CHAPTER 5

AN INTEGRATED INVENTORY MODEL OF RETURNS-QUANTITY DISCOUNTS CONTRACT

In the traditional inventory problem, to secure demand risk a retailer often requests the right to return unsold goods, although this is associated with higher wholesale prices. Various studies have attempted to illustrate the returns scenario. However, these studies have focused on optimization from the retailer's perspective only, and have thus ignored the fact that the manufacturer might have no incentive to accept returns.

5.1 Problem description

This study takes account of the self-interest of both the retailer and the manufacturer, and demonstrates that a quantity discount scheme should provide the manufacturer with incentive to accept returns. A three-stage theoretical model is developed and presented to illustrate the returns-quantity discounts contract, and demonstrates that the contract is self-enforcing. Furthermore, it is demonstrated that Pareto efficiency can be attained in the model. The scenarios are illustrated through a numerical example.

5.2 The basic model

This study models the scenario by modifying the returns framework to represent an independent manufacturer and an independent retailer. According to the scenario, an item, such as a newspaper or an airline seat, is assumed to perish if it is not sold during the selling season. In addition, in this scenario, backup is prohibited. The market consumer demand is stochastic, and both parties know their respective demand distribution. The retailer decides the quantity to be ordered from the supplier when customer demand is uncertain. If the retailer orders too much, he or she will be left with unsold stock that has no value at the end of the selling season. However, if the retailer orders too little, he or she will end up with unsatisfied demand that could otherwise have generated more revenue. To secure the revenue earned from the market, the retailer requests that all unsold items be returned to the manufacturer at a predetermined buyback price. However, if the manufacturer accepts the returns contract, the implication is that he or she accepts the demand risk. It is clear that, unless compensated by the retailer, the manufacturer will not accept such a contract. Therefore, under such a returns contract, the manufacturer often asks for a higher wholesale price. That is, the retailer is required to pay an insurance premium to be shielded from risks in demand, and this insurance premium is equivalent to the expected loss due to unsold goods destroyed. The model also assumes that the production is 'make-to-order', and that the retailer's order quantity therefore equals the manufacturer's production. The manufacturer and the retailer are assumed to apply constant marginal cost techniques to manufacturing and retailing respectively.

The basic model assumes that the retailer places order quantity, Q. If demand is greater than Q, the retailer will sell the entire order quantity and make a profit of $p \times Q$ and lose sales of $s(D-Q)^{+}$. Otherwise, if demand is less than Q, the retailer will sell only a quantity equivalent to the demand (D) for full price, and receive a return premium of $u(Q-D)^{+}$. The unsold stock will be left with no value. We assume the inverse of the demand function exist. Furthermore, to assure internal consistency, the cost parameters follow some straightforward assumptions: (a) $p > w_0 > m > 0$, (b) $u < w_0$, (c) $s > 0$.

Retailer’s profit function

The retailer's profit can be expressed as profits from the market minus wholesale costs, goodwill loss and plus returns profit.

$$\pi_r = pMin(Q,D) - wQ - s(D-Q)^+ + u(Q-D)^+ \tag{5-1}$$

Take expectation for all possible demand, the retailer’s expected profit can be expressed as:

$$E(\pi_r) = pE\{Min(Q,D)\} - wQ - sE(D-Q)^+ + uE(Q-D)^+ \tag{5-2}$$

Manufacturer’s profit function

The manufacturer's profit can be expressed as wholesale profit minus manufacturing costs and returns payoff.

$$\pi_m = w_0 Q - mQ - u(Q-D)^+ \tag{5-3}$$

Take expectation for all possible demand, the manufacturer’s expected profit can be expressed as:

$$E(\pi_m) = w_0 Q - mQ - uE(Q-D)^+ \tag{5-4}$$

Earlier studies stated the manufacturer will accept the returns contract if the manufacturer’s profit after option premium is not less than that of a no-returns scenario. That is, a manufacturer’s profit after returns should be equivalent to that of no-returns model. $Eq.$(5-5) represents this scenario, and the equation can be used to characterize the relationship between wholesale price and returns premium:

$$\pi_m(w,u,Q) - \pi_m(w_0, u=0, Q) = 0 \tag{5-5}$$

Substituting the manufacturer's profit function into the equation, *Eq.* (5-5) can be expressed as:

$$\left(wQ - mQ + uE(Q-D)^+\right) - \left(w_0 Q - mQ\right) = 0 \tag{5-6}$$

which can be rewritten as follows:

$$w^* = w_0 + \frac{1}{Q}\left\{uE(Q-D)^+\right\} \tag{5-7}$$

Eq. (5-7) represents the wholesale price in the case in which the retailer requests to return unsold goods by paying the price premium.

