國立交通大學

資訊科學與工程研究所

碩 士 論 文

在無線隨意網路中格狀繞境之臨界格子大小及傳輸半徑

The Critical Grid Size and Transmission Radius for Local-Minimum-Free Grid Routing in Wireless Ad Hoc Networks

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摘要

格狀繞徑法用於無線隨意網路，是一種以地理資訊為基礎的繞徑協定。在格狀繞徑法中，平面被分割成相同大小正方形網格。兩個網格被稱為鄰近網格倘若他們有共同的邊，兩個節點被稱做鄰近節點倘若他們位於鄰近網格且在彼此的傳輸範圍之內。除了最後一次的跳躍外，封包每次被送至一個較目前更為接近內含目的地節點的網格。

作為一個貪婪式的策略，最大的困難與挑戰在於如何克服局部最小點的存在，也就是當目前封包所在的節點，找不到一個鄰近節點比自己更為靠近目的地節點，可以代為繼續傳送封包。本論文主要探討如何藉由適當調整格狀繞徑法中兩個重要的參數—格網大小與傳輸半徑—來保證封包的可到達性。

本文假設節點是依照平均值為 n 的普瓦松點過程分布在一個單位面積的正方形上，令 l 代表格網大小，r 代表傳輸半徑。若 $l=\sqrt{\frac{\beta \ln n}{n}}$，首先我們證明$\beta$=*1*為保證封包可到達性下，格網長度最小的臨界值。 換句話說，若β>*1*，則幾乎可保證不存在局部最小點；若β<*1*，則幾乎保證會存在局部最小點。其次，在給定任一個大於一之常數β，對於傳輸半徑r，我們找到一個充分條件來保證封包可到達性。最後，我們找出最小的傳輸半徑 $r \cong 2.09\sqrt{\frac{\ln n}{n}}$ 對應於 $\beta \cong 1.092$。本文亦討論了自來源到終點之間平均需要跳躍的次數，及每個網格平均交通流量。最後以模擬的方式驗證理論推導值的正確性。

關鍵字: 無線隨意網路、地理貪婪式繞徑、格網繞徑、局部最小點、隨機佈署

The Critical Grid Size and Transmission Radius for Local-Minimum-Free Grid Routing in Wireless Ad Hoc Networks

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Abstract

Grid routing is a geographic-based routing protocol for wireless ad hoc networks. In grid routing, the plane is tessellated into equal-sized square cells. Two cells are called neighbor cells if they share a common boundary, and two nodes are called neighbor nodes if they are located in neighboring cells and within each other's transmission range. In each hop excepting the last one, each packet is forwarded to a neighbor node which is in a cell closer to the cell containing the destination node.

As a greedy local strategy, nodes using grid routing may not be able to forward packet when none of its one-hop neighbors lies in a neighboring cell that is closer to the destination; this phenomenon is called local minimum. To guarantee the deliverability, in this thesis, we investigated two vital parameters of grid routing, the grid size and the transmission radius.

Assume nodes are represented by a Poisson point process with rate n over a unit-area square, and let l denote the grid size and r denote the the transmission radius. First, we show that if $l = \sqrt{\frac{\beta \ln n}{n}}$ for some constant β and $r = \sqrt{5}l$, then $\beta=1$ is the threshold for deliverability. In other word, there almost surely don't exist local minima if $\beta>1$ and there almost surely exist local minima if $\beta<1$. Next, for any give $\beta>1$, we gave a sufficient and necessary condition to determine the critical transmission radius for deliverability. Then we showed that the minimum critical transmission radius is $r \cong 2.09\sqrt{\frac{lnn}{n}}$ which happens as $\beta \cong 1.092$. In addition, we derived the average hop-count between any source to destination and the average traffic flow through the cells. Finally, we conducted two simulations to verify our theoretical results.

Keywords: *Wireless Ad Hoc Network, Geographic Greedy Routing, Grid Routing, Local Minima, Random Deployment*

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Chapter 1

Introduction

1.1 Wireless Ad Hoc Networks

The wireless communication devices permeate throughout our lives as the technology advances and the price drops at the same time. People from both industrial and research communities are highly motivated to explore on the possible applications that utilize connections over these autonomous devices to supplement or even eliminate the necessity of a pre-established network infrastructure. The devices form a self-organizing network and autonomously collaborate with each other to transport information packets by serving as both end systems and routers at the same time. A communication session is established either through a single-hop radio transmission if the communication parties are close enough, or through relaying by intermediate devices otherwise. Since devices may move, join or leave the network freely and without prior notice, routing in such dynamic environment has to be robust and scalable.

Different routing protocols have been proposed, which can be broadly categorized as topology-based and geographic-based routing protocols. Topology-based routing protocols use the logical link information in the network to determine packet forwarding routes. They can be further distinguished into proactive and reactive from how they respond to routing requests. Proactive routing protocol, such as Dynamic Destination-Sequenced Distance-Vector Routing (DSDV) [1], Optimized Link State Routing (OLSR) [2], and Topology Broadcast based on Reverse-Path Forwarding (TBRPF) [3] [4], maintains all the routes available in the network even if the routes are not used currently. The major drawback is the high overhead, in term of traffic and computation, of maintaining up-to-date routing information in face of the frequently changing topology. To alleviate the burden, some reactive routing protocols have been proposed, such as Dynamic Source Routing (DSR) [5] and Ad hoc On Demand Distance Vector (AODV) [6], to maintain only routes currently in use. However, they still have some inherent drawbacks. First since the routes are maintained as demanded, the first packet will experience some delay while performing route discovery. Second, the number of currently in used routes can be huge when traffic pattern is scattered which hinders the saving of computation and storage as envisaged.

Recent researches [7] [8], however, have shown that these routing strategies without using geographical information in routing decisions, are not scalable. Since the cost of maintaining updated routing information about the entire route is expensive, researchers are instead seeking ways to route with partial knowledge of the route, for example only the directly reachable neighbors. Each node uses its best knowledge of the network in making routing decisions, often guided by the location information, to reach the desired destination. However, there are certain difficulties that have to be conquered while using geographic-based routing algorithms which will be visited in chapter 2.

In this thesis, we will assume some location service systems are available. In [9], a survey on this topic is available.

1.2 Goal and Organization of the Thesis

In this thesis, we will study on combating the existence of local minimum by topology control approach to ensure packet delivery using grid routing protocol. In grid routing, the plane is tessellated into equal sized square cells; two cells are called neighbors if they share a common edge and two nodes are neighboring nodes if they are located in neighboring cells and within each other's transmission range. For a communication session, the cell contains the source node is call source cell and the cell contains the destination node is called destination cell. As one node send or relay packets, if the destination node is in the same cell, packets are delivered directly to its destination; otherwise, packets are greedily forwarded to one node in a neighboring cell that is closer to the destination cell measured by the Manhattan distance, that is the sum of the cell distance in both vertical and horizontal direction. Thus, a local minimum occurs as there is no neighboring node in the neighboring cells closer to the destination cell.

In such a scheme, there are two vital parameters to be decided, namely the grid size and the transmission radius. Instead of proposing a new remedial strategy, to guarantee the deliverability of packets, we seek to eliminate the existence of local minimum via properly setting these two parameters. In particular, we will discuss three asymptotic from the probabilistic aspect, including the critical grid size that asymptotic almost surely (abbreviated by a.a.s.) guarantees there does not exist empty grid, the critical transmission radius that a.a.s. guarantees there always exit relayers, and the smallest critical transmission radius over all grid size configurations. In addition, we also derive the average hop count and traffic load of each cell.

The rest of the thesis is organized as follows. In Chapter 2, we collected several related issue and present a brief discussion. In Chapter 3, we presented several useful geometric and probabilistic results, including asymptotic scan statistics over Poisson point processes. In Chapter 4, we derive the critical grid size and critical transmission radius for local-minimum-free grid routing. Also, we give the average routing distance and cell traffic load in networks applying grid routing in section 4.4. In Chapter 5 some simulation results on the grid cell length and minimum transmission radius are presented. Finally we summarize the thesis in

Chapter 5.3.

1.3 Notations

Notice that due to the randomness of locations of nodes that may be caused by the mobility or random deployment, we assume that wireless devices are represented by a Poisson point process with rate n over a unit-area square, and every nodes have with the same transmission radius r_n. In what follows, the disk of radius r centered at x is denoted by $B(x, r)$. The unit-area square centered at the origin is denoted by D. An event is said to be asymptotic almost sure (abbreviated by a.a.s.) if it occurs with a probability converges to one as $n \to \infty$. The symbols $O, \Theta, \Omega, o, \sim$ always refer to the limit $n \to \infty$. To avoid trivialities, we tacitly assume n to be sufficiently large if necessary. For simplicity of notation, the dependence of sets and random variables on n will be frequently suppressed.

Chapter 2

Related Issues

Geographic-based routing algorithms for wireless ad hoc networks are proposed to lessen the maintenance burden of conventional topology-based routings by utilizing location information. The recent advancement of technologies in GPS and relative coordinate positioning system using signal strength or topology information [10] [11] shows the feasibility of applying geographic information in routing decisions. Yet several supporting techniques still have to be devised such as location service, forwarding strategy, and remedial strategy when encounter difficulty while forwarding to guide on selecting the appropriate next hop neighbor intelligently.

2.1 Location Services

The hierarchical nature of IP address conventionally reveals geographic location information which also enhances scalability through route entries aggregation. The topological variation is handled in a stratified fashion without propagating to the outside. In a network where topology rarely changes, the topological information can be proactively distributed and each router can pre-compute the routes using relatively inexpensive algorithms. In contrast the identifiers no longer serve any physical meaning of revealing the geographical location in the wireless mobile ad hoc network. Hence some form of location service system has to provide the missing link.

To be fully self-content, devices have to take part in serving the role of location servers, responsible for recording registration and answering queries about peers' locations. Hence it is important to consider the extra burden placed upon the capacity, lifetime, and responding time of the network and participating devices.

The topology-based strategies mentioned in the sections 1 used the relatively expensive way of sending updates and query traffic by broadcast where each node in the network practically has the location information of every nodes. To provide better spatial network usage, Hubaux et al proposed a location service framework [12] by assigning a fixed geographical region as the Virtual Home Region (VHR) for each node by hashing its identifier. Node reports its location information and queries destination node location to the nodes in the corresponding VHR. This approach isolates the update traffic into geocasting, however, it may suffers the possibility of detour update and queries resulting in longer response time.

Imitating the conventional wisdom of overlaying a logical hierarchy on top of the physical network, Li et al [8] proposed a distributed and scalable location services framework called Grid Location Service, GLS. To elongate the network lifetime, GLS balances the location server work evenly across all the nodes assuming the random distribution of node IDs across the network. For a particular node, the distribution of number of location servers is denser at the nearby grids while sparser further away. This approach effectively resolves the detour update and query with guarantee on the performance and confined region of traffic.

2.2 Forwarding and Remedial Strategies

Devices now have to make routing decision based on partial knowledge of the network with guidance of location information provided by the location service system. Generally each node takes the local greedy approach in hoping of a successful packet delivery. The common challenge awaited is the possible existence of physical void along the routing path where there is no benefit can be obtained according to the greedy criteria this phenomenon is also know as local minimum. Thus, even if there exist paths between source and destination nodes, packets can't be delivered as local minimum are met. Thus to achieve high deliverability, we need to device both forwarding mechanism on choosing the good relayer and remedial strategy when local minimum is encountered.

