國立交通大學

資訊科學與工程研究所

碩 士 論 文

在 PMC 模式下對星狀圖局部診斷能力之研究

Local Diagnosability of Star Network under PMC Model

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中 華 民 國 九 十 五 年 六 月

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Abstract

Under the PMC model, we introduce a new measure of diagnosability, called *local diagnosability*, and derive a structure for determining whether a node of a system is locally t-diagnosable. For star graph, we prove that the local diagnosability of each node is equal to its degree under the PMC model. Then, we propose a concept for system diagnosis, called strong local diagnosability property. A system $G = (V,E)$ is said to have a strong local diagnosability property, if the local diagnosability of each node is equal to its degree. We show that an n-dimensional star graph S_n has this strong property, $n \geq 3$. Next, we study the local diagnosability of a faulty star graph. We prove that S_n keeps this strong property even if it has up to $n-3$ faulty edges. Furthermore, we prove that S_n keeps this strong property no matter how many edges are faulty, provided that each node of a faulty star graph S_n is incident with at least two fault-free edges. Besides, we propose a new diagnosasis algorithm whose time complexity is $O(NlogN)$ where N is the number of nodes in a system.

Keywords **:local diagnosability, PMC model, strong local diagnosability property.**

在 PMC 模式下對星狀圖局部診斷能力之研究

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摘要

在PMC模式下面，我們介紹了一個新的診斷模式較做局部診斷能力，並且利用了一個結構來決定一個點的t-局部診斷能力。以星狀圖為例子來說，我們證明出在PMC模式下它的每個點的局部診斷能力和它的分支相同。於是我們提出了新的系統診斷觀念，叫做強局部診斷性質。一個系統若是其每點的局部診斷能力和它的分支相同則稱之具有強局部診斷性質。我們證明出星狀圖S_n在$n \geq 3$ 的時候有此性質，甚至在$n-3$個邊壞掉的情況下此性質仍然存在。此外，若是我們保持每個點都有兩個好邊的話，則無論壞多少邊性質還是會存在。最後我們提出了一個局部診斷演算法其時間複雜度是$O(NlogN)$，而N是代表一個系統內點的數目。

關鍵字：局部診斷能力、PMC 模式、強局部診斷性質。

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Chapter 1

Introduction

Recently, high-speed multiprocessor systems have become more and more popular in computer technology. The reliability of the processors in a system is significant since even few faulty processors may cause the system failure. Whenever processors are found faulty, we should replace the faulty ones with fault-free ones to maintain the reliability of the system. Identifying all the faulty processors of a system is called diagnosis of the system. The maximum number of faulty processors that can be ensured to be identified is called the *diagnosability* of the system. A system G is t-diagnosable if all the faulty processors can be precisely pointed out given that the number of faulty processors is at most t. The maximum number t for which G is t-diagnosable is called the *diagnosability* of G.

The problem of identifying faulty processors in a multiprocessor system has been widely studied in literatures [1] [10] [12]. The base of this area and the original diagnostic model were established by Preparata, Metze and Chien [1]. This model, known as the PMC model, has been extensively studied [1], [7], [8], [9]. In [13], Hakimi and Amin proved that a system is t-diagnosable if it is t-connected with at least $2t + 1$ nodes. They also gave a necessary and sufficient condition for verifying if a system is t-diagnosable under the PMC model.

Following the diagnosis model above, most previous studies focused on the diagnosisability of a system in a global sense, but ignored some local systematic details. A system is t-diagnosable if all the faulty processor can be identified whenever the number of faulty processors is at most t. However, it is possible to correctly point out all the faulty processors in a t-diagnosable system when the number of faulty processors is greater that t. For example, consider two arbitrary multiprocessor systems which are m-diagnosable and n-diagnosable, respectively, where m and n are integers and $m \gg n$. A new system can be generated by integrating these two systems with few communication links is some way. The diagnosability of this new system is upper bounded by n, but it is possible to identify all the faulty processors if the number of the faulty ones is between m and n. Thus, if only considering the global status, we lose some local details of the system.