Proposition 5-1. *According to the model:*

(a) A higher returns premium correlates with a higher wholesale price, and vice versa.

(b) The wholesale price will be higher than that of the extreme no-returns scenario.

(a) Consider *Eq.* (5-7). The wholesale price, w^*, will be positively related to the returns premium, u, only if $\frac{1}{Q}E(Q^*-D)^+$ is positive. Since $Q > 0$ and $E(Q^*-D)^+$ is also positive by integral theorem, it is self-evident that $\frac{1}{Q}E(Q^*-D)^+$ is positive. That is, the higher returns premium will correlate with higher wholesale price, and vice versa.

(b) When no returns are allowed, u will be zero. Take $u=0$ into *Eq.* (5-7). The wholesale price equals w_0, that is lower bound in the equation since $\frac{1}{Q}\left(uE(Q^*-D)^+\right)$ is positive. Notably, $uE(Q^*-D)^+$, returns unit price product expected returns units, and this could be considered intuitively to be the insurance premium that a retailer needs to pay for a returns contact.

Proposition 5-2. *The retailer's optimal order quantity in the model will be the same as that in the no-returns scenario:* $Q^* = F^{-1}\{(p+s-w_0)/(p+s)\}$.

A comparison of the retailer's profit function before and after returns can establish this proposition. *Eq.* (5-5) stated that the retailer's profit will remain the same with any given order quantity, Q. And the optimal order quantity is established by partial differentiation of the retailer's profit function with Q. Because the retailer's profit function will not change with Q, the outcome of the differentiation will remain unchanged. Therefore, the optimal order quantity will remain unchanged before and after returns. And the optimal order quantity in the no-returns scenario, $Q^* = F^{-1}\{(p+s-w_0)/(p+s)\}$, can be established by differentiating the retailer's profit function, which is also the optimal solution in the 'newsboy problem'.

At this stage, a retailer is satisfied because the demand risk is secured by offering insurance. However, problems still exist. If the manufacturer accepts returns, this means that he or she will take the demand risk whereas the expected profit will be no different from a no-returns scenario. It is clear that the manufacturer might have no incentive to accept returns. In the next section, it is proposed that a quantity discount scheme should be a solution for such a situation. This represents the distinctive contribution of the present study.

5.3 The Subgame Nash-perfect equilibrium

The previous section presented the basic model in which a retailer satisfies self-interest to secure demand risk by a price premium. In this section, the basic model is developed with further consideration of the manufacturer's interest in terms of quantity discounts.

Quantity discount schemes are characteristic of an efficient supply chain by providing an incentive to the buyer to order quantities greater than the economic order quantity (EOQ). In other words, the objective of quantity discounts is to induce the buyer to alter his or her order schedule to achieve system efficiency. This stage will verify how the manufacturer can simultaneously utilize both a quantity discount strategy and a return policy to maintain the channel efficiency.

5.3.1 Channel optimum

To achieve channel efficiency, the manufacturer and retailer first act as Nash players. The scenario can be perceived as one in which the manufacturer and the retailer are combined as a single entity. The objective is to determine the optimal inventory level by maximizing the joint profits. The manufacturer's profit can be expressed as revenue minus production and buyback costs offered to the retailer. Let π_J represent the joint profit of the manufacturer and retailer, which is $\pi_J = \pi_r + \pi_m$. Therefore, the expected joint profit in light of all possible demands can be expressed as:

$$E(\pi_J) = p\,Min\,E(Q,D) - mQ - sE(D-Q)^+ \tag{5-8}$$

To find the optimum value of Q, we set $\partial E(\pi_J)/\partial Q = 0$. That is:

$$\frac{\partial E(\pi_J)}{\partial Q} = (p+s)(1-F(Q)) - m = 0 \tag{5-9}$$

which can be rewritten as:

$$F(Q) = (p+s-m)/(p+s) \tag{5-10}$$

Differentiating $Eq.$ (5-9) yields the following second-order condition:

$$\frac{\partial^2 E(\pi_J)}{\partial Q^2} = -(p+s) f(Q) \le 0 \tag{5-11}$$

Therefore, the second-order condition is satisfied, and $Q^{**} = F^{-1}\{(p+s-m)/(p+s)\}$ denotes channel's optimal inventory level. The profit of a vertically integrated firm is the maximum attainable in the system. However, the retailer faced with uncertain demand has an incentive to order less (probably the EOQ) than the manufacturer desires. Manufacturers should then offer quantity discounts to maximize the system efficiency without harming the retailer. The manufacturer can also reduce the uncertainty facing the retailer by allowing him or her to return any unsold items (cf. Kandel 1996).