2.2.1 Forwarding Strategies

Greedy Packet Forwarding

When routing decision is to be made, based on the location information either carried in the packet or by its knowledge to the destination, current node chooses one or many of its most promising one-hop neighbors as the forwarder(s) according to some criteria.

One of the intuitive ways is to choose the neighbor that can make the most forward progress (MFP) toward the destination [13]; it tries to minimize the number of hops in the routing path. Another criteria is the nearest with forward progress (NFP) [14], the packet is transmitted to the nearest neighbor which is closer to the destination; it is commonly known that using smaller transmission radius can benefit the spatial network capacity [15]. Kranakis et al [16] suggested of selecting the node that is most aligned to the straight line between current node and the final destination in compass routing (Compass); it tries to minimize the spatial distance a packet travels under the assumption that the embedded network is compass-friendly otherwise it may suffer the risk of looping [17] which require some memorization to be done. Take the following figure 2.1 as an example, MFP will choose node c while NFP chooses node b, and Compass chooses node a as their forwarder respectively.

Finally, it is also possible for the current node to randomly choose a neighbor that is closer to the destination as the forwarder; it minimizes the accuracy of location information

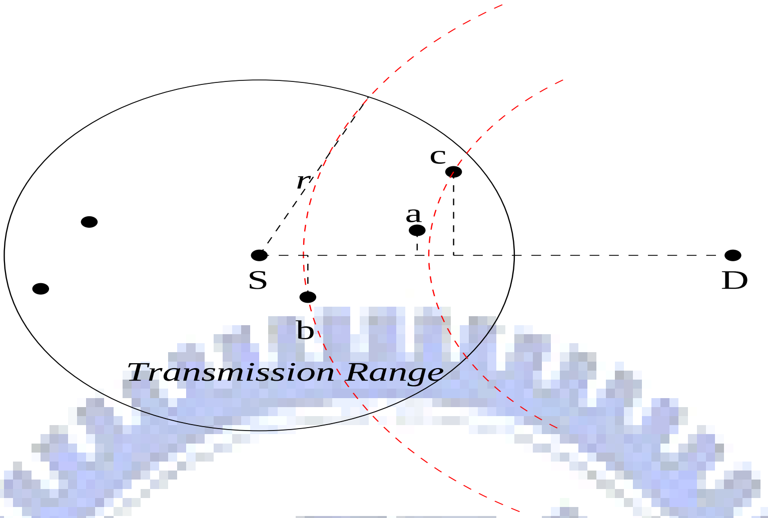

Figure 2.1: Choosing Forwarder Greedily

needed and the number of operations required to forward a packet.

Restricted Directional Flooding

Researchers proposed on selecting multiple forwarders in cope with the dynamic nature of the network. One intuitive way is to forward to all neighbors that are closer to the destination. However, the high flooding rate might cause the broadcast storm problem [18]. Bosagni et al [19] proposed to select all the neighbors that lie "in the direction" of the destination. It uses the location information and the maximum speed of the destination to calculate the expected region of the destination. All the neighbors falling within the region bounded by the two rays starting from the current node to the circular boundary of the expected region satisfy the criteria of serving as the forwarding node. Take the following fig.2.2 as an example, node b and c will be selected. The Location Aided Routing [20] also utilizes the similar concept in enhancing the route discovery by restricting the flooding to a certain area in the reactive ad hoc routing approaches.

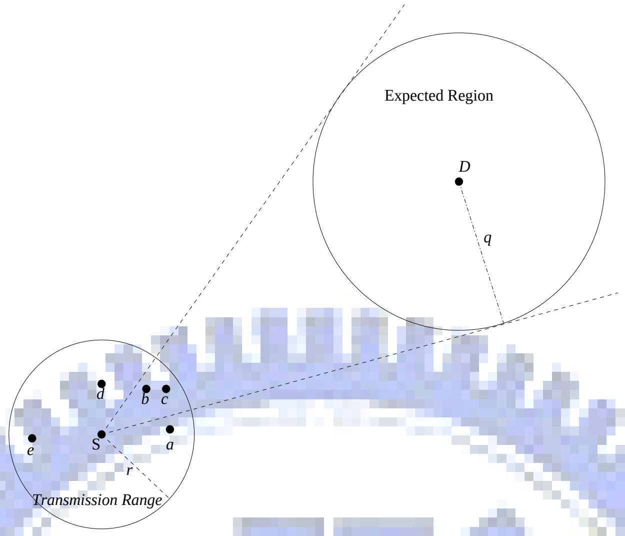

Figure 2.2: Expected Region

Hierarchical Routing

It is a common practice seen in the computer science fields to enhance the scalability by establishing some form of hierarchical structure. The deployment region is partitioned into hierarchy, and different strategies are used accordingly. For long distance routing, often a greedy strategy is used with guidance of location service system and switches to conventional topology-based routing scheme if the destination is close enough to the current node. This hierarchical approach relaxed the stringent requirement on both temporal and spatial location accuracy by distance effect also the network appears to be less dynamic in a larger resolution.

Several two-level hybrid geographical routing strategies have been proposed. In terminode project [21] [12], the proactive method is used for local traffic while the greedy anchored geodesic packet forwarding is used for remote traffic. Some zone-based routing protocols were proposed separately by Ng et al [22], Li et al [23], and Liao et al [24] by partitioning the network into fixed size zones. For intrazonal traffic, traditional topology-based methods are used to enhance routing efficiency with affordable overhead. For inter-zone traffic, either link-state protocols or greedy geographic-based protocols can be used as demanded by different communication patterns. For example, if the traffic is evenly spread throughout the entire network, using link-state protocol can reduce the latency; while if the traffic pattern reveals some form of locality then the greedy local routing algorithm may be better to minimize the routing maintenance. However, even if hierarchy is introduced into the greedy geographical routing algorithms, the existence of local minimum still pervades.

2.2.2 Remedial Strategies

Most previous research works are with some remedial detour strategies when encountering local minimum (see fig.2.3).

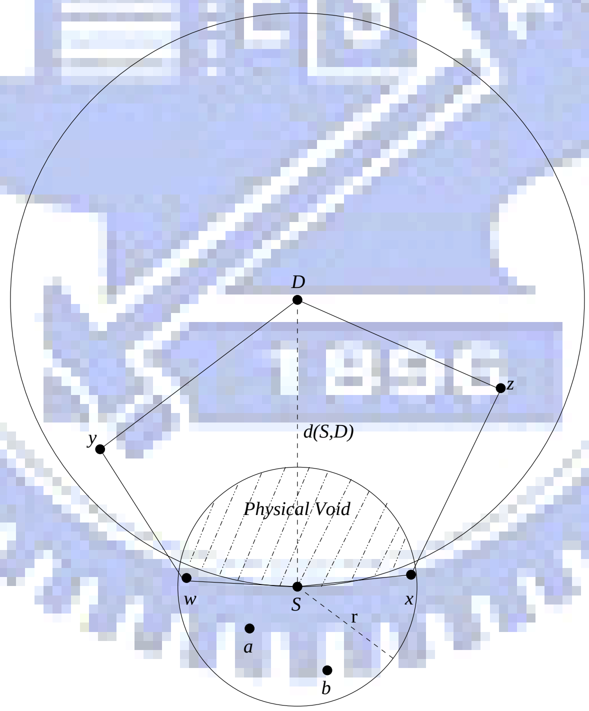

Figure 2.3: Local Minimum

The simplest way is to discard the packet when no positive progress can be made. This is applicable for real-time applications since late arrival of retransmission doesn't mean too much. The current node can also switch to route discovery mode when encountering local minimum by bread first search or depth first search. An intuitive way is to select a node that leads to least retreat; however, this may introduce the problem of looping packets which does not happen if only positive progress is allowed. Still another simple way is to flood the packet to all the neighbors who have not received the packet; this approach ensures deliverability provided a path exists between source and destination with the cost of requiring either the sending or receiving node to maintain temporal memory about the packets forwarded/received.

To decrease the flooding problem, Lin et al [25] proposed an approach of forwarding the message to only one neighbor in each connected components of the current node. Since there are only at most four such components of any concave node in the unit disk graph the degree of flooding can be largely decreased.

Several face routing algorithms were proposed to combat the occurrence of local minima on the embedded planar graph [16] [26] [27] [28]. However as pointed out by Kim et al [29], face routing algorithms may fail with realistic, non-ideal radios when the idealized unit-disk graph assumption no longer holds. Leong et al proposed [30] a geographical routing algorithm without planarization by constructing a spanning tree.

Chapter 3

Preliminaries

For the completeness of this thesis, we have listed here some necessary background knowledge before deriving our main results.

3.1 Geometric Preliminaries

If C is a convex compact set, we use C_{-r} to denote the set of points of C that are part from ∂C by at least r. According to the isodiametric inequality [31], the disk with diameter d has the largest area $\frac{1}{4}\pi d^2$ over all measurable sets with diameter d and the longest perimeter πd over all convex compact sets with diameter d.

Lemma 1 *$C \subset R^2$ is a convex compact set with diam $(C) \leq d$. We have*

$$|C_r| \leq |C| + \pi dr + \pi r^2, \text{ and}$$
$$|C_{-r}| \geq |C| - \pi dr.$$

Proof. First, we assume C is a polygon. For C_r, we draw two perpendicular lines to the edges of C at each vertex of C. $C_r - C$ is divided into disjoint rectangles and sectors. See

Fig. 3.1.

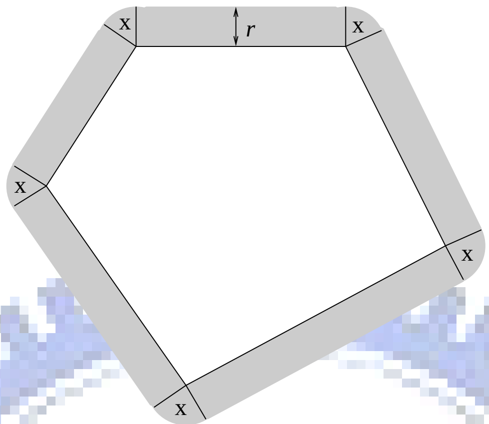

Figure 3.1: C_r

All rectangles are with width r, and the sum of their length is equal to the perimeter of C. All sectors (marked by "x") are with radius r, and the sum of their angles is equal to 2π since the angle of the sector is supplementary to interior angle. Therefore, $|C_r| = |C| + peri\,(C)\,r + \pi r^2$. Since $diam\,(C) \leq d$, we have $peri < \pi d$, and the inequality follows. For C_{-r}, we draw a rectangle by each edge with width r toward the inner. See Fig. 3.2

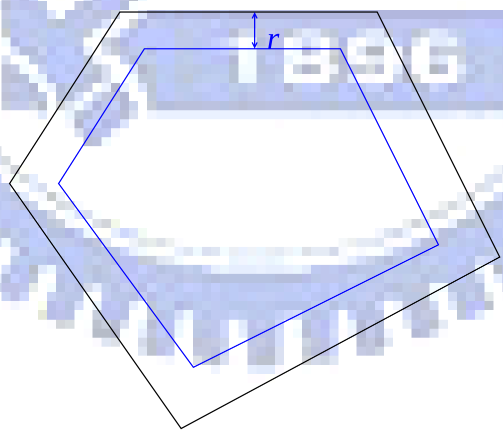

Figure 3.2: C_{-r}

Since $C - C_{-r}$ are fully covered by these rectangles, we have $|C_{-r}| \geq |C| - peri\,(C)\,r = |C| - \pi dr$. If C is a convex compact set, based on the factor that C can be approximated by a sequence of polygons contained in C, the lemma can be proved. ■

An ε-tessellation is to divide the plane by vertical and horizontal lines into grids with width ε. Without losing generality, we assume the origin is a corner of grids. In a tesselation, a polyquadrate is a collection of grids intersecting with a polygon. For example, in Fig. 3.3, the shaded grids form a polyquadrate.

