國立交通大學

資訊科學與工程研究所

碩 士 論 文

在 PMC 模式下對 k 元 n 維立方體局部診斷能力之研究

Local Diagnosability of k-ary n-cube Networks under the PMC Model

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中 華 民 國 九 十 五 年 六 月

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碩 士 論 文

A Thesis
Submitted to Institute of Computer Science and Engineering
College of Computer Science
National Chiao Tung University
in partial Fulfillment of the Requirements
for the Degree of
Master
in

Computer Science

June 2006

Hsinchu, Taiwan, Republic of China

中華民國九十五年六月

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摘要

在這篇論文裡，我們介紹了一個新的衡量錯誤診斷能力的方法，稱為局部診斷能力。依據不同的觀點，來做不同的詮釋研究，從原本廣域的觀點轉換成局部的觀點來觀察。在新的觀點下，我們展示了一個簡單的方法來診斷一個多處理機系統。我們把局部診斷能力的研究應用在連線無損壞的 k 元 n 維立方體上，並觀察得每個點的價數(degree)即為其局部診斷能力。接著我們把局部診斷能力的研究應用在有任意連線損壞的 k 元 n 維立方體上。根據我們在這篇論文裡的証明，在任意的損壞連線數不超過 2n−2 個數時，每點的局部診斷能力仍為每點的價數。此外，我們提出了一個更為有效率的演算法來診斷錯誤的發生。

關鍵字：診斷能力，局部診斷能力，k 元 n 維立方體，價數。

Local Diagnosability of k-ary n-cube Networks under the PMC Model

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Abstract

In this thesis, we introduce a new measure for diagnosability, called local diagnosability, by changing the original global viewpoint to a local viewpoint. With this new viewpoint, we yield an easy way to diagnose a multiprocessor system. We apply the concept of local diagnosability to k-ary n-cube with no missing links and the local diagnosability of each node is exactly the degree of each node. Then we investigate the local diagnosability of k-ary n-cube with arbitrarily distributed missing links. Based on the result proved in this thesis, the number of missing links can be up to $2n-2$ and the local diagnosability of each node is the remaining degree of each node. Moreover, we propose a more efficient diagnosis algorithm.

Keywords : diagnosability, local diagnosability, k-ary n-cube, degree.

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Chapter 1

Introduction

As the rapid development of digital technology, the architecture of multiprocessor system has become more and more complex today, and often with a large number of processors (nodes). It is important to maintain the reliability of such system. Therefore, fault diagnosis has become an important issue in the design of multiprocessor systems.

It is impractical to test each processor (node) individually in a large multiprocessor system, when there are faulty processors (nodes). Preparata et al. [1] first introduced a fault diagnosis model (called *PMC model*) for system level diagnosis. This is much more efficient than testing one node by one node. Also several different models have been proposed in the literature [2], [3].

Hakimi and Amin [4] proved that a multiprocessor system is t-diagnosable if it is t-connected with at least $2t + 1$ nodes. Besides, a necessary and sufficient condition was given for verifying if a system is t-diagnosable under the PMC model.

In this thesis, we adopt PMC model as the fault diagnosis model and proposed a new measure of diagnosability, called local diagnosability. Besides we investigate the local diagnosability of *k-ary n-cube* [5] with no missing links and learn that the local diagnosability of each node is exactly the degree of each node, and the local diagnosability of *k-ary n-cube* with arbitrarily distributed missing links, as the number of missing links is not exceed $2n-2$, the local diagnosability of each node is the remaining degree of each node.

The rest of this thesis can be categorized as follows: Section 2 provides terminology and preliminaries. Section 3 introduces the concept of local diagnosability and defines the local t-diagnosability of a system. We then study the local diagnosability of *k-ary n-cube* (Q_n^k) in Section 4. Moreover, we propose a fault diagnosis algorithm in Section 5. Finally, our conclusions are given in Section 6.

Chapter 2

Preliminaries

2.1 Terminology and Preliminaries

The architecture of an interconnection network can be represented as a graph in which the nodes correspond to processors and the edges to communication links.

