

# GA-QP Model to Optimize Sewer System Design

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**Abstract:** Sanitary sewer systems are fundamental and expensive facilities for controlling water pollution. Optimizing sewer design is a difficult task due to its associated hydraulic and mathematical complexities. Therefore, a genetic algorithm (GA) based approach has been developed. A set of diameters for all pipe segments in a sewer system is regarded as a chromosome for the proposed GA model. Hydraulic and topographical constraints are adopted in order to eliminate inappropriate chromosomes, thereby improving computational efficiency. To improve the solvability of the proposed model, the nonlinear cost optimization model is approximated and transformed into a quadratic programming (QP) model. The system cost, pipe slopes, and pipe buried depths of each generated chromosome are determined using the QP model. A sewer design problem cited in literature has been solved using the GA-QP model. The solution obtained from the GA model is comparable to that produced by the discrete differential dynamic programming approach. Finally, several near-optimum designs produced using the modeling to generate alternative approach are discussed and compared for improving the final design decision.

**DOI:** 10.1061/(ASCE)0733-9372(2009)135:1(17)

**CE Database subject headings:** Sewer design; Algorithm; Optimization.

## Introduction

Sanitary sewer systems are essential for the protection of water quality and the improvement of sanitation. The construction of a new sewer system in a city is quite an expensive task, especially in a highly populated city. However, the cost of establishing a sewer system can be significantly reduced if the system configuration (e.g., pipe diameters, pipe slopes, pipe buried depths, etc.) can be effectively optimized (Mays et al. 1976; Li and Matthew 1990). Unfortunately, an optimal design is difficult to obtain because a design optimization process offers numerous alternative solutions (Liebman 1967) and involves many complex hydraulic and engineering constraints (Argaman et al. 1973; Kulkarni and Khanna 1985; Swamee 2001). Typical constraints include maintaining a minimum velocity for self-cleaning capability, preventing pipes from scouring with maximum velocity, placing upstream pipes at elevations higher than those downstream, maintaining a maximum water depth to prevent the flow exceeding the design capacity, accommodating the diameters of commercially available pipes, maintaining a minimum buried depth, and ensuring that the diameters of downstream pipes are greater than or equal to those of upstream ones. These constraints are often nonlinear or discrete and the size of a sewer network is generally large; therefore, these complexities make this design optimization problem difficult to solve.

Several optimization approaches (Walsh and Brown 1973; Elimam et al. 1989; Liang et al. 2004) have been developed to

solve sewer design problems. Unfortunately, these approaches generally require excessive computational time to solve and this makes them impractical. Although the discrete differential dynamic programming (DDDP) approach provides a significant improvement (Heidari et al. 1971; Mays and Yen 1975; Mays and Wenzel 1976; Mays et al. 1976; Li and Mathew 1990), it restricts the search space and reduces the number of opportunities to locate the global optimum. The DDDP stages must be manually divided for each individual problem and this reduces its practicality. Furthermore, the computer program needed to implement DDDP is not widely available.

The genetic algorithm (GA), a search algorithm based on natural selection and the mechanisms of population genetics (Goldberg 1989), has been widely applied to civil and environmental problems (Goldberg and Kuo 1987; Simpson et al. 1994; Lippai et al. 1999; Dandy and Engelhardt 2001; Liang et al. 2004; Babayan et al. 2005; Afshar et al. 2006). Since this algorithm uses a code of model parameters instead of the parameters themselves and searches from multiple points instead of a single point, it is efficient for finding good solutions.

Although the GA has many advantages in solving a difficult optimization problem, randomly generated GA variables may reduce its efficiency, even for solving a simple linear programming model. If a problem is large or involves many variables, the solving efficiency of the GA will be reduced because the increased number of variables also increases the number of alternatives. The other major reason is that many of the randomly produced alternatives are unreasonable or inappropriate. Therefore, some researchers have tried to combine the GA with linear programming (LP) in order to improve the solving efficiency of a GA-based model. For instance, in solving a nonlinear water management model, Cai et al. (2001) chose some complex variables as the GA variables. When the values of these complex variables are generated from a GA model, the nonlinear model becomes linear and easier to solve using LP. A GA-LP model can significantly avoid many unreasonable solutions being produced randomly. Berry et al. (1999) also combined a GA with a LP to solve the optimization problem for a communication network design, and El-

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Note. Discussion open until June 1, 2009. Separate discussions must be submitted for individual papers. The manuscript for this paper was submitted for review and possible publication on August 20, 2007; approved on June 20, 2008. This paper is part of the *Journal of Environmental Engineering*, Vol. 135, No. 1, January 1, 2009. ©ASCE, ISSN 0733-9372/2009/1-17-24/\$25.00.

Araby et al. (2003) combined a GA with a successive LP to solve a flexible alternating-current transmission allocation problem. However, one point should be noted. Unlike with the traditional formulation of LP, there is no guarantee of global optimality for the result obtained from the proposed GA-LP.

