國立交通大學

資訊科學與工程研究所

碩 士 論 文

藉 由 分 享 而 得 的 影 像 還 原 技 術

Image Recovery by using Sharing

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摘 要

數位影像在 WWW 的環境中可能會被攻擊者竄改資料，或者在傳輸之中發生傳輸錯誤的情形。在本篇論文中，我們提出三種基於「資訊分享」技術的影像還原方法。在第一個方法之中，我們對每個 8x8 影像區塊進行編碼；其後，只要未被破壞的影像區塊數目大於一門檻值，我們就可以用它們來把其他已被破壞的影像區塊給還原回來。由於第一種方法若整張影像都被拿掉，則無還原機會，所以我們又提出第二種方法「多張影像交叉還原」技術。對於 n 張編碼後的影像，只要有 t 張影像是未被破壞的，我們就可以利用這 t 張影像來還原其他$(n-t)$張已被破壞的影像。第三種方法是第二種方法的變形；由於管理者可能希望影像具備自我修復的能力，但又不希望降低影像品質，所以我們的第三種方法以增加儲存空間的方式，對於每張輸入的影像會附加一塊類似雜訊的分存影像，而其功能亦可達到和第二種方法相同的效果。

除了上述三種方法外，我們也觀察到視覺密碼學(Visual Cryptography)和利用多項式的影像分享(Image Sharing by Polynomials)各有其優劣點；所以本論文最後提出一個結合此二者的新的分享機制；其分存影像可以依據環境狀況來採用其中一種回復方法。

Image Recovery by using Sharing

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Abstract

In WWW environment, digital images may be tampered by attackers, or get lost during transmitting. In this thesis, we propose three kinds of image recovery methods, based on sharing techniques. In the first method, the given image is partitioned into 8-by-8 blocks, and we encode each 8-by-8 image block. Later, if the number of the non-attacked blocks is greater than a threshold value, then we can recover all the tampered blocks by using them. Because the first method cannot recover the image if it is deleted completely, we propose the second method which is the cross recovery of multiple images. For *n* encoded images, if *t* out of the n images are not-attacked, we can recover the other (*n*-*t*) images by the *t* non-attacked images. The second method is not error-free when each image is embedded with recovery information. The third method is a modification of the second method; and it is for the users who require the images to have cross recovery ability without degrading the images' qualities. This method appends a noisy shadow image to each input image.

Besides the three methods described above, it is also observed that Visual Cryptography and polynomial-style image sharing both have their own advantages and disadvantages. So, this thesis also proposes a novel sharing mechanism combining the two methods. The recovery can be according to the computer resources available at the scene.

ACKNOWLEDGEMENTS

I would sincerely thank my advisor, Prof. Ja-Chen Lin of his teaching and instruction. He guided me patiently and kindly during my study of the Master’s degree. Without him, it is impossible for me to complete this thesis.

Besides, the whole group of the Computer Vision Laboratory at National Chiao-Tung University gave me much help and suggestion. Thanks are also given to them.

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Introduction

1.1 Motivation

Due to the popularity of Internet, people can exchange information conveniently. Many images are placed on Web pages, and authorized people can download them. Unfortunately, it’s hard to ensure that received data is identical to the original data; because Internet is an open environment, anyone can access the data, including attackers. Therefore, how to protect Internet data is a critical issue.

In the thesis, we proposed some recovery methods to protect images. Those methods are based on the polynomial sharing technology. This technology contains two phases: the sharing phase and the revealing phase. In sharing phase, the given data are divided into n shadows; later, if anyone get any t of the n shadows (t is a given threshold value), then he can reconstruct the data; but if he gets t-1 or fewer shadows, he can’t get any information about the data. This is a very useful tool for image processing; in recent years, many literatures about this topic have been published [5-22].

With Fig. 1.1, we can realize the procedure of how to recover a tampered image. We roughly separate the process into three parts; the pre-processing step, the tampering step, and the recovery step. For these steps, the pre-processing step and the recovery step are done by us. How ever, the tampering is done by the attacker. Thus, how to design a good algorithm in the pre-processing step and the recovery step is our concern.

In Chapter 2, we present a method that can recover a single embedded image based on sharing technology. Since the ability of single image recovery technology is very limited, we also present in Chapter 3 a novel image recovery method for multiple images, and this method can recover the tampered images, even some of the images are destroyed completely.

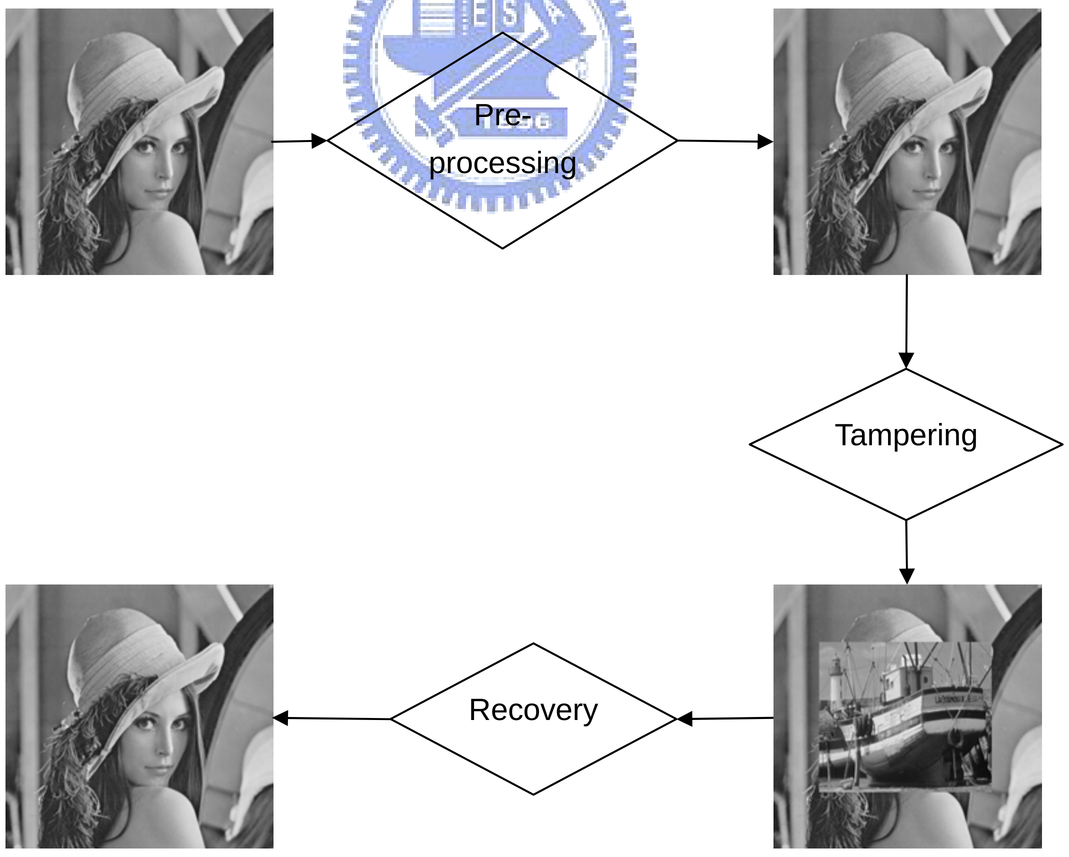


Fig. 1.1. The course of pre-processing, tampering and recovering an image.

Visual Cryptography was introduced by Naor and Shamir. In (t, n) sharing, the secret data are dispersed into n transparencies, and we can't get any message about the secret data from each transparency (except for the transparency size). If we stack any t out of n transparencies, then we can "see" the secret message; however, if we stack t-1 or fewer transparencies, then we can't get any message about the secret data. Some comparisons between polynomial-style secret sharing and Visual Cryptography are described in Chapter 4. Each of the two technologies has its own advantage, thus, we try to combine VC with polynomial-style secret sharing in Chapter 4. Certain transparencies are generated by our method there. If the transparencies can be transmitted via network, the revealing mechanism can get a high quality image (by polynomial-style sharing) with the aid of a computer. But if there is a no computer, then, we still can get "view" a rough look of the secret image by stacking the transparencies.

1.2 Related Works

In this section, some previous works about Secret Sharing and Image Hiding technology will be reviewed. In Section 1.2.1, Shamir's secret sharing method [1], Thien and Lin's image sharing algorithm [7] and Galois field will be described briefly. In Section 1.2.2, we will introduce a simple method to hide a base-k digit stream in a host image.

Because base transform is commonly used repeatedly in this thesis, it is also reviewed in Section 1.2.3.

1.2.1 Review of Secret Sharing

Sharing technology is one of the commonly used methods to protect secret data. Shamir's secret sharing [1] is a threshold method based on the polynomial interpolation. The (t, n)-threshold secret sharing scheme can disperse the secret data into n shadows, and each shadow has the properties below:

1. Any t (or more) out of n shadows can be used to reconstruct the secret data.
2. Any t-1 (or fewer) shadows cannot be get any information about the secret data.

The secret data could be text files, images, multimedia data, or encryption/decryption keys. The polynomial used in Shamir's method is described below:

Shamir's (*t*, *n*)-threshold secret sharing:

For (t, n)-threshold sharing, let

$$Q(x) = a_0 + a_1x + a_2x^2 + ... + a_{t-1}x^{t-1}$$

where a_0 is the secret data, $a_1, a_2, ..., a_{t-1}$ are random numbers, and the *n* shadows are $\{1, Q(1)\}, \{2, Q(2)\}, ..., \{n, Q(n)\}$. Any *t*-1 or fewer shadows can't get any useful information about the secret data. However, when people receive any *t* of those shadows, they can evaluate the secret data a_0 by using Lagrange interpolation.

In order to save space, Thien and Lin [7] modify the above idea and apply it to the sharing of secret images. For (*t*, *n*)-threshold scheme of secret image sharing, an image *f* of size *L* is breaks into *n* shadows, and each shadow has following properties:

1. Any *t* (or more) out of *n* shadows can be used to reconstruct the secret image.
2. The size of each shadow is *L*/*t*.

The algorithm is very similar to Shamir's secret sharing scheme, and we introduce the method below.

(*t*, *n*)-threshold Secret Image Sharing:

Steps:

1. For whole secret image, if a pixel value ***g*** is larger than 250, then change the value into 250, and creates one more pixel whose gray value is ***g***-250.
2. Encrypt the secret image by a encryption key.

3. Divide the secret image into several sections, each section has *t* pixels.

4. For each section, we define the polynomial

   $P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_{t-1} x^{t-1} (\bmod 251)$

   where $a_0, a_1, \ldots, a_{t-1}$ are *t* pixels of the section.

5. Evaluate $Share_1 = P(1)$, $Share_2 = P(2)$, ..., $Share_n = P(n)$. Each $Share_i$ is assigned to each shadow image *i*.

When we receive any *t* of those shadow images, we can evaluate the secret pixels by using Lagrange interpolation.

If some pixel values of the secret image are larger than 250, the coding, decoding, and share size will all be influenced a little. A better method is that, we change the finite field of the sharing polynomial, as below:

Let Galois field be a finite field, denoted by $GF(p^n)$, where p is a prime number, and n is a positive integer. In this thesis, unless stated otherwise, the polynomial sharing technology works on $GF(2^8) = GF(256)$, rather than GF(251).

1.2.2 Review of Image Hiding

Image hiding is a method to embed some secret data in a cover image. For the same hiding capacity, the goal is usually to reduce the difference between the cover image and the stego image for human's sense. The simplest hiding method is the least significant bit (LSB) method; the data are embedded in the least k bits of the pixels. In order to improve the image quality of the cover image, Thien and Lin present a method to embed the secret data by using modulo operation [28]. Thien and Lin's algorithm is more complex than the LSB method; in this thesis, we use a simpler method [30] whose performance is identical to Thien & Lin's, as below:

Algorithm of module-based data hiding

Let p be a pixel value, $d \in \{0,1,\ldots,b\text{-}1\}$ a secret digit to be hidden, and b the module-base. To embed d in p, we replace p by $p' = d + b \times rounding[\frac{p-d}{b}]$. If $p' < 0$, then $p' \leftarrow p' + b$. Similarly, if $p' > 255$, then $p' \leftarrow p' - b$.

Here, the *rounding* operator means rounding to the "nearest" integer. Also, we assume the pixel value is at least 0 and at most 255. Later, if we want to extract the value d from p', just use $d \leftarrow p' (\bmod b)$.

1.2.3 Base Transform

In this thesis, changing the base of a digital stream is a tool used often. We propose a simple method to do this. An example of our method is shown in Fig. 1.2. In this example, we change the base of a digit stream from 5 to 7. The method can be divided into two stages. In the first stage, we generate a look up table, as shown in Fig. 1.2(a). This table can map each digit to a binary code. Thus, a binary stream could be yielded. In the second stage, we generate a 7-base look up table, as shown in Fig. 1.2(b). By using the table, we can transform the binary stream into a base-7 digit stream. Later, if we want to convert the digital stream into the original stream, we only reverse this procedure.

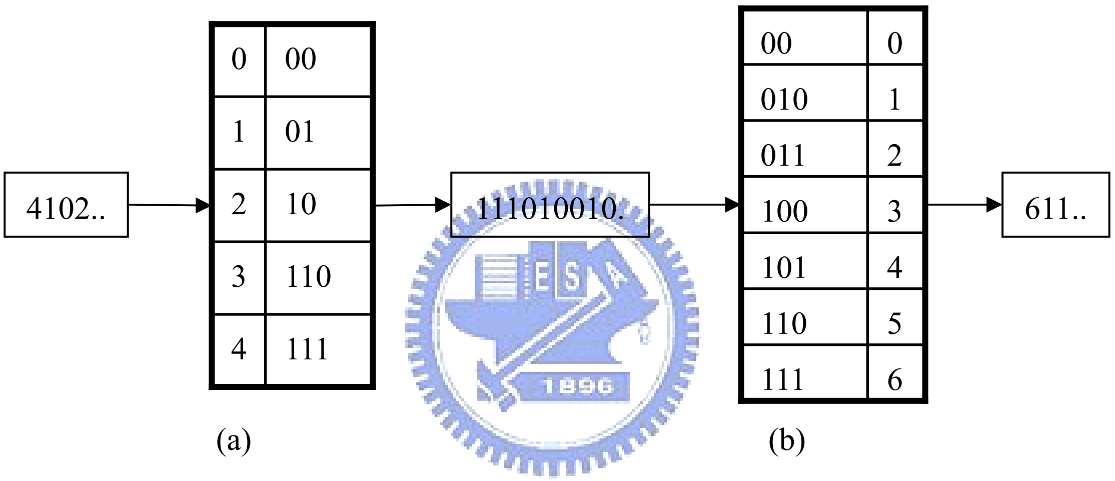


Fig. 1.2. An example to change the base of a digital stream from 5 to 7. (a) base-5 look up table. (b) base-7 look up table.

No matter in stages 1 or 2, we need a look up table to map the base from 5(or 7) to 2.

Algorithm of generating a base-*k* look up table

Input: The base *k*.

Output: A look up table.

Steps:

1. Evaluate integers *c* and *b* so that $c = \lceil \log_2 k \rceil$ and $b = 2^c - k$.

2. For each digit from 0 through *b*-1, the corresponding binary code is its (*c*-1)-bit binary expression 0(for example, $0 \to \underbrace{00...00}_{c-1 \quad bits}$, $1 \to \underbrace{00...01}_{c-1 \quad bits}$, $2 \to \underbrace{00...10}_{c-1 \quad bits}$, …).

3. Let ***B*** denote the (*c*-1)-bit binary expression of *b*, then append a zero to the right of ***B***, which is the corresponding binary code of b.

4. Each digit from *b*+1 to *k*-1 is obtained by adding value to ***B*** accordingly.

1.3 Overview of the Proposed Methods

In this section, we will describe the main methods that will be proposed in this thesis briefly. In this thesis, Single-Image Recovery is our first application of the polynomial sharing technology. In the embedding procedure, we use the polynomial sharing technology to share the compressed version (JPEG) of the host image, and then hide each shadow and embed the checksum (CRC64) in each block of the host image. If the image had been tampered, our verification procedure can detect which blocks are altered. Finally, our recover procedure can reconstruct the altered blocks by extracting the compressed version of the original image.

However, if the attackers destroy the image completely or replace it with other image, the Single Image Recovery technology cannot handle this case. Hence, we propose a Multiple Images Recovery method to resolve the problem. If (n-t) or fewer images of the n images have been deleted, we still can reconstruct those deleted images by the remaining images.

Multiple Image Recovery method needs to alter the input images. However, if some people don't want the images be changed; then they can use a different method of ours to achieve the requirement. The mechanism doesn't alter any pixel value of the input image, but each image is appended with a shadow file, whose size depends on the threshold value and the size of the input images.

Visual Cryptography (VC) is also a secret sharing technology; the most special characteristic of VC is that its is the decoding phase doesn't need any computer, all we have to do is just to stack those transparencies. Our two-layer Image Sharing method is a method which combines VC with polynomial sharing technology. At the revealing phase, if the decoding-computer is temporarily not available in the decoding sense, we can stack those transparencies to "see" the secret image of rough version. In the other case, when the computer is available, we can get a much finer gray-valued secret image using the information hidden earlier in the shadows by using polynomial sharing algorithm.

1.4 Thesis Organization

Fig. 1.3 illustrates the framework of this thesis. polynomial secret sharing is the main tool of our whole thesis. In Chapter 2, we will present the scheme of image recovery, which is based on the polynomial sharing approach. The method of cross recovery of multiple images is specified in Chapter 3. Chapter 4 proposes a novel secret sharing method, which can decode the secret image either by a computer or by stacking the transparencies.

