-j2\pi f_c t}+\varphi_0 e^{j2\pi f_c t} \end{array}\right)$$

We define the received signal as

$$\tilde{y}(t)\triangleq 2\,\mathrm{Re}\left\{y(t)e^{j2\pi f_c t}\right\}=y(t)e^{j2\pi f_c t}+y^*(t)e^{-j2\pi f_c t} \tag{2.3}$$

$y(t)$ = $s(t)\otimes h_{CH}(t)+n_0(t)$, $\otimes$ denotes convolution

$y(t), s(t), h_{CH}(t)$ is the baseband representation of the received signal, the transmitted signal, and the channel impulse response, and $n_0(t)$ is an AWGN.

The received signal $\tilde{y}(t)$ is multiplied by the output of LO $C_{LO}(t)$, which is not orthogonal perfectly.

$$\begin{aligned} v(t)&=\tilde{y}(t)\times C_{LO}(t) \\ &=\left[y(t)e^{j2\pi f_c t}+y^*(t)e^{-j2\pi f_c t}\right]\times\left[\gamma_0 e^{-j2\pi f_c t}+\varphi_0 e^{j2\pi f_c t}\right] \\ &=\gamma_0 y(t)+\varphi_0 y^*(t)+\underbrace{\gamma_0 y^*(t)e^{-j4\pi f_c t}+\varphi_0 y(t)e^{j4\pi f_c t}}_{\text{will be removed by the following LPFs}} \end{aligned} \tag{2.4}$$

$$= \gamma_0 y(t) + \varphi_0 y^*(t) \tag{2.5}$$

If the last two terms in (2.4) will be completely removed by the following LPF, we would find out that the desired signal is attenuated by a factor γ_0 and interfered by its image signal. This kind of I/Q imbalance effect due to the imperfect LO introduce the two factors γ_0 and φ_0 constant over all frequencies. Thus, that is why it is called frequency-independent I/Q imbalance.

Now, we rewrite (2.5) into (2.6) to express I branch and Q branch signal of $v(t)$ more clearly.

$$
\begin{aligned}
v(t) &\triangleq v_I(t) + j \cdot v_Q(t) \\
&= \gamma_0 y(t) + \varphi_0 y^*(t) \\
&= \frac{1}{2}\left(1 + g e^{-j\theta}\right) y(t) + \frac{1}{2}\left(1 - g e^{j\theta}\right) y^*(t) \\
&= \frac{1}{2}\left[y(t) + y^*(t)\right] + \frac{1}{2} g \left[e^{-j\theta} y(t) - e^{j\theta} y^*(t)\right] \\
\Rightarrow v_I(t) &= \frac{1}{2}\left[y(t) + y^*(t)\right],\ \ j v_Q(t) = \frac{1}{2} g \left[e^{-j\theta} y(t) - e^{j\theta} y^*(t)\right]
\end{aligned}
\tag{2.6}
$$

To analyze the effect of branch mismatches, the I part and Q part of signal $r(t)$ are expressed as following :

$$
\begin{aligned}
r(t) &= r_I(t) + j r_Q(t) \\
&= v_I(t) \otimes h_I(t) + j v_Q(t) \otimes h_Q(t) \\
&= \frac{1}{2}\left[y(t) + y^*(t)\right] \otimes h_I(t) + \frac{1}{2} g \left[e^{-j\theta} y(t) - e^{j\theta} y^*(t)\right] \otimes h_Q(t) \\
&= y(t) \otimes \underbrace{\frac{1}{2}\left[h_I(t) + h_Q(t) g e^{-j\theta}\right]}_{\triangleq h_+(t)} + y^*(t) \otimes \underbrace{\frac{1}{2}\left[h_I(t) - h_Q(t) g e^{j\theta}\right]}_{\triangleq h_-(t)} \\
&= y(t) \otimes h_+(t) + y^*(t) \otimes h_-(t) \\
&= \left[s(t) \otimes h_{CH}(t) + n_0(t)\right] \otimes h_+(t) + \left[s(t) \otimes h_{CH}(t) + n_0(t)\right]^* \otimes h_-(t)
\end{aligned}
$$

After $r(t)$ is digitized at a rate that satisfies the Nyquist sampling theorem, the resulting baseband discrete-time signal model can be represented as

$$r[n]=\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)\otimes h_+[n]+\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)^*\otimes h_-[n] \tag{2.7}$$

where
$$\begin{aligned} h_+[n] &\triangleq \frac{1}{2}\left[h_I[n]+h_Q[n]ge^{-j\theta}\right], \\ h_-[n] &\triangleq \frac{1}{2}\left[h_I[n]-h_Q[n]ge^{j\theta}\right] \end{aligned} \tag{2.8}$$

Equation (2.7) says the joint frequency-independent and frequency-dependent I/Q imbalance effect equivalent to introducing a conjugate interference in time-domain and an interfering image signal in frequency-domain. Furthermore, observing the interference power due to $h_-[n]$ and attenuation due to $h_+[n]$, the frequency-dependent I/Q imbalance is indeed caused by the degree of different response effect between $h_I[n]$ and $h_Q[n]$ in time domain.

2.2 Proposed I/Q Imbalance Compensation Scheme

The received analog baseband OFDM signal $r(t)$ after digitalized and removed CP (Cyclic Prefix) can be expressed as Equation (2.7). Here we use small letters to denote time-domain signal and system response, their corresponding large letters to denote their representation in frequency domain.

Based on equation (2.7), we can remove conjugate interference by introducing $\varphi[n]$ and applying conjugate cancellation (see Equation (2.9)). After conjugate cancellation, the remaining term of Equation (2.10) can be modeled as convolution between signal and overall channel effect plus noise.

$$\begin{aligned} r[n]-\varphi[n]\otimes r^*[n] &= \left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)\otimes\left(s[n]\otimes h_{CH}[n]+n_0[n]\right) \\ &+\underbrace{\left(h_-[n]-\varphi[n]\otimes h_+^*[n]\right)}_{\substack{\text{This term must be 0 to} \\ \text{cancel the conjugate interference}}}\otimes\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)^* \end{aligned} \tag{2.9}$$

$$\Rightarrow r[n]-\varphi[n]\otimes r^*[n]=\left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)\otimes\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)$$

$$=s[n]\otimes h[n]+n[n] \tag{2.10}$$

where $h[n]=\left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)\otimes h_{CH}[n]$ overall channel effect ,

$\mathrm{n}[n]=\left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)\otimes n_0[n]$

and $h_-[n]-\varphi[n]\otimes h_+^*[n]=0$

$$\Rightarrow \varphi[n]=\left(h_+^*[n]\right)^{-1}\otimes h_-[n] \tag{2.11}$$

We suppose that the effective length of $\varphi[n]$ plus effective channel length after I/Q imbalance effect (see (2.10), i.e. $\left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)\otimes h_{CH}[n]$) is smaller than GI (guard interval) to maintain the orthogonality of OFDM symbol. Then, we can take FFT (Fast Fourier Transform) on Equation (2.10) and (2.11) and assume total l subcarriers per OFDM symbol. We can obtain $k'th$ subcarrier :

$$R[k]-\Phi[k]R^*[l-k]=S[k]H[k]+N[k] \quad \text{for } k=1\sim l-1 \tag{2.12}$$

We discard the DC tone $k=0$ in our algorithm analysis since it does not carry any information due to implementation issues, such as in 802.11a standardized OFDM systems.

Note that from (2.8)

$$h_+[n] \triangleq \frac{1}{2}\left[h_I[n]+h_Q[n]ge^{-j\theta}\right] \;\Rightarrow\; FFT\left(h_+[n]\right)=\frac{1}{2}\left(H_I[k]+ge^{j\theta}H_Q[k]\right)$$

$$h_-[n] \triangleq \frac{1}{2}\left[h_I[n]-h_Q[n]ge^{j\theta}\right] \;\Rightarrow\; FFT\left(h_-[n]\right)=\frac{1}{2}\left(H_I[k]-ge^{j\theta}H_Q[k]\right)$$

$$\Phi[k]= FFT\left(\varphi[n]\right)=FFT\left(\left(h_+^*[n]\right)^{-1}\otimes h_-[n]\right)$$

$$
= \frac{H_{-}[k]}{H_{+}^{*}[N-k]} = \frac{\frac{1}{2}\left(H_I[k] - ge^{j\theta}H_Q[k]\right)}{\frac{1}{2}\left(H_I[N-k] + ge^{-j\theta}H_Q[N-k]\right)^{*}}
$$

$$
= \frac{\Psi_0[k]}{\Gamma_0^{*}[N-k]} \tag{2.13}
$$

where $\Psi_0[k] = \frac{1}{2}\left(1 - ge^{j\theta}\frac{H_Q[k]}{H_I[k]}\right)$, $\Gamma_0[k] = \frac{1}{2}\left(1 + ge^{-j\theta}\frac{H_Q[k]}{H_I[k]}\right)$

Note that $\Psi_0[k] \simeq 0$ and $\Gamma_0[k] \simeq 1$.

Here, we define some notation and operation for simplicity. For a vector X of size $l-1$, we write its vector form as

$X = \begin{bmatrix} X[1] \\ X[2] \\ \vdots \\ X[k] \\ \vdots \\ X[l-1] \end{bmatrix}$ and its mirror vector by the $\widehat{\ }$, $\widehat{X} = \begin{bmatrix} X[l-1] \\ X[l-2] \\ \vdots \\ X[k] \\ \vdots \\ X[1] \end{bmatrix}$

Matrix $\mathbf{D}_X$ with a vector X will denote a diagonal matrix with X on its main diagonal, and $\odot$ denotes elementwise product. Then, the subcarrier index k can be dropped and thus Equation (2.12) can be written in its equivalent vector form Equation (2.14).

$$
R - \Phi \odot \widehat{R}^{*} = R - \mathbf{D}_{\Phi}\widehat{R}^{*} = \mathbf{D}_S H + N \tag{2.14}
$$

where $\mathbf{D}_{\Phi} = \mathbf{D}_{\Psi_0}\left(\mathbf{D}_{\widehat{\Gamma}_0^{*}}\right)^{-1}$

Some characteristics of proposed compensation scheme in (2.14) can be stated as follows. The perfect compensation coefficients Φ in (2.14) can be physically viewed as being constructed from attenuation factor $\Psi_0[k]$ and $\Gamma_0[k]$ which quantifies the severity of I/Q imbalance effect on each subcarier. For more detail, the variation of Φ between subcarriers reflect on $H_Q[k] \neq H_I[k]$ caused by severity of frequency dependent I/Q imbalance effect, and the DC constant term of Φ on each subcarrier reflect on g and θ which represent the frequency independent I/Q imbalance effect. Therefore, if only frequency independent I/Q imbalance exists, Φ will be its constant term of frequency-independent contribution and reduced to single tap coefficient. Moreover, if there is no I/Q imbalance effect, i.e., $g = 1$, $\theta = 0^0$ and $H_Q[k] = H_I[k]$, then $\Psi_0[k] = 0$ and $\Gamma_0[k] = 1$, thus Φ is reduced to 0 indicating no need for I/Q imbalance compensation.

The difference between time domain compensation method (2.10) and frequency domain compensation method (2.14) can be compared as follows. For time domain compensation method, it needs convolution operation, but for frequency domain compensation, it only needs elementwise product operation. Thus, if we perform L_φ taps to compensate in time domain, the complexity of time domain method is about L_φ multiplications and $\left(L_\varphi - 1\right)$ additions per subcarrier, whereas the complexity of frequency domain method is always only 1 multiplication and 1 addition per subcarrier, which is independent of L_φ. In conclusion, the compensation complexity of frequency domain method is much lower than time domain method.

Chapter 3 Joint ML Estimation of I/Q Imbalance and Channel

In chapter 2, we have proposed an easier frequency domain method to compensate I/Q imbalance effect and demodulate signal in (2.14), once Φ and H are estimated. In this chapter, Φ and H can be estimated in an ML sense using (2.14) when the training symbols s[n] are transmitted.

We now analyze the statistical property of the noise N after compensation of the received signal in (2.14).

$$h_+[n] \triangleq \frac{1}{2}\left[h_I[n]+h_Q[n]ge^{-j\theta}\right] \Rightarrow FFT\left(h_+[n]\right)=\frac{1}{2}\left(H_I[k]+ge^{j\theta}H_Q[k]\right)=\Gamma_0[k]H_I[k]$$

$$h_-[n] \triangleq \frac{1}{2}\left[h_I[n]-h_Q[n]ge^{j\theta}\right] \Rightarrow FFT\left(h_-[n]\right)=\frac{1}{2}\left(H_I[k]-ge^{j\theta}H_Q[k]\right)=\Psi_0[k]H_I[k]$$

where $\Psi_0[k]=\frac{1}{2}\left(1-ge^{j\theta}\frac{H_Q[k]}{H_I[k]}\right)$, $\Gamma_0[k]=\frac{1}{2}\left(1+ge^{-j\theta}\frac{H_Q[k]}{H_I[k]}\right)$

Note that $n_0[n]$ is AWGN and its frequency domain N_0 is also AWGN.

