

國立交通大學

機械工程學系

碩士論文

整數階與分數階廣義心搏系統的
渾沌及其同步與反控制

Chaos, Its Synchronization and Anticontrol of Integral and
Fractional Order Generalized Heartbeat Systems

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摘要

本篇論文研究整數階與分數階的廣義心搏系統，其形式包括非自治與自治系統。首先研究受正弦時間函數激發的整數階與分數階非自治廣義心搏系統與自治廣義心搏系統的渾沌現象。透過數值分析，如相圖，Poincaré映射，分叉圖，可以觀察到週期與渾沌運動。發現分數階系統的總階數少於或多於原系統的狀態數目時皆存在渾沌現象。接著研究該系統的渾沌激發的渾沌同步。兩個相同的整數階或分數階自治廣義心搏系統的激發項的振幅及正弦函數，分別以第三個同樣階數的自治或是非自治廣義心搏系統狀態項所取代後皆可成功地產生同步現象。透過數值分析，即相圖與狀態誤差圖可以觀察到兩系統的渾沌同步。最後研究該系統的反控制。只需在該系統中加入常數項即可順利地獲得反控制效果，可藉數值分析，如相圖，分叉圖，得到驗證。

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ABSTRACT

In this thesis, chaos of a generalized heartbeat system with integral and fractional orders is studied. Both nonautonomous and autonomous systems are considered in detail. Chaos in the nonautonomous generalized heartbeat system excited by a sinusoidal time function with integral and fractional orders is studied. Next, chaos in the autonomous generalized heartbeat system with integral and fractional orders is considered. Numerical analyses, such as phase portraits, Poincaré maps and bifurcation diagrams are observed. Chaos can be successfully obtained in the fractional order system with the total order both less than and more than the number of the states of the integral order generalized heartbeat system. Chaos excited chaos synchronizations of generalized heartbeat systems with integral and fractional order are studied. Synchronizations of two identical autonomous generalized heartbeat chaotic systems are obtained by replacing the amplitude or the sine time function of their corresponding exciting terms by the same function of chaotic states of a third nonautonomous or autonomous generalized heartbeat system, respectively. Numerical simulations, such as phase portraits, Poincaré maps and state error plots are given. Chaos excited chaos synchronizations can be successfully obtained for the fractional order systems with the total fractional order both less than and more than the number of the states of the integral order generalized heartbeat system. Anticontrol of chaos for integral and fractional order generalized nonautonomous heartbeat system is obtained effectively by adding a constant term to the system. By numerical analyses, such as phase portraits, Poincaré maps and bifurcation diagrams, anticontrol can be observed evidently.

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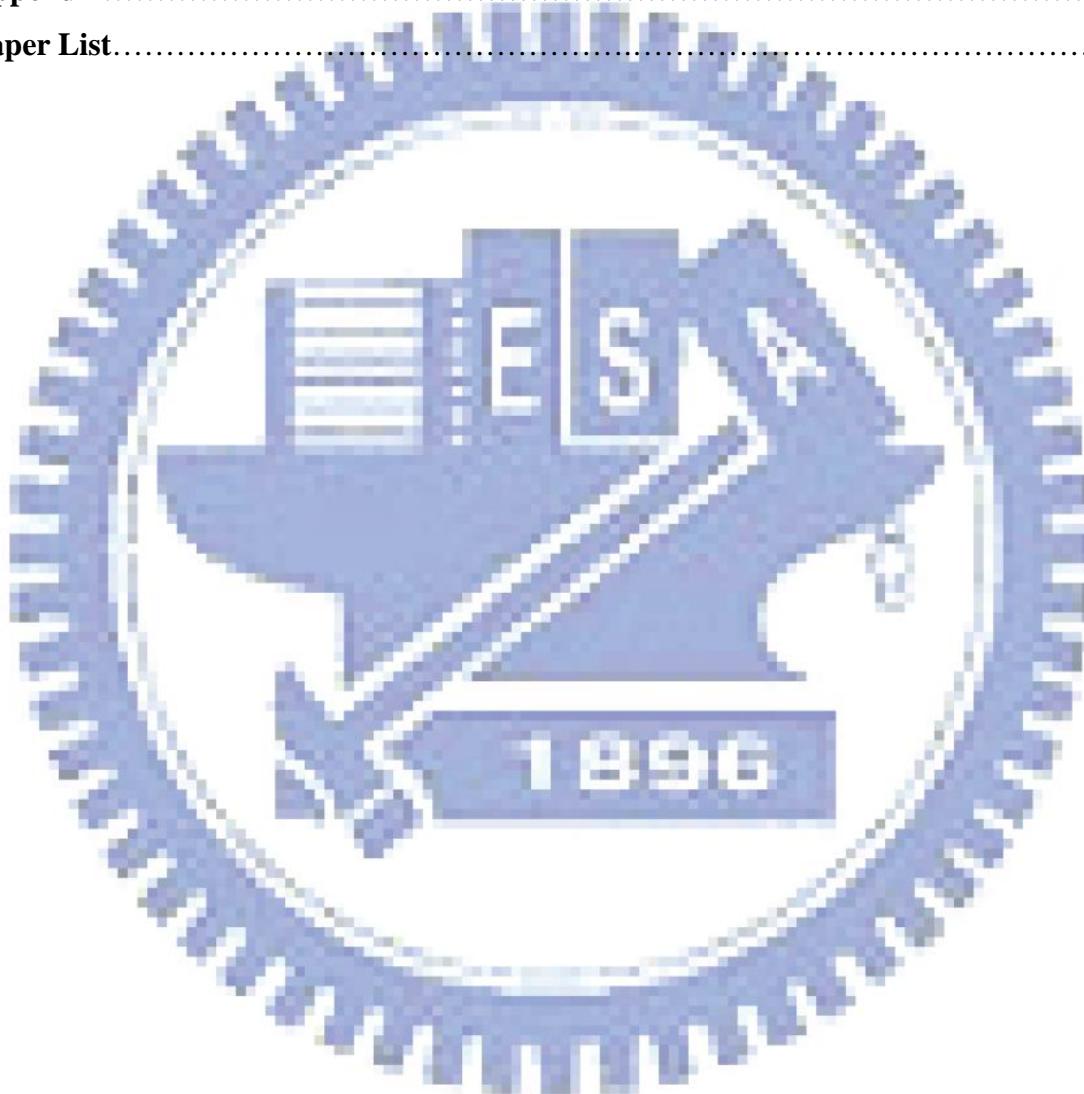
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