In the paper, we aim to propose a new definition called *local dianosability*. There is a strong relationship between the local diagnosability and the traditional global one. For this local sense, we focus more on a single processor, and require only identifying the status of this particular processor correctly. More specifically, every processor in a system has its own local diagnosability. Under the PMC model, we propose a suffucient condition to determine the local diagnosability of a given processor x. For most practical multiprocessor systems, the number of links connecting to each processor is in the order of $\log N$, where N is the total number of processors. The time complexity of our algorithm to diagnose a given processor is bound by $O(\log N)$ and to diagnose all the faulty processors in a system with N processors is bound by $O(\log N)$ under the PMC model, provided that there is a substructure $H(x; n)$ we will introduce.

Chapter 2

Preliminaries

2.1 Star graph

An n-dimensional star graph, also referred to as an n-star or S_n, is an undirected graph consisting of n! nodes and $(n-1)n!/2$ edges [5]. Each node is uniquely assigned a label $a_1a_2\ldots a_m\ldots a_n$, which is a distinct permutation of the set of symbols $\{a_1, a_2, \ldots, a_n\}$. Without loss of generality, let this symbol set be the set of integers $\{1, 2, \ldots, n\}$. Two nodes are linked by an edge labeled i if and only if the label of one node can be obtained from the label of another node by interchanging the first symbol with the i th symbol, $2 \leq i \leq n$. For example, in a $4-star$ containing 4! nodes, two nodes 1234 and 4231 are neighbors and joined through an edge labeled 4. In S_n each node is connected to $n-1$ neighbors by $n-1$ edges. Each S_n can be decomposed into $n(n-1)$-dimensional star graph S_{n-1}. We denote the $(n-1)$-dimensional star graph by S^i_{n-1} whose n th symbol is i, for $0 \leq i \leq n-1$. A 4-star graph S_4 is shown in Fig 2.1.

Lemma 1 *[14] There are no* 3*-cycles and* 4*-cycles in a star graph.*

Figure 2.1: The 4-star graph S_4

2.2 The PMC Model

For all of the *multi-processor systems*, we wish we could search all the faulty nodes in the *multi-processor system* efficiently. Several diagnosis models were proposed to identify those faulty nodes in the multi-processor system. *PMC* model is one of them provided with the self-diagnosed ability. Every node in the system can send a signal to each of his neighbors, and receive the return signal from them. For instance, there is an edge (u, v) implies that u can test v by checking the response send by v, and v also can test u. After the testing, the result is either 0 or 1 of u testing v. We assume that u is fault-free, then the result would be 0 if v is fault-free. Otherwise, the result is 1. However, the result is undependable if u is a originally faulty node. Here are all possible results of u test v in Table 2.1.

Figure 2.2: $r(u, v)$ is the result of u testing v

After the whole process, we have a binary sequence called *syndrome*. The first thing we want to ask is how to find out the faulty nodes from the *syndrome*. According to previous researches, we can make it if there are not so many faulty nodes in the system.

Theorem 1 *For every two distinct subsets of nodes* F_1 *and* F_2, (F_1, F_2) *is a distinguishable pair if and only if at least one of the following conditions is satisfied (as illustrated in Fig. 1):*

$\exists u \in V - F_1 - F_2$ *and* $\exists v \in F_1 \Delta F_2$ *such that* $(u, v) \in E$.

Figure 2.3: illustration of Theorem 1 - the distinguishability of two distinct subsets of nodes.

Theorem 2 *A system with N nodes is t-diagnosable if*

1) $N \geq 2t + 1$,

2) each node has order at least t, and

3) for each $U \subset V$ *such that* $\mid U \mid = N - 2t + p$ *and* $0 \leq p \leq t - 1$, $\mid T(G, U) \mid > p$.