Although the integration of a GA and LP is an attractive method, the transformation of nonlinear functions into linear ones by piecewise linearization is complex. Moreover, the resulting piecewise linear functions may be significantly different from the original nonlinear functions. Although increasing the number of pieces can reduce the difference, it would also make the model more complicated. Therefore, instead of piecewise linearization, this study has transformed nonlinear functions into quadratic forms and solved the problem by using quadratic programming (QP). A nonlinear programming model generally has multiple local optima and the global optimum is difficult to find. By contrast, a QP model, although nonlinear, is unimodal and has only one optimum, like a LP model, and the necessary algorithms are available to find its global optimum. Furthermore, a quadratic function maintains nonlinearity and is more appropriate than a piecewise linear function for the representation of a nonlinear cost function. Hence, this study has combined GA with QP, instead of LP, to solve a sewer design model.

Some important design factors (for example, construction, geology, and traffic impact during construction) and unquantifiable issues [for example, public preference, not in my backyard (NIMBY) consensus, and land availability] are not included within the scope of this study because they are not easily formulated mathematically into the proposed model. Thus, the solution obtained from existing sewer design models may not be the best one or even feasible when unmodeled factors or issues are simultaneously evaluated. Generating near-optimum alternatives, therefore, is desirable in the exploration of a good alternative design. Modeling to generate alternatives (MGAs) (Chang and Brill 1982; Chang and Liaw 1985; Gupta et al. 2005) is an effective approach developed to generate alternatives that are good and different. In this study, a special MGA function has been developed to evaluate the differences among the designs generated by the GA-QP and DDDP models. As a result, several MGA alternatives are generated and illustrated.

In the following sections, the chromosome and the fitness function of the proposed sewer GA model are discussed first. Next, the descriptions of the cost function and constraints of the proposed QP model and then the transformations of the nonlinear constraints into linear constraints follow. Finally, the applicability and efficiency of the GA-QP model is demonstrated using the example cited in Li and Matthew (1990). The results, including the MGA alternatives, are discussed and compared with that obtained using DDDP.

## GA Model

The GA model developed for the sewer design optimization problem utilizes a GA search algorithm simulating natural genetic evolution (Goldberg 1989). In the search algorithm, each decision variable is taken as a gene, and a set of all decision variables is treated as a chromosome. With the reproduction mechanisms including selection, crossover, and mutation, the unfit chromosomes, determined by a fitness function, are eliminated, and the other chromosomes survive and continue to the next generation until an acceptable solution is obtained. The details concerning

chromosomes, fitness, and the three operators are described below.

## Chromosome

Each set of decision variables is coded as a chromosome in the proposed GA model. Each chromosome represents one design, and the associated decision variables are coded as genes. The diameters of all the pipes and the locations of the pumping stations are expressed by genes with binary codes. A chromosome consists of the genes of the pipe diameters and pumping station locations which represent a design layout of the sewer system studied.

The values of the genes are randomly produced and thus the generated chromosomes sometimes may be obviously unacceptable. For the sewer design problem, it is not easy to repair an unacceptable chromosome automatically because upstream and downstream pipes are strongly correlated, and changing one of the pipes requires simultaneous changes of other related pipes. Therefore, this study eliminates those unacceptable chromosomes, instead of repairing them, and generates another new chromosome. To eliminate some of the unacceptable chromosomes and enhance computational efficiency, this study adopts two simple constraints: (1) the diameter must be large enough to transport the accumulated flow from the upstream pipes; and (2) the diameter of a downstream pipe must be larger than or equal to the diameter of its upstream counterpart. The first constraint assures that a pipe has enough flow capacity and is expressed by the following equation:

$$\frac{Q_i}{V_{\max}} \leq \left( \frac{A}{A_0} \right)_{(h/D)_{\max}} \times \left( \frac{D_i^2}{4} \times \pi \right) \quad (1)$$

where  $Q_i$ =flow rate of pipe  $i$ ;  $V_{\max}$ =maximum velocity;  $(h/D)_{\max}$ =maximum water depth ratio;  $(A/A_0)_{(h/D)_{\max}}$ =cross-sectional area ratio of the flow to the pipe at maximum water depth; and  $D_i$ =diameter of pipe  $i$ . On the left hand side of Eq. (1) is the minimum cross-sectional area required to transport the desired inflow and on the right hand side is the available flow cross-sectional area given the constraint of maximum allowable water depth ratio. Finally, the available cross-sectional flow area of the sewer must be larger than the minimum cross-sectional area.