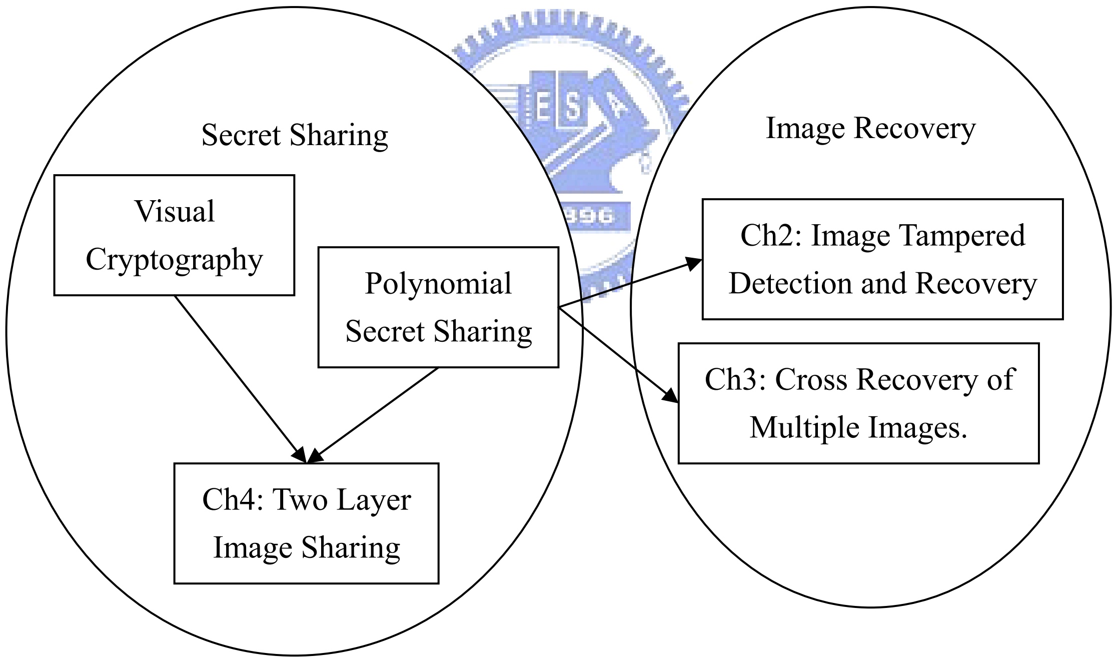

Secret Sharing
Visual Cryptography
Polynomial Secret Sharing
Ch4: Two Layer Image Sharing
Image Recovery
Ch2: Image Tampered Detection and Recovery
Ch3: Cross Recovery of Multiple Images.

Fig. 1.3. The framework of this thesis.

Chapter 2
Detection and Recovery of Tampered Images

This chapter presents an image recovery technology. We use polynomial sharing to share the critical information of the original image into many shadows, and the size of each shadow is very small. Then use data hiding technology to embed those shadows in each block of the original image. Finally, the checksum is also embedded in the block. If the image is tampered, the checksum of each block can determine whether the blocks are altered. If the block is integrity, then the embedded information can be extracted and be used to reconstruct the original image; otherwise, we mark the block as an error block and will be recover by the other integrity blocks.

2.1 Introduction

With the popularity of Internet, we can get multimedia data conveniently. However, attackers can modify those multimedia data easily, via Internet. Therefore, how to protect the integrity of multimedia data is an important issue. Image recovery technology [23-27] is one of the methods to protect important web images. In this chapter, an image recovery method is proposed.

2.2 The Proposed Method

2.2.1 The Main Algorithm

In this section, the host image is a 512-by-512, gray-level image, the range of the gray value is 0~255. Fig. 2.1 shows the embedding procedure of our method.

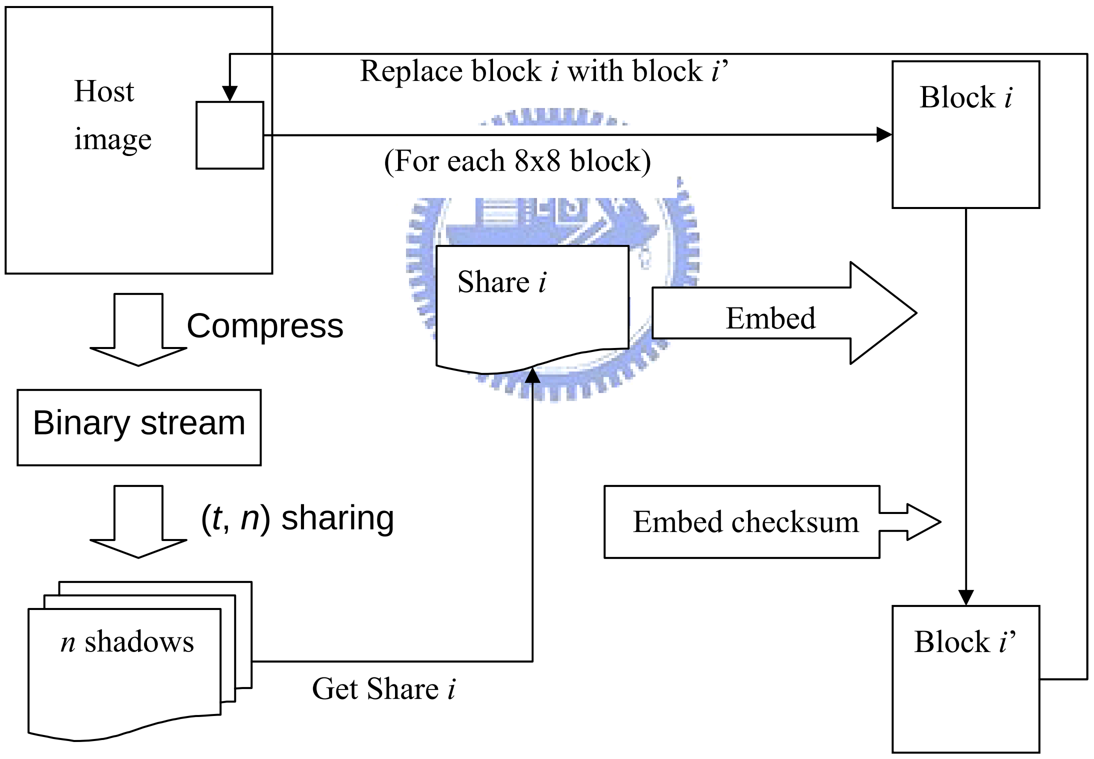


Fig. 2.1. The flow chart of the embedding procedure

The process of our embedding procedure consists of four stages. The first stage is called "image compression stage"; in this stage, we compress the host image by using JPEG method (actually, we can use any image compression method). The output is a bit stream, and the resulting size is smaller than the original image file. The second stage is called the "sharing stage". In this stage, we use polynomial sharing technology to share the binary stream, which was got from the first stage. The final stage is called the "embedding stage". For each 8-by-8 block, we embed a share and the checksum in it. The details of our method is described in the following algorithm.

Embedding Algorithm:

Input: A grey-value host image ***S*** of size *Width* ×*Height*, a decimal value *d*, $0<d<1$, which indicates the minimal percentage of the integrity blocks needed in order to recover the image if the stego image is tampered later.

Output: a stego image.

Steps:

1. Produce a grey-value image ***J*** by compressing the host image ***S*** using JPEG compression technology. Hereinafter, the data file representing the compressed image ***J*** will be treated as a bit-stream data-file.
2. Then, the bit stream of ***J*** is shared using an (t, n) sharing algorithm. Where n is equal to the number of 8-by-8 blocks in the host image; that is to say, $n = Width/8 \times Height/8$. The threshold value t means the percentage of integrity shares could be recovered, so $t = d\times n$. Here the value of n may be

very large; for example, if the image size is 512-by-512, then n=(512/8)×(512/8)=4096, this is a large number. In order to get those shadows, the sharing algorithm must works on $\mathrm{GF}(2^{12}) = \mathrm{GF}(4096)$ or modulo 4099(this a minimal prime number which larger than 4096).

3. Partition the host image ***S*** into non-overlapping 8-by-8 blocks, then set i=0 initially. For each 8-by-8 block i, doing following steps.
4. Embed the share i was produced at step 2 and a checksum in the block i. The embedding algorithm is described in Embedding-Sub Algorithm. Let the output is a block i'.
5. Replace the block i by the block i', then increment the value of i by 1. Finally, go to Step 4 unless all of the blocks are replaced.
6. The image ***S*** is our desired output.

<u>Embedding-Sub Algorithm</u>:

Purpose: To embed a share data and checksum in an 8-by-8 block.

Input: An 8-by-8 block ***B***, and a share data ***D***.

Output: An 8-by-8 block ***B'*.**

Steps:

1. Discard all of the least significant bits in the block ***B***. Let ***T*** the output block.

We use a pseudo-code to represent it,

```
For all of pixels in T,
    $T[x, y] \leftarrow B[x, y]/2$;
```

Where ***B***[x, y] (resp. ***T***[x, y]) denotes the pixel value at position (x, y) of the block ***B*** (resp. block ***T***).

2. Get the block ***T'*** by using **module-based data hiding** method (see section 1.2) to hide the share data ***D*** in block ***T***.
3. Get the checksum ***C*** by evaluating the CRC64 of the block ***T'*** (each pixel of the block ***T'*** is consist of 7 bits).
4. Append the checksum ***C*** to the block ***T'***. Our desired block ***B'*** consists of block ***T'*** and checksum ***C***. We can use the following pseudo-code to get block ***B'***.

```
For all of pixels in B',
    $B'[x, y] \leftarrow T[x, y]+C[x \times 8+y]$;
```

Where ***C***[X] denotes the **X**-th bit value of the checksum ***C*** (***C*** contains 64 bits). Fig. 2.2 shows the bit plane of the block ***B'***.

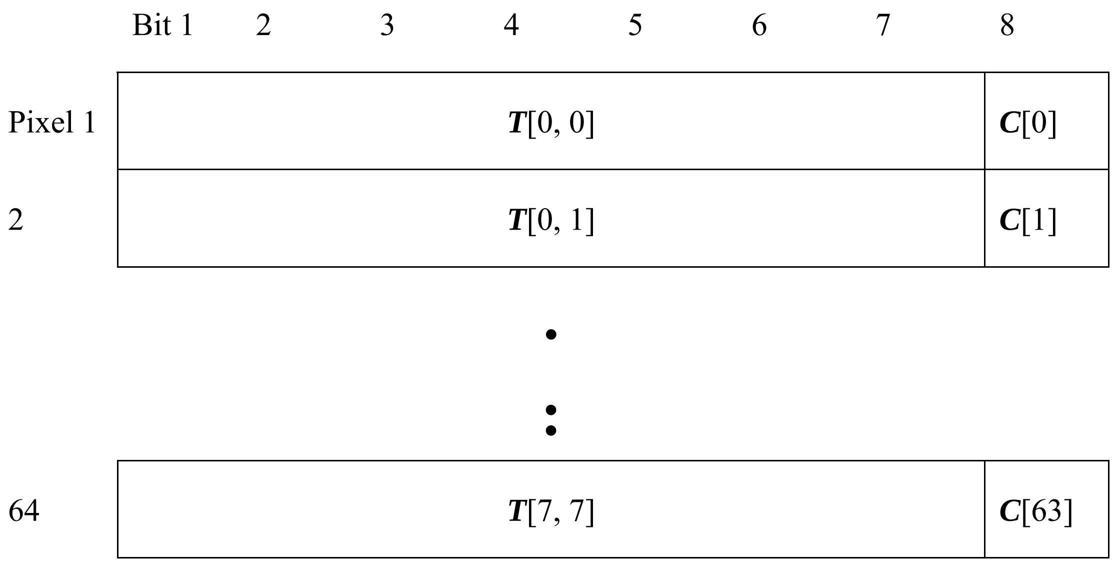


Fig. 2.2. The bit plane of the block ***B'***.

2.2.2 The Verification and Recovery Algorithm

The verification method is based on CRC64. For each 8-by-8 block, we use CRC64 algorithm to check whether the block is integrity or not. If the block is integrity, then we color the corresponding location of the verified image V white; otherwise, color black. The details are described at below.

<u>Verification Algorithm</u>:

Input: A query image ***Q***.

Output: A verified image ***V***. If a verified block is a black block, it means the block had been altered. The color is white, if the block is not altered.

Steps:

1. Divide the query image ***Q*** into 8-by-8, non-overlapping blocks.
2. For each block ***B*** in the image ***Q***, extracting all of LSBs in the block ***B***. Combine those bits, we can get a value called *CRCChecksum*.
3. Evaluating CRC64 of the block ***B*** (eliminate all of LSBs in the block ***B***). If the value is equal to *CRCChecksum*, than we can say this block ***B*** is an integral block. Otherwise, the block ***B*** is an altered block.
4. If the block B is an altered block, we color the block of verified image ***V*** black at the corresponding location. Otherwise, color white.
5. If there are some blocks un-processed, go to step 2.Otherwise, terminate this procedure, then output the verified image ***V***.

Fig.2.3 illustrates the **verification algorithm**. With this algorithm, we can know which blocks are tampered. With the verified image, we can even correct those blocks by using recovery algorithm.

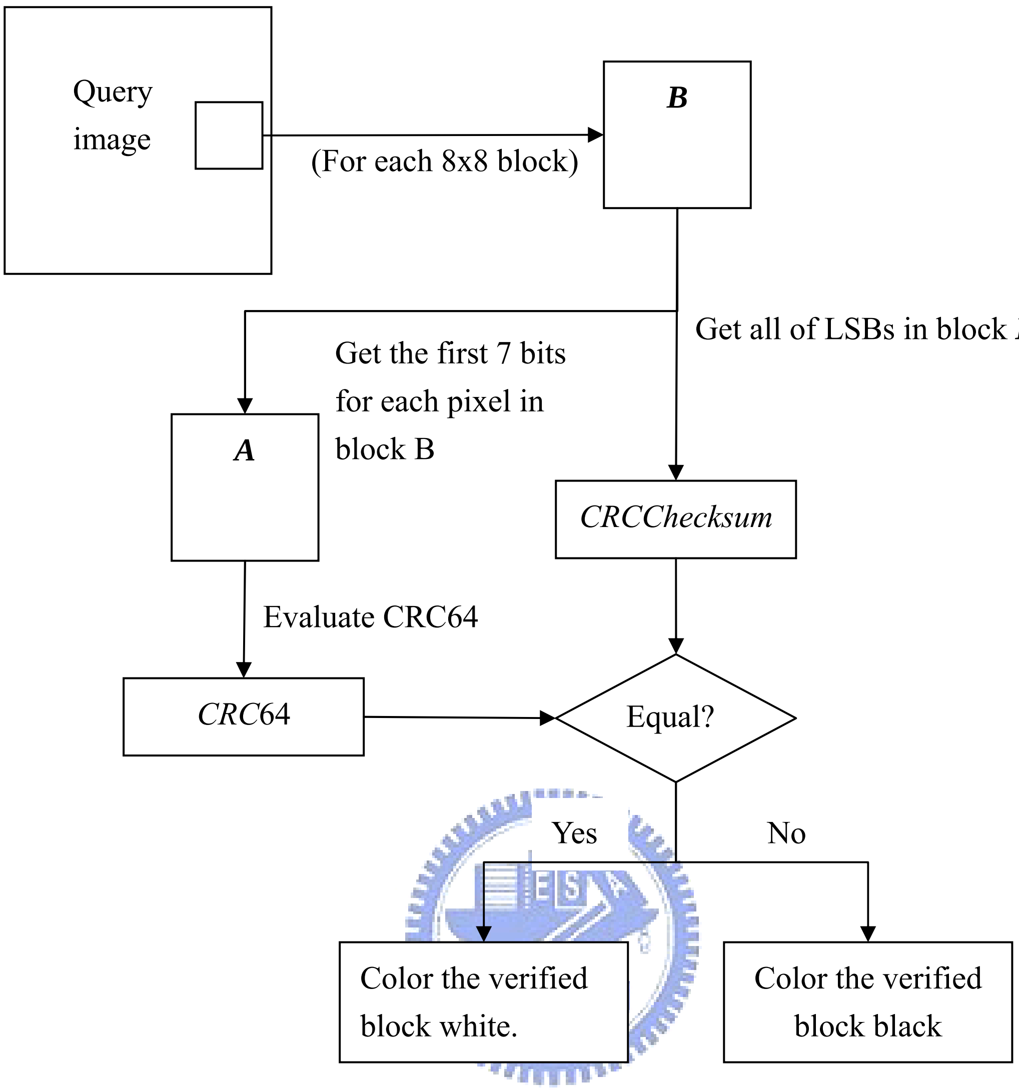


Fig. 2.3 The flow chart of the verification procedure

After the image verification procedure, we know which blocks were tampered. In order to recover the tampered blocks, the JPEG version of the original image will be extracted to restore those tampered blocks. The Recovery Algorithm below describes the details of this method.

Recovery Algorithm:

Input: A query image ***Q*** and the verified image ***V***.

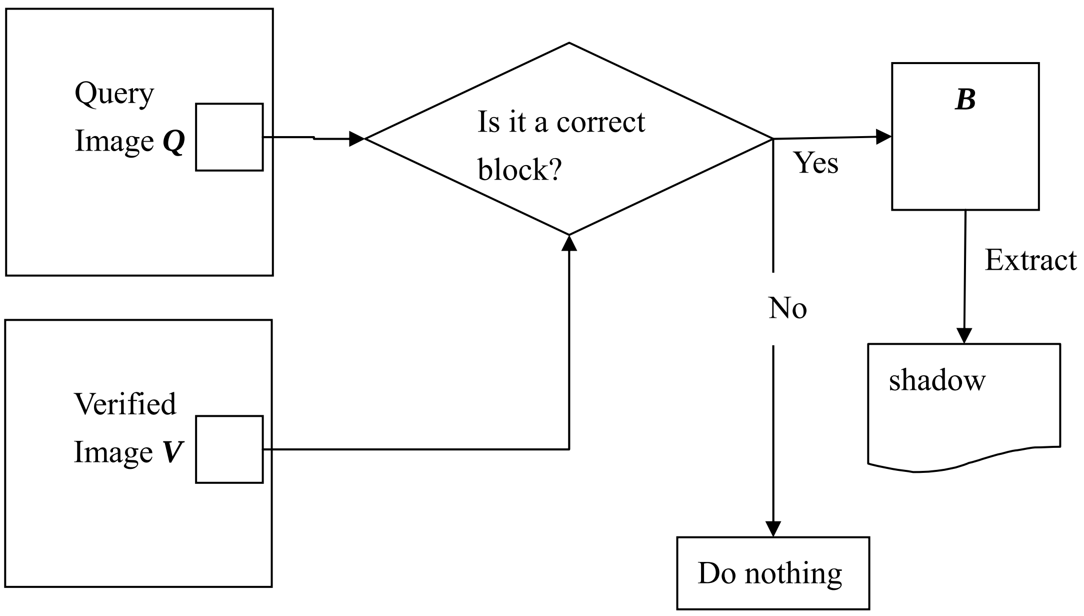
Output: a recovered image ***R***, whose altered blocks had been restored.