Chapter 3

Local diagnosability

The traditional diagnosability of a system is discussed in a global point of view. We can also call it *global diagnosabiliy*. There were some researches which neglected the local viewpoint discussing the *global diagnosability* on well-known systems. In this paper, we propose a new definition called *local diagnosability* emphasizing the diagnosability of a node. For example, for any two integer m and n with $m \gg n \geq 4$, the diagnosability of two hypercube Q_m and Q_n is m and n, respectively. However, the diagnosabiliy of the graph which is combined by Q_m and Q_n is n at most. We observe that the diagnosability of a integral system is the worst cast of two previous systems. It is desirable to take notice of some local part or one single node of a system, where the local diagnosability of the node is not as low as the global diagnosability of the whole system.

Definition 1 *A system $G(V, E)$ is locally t-diagnosable at node $x \in V(G)$ if, given a test syndrome σ_F produced by the system under the presence of a set of faulty nodes F containing node x with $|F| \leq t$, every set of faulty nodes F' consistent with σ_F and $|F'| \leq t$, must also contain node x.*

As expected, the main purpose of the following property is to identify whether a system is locally t-diagnosable at a particular node.

Proposition 1 *A system $G(V, E)$ is locally t-diagnosable at node $x \in V(G)$ if, for each pair of distinct sets $F_1, F_2 \subset V(G)$ such that $F_1 \neq F_2$, $|F_1|, |F_2| \leq t$, and $x \in F_1 \Delta F_2$, (F_1, F_2) is a distinguishable pair.*

Then, we define the *local diagnosability* of a given node as follows.

Definition 2 *The local diagnosability $t_l(x)$ of a node $x \in V(G)$ in a system $G(V, E)$ is defined to be the maximum number of t for G being locally t-diagnosable at x, that is,*

$$t_l(x) = max\{t \mid G\ is\ locally\ t - diagnosable\ at\ x\}.$$

The concept of a system being local diagnosability is consistent with the global one. The relationship between them is as follows.

Proposition 2 *A system $G(V, E)$ is t-diagnosable if and only if G is locally t-diagnosable at x, for every $x \in V(G)$.*

Obviously the local diagnosability is discussed in much more detail than the global one. It is not enough that only the global diagnosability of the whole system is decided, while the local diagnosability of every single node can be observed. In the following, we propose a condition to verify whether a system G is locally t-diagnosable at a given node x.

Theorem 3 *A system $G(V, E)$ is locally t-diagnosable at a given node $x \in V(G)$ if, for every set of nodes $S \subset V(G)$, $|S| = p$, $0 \leq p \leq t-1$, and $x \notin S$, the component C_x of $G - S$ contains x satisfying $|V(C_x)| \geq 2(t-p)+1$*

Proof. To prove that $|V(C_x)| \geq 2(t-p)+1$ is necessary, we show this by contradiction. That is, $|V(C_x)| \leq 2(t-p)$. We then arbitrarily partition C_x into two disjoin subsets, $C_x = A_1 \cup A_2$ with $|A_1| \leq t-p$ and $|A_2| \leq t-p$. Let $F_1 = A_1 \cup S$ and $F_2 = A_2 \cup S$. Then, $|F_1| < t$ and $|F_2| < t$. It is clear that there is no edge between $V - (F_1 \cup F_2)$ and $F_1 \Delta F_2$. By Proposition 1, F_1 and F_2 are indistinguishable. This contradicts the assumption that G is locally t-diagnosable at a given node $x \in V(G)$.

Furthermore, on the contrary, that G is not t-diagnosable at $x \in V(G)$. That is, there exists an indistinguishable pair (F_1, F_2) with $F_1 \leq t$ and $F_2 \leq t$, and $x \in F_1 \Delta F_2$. By proposition 1, there is no edge between $V-(F_1 \cup F_2)$ and $F_1 \Delta F_2$. Let $S = F_1 \cap F_2$. Thus, in $G-S$, $F_1 \Delta F_2$ is disconnected from from other parts. We observe that $|F_1 \Delta F_2| \leq 2(t-p)$, where $|S| = p$ and $0 \leq p \leq t-1$. Therefore, there is at least one component C_x of $G-S$ contains x with $V(C_x) \leq 2(t-p)$, which is a contradiction. This completes the proof of the theorem.