Therefore, $N[k]=FFT\left\{\left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)\otimes n_0[n]\right\}$ (3.1)

$$\begin{aligned}&\simeq\left(\Gamma_0[k]-\Phi[k]\Psi_0^*[l-k]\right)H_I[k]N_0[k]\\&\simeq\left(1-\Phi[k]\Phi^*[l-k]\right)\Gamma_0[k]H_I[k]N_0[k]\end{aligned} \quad (3.2)$$

$$\Rightarrow N\simeq\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\hat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}N_0 \quad (3.3)$$

The AWGN $n_0[n]$ is not a periodic signal, thus the convolution of AWGN $n_0[n]$ with $\left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)$ in (3.1) is not equivalent to circular convolution. Then, $N[k]$ is indeed a linear combination of $N_0[k]$ to become a color noise in (3.1). However, the correlation between $N[k]$ is very low, hence can be approximated to be independent in (3.2). Therefore, vector N is approximated as equation (3.3) and its covariance matrix is also expressed as

$$\begin{aligned}\mathbf{\Sigma} &\triangleq E\left\{NN^H\right\}\\ &= \sigma_0^2\left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\mathbf{D}_{H_I}^H\mathbf{D}_{\Gamma_0}^H\left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\widehat{\Phi}}^*\right)^H\end{aligned}$$

For given Φ and H, the vector $R - \mathbf{D}_\Phi \widehat{R}^*$ in (2.14) is Gaussian with mean $\mathbf{D}_S H$, and covariance matrix $\mathbf{\Sigma}$ derived and approximated from above. Thus, the conditional probability density function for $R - \mathbf{D}_\Phi \widehat{R}^*$ is written as

$$p\left\{R - \mathbf{D}_{\widehat{R}^*}\Phi \middle| \Phi, H\right\} \simeq \frac{1}{\pi^n \det\left(\mathbf{\Sigma}\right)} \exp\left\{-\left(R - \mathbf{D}_{\widehat{R}^*}\Phi - \mathbf{D}_S H\right)^H \mathbf{\Sigma}^{-1}\left(R - \mathbf{D}_{\widehat{R}^*}\Phi - \mathbf{D}_S H\right)\right\} \tag{3.3}$$

The ML estimates of Φ and H can be obtained by maximizing the following likelihood function (3.4) over different trial value of Φ and H :

$$\begin{aligned}&\Lambda(\Phi, H)\\ &= -\left(R - \mathbf{D}_{\widehat{R}^*}\Phi - \mathbf{D}_S H\right)^H \Sigma^{-1}\left(R - \mathbf{D}_{\widehat{R}^*}\Phi - \mathbf{D}_S H\right)\\ &= -\left(R - \mathbf{D}_{\widehat{R}^*}\Phi - \mathbf{D}_S H\right)^H \left[\left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\mathbf{D}_{H_I}^H\mathbf{D}_{\Gamma_0}^H\left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\widehat{\Phi}}^*\right)^H\right]^{-1}\left(R - \mathbf{D}_{\widehat{R}^*}\Phi - \mathbf{D}_S H\right)\end{aligned}$$

(3.4)

3.1 Smoothing Property in Time Domain

Although direct estimation for Φ and H based on Equation (3.4) is possible, it does not make use of frequency correlation properties since it treats every carrier independently. It has been reported that due to the structure of OFDM symbol, time and frequency correlation can attain significant gain for channel estimation [20]. Actually, time domain I/Q imbalance coefficients φ and impulse response channel h have the part of "Fourier matrix" relation with their frequency domain Φ and H shown below :

$$\Phi = \mathbf{F}_{\varphi}\varphi \quad , \quad H = \mathbf{F}_h h$$

where $\{\mathbf{F}_h\}_{i,n} \triangleq \exp\left(-\dfrac{2\pi \cdot i \cdot n}{N}\right),\ 1 \le i \le l-1,\ 0 \le n \le L_h - 1$

and $\{\mathbf{F}_{\varphi}\}_{i,n} \triangleq \exp\left(-\dfrac{2\pi \cdot j \cdot m}{N}\right),\ 1 \le j \le l-1,\ 0 \le m \le L_{\varphi} - 1$

L_h is effective length of channel impulse response.

L_{φ} is degree of I/Q imbalance effect.

That means Φ and H just have L_{φ} and L_{φ} degrees of freedom instead of $l-1$ degrees of freedom in $l-1$ dimension vector; therefore, only much fewer independent parameters in time domain instead of frequency domain need to be estimated, which can improve estimation performance drastically. Equation (3.5) enables us to express Φ and H in terms of their time-domain forms and use frequency-domain received data to do time domain estimation.

$$R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_{\varphi}\varphi = \mathbf{D}_S\mathbf{F}_h h + N \tag{3.5}$$

3.2 ML Estimation of Channel

Due to the smoothing criterion indicated in section 3.1, Φ is replaced with $\mathbf{F}_{\varphi}\varphi$ 、H is replaced with $\mathbf{F}_h h$ in (3.4), then the modified likelihood function become

$$\begin{aligned}
&\Lambda(\varphi, h) \\
&= -\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_{\varphi}\varphi - \mathbf{D}_S\mathbf{F}_h h\right)^H \left[\left(\mathbf{I}_N - \mathbf{D}_{\Phi}\mathbf{D}_{\hat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\mathbf{D}_{H_I}^H\mathbf{D}_{\Gamma_0}^H\left(\mathbf{I}_N - \mathbf{D}_{\Phi}\mathbf{D}_{\hat{\Phi}}^*\right)^H\right]^{-1}\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_{\varphi}\varphi - \mathbf{D}_S\mathbf{F}_h h\right) \\
&= -\left\|\left[\left(\mathbf{I}_N - \mathbf{D}_{\Phi}\mathbf{D}_{\hat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\right]^{-1}\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_{\varphi}\varphi - \mathbf{D}_S\mathbf{F}_h h\right)\right\|^2 \\
&= -\left\|\mathbf{W}(\varphi)\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_{\varphi}\varphi - \mathbf{D}_S\mathbf{F}_h h\right)\right\|^2
\end{aligned} \tag{3.6}$$

where $\mathbf{W}(\varphi) = \left[\left(\mathbf{I}_N - \mathbf{D}_{\Phi}\mathbf{D}_{\hat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\right]^{-1}$

The I/Q imbalance coefficients φ are fixed first in (3.6) and the estimated time-domain response channel response $\hat{h}_{ML}$ can be treated as maximum of Equation (3.6) in least square method. It can be expressed as

$$\hat{h}_{ML}(\varphi) = \left(\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h\right)^{\dagger}\mathbf{W}(\varphi)\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi\right) \tag{3.7}$$

where $(\ )^{\dagger}$ denotes Psudo Inverse

Then, the frequency–domain channel $\hat{H}_{ML}$ can be computed through Fourier matrix.

$$\begin{aligned}\hat{H}_{ML}(\varphi) &= \mathbf{F}_h\hat{h}_{ML}(\varphi)\\ &= \mathbf{F}_h\left(\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h\right)^{\dagger}\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi\right)\end{aligned} \tag{3.8}$$

3.3 ML Estimation of I/Q Imbalance

Equation (3.8) expresses the solution of $\hat{H}_{ML}$ as function of φ. To obtain optimal φ, we substitute $\hat{H}_{ML}(\varphi)$ back to Equation (3.6)

$$\begin{aligned}\Lambda(\varphi) &\triangleq \Lambda(\varphi, h)\big|_{H=\hat{H}_{ML}(\varphi)}\\ &= -\left\|\mathbf{W}(\varphi)\left[R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi - \mathbf{D}_S\mathbf{F}_h\left(\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h\right)^{\dagger}\mathbf{W}(\varphi)\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi\right)\right]\right\|^2\\ &= -\left\|\left(\mathbf{I}_N - \mathbf{P}_{\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{W}(\varphi)\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi\right)\right\|^2\end{aligned}$$

where $\mathbf{P}_{\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h} \triangleq \mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h\left[\left(\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h\right)^H\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h\right]^{-1}\left(\mathbf{W}(\varphi)\mathbf{D}_S\mathbf{F}_h\right)^H$
is a projection matrix

$\mathbf{W}(\varphi) = \left[\left(\mathbf{I}_N - \mathbf{D}_\Phi\mathbf{D}_\Phi^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\right]^{-1}$ is functions of φ, so it is difficult to maximize

$\Lambda(\varphi) \triangleq \Lambda(\varphi, h)\big|_{H=\hat{H}_{ML}(\varphi)}$ over φ. Some approximation on $\mathbf{W}(\varphi)$ is needed to derive the ML estimate of I/Q imbalance Φ in a close form.

1. First, express $\mathbf{D}_{\Gamma_0}$ as function of $\mathbf{D}_{\Phi}$

$$\because \Psi_0[k] = \frac{1}{2}\left(1 - ge^{j\theta}\frac{H_Q[k]}{H_I[k]}\right) \quad , \quad \Gamma_0[k] = \frac{1}{2}\left(1 + ge^{-j\theta}\frac{H_Q[k]}{H_I[k]}\right)$$

$$\therefore \Psi_0[k] = 1 - \Gamma_0^*[l-k]$$

$$\Rightarrow \mathbf{D}_{\Psi_0} = \mathbf{I} - \mathbf{D}_{\hat{\Gamma}_0^*}$$

$$\Rightarrow \mathbf{D}_{\Phi} = \mathbf{D}_{\Psi_0}\left(\mathbf{D}_{\hat{\Gamma}_0^*}\right)^{-1} = \left(\mathbf{I} - \mathbf{D}^*_{\hat{\Gamma}_0}\right)\left(\mathbf{D}_{\hat{\Gamma}_0^*}\right)^{-1} = \left(\mathbf{D}_{\hat{\Gamma}_0^*}\right)^{-1} - \mathbf{I}$$

$$\Rightarrow \mathbf{D}_{\Gamma_0}^{-1} = \mathbf{I} + \mathbf{D}_{\hat{\Phi}^*} \tag{3.13}$$

2. Because the magnitude of $\Phi[k]$ is around 0.1 in moderate I/Q imbalance level shown in fig.3-1, the second order of $\Phi[k]$ can be eliminated.

$$\left(\mathbf{I} - \mathbf{D}_{\Phi}\mathbf{D}_{\hat{\Phi}^*}\right)^{-1} \simeq \mathbf{I}$$

3. From 1 and 2, we can approximate $\mathbf{W}(\varphi)$ as

$$\begin{aligned}\mathbf{W}(\varphi) &= \left[\left(\mathbf{I}_N - \mathbf{D}_{\Phi}\mathbf{D}^*_{\hat{\Phi}}\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\right]^{-1} \\ &\simeq \left(\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\right)^{-1} \\ &= \left(\mathbf{I} + \mathbf{D}_{\hat{\Phi}^*}\right)\mathbf{D}_{H_I}^{-1}\end{aligned}$$

4. However, this approximation is not enough, approximate again

$$\mathbf{W}(\varphi) \simeq \mathbf{D}_{H_I}^{-1}$$

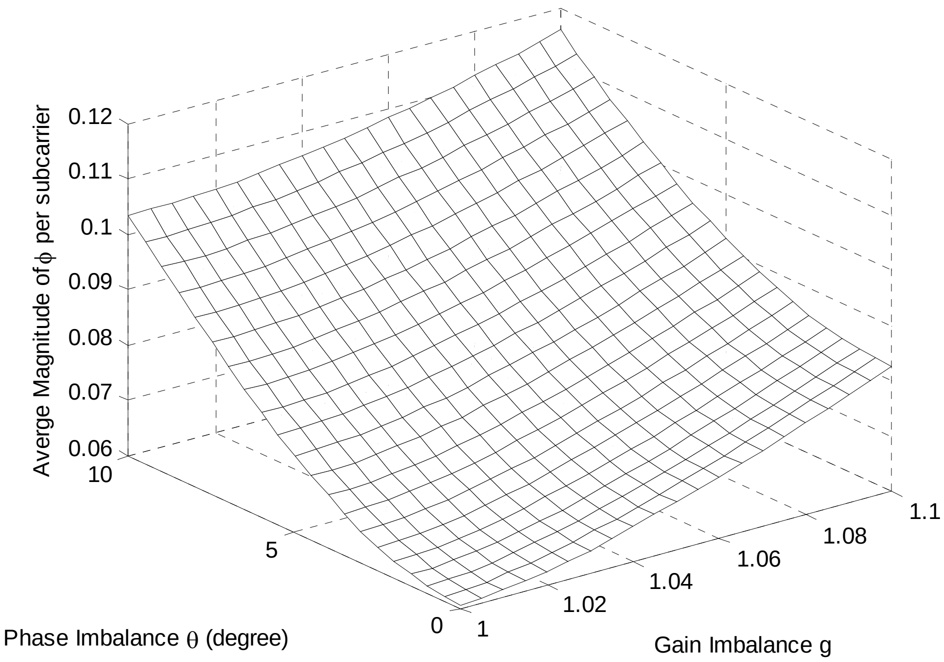


Fig.3-1 The average magnitude of Φ per subcarrier

Replace $\mathbf{W}(\varphi)$ with $\mathbf{D}_{H_I}^{-1}$, hence, the likelihood function is finally approximated as

$$\tilde{\Lambda}(\varphi) \approx -\left\| \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1} \left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \varphi \right) \right\|^2 \tag{3.9}$$

where $\tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h} \triangleq \mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h \left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h \right)^H \mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h \right]^{-1} \left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h \right)^H$

Similar to Equation (3.6), the maximum of Equation (3.9) can be expressed as

$$\begin{aligned} \hat{\varphi}_{ML} &= \left[\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \right]^\dagger \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1} R \\ &= \left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \right]^{-1} \left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1} R \end{aligned}$$

(3.10)

The frequency domain of I/Q imbalance coefficients can be obtained through FFT matrix.

