

CHAPTER 4

A MANUFACTURER'S OPTIMAL QUANTITY DISCOUNT STRATEGY AND RETURN POLICY

This study generalized traditional quantity discount problem with return contracts, in which a manufacturer promises to refund some fraction of the retailer's wholesale price if an item is returned, as a two-stage game. In the first stage the manufacturer and retailer determine the inventory level cooperatively. In the second stage, the manufacturer bargains with the retailer for quantity discount and return schemes to maintain channel efficiency.

4.1 Problem description

The developed model will demonstrate that the return policy can be considered as mirror-images of quantity discount strategy. A menu of discount-return combinations is proposed for the manufacturer to make inventory decisions. That is, options with more generous return privileges are coupled with higher wholesale prices, whereas the lowest wholesale price comes with very strict limits on returns and a restocking fee for any returned goods.

4.2 The general model

Consider a supply chain including an independent manufacturer and an independent retailer, consumer demand is stochastic and both parties know their respective demand distribution. In addition, an item, such as a newspaper or an airline seat, is assumed to perish if it is not sold during the selling season. The retailer's order quantity equals the manufacturer's production since the production is make-to-order. The proposed scenario is a one-period inventory model in which backup is prohibited. The retailer can return all

unsold items to the manufacturer at a pre-determined buyback price at the end of the selling season. The manufacturer and the retailer use constant marginal cost manufacturing and retailing technologies. We also assume the inverse of the demand function exist. Furthermore, to assure internal consistency, the cost parameters follow some straightforward assumptions: (a) $p > w_0 > m > 0$, (b) $u < w_0$, (c) $s > 0$.

4.2.1 Stage one: the optimal inventory Level

To achieve channel efficiency, the manufacturer and retailer first act as Nash players. The scenario can be viewed such that the manufacturer and the retailer are combined as a single entity. The objective is to determine the optimal inventory level by maximizing the joint profits. The manufacturer's profit can be expressed as revenue minus production and buyback costs offered to the retailer.

$$\pi_m = (w_0 - m)Q - u(Q - D)^+ \quad (4-1)$$

Thus, the manufacturer's expected profit in light of all possible demands can be written as

$$E(\pi_m) = (w_0 - m)Q - uE(Q - D)^+ \quad (4-2)$$

The retailer's profit can be expressed as profits from the market minus wholesale costs and goodwill loss plus return credit from the manufacturer.

$$\pi_r = p\text{Min}(Q, D) - w_0Q - s(D - Q)^+ + u(Q - D)^+ \quad (4-3)$$

Take expectation for all possible demand, the retailer expected profit can be expressed as

$$E(\pi_r) = pE\{\text{Min}(Q, D)\} - w_0Q - sE(D - Q)^+ + uE(Q - D)^+ \quad (4-4)$$

Let π_j represent the joint profit of the manufacturer and retailer which can be expressed as $\pi_j = \pi_r + \pi_m$, or

$$\pi_j = p \text{Min}(Q, D) - mQ - s(D - Q)^+ \quad (4-5)$$

The manufacturer's expected profit in light of all possible demands is

$$E(\pi_j) = p \text{Min} E(Q, D) - mQ - sE(D - Q)^+ \quad (4-6)$$

To find the optimal inventory level, Q , we set $\partial E(\pi_j) / \partial Q = 0$. That is:

$$(p + s)(1 - F(Q)) - m = 0 \quad (4-7)$$

which can be rewritten as:

$$F(Q) = (p + s - m) / (p + s) \quad (4-8)$$

Differentiating Eq. (4-7) yields the following second-order condition.

$$\frac{\partial^2 E(\pi_j)}{\partial Q^2} = -(p + s)f(Q) \leq 0 \quad (4-9)$$

Therefore, the second-order condition is satisfied and $Q^* = F^{-1}\{(p + s - m) / (p + s)\}$ denotes channel's optimal inventory level. The profit of a vertically integrated firm is the maximum attainable in the system. However, the retailer faced with uncertain demand has an incentive to order less (probably the EOQ) than the manufacturer desires. Manufacturers should then offer quantity discounts to maximize the system efficiency without harming the retailer. The Manufacturer can also reduce the uncertainty facing the retailer by allowing him to return any unsold items (cf. Kandel 1996). The second stage will verify how the manufacturer can simultaneously utilize both a quantity discount strategy and a return policy to maintain the channel efficiency.

