

國立交通大學

機械工程學系

碩士論文

整數階與分數階廣義心搏系統的
渾沌及其同步與反控制

**Chaos, Its Synchronization and Anticontrol of Integral and
Fractional Order Generalized Heartbeat Systems**

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國 立 交 通 大 學
機 械 工 程 學 系
碩 士 論 文

A Thesis

Submitted to Department of Mechanical Engineering

College of Engineering

National Chiao Tung University

In Partial Fulfillment of the Requirements

For the Degree of Master of Science

In

Mechanical Engineering

June 2006

Hsinchu, Taiwan, Republic of China

中華民國九十五年六月

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摘要

本篇論文研究整數階與分數階的廣義心搏系統，其形式包括非自治與自治系統。首先研究受正弦時間函數激發的整數階與分數階非自治廣義心搏系統與自治廣義心搏系統的渾沌現象。透過數值分析，如相圖，Poincaré映射，分叉圖，可以觀察到週期與渾沌運動。發現分數階系統的總階數少於或多於原系統的狀態數目時皆存在渾沌現象。接著研究該系統的渾沌激發的渾沌同步。兩個相同的整數階或分數階自治廣義心搏系統的激發項的振幅及正弦函數，分別以第三個同樣階數的自治或是非自治廣義心搏系統狀態項所取代後皆可成功地產生同步現象。透過數值分析，即相圖與狀態誤差圖可以觀察到兩系統的渾沌同步。最後研究該系統的反控制。只需在該系統中加入常數項即可順利地獲得反控制效果，可藉數值分析，如相圖，分叉圖，得到驗證。

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ABSTRACT

In this thesis, chaos of a generalized heartbeat system with integral and fractional orders is studied. Both nonautonomous and autonomous systems are considered in detail. Chaos in the nonautonomous generalized heartbeat system excited by a sinusoidal time function with integral and fractional orders is studied. Next, chaos in the autonomous generalized heartbeat system with integral and fractional orders is considered. Numerical analyses, such as phase portraits, Poincaré maps and bifurcation diagrams are observed. Chaos can be successfully obtained in the fractional order system with the total order both less than and more than the number of the states of the integral order generalized heartbeat system. Chaos excited chaos synchronizations of generalized heartbeat systems with integral and fractional order are studied. Synchronizations of two identical autonomous generalized heartbeat chaotic systems are obtained by replacing the amplitude or the sine time function of their corresponding exciting terms by the same function of chaotic states of a third nonautonomous or autonomous generalized heartbeat system, respectively. Numerical simulations, such as phase portraits, Poincaré maps and state error plots are given. Chaos excited chaos synchronizations can be successfully obtained for the fractional order systems with the total fractional order both less than and more than the number of the states of the integral order generalized heartbeat system. Anticontrol of chaos for integral and fractional order generalized nonautonomous heartbeat system is obtained effectively by adding a constant term to the system. By numerical analyses, such as phase portraits, Poincaré maps and bifurcation diagrams, anticontrol can be observed evidently.

誌

謝

師恩浩瀚，教澤永銘，走過必留下痕跡，感謝我的恩師 戈正銘教授。

終於要畢業了，精采且難忘；這兩年來的研究生涯，非自己一人可完成；首先感謝啟蒙我研究生涯的恩師 戈正銘教授，教授的虛懷若谷與博學多聞，除了教導學生在專業的研究領域上有著深入的了解外，老師的深厚文學、史學、哲學素養甚至達觀的待人處世與生活態度，都深深地善誘引導著學生，使學生受益匪淺。

在兩年的研究生活，承蒙學長的教導與照顧。感謝博士班陳炎生、張晉銘與楊振雄學長的種種建議與熱心指導，給予我新的啟發，使我的論文得以順利完成；謝謝已畢業的碩士班學長莊為任，楊坤偉與林國樺學長幫助我在青澀的小學弟時帶領我進入這個渾沌研究的大家庭，謝謝你們的照顧與提攜；以及親愛的同學歐展義、易昌賢與張安瑞，感謝大家在課業、研究及生活上的互相扶持，共同打拼，每一晚的討論聲與歡笑聲都將是我最美好的回憶；並且感謝學弟宗訓、乾豪、式中與學妹郁婷，謝謝活潑可愛的你們帶給我愉快的研究生涯且美好的回憶。

最要感謝我最愛的家人，在家裡最辛苦時仍舊支持我繼續走在研究的道路上；辛苦的母亲與兄長默默地培養與支持著我成長，以青春歲月與大好前程培育我成為一個社會的中堅青年，謝謝大姐在求學的道路上與我互相砥礪與鼓勵；特別謝謝我心愛的女友LOLO的娘，謝謝妳的細心體貼與支持鼓勵，讓我每天生活充滿力量，是你支持著我在即將走出象牙塔的艱澀時期勇往直前。