Steps:

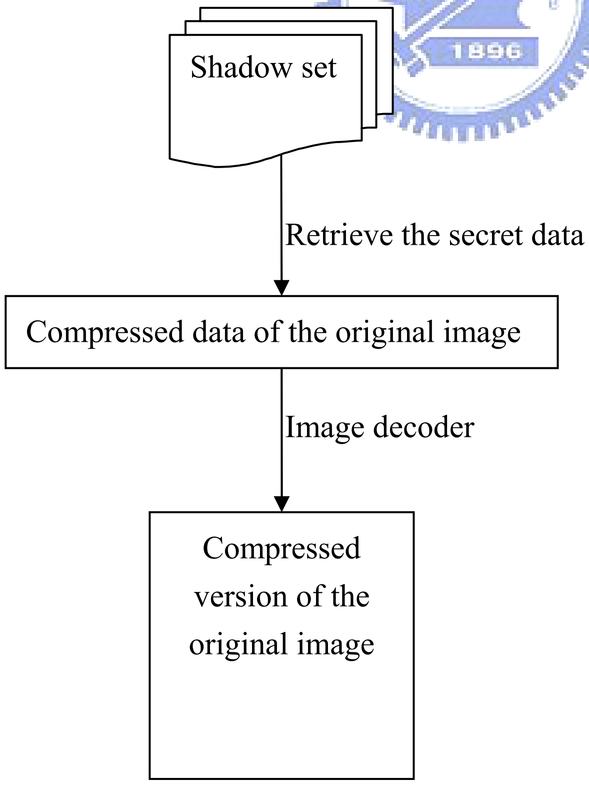
1. Divide the image ***Q*** into 8-by-8, non-overlapping blocks.
2. For each block ***B***, if it is a integral block, extracting the share data which be embedded in the block ***B***.
3. Use revealing algorithm to get the secret data from those extracted share files. (The secret data is a JPEG-format file of the original image). From the secret data, we can know a JPEG version of the original image, denote the image is ***J***.
4. For each 8-by-8 block of the verified image ***V***, if the block color is black (this means the block had been tampered), we use a corresponding block of the image ***J*** to replace the block of the image ***Q***.
5. The image ***Q*** is our desired output.

Fig. 2.4. Illustrates the three passes of the Recovery Algorithm.

Pass 1:


Query
Image ***Q***
Is it a correct block?
Yes
B
Extract
No
shadow
Verified
Image ***V***
Do nothing

Pass 2:


Shadow set
Retrieve the secret data
Compressed data of the original image
Image decoder
Compressed version of the original image

Pass 3:

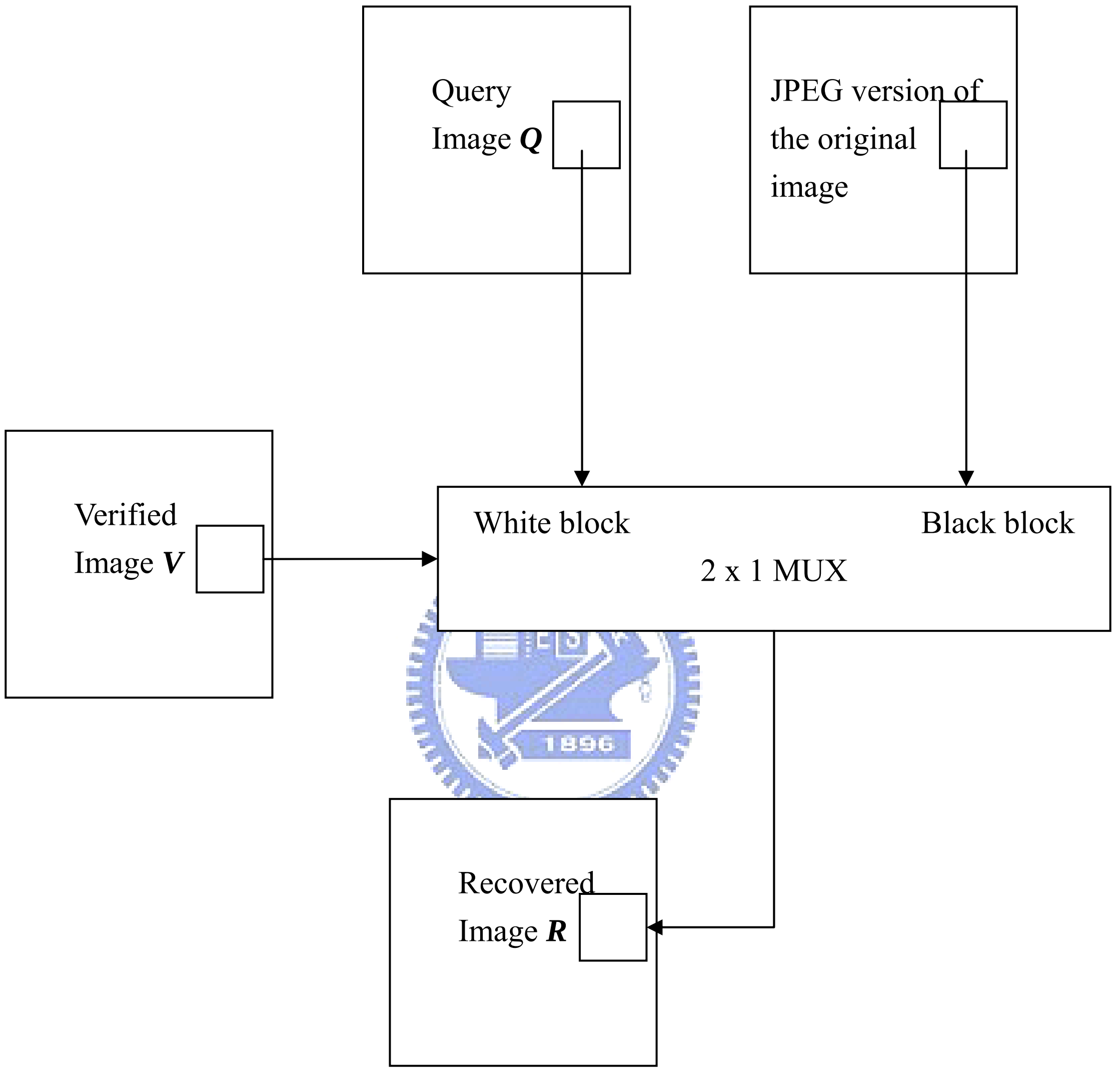


Fig. 2.4. The flow chart of the recovery procedure.

2.3 Experimental Results

In order to evaluate the ability of our method, in this section, we design some experiments, and the results are show below.

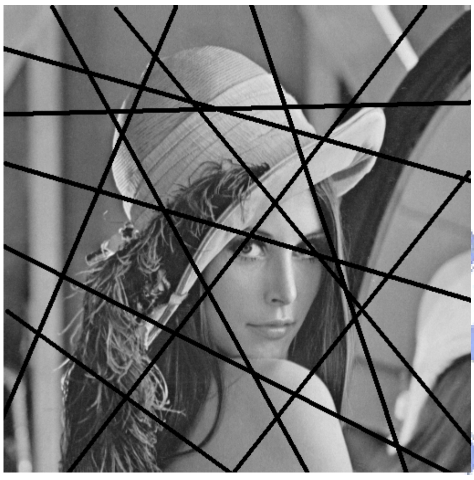
Fig. 2.5. shows our first experiment; Lena is the host image, as shown in Fig. 2.5(a). The decimal value *d* of embedding algorithm is set 0.5. Fig. 2.5(b) shows the embedded image, which the PSNR is 40.61 d.b.. Fig. 2.5(c) shows that some straight lines had been added on the embedded image. Fig. 2.5(d) and Fig. 2.5(e) show the verification result and the recovered result. The PSNR of Fig. 2.5(e) is 38.71 d.b. (This result is compared with the original image, the following two experiments are also the same).

Fig. 2.6. shows our second experiment; Fruit is the host image, as shown in Fig. 2.6(a). The decimal value *d* of embedding algorithm is 0.25. Fig. 2.6(b) shows the embedded image, which the PSNR is 37.14 d.b.. Fig. 2.6(c) shows some handwriting had been added on the embedded image. Fig. 2.6(d) and Fig. 2.6(e) show the verification result and the recovered result. The PSNR of Fig. 2.6(e) is 34.72 d.b..

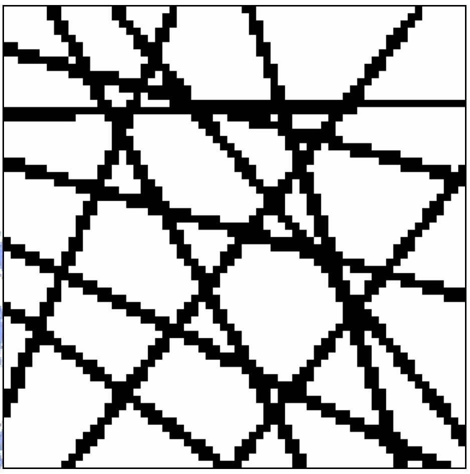
Fig. 2.7. shows our third experiment; Jet is the host image, as shown in Fig. 2.7(a). The decimal value *d* of embedding algorithm is 0.25. Fig. 2.7(b) shows the embedded image, which the PSNR is 37.15 d.b.. Fig. 2.7(c) shows a picture "Car" had been added on the embedded image. Fig. 2.7(d) and Fig. 2.7(e) show the verification result and the recovered result. The PSNR of Fig. 2.7(e) is 34.33 d.b..

(a)

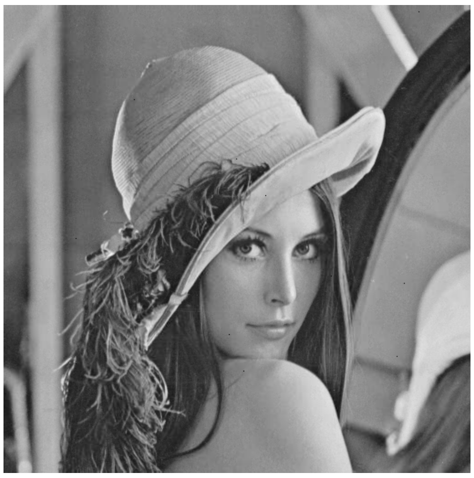
(b)



(c)

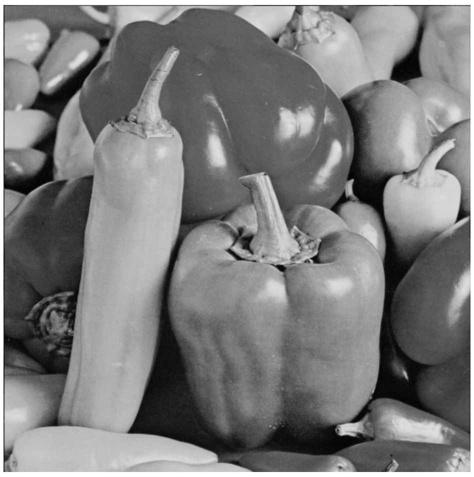


(d)

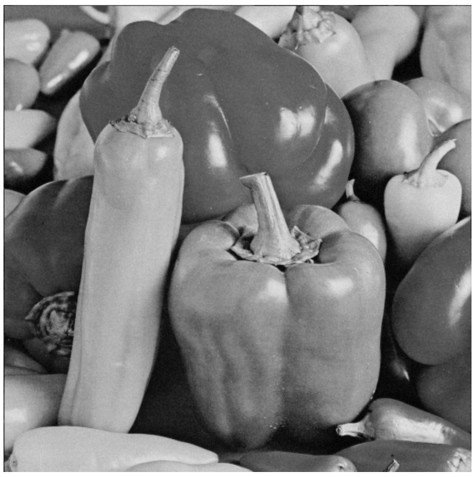


(e)

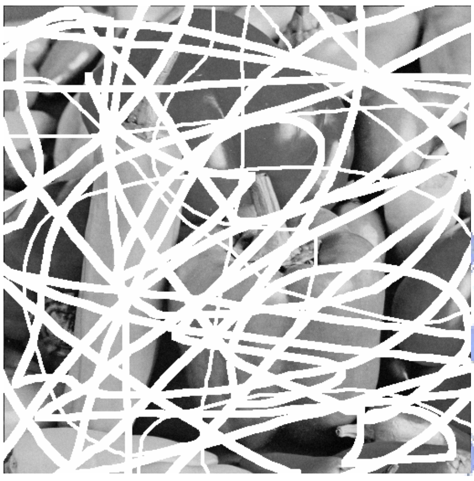
Fig. 2.5. (a) the host image, (b) the stego image (PSNR=40.61 d.b.), (c) a tampered version, (d) the verification image, (e) the recovered image (PSNR=38.71 d.b.)



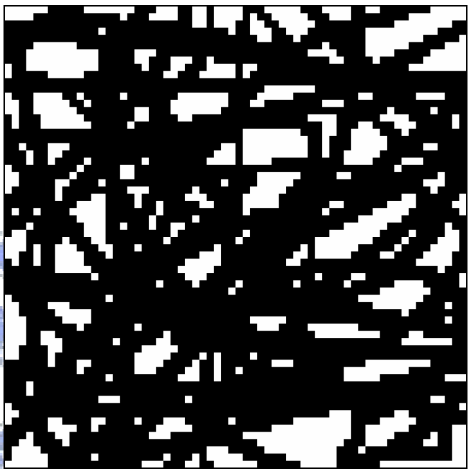
(a)



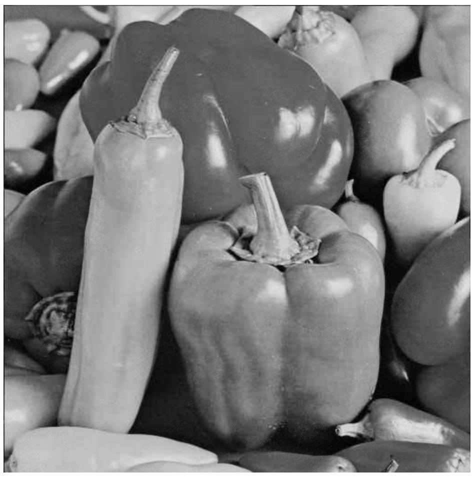
(b)



(c)



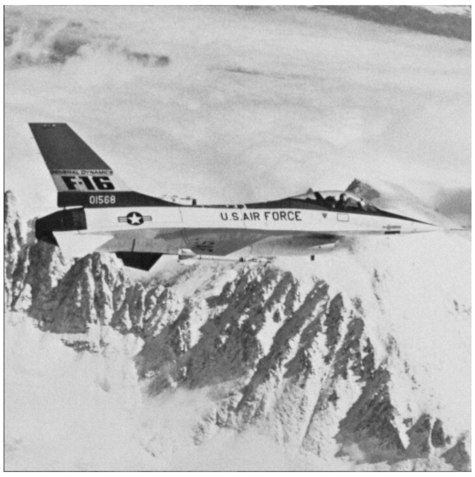
(d)



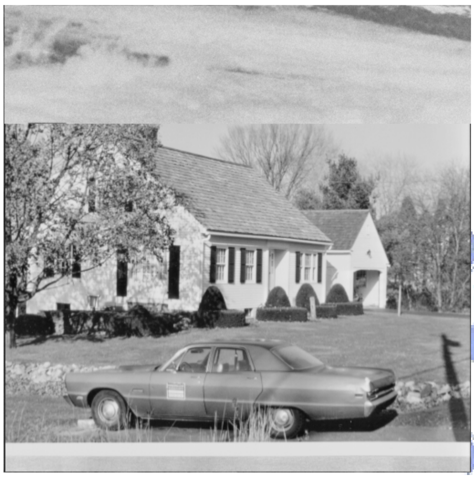
(e)

Fig. 2.6. (a) the host image, (b) the stego image (PSNR=37.14 d.b.), (c) a tampered version, (d) the verification image, (e) the recovered image (PSNR=34.72 d.b.).