Though the theorem given above seems very strong, it is not useful to calculate the local diagnosability of a node in a system. We propose a effective algorithm which has time complexity $O(N \log N)$ is better than the previous result. First of all, we introduce a structure that we will use frequently.

Definition 3 *Let x be a node in a graph $G(V, E)$. For $n \leq deg_G(x)$, an substructure $H(x; n)$ of order n at node x is defined as $H(x; n) = (V(x; n), E(x; n))$, where the set*

Figure 3.1: subgraph $H(x;n)$ of G of order n at node x.

of nodes $V(x;n) = \{x\} \cup \{v_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq 2\}$ *and the set of edges* $E(x;n) = \{(x, v_{k1}), (v_{k1}, v_{k2}) \mid 1 \leq k \leq n\}$.

The structure is common in many well-known systems like hypercube and star graph. We say that there is a substructure $H(x;n) \subseteq G$ at node x if G contains $H(x;n)$ of order n at node x as a subgraph.

Theorem 4 *Let x be a node in a system $G(V,E)$. The local diagnosability of x is at least n if there exists $H(x;n) \subseteq G$ at x.*

Proof. We use Theorem 3 to prove this result. It is clear that $|V(H(x;n))| \geq 2n+1$. $H(x;n)$ will lose one nodes when v_{k2} is removed but two nodes when v_{k2} is removed, for $1 \leq k \leq n$. It means $H(x;n)$ will lose two nodes at most when a node which is not x is removed. In other words, for any set of nodes $S \subset V(G)$, $|S| = p$, $0 \leq p \leq n-1$, and $x \notin S$. After S is removed, the number of nodes of $H(x;n)$ is not less than $2n+1-2p = 2(n-p)+1$. By Theorem 3, the local diagnosability of x is at least n.

Proposition 3 *Let x be a node in a system $G(V,E)$ with $deg_G(x) = n$. The local diagnosability of x is at most n.*

Proof. We prove it by contradiction. That is, the local diagnosability of x is $n+1$ at least. Consider two sets of nodes S_1 and S_2, where S_1 contains node x and all neighbors of x, and S_2 contains all neighbors of x. There is no edge between $G - S_1 \cup S_2$ and $S_1 \Delta S_2$. We can see that S_1 and S_2 are indistinguishable since $|S_1| = n + 1$ and $|S_2| = n$, which is a contradiction. Therefore, the proposition is hold.

By Theorem 4 and Proposition 3, we have the following result.

Theorem 5 *Let x be a node in a system $G(V, E)$ with $deg_G(x) = n$. The local diagnosability of x is n if there exists an substructure $H(x; n) \subseteq G$ at x.*

Theorem 5 is more useful than Theorem 3 to estimate the local diagnosability of a node in a system, while the substructure $H(x; n)$ exists. However, the substructure $H(x; n)$ does exist in most well-known systems even the systems lose some links. This is very comforting that we can easily estimate the local diagnosability of many multiprocessor systems and interconnection networks. In the paper, we show that the diagnosability of every node in star graph S_n is equal to its degree since missing links are not greater than $n - 3$.

Definition 4 *Let $G = (V, E)$ be a graph and $v \in V$ be a vertex. The vertex v has the strong local diagnosability property if the following condition holds:*

$t_1(v) = deg(v)$

Definition 5 *Let $G(V, E)$ be a graph. G has the strong local diagnosability property if the following condition holds:*

$\forall v \in V(G)$, the vertex v has the strong local diagnosability property.

By Definition 5 and Definition 6, we have the following proposition.