Proposition 5-3. *The order quantity to reach system optimum will be larger than that in the basic model.*

In Proposition 5-2, the optimal order quantity in the basic model was characterized as $Q^* = F^{-1}\{(p+s-w_0)/(p+s)\}$, and the optimal order quantity, $Q^{**} = F^{-1}\{(p+s-m)/(p+s)\}$, in Nash-perfect equilibrium was demonstrated in *Eq.* (5-10). Because $w_0 > m$, $Q^{**} > Q^*$ is established.

Proposition 5-4. *According to the model:*

(a) Joint profit of the Nash-perfect equilibrium model is higher than that of the basic model—that is, extra profit will result from the Nash-perfect equilibrium model.

(b) Pareto efficiency is also attained herein.

(a) Because the objective of the Nash-perfect equilibrium model is to maximize the joint profit, this proposition is self-evident. That is, the system joint profit will be optimized in the Nash-perfect equilibrium and it will be higher than, or equivalent to, that in the sub-optimal model (such as the basic model). Furthermore, once the optimal quota within the sub-optimal model is ascertained to be different from that within the Nash-perfect equilibrium, the Nash-perfect equilibrium model dominates the sub-optimal model. Compare Q^{**} and Q^{*}. The difference between them is apparent. The Nash-perfect equilibrium model thus dominates the basic model, or, in other words, extra profit results from the Nash-perfect equilibrium model, and a retailer could bargain with the manufacturer to share this extra profit produced through channel cooperation.

(b) A feasible allocation, X, is a Pareto-efficient allocation if there is no feasible allocation, $X^{'}$, such that all agents prefer $X^{'}$ to X (cf. Varian 1984). There is no feasible inventory level at which both the manufacturer and retailer will be satisfied, and one of them will be significantly better off since the model strives to maximize the joint profit. That is, if such a quantity exists, then the joint profit can be improved. However, joint profit is maximized in the channel. As a result, it can be concluded that Pareto efficiency will result from the model.

5.3.2 Modified wholesale prices

It will be recalled that the retailer's optimal quantity is $Q^{*}=F^{-1}\{(p+s-w_0)/(p+s)\}$. To induce the retailer to increase the order quantity to $Q^{**}=F^{-1}\{(p+s-m)/(p+s)\}$, a manufacturer needs to offer the retailer quantity discounts to compensate for the loss due to altering the order scheme. To determine the wholesale price at this stage, the model assumes that a retailer will increase order only when the profit after returns is no less than its counterpart in the basic model, as expressed in $Eq.$(5-12).

$$\pi_r\left(w=w^{**},u,Q=Q^{**}\right)-\pi_r\left(w=w^{*},u,Q=Q^{*}\right)\geq 0 \tag{5-12}$$

Let $Eq.$(5-12) equal zero, so the retailer's profit after returns equals its counterpart in the basic model.

$$\begin{aligned}&\left(pEMin\left(Q^{**},D\right)-w^{**}Q^{**}-sE\left(D-Q^{**}\right)^{+}+uE\left(Q^{**}-D\right)^{+}\right)\\&-\left(pEMin\left(Q^{*},D\right)-w^{*}Q^{*}-sE\left(D-Q^{*}\right)^{+}+uE\left(Q^{*}-D\right)^{+}\right)=0\end{aligned} \tag{5-13}$$

which can be rewritten as:

$$\begin{aligned}w^{**}=&\frac{1}{Q^{**}}\left\{p\left(EMin\left(Q^{**},D\right)-EMin\left(Q^{*},D\right)\right)-s\left(E\left(D-Q^{**}\right)^{+}-E\left(D-Q^{*}\right)\right)+u\left(E\left(Q^{**}-D\right)^{+}-E\left(Q^{*}-D\right)^{+}\right)\right\}\\&+w^{*}\frac{Q^{*}}{Q^{**}}\end{aligned} \tag{5-14}$$

$Eq.$(5-14) represents the wholesale price, w^{**}, in the case in which the retailer agrees to increase order volume since his or her profit will not be worse off. Notably, w^{**} will be lower than w^{*} caused quantity discount is taken into consideration.