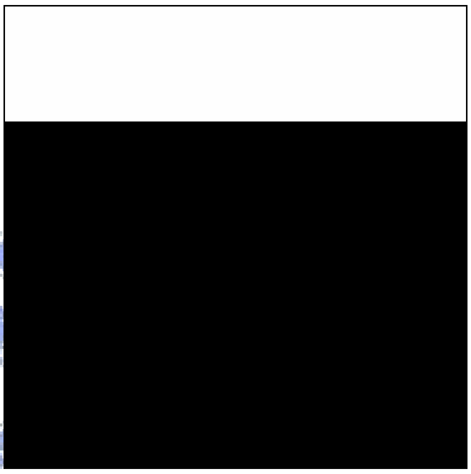
(a)



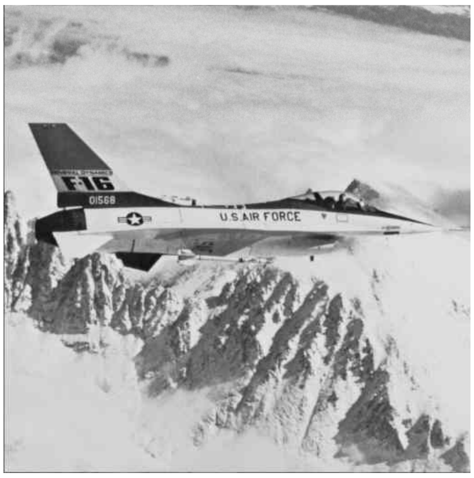
(b)



(c)



(d)



(e)

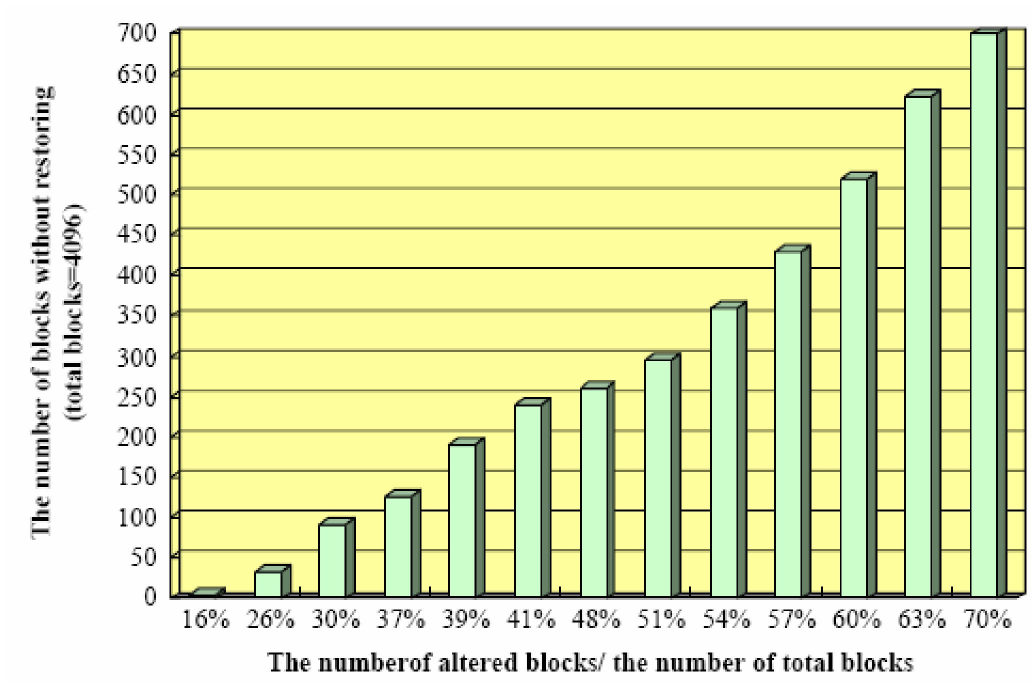
Fig. 2.7. (a) the host image, (b) the stego image (PSNR=37.15 d.b.), (c) a tampered version, (d) the verification image, (e) the recovered image (PSNR=34.33 d.b.).

2.4 Discussion

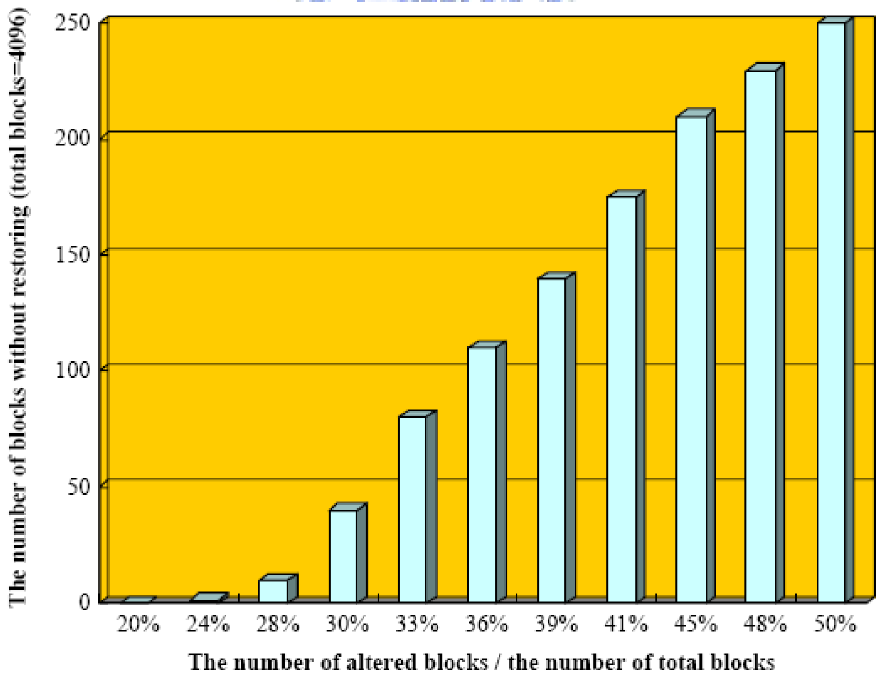
In this chapter, our recovery method is based on polynomial sharing technology. A secret worry is that, if any a mis-judgment occurs in the verification phase, some portions of the image will be lost. Because if any one of the shadows had been altered (or the shadow is a fake), then we cannot get any information at revealing phase! Therefore, the verification phase is very important, we cannot allow any one mis-judgment, and a strict verification algorithm is much needed.

Review the verification algorithm of our method, the probability of the correct judgment for a block is $1-2^{-64}$. In our experiment, there are 4096 blocks in a 512-by-512 image. So, the probability of the correct judgment upon all of those blocks is $(1-2^{-64})^{4096} \approx 1-4096\times 2^{-64} = 1-2^{-52}$, this number is very close to 1, so, we can believe that, the case almost doesn't happen.

Lin, Hsieh and Huang [23] present an image recovery method. They use a watermarking technology to embed some recover data in the host image. Table 2.1 shows the relation between restoration performance and the number of un-tampered blocks. From this table, we can know the restoration performance very depend on the ratio of integral blocks to whole blocks. Compare with our method, if the correct blocks is only exceed the threshold, then we can reconstruct the image completely.



(a)



(b)

Table. 2.1. The performance of restoration ratio by Lin's method. (a) The number of un-recovered blocks for single-tampered-chunk. (b) The number of un-recovered blocks for spread-tampered blocks.

2.5 Remark

In our experiment, the number of blocks and the threshold is very large (Actually, in general case, the two numbers should very large). How to increase the speed of encoding/decoding time is a very important issue. Rabin [20] provides a method to design the sharing matrix. For (t, n)-threshold sharing, the time complexity of the revealing phase is $\theta(t^2 + tL)$, where L is the size of the sharing file. However, Rabin's method requires designing a sharing matrix. This section we introduce a method to compute the inverse matrix, and the time complexity is equal to Rabin's method. Our method is based on Lagrange's interpolation.

Firstly, the sharing/revealing algorithm will be introduced. We use matrix operator to represent the sharing algorithm.

In (t, n) sharing, $\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{bmatrix} = \begin{bmatrix} 1^0 & 1^1 & \cdots & 1^{t-1} \\ 2^0 & 2^1 & \cdots & 2^{t-1} \\ \vdots & \vdots & \ddots & \vdots \\ n^0 & n^1 & \cdots & n^{t-1} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_t \end{bmatrix}$, where $s_1, s_2, \ldots, s_t$ are the secret message, and $h_1, h_2, \ldots, h_n$ are the n shadows. If we get any t of the n shadows, then we also get the secret message by evaluating the matrix operation below.

$\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_t \end{bmatrix} = \begin{bmatrix} k_1^0 & k_1^1 & \cdots & k_1^{t-1} \\ k_2^0 & k_2^1 & \cdots & k_2^{t-1} \\ \vdots & \vdots & \ddots & \vdots \\ k_t^0 & k_t^1 & \cdots & k_t^{t-1} \end{bmatrix}^{-1} \begin{bmatrix} h_{k_1} \\ h_{k_2} \\ \vdots \\ h_{k_t} \end{bmatrix}$, where $h_{k_1}, h_{k_2}, \ldots, h_{k_t}$ are the t received shadows.

Let L be the size of the secret file, then the time complexity is $\theta(t^3 + Lt)$, where the term t^3 is the time complexity of computing the inverse matrix. If the threshold t is a large number (In section 2.3, the threshold is 1024 or 2048), the time cost is very much. In order to reduce the time complexity of the revealing algorithm, we proposed a $\theta(t^2)$ time complexity of evaluating the inverse matrix method. This method is based on Lagrange's interpolation.

Firstly, we rewrite the Lagrange's interpolation below:

$$F(x) = \sum_{i=0}^{t-1} h_{k_i} \prod_{\substack{j=0 \\ j\neq i}}^{t-1} \frac{x-k_j}{k_i-k_j} = \sum_{i=0}^{t-1} h_{k_i} \sum_{j=0}^{t-1} b_{i,j} x^j$$
$$= \sum_{i=0}^{t-1}\sum_{j=0}^{t-1} h_{k_i} b_{i,j} x^j = \sum_{j=0}^{t-1}\sum_{i=0}^{t-1} h_{k_i} b_{i,j} x^j = \sum_{j=0}^{t-1} x^j \sum_{i=0}^{t-1} h_{k_i} b_{i,j},$$

The secret message is the coefficient of the variable x^j, so if we use a matrix operation to represent the secret message, then

$$\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_t \end{bmatrix} = \begin{bmatrix} b_{0,0} & b_{1,0} & \cdots & b_{t-1,0} \\ b_{0,1} & b_{1,1} & \cdots & b_{t-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ b_{0,t-1} & b_{1,t-1} & \cdots & b_{t-1,t-1} \end{bmatrix} \begin{bmatrix} h_{k_1} \\ h_{k_2} \\ \vdots \\ h_{k_t} \end{bmatrix}$$, this means $b_{i,j}$ is the coefficient of the inverse matrix. Therefore, we can get the inverse matrix by evaluating the polynomial, and the method is described below.

Computing the inverse matrix method:

Input: t numbers, i.e. $\{k_i \mid 1 \le i \le t\}$.

Output: a t-by-t matrix of $\begin{bmatrix} k_1^0 & k_1^1 & \cdots & k_1^{t-1} \\ k_2^0 & k_2^1 & \cdots & k_2^{t-1} \\ \vdots & \vdots & \ddots & \vdots \\ k_t^0 & k_t^1 & \cdots & k_t^{t-1} \end{bmatrix}^{-1}$.

Notation:

$B = \begin{bmatrix} b_{0,0} & b_{1,0} & \cdots & b_{t-1,0} \\ b_{0,1} & b_{1,1} & \cdots & b_{t-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ b_{0,t-1} & b_{1,t-1} & \cdots & b_{t-1,t-1} \end{bmatrix}$ is the output. i.e., $b_{i,j}$ means the (i+1)th column and (j+1)th row of the output matrix.

Steps:

1. Expand the polynomial $G(x) = \prod_{i=0}^{t-1} x - k_i = g_0 + g_1 x + ... + g_{t-1}x^{t-1} + x^t$.

2. Let the initial value l be 0.

3. Evaluate $c = \prod_{\substack{j=0 \\ j \neq l}}^{t-1} k_l - k_j$.

4. Expand the polynomial $D(x) = \dfrac{G(x)}{c(x - k_l)} = d_0 + d_1 x + ... + d_{t-1}x^{t-1}$, then $b_{l,0} = d_0, b_{l,1} = d_1, ..., b_{l,t-1} = d_{t-1}$.

5. If value l is t-1, then the matrix B is our desired output; otherwise, go to step 3.

The time complexity of this method is $\theta(t^2 + t(t+t)) = \theta(t^2)$.

Chapter 3
Cross Recovery of Multiple Images

This chapter presents a novel cross recovery system of multiple images. We use polynomial-sharing and module-based data-hiding techniques to hide the recovery message. If any (*n*-*t*) of the *n* stego images are lost, then the system can recover these stego images by using other *t* stego images.

3.1 Introduction

In a (distributed) storage system of *n* images, if some images were lost, a trivial way to recover them is by the back-up copies (identical to the lost images but stored elsewhere). Recently, there are some non-trivial approaches [23-27] that recover a tampered image gorgeously. In this chapter, instead of dealing with a single image, we try to consider several images simultaneously (so that the recovery of any member images in this group can be done through the mutual support of the remaining member images). More specifically, we present a novel cross-recovery method that modify the *n* input images and obtain *n* stego images in which the cross-recovery information is hidden; later, up to (*n*-*t*) of the stego images are allowed to get lost, for we can still recover the lost stego images using *t* of the non-lost stego images. Here,

$t \geqq n/2$ is a threshold value pre-specified by the reader.

3.2 The Proposed Method

Sec. 3.2.1 presents a novel polynomial sharing method. On basis of this sharing method, our main algorithm is described in Sec. 3.2.2.

3.2.1 The Tool to Share *n* Pixels

The sharing phase and the revealing phase are described below. Fig. 3.1 shows the flowchart of the sharing/revealing phase. By using the sharing method, the first t shadows is equal to the input message.

The sharing phase:

Input: the t pixels $\{p_i \mid 1 \le i \le t\}$ where $0 \le p_i \le 255$.

Output: n numbers $\{\{h[1], h[2], \ldots, h[t]\}; h[n+1], \ldots, h[n]\}$, where $0 \le h[i] \le 255$ for each i, and $h[1] = p_1, h[2] = p_2, \ldots, h[t] = p_t$.

Steps:

1. Use Lagrange's interpolation (see Ref. [3], the finite Galois Field used here is $GF(256)$) to get an n-points interpolation polynomial $f(x)$ satisfying $f(1) = p_1, f(2) = p_2, \ldots, f(t) = p_t$.

2. Compute $f(t+1)$, $f(t+2)$, ..., $f(n)$; then the u numbers are { {$h[1]=f(1)=p_1$, $h[2]=f(2)=p_2$, ..., $h[t]=f(t)=p_t$}; $h[t+1]=f(t+1)$,..., $h[n]=f(n)$}.

The revealing phase:

Input: any t of the n shadow-numbers, i.e. $\{h[k_i] \mid 1 \le i \le t\}$ where $1 \le k_i \le n$ for each k_i.

Output: the t pixels $\{p_i \mid 1 \le i \le t\}$.

Steps:

6. Use Lagrange's interpolation to get the t-points interpolation polynomial $f(x)$ satisfying $f(k_1)=h[k_1]$, $f(k_2)=h[k_2]$, ..., $f(k_t)=h[k_t]$.

7. The t pixels are$\{p_1=f(1), p_2=f(2), ..., p_t=f(t)\}$.

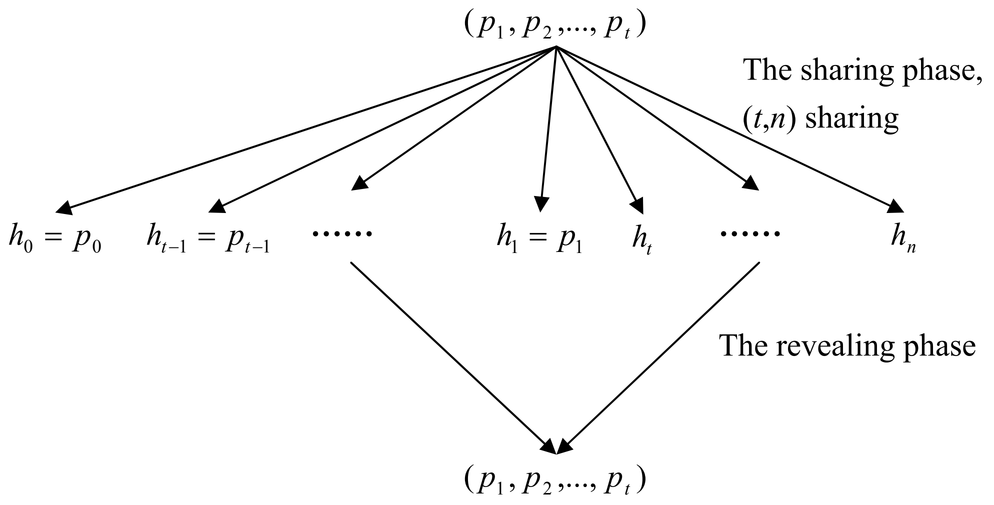


Fig. 3.1. The flowchart of the sharing/revealing phase.

3.2.2 The Main Algorithm

<u>The Main Algorithm:</u>

Input: *n* gray-level images $\{M_i \mid 1 \le i \le n\}$ (each is, say, 512-by-512); and a specified integer *t*≧*n/2*.

Output: *n* stego images.

Goal: Later, if any (*n*-*t*) of the *n* stego images are lost, we may recover the lost stego images by using other *t* stego images.

Steps:

1. Let the initial value of *j* be 1.
2. Let $M_i[j]$ denote the j^{th} pixel value of the i^{th} image M_i. Then use the sharing tool in Sec. 2.2 to share the *n* pixel values $\{p_i \mid 1 \le i \le n; p_i = M_i[j]\}$ and thus generate $u = 2n - t$ shadow-numbers $\{h[1], h[2], \ldots, h[n], \ldots, h[2n-t]\}$.
3. Among the 2*n*-*t* shadow numbers, the final (*n*-*t*) shadow-numbers {*h*[*n*+1], *h*[*n*+2], ..., *h*[2*n*-*k*]} are the auxiliary data. In order to hide the auxiliary data in *n* images, we need to split the data into *n* units. Therefore, before hiding, treat these (*n*-*t*) numbers (each is in the range 0-255) as a value of (*n*-*t*) digits in the base-256 system; then, transform this value from base-256 to a

n-digits value $d_1d_2\dots d_n$ in the base-b system (where b= $\left\lceil 256^{\frac{n-k}{n}} \right\rceil$=*ceiling*($256^{(n-t)/n}$) is the smallest integer not less than $256^{(n-t)/n}$).