$$\hat{\Phi}_{ML} = \mathbf{F}_{\varphi}\hat{\varphi}_{ML}$$

3.4 Algorithm with Frequency Offset Compensation

So far, we haven't considered the carrier frequency offset (CFO) effect at the receiver. In this section, we propose the CFO compensation method combined with our algorithm. The received signal path considering CFO effect can be derived as following

$$\tilde{y}(t) \triangleq 2\,\mathrm{Re}\left\{ y(t)e^{j2\pi(f_c+\Delta f)t} \right\} = y(t)e^{j2\pi(f_c+\Delta f)t} + y^*(t)e^{-j2\pi(f_c+\Delta f)t} \quad \text{where } \Delta f \text{ is the CFO}$$

$$\begin{aligned}
v(t) &= \tilde{y}(t)\times C_{LO}(t) \\
&= \left[y(t)e^{j2\pi(f_c+\Delta f)t} + y^*(t)e^{-j2\pi(f_c+\Delta f)t} \right] \times \left[\gamma_0 e^{-j2\pi f_c t} + \varphi_0 e^{j2\pi f_c t} \right] \\
&= \gamma_0 y(t)e^{j2\pi\Delta f t} + \varphi_0 y^*(t)e^{-j2\pi\Delta f t} + \underbrace{\gamma_0 y^*(t)e^{-j(4\pi f_c+2\pi\Delta f)t} + \varphi_0 y(t)e^{j(4\pi f_c+2\pi\Delta f)t}}_{\text{will be removed by the following LPFs}} \\
&= \gamma_0 y(t)e^{j2\pi\Delta f t} + \varphi_0 y^*(t)e^{-j2\pi\Delta f t}
\end{aligned}$$

$$\begin{aligned}
r(t) &= r_I(t) + r_Q(t) \\
&= v_I(t)\otimes h_I(t) + v_Q(t)\otimes h_Q(t) \\
&= \frac{1}{2}\left[y(t)e^{j2\pi\Delta f t} + \mathrm{y}^*(t)e^{-j2\pi\Delta f t} \right]\otimes h_I(t) + \frac{1}{2}g\left[e^{-j\theta}y(t)e^{j2\pi\Delta f t} - e^{j\theta}y^*(t)e^{-j2\pi\Delta f t} \right]\otimes h_Q(t) \\
&= y(t)e^{j2\pi\Delta f t}\otimes \underbrace{\frac{1}{2}\left[h_I(t) + h_Q(t)ge^{-j\theta} \right]}_{\triangleq h_+(t)} + y^*(t)e^{-j2\pi\Delta f t}\otimes \underbrace{\frac{1}{2}\left[h_I(t) - h_Q(t)ge^{j\theta} \right]}_{\triangleq h_-(t)} \\
&= y(t)e^{j2\pi\Delta f t}\otimes h_+(t) + y^*(t)e^{-j2\pi\Delta f t}\otimes h_-(t) \\
&= e^{j2\pi\Delta f t}\left[s(t)\otimes h_{CH}(t) + n_0(t) \right]\otimes h_+(t) + e^{-j2\pi\Delta f t}\left[s(t)\otimes h_{CH}(t) + n_0(t) \right]^*\otimes h_-(t)
\end{aligned}$$

The finally resulting baseband signal model is given by

$$r[n]=e^{j2\pi\Delta n}\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)\otimes h_+[n]+e^{-j2\pi\Delta n}\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)^*\otimes h_-[n]$$

After conjugate cancellation and CFO compensation in time domain, the modified algorithm is shown in equation (3.11).

$$\begin{aligned}r[n]-\varphi[n]\otimes r^*[n]&=\left(h_+[n]-\varphi[n]\otimes h_-^*[n]\right)\otimes\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)e^{j\Delta\omega n}\\&=\left(s[n]\otimes h[n]+n[n]\right)e^{j\Delta\omega n}\end{aligned}$$

$$\Rightarrow\ e^{-j\Delta\omega n}\left(r[n]-\varphi[n]\otimes r^*[n]\right)=s[n]\otimes h[n]+n[n]$$

For frequency domain compensation, we must modify the equation as following :

(Note that $\varphi[n]$ is not $\tilde{\varphi}[n]$ at all)

$$\left(e^{-j\Delta\omega n}r[n]\right)-\tilde{\varphi}[n]\otimes\left(e^{-j\Delta\omega n}r^*[n]\right)=s[n]\otimes h[n]+n[n] \qquad (3.11)$$

Take FFT of (3.11)

$$R_{Comp}-\tilde{\Phi}\odot R_{\mathrm{Im}age-Comp}=R_{Comp}-\mathbf{D}_{R_{\mathrm{Im}age-Comp}}\tilde{\Phi}=SH+N \qquad (3.12)$$

where $R_{Comp}=FFT\left(e^{-j\Delta\omega n}r[n]\right)$, $R_{\mathrm{Im}age-Comp}=FFT\left(e^{-j\Delta\omega n}r^*[n]\right)$

Equation (3.12) similar to equation (2.14) illustrates how to apply proposed algorithm in frequency domain with CFO compensation. Note that we assume CFO has been estimated prior to our joint estimation. Therefore, we can get R_{Comp} and $R_{\mathrm{Im}age-Comp}$ to pre-compensate CFO effect in (3.12) and then do joint estimation based on the signal model the same with (2.14) derived previously.

Chapter 4 Performance Analysis

In this chapter, the mean and mean square error (MSE) of the ML estimators are examined under the assumption of a high signal-to-noise ratio (SNR). Section 4.1 provides the analysis of the mean and MSE of our proposed estimator of I/Q imbalance Φ, and the mean and MSE of our proposed estimator of channel H are analyzed in section 4.2.

4.1 Mean and MSE of I/Q Imbalance Φ

In this section, the mean and MSE of $\hat{\Phi}_{ML}$ defined in (3.10) are analyzed.

Following from (2.14)

$$R - \mathbf{D}_{\hat{R}^*}\Phi = \mathbf{D}_S H + N \quad \Rightarrow \quad R = \mathbf{D}_{\hat{R}^*}\Phi + \mathbf{D}_S H + N = \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \varphi + \mathbf{D}_S \mathbf{F}_h h + N \tag{4.1}$$

and (3.10)

$$\hat{\varphi}_{ML} = \left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}R$$

$$\hat{\Phi}_{ML} = \mathbf{F}_\varphi \hat{\varphi}_{ML}$$

$$= \mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\left(\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi + \mathbf{D}_S\mathbf{F}_h h + N\right)$$

$$= \mathbf{F}_\varphi \underbrace{\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi}_{\mathbf{I}_{L_\varphi \times L_\varphi}}\varphi$$

$$+ \mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H\underbrace{\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_S\mathbf{D}_{H_I}^{-1}\mathbf{F}_h}_{\mathbf{0}_{N \times L_h}} h$$

$$\left(\because\ \mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h} \text{ is a projection matrix of } \mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h\right)$$

$$+ \mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}N$$

$$= \mathbf{F}_\varphi\varphi + \mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}N$$

$$= \Phi_{N\times 1} + \mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}N$$

The relation between the true I/Q imbalance $\Phi_{N\times1}$ and its estimates $\hat{\Phi}_{ML}$ can be expressed as (4.2) derived from above equation.

$$\therefore\ \hat{\Phi}_{ML} = \Phi_{N\times1} + \mathbf{F}_\varphi \left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right) \mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \right]^{-1} \left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right) \mathbf{D}_{H_I}^{-1} N$$

(4.2)

To proceed, we define R_a which is the deterministic part of R in (2.14)

$$\begin{aligned}
R &= FFT\left\{\left(s[n]\otimes h_{CH}[n] + n_0[n]\right)\otimes h_+[n] + \left(s[n]\otimes h_{CH}[n] + n_0[n]\right)^* \otimes h_-[n]\right\} \\
&= \underbrace{FFT\left\{s[n]\otimes h_{CH}[n]\otimes h_+[n] + \left(s[n]\otimes h_{CH}[n]\right)^* \otimes h_-[n]\right\}}_{R_a} \\
&\quad + FFT\left\{n_0[n]\otimes h_+[n] + \left(n_0[n]\right)^* \otimes h_-[n]\right\} \\
&\simeq R_a + \mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I} N_0 + \mathbf{D}_{\Psi_0}\mathbf{D}_{H_I} \widehat{N}_0^*
\end{aligned}$$

$$R_a = FFT\left\{s[n]\otimes h_{CH}[n]\otimes h_+[n] + \left(s[n]\otimes h_{CH}[n]\right)^* \otimes h_-[n]\right\}$$

The unbiasedness of estimators in (4.1) can be seen if R is replaced with R_a. Therefore, the estimators are approximately unbiased when $\mathrm{SNR} \gg 1$. Specifically, the bias of $\hat{\Phi}_{ML}$ may be approximated by

$$\begin{aligned}
E\left[\hat{\Phi}_{ML} - \Phi\right] &= E\left\{\mathbf{F}_\varphi \left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right) \mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \right]^{-1} \left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right) \mathbf{D}_{H_I}^{-1} N\right\} \\
&\simeq \mathbf{F}_\varphi \left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}_a^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right) \mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}_a^*}\mathbf{F}_\varphi \right]^{-1} \left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\hat{R}_a^*}\mathbf{F}_\varphi\right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right) \mathbf{D}_{H_I}^{-1} E\left[N\right]
\end{aligned}$$

From (3.3) $N \simeq \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^*\ \right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I} N_0$

Because $E\left[N\right] \simeq E\left[\left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^*\ \right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I} N_0\right] = 0$

$\therefore E\left[\hat{\Phi}_{ML} - \Phi\right] = 0$ in high SNR

The MSE of $\hat{\Phi}_{ML}$ can be approximated as following

$$E\left[\left|\hat{\Phi}_{ML}-\Phi\right|^2\right]$$

$$=E\left\{\left|\mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N-\tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N-\tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}N\right|^2\right\}$$

$$\simeq E\left\{\left|\mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}_a^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N-\tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}_a^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}_a^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N-\tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}N\right|^2\right\}$$

$$=E\left\{\left|\mathbf{Q}N_0\right|^2\right\}$$

$$=tr\left\{\mathbf{Q}E\left[N_0N_0^H\right]\mathbf{Q}^H\right\}$$ Note that N_0 is AWGN noise in frequency domain

$$=\sigma_0^2 tr\left\{\mathbf{Q}\mathbf{Q}^H\right\}$$

$$=\sigma_0^2\left\|\mathbf{Q}\right\|_F$$ $\left\|\ \right\|_F$ denotes Fronbeinus norm

where $\mathbf{Q}=\mathbf{F}_\varphi\left[\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}_a^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N-\tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}_a^*}\mathbf{F}_\varphi\right]^{-1}\left(\mathbf{D}_{H_I}^{-1}\mathbf{D}_{\widehat{R}_a^*}\mathbf{F}_\varphi\right)^H\left(\mathbf{I}_N-\tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1}\mathbf{D}_S\mathbf{F}_h}\right)\mathbf{D}_{H_I}^{-1}\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\hat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}$

4.2 Mean and MSE of Channel H

The mean and MSE of H defined in (3.8) can be derived in the way similar to the above procedure. Start from (3.8) :

$$\hat{H}_{ML}=\mathbf{F}_h\hat{h}_{ML}$$
$$=\mathbf{F}_h\left[(\mathbf{D}_S\mathbf{F}_h)^H\mathbf{W}^H(\varphi)\mathbf{W}(\varphi)(\mathbf{D}_S\mathbf{F}_h)\right]^{-1}(\mathbf{D}_S\mathbf{F}_h)^H\mathbf{W}^H(\varphi)\mathbf{W}(\varphi)\left(R-\mathbf{D}_{\widehat{R}^*}\mathbf{F}_\varphi\hat{\varphi}_{ML}\right)$$
$$=\mathbf{F}_h\left[(\mathbf{D}_S\mathbf{F}_h)^H\mathbf{W}^H(\varphi)\mathbf{W}(\varphi)(\mathbf{D}_S\mathbf{F}_h)\right]^{-1}(\mathbf{D}_S\mathbf{F}_h)^H\mathbf{W}^H(\varphi)\mathbf{W}(\varphi)\left(R-\mathbf{D}_{\widehat{R}^*}\hat{\Phi}_{ML}\right)$$

(4.3)

Substitute (4.1) into (4.3)

$$
\begin{aligned}
\hat{H}_{ML} &= \mathbf{F}_h \left[(\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{-1} (\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) \left[\left(\mathbf{D}_{\hat{R}^*} \Phi + \mathbf{D}_S H + N \right) - \mathbf{D}_{\hat{R}^*} \hat{\Phi}_{ML} \right] \\
&= \mathbf{F}_h \left[(\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{-1} (\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) \left[\mathbf{D}_S H + \mathbf{D}_{\hat{R}^*} \left(\Phi - \hat{\Phi}_{ML} \right) + N \right] \\
&= \mathbf{F}_h \underbrace{\left[(\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{-1} (\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) \mathbf{D}_S \mathbf{F}_h}_{=\mathbf{I}_{L_h \times L_h}} h \\
&\quad + \mathbf{F}_h \left[(\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{-1} (\mathbf{D}_S \mathbf{F}_h)^H \mathbf{W}^H(\varphi) \mathbf{W}(\varphi) \left[\mathbf{D}_{\hat{R}^*} \left(\Phi - \hat{\Phi}_{ML} \right) + N \right] \\
&= \mathbf{F}_h h + \mathbf{F}_h \left[\mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{\dagger} \mathbf{W}(\varphi) \left[\mathbf{D}_{\hat{R}^*} \left(\Phi - \hat{\Phi}_{ML} \right) + N \right]
\end{aligned}
$$

