# CHAPTER 3

# IMPROVING SUPPLY CHAIN EFFICIENCY VIA OPTION PREMIUM INCENTIVE

This study considers a supply chain, including one manufacturer and one retailer, in which stochastic demand and return policy is considered, focused upon. When each site aims to maximize its individual profitability, decentralized control may arise in the system. From the overall system's point of view, the decentralized supply chain will not be as efficient as the centralized one. Various studies have attempted to resolve the inefficiency of supply chains. However, said studies have focused on the optimization from retailer's perspective only and thus ignored manufacturer's interest. This study, instead, emphasizes manufacture's self-interested situation and determines optimal production quantity. Furthermore, we will contribute to decentralized control with the retailer offering option premium. We will also demonstrate that the Pareto efficiency can be attained in the supply chain by employing the option premium incentive.

## 3.1 Problem description

Let us describe the problem as follows. In response to a given wholesale price and production cost, the manufacturer determines the production quantity by realizing the company's best interest (*i.e.* maximizing profit). During the selling season, the retailer sells the items in the market at a constant retail price. At the end of the season, the retailer returns unsold items to the manufacturer for full credit.

The relevant assumptions are twofold. (1) The distribution of market demand is available. Although this is a simplification, it is known that the distribution can be found by analyzing history data. Furthermore, we assume the inverse of the demand function exist; (2) Under the circumstance with return policy, prior to the selling season, the retailer makes commitment in terms of order quantity and wholesale price, and at the end of the season, is allowed to return unsold items. Pasternack (1985) assumed two forms of return policy, which include partial credit to the retailer for all unsold items and full credit for the return of a certain portion of the original order. To simplify it, the retailer is assumed to have received full credit for all unsold items in this study. Thus, we can focus on our model, which deals with a profit-oriented manufacturer. To assure internal consistency, the cost parameters follow some straightforward assumptions: (a) $p > w_0 > m > 0$, (b) $u < m$, (c) $s > 0$.

**Manufacturer's profit function**

The manufacturer's profit can be expressed as wholesale revenue minus production costs plus salvage value. Notably, wholesale revenue only counts sold items because unsold items are returned with full credit.

$$\pi_m = w_0 Min(Q, D) - mQ + u(Q - D)^+ \tag{3-1}$$

Take expectation for all possible demand, then the manufacturer's average profit can be written as:

$$E(\pi_m) = w_0 E\{Min(Q, D)\} - mQ + uE(Q - D)^+ \tag{3-2}$$

Differentiation of $Eq.$ (3-2) yields the following first-order condition:

$$\frac{\partial}{\partial Q}E(\pi_m) = w_0\frac{\partial}{\partial Q}\{E(D) - E(D-Q)^+\} - m + u\frac{\partial}{\partial Q}E(Q-D)^+ \tag{3-3}$$
$$= w_0(1 - F(Q)) - m + uF(Q)$$
$$= (w_0 - m) + (w_0 - u)F(Q) = 0$$

or it can be rewritten as

$$F(Q) = (w_0 - m)/(w_0 - u) \tag{3-4}$$

Differentiating $Eq.$ (3-3) yields the second-order condition.

$$-(w_0 - u)f(Q) \le 0 \tag{3-5}$$

Hence, the second-order condition is satisfied and $Q_d = F^{-1}\{(w_0 - m)/(w_0 - u)\}$ is the manufacturer's optimal production quantity.

**Retailer's profit function**

The retailer's profit can be expressed as revenue minus wholesale cost and goodwill loss. Notably, wholesale cost only counts sold items because unsold items can be returned for full credit.

$$\pi_r = (p - w_0)Min(Q, D) - s(D-Q)^+ \tag{3-6}$$

Take expectation for all possible demand, then the retailer's average profit becomes:

$$E(\pi_r) = (p - w_0)E\{Min(Q, D)\} - sE(D-Q)^+ \tag{3-7}$$

Differentiation of $Eq.$ (3-7) yields the following first-order condition:

$$(p + s - w_0)(1 - F(Q)) = 0 \tag{3-8}$$

It concludes that the retailer's optimal ordering quantity is infinite. Restated, the retailer hopes the manufacturer produces as many as possible so that shortage costs can be reduced significantly. Furthermore, the retailer does not need to worry about ordering too much because unsold items can be returned for full credit.