4. For i=1,2,..,n, respectively, hide in $M_i[j+1]$ the i^{th} digit d_i of the base-b auxiliary data $(d_1d_2\dots d_n)_b$ by the module-based data-hiding algorithm mentioned in Section 1.2.2.

5. Let $j \leftarrow j+1$. Then go to step 2 if j < 512×512=262144 (the image size).

6. The images $\{M_i \mid 1 \le i \le n\}$ are now the desired output. Also store in a safe place (or attach to each stego image) the n numbers $\{M_i[262144] \mid 1 \le i \le n\}$ where 262144=512×512 is the final pixel-position. Use these n numbers as a recovery seed later.

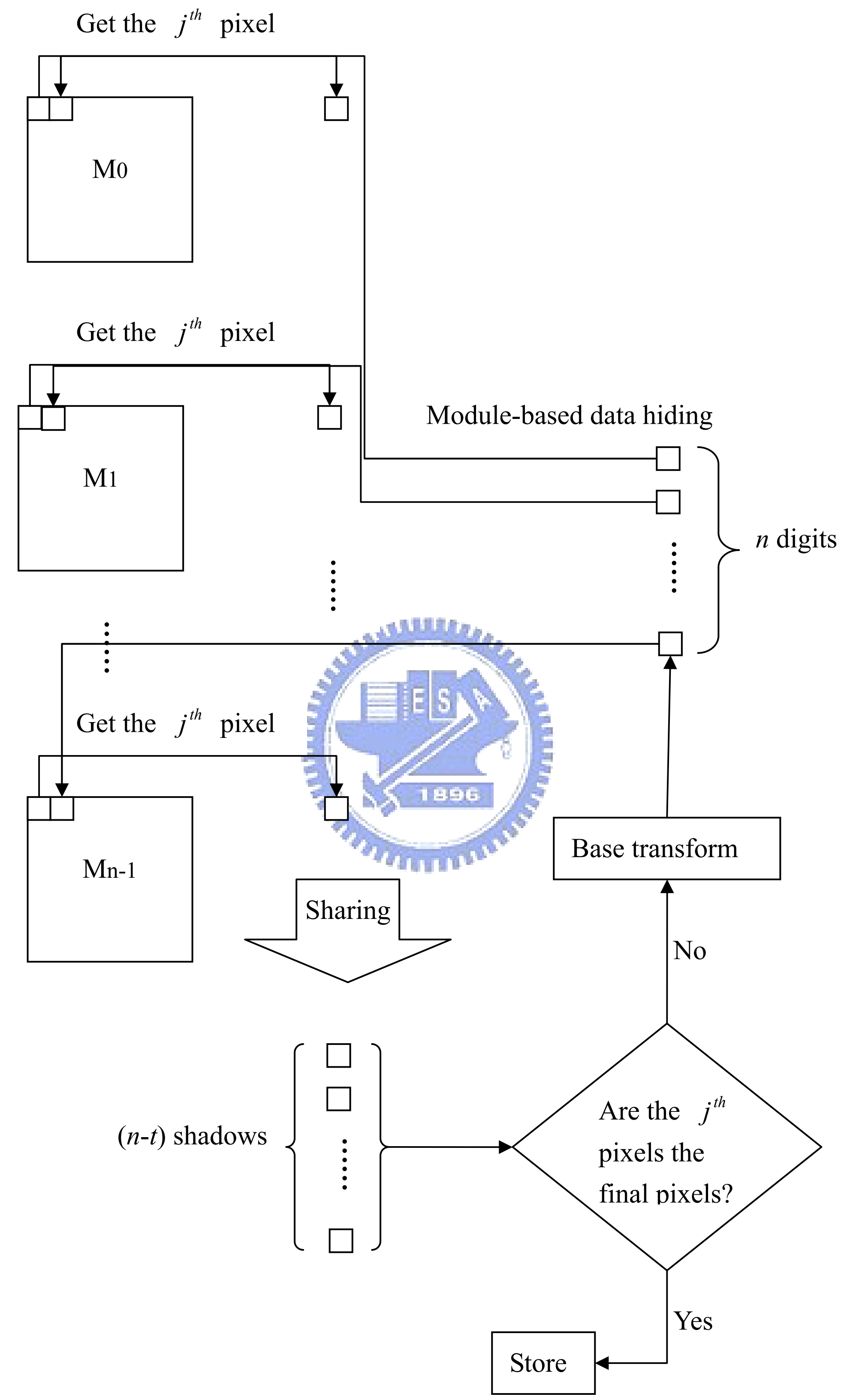


Fig. 3.2. The main algorithm.

<u>The Recovery Algorithm:</u>

Input: *t* gray level stego images $\{M_{k_i} \mid 1 \le i \le t\}$.

Output: *n* recovered images.

Goal: Recover the other *(n-t)* tampered images by using the input *t* images.

Steps:

1. Let the value *j* be *S*, where S is one of the size of the input images (each image size is the same value).
2. Set the *t* shadow values are $\{h[1] = M_{k_1}[j], h[2] = M_{k_2}[j], ..., h[t] = M_{k_t}[j]\}$. Then set $\{h[t+1], h[t+2], ..., h[n]\}$ be the store values generated at the main algorithm step 6 (must revealing the base).
3. Recover the *j*th deleted pixels by revealing the *n* shadows $\{h[1], h[2], ..., h[n]\}$ described in Sec. 3.2.2. Then Decrease the value *j* by 1. If remain any pixel not to recover (In other words, *j*>0), then go to step 4, else go to step 5.
4. Set the *t* shadow values are $\{h[1] = M_{k_1}[j], h[2] = M_{k_2}[j], ..., h[t] = M_{k_t}[j]\}$. Then set $\{h[t+1], h[t+2], ..., h[n]\}$ be the hiding values embedded in the $\{M_{k_1}[j+1], M_{k_2}[j+1], ..., M_{k_n}[j+1]\}$ (must revealing the base). Finally, go to step 3.
5. The images $\{M_i \mid 1 \le i \le n\}$ are now the desired output.

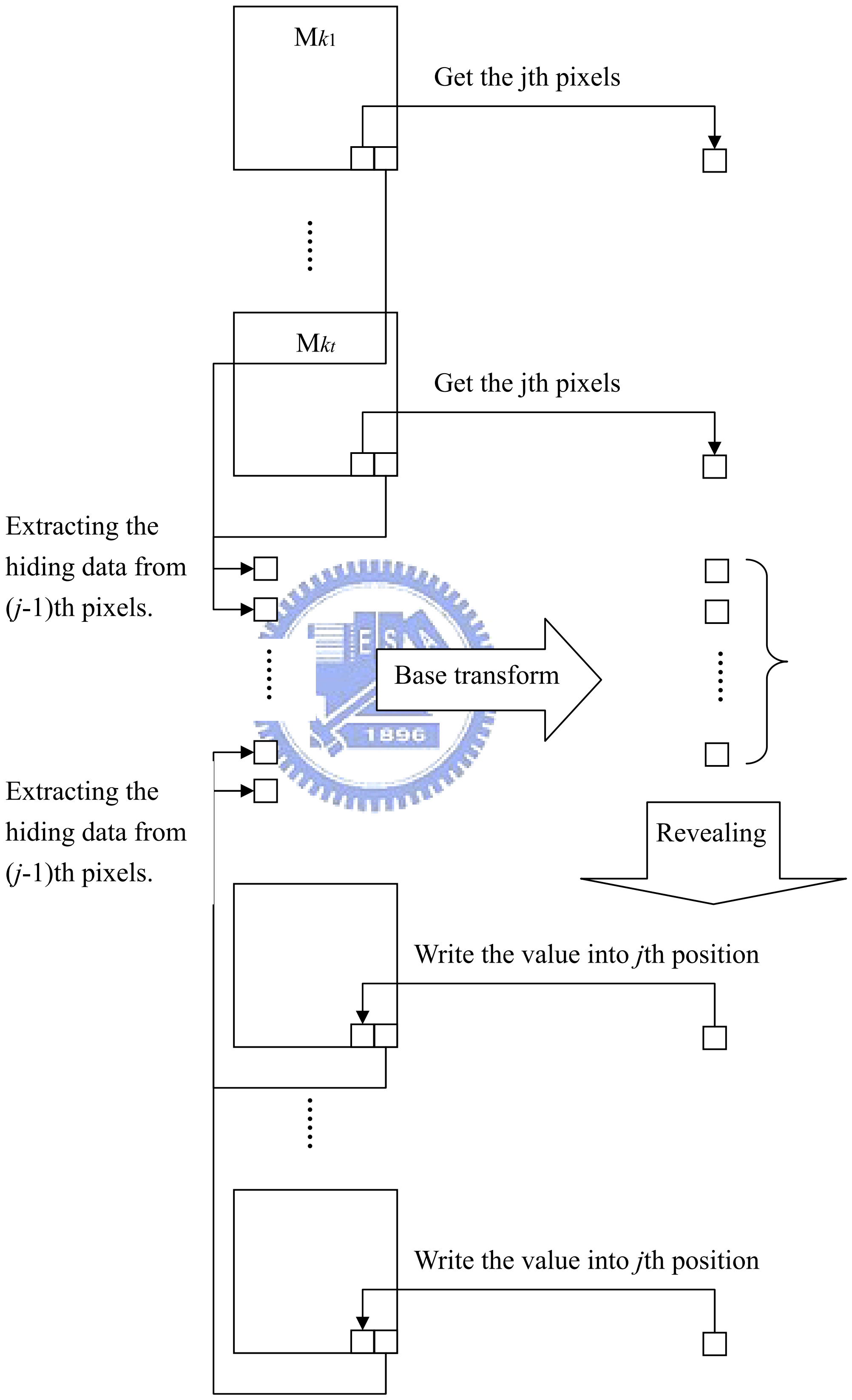

M_{k1}
Get the jth pixels
M_{kt}
Get the jth pixels
Extracting the hiding data from (*j*-1)th pixels.
Base transform
Extracting the hiding data from (*j*-1)th pixels.
Revealing
Write the value into *j*th position
Write the value into *j*th position

Fig. 3.3. The recovery algorithm.

3.3 Experimental Results

Fig. 3.4 shows an experiment using (t, n)=(3, 4). If 4-3=1 of the four 512-by-512 stego images in Fig. 3.4 is lost, we can still recover it by using the other three stego images. Without the loss of generality, assume that stego image M_1 is the one disappears. So, we only have M_2 , M_3 , M_4 , and the 4-numbers seed $\{M_i[262144] | 1 \le i \le 4\}$ mentioned in Step 6. We first use Sec 2.3 to extract the n=4 hidden digits $(d_1 d_2 \cdots d_4)_b$ from the seed, where b=$ceiling$($256^{(n-t)/n}$)=4, then covert $(d_1 d_2 \cdots d_4)_b$ to a value $h[5]$. Together with $h[2]=M_2[262143]$, $h[3]=M_3[262143]$, $h[4]=M_4[262143]$, we have four $h[t_i]$, enough to do inverse sharing to recover the value $M_1[262143]$ (see Sec. 2.2). Then, in next iteration, extract analogously the next n=4 hidden digits $(d_1 d_2 \cdots d_4)_b$ from $\{M_i[262143] \mid 1 \leqq i \leqq 4\}$, and convert $(d_1 d_2 \cdots d_4)_b$ to a new value of $h[5]$. Together with $h[2]=M_2[262142]$, $h[3]=M_3[262142]$, $h[4]=M_4[262142]$, we can do inverse sharing to recover $M_1[262142]$. The process repeats to find all values of M_1. The recovered image is exactly the once-lost Fig. 3.4(a). The embedding (to achieve cross-recovery goal) did not give the images big impact; for all four stego images shown in Fig. 3.4 have PSNR about 46.3 db.

Fig. 3.5 shows another experiment using (t, n) = (4, 6). If 6-4=2 of the six 512-by-512 stego images in Fig. 3.5 is lost, we can still recover it by using the other four stego images. The six stego images have PSNR around 42 db.

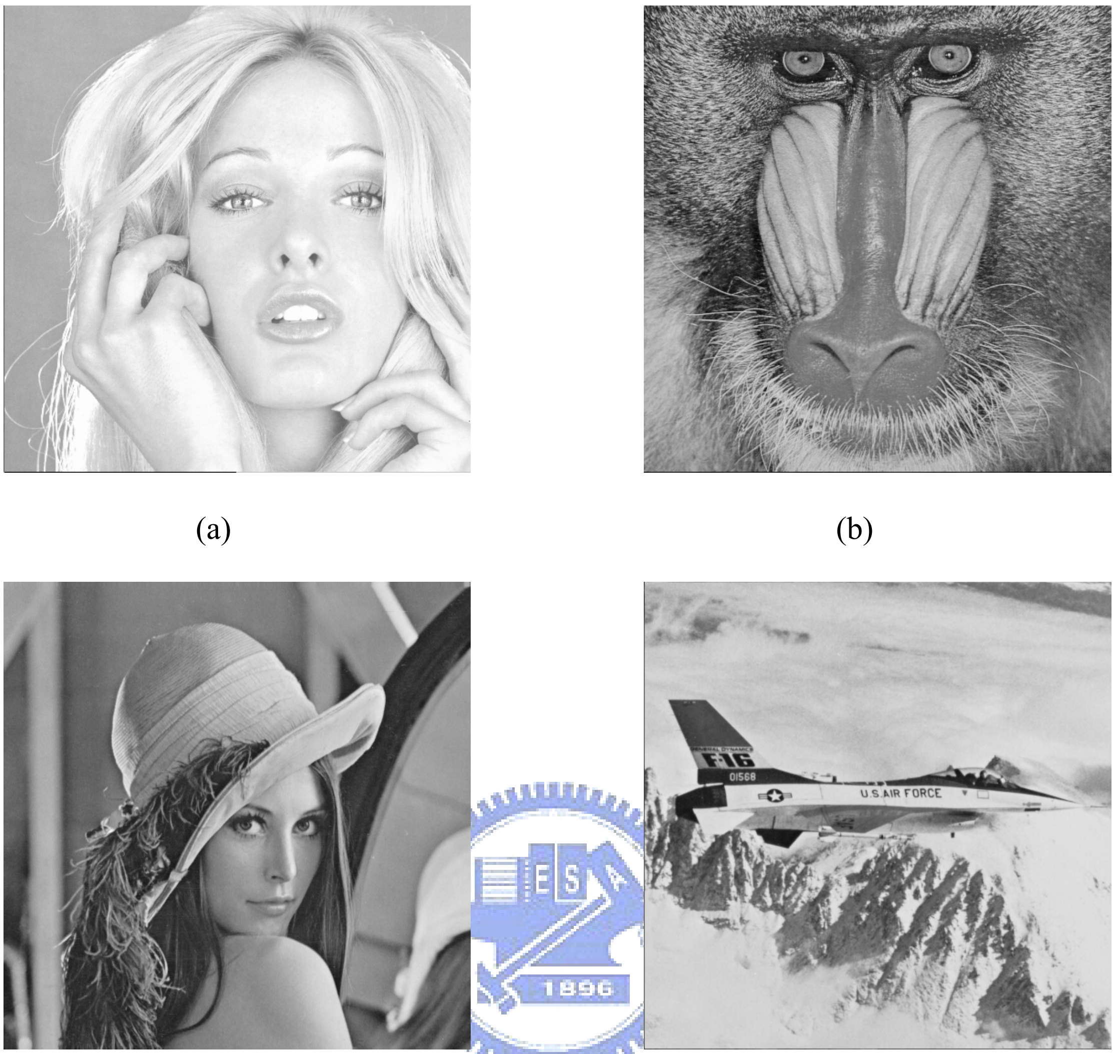
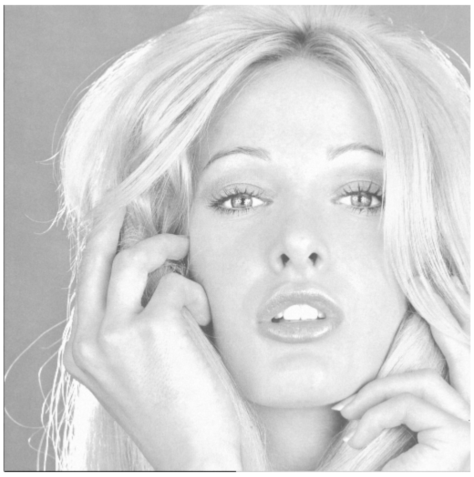
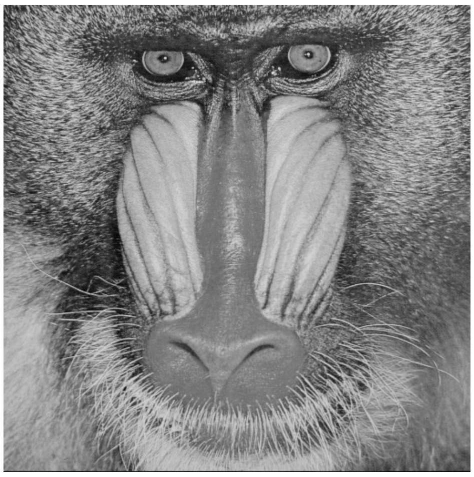
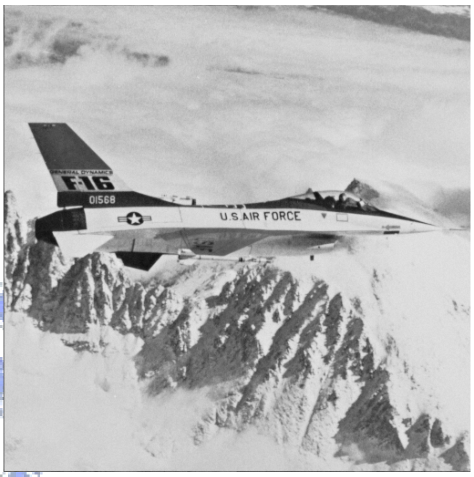


Fig. 3.4. (3, 4) sharing scheme by our method. (a) PSNR = 46.34 db (b) PSNR=46.36 db (c) PSNR=46.37 db (d) PSNR=46.37 db.