The relation between the true channel H and its estimates $\hat{H}_{ML}$ can be expressed as (4.3) derived from above equation.

$$
\hat{H}_{ML} = H + \mathbf{F}_h \left[\mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{\dagger} \mathbf{W}(\varphi) \left[\mathbf{D}_{\hat{R}^*} \left(\Phi - \hat{\Phi}_{ML} \right) + N \right] \tag{4.3}
$$

Again, R is replaced with R_a in high SNR in (4.2) :

$$
\begin{aligned}
\hat{\Phi}_{ML} &= \Phi_{N \times 1} + \mathbf{F}_\varphi \left[\left(\mathbf{D}_{H_I}^{-1} \mathbf{D}_{\hat{R}^*} \mathbf{F}_\varphi \right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1} \mathbf{D}_S \mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1} \mathbf{D}_{\hat{R}^*} \mathbf{F}_\varphi \right]^{-1} \left(\mathbf{D}_{H_I}^{-1} \mathbf{D}_{\hat{R}^*} \mathbf{F}_\varphi \right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1} \mathbf{D}_S \mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1} N \\
&\simeq \Phi_{N \times 1} + \mathbf{F}_\varphi \left[\left(\mathbf{D}_{H_I}^{-1} \mathbf{D}_{\hat{R}_a^*} \mathbf{F}_\varphi \right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1} \mathbf{D}_S \mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1} \mathbf{D}_{\hat{R}_a^*} \mathbf{F}_\varphi \right]^{-1} \left(\mathbf{D}_{H_I}^{-1} \mathbf{D}_{\hat{R}_a^*} \mathbf{F}_\varphi \right)^H \left(\mathbf{I}_N - \tilde{\mathbf{P}}_{\mathbf{D}_{H_I}^{-1} \mathbf{D}_S \mathbf{F}_h} \right) \mathbf{D}_{H_I}^{-1} N \\
&= \Phi_{N \times 1} + \mathbf{Q} N_0
\end{aligned}
$$

$$
\Rightarrow \quad \hat{\Phi}_{ML} - \Phi_{N \times 1} = \mathbf{Q} N_0 \tag{4.4}
$$

Substitute (4.4) and (3.3) into (4.3)

Therefore,

$$
\begin{aligned}
E\left[\hat{H}_{ML} - H \right] &\simeq E\left\{ \mathbf{F}_h \left[\mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{\dagger} \mathbf{W}(\varphi) \left[\mathbf{D}_{\hat{R}_a^*} \mathbf{Q} + \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right] N_0 \right\} \\
&= \mathbf{F}_h \left[\mathbf{W}(\varphi) (\mathbf{D}_S \mathbf{F}_h) \right]^{\dagger} \mathbf{W}(\varphi) \left[\mathbf{D}_{\hat{R}_a^*} \mathbf{Q} + \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right] E\left[N_0 \right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
E\left[\left|\hat{H}_{ML}-H\right|^{2}\right] &\simeq E\left\{\left|\underbrace{\mathbf{F}_{h}\left[\mathbf{W}(\varphi)(\mathbf{D}_{S}\mathbf{F}_{h})\right]^{\dagger}\mathbf{W}(\varphi)\left[\mathbf{D}_{\widehat{R}_{a}^{*}}\mathbf{Q}+\left(\mathbf{I}_{N}-\mathbf{D}_{\Phi}\mathbf{D}_{\widehat{\Phi}}^{*}\right)\mathbf{D}_{\Gamma_{0}}\mathbf{D}_{H_{I}}\right]}_{\text{Define as P}}N_{0}\right|^{2}\right\} \\
&= E\left\{\left|\mathbf{P}N_{0}\right|^{2}\right\} \\
&= \sigma_{0}^{2}\left\|\mathbf{P}\right\|_{F}
\end{aligned}
$$

Chapter 5 Derivation of Cramer-Rao Lower Bounds

The CRLBs of H and Φ can be derived following the procedure in [19].

Define vectors $\theta = \left(\varphi^T \quad \mathrm{h}^T\right)^T$ and $\omega = \left(\Phi^T \quad \mathrm{H}^T\right)^T = \mathbf{F}_{\varphi,h}\theta$

where $\mathbf{F}_{\varphi,h} = \begin{pmatrix} \mathbf{F}_\varphi & 0_{(l-1)\times(l-1)} \\ 0_{(l-1)\times(l-1)} & \mathbf{F}_h \end{pmatrix}$, $\mathbf{F}_\varphi = \begin{bmatrix} f_1 \\ \vdots \\ f_{l-1} \end{bmatrix}$

f_i is the $i'th$ row vector of $\mathbf{F}_\varphi$, and $\Phi[i] = f_i\varphi$

The approximated conditional probability density function is shown below:

$$p\left\{R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\rho\varphi \middle| \varphi, h\right\} \simeq \frac{1}{\pi^n \det(\mathbf{\Sigma})} \exp\left\{-\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi - \mathbf{S}\mathbf{F}_h h\right)^H \mathbf{\Sigma}^{-1}\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi - \mathbf{S}\mathbf{F}_h h\right)\right\}$$

$$\text{where } \mathbf{\Sigma} = E\left\{NN^H\right\} = \sigma_0^2\left(\mathbf{I}_N - \mathbf{D}_\Phi\mathbf{D}_{\hat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\mathbf{D}_{H_I}^H\mathbf{D}_{\Gamma_0}^H\left(\mathbf{I}_N - \mathbf{D}_\Phi\mathbf{D}_{\hat{\Phi}}^*\right)^H$$

$$\simeq \sigma_0^2\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\mathbf{D}_{H_I}^H\mathbf{D}_{\Gamma_0}^H \quad (\text{First order approximation})$$

From (3.13) $\Gamma_0[k] = \dfrac{1}{1 + \Phi^*[N-k]}$

Hence, the k'th diagonal of $\mathbf{\Sigma}$ refers to $\mathbf{\Sigma}_{kk} \simeq \sigma_0^2\left|H_I[k]\right|^2 \dfrac{1}{\left|1 + \Phi[N-k]\right|^2}$

Define log-likelihood function $f(\theta) = \ln p\left\{R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi\varphi \middle| \varphi, h\right\}$. Because ω is a function of θ, the CRLBs of ω in frequency domain can be derived from $f(\theta)$ by (5.1)

$$\mathrm{CRLB}(\omega_k) = \left(\mathbf{F}_{\varphi,h}\mathbf{I}^{-1}\mathbf{F}_{\varphi,h}^H\right)_{kk} \tag{5.1}$$

$$\mathbf{I} = E\left[\frac{\partial f(\omega)}{\partial\theta^*}\left(\frac{\partial f(\omega)}{\partial\theta^*}\right)^H\right]$$

$$\frac{\partial f(\omega)}{\partial\theta^*} = \begin{bmatrix} \dfrac{\partial f(\omega)}{\partial\varphi^*} \\ \dfrac{\partial f(\omega)}{\partial h^*} \end{bmatrix}$$

A. We express $f(\theta)$ in scalar form :

$$
\begin{aligned}
f(\theta) &= \ln p\left\{R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \varphi \middle| \varphi, h\right\} \\
&= \ln \frac{1}{\pi^n \det(\mathbf{\Sigma})} \exp\left\{-\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \varphi - \mathbf{S}\mathbf{F}_h h\right)^H \mathbf{\Sigma}^{-1}\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \varphi - \mathbf{S}\mathbf{F}_h h\right)\right\} \\
&= \ln\left(\pi^{-n}\det{}^{-1}(\mathbf{\Sigma})\right) - \left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \varphi - \mathbf{D}_S\mathbf{F}_h h\right)^H \mathbf{\Sigma}^{-1}\left(R - \mathbf{D}_{\hat{R}^*}\mathbf{F}_\varphi \varphi - \mathbf{D}_S\mathbf{F}_h h\right) \\
&= \sum_{k=1}^{l-1} \ln\left(\pi^{-n}\sigma_0^{-2}\left|H_I[k]\right|^{-2}\left|1+\Phi[l-k]\right|^2\right) \\
&\quad - \frac{1}{\sigma_0^2}\sum_{k=1}^{l-1}\left(\left|H_I[k]\right|^{-2}\left|1+\Phi[l-k]\right|^2\left|R[k] - R^*[l-k]\Phi[k] - S[k]H[k]\right|^2\right)
\end{aligned}
$$

B. Take derivative of $f(\theta)$ with respect to φ^* using chain rules (φ is a function of Φ). Then, we obtain (5.2) from

$$
\begin{aligned}
\frac{\partial f(\omega)}{\partial \varphi^*} &= \sum_{k=1}^{l-1} \frac{\partial \ln\left(\left|1+\Phi[l-k]\right|^2\right)}{\partial \Phi^*[l-k]} \frac{\partial \Phi^*[l-k]}{\partial \varphi^*} \\
&\quad - \frac{1}{\sigma_0^2}\sum_{k=1}^{l-1}\left(
\begin{array}{l}
\left|H_I[k]\right|^{-2}\left|R[k]-R^*[l-k]\Phi[k]-S[k]H[k]\right|^2 \dfrac{\partial\left(\left|1+\Phi[l-k]\right|^2\right)}{\partial \Phi^*[l-k]} \dfrac{\partial \Phi^*[l-k]}{\partial \varphi^*} \\
+\left|H_I[k]\right|^{-2}\left|1+\Phi[l-k]\right|^2 \dfrac{\partial\left(\left|R[k]-R^*[l-k]\Phi[k]-S[k]H[k]\right|^2\right)}{\partial \Phi^*[k]} \dfrac{\partial \Phi^*[k]}{\partial \varphi^*}
\end{array}
\right) \\
&= \sum_{k=1}^{l-1} \frac{f_{l-k}^H}{1+\Phi^*[l-k]} \\
&\quad - \frac{1}{\sigma_0^2}\sum_{k=1}^{l-1}\left(
\begin{array}{l}
\left|H_I[k]\right|^{-2}\left|R[k]-R^*[l-k]\Phi[k]-S[k]H[k]\right|^2\left(1+\Phi[l-k]\right) f_{l-k}^H \\
-\left|H_I[k]\right|^{-2}\left|1+\Phi[l-k]\right|^2\left(R[k]-R^*[l-k]\Phi[k]-S[k]H[k]\right) R[l-k]\, f_k^H
\end{array}
\right) \\
&= \mathbf{F}_\varphi^H \Phi_{\frac{1}{1+\Phi^*}} - \frac{1}{\sigma_0^2}\mathbf{F}_\varphi^H \mathbf{D}^{-1}_{\hat{H}_I \odot \hat{H}_I^*}\mathbf{D}_{\Phi_{1+\Phi}} \hat{N} \odot \hat{N}^* + \frac{1}{\sigma_0^2}\mathbf{F}_\varphi^H \mathbf{D}^{-1}_{H_I \odot H_I^*}\mathbf{D}_{\hat{\Phi}_{1+\Phi}\odot \hat{\Phi}^*_{1+\Phi}} \hat{R} \odot N
\end{aligned}
$$

(5.2)

where $\odot$ denotes elementwise product

$$
\Phi_{\frac{1}{1+\Phi^*}} = \left[\frac{1}{1+\Phi^*[1]} \cdots \frac{1}{1+\Phi^*[l-1]}\right]^T \quad , \quad \Phi_{1+\Phi} = \left[1+\Phi[1] \cdots 1+\Phi[l-1]\right]^T
$$

C. Define R_a as the deterministic part of R :

Take FFT of (2.7) :

$$
\begin{aligned}
R &= FFT\left\{\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)\otimes h_+[n]+\left(s[n]\otimes h_{CH}[n]+n_0[n]\right)^*\otimes h_-[n]\right\} \\
&= \underbrace{FFT\left\{s[n]\otimes h_{CH}[n]\otimes h_+[n]+\left(s[n]\otimes h_{CH}[n]\right)^*\otimes h_-[n]\right\}}_{R_a} \\
&\quad + FFT\left\{n_0[n]\otimes h_+[n]+\left(n_0[n]\right)^*\otimes h_-[n]\right\} \\
&\simeq R_a+\mathbf{D}_{\Gamma_\mathbf{0}}\mathbf{D}_{H_I}N_0+\mathbf{D}_{\Psi_\mathbf{0}}\mathbf{D}_{H_I}\widehat{N}_0^*
\end{aligned}
$$

From (3.3) $\quad N\simeq\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}N_0$

$$
\begin{aligned}
\widehat{R}\odot N &\simeq\left(\widehat{R}_a+\mathbf{D}_{\widehat{\Gamma}_\mathbf{0}}\mathbf{D}_{\widehat{H}_I}\widehat{N}_0+\mathbf{D}_{\widehat{\Psi}_\mathbf{0}}\mathbf{D}_{\widehat{H}_I}N_0^*\right)\odot\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}N_0 \\
&=\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}\mathbf{D}_{\widehat{R}_a}N_0 \\
&\quad+\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{\widehat{\Gamma}_\mathbf{0}}\mathbf{D}_{H_I}\mathbf{D}_{\widehat{H}_I}\widehat{N}_0\odot N_0 \\
&\quad+\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{\widehat{\Psi}_\mathbf{0}}\mathbf{D}_{H_I}\mathbf{D}_{H_I^*}N_0\odot N_0^* \quad , \quad \left(\widehat{H}_I=H_I^*\right)
\end{aligned}
\tag{5.3}
$$