5.4 Bargain to share system extra profit

Proposition 4-4 stated that extra profit will accrue in the Nash-perfect equilibrium through channel cooperation. This section describes how the retailer and the manufacturer share the extra profit, and how the bargaining outcome affects the wholesale price. Moreover, we will characterize the factor that causes the model self-enforcing. The extra profit of the manufacturer can be expressed as:

$$\Delta\pi_m=\pi_m\left(w,u,Q^{**}\right)-\pi_m\left(w^{*},u,Q^{*}\right) \tag{5-15}$$

which can be rewritten as:

$$\Delta\pi_m=\left(w^{**}Q^{**}-w^{*}Q^{*}\right)+m\left(Q^{*}-Q^{**}\right)+u\left(E\left(Q^{*}-D\right)^{+}-E\left(Q^{**}-D\right)^{+}\right) \tag{5-16}$$

The bargaining outcome will be entirely dependent on the relative negotiating power of the manufacturer and the retailer. If the negotiating power of the manufacturer is much larger than that of the retailer, the extra gain will be largely taken by the manufacturer, and vice versa. Let p denote the bargaining power between the manufacturer and the retailer, with $p = 1$ implying that the manufacturer has a monopoly in the wholesale market. The following proposition can then be stated.

Proposition 5-5. *The returns-quantity discounts model is self-enforcing with any bargaining outcome* (Q^{**}, w^{**}), $w^{**} = \frac{1}{Q^{**}}\left(Q^{*}w^{*} - p\Delta\pi_m\right)$ *for* $0 \le p \le 1$.

According to model, a profit increase of $\Delta\pi = p\Delta\pi_m$ for the retailer compared with that in the basic model; and a profit increase of $\Delta\pi = (1 - p)\Delta\pi_m$ for the manufacturer compared with that in the basic model. That is, the returns-quantity discounts contact benefits both the retail and the manufacturer, and the contact is therefore self-enforcing.

Proposition 5-5 indicates that both the retailer and the manufacturer can benefit through channel cooperation. In other words, this model satisfies both the retailer’s interest and the manufacturer’s interest, and aspires to a ‘win-win’ status.

5.5 Numerical illustration

To illustrate the above arguments, a numerical example is now presented. The problem scenario is that of a retailer ordering a specific commodity from a single manufacturer at a wholesale price $w_0 = 8$. The retailer, in turn, sells the commodity in the retail market at a retail price of $p = 15$. Consumer demand in the market is uniformly distributed within [0, 100]. The manufacturer produces at the time when the retailer places an order, with a variable production cost per unit, $m = 6$. If shortage occurs in the market, the retailer serves cost with $s = 3$.

Table 5-1: Model results (The basic model extreme case—no returns scenario)

	Manufacturer	Retailer
Revenue	424.00	581.10
Production cost	(318.00)	
Wholesale cost		(424.00)
Goodwill loss		(28.65)
Profit	106.00	128.45
Joint profit	234.45	

The extreme case in the basic model—the no returns scenario, which is used as the benchmark for the developed model—is considered first. The retailer's optimal order quantity is determined to be 55.56. Notably, every individual cost and the total profit are computed when the optimal ordering quantity is substituted into the retailer's profit function. Furthermore, with this quota, the manufacturer's cost is also computed. Table 5-1 illustrates the results.

Table 5-2: Model results (The basic model)

	Manufacturer	Retailer
Revenue	466.78	581.10
Production cost	(318.00)	
Wholesale cost		(466.78)
Goodwill loss		(28.65)
Returns cost/profit	(42.78)	42.78
Profit	106.00	128.45
Joint profit	234.45	

Next, the basic model is utilised with returns involved. At the end of the selling season, the retailer can return all unsold items to the manufacturer with a predetermined buyback price $u = 3$. The optimal order quantity is 55.56, which equals the optimal quota in the no returns scenario, and the wholesale price, w^*, is 8.8072, which is higher than that obtained from the no returns scenario. The higher wholesale price that a manufacturer charges is to compensate for his or her additional cost of returns. This result is consistent with the conclusion reached by Padmanabhan and Png—that the wholesale price when returns are acceptable should incorporate an insurance premium, and hence should be higher than that when returns are not acceptable. The results are illustrated in Table 5-2, and show that the manufacturer's revenue increases by 42.78—that is, the retailer's wholesale cost also increases by 42.78. From the retailer's view, he or she pays an insurance premium in an amount of 42.78, to avoid the inventory risk. For the manufacturer, he or she charges a higher wholesale price to compensate for the additional cost of returns.