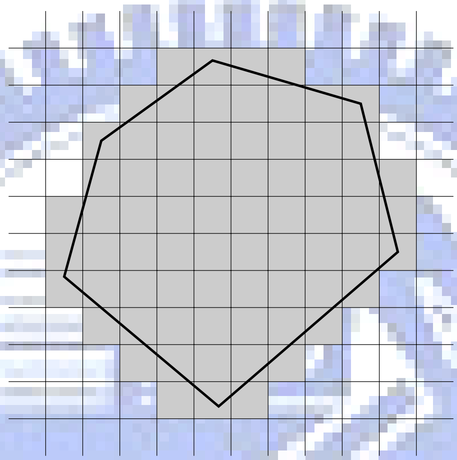

Figure 3.3: Polyquadrate

The horizontal span of a polyquadrate is the horizontal distance measured by the number of grids from the left to the right. The vertical span of a polyquadrate is with similar definition but for the vertical direction. If the span of a polygon is s and each square is with edge l, the span of the corresponding polyquadrate is at most $\lceil s/l \rceil + 1$.

Lemma 2 *If S consists of m cells and τ is a positive integer constant, the number of polyquadrates with span at most τ and intersecting with S is $\Theta(m)$.*

Proof. For a specified cell, since τ is a constant, the number of polyquadrates that contain the cell and have span at most τ is also a constant (depending on τ). For each cell in S, the number of polyquadrates that contain the cell and have span at most τ is $\Theta(1)$. Therefore, since there are m cells in S, the total number of polyquadrates with span at most τ and intersecting with S is $\Theta(m)$. ■

3.2 Extremes of a Collection of Poisson RVs

For the brevity of later discussions, we will define a function that will be often seen in the subsequent derivations. Let ϕ be the function over $(0, \infty)$ defined by $\phi(\mu) = 1 - \mu + \mu \ln \mu$. A straightforward calculation yields $\phi'(\mu) = \ln \mu$ and $\phi''(\mu) = 1/\mu$. Thus, ϕ is strictly convex and has the unique minium zero at $\mu = 1$ (see Figure 3.4). Let $\phi_-^{-1} : [0, 1) \to (0, 1]$

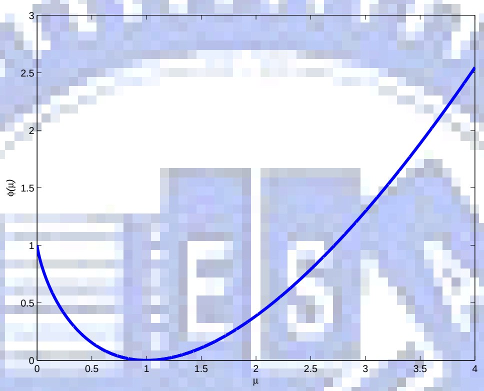

Figure 3.4: $\phi(\mu)$

be the inverse of the restriction of ϕ to $(0, 1]$ see Fig. 3.5. Here we present the upper and

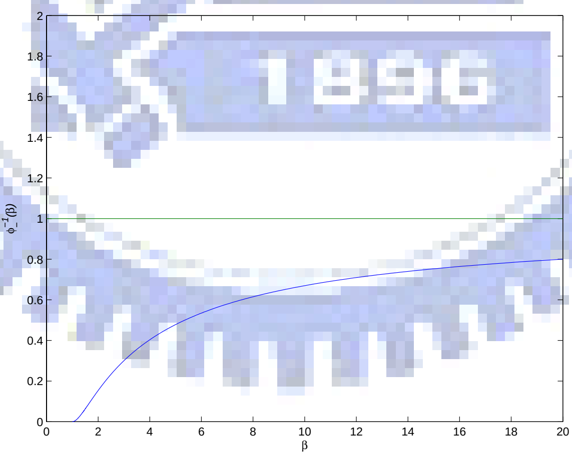

Figure 3.5: $\phi_-^{-1}(1/\beta)$

lower tail distribution of a Poisson random variable.

Lemma 3 *Given any Poisson random variable X with rate λ. We have*

For any $\mu \in (0,1)$, as $\lambda \to \infty$,

$$\Pr(X \leq \mu\lambda) \sim \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\mu}(1-\mu)} \frac{1}{\sqrt{\lambda}} e^{-\lambda\phi(\mu)}.$$

Proof. Assume that $\mu \in (0,1)$. By Lemma 3.2.5.(i) in [32] and Sterling's formula,

$$\begin{aligned}
\Pr(Po(\lambda) \leq \mu\lambda) &\sim \frac{1}{1-\mu} \frac{\lambda^{\mu\lambda}}{(\mu\lambda)!} e^{-\lambda} \\
&\sim \frac{1}{1-\mu} \frac{\lambda^{\mu\lambda}}{\sqrt{2\pi\mu\lambda}(\mu\lambda)^{\mu\lambda} e^{-\mu\lambda}} e^{-\lambda} \\
&= \frac{1}{1-\mu} \frac{1}{\sqrt{2\pi\mu\lambda}\mu^{\mu\lambda}} e^{-\lambda+\mu\lambda} \\
&= \frac{1}{1-\mu} \frac{1}{\sqrt{2\pi\mu\lambda}} e^{-\lambda+\mu\lambda-\mu\lambda\ln\mu} \\
&= \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\mu}(1-\mu)} \frac{1}{\sqrt{\lambda}} e^{-\lambda(1-\mu+\mu\ln\mu)} \\
&= \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\mu}(1-\mu)} \frac{1}{\sqrt{\lambda}} e^{-\lambda\phi(\mu)}
\end{aligned}$$

■

The next lemma gives an a.a.s. lower bound on the minimum of a collection of Poisson RV's.

Lemma 4 *Assume that* $\lim \frac{\lambda_n}{\ln n} = \beta$ *for some* $\beta > 0$. *Let* $Y_1, Y_2, \cdots, Y_{I_n}$ *be* I_n *Poisson RVs with means* λ_n.

1. *If* $I_n = o\left(n\sqrt{\ln n}\right)$, *then for any* $0 < \mu < \phi_-^{-1}(1/\beta)$, $\min_{i=1}^{I_n} Y_i > \mu\lambda_n$ *a.a.s..*

2. *If* $Y_1, Y_2, \cdots, Y_{I_n}$ *are independent and* $I_n = \Omega\left(\frac{n}{\ln n}\right)$, *then for any* $\phi_-^{-1}(1/\beta) < \mu < 1$, $\min_{i=1}^{I_n} Y_i \leq \mu\lambda_n$ *a.a.s..*

Proof. Let Y be a Poisson RV with mean λ_n. We first show that for any $\mu > 0$,

$$\Pr\left[\min_{i=1}^{I_n} Y_i \leq \mu\lambda_n\right] \leq I_n \Pr[Y \leq \mu\lambda_n].$$

Let X_i be the indicator of the event $Y_i \leq \mu\lambda_n$. Then X_i is a Bernoulli RV with probability $\Pr[Y \leq \mu\lambda_n]$. Let $X = X_1 + \cdots + X_{I_n}$. Then, $\min_{i=1}^{I_n} Y_i \leq \mu\lambda_n$ if and only if $X \geq 1$. By Markov's inequality,

$$\Pr\left[\min_{i=1}^{I_n} Y_i \leq \mu\lambda_n\right] = \Pr[X \geq 1]$$

$$\leq E[X] = \sum_{i=1}^{I_n} E[X_i] = I_n \Pr[Y \leq \mu\lambda_n].$$

1. First, assume that $I_n = o\left(n\sqrt{\ln n}\right)$ and $0 < \mu < \phi_-^{-1}(1/\beta)$. By Lemma 3.2(2),

$$\Pr\left[\min_{i=1}^{I_n} Y_i \leq \mu\lambda_n\right]$$

$$\lesssim \frac{1}{\sqrt{2\pi\beta}} \frac{1}{\sqrt{\mu}(1-\mu)} \frac{I_n}{n\sqrt{\ln n}} n^{1-(\lambda_n/\ln n)\phi(\mu)}.$$

Since

$$1 - (\lambda_n/\ln n)\phi(\mu) \to 1 - \beta\phi(\mu) < 0,$$

we have

$$\Pr\left[\min_{i=1}^{I_n} Y_i \leq \mu\lambda_n\right] = o(1),$$

hence $\min_{i=1}^{I_n} Y_i > \mu\lambda_n$ a.a.s..

2. Finally, assume that Y_i's are iid, $I_n = \Omega\left(\frac{n}{\ln n}\right)$ and $\phi_-^{-1}(1/\beta) < \mu < 1$. Then,

$$\begin{aligned}
&\Pr\left[\min_{i=1}^{I_n} Y_i \le \mu\lambda_n\right] \\
&= 1 - \Pr\left[\min_{i=1}^{I_n} Y_i > \mu\lambda_n\right] = 1 - \prod_{i=1}^{I_n} \Pr\left[Y_i > \mu\lambda_n\right] \\
&= 1 - \prod_{i=1}^{I_n} \left(1 - \Pr\left[Y_i \le \mu\lambda_n\right]\right) \\
&= 1 - \left(1 - \Pr\left[Y \le \mu\lambda_n\right]\right)^{I_n} \\
&\ge 1 - e^{-I_n \Pr[Y \le \mu\lambda_n]}.
\end{aligned}$$

By Lemma 3,

$$\begin{aligned}
&I_n \Pr\left[Y \le \mu\lambda_n\right] \\
&\sim \frac{1}{\sqrt{2\pi\beta}} \frac{1}{\sqrt{\mu}\,(1-\mu)} \frac{I_n}{n\sqrt{\ln n}} n^{1-(\lambda_n/\ln n)\phi(\mu)}.
\end{aligned}$$

Since

$$1 - (\lambda_n/\ln n)\,\phi(\mu) \to 1 - \beta\phi(\mu) > 0,$$

we have

$$I_n \Pr\left[Y \ge \mu\lambda\right] \to \infty,$$

hence

$$\Pr\left[\min_{i=1}^{I_n} Y_i \le \mu\lambda_n\right] \to 1.$$

Thus, $\min_{i=1}^{I_n} Y_i \le \mu\lambda_n$ a.a.s..

■

3.3 Scan Statistics

For the following discussion, we define a functions L over $(0,\infty)$ by

$$\mathcal{L}(\beta) = \begin{cases} \beta\phi_{-}^{-1}(1/\beta) & \text{if } \beta \geq 1, \\ 0 & \text{otherwise.} \end{cases}.$$

The curves of L is illustrated in Fig. 3.6. $L(\beta)$ is a monotonic increasing function of β.