Theorem 6 *A n-star-structured system S_n has the strong local diagnosability property, $n \geq 3$.*

Proof. We prove this by induction on n. Since an n-dimensional star graph S_n is vertex-symmetric, we can concentrate on the construction of $H(x; n-1)$ structure at a given node x. For $n = 3, deg(v) = 2$ and it is clear that S_3 contains $H(x; n-1)$ of order 2 at node x. As the inductive hypothesis, we assume that S_{n-1} contains $H(x; n-2)$ of order $n-2$ at each node, for some $n \geq 4$. Now we consider S_n, S_n can be decomposed into $(n-1)$ subcomponents S^0_{n-1}, S^1_{n-1}, ..., and S^{n-1}_{n-1} by some dimension. Without loss of generality, we may assume that the node $x \in S^0_{n-1}$. By the inductive hypothesis, S^0_{n-1} contains $H(x; n-2)$ of order $n-2$ at node x. There exists a node $x^{(1)}$ n another subcomponent which is the neighbor of x i. Node $x^{(1)}$ has an adjacent neighbor that is its subcomponent due to $deg(x^{(1)}) = n$, where $n \geq 3$. Thus, S_n contains $H(x; n-1)$ of order $n-1$ at node x. By Theorem 5, Definition 4 and Definition 5, S_n has the strong local diagnosability property.

Theorem 7 *Let S_n be an n-dimensional star graph with $n \geq 3$, and $S \subseteq E(S_n)$ be a set of edges, $0 \leq |S| \leq n-2$. Removing all the edges in S from S_n, the local diagnosability of each node is still equal to its remaining degree.*

Proof. We prove this by induction on n. For $n = 4, 0 \leq |S| \leq 1$, if $|S| = 0$, it is clear that S_4 contains $H(x; 3)$ of order 3 at every node. It is a routine work to see that every node has a structure $H(x; k)$ of order k at it, where k is the remaining degree of the node. As the inductive hypothesis, we assume that the result is true for $S_{n-1}, 0 \leq |S| \leq (n-2)-2$, for some $n \geq 4$. Now we consider $S_n, 0 \leq |S| \leq n-2$. If $|S| = 0$, refer to the proof of Theorem 6, S_n contains a structure $H(x; n-1)$ of order $n-1$ at every node. If $1 \leq |S| \leq n-2$, we have two cases to discuss.

Case 1: $S \subseteq S^q_{n-1}$, for some $0 \leq q \leq n-1$. It means that S is totally in a $n-1$-dimensional star graph component and other components are fault-free. Without loss of generality, we assume that $S \subseteq S^0_{n-1}$. Let x be a node in S^0_{n-1}. Consider $H(x; n-1) = (V(x; n-1), E(x; n-1))$, where the set of nodes $V(x; n-1) = \{x\} \cup \{v_{ij} \mid 1 \leq i \leq n-1, 1 \leq j \leq 2\}$ and the set of edges $E(x; n-1) = \{(x, v_{k1}), (v_{k1}, v_{k2}) \mid 1 \leq k \leq n-1\}$. If there are q edges of $N = \{(x, v_{k1}) \mid 1 \leq k \leq n-1\}$ which are removed, for $1 \leq q \leq n-3$, $H(x; n-1-q)$ still exist. It means that we don't have to care about the edges which are subset of N. Let M be the set $\{(v_{k1}, v_{k2}) \mid 1 \leq k \leq n-1\}$, and let $R \subseteq M$ be another set, $1 \leq |R| \leq n-3$. If R is removed, we do care about if $H(x; b)$ is destroyed or not, for some $1 \leq b \leq n-3$. Without loss of generality, R contains (v_{11}, v_{12}) and (v_{21}, v_{22}). After removing (v_{11}, v_{12}) and (v_{21}, v_{22}), there exists one node i adjacent v_{11} in S^c_{n-1}, and j adjacent v_{11} in S^d_{n-1}, for $1 \leq c \leq n-1$, $1 \leq c \leq n-1$. By lemma xx, there is no 4 cycle in star graph, so $i \neq j$. Thus, the structure $H(x; b)$ is hold.

Case 2: Otherwise, let the the number of missing edges in each S^i_{n-1} is $M_i, 0 \leq M_i \leq n-4, i \leq 0 \leq n-1$.Without loss of generality, consider the node d in S^0_{n-1}, and we ignore the edges which are adjacent to another S_{n-1} component. By the inductive hypothesis, there exists $H(d; q)$ in S^0_{n-1}, $1 \leq q \leq n-2$. If there is no node in S^i_{n-1} adjacent to d, $1 \leq i \leq n-1$, the local diagnosability of d is equal to its degree.Otherwise, there is a node b in $S^i n-1$ adjacent to d, for some $1 \leq i \leq n-1$. Without loss of generality, d is in S^1_{n-1} and $0 \leq M_1 \leq n-4$. We can sure that there are at least two nodes adjacent b.Thus, $H(x; q+1)$ does exist and the local diagnosability of d is equal to its degree.This completes the proof of the theorem.