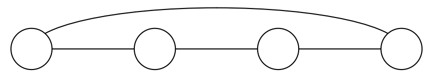
Let $G = (V, E)$ be a *graph* if V is a finite set and E is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. We say that V is the *vertex set* and E is the *edge set*. Two vertices u and v are *adjacent* if (u, v) is an edge of G. The *neighborhood* of u, denote by $N(u)$, is $\{v \mid (u, v) \in E\}$. The *degree* $deg(u)$ of a vertex u of G is the number of edges incident with u. The components of a graph G are its maximal connected subgraphs. A component is trivial if it has no edges; otherwise, it is nontrivial. The set of nodes in component C is denoted by V_c and C_x is the component which contains node x. For a set $F \subset E$, the notation $G - F$ represents the graph obtained by removing the edges F from G.

There are many mutually conflicting requirements in designing the topology for computer networks. The n-cube is one of the most popular topologies [6], and the class of *k-ary* *n-cubes* is another commonly used interconnection topology for parallel and distributed systems. In this thesis, we study the local diagnosability of *k-ary n-cube*.

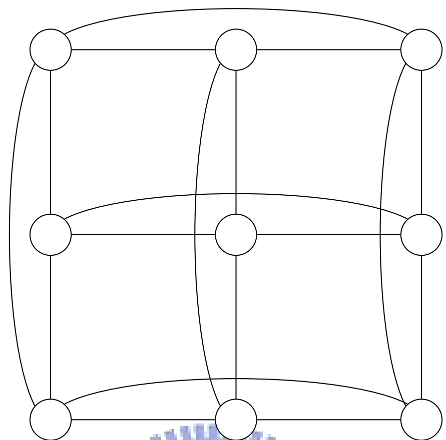
2.2 Basic properties of Q_n^k

The *k-ary n-cube* Q_n^k is a $2n$ regular graph consists of $N = k^n$ nodes, and is highly symmetric. Each node has the form $X = x_{n-1}, x_{n-2}, ..., x_0$, where $0 \le x_i \le k-1$, for all $0 \le i \le n-1$. Two nodes $X = x_{n-1}, x_{n-2}, ..., x_0$ and $Y = y_{n-1}, y_{n-2}, ..., y_0$ are interconnected if and only if there exists an i, $0 \le i \le n-1$, such that $x_i = y_i \pm 1$ (mod k) and $x_j = y_j$, for $i \neq j$. Figure 2.1. and Figure 2.2. shows some examples of the *k-ary n-cubes*.

A k-ary n-cube Q_n^k can be decomposed into k copies of Q_{n-1}^k or in general k^β copies of $Q_{n-\beta}^k$ subcubes for all $\beta \le n$. If we refer to $d^*(x, y) \in E(Q_n^k)$ where x differs from y in the dth position of bitwise representation, for $0 \le d \le n-1$, we then have an edge of dimension d. We say that Q_n^k is divided into k copies of subgraph, $Q_n^k[0]$, $Q_n^k[1]$, ..., $Q_n^k[k-1]$ (abbreviated as $Q[0]$, $Q[1]$, ..., $Q[k-1]$, if there are no ambiguities), along dimension d for some $0 \le d \le n-1$, and the edges loop around these subgraphs above is the so called, *dimension edges*. The bitwise represent of $Q_n^k[l]$ is labeled by $x_{n-1}...x_{d+1}lx_{d-1}...x_0$, for every $0 \le l \le k-1$ (see Figure 2.3). It is clear that each $Q_n^k[l]$ is isomorphic to Q_{n-1}^k for $0 \le l \le k-1$. As a result, there are n ways that a Q_n^k can be divided into k copies of Q_{n-1}^k along n different dimensions.