The second constraint is used to maintain flow continuity by assuring that the diameter of a pipe is larger than that of the corresponding pipe upstream. The model checks all the diameters of pipes selected in each generation. In order to reduce the possibility of generating too many unacceptable chromosomes, the diameter of the outlet pipe is determined first, according to total design flow rate, and this measurement serves as the maximum diameter for the pipes upstream of the outlet pipe in order to avoid choosing obviously unreasonable diameters. Once the diameter of a pipe upstream to the outlet pipe is determined, it becomes the maximum limit for the diameters of the corresponding pipes upstream. This process is repeated until the diameters of all the pipes are determined.

Other than the diameter, each candidate pumping station location is also coded as a gene. If the code value is 1, a pumping station is installed at the upstream end of the corresponding pipe. Since a pumping station is quite expensive, a sewer system generally does not install it. Therefore, in order to improve the efficiency of the search, all pumping station code values for the pumps for all manholes were initially set at 0.

## Fitness Function

The fitness function of the GA model is used to evaluate the fitness of a chromosome. In general, a chromosome with a high fitness value is superior to a chromosome with a low fitness value. In this study, fitness value is defined by the reciprocal construction cost of a sewer network

$$f_n = \frac{1}{C_n} \quad (2)$$

where  $f_n$ =fitness of chromosome  $n$ ; and  $C_n$ =cost of chromosome  $n$ . The cost of each chromosome is computed by using the same cost function as that used by Li and Matthew (1990).

## Selection, Crossover, and Mutation

Three processes of reproduction, crossover, and mutation are applied to generate new chromosomes in the GA model. In the reproduction process, the probability of a chromosome being selected for the crossover pool is determined based on its fitness value. The selection process adopted in this study, as described in Simpson et al. (1994), is similar to playing a roulette wheel in that the greater the area the higher the probability of being chosen. The associated probability of each chromosome is determined by the following equation:

$$p_n = \frac{f_n}{\sum_{n=1}^N f_n} \quad (3)$$

where  $p_n$ =selection probability of chromosome  $n$ , and  $N$ =number of chromosomes. In each generation, the selection probability of a chromosome depends on its fitness value. As the construction cost decreases, the fitness and associated area on the wheel both increase. The ratio of the wheel area for each chromosome indicates the probability of the chromosome being selected for the crossover pool.

In the crossover process, two chromosomes in a mother generation are selected from the crossover pool to exchange their binary (gene) information. This process randomly selects parts of genes from two chromosomes and makes up a new chromosome. In this study, the crossover process exchanges the diameters and pumping station locations of two mother chromosomes to generate two child chromosomes.

The mutation process randomly selects the mutation point in the chromosome and changes its binary information. The probability of a mutation is set by the user to indicate the frequency of mutations that may happen in the GA model. The mutation process changes the pipe diameters and pumping station locations in order to increase the search range and to avoid being trapped at a local optimum.

## QP Model

To improve solving efficiency, this study has established a QP model to determine the fitness value of a chromosome. The decision variables associated with the QP model are the slopes and buried depths at the downstream ends of pipes. The objective function and constraints of the QP model are listed below

$$\text{Min } C_n = \sum_{i=1}^{NL} (C_{n\_Li} + C_{n\_Pi}) + \sum_{k=1}^{NM} C_{n\_Mk} \quad (4)$$

$$C_{n\_Li}(D_{n\_i}, h_{n\_i}, L_i) = (\alpha_{n\_1i} + \alpha_{n\_2i} \cdot D_{n\_i}^2 + \alpha_{n\_3i} \cdot D_{n\_i} \cdot h_{n\_i} + \alpha_{n\_4i} \cdot h_{n\_i}^2) \cdot L_i \quad (5)$$

$$h_{n\_i} = GD_i - b_{n\_i} \quad (6)$$

$$C_{n\_Mk}(D_{n\_k}, h_{n\_k}) = (\beta_{n\_1k} + \beta_{n\_2k} \cdot D_{n\_k}^2 + \beta_{n\_3k} \cdot D_{n\_k} \cdot h_{n\_k} + \beta_{n\_4k} \cdot h_{n\_k}^2) \quad (7)$$

$$h_{n\_k} = GU_k - (b_{n\_k} + s_{n\_k} \cdot L_k) \quad (8)$$

$$C_{Pi}(P_{n\_i}, Q_i) = P_{n\_i} \cdot (\gamma_1 + \gamma_2 \cdot Q_i + \gamma_3 \cdot Q_i^2) \quad (9)$$

subject to

$$S_{n\_i\_min} \leq s_{n\_i} \leq S_{n\_i\_max} \quad (10)$$

$$b_{n\_i} + s_{n\_i} \cdot L_i \leq b_{n\_j} \quad (\text{no pumping station at pipe } i) \quad (11)$$

$$b_{n\_i} + D_{n\_i} + s_{n\_i} \cdot L_i \leq b_{n\_j} + D_{n\_j} \quad (\text{a pumping station exists at pipe } i) \quad (12)$$