If the manufacturer and the retailer make decisions independently, a rational manufacturer will determine the optimal production quantity by maximizing the profit. Since backup is prohibited in this study, once the production quantity is determined, regardless of shortage during a selling season, the retailer cannot acquire any extra quantity. Explicitly the shortage does not affect the manufacturer but only matters to the retailer. Thus, the retailer encounters a great potential risk that is a shortage might destroy business honest. The situation mentioned above is also referred to as decentralized dilemma. If the profits of the manufacturer and the retailer are considered together, the quota ascertained in the model will not allow for too much shortage to occur. This is called a centralized scenario. The next section attempts to model the centralized scenario.

## 3.2 Centralized model

Let $\pi_J$ be the joint profit of the manufacturer and the retailer, which can be written as $\pi_J = \pi_r + \pi_m$, or

$$\pi_J = p\,Min(Q, D) - mQ + u(D - Q)^+ - s(D - Q)^+ \tag{3-9}$$

Take expectation for all possible demand, the manufacturer average profit is:

$$E(\pi_J) = p\,Min\,E(Q, D) - mQ + u(Q - D)^+ - sE(D - Q)^+ \tag{3-10}$$

Differentiation of $Eq.$ (3-10) yields the following first-order condition:

$$(p + s - m) - (p + s - u)F(Q) = 0 \tag{3-11}$$

or rewritten as

$$F(Q) = (p + s - m)/(p + s - u) \tag{3-12}$$

The second-order condition is produced by differentiating *Eq.* (3-11).

$$-(p + s - u) f(Q) \le 0 \tag{3-13}$$

Therefore, the second-order condition is satisfied and $Q^* = F^{-1}\{(p + s - m)/(p + s - u)\}$ is the system's optimal production quantity.

**Proposition 3-1.** *Joint profit of the centralized model is higher than that of the decentralized model.*

As the objective of the centralized model is to maximize the joint profit, this proposition is self-explanatory. Therefore, the optimal joint profit of the centralized model is always higher than or equivalent to that of the decentralized model. Furthermore, once the optimal quota within the decentralized model is ascertained to be different from that within the centralized one, the centralized model dominates the decentralized model. Compare *Eqs.* (3-12) and (3-4), the difference of their optimal quota is apparent. So the centralized model dominates the decentralized model, or restated, extra profit results in the centralized model.

**Proposition 3-2.** *The manufacturer's profit of the centralized model is less than that of the decentralized one.*

The objective of the decentralized model is to maximize the manufacturer's profit, so that when optimum, the manufacturer's profit of the decentralized model is always higher than or equivalent to that of the centralized model. Furthermore, once the optimal quota within the centralized model differs from that within the decentralized one, the decentralized model dominates the centralized model. Proposition 3-1 confirms that the optimal quota between two models is distinct, therefore, when the centralized model is implemented, the manufacturer's profit decreases.

**Proposition 3-3.** *The retailer's profit of the centralized model is higher than that of the decentralized one.*

From proposition 3-1, when optimum the joint profit of the centralized model is higher than that of the decentralized one. In addition, from proposition 3-2, the manufacturer's cost of the centralized model is less than that of the decentralized model. Thus, it is explicit that the retailer's profit of the centralized model is higher than that of the decentralized one.

**Proposition 3-4.** *The manufacturer's production quantity of the centralized model is greater than that of the decentralized situation.*

This proposition is proven by comparing $Q_d = F^{-1}\{(w_0 - m)/(w_0 - u)\}$ and $Q^* = F^{-1}\{(p + s - m)/(p + s - u)\}$. Since $p > w_0$ based on the assumption, and $F^{-1}(\cdot)$ is a monotonous increasing function, then $Q^* > Q_d$ can be concluded.

**Proposition 3-5.** *Pareto efficiency is attained in the centralized model.*

A feasible allocation $X$ is a Pareto efficient allocation if there is no feasible allocation $X'$ such that all agents prefer $X'$ to $X$ (cf. Varian 1984). There is no feasible production quantity where the manufacturer and the retailer are at least both well off and at least one of them is strictly better off. That is, if such a quantity exists, then the joint profit of the centralized model can be improved. However, the joint profit of the centralized model is maximized in the supply chain. As a result, we can conclude that Pareto efficiency is attained in the centralized model.