Substitute (3.3) N and (5.3) $\widehat{R}\odot N$ into (5.2), we get :

$$
\frac{\partial f(\omega)}{\partial\varphi^*}=\mathbf{F}_\rho^H\left[\Phi_{\frac{1}{1+\Phi^*}}-\frac{1}{\sigma_0^2}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*+\frac{1}{\sigma_0^2}\mathbf{V}_2\mathbf{D}_{R_a}N_0+\frac{1}{\sigma_0^2}\mathbf{V}_3N_0\odot\widehat{N}_0+\frac{1}{\sigma_0^2}\mathbf{V}_4N_0\odot N_0^*\right]
$$

(5.4)

$$
\begin{aligned}
\text{where }\ \mathbf{V}_1 &=\mathbf{D}_{\Phi_{1+\Phi}}\left(\mathbf{I}_N-\mathbf{D}_{\widehat{\Phi}}\mathbf{D}_\Phi^*\right)\mathbf{D}_{\widehat{\Gamma}_0}\left[\left(\mathbf{I}_N-\mathbf{D}_{\widehat{\Phi}}\mathbf{D}_\Phi^*\right)\mathbf{D}_{\widehat{\Gamma}_0}\right]^* \\
\mathbf{V}_2 &=\mathbf{D}_{H_I}^{-H}\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{\widehat{\Phi}_{1+\Phi}\odot\widehat{\Phi}_{1+\Phi}^*} \\
\mathbf{V}_3 &=\mathbf{D}_{\widehat{H}_I}\mathbf{D}_{H_I}^{-H}\mathbf{D}_{\widehat{\Phi}_{1+\Phi}\odot\widehat{\Phi}_{1+\Phi}^*}\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{\widehat{\Gamma}_0}\quad , \\
\mathbf{V}_4 &=\mathbf{D}_{\widehat{\Phi}_{1+\Phi}\odot\widehat{\Phi}_{1+\Phi}^*}\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{\widehat{\Psi}_\mathbf{0}}
\end{aligned}
$$

D. Take derivative of $f(\theta)$ with respect to h^* :

$$
\frac{\partial f(\omega)}{\partial h^*}=\mathbf{F}_h^H\mathbf{D}_S\Sigma^{-1}\left(\mathbf{I}_N-\mathbf{D}_\Phi\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}N_0
\tag{5.5}
$$

E. From the derivation of *C.* and *D.*, we have expressed $\dfrac{\partial f(\omega)}{\partial\varphi^*}$ and $\dfrac{\partial f(\omega)}{\partial h^*}$ as function of AWGN N_0 in frequency domain. Then, we can use (5.4) and (5.5) to calculate $E\left[\frac{\partial f(\omega)}{\partial \varphi^*}\left(\frac{\partial f(\omega)}{\partial \varphi^*}\right)^H\right]$, $E\left[\frac{\partial f(\omega)}{\partial \varphi^*}\left(\frac{\partial f(\omega)}{\partial h^*}\right)^H\right]$ and $E\left[\frac{\partial f(\omega)}{\partial h^*}\left(\frac{\partial f(\omega)}{\partial h^*}\right)^H\right]$. In order to compute the above three terms, we should first compute the expectation of the following functions of AWGN N_0 terms :

Note that for complex value $N_0[k]$

$\forall k \;\; E\{N_0[k]\} = 0$,

$E\left\{|N_0[k]|^2\right\} = \sigma_0^2$,

$E\left\{|N_0[k]|^4\right\} = 2\sigma_0^4$

$\forall k, j$ and $k \neq j \quad E\{N_0[k]N_0[j]\} = 0$

and all the third moment of $N_0[k]$ is 0 ,

i.e. $E\{N_0^3[k]\} = E\left\{\left(N_0^*[k]\right)^3\right\} = E\left\{|N_0[k]|^2 N_0^*[k]\right\} = E\left\{|N_0[k]|^2 N_0[k]\right\} = 0$

To be more detail, we express those functions of N_0 as scalar form :

$$N_0 = \left[N_0[1], N_0[2], \cdots, N_0[l-1]\right]^T$$

$$\widehat{N}_0 \odot \widehat{N}_0^* = \left[|N_0[l-1]|^2, |N_0[l-2]|^2, \cdots, |N_0[1]|^2\right]^T \;,\; \left(\widehat{N}_0 \odot \widehat{N}_0^*\right)_k = |N_0[l-k]|^2$$

$$N_0 \odot \widehat{N}_0 = \left[N_0[1]N_0[l-1], N_0[2]N_0[l-2], \cdots, N_0[l-1]N_0[1]\right]^T \;,\; \left(N_0 \odot \widehat{N}_0\right)_k = N_0[k]N_0[l-k]$$

$$N_0 \odot N_0^* = \left[|N_0[1]|^2, |N_0[2]|^2, \cdots, |N_0[l-1]|^2\right]^T \;,\; \left(N_0 \odot N_0^*\right)_k = |N_0[k]|^2$$

Then, use the above scalar form to calculate their self and cross expectation as following :

$$\left(\textit{Define} \;\; \mathbf{1} = [1,1,\cdots,1]_{l\times 1}^T \;\;,\;\; \mathbf{P} = \begin{bmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix} \right)$$

Self expectation :

a. $\quad E\left\{N_0 N_0^H\right\} = \sigma_0^2 \mathbf{I}_{l-1}$

b. $\because\ E\left\{ \hat{N}_0 \odot \hat{N}_0^* \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \right\}_{kj} = \begin{cases} E\left\{ |N_0[l-k]|^2 \, |N_0[l-j]|^2 \right\} = \sigma_0^4 & \text{for } k \neq j \\ E\left\{ |N_0[l-k]|^4 \right\} = 2\sigma_0^4 & \text{for } k{=}j \end{cases}$

$$\Rightarrow E\left\{ \hat{N}_0 \odot \hat{N}_0^* \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \right\} = \sigma_0^4 \left(\mathbf{1}\mathbf{1}^T + \mathbf{I}_{l-1} \right)$$

where $E\{\ \}_{kj}$ is its k'th row and j'th column value

c. $\because\ E\left\{ N_0 \odot \hat{N}_0 \left[N_0 \odot \hat{N}_0 \right]^H \right\}_{kj} = E\left\{ N_0[k] N_0[l-k] N_0^*[j] N_0^*[l-j] \right\}$

$$= \begin{cases} E\left\{ |N_0[k]|^2 \, |N_0[l-k]|^2 \right\} = \sigma_0^4 & \text{for } k{=}j \text{ and } j{=}l{-}k \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow E\left\{ N_0 \odot \hat{N}_0 \left[N_0 \odot \hat{N}_0 \right]^H \right\} = \sigma_0^4 \left(\mathbf{I}_{l-1} + \mathbf{P} \right)$$

d. $\because\ E\left\{ N_0 \odot N_0^* \left[N_0 \odot N_0^* \right]^H \right\}_{kj} = \begin{cases} E\left\{ |N_0[k]|^2 \, |N_0[j]|^2 \right\} = \sigma_0^4 & \text{for } k \neq j \\ E\left\{ |N_0[k]|^4 \right\} = 2\sigma_0^4 & \text{for } k{=}j \end{cases}$

$$\Rightarrow E\left\{ N_0 \odot N_0^* \left[N_0 \odot N_0^* \right]^H \right\} = \sigma_0^4 \left(\mathbf{1}\mathbf{1}^T + \mathbf{I}_{l-1} \right)$$

Cross expectation :

e. $\because\ E\left\{ N_0 \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \right\}_{kj} = E\left\{ N_0 \left[N_0 \odot \hat{N}_0 \right]^H \right\}_{kj} = E\left\{ N_0 \left[N_0 \odot N_0^* \right]^H \right\}_{kj} = 0$

(All Third moment of N_0 is 0)

$$\Rightarrow E\left\{ N_0 \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \right\} = E\left\{ N_0 \left[N_0 \odot \hat{N}_0 \right]^H \right\} = E\left\{ N_0 \left[N_0 \odot N_0^* \right]^H \right\} = 0$$

f. $\because\ E\left\{ \hat{N}_0 \odot \hat{N}_0^* \left[N_0 \odot \hat{N}_0 \right]^H \right\}_{kj} = E\left\{ |N_0[l-k]|^2 \, N_0^*[j] N_0^*[l-j] \right\} = 0$

$$\Rightarrow E\left\{ \hat{N}_0 \odot \hat{N}_0^* \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \right\} = 0$$

g. $\because\ E\left\{ \hat{N}_0 \odot \hat{N}_0^* \left[N_0 \odot N_0^* \right]^H \right\}_{kj} = E\left\{ \left|N_0[l-k]\right|^2 \left|N_0[j]\right|^2 \right\}$

$$= \begin{cases} E\left\{ \left|N_0[l-k]\right|^4 \right\} = 2\sigma_0^4 & \text{for } j = l-k \\ E\left\{ \left|N_0[l-k]\right|^2 \left|N_0[j]\right|^2 \right\} = \sigma_0^4 & \text{otherwise} \end{cases}$$

$$\Rightarrow E\left\{ \hat{N}_0 \odot \hat{N}_0^* \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \right\} = \sigma_0^4 \left(\mathbf{1}\mathbf{1}^T + \mathbf{P} \right)$$

h. $\because\ E\left\{ N_0 \odot \hat{N}_0 \left[N_0 \odot N_0^* \right]^H \right\}_{kj} = E\left\{ N_0[k] N_0[l-k] \left|N_0[j]\right|^2 \right\} = 0$

$$\Rightarrow E\left\{ N_0 \odot \hat{N}_0 \left[N_0 \odot N_0^* \right]^H \right\} = 0$$

F. Now we can calculate $E\left[\frac{\partial f(\omega)}{\partial \varphi^*} \left(\frac{\partial f(\omega)}{\partial \varphi^*} \right)^H \right]$, $E\left[\frac{\partial f(\omega)}{\partial \varphi^*} \left(\frac{\partial f(\omega)}{\partial h^*} \right)^H \right]$ and $E\left[\frac{\partial f(\omega)}{\partial h^*} \left(\frac{\partial f(\omega)}{\partial h^*} \right)^H \right]$ using above *a.* ~ *h.* expectation

1. $E\left[\frac{\partial f(\omega)}{\partial \varphi^*} \left(\frac{\partial f(\omega)}{\partial \varphi^*} \right)^H \right]$

$$= E\left\{ \begin{array}{l} \mathbf{F}_\varphi^H \left[\Phi_{\frac{1}{1+\Phi^*}} - \frac{1}{\sigma_0^2} \mathbf{V}_1 \hat{N}_0 \odot \hat{N}_0^* + \frac{1}{\sigma_0^2} \mathbf{V}_2 \mathbf{D}_{R_a} N_0 + \frac{1}{\sigma_0^2} \mathbf{V}_3 N_0 \odot \hat{N}_0 + \frac{1}{\sigma_0^2} \mathbf{V}_4 N_0 \odot N_0^* \right] \\ \cdot \left[\Phi_{\frac{1}{1+\Phi^*}} - \frac{1}{\sigma_0^2} \mathbf{V}_1 \hat{N}_0 \odot \hat{N}_0^* + \frac{1}{\sigma_0^2} \mathbf{V}_2 \mathbf{D}_{R_a} N_0 + \frac{1}{\sigma_0^2} \mathbf{V}_3 N_0 \odot \hat{N}_0 + \frac{1}{\sigma_0^2} \mathbf{V}_4 N_0 \odot N_0^* \right]^H \mathbf{F}_\varphi \end{array} \right\}$$

$$= E\left\{ \mathbf{F}_\varphi^H \left[\begin{array}{l} \Phi_{\frac{1}{1+\Phi^*}} \Phi^H_{\frac{1}{1+\Phi^*}} + \frac{1}{\sigma_0^4} \mathbf{V}_1 \hat{N}_0 \odot \hat{N}_0^* \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \mathbf{V}_1^H + \frac{1}{\sigma_0^4} \mathbf{V}_2 \mathbf{D}_{R_a} N_0 N_0^H \mathbf{V}_2^H \mathbf{D}_{R_a}^H \\ + \frac{1}{\sigma_0^4} \mathbf{V}_3 N_0 \odot \hat{N}_0 \left[N_0 \odot \hat{N}_0 \right]^H \mathbf{V}_3^H + \frac{1}{\sigma_0^4} \mathbf{V}_4 N_0 \odot N_0^* \left[N_0 \odot N_0^* \right]^H \mathbf{V}_4^H \\ - \left\{ \frac{1}{\sigma_0^2} \Phi_{\frac{1}{1+\Phi^*}} \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \mathbf{V}_1^H \right\} - \left\{ \frac{1}{\sigma_0^2} \Phi_{\frac{1}{1+\Phi^*}} \left[\hat{N}_0 \odot \hat{N}_0^* \right]^H \mathbf{V}_1^H \right\}^H \\ + \left\{ \frac{1}{\sigma_0^2} \Phi_{\frac{1}{1+\Phi^*}} N_0^H \mathbf{V}_2^H \mathbf{D}_{R_a}^H \right\} + \left\{ \frac{1}{\sigma_0^2} \Phi_{\frac{1}{1+\Phi^*}} N_0^H \mathbf{V}_2^H \mathbf{D}_{R_a}^H \right\}^H \end{array} \right] \mathbf{F}_\varphi \right\}$$