Corollary 1 *Let S_n be an n-dimensional hypercube with $n \leq 3$, and $S \subset E(S_n)$ be a set of edges, $0 \leq |S| \leq n-2$. Then $S_n - S$ has the strong local diagnosability property.*

Chapter 4

Conditional Fault Local Diagnosability

In previous section, we know that S_n does not have the strong local diagnosability porperty, if there are $n-1$ faulty edges, all these faulty edges are incident with a single vertex and this vertex is incident with only fault-free edge. Therefore, we are led to the following question: How many edges can be removed from S_n such that S_n keeps the strong local diagnosability property under the conditional that each vertex of the faulty star graph S_n is incident with at least two fault-free edges?

Theorem 8 *Let S_n be an n-dimensional hypercube with $n \geq 3$, and $S \subseteq E(S_n)$ be a set of edges. Assume that each vertex of $S_n - S$ is incident with at least two fault-free edges. Removing all the edges in S from S_n, the local diagnosability of each vertex is still equal to its remaining degree.*

Proof. For any node x in S_n, we assume the degree of x is g and all its neighbors are denoted by $\{a_1, a_2, ..., a_g\}$. Each of a_i has at last two neighbors, for $1 \leq i \leq g$. Let b_i be the neighbor of a_i, and $b_i \neq x$. By lemma 1, we can sure that b_p and b_q have no common neighbor and $b_p \neq b_q$, for $1 \leq p \leq g$, $1 \leq q \leq g$, $p \neq q$. As the result, there must exist $H(x; g)$ in S_n. Therefore, the diagnosability of every single node is equal to its remaining degree.

Chapter 5

A Diagnosis Algorithm

In the section, we aim to propose a diagnosis algorithm whose time complexity is $O(N \log N)$, where N is the number of nodes. Given a substructure $H(x;n)$ at node x, we shall present a diagnosis algorithm to determine whether this node is faulty or not for a given syndrome under the PMC model. Let $H(x;n)$ be the substructure at a given node x in $V(G)$, the diagnosing signal are sent back and forth inside $H(x;n)$. Since there are communication links between x and v_{k1}, v_{k1} and v_{k2}, for all $1 \leq k \leq n$. Given a substructure $H(x;n)$ at a node x, we define $r_k = (r^1, r^2)$, where r^1 is the result of v_{k1} testing x and r^2 is the result of v_{k2} testing v_{k1}. Then, r_k can be in one of the four different states which are $r(0) = (0,0)$, $r(1) = (0,1)$, $r(2) = (1,0)$, $r(2) = (1,1)$. Let $R(i)$ be the set of collection of all $r(i)$, for all $0 \leq i \leq 3$. Obviously, the summation of the cardinality of $R(0)$ to $R(3)$ is n, that is, $\sum_{i=0}^{3} |R(i)| = n$.

Let x be a node in a system. Suppose that the degree of x is n and suppose that there is an substructure $H(x;n)$ at x. Then the local diagnosability of x is n, which means we maybe not be able to identify all the faulty nodes, if the number of faulty nodes in $H(x;n)$ is $(n+1)$ or more. Therefore, we assume that the number of faulty nodes is at most n. Under this assumption, we have an efficient algorithm to determine whether node x is faulty or not.