$$GU_i - (b_{n\_i} + s_{n\_i} \cdot L_i + D_{n\_i}) \geq AC_{min} \quad (13)$$

$$GD_i - (b_{n\_i} + D_{n\_i}) \geq AC_{min} \quad (14)$$

where  $C_n$ =cost of chromosome  $n$ ;  $NL$ =number of pipes;  $NM$ =number of manholes;  $C_{n\_Li}$ =construction cost of pipe  $i$ ;  $C_{n\_Mk}$ =construction cost of manhole  $k$ ;  $C_{n\_Pi}$ =construction cost of the pumping station located at the upstream end of pipe  $i$ ;  $h_{n\_i}$  and  $L_i$ =buried depth and length of pipe  $i$ , respectively; the values of pipe cost function coefficients  $\alpha_{n\_1i}$ ,  $\alpha_{n\_2i}$ , and  $\alpha_{n\_3i}$  vary depending on the diameter and buried depth of pipe  $i$ ;  $GD_i$  and  $b_{n\_i}$ =respectively, ground surface elevation and bottom elevation of the downstream end of pipe  $i$ ;  $D_{n\_k}$  and  $h_{n\_k}$ =diameter and depth of manhole  $k$ , respectively; the values of manhole cost function coefficients  $\beta_{n\_1k}$ ,  $\beta_{n\_2k}$ , and  $\beta_{n\_3k}$  are set according to the depth of manhole  $k$  and the diameter of its outlet pipe;  $GU_k$ ,  $b_{n\_k}$ ,  $s_{n\_k}$ , and  $L_k$ =respectively, ground surface elevation of the upstream end, bottom elevation of the downstream end, slope, and length of the pipe connected to the outlet of manhole  $k$ ;  $P_{n\_i}$ =0–1 binary variable indicating if a pumping station exists and located at the upstream end of pipe  $i$ ;  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_3$ =pumping station cost function coefficients;  $s_{n\_i}$ =slope of pipe  $i$ ;  $S_{n\_i\_min}$  and  $S_{n\_i\_max}$ =minimum and maximum allowed slopes of pipe  $i$ , respectively;  $b_{n\_j}$  and  $D_{n\_j}$ =respectively, bottom elevation and diameter of the downstream end of pipe  $j$  (pipe  $j$  is immediate upstream to pipe  $i$ );  $GU_i$ =ground surface elevations of upstream ends of pipe  $i$ ; and  $AC_{min}$ =minimum buried depth.

Eq. (4) is the total cost function, as proposed by Li and Matthew (1990), which is used as the objective function of the QP model. The construction cost of a sewer system includes all the costs of its components, including pipes, manholes, and pumping stations. This objective function is quadratic. Eq. (5) is the computed construction cost of a pipe based on the diameter, buried depth, and length of a pipe. Eq. (6) is used to determine the buried depth of a pipe which is the difference between the bottom elevation ( $b_{n\_i}$ ) and ground surface elevation ( $GD_i$ ) of its downstream end. Eq. (7) estimates the construction cost of a manhole. The cost of a manhole is computed based on the buried depth of a manhole and the diameter of its outflow pipe. In Eq. (8), the buried depth of a manhole is computed based on the elevations of

**Table 1.** Design Parameters for Case Studied (Li and Matthew 1990, ASCE)

Parameter name	Parameter value
Maximum velocity	5.0 m/s
Minimum velocity	0.7 m/s (if $D_{n_i} \leq 0.5$ m, $Q_i > 15$ L/s) 0.8 m/s (if $D_{n_i} > 0.5$ m, $Q_i > 15$ L/s)
Minimum slope	0.003 (if $Q_i \leq 15$ m)
Maximum proportional water depth	0.60 (if $D_{n_i} \leq 0.30$ m) 0.70 (if $D_{n_i} = 0.35-0.45$ m) 0.75 (if $D_{n_i} = 0.50-0.90$ m) 0.80 (if $D_{n_i} \geq 1.0$ m)
Minimum cover depth	1 m

the upstream pipe it connects and the outlet of a manhole. Eq. (9) is used to calculate the construction cost of a pumping station, mainly depending on the volume of flow. A pumping station is assumed to be located at the manhole of the upstream end of the pipe.