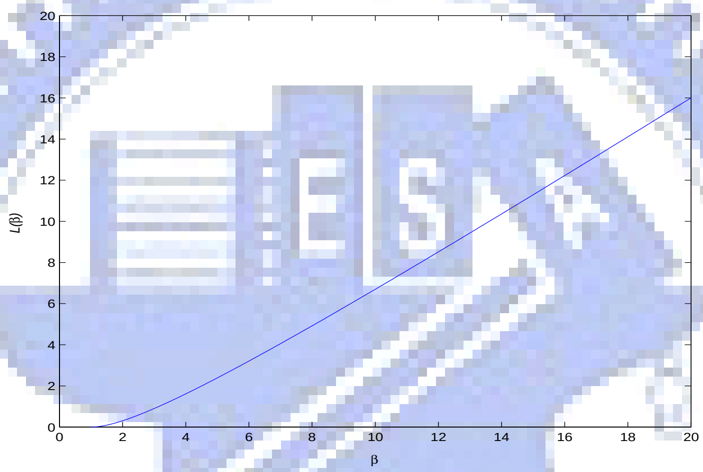

Figure 3.6: $\mathcal{L}(\beta)$

Let $V \subset \mathbb{D}$ be a finite point set, C be a convex compact set, and $P_n(\mathbb{D})$ represent a Poisson point process with mean n over $\mathbb{D}$. The minimum scan statistic for V with scanning set C, denoted by $S_m(V,C)$, is defined as

$$S_m(V,C) = \min_{C' \cong C, C' \subset \mathbb{D}} |V \cap C'|.$$

Here $C' \cong C$ means C' and C are congruent. Let β be a constant and $\mathbf{C}_n$ be a collection of convex compact sets such that for any $C_n \in \mathbf{C}_n$, $|C_n| = (\beta + o(1))\frac{\ln n}{n}$ and $diam(C_n) \leq c_0\sqrt{|C_n|}$ for some constant c_0. Let $S_m(P_n(\mathbb{D}), \mathbf{C}_n) = \min_{C_n \in \mathbf{C}_n} S_m(P_n(\mathbb{D}), \mathbf{C}_n)$. The following theorem was given in Yi et al [33].

Theorem 5 *For the asymptotic of $S_m(P_n(\mathbb{D}), \mathbf{C}_n)$, it is almost sure that*

$$\Pr\left[\frac{\mathcal{S}_m(P_n(\mathbb{D}), \mathbf{C}_n)}{\ln n} \sim \mathcal{L}(\beta)\right] \to 1.$$

Furthermore, if $\beta < 1$, we almost surely have

$$\Pr[\mathcal{S}_m(P_n(\mathbb{D}), \mathbf{C}_n) = 0] \to 1.$$

The proof of Theorem 5 is separated into three parts. In Lemma 6, a.a.s. upper bounds for $S_m(P_n(\mathbb{D}), \mathbf{C}_n)$ are given; and in Lemma 7, a.a.s. lower bounds are given.

Lemma 6 *For any constant $\beta' \in (\beta, \infty)$, it is almost sure that*

$$\Pr(\mathcal{S}_m(P_n(\mathbb{D}), \mathbf{C}_n) \le \mathcal{L}(\beta') \ln n) \to 1.$$

Especially, if $\beta < 1$, we almost surely have

$$\Pr(\mathcal{S}_m(P_n(\mathbb{D}), \mathbf{C}_n) = 0) \to 1.$$

Proof. For any $C_n \in \mathbf{C}_n$, let I_n be the number of copies of C_n that can be packed into $\mathbb{D}$, and Y_i denote the number of nodes in the i-th copy of C_n. Then, $Y_1, Y_2, \cdots, Y_{I_n}$ are i.i.d. Poisson RV's with rate $\beta \ln n$, and $S_m(P_n(\mathbb{D}), \mathbf{C}_n) \le \min_{1 \le i \le I_n} Y_i$. Since $diam(C_n) \le c_0\sqrt{|C_n|}$, we can tile $\Theta\left(\left(\frac{1}{c_0\sqrt{|C_n|}}\right)^2\right) = \Theta\left(\frac{n}{\ln n}\right)$ squares with width $c_0\sqrt{|C_n|}$ in $\mathbb{D}$. Then, one copy of C_n can be placed in each square, and these copies are pairwise disjoint. Thus, $I_n = \Omega\left(\frac{n}{\ln n}\right)$. By Lemma 4 (2), we have

$$\mathcal{S}_m(P_n(\mathbb{D}), \mathbf{C}_n) \le \min_{1 \le i \le I_n} Y_i \le \mathcal{L}(\beta') \ln n.$$

If $\beta < 1$, we may choose a $\beta' \in (\beta, 1)$, and then $S_m(P_n(\mathbb{D}), \mathbf{C}_n) = 0$ is a.a.s. implied since

$L(\beta') = 0$. Thus, the lemma is proved. ∎

Lemma 7 *Assume $V_n = P_n(\mathbb{D})$. For any constant $0 < \beta' < \beta$, it is almost sure that*

$$\Pr\left(\mathcal{S}_m\left(P_n\left(\mathbb{D}\right), \mathbf{C}_n\right) \geq \mathcal{L}(\beta') \ln n\right) \to 1.$$

Proof. Let r_n be the inferior over any $C_n \in \mathbf{C}_n$ of the (smallest) distance from the mass center of C_n to ∂C_n, $\varepsilon_n = \frac{1}{\sqrt{2}}\left(1 - \sqrt{\frac{\beta'}{\beta}}\right) r_n$, and $M_n = 1/\varepsilon^2$. We have $r_n = \Theta\left(\sqrt{\frac{\ln n}{n}}\right)$, $\varepsilon_n = \Theta\left(\sqrt{\frac{\ln n}{n}}\right)$, and $M_n = \Theta\left(\frac{n}{\ln n}\right)$. Divide $\mathbb{D}$ by an ε_n-tessellation. The distance between any two points in a cell is at most $\sqrt{2}\varepsilon_n$. First, we claim that any copy of C_n that is fully contained in $\mathbb{D}$ must contain a polyquadrate that is with span at most $\Theta(1)$ and with area at least $(\beta' + o(1)) \frac{\ln n}{n}$. Let A be a copy of C_n fully contained in $\mathbb{D}$, and P be the maximal polyquadrate contained in A. P contains $\sqrt{\frac{\beta'}{\beta}} A$ since the (smallest) distance between ∂A and $\partial \sqrt{\frac{\beta'}{\beta}} A$ is at least $\left(1 - \sqrt{\frac{\beta'}{\beta}}\right) r_n = \sqrt{2}\varepsilon_n$. Thus,

$$|P| \geq \left|\sqrt{\frac{\beta'}{\beta}} A\right| = \frac{\beta'}{\beta} |C_n| = (\beta' + o(1)) \frac{\ln n}{n},$$

and the span of P is at most $m = \left\lceil \frac{diam(C_n)}{\varepsilon_n} \right\rceil + 1 = \Theta(1)$, asymptotically depending only on β, β', and c_0. In addition, if Y is the number of nodes in P, Y is a Poisson RV with rate at least $(\beta' + o(1)) \ln n$. So, our claim is true. Now, consider all polyquadrates that are contained in $\mathbb{D}$ with span at most m and area at least $(\beta' + o(1)) \frac{\ln n}{n}$. Let I_n denote the number of those polyquadrates, and Y_i denote the the number of nodes in the i-th polyquadrate. Then, we have $S_m\left(P_n\left(\mathbb{D}\right), \mathbf{C}_n\right) \geq \min_{i=1}^{I_n} Y_i$. In addition, from Lemma 2, $I_n = \Theta(M) = \Theta\left(\frac{n}{\ln n}\right)$. Then, applying Lemma 4(1), we a.a.s. have

$$\mathcal{S}_m\left(P_n\left(\mathbb{D}\right), \mathbf{C}_n\right) \geq \mathcal{L}\left(\beta'\right) \ln n.$$

Thus, the lemma is proved. ∎

So, for any constant $\beta > 1$, since $L(\beta) > 0$, Theorem 5 implies that any copy of C_n contained in D a.a.s. covers at least one point of V_n.

Chapter 4

Critical Grid Size and Transmission Radius

In the grid routing, the plane is tessellated into equal sized grids. Two cells are called neighboring cells if they share a common edge and two nodes are called neighboring nodes if they are located in the neighboring cells and within each other's transmission range. In what follows, we assume nodes are represented by a Poisson random point process with mean n over the unit-area square region D. Let $N = \left\lceil \sqrt{\frac{n}{\beta \ln n}} \right\rceil$ for some constant β and $l = \frac{1}{N}$. The deployment region D is divided into N^2 equal sized square cells with length l. So, the area of each cell is $l^2 = (\beta + o(1)) \frac{\ln n}{n}$, and the number of nodes in each cells is a Poisson RV with rate $nl^2 = (\beta + o(1)) \ln n$. Each cells is given a grid coordinate (i, j) where $1 \le i, j \le N$. We assume each node knows the cell in which it is located by utilizing geometric information. The routing distance between two nodes is measured by L_1 grid distance. In other words, if node u is in the cell (i_u, j_u) and node v is in the cell (i_v, j_v), the routing distance between them is given by $|i_u - i_v| + |j_u - j_v|$, the sum of vertical and horizontal distance between two cells measured by the number of cells. Excepting the last hop to the destination node, packets are forwarded to one neighboring node with smaller routing distance to the destination node, and thus, the routing distance is deducted by 1. A local minimum is encountered as none of the neighboring nodes are closer to the final destination than the current node in routing distance.

In this chapter, we present main results of this paper namely the critical grid size and critical transmission radius. In section 4, we give the result of the critical grid size. If the constant β for tessellation is smaller than the critical value, there must a.a.s. exist empty cells in which there is no node. In such a case, we shall prove that some traffic need to across these empty cells by grid routing, and thus, local minima can not be avoided. On the other hand, if the constant β is larger than the critical value, every cells a.a.s. contain at least one node. So, if we set the transmission radius to be $\sqrt{5}$ times of the grid size or larger, every nodes can reach any nodes in neighboring cells, and therefore, deliverability can be a.a.s. guaranteed. Furthermore in section 4.1, as the grid size is larger than the critical grid size, it is not necessary always to set the transmission radius as large as $\sqrt{5}$ times of the grid size. So, the second result is of the critical transmission radius. For a given tessellation, the critical transmission radius is the smallest transmission radius that a.a.s. guarantees that every nodes have at least one neighboring node in each neighboring cell. In section 4.3, for the sake of power saving, we also derive the minimum critical transmission radius. Finally, we derive the average hop count between a source-destination pair and the traffic load per cell in section 4.3.

4.1 The Critical Grid Size

Since β is the parameter used to tessellate the deployment region, we will derive the critical grid size in terms of β in the next theorem.

Theorem 8 *If $\beta > 1$ and $r = \sqrt{5}l$, it is a.a.s. local-minium-free; if $\beta < 1$, it is a.a.s. that there exist local minima;*

Proof. First, we consider the case in which $\beta > 1$ and $r = \sqrt{5}l$. Let $Y_1, \cdots, Y_{N^2}$ denote the number of nodes in each cells. Since $N^2 = \Theta\left(\frac{n}{\ln n}\right)$ and $Y_1, \cdots, Y_{N^2}$ are Poisson RVs with mean $(\beta + o(1)) \ln n$, according to Lemma 4 (1), for $\phi_1^{-1}(1/\beta) > 0$,

$$\lim_{n\to\infty} \Pr\left[\min_{1\leq i\leq I_n} Y_i > 0\right] = 1.$$

So, every cells a.a.s. contain at least one node. In addition, since $r = \sqrt{5}l$, every nodes have neighboring nodes in each of the four directions. Thus, the network is local-minimum-free.