Theorem 9 *Let x be a node with degree n in a system $G = (V, E)$. Suppose that there is an substructure $H(x; n) \subseteq G$ at x. Define $r_k = (r^1, r^2)$ to be the testing result, where r^1 is the result of v_{k1} testing x and r^2 is the result of v_{k2} testing v_{k1}. Then, r_k can be in one of the four states (as illustrated in Fig):*

$r(0) = (0, 0)$, $r(1) = (0, 1)$, $r(2) = (1, 0)$, $r(3) = (1, 1)$

Let $R(i)$ be the set of the collection of all $r(i)$, and $|R(i)|$ be the cardinality of $R(i)$. Then, under the assumption that the number of faulty nodes is at most n,

i) x is fault-free, if$|R(0)| \geq |R(2)|$; or,

ii) x is faulty, if$|R(0)| < |R(2)|$

Figure 5.1: four different output states

Proof. Let $l_k = (v_{k1}, v_{k2})$ be an ordered double, $1 \leq k \leq n$, with respect to $H(x;n)$. We prove the first part of this theorem by contradiction. Suppose that the number of faulty nodes in $H(x;n)$ is at most n and suppose that x is faulty, the counting of all the other faulty nodes is as follows:

For those l_k with result $r(0)$, there are at least 2 faulty nodes which are v_{k1}, v_{k2}.

For those l_k with result $r(1)$, there is at least 1 faulty node which is v_{k1}.

For those l_k with result $r(2)$, the number of faulty nodes is uncertain.

For those l_k with result $r(3)$, there is at least 1 faulty node which is either v_{k1} or v_{k2}.

Thus, the number of faulty nodes is at least

$1 + 2|R(0)| + |R(1)| + |R(3)| = \sum_{i=0}^{3} |R(i)| + (1 + |R(0)| - |R(2)|)$

By the assumption that $|R(0)| \leq |R(2)|$, the number of faulty nodes is strictly more than $\sum_{i=0}^{3} |R(i)|$ which is equal to n. This contradicts to the assumption that the number of faulty nodes in $H(x;n)$ is at most n. Therefore, x is fault-free.

Now, we prove the second part of the theorem. Suppose that the number of faulty nodes in $H(x;n)$ is at most n and suppose that x is fault-free, the counting of all the other faulty nodes is as follows:

For those l_k with result $r(0)$, the number of faulty nodes is uncertain.

For those l_k with result $r(1)$, there is at least 1 faulty node which is either v_{k1} or v_{k2}.

For those l_k with result $r(2)$, there are at least 2 faulty nodes where are v_{k1} and v_{k2}.

For those l_k with result $r(3)$, there is at least 1 faulty nodes where is v_{k1}.

Thus, the number of faulty nodes is at least

$$|R(1)| + 2|R(2)| + |R(3)| = \sum_{i=0}^{3} |R(i)| + (|R(2)| - |R(0)|)$$

By the assumption that $|R(0)| < |R(2)|$, the number of faulty nodes is larger than $\sum_{i=0}^{3} |R(i)|$ which is equal to n. This contradicts to the assumption that the number of faulty nodes in $H(x; n)$ is at most n. Therefore, x is faulty.

Chapter 6

Conclusions

The issue of identifying all the faulty processors is important in the design of interconnection networks or multiprocessor systems, which is implementable is very large scale integration (VLSI) or wafer-scale integration (WSI). The process of identifying all the faulty processors is called diagnosis of a system.

In this paper, we propose a new concept of local diagnosability for a system and derive a structure for determining whether a system is locally t-diagnosable at a given node. Through this concept, the diagnosability of a system can be determined by computing the local diagnosability of each node. We also introduce a concept for system diagnosis, called strong local diagnosability property. Then, we prove that the star graph has this strong property. Next, we consider a faulty star graph S_n with $n \geq 3$. We prove that S_n keeps this strong property even if it has up to $n-3$ edges. Furthermore, we prove that S_n keeps this strong property no matter how many edges are faulty, provided that each node of a faulty star graph S_n is incident with at least two fault-free edge.Finally, we propose a local diagnosis algorithm whose time complexity is $O(N \log N)$ where N is the number of nodes in a system.

There are several different fault diagnosis model in the area of diagnosability. It is worth investigating, under various models, whether a system has this strong local diagnosability property after removing some edges. It is also an attractive work to develop more different measures or diagnosability based on network reliability, network topology, application environment and statistics related to fault patterns.

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