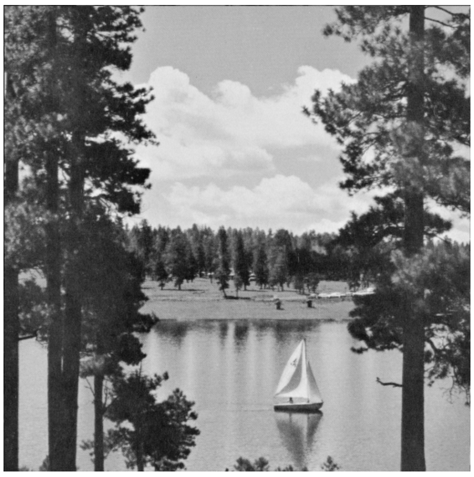

(a)


(b)

(c)



(d)


(e)

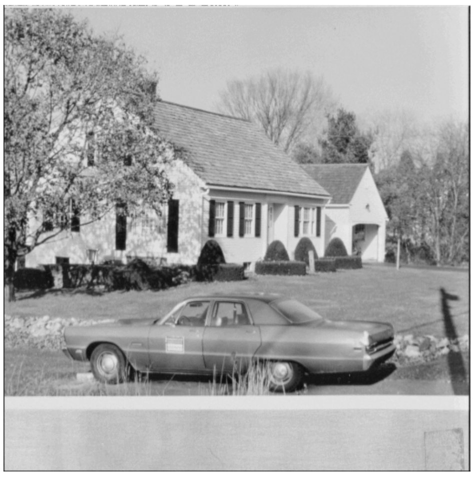

(f)

Fig. 3.5. (4, 6) sharing scheme by our method. (a) PSNR = 42.03 db. (b) PSNR = 42.11 db. (c) PSNR = 42.10 db. (d) PSNR = 42.10 db. (e) PSNR = 42.12 db. (f) PSNR = 42.09 db.

3.4 Discussion

In this chapter, we have presented a cross recovery system for multiple images. Since we required in the introduction that $t \geqq n/2$, i.e. at least one half of the images are not lost, the value of $b=ceiling(256^{(n-t)/n})$ is at most 16. The p and $p' = d + b \times rounding[(p-d) / b]$ in Sec. 2.2.2 has difference at most 16/2=8 (see Ref. [4]). So the PSNRs of the stego images are at least 10×log(255×255/(8×8))=30.07 db. (Indeed, our experimental values are about 34 db, when $(t, n)=(2, 4)$ and the four original images utilized to produce Fig. 3.4 are used again as input.) On the other hand, if $t \geqq 3n/4$ (the case in Fig. 3.4), then $b \geqq 4$ and the difference between the input pixel value p and stego pixel value p' is at most 4/2=2. The PSNRs of the stego images are therefore at least 10×log(255×255/(4×4))=42.11 db (indeed, the experiment in Fig. 3.4 has PSNRs around 46.3 db).

3.5 Extra Topic - a Lossless Version

The yielded images by the method that be presented in this chapter are loss version images. In this section, we present a loss-less version of image recovery method. However, the dealer must consume extra memory space to store the shadows. Each shadows append to each image, the size of each shadow is depend the original images size and the threshold coefficient.

The Encoding Algorithm:

Input: n gray-value images $\{M_i \mid 1 \le i \le n\}$, and those images are all the same size; and a specified integer t.

Output: n shares, each share consists of a visible image (identical with the input image) and a shadow.

Goal: Later, if any (n-t) of the n image are lost, we may recover the lost images by using other t images.

Steps:

1. Use the sharing tool in Sec. 3.2.1 to share the n gray-value images and thus generate $2n-t$ shadows, then combine those shadows as a large shadow H.
2. Use (t, n)-threshold sharing mechanism to share the shadow file H into n sub- shadows $\{h_1, h_2, ..., h_n\}$.
3. For each input image M_i, append a sub-shadow h_i. Then output those images.

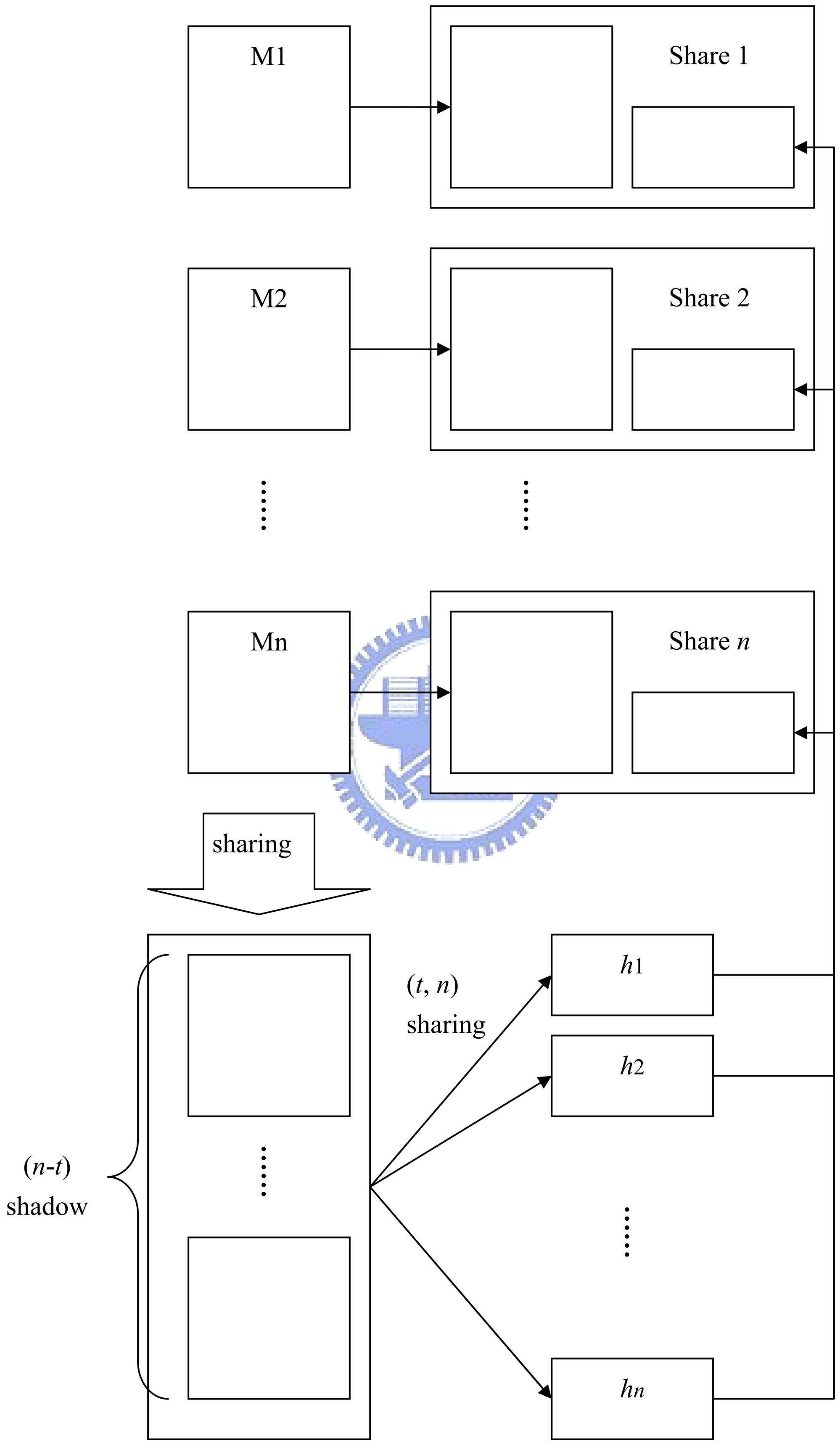


Fig. 3.5. The encoding algorithm.

If we get any t of the shadow images (or there are (n-t) shadow images have been destroy), then we can recover the miss images by inverse the encoding algorithm.

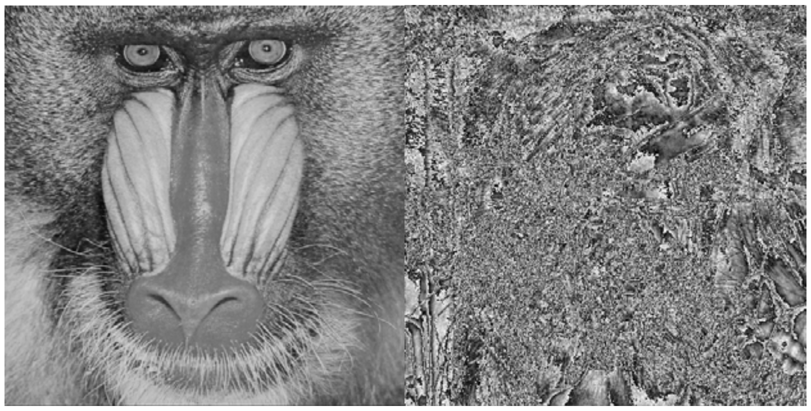
Step 1: Get the secret shadow image H by revealing the noise-like shadow images.

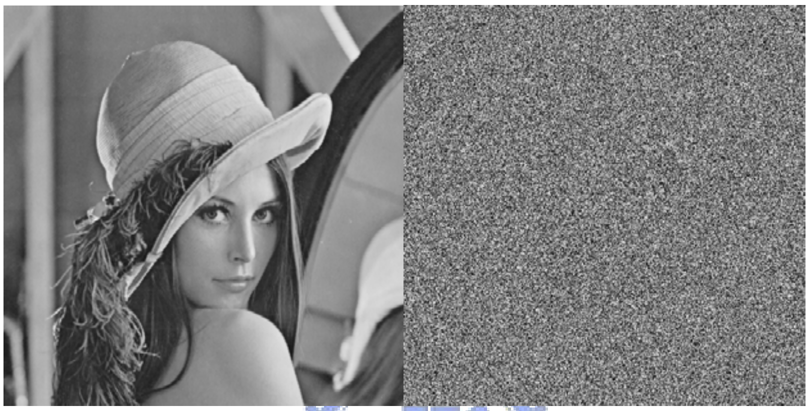
Step 2: Recover the images by the revealing algorithm described in Sec. 3.2.1.

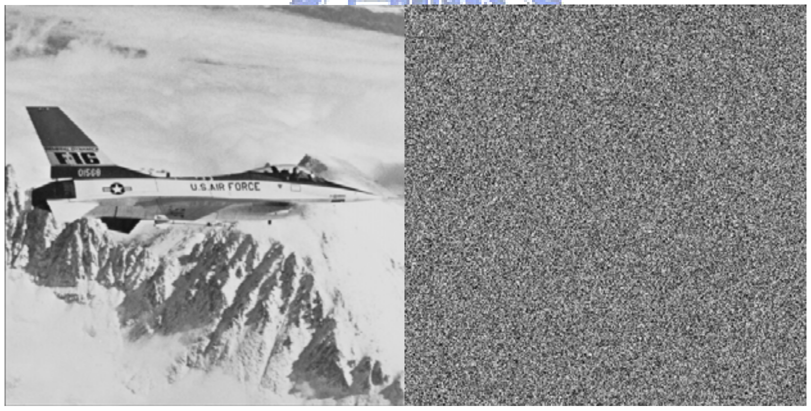
Actually, because this sharing method can be applied on any binary file, we also can apply this method on any compressed image file (for example, JPEG, JPEG2000,…,etc). The critical advantage is to reduce the memory space of the appended shadow.

Experimental Results

Fig. 3.6 shows an experiment in (t, n)=(2, 4). Each input image appends a noise-like shadow image. If 4-2=2 of the four 512-by-512 appended images in Fig. 3.6 is lost, we can still recover it by using the other two appended images. The size of each shadow image is (n/t-1), therefore, if we want the shadow size smaller, the value t must close to n.


(a)


(b)


(c)

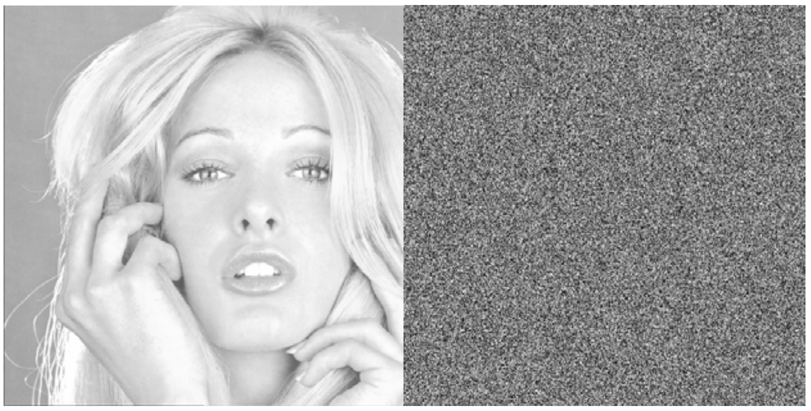

(d)

Fig. 3.6. (2, 4) sharing scheme by our method.

Chapter 4
Two-Layer Image Sharing

This chapter presents a novel method to combine two major branches of image sharing: Visual Cryptography and Polynomial-style Sharing. n transparencies are created for a given gray-valued secret image. If the decoding-computer is temporarily not available at (or, not connected to) the decoding scene, we can still physically stack any t received transparencies ($t \leqq n$ is a threshold value) to get a vague black-and-white view of the secret image immediately. On the other hand, when the decoding-computer is finally available later, then we can get a much finer gray-valued view of the secret image using the information hidden in the transparencies. In summary, each transparency is a two-in-one carrier of the information, and the decoding has two options. The case of virtual-transparencies (electronic-files) is also discussed.

4.1 Introduction

Image sharing can be used in a team when no member alone should be trusted. Visual Cryptography (VC) [1-4] and Polynomial-Style Sharing (PSS) [5-19] are both well-known branches to share images. Both can be designed as (t, n) schemes. (In this chapter, we say that a sharing technique is (t, n) if and only if it shares a secret image ***S*** among n shadows so that any t of the n shadows ($n \geqq t$) can unveil the secret image ***S*** (or a compressed version of ***S***), whereas less than t shadows cannot.) Although both VC and PSS can share images, they are quite different in many manners. The table below compares VC and PSS.

Table 4.1: A comparison between VC and PSS.

	Visual Cryptography	Polynomial-Style Sharing
The Secret image **S** being shared	Black-and-white	Gray or color
Decoding speed (and decoding method)	Instant (by using eyes after stacking shadows)	Slow (by computation)
Is a computer needed in decoding?	No	Yes
Recovered image's perceptual quality	vague	fine
Size of each shadow	Larger than that of **S**	Smaller than that of **S**

From Table 1, we can see that VC is simple and fast, while PSS gives good image quality. A question arises naturally: "Can VC be combined with PSS?" To certain extent, the answer is positive, as is shown here. In this chapter, we present a method to combine these two techniques and achieve a goal: if the decoding-computer is temporarily not available in (or, not connected to) the decoding scene, we can still physically stack the *t* received shadows to get a vague black-and-white view of the secret image immediately; later, when the computer is finally available, we can get a much finer gray-valued view of the secret image using the information hidden earlier in the shadows by using PSS. (In this chapter, our generated "shadows" will be called as "transparencies" because, as mentioned above, one of the two decoding manners is that the shadows can be stacked physically for viewing, just like ordinary transparencies can be stacked and view.)

Below in Sec. 4.2 we first review the basis matrices [B0] and [B1] used in VC, and then in Sec. 4.3 we introduce our method. The experimental results are in Sec. 4.4, and the conclusions are in Sec. 4.5. The case that the transparencies are virtual (electronic files) rather than physical is also discussed in Sec. 4.5.

4.2 A Review of the Basis Matrices [B0] and [B1]

Below we review the two basis matrices **[B0]** and **[B1]** often mentioned in VC field (e.g. see Reference [2]). The matrix **[B0]** is called a "white matrix" because it is useful to produce blocks whose stacking result will represent white pixels of a black-and-white (e.g. halftone) image. Matrix **[B1]** is called a "black matrix" for analogous reason. Without the loss of generality, below we only show the case (*t*, *n*)=(2,4), i.e. only 2 out of 4 shares are needed in recovering. For a general pair of given values (*t*, *n*), the readers may either design their own **[B0]** and **[B1]**, or use the appendix to create some pairs of **[B0]** and **[B1]** by setting parameters there (see Step 1 of the appendix**).** In fact, even if the values of *u* and *n* are fixed, the choice of **[B0]** and **[B1]** is still not unique. To apply the proposed VCPSS two-in-one sharing method, people can use any pair of **[B0]** and **[B1] s**atisfying the requirements (i)-(iii) stated in next paragraph (these three requirements also appear in the appendix). In summary, the pair **[B0]** and **[B1]** is not necessarily generated from the appendix; the appendix is just to let the readers know that there always exists at least one solution to find out **[B0]** and **[B1]**.

In the (*t*, *n*)=(2,4) case, one of the several possible choices for the white matrix **[B0]** and the black matrix **[B1]** is to use

$$[B_0] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{; and } [B_1] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}. \qquad \text{(Equation 1)}$$

Both matrix has *n*=4 rows. (In general, no matter how we assign the two matrices, each matrix must have *n* rows if *n* transparencies are to be created. This is the so-called requirement (i).) In both matrices, each 0 means that a white element is painted there, and each 1 means a black element is painted there. As we can see, both **[B0]** and **[B1]** have 2 black elements per row. (In general, the number of 1s appearing in each row of **[B0]** must be identical to that of **[B1].** This is the so-called requirement (ii).) It is also obvious that if we stack any two (=*t*) rows of our **[B0]**, the stacking result has 2 black elements and 2 white elements. On the other hand, if we stack any two(=*t*) rows of our **[B1]**, the stacking result has at least 3 black elements. (In general, no matter how we choose **[B0]** and **[B1],** the number of 1s contained in the result of stacking any *t* rows of **[B1]** must exceed that of stacking any *t* rows of **[B0]**. This is the so-called requirement (iii).)