Table 5-3: Model results (The Nash-perfect equilibrium model)

	Manufacturer	Retailer
Revenue	554.83	639.15
Production cost	(378.00)	
Wholesale cost		(554.83)
Goodwill loss		(17.04)
Returns cost/profit	(61.17)	61.17
Profit	115.66	128.45
Joint profit	244.11	

The Nash-perfect equilibrium is attained when the retailer increases order quantity to 66.67, which is greater than that obtained in the basic model. And the wholesale price, w^{**}, is 8.8068, which is lower than that in the basic model. The wholesale price is brought down by the quantity discount that the manufacturer offers to the retailer—because a larger order quantity will mitigate the manufacturer's cost decline. If the optimal quota and wholesale price are substituted into the profit function of both the retailer and the manufacturer, every individual cost, and the total profit, can be computed. The results are illustrated in Table 5-3, which shows that the retailer's profit, 128.45, equals that of the returns scenario. However, the manufacturer's profit, at 115.66, is higher than that of the returns scenario, 106. That is, an excess profit, 9.66, is produced in the system, which increases by about 4.1%.

Table 5-4: Model results (The bargaining outcome)

	Manufacturer	Retailer
Revenue	550.00	639.15
Production cost	(378.00)	
Wholesale cost		(550.00)
Goodwill loss		(17.04)
Returns cost/profit	(61.17)	61.17
Profit	110.83	133.28
Joint profit	244.11	

Thereafter, the retailer and the manufacturer negotiate to share the system extra profit. This scenario assumes equal bargaining power between the retailer and the manufacturer, and therefore, that the parties will share equally in the extra profit (cf. Table 5-4).

Figure 5-1 displays how wholesale prices and order quantities are allocated when distinct scenarios are employed. The wholesale price of the basic model with a returns scenario is higher than that of no-returns scenario due to the additional cost incurred by the manufacturer for accepting returns. Thereafter, the Nash-perfect equilibrium model takes this fact into consideration and demonstrates that the wholesale price should be lower when the retailer accepts quantity discounts contracts.

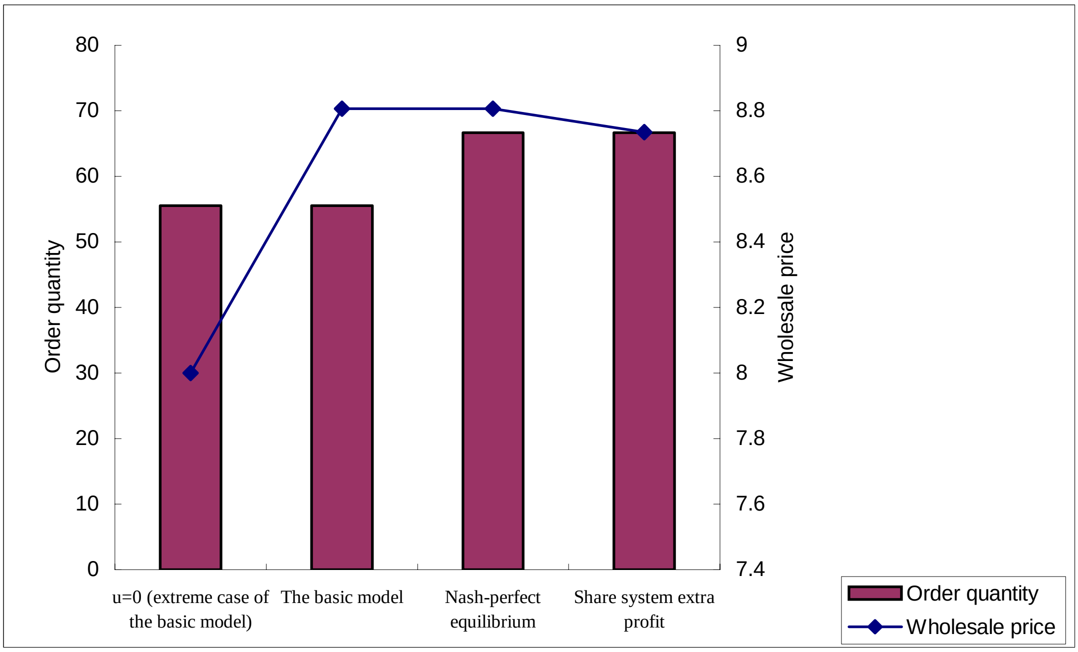

Figure 5-1: Wholesale price and optimal order quantity summary

Finally, the wholesale price will decline again after the retailer bargains to share the system profit. The optimal order quantity will increase only when a manufacturer offers quantity discounts to encourage the retailer to place more orders in the Nash-perfect equilibrium model.