In designing a sewer system, the flow velocity and water depth in a pipe are important hydraulic constraints. However, the relationship among flow velocity, water depth, slope, and diameter is nonlinear. If the flow velocity, water depth, and slope were decision variables in the model, the model would be too complex to solve. When the diameter and flow volume are both determined, the primary factor that influences the flow velocity and ratio is the pipe slope. Therefore, as can be seen from Eq. (10), this study constrains the flow velocities and water depths in the pipes by limiting the pipe slope range. Transformation of the constraints of maximum and minimum flow velocities into the constraints of maximum and minimum slopes is explained as follows:

1. Determine the flow cross-sectional area ( $A$ ) by the following equation for the maximum or minimum flow velocity ( $V$ ) under a specific flow ( $Q_i$ ) of pipe  $i$ :

$$\text{cross-section area: } A = Q_i/V \quad (15)$$

2. After the diameter ( $D_i$ ) of a pipe is set in the GA model, the pipe cross-sectional area ( $A_0$ ) is known. When both the pipe cross-sectional area and flow are known, the central angle ( $\phi$ ) from the center of the pipe circle to the water surface can be theoretically obtained using the following equation:

$$\text{area ratio: } \frac{A}{A_0} = \frac{\phi - \sin \phi}{2\pi} \quad (16)$$

However, Eq. (16) is difficult to solve for  $\phi$ . To improve solving efficiency, this study has developed a compatible regression function

$$\begin{aligned} \phi = f\left(\frac{A}{A_0}\right) = & 92.88 \cdot \left(\frac{A}{A_0}\right)^5 - 232.13 \cdot \left(\frac{A}{A_0}\right)^4 \\ & + 214.73 \cdot \left(\frac{A}{A_0}\right)^3 - 90.026 \cdot \left(\frac{A}{A_0}\right)^2 + 20.044 \cdot \left(\frac{A}{A_0}\right) \\ & + 0.39 \end{aligned} \quad (17)$$

The  $R$ -squared value for Eq. (17) is 0.9961;

3. When  $\phi$  is known, the hydraulic radius ( $R$ ) can be determined using Eq. (18)

$$\text{hydraulic radius: } R = \frac{D_i}{4} \cdot \frac{\phi - \sin \phi}{\phi} \quad (18)$$

Then, the maximum and minimum slopes can be obtained by substituting  $V$  in the following Manning equation with the maximum and minimum flow velocities, respectively;

$$\text{Manning's equation: } V = \frac{1}{n} \cdot R^{2/3} \cdot S^{1/2} \quad (19)$$

Maximal proportional water depth,  $(h/D)_{\max}$ , is an important sewer design constraint. However, the water depth ( $h$ ) of a pipe is not an appropriate decision variable because the relationship among water depth, diameter, and slope is too complex to be formulated into the QP model. Therefore, the minimum pipe slope is computed by using the following procedure in the QP model instead:

1. The diameter of each pipe is determined during the application of the QP model, and thus the  $(h/D)_{\max}$  for each pipe can be determined by using the associated value listed in Table 1;
2. Then, the central angle,  $\phi$ , for each pipe is determined using the following equation:

maximum proportional water depth:

$$(h/D)_{\max} = \frac{1}{2} \cdot \left(1 - \cos \frac{\phi}{2}\right) \quad (20)$$

3. Then by substituting  $\phi$  into Eqs. (18) and (19), the minimum pipe slope can be computed.

In this study, the minimum flow velocity and maximum proportional water depth are transferred to two separate limits on the possible pipe slopes. The larger one of these limits is used as the minimum slope constraint in the QP model.

The wastewater in a sewer system is generally collected using gravity rather than a pumping station. Therefore, the bottom and top elevations of the downstream end of a pipe should be higher than or equal to those of the upstream end of the corresponding downstream pipe. These two constraints can be expressed by the following two equations:

$$ub_{n_i} \leq b_{n_j} \quad (21)$$

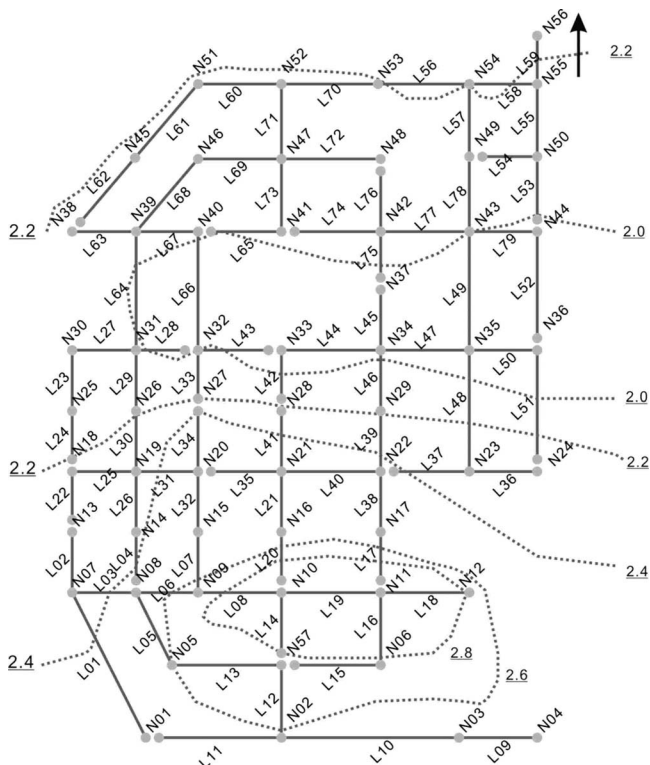
$$ub_{n_i} + D_{n_i} \leq b_{n_j} + D_{n_j} \quad (22)$$