Now, assume that we want to create 4(=n) blocks, each is 2-by-2 in size, so that stacking any 2(=*t*) of them will yield a 2-by-2 so-called "white block" (defined here as a block in which only two of the four elements are 1s [i.e. only two black elements]). All we have to do is to permute the columns of **[B0]** randomly, and then distribute the 4(=*n*) rows of the permuted **[B0]** to 4 customers. After that, each customer uses the first two elements as the first row of his block, and next two elements as the 2nd row of his block. As a result, each of the 4(=*n*) customers has his own 2-by-2 block, and any two of these four 2-by-2 blocks can be stacked to yield a 2-by-2 white block (only two of its 2x2=4 elements are 1s).

Similarly, if we want that any *t*(=2) of the *n*(=4) created blocks (each is still 2-by-2 in size) can be stacked to yield a so-called "black block" (defined as a 2-by-2 block in which at least three of the four elements are 1s [i.e. at least three black elements]), then we only have to replace the role of **[B0]** by **[B1]** in the above argument, and obtain four blocks corresponding to **[B1]**. Then distribute these four blocks arbitrarily to the four customers (one block per customer.)

In the above example, each block has w=2 white elements and b=2 black elements, (or equivalently, each row of **[B0]** or **[B1]** has 2 white elements and 2 black elements), and the permutation of the columns of **[B0]** or **[B1]** will not affect the stacking result's brightness (i.e. number of black elements of the stacking result). Moreover, when we do column permutation of the matrix **[B0]** (or **[B1]**), if we look at (concentrate on) the first row, we can see that the first row can be either [0011], or [0101], or [1001], or [0110], or [1010], or [1100]. Therefore, all $C_2^{2+2} = C_2^4 = 6$ types of row vectors can appear. The $C_b^{w+b} = C_2^4 = 6$ types of blocks represented by these $C_b^{w+b} = C_2^4 = 6$ types of row vectors will be called as fundamental blocks. (In general, if a VC system uses blocks of size L-by-L, and each block of each transparency has w white elements and b (b=L×L-w) black elements, then there C_b^{w+b} types of fundamental blocks. For example, if 3-by-3 blocks are to be used, and if each block of each transparency has 5 white elements and 4=3x3-5 black elements, then there will have C_5^{5+4} =126 types of fundamental blocks.)

As a remark of this section and the appendix, note that the appendix is a self-explained appendix extracted from Ref. [2] which is a graceful paper proposed by

G. Ateniese *et. al.* In order to reduce current chapter length and concentrate on our topic, we did not intend to discuss in our appendix the many materials mentioned in Ref. [2]. Interested readers should refer to Ref. [2] for further details. As stated earlier, the appendix here is just to show the readers that they can always create two basis matrices **[B0]** and **[B1]** for any pair of given t and n ($2 \leqq t \leqq n$).

4.3 The Proposed Method

The n expected transparencies can be created using the main algorithm given below in Section 4.3.1, which is supported by some other supportive algorithms described in Section 4.3.2.

4.3.1 The Main Algorithm

<u>Main Algorithm</u> (the main algorithm for (*t*, *n*) sharing scheme):

Input: A grey-value secret image ***S*** of size *Width* ×*Height*, an integer threshold value *t*, and an integer n ($n \geqq t$) indicating the number of produced transparencies.

Output: n transparencies $R=\{r_0, r_1, \ldots, r_{n-1}\}$ are produced.

Steps:

Step 1. Use the appendix to create two basic matrices [B0] and [B1] for the given t and n (see Sec. 4.2 to understand [B0] and [B1]; see appendix to know how to create [B0] and [B1].) Then, from the [B0] and [B1], the corresponding values of w and b can be found (each block in each transparency has w white elements and b black elements).

Step 2. Since each block of each share is required to have w white elements and b black elements, there are C_b^{w+b} fundamental blocks. Label all fundamental blocks, i.e. create a mapping table L that maps each number in {0, 1, …, (C_b^{w+b}-1)} to its corresponding fundamental block.

Step 3. Produce an image H (the halftone binary version of the image S).

Step 4.Produce another grey-value image ***J*=*S**** by compressing S using some image compression techniques (e.g. JPEG, GIF…); and the compression ratio **C** must be at least $\frac{8\times n}{u\times \log C_b^{w+b}}$ so that the space needed to store ***J*** is small enough. Hereinafter, the data file representing the compressed image ***J*** will be treated as a bit-stream data-file.

Step 5. Due to security concern, we may encrypt ***J*** to get an encrypted bit stream ***J'*** **.** This can either be done by using a security key, or by using some very simple functions. (For example, to get ***J'***, we may just use XOR function in a bit-by-bit manner on the bit stream ***J*** and the bit stream of the image H

(re-read the image *H* several times if the size of *J* is larger than that of *H*).

Step 6. Then, the bit stream of ***J'*** is shared using an (*t*, *n*) sharing method that share a sequence of numbers by polynomials (see Pages 766-767 of Ref. [7]. But we use mod-257 function here to replace the mod-251 function used in Ref. [7]). Let $\{s_0, s_1, ..., s_{n-1}\}$ be the *n* created bit-string-shares. Notably, according to Ref. [7], each bit-string-share is *t* times smaller than ***J'***, so the *n* so-called bit-string-shares together have n[(*Width* ×*Height* × *8* / C)/*t*] bits.

Step 7. For each s_i in $\{s_0, s_1, ..., s_{n-1}\}$, transform the data in s_i from base 257 to base C_b^{w+b}. This step is for the hiding purpose later in Step 8. (Because we only have C_b^{w+b} types of fundamental blocks, we can only embed digits in the range {0, 1, …, (C_b^{w+b} -1)} according to Step 2.)

Step 8. Use the TC Algorithm below to create n transparencies $\{r_0, r_1, ..., r_{n-1}\}$ so that the n shares $\{s_0, s_1, ..., s_{n-1}\}$ are hidden in them, and any *t* of the *n* transparencies can be stacked to obtain an image looks like the halftone image ***H.***

Remark: In Step 4 above, it was stated that the compression ratio (C) must be greater than $\frac{8 \times n}{u \times \log C_b^{w+b}}$. We explain why. Since ***J*** is the compressed version of ***S***, which has *Width* × *Height* pixels, the number of bits contained in image ***J*** (or ***J')*** is *Width* × *Height* × *8* / C. Then, according to Ref. [TL02], each bit-string-share will have (*Width*

×*Height* × *8 /* C)/*t* bits after (*t*, *n*) sharing of ***J'*** by using polynomial-style sharing. Therefore the n bit-string-shares together will have (*Width* × *Height* × *8 /* C) x n/*t* bits. On the other hand, since we will use the indices of fundamental blocks to hide the bit-string of each bit-string-share of ***J'*** , and since there are C_b^{w+b} kinds of fundamental blocks, each index of a block can represent a value in the range from 0 through (C_b^{w+b}-1). As a result, each block can use its index (the block type) to recover a value in the range 0~ (C_b^{w+b}-1). In other words, if we grab the *n* so-called bit-string-shares, and divide their n[(*Width* × *Height* × *8 /* C)/*t*] bits into many small segments (each segment has $\log C_b^{w+b}$ bits), then each segment can be hidden in (i.e. represented by the block type of) a suitably chosen block whose block index happens to be this value. In summary, the *n* so-called bit-string-shares can be represented by n[(*Width* ×*Height* × *8 /* **C**) /*t*]/$\log C_b^{w+b}$ blocks. Since *S* has *Width* × *Height* pixels, there are *Width* × *Height* binary pixels in *H*. Since each binary pixel is extended to a block when VC creates transparencies, there will exist *Width* × *Height* blocks in the enlarged recovered binary image of *H* after stacking. So we require n[(*Width* × *Height* × *8 /* **C**) /*t*] /$\log C_b^{w+b}$ ≦ *Width* × *Height* .That is why we require **C** ≧ $\frac{8 \times n}{u \times \log C_b^{w+b}}$.

4.3.2 Some Supportive Algorithms

The main algorithm above requires the use of the following supportive algorithms. The TC algorithm is to create n transparencies. The step 2a of this algorithm use a subroutine called TC-SUB. (Notably, TC-SUB is to create n blocks [each transparency will receive one block], and the first of these n blocks can be used to hide a value in the range {0 ,1,…, (C_b^{w+b} -1)}.)

<u>TC Algorithm</u> (Transparencies-creating Algorithm):

Purpose: to create n transparencies in which the n shares $\{s_0, s_1, ..., s_{n-1}\}$ are hidden.

Input: the n shares $\{s_0, s_1, ..., s_{n-1}\}$, and the halftone image *H*.

Output: n transparencies $\{r_0, r_1, ..., r_{n-1}\}$.

Steps:

Step1**:** Divide the halftone image *H* into *n* parts of equal size. Then, for each *i*= 0,1,…,n-1, the Step 2 below will be utilized to hide the encrypted share s_i with the help of Part *i* of *H*. Therefore, set *i*=0 and *j*=0 initially, then go to Step 2.

Step 2: (To hide the encrypted share s_i)

(2a): Read the next not-yet-processed integer (the *j*-th integer $s_i(j)$) from the share s_i; also read the next not-yet-processed binary pixel (the *j*-th pixel) value of Part *i* of *H*; then use them to create n blocks $\{b_0, b_1, ..., b_{n-1}\}$ by the sub-algorithm TC-SUB below. Among $\{b_0, b_1, ..., b_{n-1}\}$, the generated block

b_0 is particularly assigned to the transparency r_i at location *j*, and other n-1 blocks $\{b_1,...,b_{n-1}\}$ are arbitrarily assigned to the *n*-1 remaining transparencies $\{r_l \mid 0 \le l \le n-1, \quad l \ne i\}$ at location j.

(2b): *j*←*j*+1, and go to (2a) unless all integer values $\{s_i(j)\}$ of s_i have been processed and embedded in Transparency r_i (in that case, go to Step 3 instead).

Step 3: Let *j*=0 again. Increment the value of *i* by 1. Then go to Step 2 unless *i* =n. (When *i*=n, all n shares $\{s_0, s_1,..., s_{n-1}\}$ have been hidden in the n transparencies $\{r_0, r_1,..., r_{n-1}\}$.) □

Notably, in the Step (2a) above, s_i is hidden in r_i; some readers might wonder: at the decoding phase, if r_i is missed, how can s_i be recovered? To answer this question, note that s_i is an (*t*, *n*) sharing result (see Step 6 of the Main Algorithm), so any *t* of *n* encrypted shares $\{s_0, s_1,..., s_{n-1}\}$ can recover the secret information (i.e. the encrypted bit-stream ***J'*** of the JPEG image *J*).

<u>Sub-algorithm TC-SUB</u>

Purpose: to hide an integer $s_i(j)$.

Input: an integer value $s_i(j)$ and a pixel q (either black or white) of the halftone image *H*.

Output: n blocks $\{b_0, b_1, ..., b_{n-1}\}$ (one block per transparency). Notably, the secret value $s_i(j)$ is hidden in b_0; meanwhile, stacking together any t of these n blocks can reveal the pixel value (black vs. white) of q.

Steps:

Step 1: Use the mapping table L to map the value $s_i(j)$ to a fundamental block. Let b_0 be that block. In other words, $b_0 = L\,(s_i(j))$.

Step 2: If q is a white pixel, set matrix **[B]** to the basic matrix **[B0]** produced in Step 1 of the Main Algorithm. If q is black, set matrix **[B]** to the basic matrix **[B1]** .

Step 3: Permute each column of **[B]** so that the first row of **[B]** is the same as b_0 (treat the 2-by-2 matrix b_0 as a row vector of 4 elements). Then, for each d= 1,2,…,n-1, transform Row d of **[B]** to a 2-by-2 block b_d (by sequentially assigning the elements of Row d to elements of block b_d) .

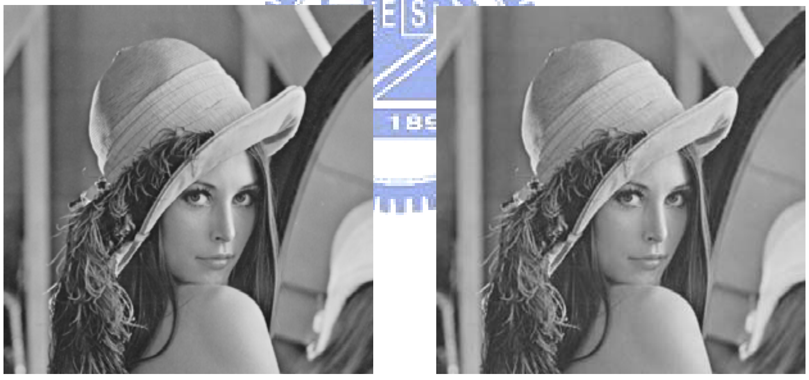
Step 4: $\{b_0, b_1, ..., b_{n-1}\}$ is now the desired output. □

Remark (about the TC-SUB algorithm): At the decoding phase, when we stack any t of the n generated blocks, then, due to Steps 2 and 3 of TC-SUB algorithm, the stacking result will be a black block if and only if the pixel q of the halftone image H is a black pixel.

4.4 Experimental Results

In our first experiment, the secret image is the image 512-by-512 Lena shown in Fig. 4.1(a). Its JPEG-compressed version, the so-called image ***J*** in Step 4 of the main algorithm, is the 512-by-512 Lena* shown in Fig. 4.1(b); and the PSNR of Lena* is 39.31 dB. The 512-by-512 halftone image ***H*** of Lena is shown in Fig. 4.2. Assuming (t, n)=(2,4). In other words, the target is that any two of the four generated transparencies can be used for image reconstruction. Then, using Fig. 4.1(b) and Fig. 4.2, the n=4 transparencies are created, as shown in Fig. 4.3. Each transparency is 2×2 times bigger than each image in Fig. 4.1 and .42, for each fundamental block used to expand a pixel (see Sec. 2) is 2-by-2 in this experiment. Now, if we stack any two of the four generated transparencies, as shown in Fig. 4.4, we get a 1024-by-1024 black-and-white image which looks like a 2×2-times enlarged version of the halftone image ***H*** shown in Fig. 4.2. On the other hand, if we use the two transparencies to extract the information hidden in them, we can recover exactly the 512-by-512 grey-value compressed image Lena* shown in Fig. 4.1(b), as is shown in Fig. 4.5.

In the second experiment, the secret image is the 512-by-512 image “Jet” shown in Fig. 4.6(a). Assuming (t, n)=(3, 4). In other words, the goal is that any three of the four generated transparencies can be used for image reconstruction. The corresponding experimental results are shown in the remaining parts of Fig. 4.6. Note that 6(d) is 2-by-3 times larger than the halftone image *H* because we use 2-by-3 blocks to expand pixels of the halftone image *H* there. Also note that any two transparencies together ($2<3=t$) cannot reveal the JPEG version (Jet*) or the halftone version *H*. (Stacking two transparencies together only gets an image with nothing but noise; no trace of the image *H* can be seen. As for the reason why the JPEG version Jet* cannot be revealed, it is due to the decoding of Jet* requires the use of the recovered *H*. [See Steps 1 and 2a of the TC algorithm.])



(a) (b)

Fig. 4.1. A secret image Lena (a), and its JPEG-compressed version Lena* whose PSNR is 39.31 dB (b).

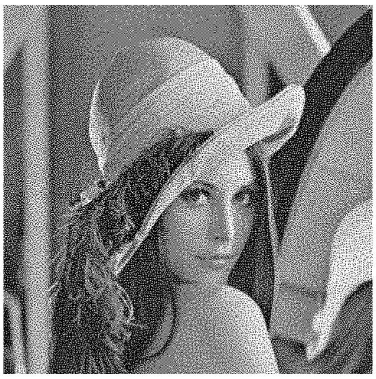


Fig. 4.2: The halftone version (the binary image *H*) of Fig. 4.1(a).

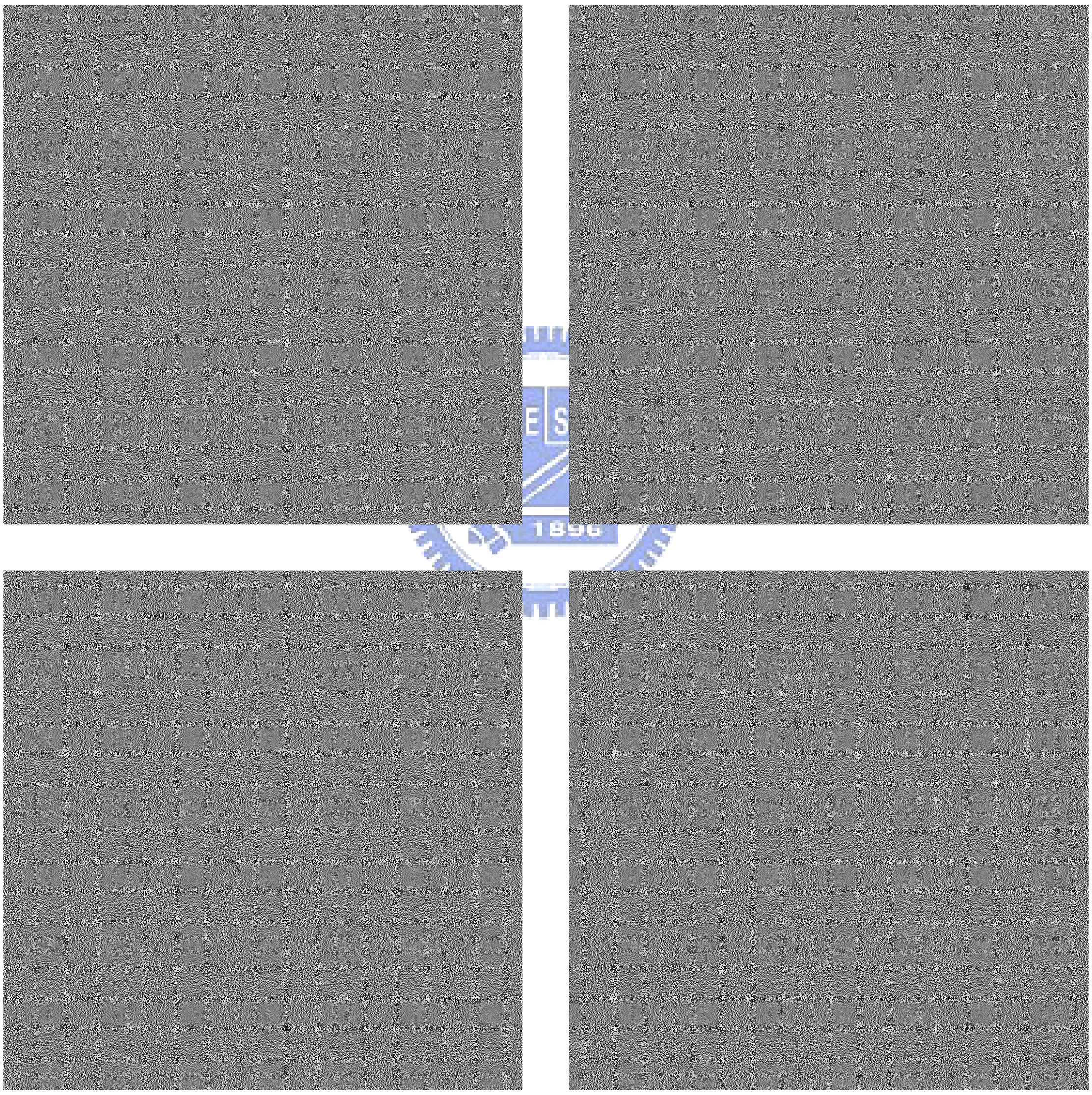


Fig. 4.3. The *n*=4 transparencies T0-T3 generated from the pair {Fig. 4.1(b), Fig. 4.2} in our (*t*=2, *n*=4) threshold scheme.