The above propositions verify that when the centralized model is implemented, the manufacturer's profit decreases, while both the retailer's profit and the joint profit increase. Notably, the retailer's profit increases more than the manufacturer's profit decreases, therefore, the manufacturer-retailer system will yield extra overall profit. Furthermore, it is demonstrated that implementation of the centralized model results in Pareto efficiency. However, problems still exist. If the manufacturer accepts a centralized model contract, it means that he must produce more but earn less. It is clear that unless compensated by the retailer, the manufacturer will not accept such a contract. This will be considered in the next section.

## 3.3 Cooperative model employing option premium as an incentive

In this section, we propose that the retailer employ an option premium as an incentive, so that the manufacturer is motivated to accept the contract. Furthermore, we will determine the amount of the option premium that the retailer should offer. Also, production quantity in such a situation will be determined.

Let the option premium be $\Delta W$ that is defined as $(W - W_0)$, and $\pi_m(Q_d)$ be the profit function before the option premium. After the option premium, the manufacturer's profit function can be expressed as $\pi_m(Q)$. If the manufacturer's profit after option premium is not less than that of the decentralized model, the motivation to accept the contract of the centralized model will exist. $Eq.$(3-14) expresses such a situation, where $Q_d$ is the optimal production quantity of the decentralized model.

$$\pi_m(w,Q) - \pi_m(w_0, Q_d) \geq 0 \tag{3-14}$$

For simplicity, let $Eq.$(3-14) equal zero, which implies that after option premium, the manufacturer's profit is equal to that of the decentralized model. After solving $Eq.$(3-14), the reaction function of the manufacturer is as follows:

$$w = \frac{\{\pi_m(Q_d) + mQ - uE(Q-D)^+\}}{E\{Min(Q,D)\}} \tag{3-15}$$

Furthermore, to determine the ordering quantity, the retailer will maximize profit under the reaction function, which is expressed in the following:

$$\begin{aligned} &Max \quad E(\pi_r) = (p-w)E\{Min(Q,D)\} - sE(D-Q)^+ \\ &s.t. \quad w = \frac{\{\pi_m(Q_d) + mQ - uE(Q-D)^+\}}{E\{Min(Q,D)\}} \end{aligned} \tag{3-16}$$

Differentiation of $Eq.$(3-16) yields the following first-order condition:

$$(p+s-m) - (p+s-u)F(Q) = 0 \tag{3-17}$$

It is evident that $Eq.$(3-17) is equivalent to $Eq.$(3-11). Thus, the ordering quantity determined herein is equal to that of the centralized model (*i.e.* $Q^*$). Substitute $Q^*$ into $Eq.$(3-15), the optimal wholesale price $w^*$ can be ascertained and option premium $\Delta w$ is obtained by $(w^* - w_0)$.

**Proposition 3-6.** *Through the employment of option premium as an incentive, the manufacturer's profit will not decrease compared with that of the decentralized model, whereas will increase compared with that of the centralized model. Meanwhile, the retailer's profit will increase compared with that of the decentralized model, whereas will decrease compared with that of the centralized model. Furthermore, Pareto efficiency is also attained herein.*

Based on $Eq.$(3-14), the manufacturer's profit is equivalent to that of the decentralized model. From proposition 3-2, the manufacturer's profit of the decentralized model is higher than that of the centralized one. Therefore, we can conclude that the manufacturer's profit herein is larger than that of the centralized model. Since option premium paid by the retailer is equivalent to the amount received by the, therefore the joint profit of the manufacturer and the retailer will be also equivalent both before and after option premium. Thus, proposition 3-1 is also correct here. From proposition 3-1, extra profit exists within the centralized model. When the manufacturer's profit equals that of the decentralized situation, it is explicit that the retailer's profit will increase in comparison with that of the decentralized model. However, since the manufacturer's profit herein is larger than that of the centralized model, the retailer's profit will decrease in comparison with that of the centralized model. Finally, Pareto efficiency is attained, because the joint profit after option premium is equivalent to that of the centralized model.