Now, we consider the case in which $\beta < 1$. For any constant $\beta < 1$, we shall show that there a.a.s. exist some empty cells and some packets need to be delivered across these cells (See fig.4.1). For each horizontal and vertical strip of cells, we group the $2\left\lceil\frac{1}{\beta}\right\rceil$ cells

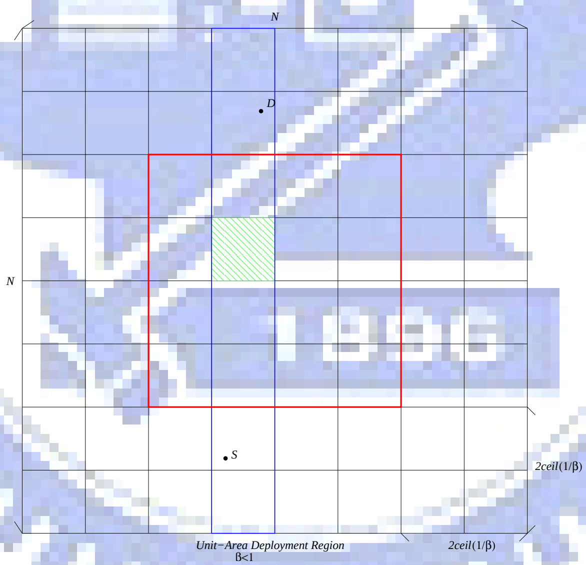

Figure 4.1: Empty cell as $\beta < 1$

at each end of the strip. Let $Y_1, \cdots, Y_{I_n}$ denote the number of nodes on each group. Since $I_n = 4N = \Theta\left(\sqrt{\frac{n}{\ln n}}\right)$ and Y_i are Poisson RVs with rate $n\left(2\left\lceil\frac{1}{\beta}\right\rceil l^2\right) = (2 + o(1)) \ln n$. According to Lemma 4 (1), since $\phi_1^{-1}(1/\beta) > 0$, we have

$$\lim_{n\to\infty} \Pr\left[\min_{1\leq i\leq I_n} Y_i > 0\right] = 1.$$

This implies that there must be traffic between two ends of each strip. So, if there are some empty cells in the middle, packets can't be delivered across these empty cells. Since the number of remaining cells is $\left(N - 4\left\lceil\frac{1}{\beta}\right\rceil\right)^2 = \Theta\left(\frac{n}{\ln n}\right)$ and the number of nodes in each cell is a i.i.d. Poisson RV with rate $nl^2 = (\beta + o(1)) \ln n$, according to Lemma 4 (2), there a.a.s. exist cells without nodes on them. Thus, the theorem is proved. ■

4.2 The Critical Transmission Radius

In each hop, packets are delivered to nodes in neighboring cells, thus one-hop distance is at most $\sqrt{5}l$. So, it is not necessary to set the transmission radius larger than $\sqrt{5}l$. Therefore, in the following, we assume $r \leq \sqrt{5}l$. To eliminate possible traps we need to ensure there is at least some nodes residing in any one of the four covered neighboring areas on upper, lower, left, and right neighboring grids. Followed by Theorem 5, we know that if covered area is not too slim, the number of nodes in these covered neighboring areas is only relevant to the size of the area as their shapes are not degenerated into shallow strips. Therefore, we will focus on conditions ensuring A_r is non-empty which is sufficient to guarantee the non-emptyness of all the covered neighboring areas.

Assume a node locates at a corner of a cell, and let $A_l(r)$ be the region covered by the transmission range of this node in a neighboring cell that doesn't share an edge with the corner. See fig.4.2. For simplicity, we also use $A_l(r)$ to the area of the covered region, i.e. $|A_l(r)|$, and we have

$$A_l(r) = \begin{cases} \frac{1}{2}r^2 \arccos\frac{l}{r} - \frac{1}{2}l\sqrt{r^2 - l^2} & \text{if } l \leq r \leq \sqrt{2}l; \\ \frac{1}{2}r^2 \arcsin\frac{l}{r} + \frac{1}{2}l\sqrt{r^2 - l^2} - l^2 & \text{if } \sqrt{2}l \leq r \leq 2l; \\ \frac{1}{2}r^2\left(\arcsin\frac{l}{r} - \arccos\frac{2l}{r}\right) - l^2 + \frac{1}{2}l\left(2\sqrt{r^2 - (2l)^2} + \sqrt{r^2 - l^2}\right) & \\ \qquad\qquad \text{if } 2l \leq r \leq \sqrt{5}l. & \end{cases}$$

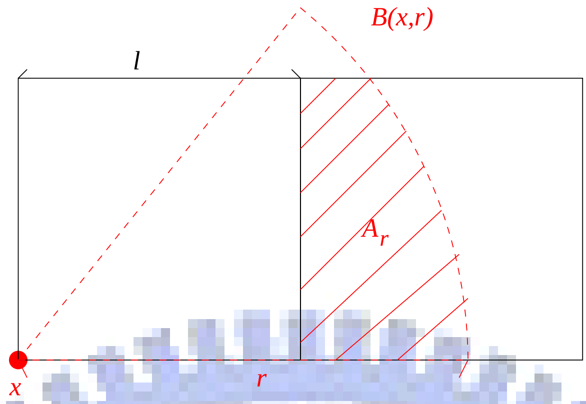

Figure 4.2: Lower bound of Covered Neighbor Area

According to Theorem 8, in what follows, we only consider the tessellation with $\beta > 1$. For a given $\beta > 1$, we shall derive the critical transmission radius.

Theorem 9 *For any constant $\beta > 1$, if the transmission radius r is implicitly given by $A_l(r) = \alpha \frac{\ln n}{n}$ for some constant α, then*

1. If $\alpha > 1$, it is a.a.s. local-minimum-free.

2. If $\alpha < 1$, there a.a.s. exist local minima.

Proof. First, we prove that if $\alpha > 1$, the network is a.a.s. local-minimum-free. For a given grid size l and transmission radius r, the area of neighboring cell covered by the transmission range of a node is minimum as the node locates at a corner opposite to the neighboring cells. The covered area in neighboring cells is at least $\alpha \frac{\ln n}{n}$. According to Theorem 5, for any constant $\alpha > 1$, every nodes a.a.s. have neighboring nodes in each neighbor cell. Thus, it is a.a.s. that packets can be delivered in all directions, i.e. the network is a.a.s. local-minimum-free.

Now, we prove that if $\alpha < 1$, there a.a.s. exist local minima. As $\beta > 1$, according to Theorem 8, we know that there is some nodes in each cell a.a.s.. So if we can show that there is some node without a neighboring node in one of its neighboring cells, local minima will occurs at these nodes when traffic is destined to this particular neighboring cells. Choose ε, which is a constant for fixed α, such that $A_l\left(r+\sqrt{2}\varepsilon l\right) < \left(\frac{1+\alpha}{2}\right)\frac{\ln n}{n}$. Due to symmetry of the four directions, we will consider the pairs of horizontal neighboring cells. Let Y_i denote the joint event of the i^{th} pair that there are some nodes in the small square with width εl by the lower half corner of the left-hand side cell but there are no nodes in the covered area $A_l\left(r+\sqrt{2}\varepsilon l\right)$ of the right-hand side cell. This event implies the occurrence of local minima at this pair of cells when there is traffic destined to the nodes in the remaining portion of the right-hand side cell via the left-hand side cell. Let $A_1 = (\varepsilon l)^2$ be the area of the small square. The probability of the event that some nodes exist in the small square is equal to

$$1 - e^{-nA_1} = 1 - e^{-n(\varepsilon l)^2} \to 1.$$

The probability of the event that there are no nodes in the covered area $A_l\left(r+\sqrt{2}\varepsilon l\right)$ of the right-hand side cell is

$$e^{-nA_l\left(r+\sqrt{2}\varepsilon l\right)} \geq e^{-n\left(\frac{1+\alpha}{2}\right)\frac{\ln n}{n.}} = n^{-\frac{1+\alpha}{2}}$$

Grouping two horizontal neighboring cells to a pair, we have $I_n = \frac{1}{2}N^2 = \Theta(\frac{n}{\ln n})$ pairs. For $1 \leq i \leq I_n$, let X_i be the indicator of the event that Y_i occurs on the i-th pair of cells(see fig.4.3(a)), and $X = X_1 + X_2 + ... + X_{I_n}$ denote the total number of events that has occurred in the deployment region (see fig.4.3(b)). We claim that at least one grid-pair will have the

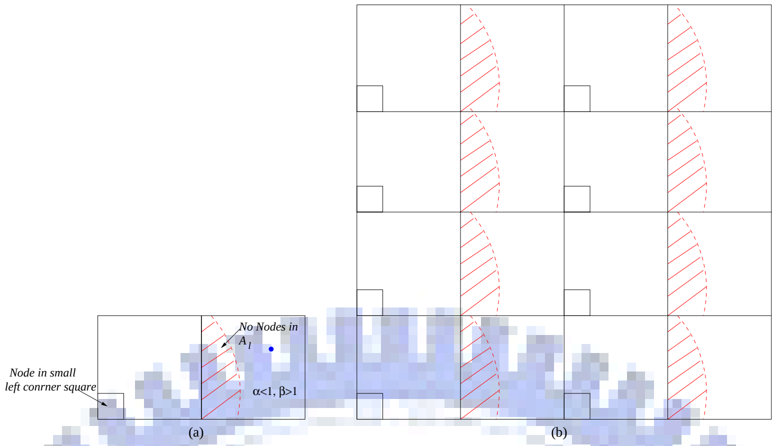

Figure 4.3: Grid-pairs in $\mathbb{D}$

event occurred by showing the probability of $X = 0$ tends to 0.

$$\begin{aligned}
\Pr(X=0) &= \Pr\left(\sum_{i=1}^{I_n} X_i = 0\right) = \Pr\left(\bigwedge_{i=1}^{I_n} X_i = 0\right) \\
&= \prod_{i=1}^{I_n} \Pr(X_i = 0) = \prod_{i=1}^{I_n} \{1 - \Pr(X_i = 1)\} \\
&\leq \prod_{i=1}^{I_n} e^{-\Pr(X_i=1)} = e^{-\sum_{i=0}^{I_n} \Pr(X_i=1)}
\end{aligned}$$

Here

$$\begin{aligned}
\sum_{i=0}^{I_n} \Pr(X_i = 1) &\geq I_n \left(1 - e^{-nA_1}\right) e^{-\alpha \ln n} = \Theta\left(\frac{n}{\ln n} e^{-\alpha \ln n}\right) \\
&= \Theta\left(\frac{n^{1-\alpha}}{\ln n}\right) \to \infty.
\end{aligned}$$

Hence, $\Pr(X = 0) \to 0$, and the event a.a.s. occurs. This implies that local minima a.a.s. exist. ∎

4.3 The Minimal Critical Transmission Radius

In this section, our objective is to derive the minimal critical transmission radius over $\beta > 1$, i.e. the smallest r such that $A_r = \frac{\ln n}{n}$ and its corresponding β. A simple corollary of Theorem 9 (1) is that there a.a.s. exist local minimum $r < l$. Thus, $l < r < \sqrt{5}l$ is a reasonable constraint.