(a)



(b)

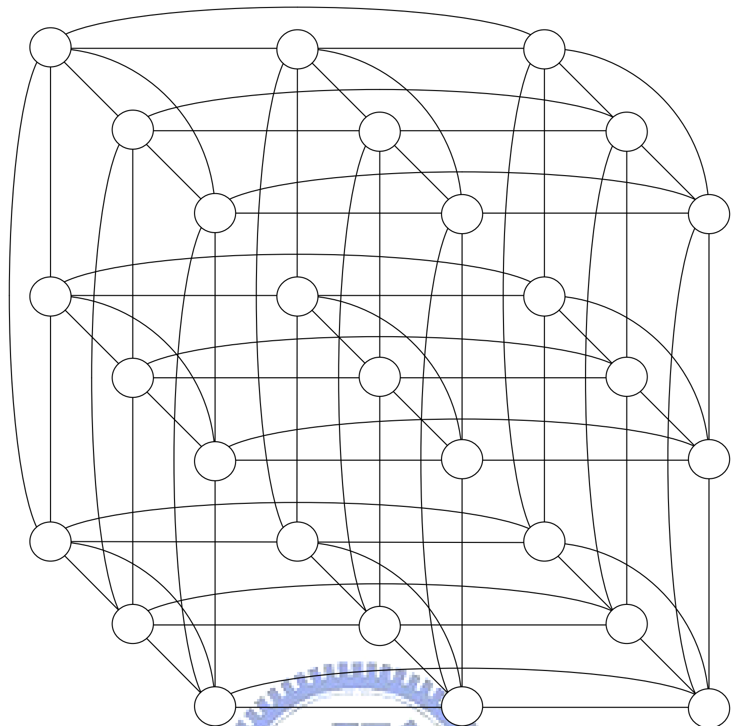
Figure 2.1: Examples of *k-ary n-cubes*; (a) Q_1^4 and (b) Q_2^3.

2.3 The PMC Model

In the study of multiprocessor systems, there are several different models of self-diagnosis, the PMC Model [1] was adopted each vertex (node) will test all its neighboring vertices (neighboring nodes) and it is assumed that there is no vertex tested by itself.

Definition 1 *Under the PMC model, a syndrome σ for system G is defined as follows: for any two distinct adjacent vertices u and v,*

$$\sigma(u,v) = \begin{cases} 0, & \text{if } v \text{ is tested by } u, \text{ that } u \text{ is fault-free and } v \text{ is fault-free.} \\ 1, & \text{if } v \text{ is tested by } u, \text{ that } u \text{ is fault-free and } v \text{ is faulty.} \\ 0/1, & \text{if } v \text{ is tested by } u, \text{ that } u \text{ is faulty.} \end{cases}$$



(c)

Figure 2.2: An example of a *k-ary n-cubes;* (*c*) Q_3^3 .

Let $\sigma(F)$ represent the set of all syndromes which could be produced if F is the set of faulty vertices.

Definition 2 *Two distinct sets* $F_1, F_2 \subset V$ *are said to be indistinguishable if* $\sigma(F_1) \bigcap \sigma(F_2) \neq \emptyset$ *; otherwise,* F_1, F_2 *are said to be distinguishable. We say* (F_1, F_2) *is an indistinguishable pair if* $\sigma(F_1) \bigcap \sigma(F_2) \neq \emptyset$*, else,* (F_1, F_2) *is a distinguishable pair.*

Definition 3 *[1] A system of n units is t-diagnosable if all faulty units can be identified without replacement, provided that the number of faults presented does not exceed t.*