where  $ub_{n_i}$ =bottom elevation of the upstream end of pipe  $i$ ; and  $D_{n_j}$ =diameter of pipe  $j$ , located immediately upstream to pipe  $i$ . The  $ub_{n_i}$  in Eqs. (21) and (22) can then be replaced by the following equation to obtain Eqs. (11) and (12):

$$ub_{n_i} = b_{n_i} + s_{n_i} \cdot L_i \quad (23)$$

In practice, sewer pipes need to be buried to a certain depth, in order to connect the household waste flow with the sewer system and also to avoid damage to the pipeline caused by such things as heavy trucks. Eqs. (13) and (14) indicate that the buried depths of both ends of a pipe must be deeper than a prespecified minimum buried depth.

As a consequence of the constraint simplification procedure described above, the proposed QP model contains only two decision variables: pipe slope and buried depth. In this way, the problem becomes easier to solve.



**Fig. 1.** System layout of case studied (Li and Matthew 1990, ASCE)

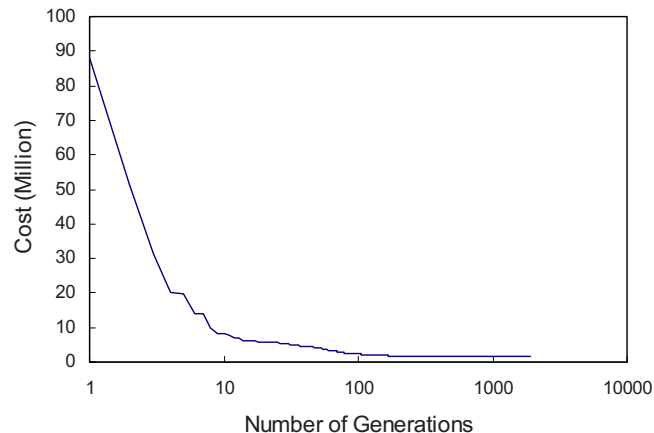
### MGA Function

Since some decision-making factors or issues, which are either hard to quantify or difficult to formulate into the proposed model, have been excluded the optimal solution obtained may not be the true best. To facilitate the analysis for the selection of an appropriate alternative, various near-optimum design alternatives were generated by applying the MGA method (Chang and Brill 1982; Chang and Liaw 1985; Gupta et al. 2005). The purpose of the MGA is to identify maximally different solutions which can still be regarded as good alternatives when compared to the mathematically optimal solution. The following procedure was applied in this study to generate MGA alternatives from chromosomes obtained by using the GA-QP model:

1. During the GA-QP search, all feasible solutions or chromosomes, with a cost less than 120% of the cost for the DDDP solution, were saved;
2. Define a cost relaxation for good alternatives. In this study, a 10% relaxation to the least cost solution is set, i.e., any solution saved in Step 1 with a cost difference of less than 10% to the least cost solution is regarded as a good alternative;
3. Form an MGA alternative set with the least cost solution as the first alternative;
4. Compute the MGA difference (MD) of each good alternative to each alternative in the MGA alternative set by using the following equation:

$$MD_n = \sum_m \left( w_D \cdot \sum_i |D_{n,i} - D_{m,i}| + w_B \cdot \sum_k |B_{n,k} - B_{m,k}| \right) + w_p \cdot (\text{Min}\{PD_{n,1}, \dots, PD_{n,m}\}) \quad (24)$$

where  $MD_n$  = MGA difference of alternative  $n$ ;  $M$  = number of alternatives in the MGA alternative set; and  $m$  = one alter-