$$
\begin{aligned}
&+E\left\{\mathbf{F}_{\varphi}^{H}\left[\begin{array}{l}
+\left\{\frac{1}{\sigma_0^2}\Phi_{\frac{1}{1+\Phi^*}}\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}+\left\{\frac{1}{\sigma_0^2}\Phi_{\frac{1}{1+\Phi^*}}\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}^H\\
+\left\{\frac{1}{\sigma_0^2}\Phi_{\frac{1}{1+\Phi^*}}\left[N_0\odot N_0^*\right]^H\mathbf{V}_4^H\right\}+\left\{\frac{1}{\sigma_0^2}\Phi_{\frac{1}{1+\Phi^*}}\left[N_0\odot N_0^*\right]^H\mathbf{V}_4^H\right\}^H\\
-\left\{\frac{1}{\sigma_0^4}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*N_0^H\mathbf{V}_2^H\mathbf{D}_{R_a}^H\right\}-\left\{\frac{1}{\sigma_0^4}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*N_0^H\mathbf{V}_2^H\mathbf{D}_{R_a}^H\right\}^H\\
+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}^H\\
+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}^H\\
+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_2\mathbf{D}_{R_a}N_0\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_2\mathbf{D}_{R_a}N_0\left[N_0\odot\widehat{N}_0\right]^H\mathbf{V}_3^H\right\}^H\\
+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_2\mathbf{D}_{R_a}N_0\left[N_0\odot N_0^*\right]^H\mathbf{V}_4^H\right\}+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_2\mathbf{D}_{R_a}N_0\left[N_0\odot N_0^*\right]^H\mathbf{V}_4^H\right\}^H\\
+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_3N_0\odot\widehat{N}_0\left[N_0\odot N_0^*\right]^H\mathbf{V}_4^H\right\}+\left\{\frac{1}{\sigma_0^4}\mathbf{V}_3N_0\odot\widehat{N}_0\left[N_0\odot N_0^*\right]^H\mathbf{V}_4^H\right\}^H
\end{array}\right]\mathbf{F}_{\varphi}\right\}\\
&=\mathbf{F}_{\varphi}^{H}\left\{\begin{array}{l}
\Phi_{\frac{1}{1+\Phi^*}}\Phi^H_{\frac{1}{1+\Phi^*}}+\mathbf{V}_1\left(\mathbf{1}\mathbf{1}^T+\mathbf{I}_{l-1}\right)\mathbf{V}_1^H+\mathbf{V}_3\left(\mathbf{I}_{l-1}+\mathbf{P}\right)\mathbf{V}_3^H+\mathbf{V}_4\left(\mathbf{1}\mathbf{1}^T+\mathbf{I}_{l-1}\right)\mathbf{V}_4^H\\
-\left[\Phi_{\frac{1}{1+\Phi^*}}\mathbf{1}^T\mathbf{V}_1^H\right]-\left[\Phi_{\frac{1}{1+\Phi^*}}\mathbf{1}^T\mathbf{V}_1^H\right]^H+\left[\Phi_{\frac{1}{1+\Phi^*}}\mathbf{1}^T\mathbf{V}_4^H\right]+\left[\Phi_{\frac{1}{1+\Phi^*}}\mathbf{1}^T\mathbf{V}_4^H\right]^H\\
-\left[\mathbf{V}_1\left(\mathbf{1}\mathbf{1}^T+\mathbf{P}\right)\mathbf{V}_4^H\right]-\left[\mathbf{V}_1\left(\mathbf{1}\mathbf{1}^T+\mathbf{P}\right)\mathbf{V}_4^H\right]^H+\frac{1}{\sigma_0^2}\mathbf{V}_2\mathbf{D}_{R_a}\mathbf{V}_2^H\mathbf{D}_{R_a}^H
\end{array}\right\}\mathbf{F}_{\varphi}
\end{aligned}
$$

2. $E\left[\frac{\partial f(\omega)}{\partial\varphi^*}\left(\frac{\partial f(\omega)}{\partial h^*}\right)^H\right]$

$$
=E\left\{\begin{array}{l}
\mathbf{F}_{\varphi}^{H}\left[\Phi_{\frac{1}{1+\Phi^*}}-\frac{1}{\sigma_0^2}\mathbf{V}_1\widehat{N}_0\odot\widehat{N}_0^*+\frac{1}{\sigma_0^2}\mathbf{V}_2\mathbf{D}_{R_a}N_0+\frac{1}{\sigma_0^2}\mathbf{V}_3N_0\odot\widehat{N}_0+\frac{1}{\sigma_0^2}\mathbf{V}_4N_0\odot N_0^*\right]\\
\cdot\left[\mathbf{F}_h^H\mathbf{D}_S\Sigma^{-1}\left(\mathbf{I}_N-\mathbf{D}_{\Phi}\mathbf{D}_{\widehat{\Phi}}^*\right)\mathbf{D}_{\Gamma_0}\mathbf{D}_{H_I}N_0\right]^H
\end{array}\right\}
$$

$$
= E\left\{ \mathbf{F}_{\varphi}^{H} \begin{bmatrix} \Phi_{\frac{1}{1+\Phi^*}} N_0^H \left[\mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \\ -\frac{1}{\sigma_0^2} \mathbf{V}_1 \hat{N}_0 \odot \hat{N}_0^* N_0^H \left[\mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \\ +\frac{1}{\sigma_0^2} \mathbf{V}_2 \mathbf{D}_{R_a} N_0 N_0^H \left[\mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \\ +\frac{1}{\sigma_0^2} \mathbf{V}_3 N_0 \odot \hat{N}_0 N_0^H \left[\mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \\ +\frac{1}{\sigma_0^2} \mathbf{V}_4 N_0 \odot N_0^* N_0^H \left[\mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \end{bmatrix} \mathbf{F}_h \right\}
$$

$$
= \frac{1}{\sigma_0^2} \mathbf{F}_{\varphi}^{H} \mathbf{V}_2 \mathbf{D}_{R_a} E\left[N_0 N_0^H \right] \left[\mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \mathbf{F}_h
$$

$$
= \frac{1}{\sigma_0^2} \mathbf{F}_{\varphi}^{H} \mathbf{V}_2 \mathbf{D}_{R_a} \mathbf{D}_S^H \sigma_0^2 \left[\Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \mathbf{F}_h
$$

$$
\left(P.S \quad \mathbf{\Sigma} = \sigma_0^2 \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \mathbf{D}_{H_I}^H \mathbf{D}_{\Gamma_0}^H \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right)^H \right)
$$

$$
= \frac{1}{\sigma_0^2} \mathbf{F}_{\varphi}^{H} \mathbf{V}_2 \mathbf{V}_{R_a} \mathbf{D}_S^H \left[\mathbf{D}_{H_I} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \right]^{-1} \mathbf{F}_h
$$

3. $E\left[\frac{\partial f(\omega)}{\partial h^*} \left(\frac{\partial f(\omega)}{\partial h^*} \right)^H \right]$

$$
= E\left\{ \left[\mathbf{F}_h^H \mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} N_0 \right] \left[\mathbf{F}_h^H \mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} N_0 \right]^H \right\}
$$

$$
= E\left\{ \mathbf{F}_h^H \mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} N_0 N_0^H \left[\mathbf{D}_S \Sigma^{-1} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H \mathbf{F}_h \right\}
$$

$$
= \mathbf{F}_h^H \mathbf{D}_S \mathbf{D}_S^H \Sigma^{-1} \Sigma^{-1} \underbrace{\sigma_0^2 \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \left[\left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \mathbf{D}_{H_I} \right]^H}_{=\Sigma} \mathbf{F}_h
$$

$$
= \mathbf{F}_h^H \mathbf{D}_S \mathbf{D}_S^H \Sigma^{-1} \mathbf{F}_h
$$

$$
= \frac{1}{\sigma_0^2} \mathbf{F}_h^H \mathbf{D}_S \mathbf{D}_S^H \left\{ \mathbf{D}_{H_I} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \left[\mathbf{D}_{H_I} \left(\mathbf{I}_N - \mathbf{D}_\Phi \mathbf{D}_{\hat{\Phi}}^* \right) \mathbf{D}_{\Gamma_0} \right]^* \right\}^{-1} \mathbf{F}_h
$$

G. Then, define $\mathbf{A} = E\left[\frac{\partial f(\omega)}{\partial \varphi^*}\left(\frac{\partial f(\omega)}{\partial \varphi^*}\right)^H\right]$

$$\mathbf{B} = E\left[\frac{\partial f(\omega)}{\partial h^*}\left(\frac{\partial f(\omega)}{\partial h^*}\right)^H\right]$$

$$\mathbf{C} = E\left[\frac{\partial f(\omega)}{\partial \varphi^*}\left(\frac{\partial f(\omega)}{\partial h^*}\right)^H\right]$$

Fisher Information Matrix of θ : $\mathbf{I} = E\left[\frac{\partial f(\omega)}{\partial \theta^*}\left(\frac{\partial f(\omega)}{\partial \theta^*}\right)^H\right]$

$$= E\left[\begin{bmatrix}\frac{\partial f(\omega)}{\partial \varphi^*} \\ \frac{\partial f(\omega)}{\partial h^*}\end{bmatrix}\begin{bmatrix}\left(\frac{\partial f(\omega)}{\partial \varphi^*}\right)^H & \left(\frac{\partial f(\omega)}{\partial h^*}\right)^H\end{bmatrix}\right]$$

$$= \begin{bmatrix}\mathbf{A} & \mathbf{C} \\ \mathbf{C}^H & \mathbf{B}\end{bmatrix}$$

$\therefore$ $\mathrm{CRLB}(\omega_k) = \left(\mathbf{F}_{\varphi,h}\mathbf{I}^{-1}\mathbf{F}_{\varphi,h}^H\right)_{kk}$ which can be numerically computed through computer simulation

where $(\ )_{kk}$ is its k'th diagnoal value

Chapter 6 Simulation Results

Computer simulations were conducted to evaluate the performance of proposed scheme. In the first part of simulation, the analytical results and CRLBs were confirmed and compared to the simulation results. Then, we examined performances and sensitivities of the proposed compensation scheme in the second and third part of simulation.

An OFDM system designed for IEEE 802.11a WLAN standards is considered in our simulation. The OFDM symbol is based on total 64 carriers (48 for data, 4 for pilot and others left open) uniformly distributed in 20 MHz channel bandwidth in RF band. The modulations on each carrier range from BPSK, QPSK, 16-QAM to 64-QAM.. Cyclic prefix is copy of the last quarter of each OFDM symbol. The 802.11a standards also specify one short preamble for synchronization and one long preamble for channel estimation. Our estimation is using the long preamble as prior information to do joint estimation. Following model described in Equation 1, a multi-path channel effect is constructed as a three-taps complex-valued FIR whose phases are uniformly random distributed and magnitudes are Rayleigh distributed with averaged power decaying exponentially.

For frequency-independent IQ imbalance, amplitude g and phase imbalance θ are set to be 1.08 and 5°. As to frequency-dependent IQ imbalance, it is modeled in terms of impulse responses of baseband IQ branches $h_I[n]$ and $h_Q[n]$. Due to variation of analog components, these two filters are modeled based on order-3 of low-pass Butterworth filters with different cutoff frequencies at 8.5 MHz and 8.2 MHz. The sampling rate is set to be 20 MHz.

6.1 MSE of ML Estimates

The MSE analytical expressions derived in chapter 4 were checked using fixed channel given by h=[0.7047 + 0.7047i , 0.0578 + 0.0578i , 0.0047 + 0.0047i] (L=3) Also, the CRLBs derived in chapter 5 were compared to our estimators. The MSE of I/Q imbalance Φ is calculated as its average MSE per subcarrier, and channel H is also calculated as its average MSE per subcarrier.

Figure 6-1 and Figure 6-2 show means-squared-error (MSE) of IQ imbalance and delay-spreading channel estimation. The estimation error is computed averagely in frequency domain per subcarrier. It was observed that the computer simulation results of proposed estimators coincide with the MSE analyses in high SNR, and there still exists slight mismatch in the low SNR level due to some approximations that can not be made in low SNR. Furthermore, the MSE were almost identical to the CRLBs.

Figure 6-2 also compare the performance of channel estimator with and without smoothing. The "Smoothing" refers to our proposed estimator which estimates channel impulse response in time domain, while the "No Smoothing" refers to estimating the channel independently between subcarriers in frequency domain. As expected, the smoothing property indeed obtains a large performance gain over channel estimation.