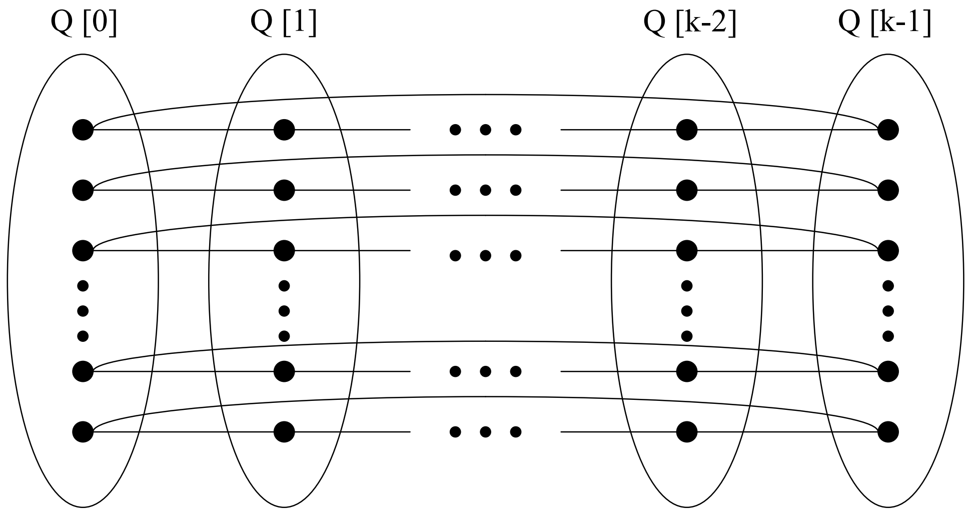


Figure 2.3: Q_n^k is divided into $Q[0]$, $Q[1]$, ..., $Q[k-1]$.

Let $F_1, F_2 \subset V$ be two distinct sets and let the symmetric difference $F_1 \triangle F_2 = (F_1 - F_2) \bigcup (F_2 - F_1)$. $|F_1|$ represent the number of nodes in F_1.

Lemma 1 *[7] A system $G(V, E)$ is t-diagnosable under PMC model if and only if for each pair F_1, $F_2 \subset V$ with $|F_1|$, $|F_2| \leq t$ and $F_1 \neq F_2$, there is at least one test from $V - (F_1 \bigcup F_2)$ to $F_1 \triangle F_2$.*

Lemma 2 *[9] For any two distinct sets $F_1, F_2 \subset V$, (F_1, F_2) is a distinguishable pair if and only if there exists a vertex $u \in V - (F_1 \bigcup F_2)$ and there exists a vertex $v \in F_1 \triangle F_2$ such that $(u, v) \in E$ (see Figure 2.4).*

Theorem 1 *[9] Let $G(V, E)$ be the graph of a system G. Then, G is t-diagnosable if and only if, for each vertex set $S \subset V$ with $|S| = p$, $0 \leq p \leq t - 1$, every component C of $G - S$ satisfies $|V_C| \geq 2(t - p) + 1$.*

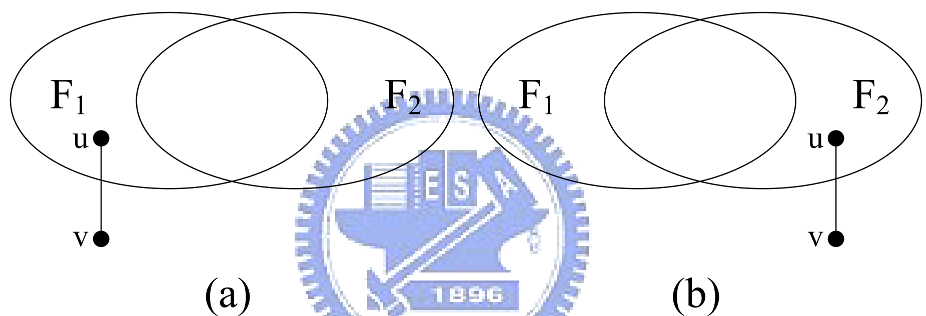


Figure 2.4: Illustrations of a distinguishable pair (F_1, F_2).

Chapter 3

Locally t-diagnosable

3.1 Local Diagnosability

Definition 4 *A system of n units is locally t-diagnosable at vertex x, F is the set of all faulty units, x can be identified without replacement, provided that the number of faults presented does not exceed t.*

Lemma 3 *A system $G(V, E)$ is locally t-diagnosable at vertex x under PMC model if and only if for each pair F_1, $F_2 \subset V$ with $|F_1|$, $|F_2| \leq t$ and $F_1 \neq F_2$, and $x \in F_1 \triangle F_2$, there is at least one test from $V - (F_1 \bigcup F_2)$ to $F_1 \triangle F_2$.*

Lemma 4 *For any two distinct sets $F_1, F_2 \subset V$, (F_1, F_2) is a local distinguishable pair if and only if there exists a vertex $u \in V - (F_1 \bigcup F_2)$, a vertex $v \in F_1 \triangle F_2$ and a vertex $x \in F_1 \triangle F_2$ such that $(u, v) \in E$ (see Figure 3.1).*