In order to derive the minimal CTR, we first find a simple fact of the differential of $A_l(r)$. Let $A_l(r)$ be fixed at $\frac{\ln n}{n}$, and Consider r is a function of l. Applying implicit differentiation, we have

$$\frac{\partial A_l(r)}{\partial r}\frac{dr}{dl} + \frac{\partial A_l(r)}{\partial l} = 0.$$

Since $\frac{\partial A_l(r)}{\partial r}$ is positive, we observe that $\frac{\partial r}{\partial l}$ is with a opposite sign $\frac{\partial A_l(r)}{\partial l}$, and $\frac{dr}{dl} = 0$ if and only if $\frac{\partial A_l(r)}{\partial l} = 0$.

Next, we informally derive the partial deviation of $A_l(r)$ over l. Fix the the transmission radius r and enlarge the the cell size by Δl, then $A_l(r)$ decrease at the left but increase at the up and right. See Fig.4.4. After some straightforward calculation, we obtain $\frac{\partial A_l(r)}{\partial l}$ as

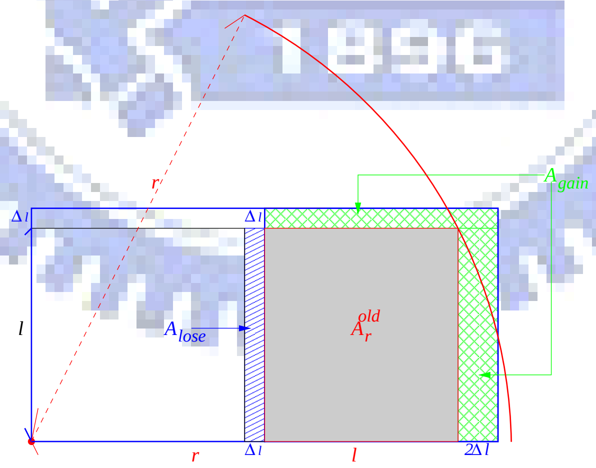

Figure 4.4: Covered Neighboring Area

following

$$\frac{\partial A_l(r)}{\partial l} = \begin{cases} -\sqrt{r^2-l^2}, \text{ if } l \le r \le \sqrt{2}l; \\ \sqrt{r^2-l^2} - 2l, \text{ if } \sqrt{2}l \le r \le 2l; \\ \sqrt{r^2-l^2} - 2l + 2\sqrt{r^2-(2l)^2}, \text{ if } 2l \le r \le \sqrt{5}l. \end{cases}$$

Informally speaking, as r is fixed and l increase by Δl, $A_l(r)$ decrease A_{loss} at the left but increase A_{gain} at the up and right. The new covered area increases by $A_{gain} - A_{loss}$. If the area gained can compensate the area lost, that is $A_{gain} > A_{loss}$, then we can decrease the transmission radius r while still have the new covered area maintaining at the critical size$\frac{\ln n}{n}$. After some calculation, we have

$$\begin{aligned} &\frac{\partial A_l(r)}{\partial l} < 0, \text{ as } r \in [l, \frac{1}{3}\sqrt{3}\sqrt{\frac{65}{3} - \frac{8}{3}\sqrt{13}l}); \\ &\frac{\partial A_l(r)}{\partial l} = 0, \text{ as } r = \frac{1}{3}\sqrt{3}\sqrt{\frac{65}{3} - \frac{8}{3}\sqrt{13}l}; \text{and} \\ &\frac{\partial A_l(r)}{\partial l} > 0, \text{ as } r \in (\frac{1}{3}\sqrt{3}\sqrt{\frac{65}{3} - \frac{8}{3}\sqrt{13}l}, \sqrt{5}l]. \end{aligned}$$

Here $\frac{1}{3}\sqrt{3}\sqrt{\frac{65}{3} - \frac{8}{3}\sqrt{13}} \approx 2.004 \in \left[2, \sqrt{5}\right]$. Replacing r by $2.004l$ in to the equation

$$\begin{aligned} A_l(r) &= \frac{1}{2}r^2\left(\arcsin\frac{l}{r} - \arccos\frac{2l}{r}\right) - l^2 + \frac{1}{2}l\left(2\sqrt{r^2-(2l)^2} + \sqrt{r^2-l^2}\right) \\ &= \frac{\ln n}{n}, \end{aligned}$$

we obtain that the cell size is corresponding to $\beta \cong 1.092$. Thus, the optimal configuration for the minimal CTR is

$$\begin{aligned} &l \cong \sqrt{\frac{1.092\ln n}{n}} \cong 1.045\sqrt{\frac{\ln n}{n}}, \text{ and} \\ &r \cong 2.004l \cong 2.094\sqrt{\frac{\ln n}{n}}. \end{aligned}$$

4.4 Average Routing Distances and Traffic Loads

In this section we will derive the average hop count between a source-destination pair of nodes and the traffic load sent or relayed by any particular cell in the deployment region by applying grid routing. We assume every node has the same probability of being a sender and every source node has the equal probability of choosing any other nodes as destinations.

4.4.1 Average Routing Distances

Recall that we use L_1 grid distance, also known as the Manhattan distance, to measure the routing distance. That is if sender S is in cell (i_s, j_s) and destination D is in cell (i_d, j_d), the routing distance between them is given by $|i_s - i_d| + |j_s - j_d|$. In each hop of grid routing, packets advance one grid cell in a zigzagged fashion toward the destination node except possibly the last hop where the sender/forwarder and the destination node lie in the same cell. So, the hop count is equal to either the routing distance or routing distance plus one, for the last hop. Thus, deriving the average hop count is equivalent to deriving the average routing distance. Note that we will use routing distance and distance interchangeably hereafter unless clarity is needed. From Poisson point process assumption, we know that the x-coordinate and y-coordinate of a node is *iid* to uniform distribution and the coordinate of two nodes are independent. Hence the distance between two nodes is the sum of the distances along respectively the x-axis and y-axis that can be calculated similarly and independently. Thus we only derive the average distance along the x-axis and the average distance is obtained by multiplying the x-axis distance by 2.

Again from the Poisson point process assumption, each node have the same and independent probability of falling into any one of the N columns, hence the x-axis distance between the source S falling into the i_s column and destination D falling into the i_d column is $|i_s - i_d|$ with equal probability $\frac{1}{N}\frac{1}{N} = \frac{1}{N^2}$ for all the N^2 combinations. We can simply enumerate all the N^2 combinations in a $N \times N$ matrix in which the entry (i_s, i_d) is $|i_s - i_d|$ where

0	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Table 4.1: X-distance between S and D.

$i_s, i_d \in \{1, 2, ..., N\}$. Take the following 7×7 matrix as an example (see Table 4.1) clearly reveals that this matrix is symmetric and for any integer $0 \leq k \leq N-1$, there are $(N-k)$ entries of value k respectively in the upper and lower triangular matrices.

Hence the average x-distance can be obtained by the following equation.

$$\frac{1}{N^2} \times 2 \sum_{k=0}^{N-1} ((N-k)k) = \frac{1}{N^2} \frac{N(N+1)(N-1)}{3} = \frac{(N+1)(N-1)}{3N} \cong \frac{N}{3}$$

Thus, the average routing distance between any source and destination pair is twice the amount and about $\frac{2N}{3}$.

4.4.2 Traffic Loads of Cells

By Poisson point process, the nodal density in each cell is the same so we can regard each cell (i_0, j_0) as a unit when calculating the cell traffic load $f(i_0, j_0)$. To simplify calculation, we will ignore packet collisions and retransmissions and assume the total traffic flow in the network is one with random traffic pattern. Then, the traffic originating from each cell is $\frac{1}{N^2}$ and each cell have the same probability of containing source or destination nodes. So the traffic flow between any two cells is $\frac{1}{N^2} \cdot \frac{1}{N^2} = \frac{1}{N^4}$. The total traffic in the deployment region can be estimated by multiplying the total traffic flow with the average hop count. By setting the transmission radius larger than the critical transmission radius, every nodes a.a.s. have neighboring nodes in any of the four neighboring cells. Let (i_s, j_s) denote the cell containing the source node and (i_d, j_d) denote the cell containing the destination node. According to locations of source and destination nodes, the traffic sent from or relayed through the cell (i_0, j_0) can be categorized into the following four case. See Figure.4.5.

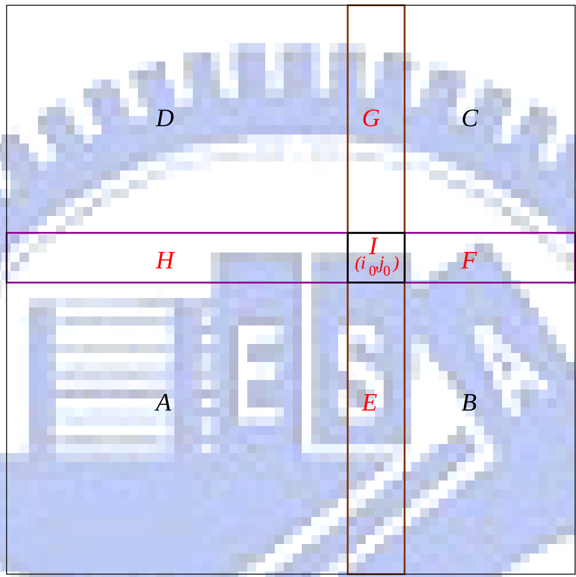

Figure 4.5: Flow partitions of $\mathbb{D}$ related to cell (i_0, j_0)

1. (i_s, j_s) is in the region $A \cup H \cup I \cup E$ and (i_d, j_d) is in the region $G \cup I \cup F \cup C$.

2. (i_s, j_s) is in the region $G \cup I \cup F \cup C$ and (i_d, j_d) is in the region $A \cup H \cup I \cup E$.

3. (i_s, j_s) is in the region $I \cup E \cup B \cup F$ and (i_d, j_d) is in the region $D \cup H \cup I \cup G$.

4. (i_s, j_s) is in the region $D \cup H \cup I \cup G$ and (i_d, j_d) is in the region $I \cup E \cup B \cup F$.