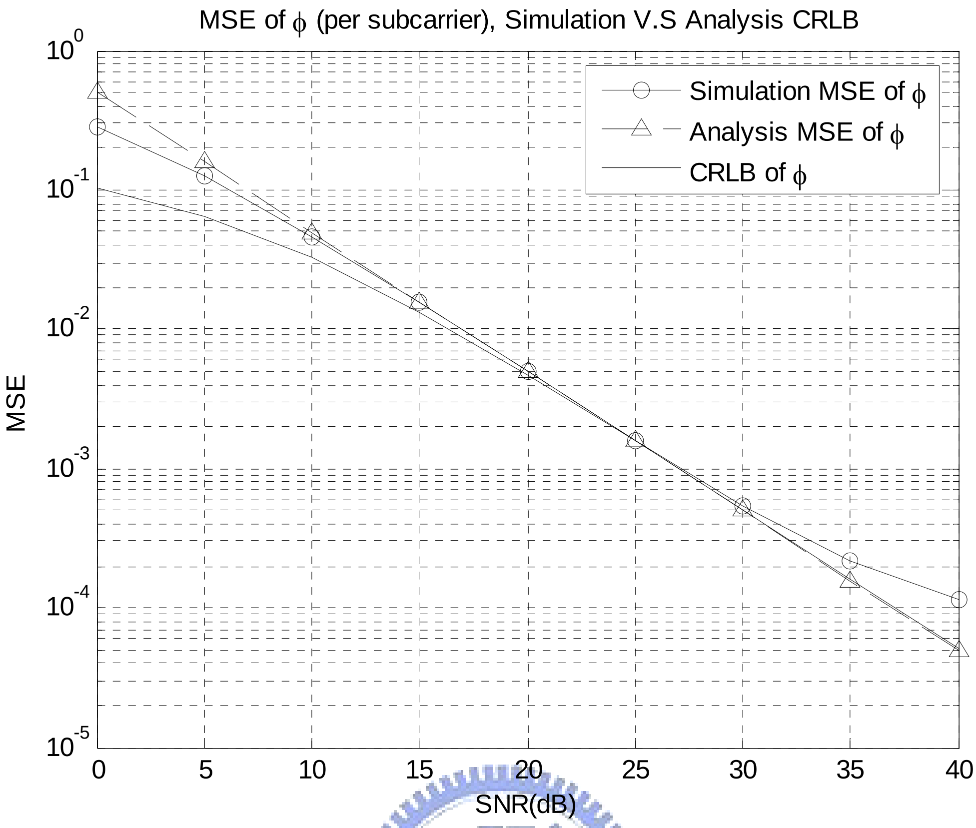


Fig.6-1 MSE of IQ imbalance estimation

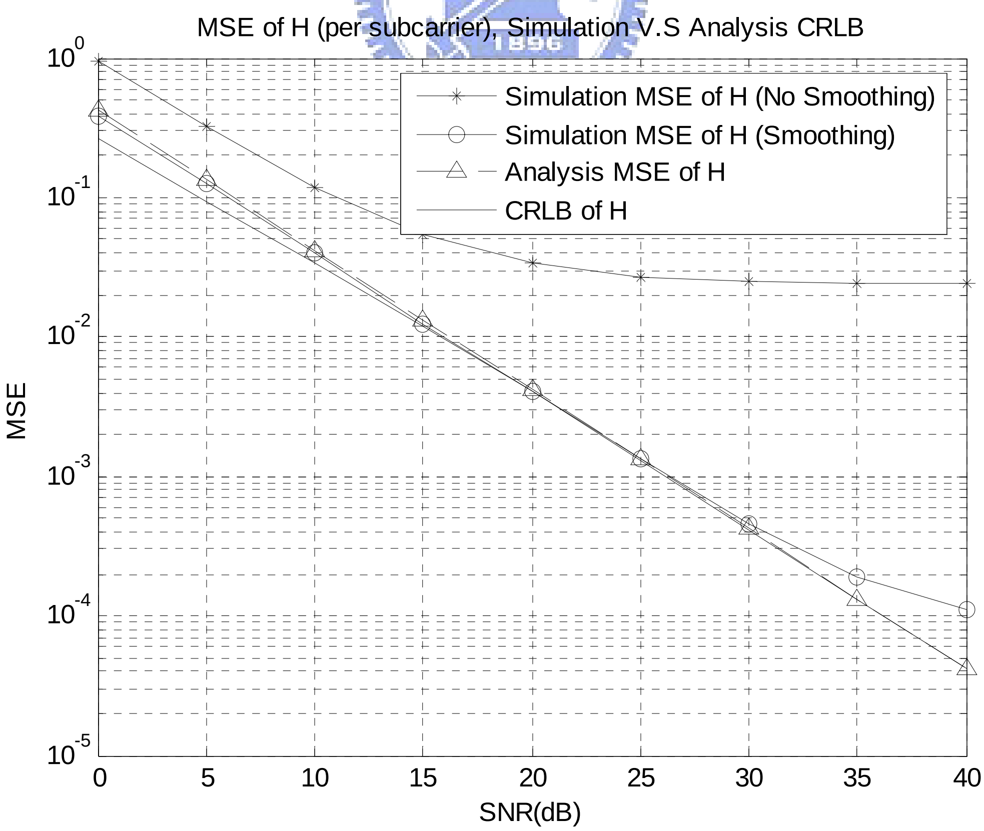


Fig.6-2 MSE of delay-spreading channel estimation

6.2 Sensitivities of Proposed Estimators

In practical situation, the effective length of channel and I/Q imbalance φ need to be estimated, thus could not be optimal and suffers from some modeling error in a real front-end filter equivalent to an infinite impulse response (IIR) filter.

It can be observed in Fig.6-3 and 6-4 that there indeed exists an optimal length $L_h = 11$ for our given I/Q imbalance and channel case. In addition, the performance will be saturate when L_h is chosen shorter than its effective optimal length, however, it suffers from just a little acceptable performance loss when L_h is chosen larger than its equivalent optimal length. Hence, this result suggests us to choose larger channel length to maintain a good performance level.

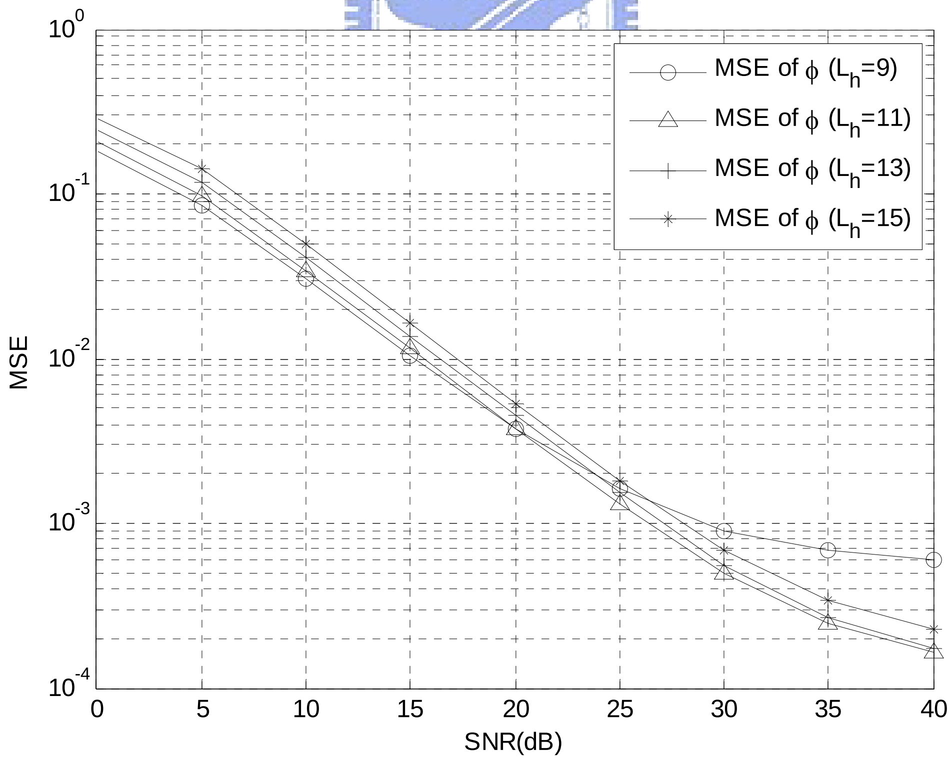


Fig.6-3 Channel estimation with various L_h

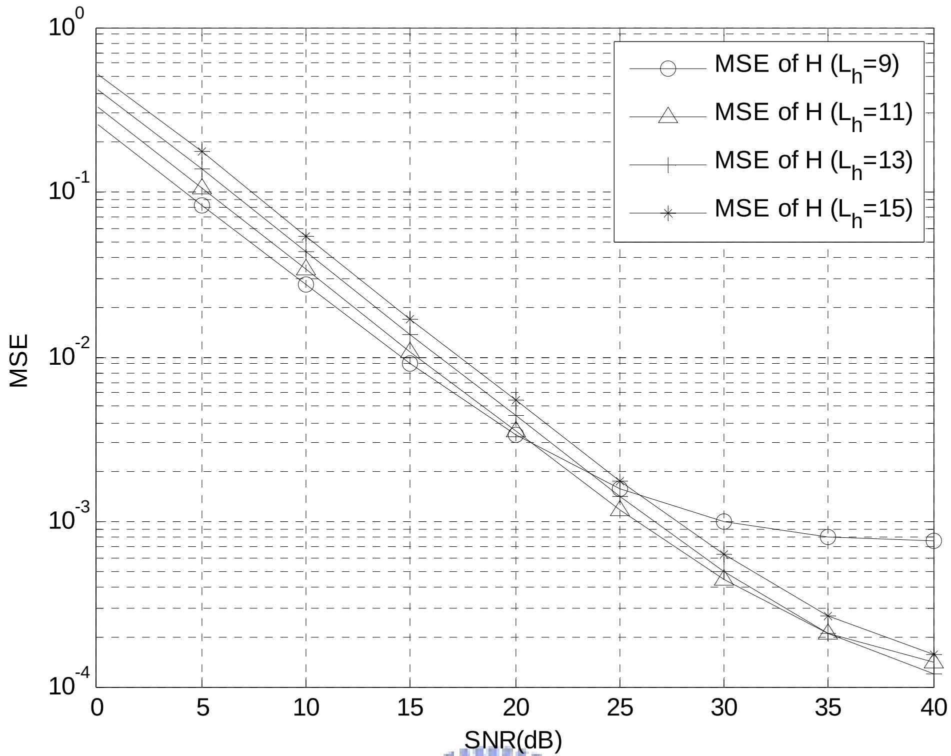


Fig.6-4 I/Q imbalance Φ estimation with various L_h

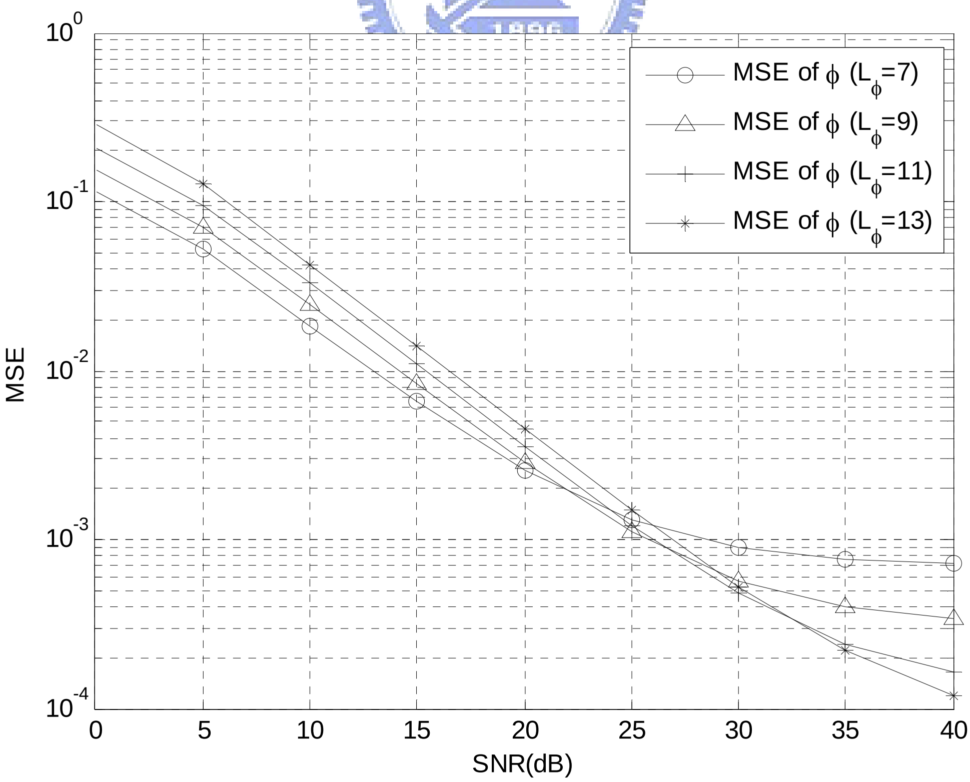


Fig.6-5 I/Q imbalance Φ estimation with various L_φ

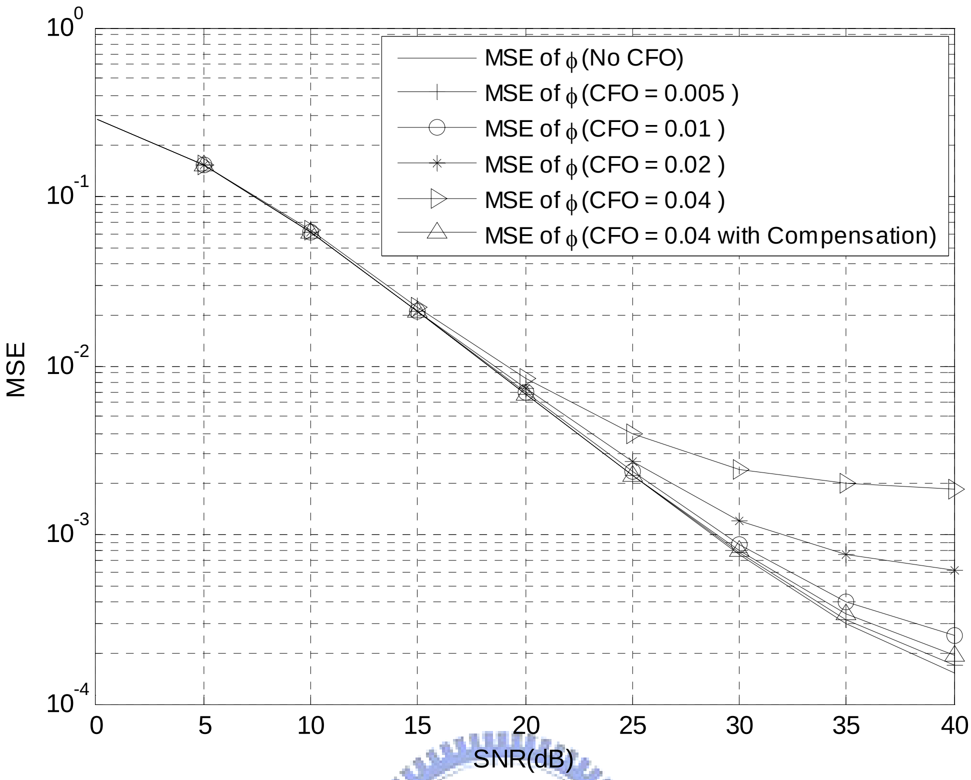


Fig.6-6 I/Q imbalance Φ estimation with various CFO effect

In Fig.6-5, different length L_φ is checked. When SNR is smaller, AWGN noise dominates interferences, and it suggests us to chose smaller length L_φ. On the contrast, when SNR is larger, I/Q imbalance may dominate interferences, then the performance will be better in choosing longer length L_φ. Hence, the length L_φ depends on operating SNR.