4.2.2 Stage two: the channel subgame

The model consists of a supply chain composed of an independent manufacturer and retailer. The joint optimal inventory level is $Q^* = F^{-1}\{(p+s-m)/(p+s)\}$ according to the specific wholesale and buyback prices detailed in stage one. However, rather than order Q^* , a rational retailer would make ordering decisions based on maximizing his profit (*i.e.* EOQ). The manufacturer must “devise a pricing discount schedule to induce the retailer to change his ordering policy so that (a) the retailer is no worse off, and (b) the manufacturer’s profit is increased” (cf. Lal and Staelin 1984, and Lee and Rosenblatt 1986) to achieve channel efficiency. This stage describes a quantity discount scheme and return policy that will encourage the retailer to change his ordering policy. The retailer’s expected profit can be written as Eq.(4-10). Notably, Eq.(4-10) is Eq.(4-4) with $u = 0$.

$$E(\pi_r) = pE\{\text{Min}(Q, D)\} - w_0Q - sE(D-Q)^+ \quad (4-10)$$

To find the optimal inventory level in the EOQ model, Q_{EOQ} , we set $\partial E(\pi_r)/\partial Q = 0$. That is:

$$(p+s-w_0) - (p+s)F(Q) = 0 \quad (4-11)$$

which can be rewritten as

$$F(Q) = \frac{(p+s-w_0)}{(p+s)} \quad (4-12)$$

Eq.(4-11) can be differentiated to yield the second-order condition.

$$\frac{\partial^2 E(\pi_r)}{\partial Q^2} = -(p+s)f(Q) \leq 0 \quad (4-13)$$

Hence, the second-order condition is satisfied and $Q_{EOQ} = F^{-1}\{(p + s - w_0)/(p + s)\}$ denotes the retailer's optimal ordering quantity (*i.e.* EOQ) without any quantity discounts or return credits.

Next, an inventory model for the manufacturer is developed to determine adequate quantity discount schemes and return policies. A retailer will order Q_{EOQ} only when the profit after quantity discounts and returns is no less than its counterpart in the EOQ model as expressed in *Eq.*(4-14) (where Q_{EOQ} denotes the optimal ordering quantity of the EOQ model).

$$\pi_r(w, u, Q^*) - \pi_r(w_0, u = 0, Q_{EOQ}) \geq 0 \quad (4-14)$$

Let *Eq.*(4-14) equal zero so retailer profit after quantity discount equals its counterpart in the EOQ model.

$$pE\{Min(Q^*, D)\} - wQ^* - sE(D - Q^*)^+ + uE(Q^* - D)^+ = \pi_r(w_0, u = 0, Q_{EOQ}) \quad (4-15)$$

which can be rewritten as

$$w = \frac{1}{Q^*} \left\{ pE\{Min(Q^*, D)\} - sE(D - Q^*)^+ - \pi_r(w_0, u = 0, Q_{EOQ}) \right\} + u \left\{ \frac{1}{Q^*} E(Q^* - D)^+ \right\} \quad (4-16)$$

By defining $w^* = \frac{1}{Q^*} \left\{ pE\{Min(Q^*, D)\} - sE(D - Q^*)^+ - \pi_r(w_0, u = 0, Q_{EOQ}) \right\}$, which can be regarded as the wholesale price when only quantity discount implemented. *Eq.*(4-16) can be further simplified as follows.

$$\begin{aligned} \Delta w &= (w_0 - w) \\ &= (w_0 - w^*) - u \left\{ \frac{1}{Q^*} E(Q^* - D)^+ \right\} \end{aligned} \quad (4-17)$$

Eq. (4-17) represents the feasible sets with a wholesale price discount and buyback price that can be designed as a menu of discount-return combinations. The idea of a discount-return menu is consistent with the return program offered by distributor to its retail customers, in which return policy trades off with a wholesale price discount (cf. Padmanabhan and Png 1995).