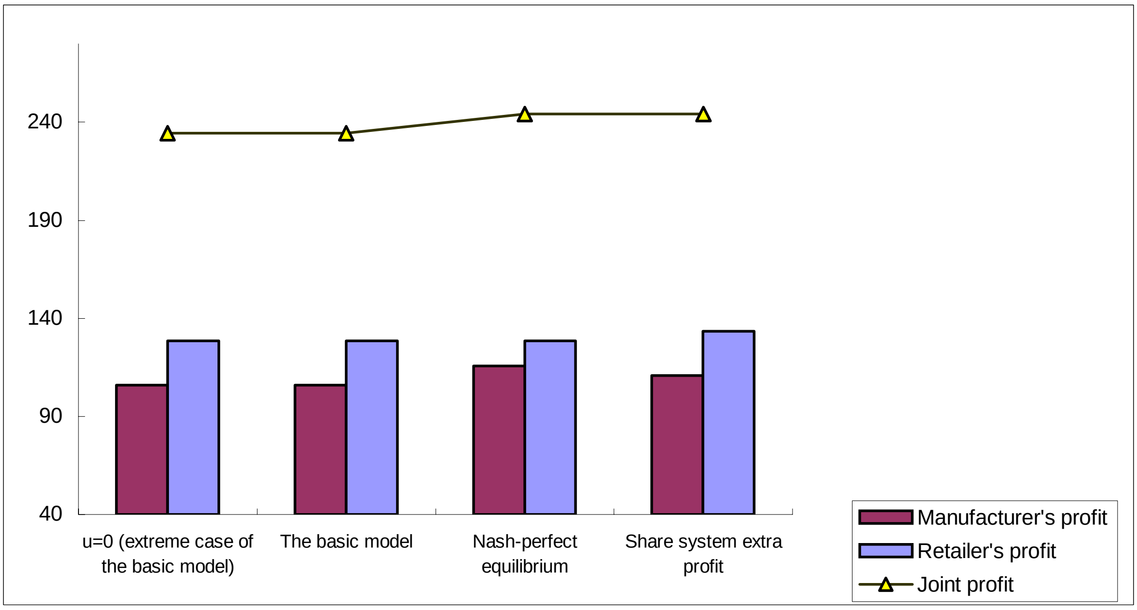

Figure 5-2: Profits summary

In Figure 5-2, the profits of manufacturer and retailer are illustrated when distinct models are employed. It can be seen that the manufacture's profit in the Nash-perfect equilibrium model is higher than in the basic model, whereas the retailer's profit remains the same. That is, extra profit is produced in the Nash-perfect equilibrium model. Moreover, the profits of both the retailer and the manufacturer increase (as compared with the basic model) after sharing the system profit that causes the returns-quantity discounts contact self-enforcing as Proposition 5-5 stated.

5.6 Discussion and conclusion Remark

The purpose of this study was to develop and present a generalized returns-quantity discounts model, which represents the interests of both the retailer and the manufacturer through a three-stage theoretical approach. First, to secure against demand risk, a retailer requests the right to return any unsold goods. The basic model was addressed and the relationship between the wholesale price and return premium was determined at this stage. Through the basic model, it was illustrated that a manufacturer will charge more to compensate for the extra cost after he or she accepts return. At the second stage, quantity discounts were incorporated into the model, and Nash-perfect equilibrium was achieved. The manufacturer's interest was satisfied at this stage, because higher order volume would definitely increase his or her profit. Finally, the retailer could ask to share the system extra profit because it was caused through channel cooperation.

The present study has shown that a quantity discounts-returns scheme is self-enforcing, that is, the retailer and the manufacturer can both benefit from the scheme. Furthermore, it has been demonstrated that, through coordination, the system can be Pareto efficient, and thus achieves a real win-win status.