Finally, the severity of CFO effect on MSE and effectiveness of our proposed CFO compensation scheme in section 3.4 are examined in fig.6-6. The CFO is normalized to subcarrier spacing. It is shown that our estimation scheme can suffer CFO effect lower than 0.01. Thus, the residue of estimated CFO should be smaller than 0.01 to make our algorithm work. “ CFO =0.04 with Compensation” legend means that applying proposed CFO pre-compensation at receiver with CFO being 0.04. It can be seen that our proposed CFO compensation is effective to combat CFO effect.

6.3 Uncoded BERs Performance

The performances of the estimators were also examined in terms of the uncoded bit error rate (BER). "IQ imbalance/No Comp" refers to a receiver with I/Q imbalance but no compensation. "IQ imbalance/Comp with No smoothing" refers to ordinary OFDM channel estimation in frequency domain; whereas, "IQ imbalance/Comp with smoothing" refers to our proposed estimators in time domain. "Ideal Receiver" refers to a receiver with no I/Q imbalance. As expected, the BER curve becomes saturated in the presence of I/Q imbalance effect; on the other hand, the performances of our proposed algorithms is close to ideal receivers and outperforms the traditional OFDM channel estimation done in frequency domain.

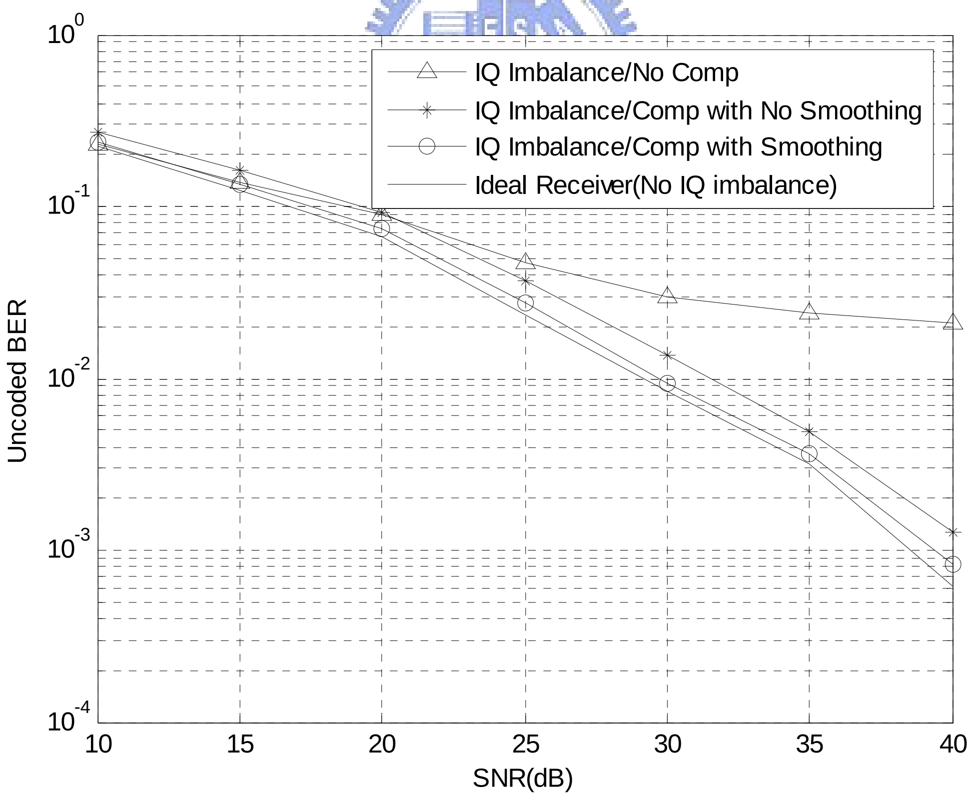


Fig.6-7 Uncoded BERs with 64 QAM

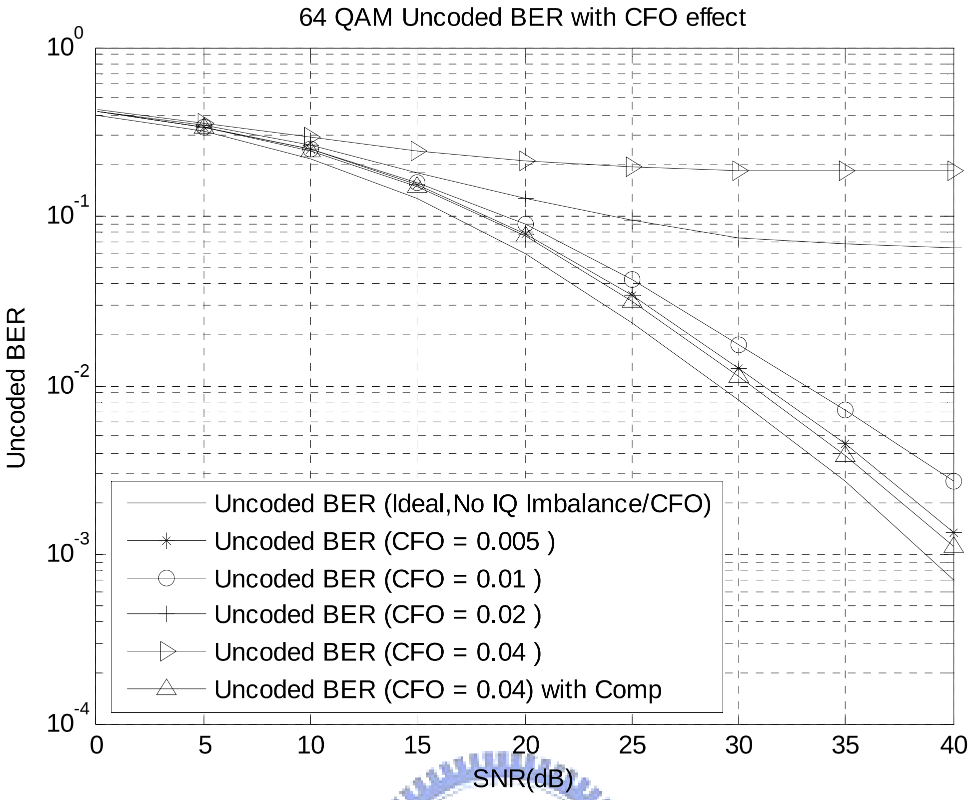


Fig.6-8 CFO sensitivities on 64 QAM Uncoded BERs

The last Fig.6-2 shows the sensitivity of CFO effect on uncoded BER performance and effectiveness of our proposed CFO compensation scheme in section 3.4. The same results with MSE performance can be seen in terms of uncoded BERs. The BERs get saturate again when estimated CFO residue error is larger than 0.01. In addition, our proposed CFO compensation scheme combined with our proposed algorithm also makes it work to achieve performance target assuming CFO has been estimated previously within certain level.

Chapter 7 Conclusions

An algorithm for joint estimation of channel and IQ imbalance effects (both of frequency-independent and frequency-dependent) for OFDM system was developed. To make use of characteristics of OFDM symbol and channel smoothing property, we construct our likelihood function using frequency data to estimate time-domain channel and IQ imbalance effects. Also, we develop carrier frequency offset compensation method combined with our algorithm assuming that carrier frequency offset has been estimated prior to our estimation. The estimation requires only one OFDM symbol as prior information to reaches performance target. No special structure for such OFDM symbol is assumed. The performance was investigated analytically and by computer simulation, which shows that the proposed algorithm reaches CRLBs as the received SNR above certain level. We also observed that our performance depends on prior information about channel length of delay-spreading channel effect and degree of difference of IQ imbalance. Some future topics about this work can be extended and researched.

(1) Apply to MIMO-OFDM systems that may take advantage of spatial diversity gains to improve performances.

(2) Take into account transmitter side I/Q imbalance and DC offset effect in MIMO technique.

(3) Develop CFO estimation method in the presence of I/Q imbalance effect.

References

[1] R. V. Nee and R. Prasad, OFDM for Wireless Multimedia Communications, Artech House Publisher, 2000.

[2] M. Engels, Wireless OFDM Systems: How to make them work, Kluwer Academic Publisher, 2002.

[3] IEEE, “Part 11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications: High speed physical layer in the 5-GHz band,” IEEE Std 802.11a-1999, Dec. 1999.

[4] Myoung-Soo Kang and Woo-Jin Song, "A robust channel equalizer for OFDM TV receivers," IEEE Transactions on Consumer Electronics, Volume 44, Issue 3, Page(s): 1129 - 1133, Aug. 1998 OFDM one tap equalizer

[5] Whang, A.J., Yi-Uan Chen and Rui-Xin Whang, "Performance evaluation of OFDM system for wideband wireless LAN with one-tap equalizer", IEEE 2002 International Conference on Communications, Circuits and Systems and West Sino Expositions, Volume 1, Page(s): 521 - 524 vol.1, 29 June-1 July 2002

[6] Sun-Wook Kim and Kyun Hyon Tchah, "Performance analysis of adaptive equalizer design for OFDM wireless LAN", IEEE Transactions on Consumer Electronics, Volume 50, Issue 2, Page(s): 512 - 516, May 2004

[7] A.A. Abidi; “Direct-conversion radio transceivers for digital communications”, IEEE Journal Solid-State Circuits, Volume 30, Issue 12, Page(s): 1399–1410, Dec. 1995.

[8] B. Razavi, “Design considerations for direct-conversion receivers”, IEEE Transactions Circuits and Systems II: Analog and Digital Signal Processing, Volume 44, Issue 6, Page(s): 428–435, June 1997

[9] Baier, A., "Quadrature mixer imbalances in digital TDMA mobile radio receivers", International Zurich Seminar on Digital Communications, 'Electronic

Circuits and Systems for Communications'. Proceedings, Page(s): 147 - 162 5-8 March 1990

[10] Chia-Ling Liu, "Impacts of I/Q imbalance on QPSK-OFDM-QAM detection", IEEE Transactions on Consumer Electronics, Volume 44, Issue 3, Page(s): 984 - 989, Aug. 1998

[11] Ykaczewski, P.; Brakensiek, J.; Jondral, F.K., "Towards an analytical model of I/Q imbalance in OFDM based direct conversion receivers", Vehicular Technology Conference, Volume 4, Page(s): 1831 - 1835 Vol.4, 17-19 May 2004

[12] M. Valkama, M. Renfors, V. Koivunen, "Compensation of frequency-selective I/Q imbalances in wideband receivers: models and algorithms", IEEE 3rd Workshop on Signal Processing Advances in Wireless Communications (SPAWC), Page(s): 42-45, March 2001.

[13] M. Valkama, M. Renfors, V. Koivunen, "Advanced methods for I/Q imbalance compensation in communication receivers", IEEE Transactions Signal Processing, Volume 49, Issue 10, Page(s): 2335–2344, Oct. 2001.

[14] T.M. Ylamurto, "Frequency domain IQ imbalance correction scheme for orthogonal frequency division multiplexing (OFDM) systems", IEEE Wireless Communications and Networking, Volume 1, Page(s): 20–25, March 2003.

[15] J. Tubbax, B. Come, L. VanderPerre, S. Donnay, M. Engels, H. DeMan, M. Moonen, "Compensation of IQ Imbalance and Phase Noise in OFDM Systems", IEEE Transactions Wireless Communications, Volume 4, Issue 3, Page(s): 872–877, May 2005.

[16] G. Gye-Tae, S. Il-Hyun, P. Jin-Kyu, Y.H. Lee, "Joint ML estimation of carrier frequency, channel, I/Q mismatch, and DC offset in communication receivers", IEEE Transactions Vehicular Technology, Volume 54, Issue 1, Page(s): 338–349, Jan. 2005.

[17] A. Tarighat, R. Bagheri, and A. H. Sayed, “Compensation schemes and performance analysis of IQ imbalances in OFDM receivers”, IEEE Trans. Signal Process., vol. 53, no. 8, pp. 3257–3268, Aug. 2005

[18] X. Guanbin, S. Manyuan, L. Hui, “Frequency offset and I/Q imbalance compensation for direct-conversion receivers”, IEEE Transactions Wireless Communications, Volume 4, Issue 2, Page(s): 673–680, March 2005.

[19] A. can den Bos, ”A Cramer-Rao bound for complex parameters,” IEEE Trans. Signal Processing, vol. 42, p. 2859, Oct. 1994

[20] Deneire, L.; Vandenameele, P.; van der Perre, L.; Gyselinckx, B.; Engels, M.; “A low complexity ML channel estimator for OFDM,” Communications, 2001. ICC 2001. IEEE International Conference on Volume 5, 11-14 June 2001 Page(s):1461 - 1465 vol.5