There are total $\binom{|i_d - i_s| + |j_d - j_s|}{|i_d - i_s|}$ routes from s to d. In particular, if the traffic is designated to go through the intermediate cell (i_0, j_0), there are $\binom{|i_0 - i_s| + |j_0 - j_s|}{|i_0 - i_s|}\binom{|i_d - i_0| + |j_d - j_0|}{|i_d - i_0|}$ routes. Let $f(i_s, j_s, i_0, j_0, i_d, j_d)$ denote the percentage of traffic going from cell (i_s, j_s) through cell (i_0, j_0)

to cell (i_d, j_d), then

$$f\left(i_s, j_s, i_0, j_0, i_d, j_d\right) = \frac{\binom{|i_0-i_s|+|j_0-j_s|}{|i_0-i_s|}\binom{|i_d-i_0|+|j_d-j_0|}{|i_d-i_0|}}{\binom{|i_d-i_s|+|j_d-j_s|}{|i_d-i_s|}}.$$

Case 1, the traffic through cell (i_0, j_0) is

$$\sum_{i_s=1}^{i_o}\sum_{j_s=1}^{j_o}\sum_{i_d=i_0}^{N}\sum_{j_d=j_0}^{N}\frac{1}{N^4}f\left(i_s, j_s, i_0, j_0, i_d, j_d\right).$$

and case 2 have the same traffic due to symmetry. For case 3, the traffic through cell (i_0, j_0) is

$$\sum_{i_s=i_o}^{N}\sum_{j_s=1}^{j_o}\sum_{i_d=1}^{i_0}\sum_{j_d=j_0}^{N}\frac{1}{N^4}f\left(i_s, j_s, i_0, j_0, i_d, j_d\right).$$

and case 4 have the same traffic due to symmetry as well. According to the inclusion-exclusion principle, the total traffic that cell (i_0, j_0) experiences is the sum of the traffics in the four cases minus the traffic between $H \cup I$ and $I \cup F$ and the traffics between $G \cup I$ and $I \cup E$. The traffic between $H \cup I$ and $I \cup F$ is $2\frac{1}{N^4}i_0(N - i_0 + 1)$ and the traffic between $G \cup I$ and $I \cup E$ is $2\frac{1}{N^4}j_0(N - j_0 + 1)$. Hence the total traffic through cell (i_0, j_0) is

$$\begin{aligned} f(i_0, j_0) = {} & \frac{2}{N^4}\left(\sum_{i_s=1}^{i_o}\sum_{j_s=1}^{j_o}\sum_{i_d=i_0}^{N}\sum_{j_d=j_0}^{N} + \sum_{i_s=i_o}^{N}\sum_{j_s=1}^{j_o}\sum_{i_d=1}^{i_0}\sum_{j_d=j_0}^{N}\right) f\left(i_s, j_s, i_0, j_0, i_d, j_d\right) \\ & - 2\frac{1}{N^4}\left(i_0(N - i_0 + 1) + j_0(N - j_0 + 1)\right) + \frac{1}{N^4}. \end{aligned}$$

Note that the last term accounts for the traffic when both source and destination lie in cell (i_0, j_0) which has been added four times and subtracted four times. The traffic distribution of each cell is illustrated in Figure 4.6 for $N = 60$. Clearly the hot spot occurs at the center of the deployment region which matches with our intuition.

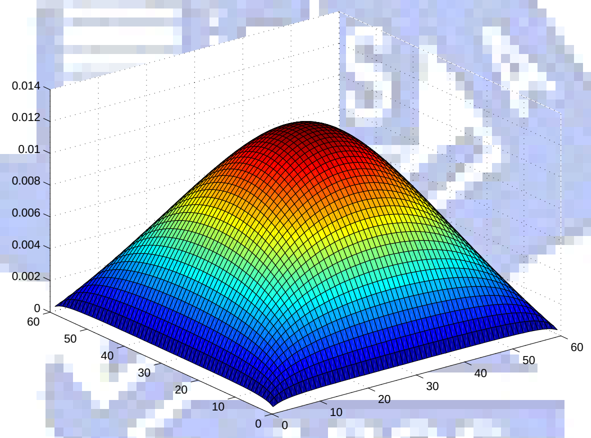

Figure 4.6: Traffic distribution over celss for 60 × 60 grids

Chapter 5

Simulations

To evaluate our theoretical derivations, we present the simulation results on critical grid size and critical transmission radius in this section.

5.1 Critical Grid Size

The first simulation we try to find the critical grid size of finite number of nodes. First, we illustrate that the critical grid size is not a monotonic property when nodes number is finite see Figure 5.1. That is smaller cell length with no empty cell does not imply larger cell length will also guarantee no empty cell. Second, we can also see from the same figure that the location of a cell is not predictable when varying the partitioning number N. With the nodes uniformly distributed in the unit-area square deployment region $\mathbb{D}$, we use the largest empty square, LES, with length l determined by the particular instance as the estimator of critical cell length. Let us see the relation of CCL to the largest empty square. It is easy to see that CCL lies within the range of $(\frac{1}{2}l, l)$. Since for any partition with cell length smaller than $\frac{1}{2}l$, there will be at least one cell falls completely in the largest empty square no matter how you move the cell boundary. On the other hand, for any partition with cell length larger than l, in the worst case the largest empty square will be completely included within a cell

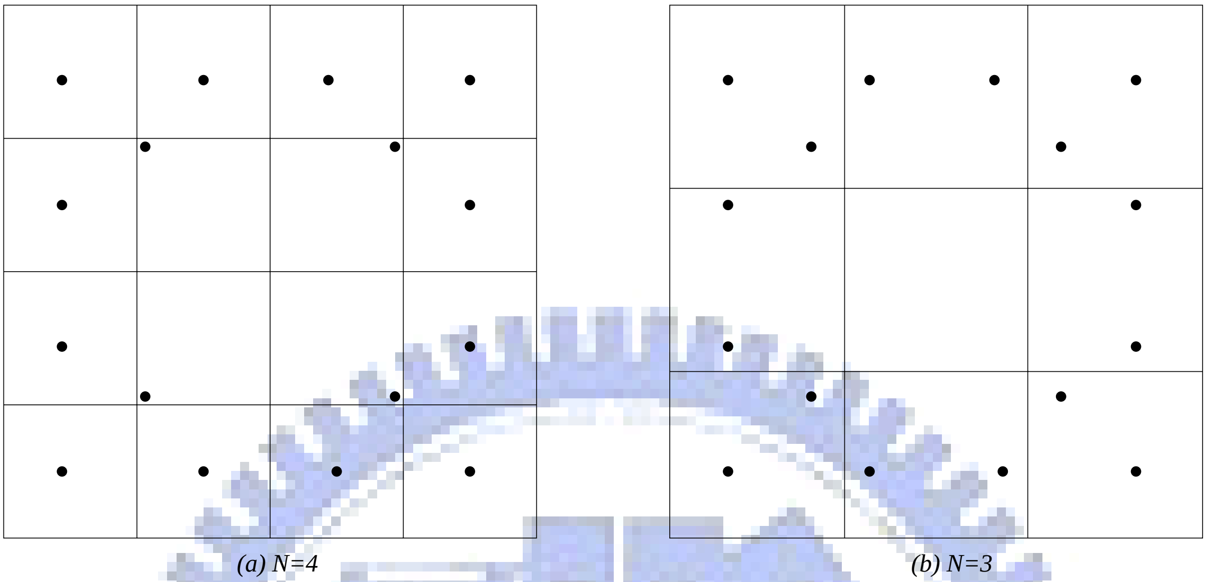

Figure 5.1: Nonmonotone of Empty Cell

with the boundary point included as well. Thus any plane partition with cell length larger than l will include some points.

The problem of finding the largest empty square has already been solved in $\theta(n \log n)$ time which is equivalent to finding the the largest empty circle in the Voronoi Diagram constructed using L_∞ distance metric [34] [35] [36]. Hence we will not delve into it further theoretically.

The largest empty square in $\mathbb{D}$ must satisfies at least of the following cases 1) one node with one boundary edge 2) one node with two boundary edge 3) two nodes 4) two nodes with a boundary edge 5) three nodes.

Figure5.2 demonstrates an example of network instance for 100 nodes and its largest empty square.

Let n be the number of nodes in the network. The simulation is done for $n = 400$, 800, and 1600 respectively. For each n, 500 instances of random point sets are generated. The $c.d.f.$ of LES is illustrated in Figure 5.3. Besides the $c.d.f.$ curve, the vertical lines is the

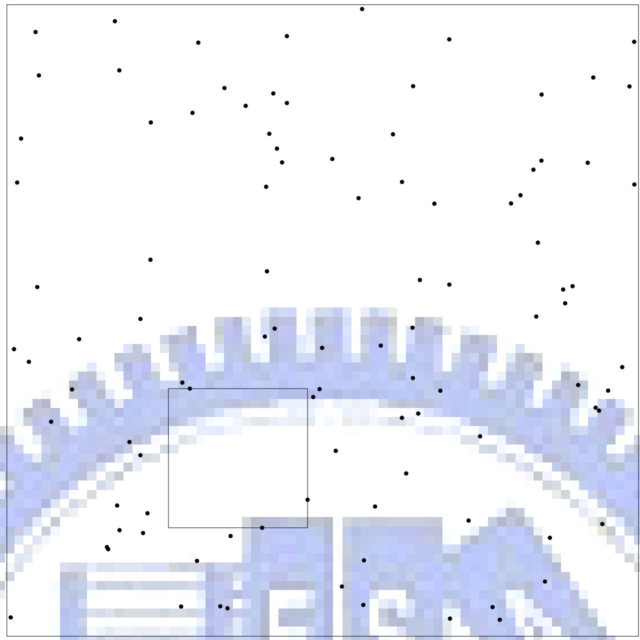

Figure 5.2: Largest Empty Square

corresponding CGS w.r.t. $n = 400$, 800, and 1600 from left to right.

We have listed some numbers corresponding to the LES length L at various $c.d.f.$ values at the following table 5.1. Three things were observed. First, as the number of nodes n increases, the LES length L decreases. Second, we can see the transition width of our data decreases as the number of nodes n increases. Since in a finite deployment region with confined boundary, although the nodal position is chosen randomly but the surrounding environment of each node is not exactly equivalent since of the boundary effect as we can recall the five different cases of determining an empty square. But this phenomenon gradually vanishes as the number of node increases, and the largest empty squares are mostly determined by three nodes.

Third, in table 5.2, to reveal the trend of convergence we have listed the degree of deviation between the CGS and LES length at cdf value of 75% and 90%. We have observed that the error rate also decreases as the number of nodes increases since our derivation is

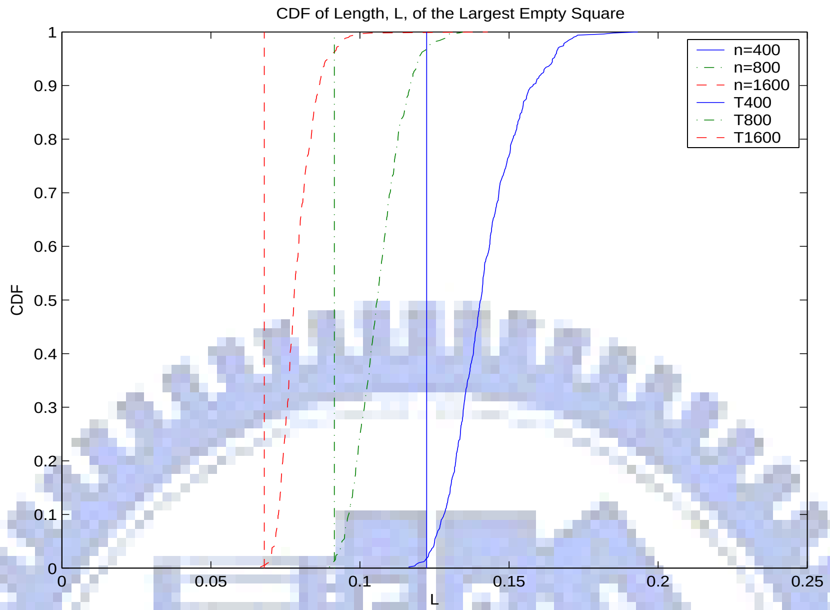

Figure 5.3: CDF of Largest Empty Square Length

CDF	5%	75%	90%	Transition Width(5% $\sim$ 90%)	TheoreticalValue
$n = 400$	0.125060	0.149030	0.158082	0.033022	0.12239
$n = 800$	0.093968	0.111445	0.116876	0.022908	0.09141
$n = 1600$	0.071694	0.081976	0.086313	0.014619	0.06791

Table 5.1: Largest Empty Square

based on asymptotic behavior, i.e. $n \to \infty$.