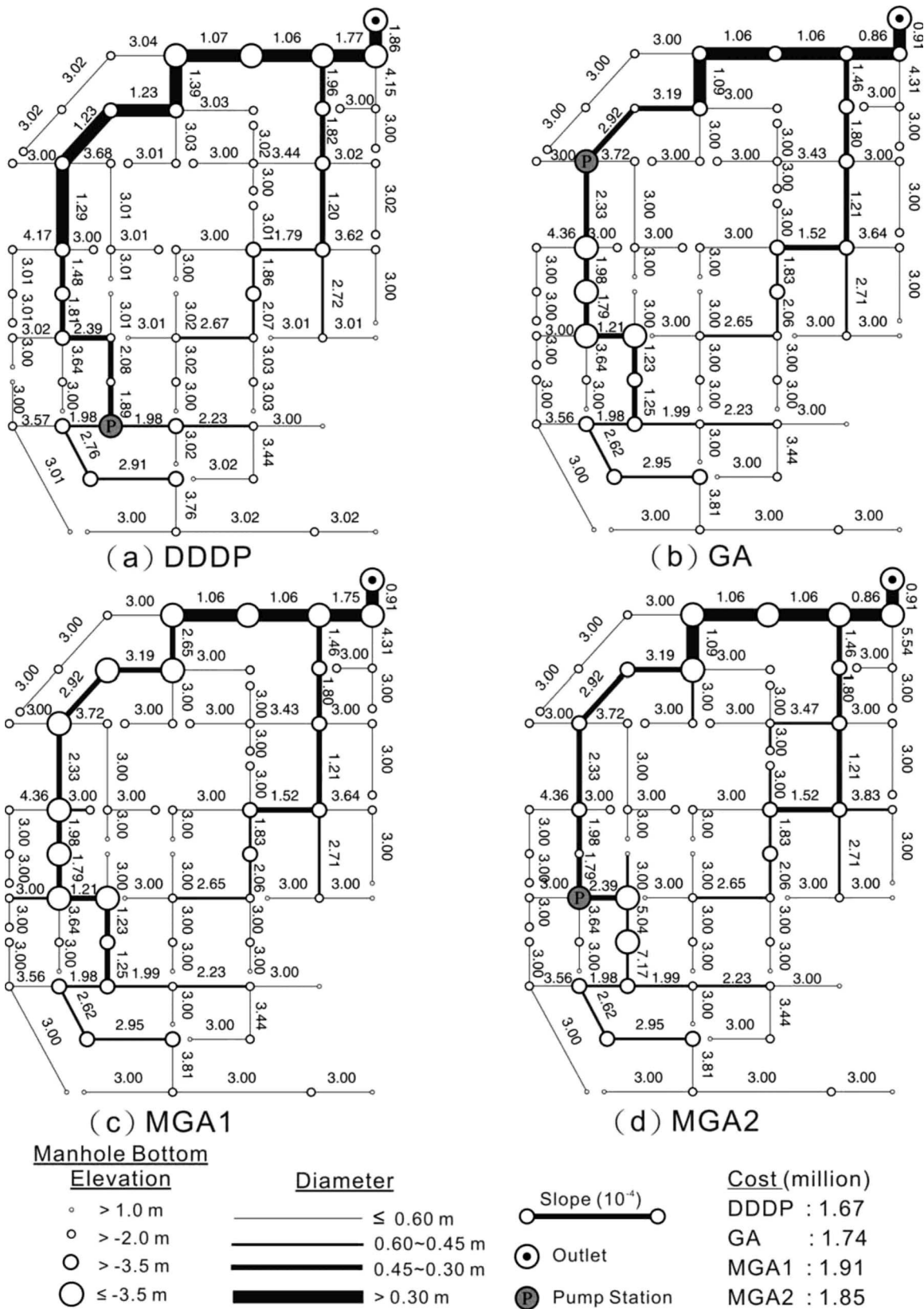
**Fig. 2.** Best design cost among alternatives generated in each GA-QP generation

native in the MGA alternative set;  $w_D$ ,  $w_B$ , and  $w_p$  = weights of diameter, buried depth, and pumping station, respectively;  $D_{n,i}$  and  $D_{m,i}$  = diameters of pipe  $i$  of good alternative  $n$  and alternative  $m$  in the MGA alternative set, respectively;  $B_{n,k}$  and  $B_{m,k}$  = buried depths of manhole  $k$  of alternatives  $n$  and  $m$ , respectively;  $PD_{n,m}$  = distance between the pumping station in alternative  $n$  and the pumping station in alternative  $m$ , and if one of the two compared alternative has no pumping station, the distance is set to be that from the pumping station in the other alternative to the outfall of the sewer system studied. The MGA difference computed by using the proposed MGA function is the sum of the differences of diameter, manhole buried depth, and location of pumping station between a good alternative,  $n$ , and all alternatives in the MGA alternative set;

5. Incorporate the best alternative with the largest  $MD_n$  and into the MGA alternative set; and
6. Repeat Steps 4 and 5 until a desired number of alternatives are generated or until the MGA difference is insignificant.

### Case Study

The applicability of the proposed GA-QP model is demonstrated by addressing the problem cited in Li and Matthew (1990). The problem is to design a sewer system for a residential area with a total drainage area of 260 ha, 56 nodes, and 79 links, as illustrated in Fig. 1. Designing such a system requires several major design parameters, including Manning coefficient, maximum and minimum velocities, maximum water depth, and minimum buried depth. The values of these parameters for this problem are listed in Table 1. The parameters of the cost function,  $\alpha_{n-1i}$ ,  $\alpha_{n-2i}$ ,  $\alpha_{n-3i}$ ,  $\beta_{n-1k}$ ,  $\beta_{n-2k}$ ,  $\beta_{n-3k}$ ,  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_3$  can be found from Li and Matthews (1990). There are 24 available commercial pipe diameters (in meters): 0.2, 0.25, 0.30, 0.35, 0.38, 0.40, 0.45, 0.50, 0.53, 0.60, 0.70, 0.80, 0.90, 1.00, 1.05, 1.20, 1.35, 1.40, 1.50, 1.60, 1.80, 2.00, 2.20, and 2.40. These diameters are represented by binary codes in the GA model. The diameter, buried depth, and pumping station weights of Eq. (24) are 200, 2, and 1, respectively. For designing such a sewer system, several other essential issues such as the system topology (or layout) and the estimation