Proposition 4-1. *All feasible sets of $(\Delta w, u)$ combinations, as represented in Eq. (4-17), will satisfy the Pareto efficiency.*

A feasible allocation, X , is a Pareto efficient allocation if there is no feasible allocation, X' , such that all agents prefer X' to X (cf. Varian 1984). There is no feasible inventory level where both the manufacturer and retailer are satisfied and one of them is significantly better off since the model strives to maximize the joint profit. That is, if such a quantity exists, then the joint profit can be improved. However, it is found that the joint profit is maximized in the supply chain. As a result, we can conclude that Pareto efficiency will result in the model. That is, all feasible sets of $(\Delta w, u)$ combinations, as represented in *Eq. (4-17)*, will result maximum channel profit.

Proposition 4-2. *The retailer's loss due to altering the order is partly offset by the return credit.*

As mentioned above, the retailer will order Q_{EOQ} when no quantity discounts or return credits is implemented. When only quantity discount implemented, the retailer's loss due to altering order is totally offset by quantity discount. Take a look into *Eq. (4-15)*, the retailer will change order decision only when quantity discount saving is higher enough to offset the loss due to altering order since $u = 0$ and p, s is constant. When both

quantity discount and return policy implemented, from Eq.(4-15), the loss due to altering the order is offset by both discount saving and return credit. Notably, from Eq.(4-17), it is obvious that the inventory model is reduced to a traditional quantity discount scenario under the extreme case, $u = 0$ (cf. Monahan 1984, Lal and Staelin 1984, Banerjee 1986, and Chiang *et al.* 1994). Restated, $(w - w^*)$ is the price discount the manufacturer should offer when returns are prohibited. Therefore, the price discount will be lower than its no return counterpart if $u > 0$ since the wholesale price discount among the return scenario is greater than the traditional quantity discount model.

Proposition 4-3. *A return policy can be considered as mirror-images of a quantity discount strategy. That is, the highest quantity discount comes with very strict limits on returns and a restocking fee for any returned goods whereas a lower quantity discount produces a more liberal returns policy.*

Eq.(4-17) can thus be rewritten as follows

$$u = \left(\frac{Q^*}{E(Q^* - D)^+} \right) \left((w_0 - w^*) - \Delta w \right) \quad (4-18)$$

Eq.(4-18) states that u must increase if Δw decreases. That is, buyback price is negatively associated with the wholesale price discount and such a relationship can be considered as mirror-images.

4.3 Numerical illustration

This section provides a numerical example to illustrate the previous developments. The case concerns a retailer who orders a specific commodity from a single manufacturer at a wholesale price, $w_0 = 80$. The retailer then sells the commodity at a retail price, $p = 150$.

Consumer demand is uniformly distributed within $[0,100]$. The manufacturer begins production at production cost $m = 60$ whenever the retailer places an order. The retailer can return any unsold items to the manufacturer at a pre-determined buyback price, u , at the end of the selling season. The retailer serves goodwill loss with $s = 30$ if the market creates any shortages.

Table 4-1: The general model results ($u = 0$, traditional quantity discount case)

	Manufacturer	Retailer
Revenue	5213.92	6568.84
Production cost	(4000.20)	
Wholesale cost		(5213.92)
Goodwill loss		(170.70)
Profit	1213.72	1184.22
Joint profit	2397.94	