Theorem 2 *Let $G(V, E)$ be the graph of a system G. Then, G is locally t-diagnosable at vertex x if and only if, for each vertex set $S \subset V$ with $|S| = p$, $0 \leq p \leq t - 1$, every component C_x of $G - S$ satisfies $|V_{C_x}| \geq 2(t - p) + 1$.*

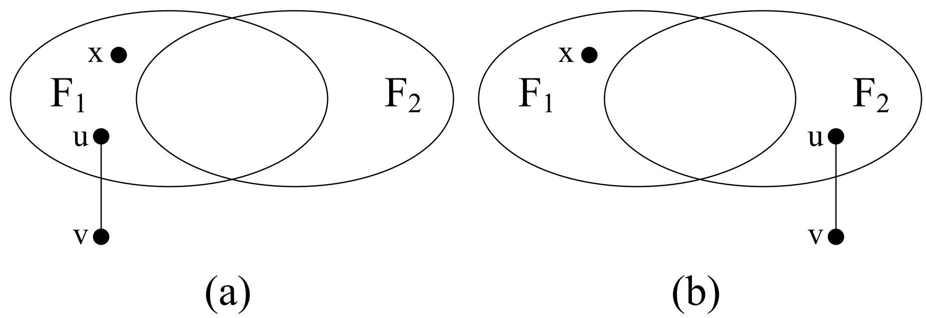


Figure 3.1: Illustrations of a local distinguishable pair (F_1, F_2).

Proof. We prove $|V_{C_x}| \geq 2(t-p)+1$ is necessary by contradiction. Suppose that there exists a set of vertices $S \subset V$ with $|S| = p$, $0 \leq p \leq t-1$ and $x \notin S$ such that, in graph G-S, the connected component which x belongs to has strictly less than $2(t-p)+1$ vertices. Let C_x be such a component with $|V_{C_x}| \leq 2(t-p)$. We then arbitrarily partition V_{C_x} into two disjoint subsets, $V_{C_x} = A_1 \bigcup A_2$ with $|A_1| \leq t-p$ and $|A_2| \leq t-p$. Let $F_1 = A_1 \bigcup S$ and $F_2 = A_2 \bigcup S$. Then $|F_1| \leq t$ and $|F_2| \leq t$. It is clear that there is no edge between $V - (F_1 \bigcup F_2)$ and $F_1 \triangle F_2$. By Lemma 4, F_1 and F_2 are indistinguishable and $x \in F_1 \triangle F_2$. This contradicts to the assumption that G is locally t-diagnosable at vertex x.

To prove the sufficiency, suppose on the contrary, that G is not locally t-diagnosable at vertex x, i.e., there exists an indistinguishable pair (F_1, F_2) with $|F_i| \leq t$, $i = 1, 2$ and $x \in F_1 \triangle F_2$. By Lemma 4, there is no edge between $V - (F_1 \bigcup F_2)$ and $F_1 \triangle F_2$. Let $S = F_1 \bigcap F_2$. Thus, in $G - S$, $F_1 \triangle F_2$ is disconnected from other parts. We observe that

$|F_1 \triangle F_2| \leq 2(t-p)$, where $|S| = p$ and $0 \leq p \leq t-1$. Therefore, there is at least one component C_x of $G-S$ with $|V_{C_x}| \leq 2(t-p)$, which is a contradiction. This completes the proof of the theorem.

A Type I graph at vertex x is defined by every neighboring node of x must has a different good neighbor to other neighboring nodes of x, (see Figure 3.2).

Theorem 3 *Let $G(V,E)$ be the graph of a system G. Then, G is locally t-diagnosable at vertex x, if there exists a Type I subgraph.*

Proof. In highly structure, the total number of nodes is $2t+1$, t is the degree of node x ($deg(x) = t$), and $|V_{C_x}| \geq 2t+1$. Each time we remove one node (not include node x) in the highly structure, the number of nodes will loss connection to x is at most two. So, if we removed a set of nodes P, $P \subseteq V$ with $|P| = p$, for $0 \leq p \leq t-1$, then the number of nodes it will loss is at most $2p$. The number of nodes of V_{C_x} is the total number of nodes $2t+1$ minus the number of losing nodes $2p$. As a result represent, $|V_{C_x}| \geq 2(t-p)+1$. By Theorem 2, G at node x is locally t-diagnosable.