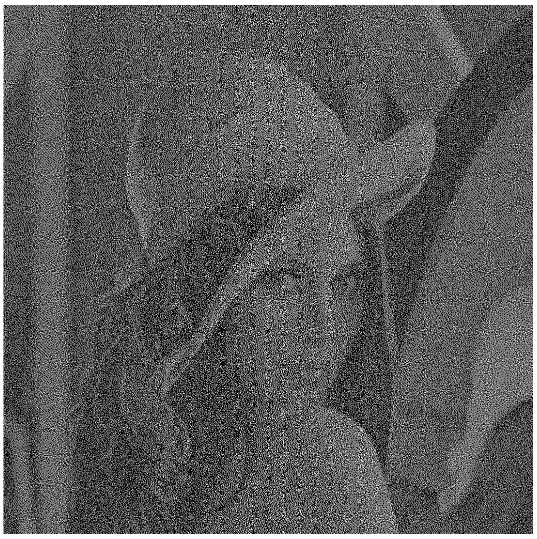


Fig. 4.4. Stacking “any” two transparencies (e.g. 1st and 3rd transparencies here) yield an enlarged binary image of Lena.

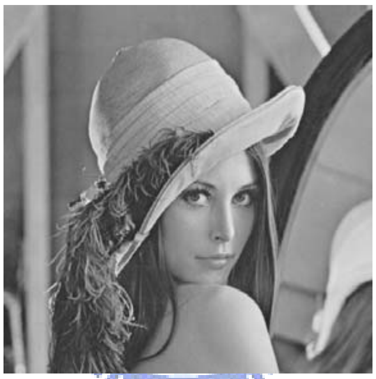
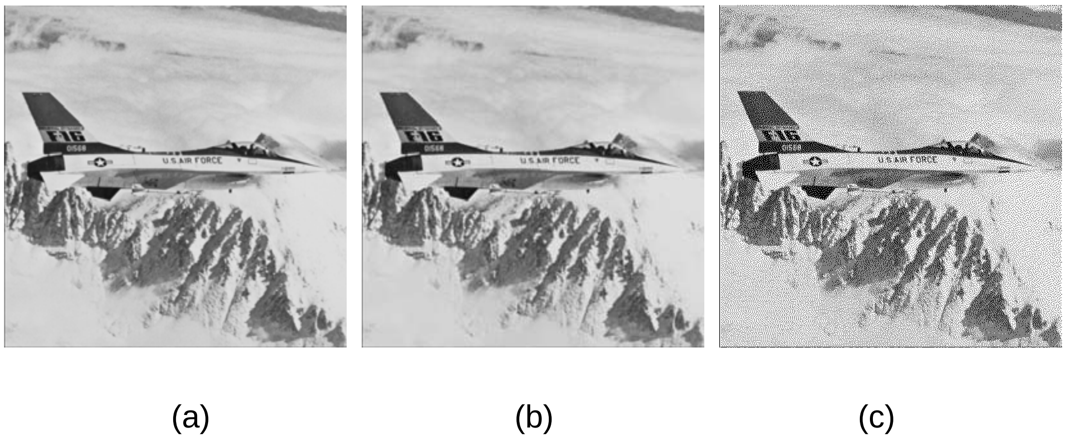
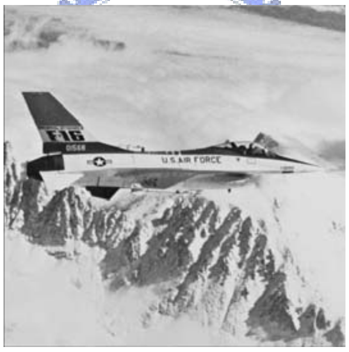


Fig. 4.5. The gray-value image Lena* (identical to Fig. 4.1(b)) reconstructed using the information embedded in any two transparencies (the 1st and 3rd transparencies here).



(d)



(e)

Fig. 4.6. The second experiment. ((*t, n*) = (3, 4), and this experiment uses 2-by-3 blocks). (a): a secret image Jet. (b): the JPEG-compressed version Jet* whose PSNR is 47.6dB. (c): the halftone version *H* of Fig. 4.6(a). (d): stacking result using "any three" of the four generated transparencies (here, 1st, 2nd and 4th transparencies). (e): the Jet* (identical to Fig. 4.6(b)) reconstructed using the information embedded earlier in the three transparencies mentioned in Fig. 4.6(d).

4.5 Concluding Discussions

In this , we have proposed a new method which combines two major branches of image sharing: visual cryptography (VC) and polynomial-style sharing (PSS). In the decoding issue, this new method is more flexible than applying VC or PSS independently, since our method provides a "two-options" decoding.

In Fig. 4.1(b) and Fig. 4.5, the PSNR of Lena* is 39.31 dB. If the readers would like to have an image of better PSNR, then they should use larger blocks (say, 2-by-3 or 3-by-3 blocks) to replace the 2-by-2 blocks here. Without the loss of generality, assume 3-by-3 blocks are to be used. Then, create the corresponding [B0] and [B1] matrices; each is n-by-9. Notably, using 3-by-3 blocks will make each transparency become 3-by-3 times larger than the input image Lena. However, according to Step 4 of the Main Algorithm, we have the constraint that the compression rate C should be at least $\frac{8\times n}{u\times\log C_b^{w+b}}$. With $C_2^4 = 6$, C_5^{5+4} =126 (assuming each 3-by-3 block of each transparency has 5 white elements and 4=3x3-5 black elements), and $\frac{\log 126}{\log 6} = 2.7$, the compression rate to create Lena* from Lena can be 2.7 times less strict than the one used to generate Fig. 4.2(a). Therefore, the quality of Lena* will be much better. Analogous analysis also holds if 2-by-3 blocks are used (see the example 1 of the appendix, and the remark after that example).

Below we discuss what happens if the transparencies are not created physically, i.e. if the transparencies are in fact electronic files. To "stack" these "virtual" transparencies, we use a computer to do a simple "OR" function on all *t* elements having identical coordinate among all t received electronic files, i.e. evaluate $y_1 \vee y_2 \vee \ldots \vee y_t$, and use the rule $(y_1 \vee y_2 \vee \ldots \vee y_t)=1$ if and only if at least one of the u elements being checked is 1. Of course, this reveals the size-enlarged version of the half tone image H very quickly. Then, the enlarged version can be transformed back to the original-size halftone image H by summing up the number of black elements (the number of 1s) appeared in each of the non-overlapping blocks (each block is 2-by-2 in Fig. 4.5 and 2-by-3 in Fig. 4.6.) Therefore, in the computer version, the halftone image H can be obtained quickly without any security obsession (except the threshold limitation: at least t of the n produced files must attend). As for the JPEG-compressed gray-value image J^*, the decoding is as before, which requires the decoded halftone image H because of Steps 1 and 2a of the TC algorithm (and the decoding of J^* may also require the use of the encryption key mentioned in Step 5 of the Main Algorithm). Therefore, compared with the reconstruction of the gray-valued image Lena*, obtaining the halftone image H is much faster; no matter the transparencies are virtual transparencies (electronic files) or physical transparencies.

4.6 An Application

From the analysis given in the final paragraph of last section, our two-layer decoding system can also be used in business in the following way that balances between convenience and security: 1) any *t* of the *n* lower-rank employees can gather together and use a computer to unveil a vague black-and-white version of the image; while 2) with the help of this decoded vague version in (1), the manager of these employees can unveil further a fine gray-valued version of the image using the key that only he knows. In the above, (1) is for the convenience of the daily meeting between the employees (the meeting does not need the attendance of the manager, although less than *t* employees cannot have a meeting); while (ii) is for the company owner to prevent a high-quality image from being sold in a black market (neither the *t* lower-rank employees nor the manager alone can obtain the JPEG version, unless the two sides cooperate).

Chapter 5
Conclusions and Future Works

5.1 Conclusions

In this thesis, we make use of the polynomial sharing technology on some topics. Single image recovery, multiple images cross recovery and Visual Cryptography.

Polynomial sharing technology is a useful tool for preserve secrets, disperse information and fault tolerance, etc. With those advantages, in recent years, in image processing domain, many literatures about secret sharing had been published. Those literatures use secret sharing technology to improve the image security or to share the image files. But apply this technology to image processing issues is fewer. Chapter 1 presents an application on image recovery, to improve the ability of the recover algorithm.

How to protect a data? In generality, if we want the message more robust, then more bits/space to store it. But in image processing domain, we can tolerate some change of the pixels. Chapter 3 uses this property to propose a method of multiple images recovery.

Visual Cryptography is also a secret sharing technology. In order to combine with VC and polynomial sharing, the critical idea is to hide the shadows in each transparency. Image compression (JPEG) resolve us the hiding capacity problem.

5.2 Future Works

There are some topics for extending and improving the proposed methods. We list them as follows:

Chapter 2: Image Tampered detection and Recovery.

This method that we propose may add some conditions. For example, consider the image contains. ROI(region of interesting) is a more important region, so we may want to improve the recover ability of this region.

Chapter 3: Cross Recovery of Multiple Images.

The method be proposed in section 3.2 isn't a very good method. We list the weaknesses as follows:

I. In inversing phase, if any an error occurs, the error can send to next decoding pixels.

II. We don't add any method to check which image had been destroy. This can add fragile watermarking or checksum on those stego images to check which images were destroyed.

Chapter 4: Two Layer Image Sharing

By using stack or computing method, the decoding images are very similar.

Some information remainders at this point. A better method is change the hiding information to the remainder between the original secret image and the stacked version (this is the halftone version of the secret image).

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Appendix

A possible way to generate a pair of basis matrices [B0] and [B1] for an (u, n) threshold system. (This self-explained appendix is a re-written part extracted from Ref. [3-4])

Input: an integer threshold *u*, and an integer *n* indicating the number of produced transparencies. (u≤n.)

Output: a pair of basis matrices [B0] and [B1] (both matrices are formed of 0s and 1s) satisfying the following requirements:

i. Each matrix has n rows.

ii. The number of 1s shown in any row of [B0] is identical to that of [B1].

iii. Stacking any u of the n rows of the matrix [B0] (or [B1], respectively) get a row vector called the White-Row-Vector (the Black-Row-Vector, respectively) corresponding to these u rows. Any Black-Row-Vector contains more 1s than any White-Row-Vector does.

Steps:

Step 1: Arbitrarily choose (*n-u*) integers $\{h_j \mid 0 \le j \le n-u-1\}$ and an integer $h_{n-u} > 0$.

Step 2: Compute the integers $\{a_i \mid 0 \le i \le n\}$ by the formula

$$a_i = \sum_{j=0}^{\min\{i,n-u\}} (-1)^{i+j} h_j C_j^i \qquad \text{(Equation } A1\text{)}$$

Step 3: Create $\{G_0, \ldots\ldots, G_n\}$. Here, each G_i is a matrix having n rows and C_i^n columns; no two columns of G_i are identical; and each of these C_i^n columns is exactly one of the C_i^n permutations of the n elements of the first column of G_i. (The first column of G_i is formed of *i* consecutive 1s followed by (*n-i*) consecutive 0s.)

Step 4: Initially, set both [B0] and [B1] to empty set; also set $a_i = a_0$. Then,

(4-i): If (a_i=0), then do nothing for this a_i, just go to (4-iv).

(4-ii): If (a_i>0), then repeatedly append G_i into [B0] (repeat $|a_i|$ times).

(4-iii): If (a_i<0), then repeatedly append G_i into [B1] (repeat $|a_i|$ times).

(4-iv): If i=n, go to Step 5; else, increment i by 1 and go to (4-i).

Step 5: [B0] and [B1] are now the desired output. $\square$

EXAMPLE 1. In the (*u*=2, *n*=4) case, if a reader uses $\{h_0, h_1, h_2\} = \{3,3,2\}$ as his arbitrary setting for the $\{h_0, h_1, h_2\}$ in Step 1 above, then he will proceed as follows:

Step 1: Let $\{h_0, h_1, h_2\} = \{3,3,2\}$.

Step 2: By formula (A1), compute and obtain $\{a_0, a_1, a_2, a_3, a_4\} = \{3,0,-1,0,3\}$.

Step 3: $G_0,...,G_4$ are, respectively,

$$G_0=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}, G_1=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}, G_2=\begin{bmatrix}1&1&1&0&0&0\\1&0&0&1&1&0\\0&1&0&1&0&1\\0&0&1&0&1&1\end{bmatrix}, G_3=\begin{bmatrix}1&1&1&0\\1&1&0&1\\1&0&1&1\\0&1&1&1\end{bmatrix}, G_4=\begin{bmatrix}1\\1\\1\\1\end{bmatrix}.$$

Step4: Since $\{a_0,a_1,a_2,a_3,a_4\}=\{\boxed{3}, 0, \underline{-1}, 0, \boxed{3}\}$; from those positive a_i, we get

[B0] = (G_0 repeats three times (for $a_0=3$), followed by repeating G_4 three times (for $a_4=3$)), i.e.

$$[\text{B0}]=\begin{bmatrix}0&0&0&1&1&1\\0&0&0&1&1&1\\0&0&0&1&1&1\\0&0&0&1&1&1\end{bmatrix};$$

similarly, from those negative a_i, we get

[B1] = (G_2 appears $|a_2|=|-1|=1$ time), i.e.

$$[\text{B1}]=G_2=\begin{bmatrix}1&1&1&0&0&0\\1&0&0&1&1&0\\0&1&0&1&0&1\\0&0&1&0&1&1\end{bmatrix}.$$

Step 5: [B0] and [B1] are now the desired output. □

Remark: In example 1 above, stacking any two rows of [B0] yields a row having three 1s; while stacking any two rows of [B1] yields a row having five 1s (and hence, "darker" than stacking any two rows of [B0]). Also note that each row of each matrix has 6 elements; therefore, each block is 2-by-3 (or 3-by-2) rather than the 2-by-2 used in Section 2 and in the experiment for Lena (Fig. 4.1 through 4.5). Now, since

$C_b^{b+w} = C_3^6 = 20 > 6 = C_2^4$, the JPEG compression rate C mentioned in Step 4 of the main algorithm is thus smaller (less strict) now, and the quality of the JPEG image Lena* in Fig. 4.1(b) and Fig 4.5 will thus becomes better (has higher PSNR value). The price of getting this better-quality gray-value image Lena* is that the stacked black-and-white image in Fig. 4.4 will become larger in width (just like the Fig 4.6(d) does) because of the use of 2-by-3 blocks. □

EXAMPLE 2. For example, in (u=2, n=5) case, if a reader uses $\{h_0, h_1, h_2, h_3\}$ = $\{1,1,1,1\}$ as his arbitrary setting for the $\{h_0, h_1, h_2, h_3\}$ in Step 1 above, then he will proceed as follows:

Step 1: Arbitrarily assign $\{h_0, h_1, h_2, h_3\}$. For example, let $\{h_0, h_1, h_2, h_3\}$ = $\{1,1,1,1\}$.

Step 2: By formula (A1), compute and obtain $\{a_0, a_1, a_2, a_3, a_4, a_5\} = \{1,0,0,0,-1,4\}$.

Step 3: $G_0, ..., G_5$ are, respectively,

$$G_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix},\ G_4 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix},\ G_5 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Step4: Since $\{a_0, a_1, a_2, a_3, a_4\} = \{\boxed{1}, 0, 0, 0, \underline{-1}, \boxed{4}, \}$; from those positive a_i, we get

[B0] = (G_0 appear once (for $a_0 = 1$), followed by repeating G_5 four times (for $a_5 = 4$)), i.e.

$$[\mathrm{B0}] = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix};$$

similarly, from those negative a_i, we get [B1] = (G_3 appears $|a_3| = |-1| = 1$ time, followed by repeating G_4 once (for $a_4 = 1$)), i.e.

$$[\mathrm{B1}] = G_4 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Step 5: [B0] and [B1] are now the desired output. (Stacking any 2 rows of [B1] always gets more 1s than stacking any 2 rows of [B0] does. Moreover, the number of 1s contained in each row of [B0] is identical to that contained in each row of [B1].)

□