5.2 Critical Transmission Radius

Next, we performed the simulation of finding the critical transmission radius, CTR, with finite number of nodes. We assume that the deployment region is tessellated into $N \times N$ cells. With the premise that there is no empty cell in the deployment region $\mathbb{D}$, we are going to find out the minimum transmission radius needed to guarantee the deliverability. From theorem 8, for a particular n, the critical grid size $l_c = \frac{1}{N_c} = \sqrt{\frac{\ln n}{n}}$. We will have N not larger than N_c, in particular we have set $N = \{4, 5, ..., N_c\}$ and 4 was chosen so that the

Degree of Deviation	75%	90%
$n = 400$	2.17%	2.91%
$n = 800$	2.19%	2.78%
$n = 1600$	2.07%	2.71%

Table 5.2: Deviation between CGS and LES.

boundary conditions will not prevail that is to prevent of having large portion of cells locates at the boundary of $\mathbb{D}$.

As demanded by the grid routing, the transmission radius must be large enough to accomplish two things: 1) Two nodes are directly reachable if they locate in the same cell 2) Each node must be able to reach at least one neighbor in all of its neighboring cell see Figure 5.4.

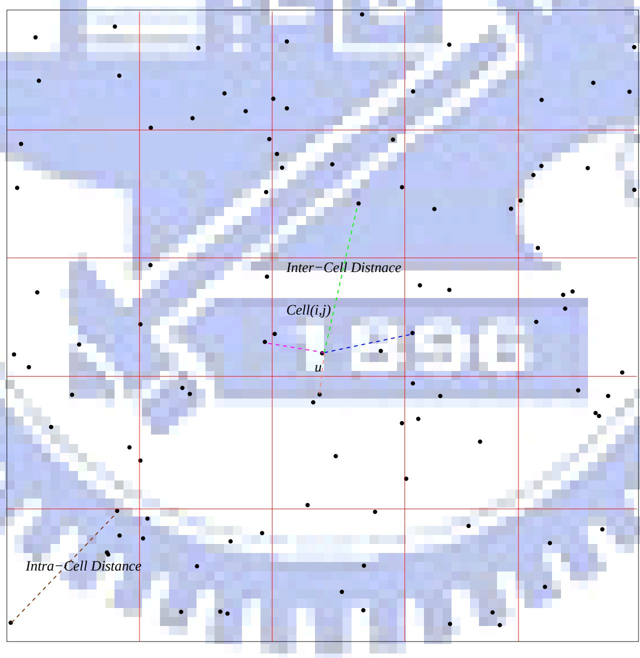

Figure 5.4: Inter & Intra-Cell Distance

Critical transmission radius is the maximum over these two cases. For the first thing, we find out the maximum distance between any two nodes lying within the same cell $\max\limits_{u,v \in \text{same cell in } \mathbb{D}} d(u,v)$. For the second part, we find out the maximum distance between any two nodes in a cell. That is we find out the the minimum transmission radius of each node needed to reach its neighboring nodes in each of the four neighboring cells, namely the upper, lower, left, and right adjacent cells. For node u in cell (i, j), the transmission radius needed to reach all its neighboring nodes is:

$$d(u) = max\{ \min_{v \in \{cell(i,j+1), cell(i,j-1), cell(i-1,j), cell(i+1,j)\}} d(u, v)\}$$

Thus the critical transmission radius for this particular instance is thus the maximum over both inter-cell and intra-cell distance over all the nodes u in $\mathbb{D}$, i.e. $max\{\max_{u \in \mathbb{D}} d(u), \max_{u,v \in \text{same cell in } \mathbb{D}} d(u, v)\}$.

In the following figure 5.5, we have plotted the simulated critical transmission radius R at c.d.f. value of 99% versus the grid cell length, l.

From figur 5.5, we can see that as the number of nodes n increases, there will be more neighboring nodes to choose from as relaying node, thus R decreases as n increases with respect to the same cell length l. Next for fixed number of nodes, the transmission radius increases as the cell length increases since the distance in the same cell as well as between two cell increases.

In the figure, we have also added two auxiliary lines corresponding to $R = \sqrt{2}l$ and $R = \sqrt{5}l$. For $R = \sqrt{2}l$, it is the maximum distance between two nodes lying in the opposite diagonal vertex point of the same cell. For fixed number of nodes n, we will have more neighboring nodes to choose from as cell length l increases by constant nodal density. Also by having more nodes, we will also have more neighboring nodes to choose from. Therefore, let's consider the extreme case when the cell is loaded full of nodes. The maximum distance between two nodes will equal to $\sqrt{2}l$ hence serves as the asymptotic line. With minor subtlety that the probability of having two nodes locate at the exact opposite diagonal vertex point is zero, hence the transmission radius governed by intra-cell communication

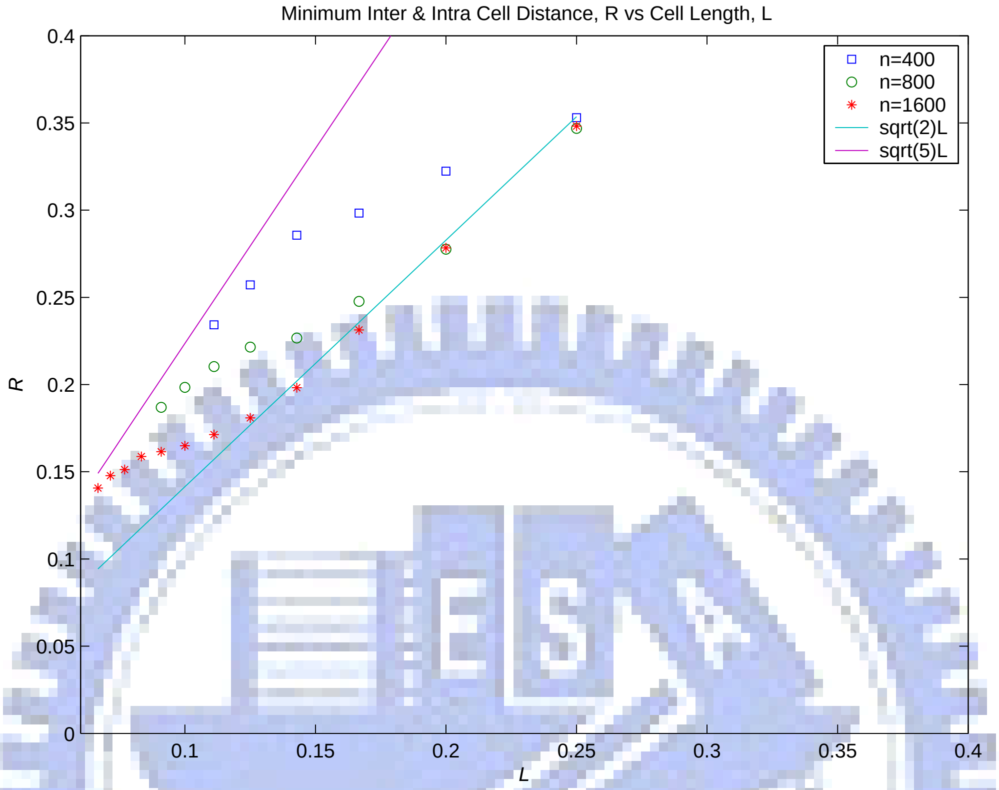

Figure 5.5: Critical Transmission Radius

approaches $R = \sqrt{2}l$ from below. As for $R = \sqrt{5}l$, it is the maximum possible distance between two neighboring nodes hence it serves as a upper bound.

Observe figure 5.6, for inter-cell communication, the transmission radius required drops below $R = \sqrt{2}l$ line as nodal density increases which coincide with our intuition that maximum distance between two neighboring nodes in the extreme case of full of nodes is L. We can obtain smaller transmission radius by relaxing the requirement of direct reachable between two co-located nodes to two hops. All of our result still remain valid with only small modification about the average hop count being incremented by the additional last one hop.

The following figure 5.6 shows the minimum transmission radius needed for every node to reach their neighboring nodes.

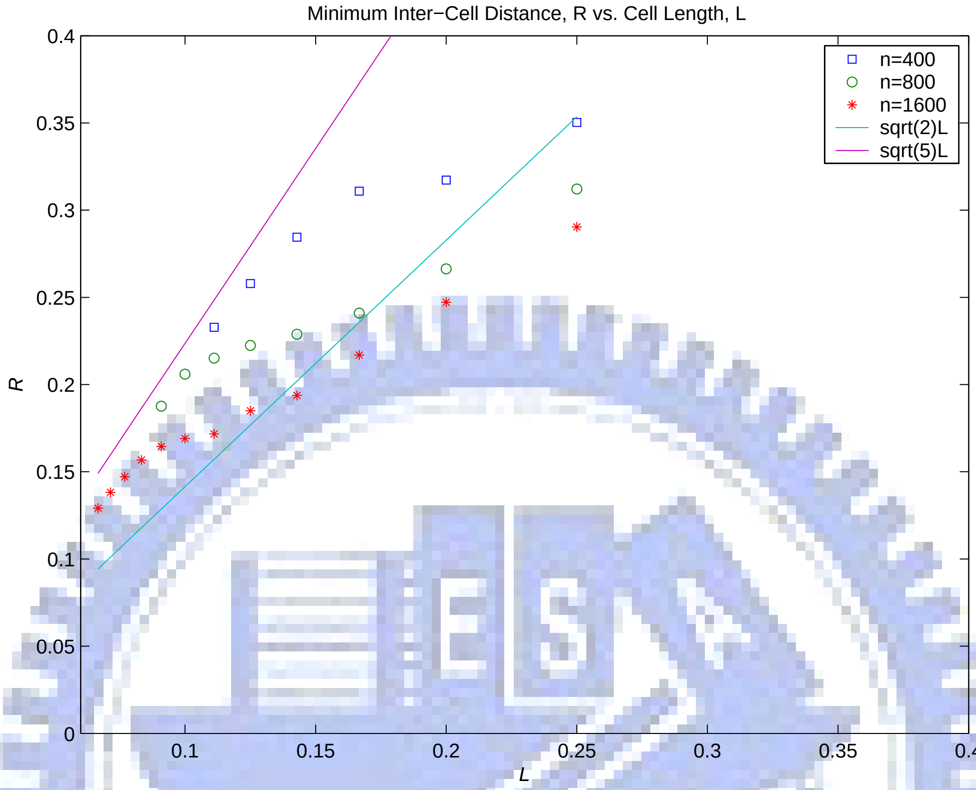

Figure 5.6: Maximum Inter-Cell Distance

5.3 Conclusion

Geographic greedy routing algorithms are localized, distributed and memoryless which make them more suitable to the dynamic network topology environment. However the existence of local minima hindering packets delivery in the greedy manner is one of the major drawbacks. Previous researches proposed many remedial recovery strategy when encountering local minima. In this thesis, we eliminate the existence of local minimum by adjusting the grid size and transmission radius directly. In addition to the critical grid size and transmission radius, we derived the minimal transmission radius guaranteeing packet delivery in favor of better power usage. In addition, we calculated the average hop-count or cell-distance between any particular source-destination node pair in the deployment region $\mathbb{D}$ which is about $\frac{2N}{3}$, and we also calculated the average traffic load experienced by any particular cell and the result showed that the hot spot is at the center.

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