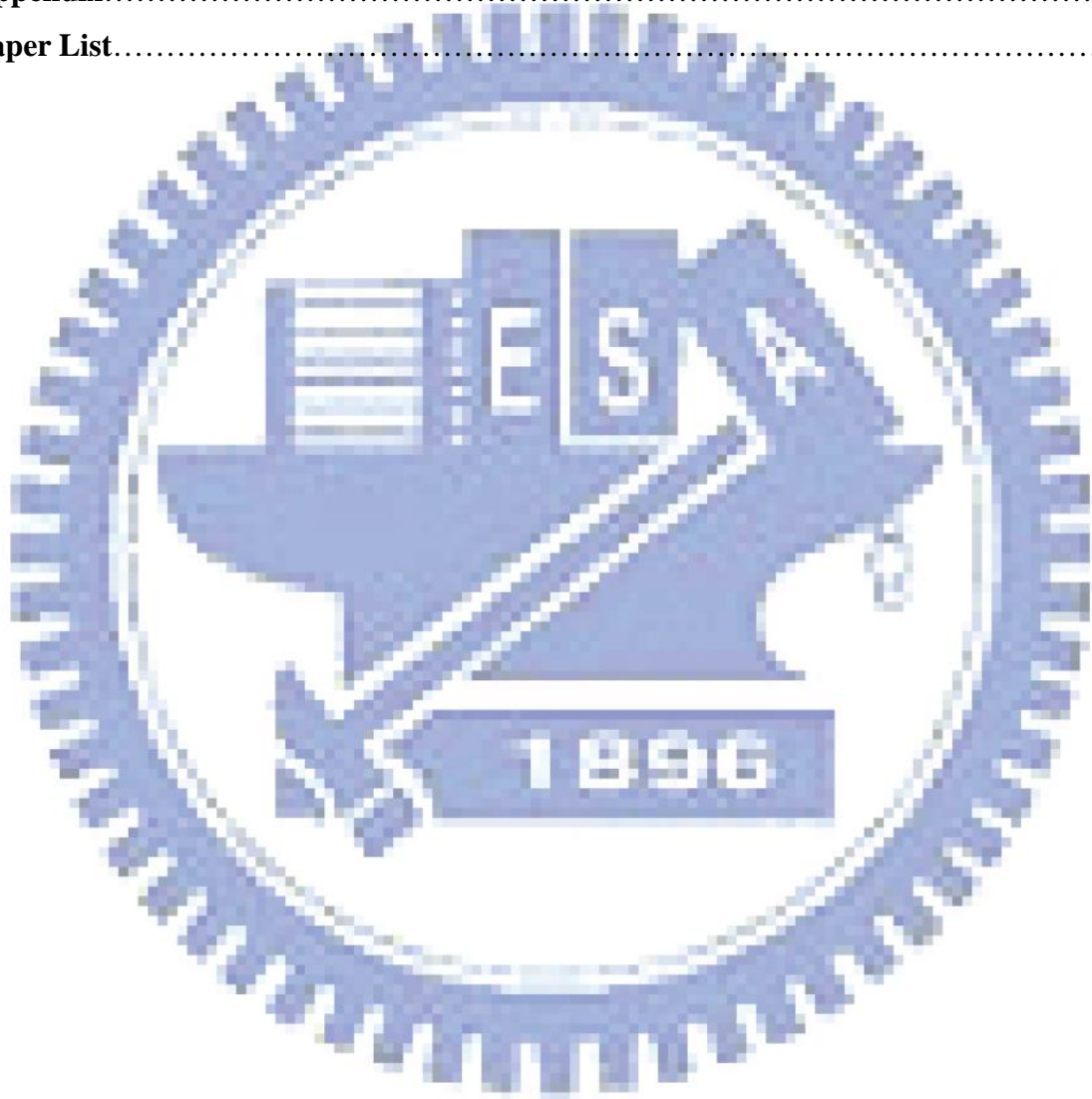
兩年的研究生活一晃眼就過去，求學的点點滴滴一一記在腦中，離情依依，在此謝謝同門師兄弟們的鼎力相助，由衷的感謝。

最後，我願以此篇論文獻給遠在天國的父親，因您永遠在我心中讓我毫無所懼絕不猶豫地向前走，我願繼承您的心志與心願，努力的讓自己發光發熱，請您為我感到驕傲，因為您我會更加茁壯。

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