At stage one, the optimal inventory level Q^* , 66.67, is determined. With this quota, the channel subgame is solved. As shown in the second stage, rather than order Q^* , a rational retailer would order EOQ, 55.56. To induce the retailer to change his ordering policy, the manufacturer uses quantity discounts and a return policy as incentive. The feasible sets of $(\Delta w, u)$ can be expressed as $(1.80 - 0.34u, u)$ by substituting the relevant parameters into the solutions. Thus, a greater wholesale price discount is associated with a lower buyback price and the quantity discount will be minus when $u > 5.29$. That is, $u = 5.29$ is the upper bound if the manufacturer wishes to use both a quantity discount strategy and a return policy. The wholesale price must be higher than the original price, if the manufacturer chooses a buyback price over 5.29 per item, in order to maintain channel efficiency. Alternatively, the developed model is reduced to a traditional quantity discount example if the manufacturer decides not to accept returns (*i.e.* $u = 0$). The wholesale price

discount offered to the retailer is 1.80 per item according to the menu. Table 4-1 illustrates the relevant results of this extreme case.

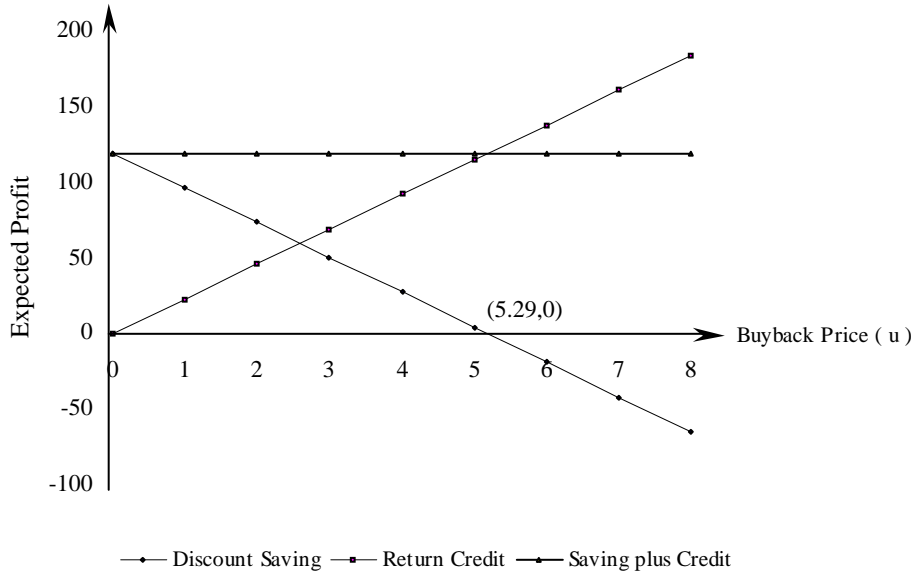


Figure 4-1: The quantity discount savings vs. the return credit

Figure 4-1 displays how profits are allocated when distinct quantity discounts and buyback prices are employed. The wholesale price discount and the saving quantity discount will decrease whenever the buyback price increases. However, the retailer and the manufacturer's profits will not change when the wholesale price and buyback price changes within feasible sets because the return credit that the retailer receives will offset any lost savings according to Proposition 4-2. Thus, the channel will remain efficient within feasible sets.

4.4 Discussion and conclusion Remark

This study evaluated manufacturer quantity discount strategies and return policies. The general model is developed by a two-stage game. At stage one, the manufacturer and retailer cooperate to determine the channel's optimal inventory level. At stage two, to maintain channel efficiency, the manufacturer design adequate incentive schemes to entice the buyer to change ordering decision.

The joint profit peaked when the retailer and manufacturer are vertically integrated within the system. However, since retailers prefer to order the EOQ rather than the quantity determined in a vertically integrated scenario, a traditional quantity discount model can be adapted so the manufacturer can accept returns in order to boost retailer orders. A menu of discount-return combinations that balanced a return policy with a quantity discount was designed to produce the optimum product price and order quantity. Furthermore, we have shown that return policy can be considered as mirror-images of quantity discount strategy. That is, the lowest wholesale price comes with very strict feasibility on return and a restocking fee for any returned goods. Otherwise, a higher wholesale price can result the most liberal return policy.