**Fig. 3.** MGA versus DDDP/GS-QP results: (a) DDDP; (b) GA-QP; (c) MGA1; and (d) MGA2

of nodal demands should be considered as well; however, they are beyond the scope of this study. In this study, the layout and the demands used by Li and Matthew (1990) were adopted for comparing our results with their DDDP solution.

## Result and Discussion

The Perl GA module developed by Kamphausen (2003) is used to solve the GA portion of the proposed model and CPLEX (ILOG 2002) is applied to solve the QP model. The parameters of population size and mutation probability are set to be 300 and 0.01, respectively. Fig. 2 shows the least cost for all designs examined in each GA generation, and the total generation is 1,903. In the first ten generations, the cost reduces from about \$87 million to \$10 million. The reproduction and crossover processes of the GA model rapidly reduce costs. With respect to the subsequent generations, the crossover and mutation processes are implemented in order to search for a better solution, and the cost finally declines to \$17 million, which is close to the cost (\$1.67 million) obtained using the DDDP model (Li and Matthew 1990). The computational time for solving the GA-QP model is about 299 min CPU time on a PC with an Intel Pentium 4 2.0 GHz CPU. This is an acceptable amount of time for solving such a complex problem on a low-cost personal computer. The proposed method was implemented again by setting the DDDP solution to be the initial chromosome, and a slightly improved solution with a cost equal to \$1.668 million was generated. Although it may be possible to increase the population size or the number of applications for obtaining a better solution from a typical initial solution, it will require significant additional computational time.

Fig. 3 shows the DDDP, GA, and two typical MGA design alternatives. The DDDP alternative (Li and Matthew 1990) [Fig. 3(a)] has one pump station in the upstream portion with deep buried depth in the downstream portion. The GA alternative [Fig. 3(b)] is a design with deeply buried manholes in the middle and shallowly buried ones in the downstream portion. The MGA1 alternative, [Fig. 3(c)], has no pump station, and the diameters for the pipes in the middle and downstream portions are smaller than for those in the other alternatives. The MGA2 alternative [Fig. 3(d)] has one pump station in the upstream portion, a shallow-average buried depth, and a small diameter pipe in the middle portion.

Some excluded factors or issues may be essential and should be evaluated. For example, due to geological characteristics or anticipated construction difficulties, some locations may not provide suitable buried depths or accommodate large diameter pipes. Furthermore, a pumping station usually requires more space than does a manhole, and a design alternative may not be able to place a pumping station at a suitable location. In such situations, the mathematically optimal solution may become less attractive or even infeasible; here, the MGA alternatives can be helpful in exploring a good design alternative. For example, the MGA1 alternative, although its cost is slightly higher than that of others, has no pumping station and this fact may make it an attractive and energy-saving alternative; and when the locations suggested for installing a pumping station indicated by the DDDP or GA alternatives are not appropriate, the location suggested in the MGA2 alternative may be a good substitute.

## Conclusions

The design of an efficient sewer collection system is important in order to save construction cost. However, finding the optimal de-

sign solution from among numerous alternatives is a difficult task. Several approaches such as piecewise linearization or dynamic programming, although available for solving this design optimization problem, may either eliminate good alternatives or require long computational time. Therefore, an efficient model based on the genetic algorithm and quadratic programming is proposed. The GA model uses coded parameters and searches from multiple points to enhance the probability of finding the global optimum. In this study, discrete variables in pipe diameters and pumping station locations are selected as decision variables to enhance model solvability. Two simple constraints are applied to the GA model in order to eliminate unacceptable chromosomes in a process designed to increase solving efficiency. To increase solving efficiency for the model while preserving the nonlinear characteristic of the original cost functions, a QP model is proposed and integrated into the GA model. The proposed GA-QP model has been demonstrated in a case study and various good design alternatives were obtained within an acceptable computational time. Furthermore, several MGA alternatives were generated, illustrated, and compared to the GA-QP and DPPP alternatives. While an excluded factor or issue may be essential, the GA-QP and DPPP alternatives may be inappropriate or infeasible. For this situation, good MGA alternatives which are significantly different can be evaluated to find a good substitute.

## Acknowledgments

The writers would like to thank the National Science Council, R.O.C., for providing partial financial support of this research under Grant No. NSC94-2211-E-009-010.

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