## 3.4 Numerical illustration

To illustrate the effectiveness of the cooperative model, the decentralized control from previous literature is solved by our proposed approach. This problem originates from Tsay (1999). Tsay assumed that market demand is uniformly distributed within $[0,100]$. He also assumed $p=15$ 、 $w_0=10$ 、 $m=6$ 、 $u=3$. We assume $s=3$ in the retailer's parameters.

First, the decentralized model is solved. The optimal quota, which manufacturer produces is 57.14. Notably, every individual cost and the total profit are computed when the optimal quota is substituted into the manufacturer's profit function. Furthermore, with this quota, the retailer's cost is also computed. Table 3-1 illustrates the results.

**Table 3-1: Results obtained from the decentralized model**

| Manufacturer | | Retailer | |
|---|---|---|---|
| Revenue | 402.57 | Revenue | 201.28 |
| Production cost | (342.84) | Shortage cost | (25.67) |
| Salvage cost | 50.64 | | |
| Profit | 110.37 | Profit | 175.61 |
| Joint profit | 285.98 | | |

Then, the centralized model is solved. The optimal quota is 80. Notably, it is greater than that of the decentralized model (*i.e.* 57.14). When the option quota is substituted into the profit function of both the retailer and the manufacturer, every individual cost and the total profit are again computed. Table 3-2 illustrates the results. That is, it shows that the retailer's shortage cost decreases, but the manufacturer's production and salvage costs increase here. As a result, the retailer's profit increases, but the manufacturer's profit decreases in comparison with that of the decentralized model. Moreover, the joint profit is 322.35, which increases by about 12.7% in comparison with that of the decentralized model.

**Table 3-2: Results obtained from the centralized model**

| Manufacturer | | Retailer | |
|---|---|---|---|
| Revenue | 472.53 | Revenue | 236.26 |
| Production cost | (480) | Shortage cost | (4.68) |
| Salvage cost | 98.24 | | |
| Profit | 90.77 | Profit | 231.58 |
| Joint profit | 322.35 | | |

Finally, the cooperative model is solved. The optimal quota of 80 is the same as it is in the centralized model. When substitute the optimal quota into the profit function of both the retailer and the manufacturer, every individual cost and the total profit are computed.

The wholesale price can be solved as 10.41, thus option premium $\Delta w$, 0.41, is obtained. That is, the retailer should offer an option premium, 0.41 per item to induce the manufacturer to increase the production quantity from 57.41 to 80. The results are illustrated in Table 3-3, which shows that due to the option premium, the manufacturer's profit, 110.37, equals that of the decentralized model. However, the retailer's profit, 212.98, is higher than that of the decentralized model. Restated, it is increased by approximately 21.3%. In addition, the joint profit, 322.35, is equivalent to that of the centralized model.

**Table 3-3: Results obtained from the cooperative model**

| Manufacturer | | Retailer | |
|---|---|---|---|
| Revenues | 492.13 | Revenues | 216.66 |
| Production cost | (480) | Shortage cost | (4.68) |
| Salvage cost | 98.24 | | |
| Profit | 110.37 | profit | 212.98 |
| Joint profit | 322.35 | | |



## 3.5 Discussion and conclusion Remark

The decentralized dilemma within a supply chain has been discussed. Stochastic demand and return policy are included in our model. Furthermore, we have revealed the decentralized effect within a supply chain, as well as modeled the decision policies of the retailer and the manufacturer. The retailer hopes the manufacturer produce as many as possible, thereby reducing the cost of shortage due to the return policy. However, if the manufacturer and the retailer make decisions independently, a rational manufacturer will determine the optimal production quantity by maximizing profit.

We have proven that when a retailer and a manufacturer are conjoined by a single entity, the system can attain the Pareto efficiency or centralization. Within the centralized model, the manufacturer must produce a greater amount, but earn less. Therefore, unless compensated, the manufacturer will not have incentive to accept this type of contract. We therefore propose that the retailer should offer an option premium to induce the manufacturer to increase production quantity. We have demonstrated that when the retailer offers the manufacturer an option premium, the system can be Pareto efficient, and thus remedy the inefficiencies caused by decentralized control within the system.