3.2 Local Diagnosability of Q_n^k with No Faulty Edges

In the following, if we not mention that a *k-ary n-cube* is missing links or with faulty edges, then we consider *k-ary n-cube* as a complete *k-ary n-cube*. On the other hand, we do not discuss the case of missing nodes.

We study case by case on the growth of the size to show that whether *k-ary n-cue* is locally t-diagnosable or not. It is clear that Q_1^4 and Q_2^2 (usually we take Q_2^2 as a hypercube

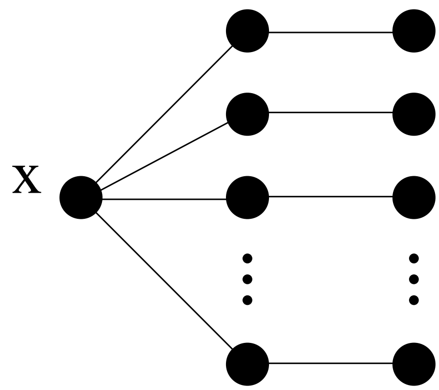


Figure 3.2: An illustration of a Type I.

of $n = 2$) is not locally 2-diagnosable, because of there exists an indistinguishable pair $S_1 \in \{x, a\}$ and $S_2 \in \{y, b\}$ at node x by Lemma 4 (see Figure 3.3).

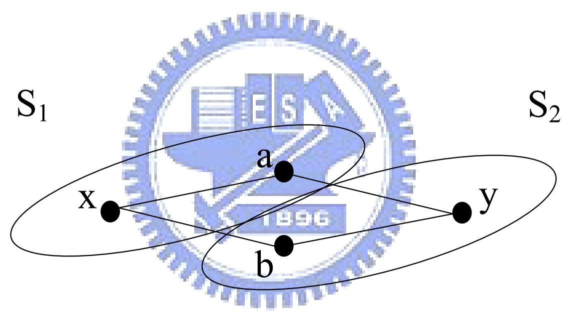


Figure 3.3: An indistinguishable pair of S_1, S_2 in Q_1^4, but also in Q_2^2.

Lemma 5 *There exists a Type I at any vertex $x \in Q_1^k$, and for all $k \geq 5$.*

Proof. With out loss of generality, let x be any node in Q_1^k for all $k \geq 5$ and the degree of each node x is 2. Therefore, every neighboring node of x ($N(x)$) has a link to test another node which has not been tested yet, for all k is greater than 5. Clearly, there exists a Type I in any node x (see Figure 3.4).

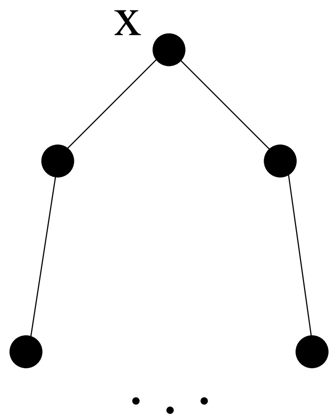


Figure 3.4: A Type I in Q_1^k, for all $k \geq 5$.

Lemma 6 *There exists a Type I at any vertex $x \in Q_2^k$, and for all $k \geq 3$.*

Proof. Since *k-ary n-cube* is highly symmetric. With out loss of generality, let node x be any node in the Q_2^k. Then, we have three different cases: 1) $k = 3$, 2) $k = 4$, and 3) $k > 4$.

Case 1: k = 3.

Because there are four different nodes which are adjacent to the neighboring nodes of x, by *Pigeonhole Principle*, each node of the $N(x)$ has at least a link to test a different node.

Case 2: k = 4.

Because there are six different nodes which are adjacent to the neighboring nodes of x, by *Pigeonhole Principle*, each node of the $N(x)$ has at least a link to test a different node.

Case 3: k $>$ 4.

Because there are eight different nodes which are adjacent to the neighboring nodes of x, by *Pigeonhole Principle*, each node of the $N(x)$ has at least two links to test two different nodes.

(See Figure 3.5)

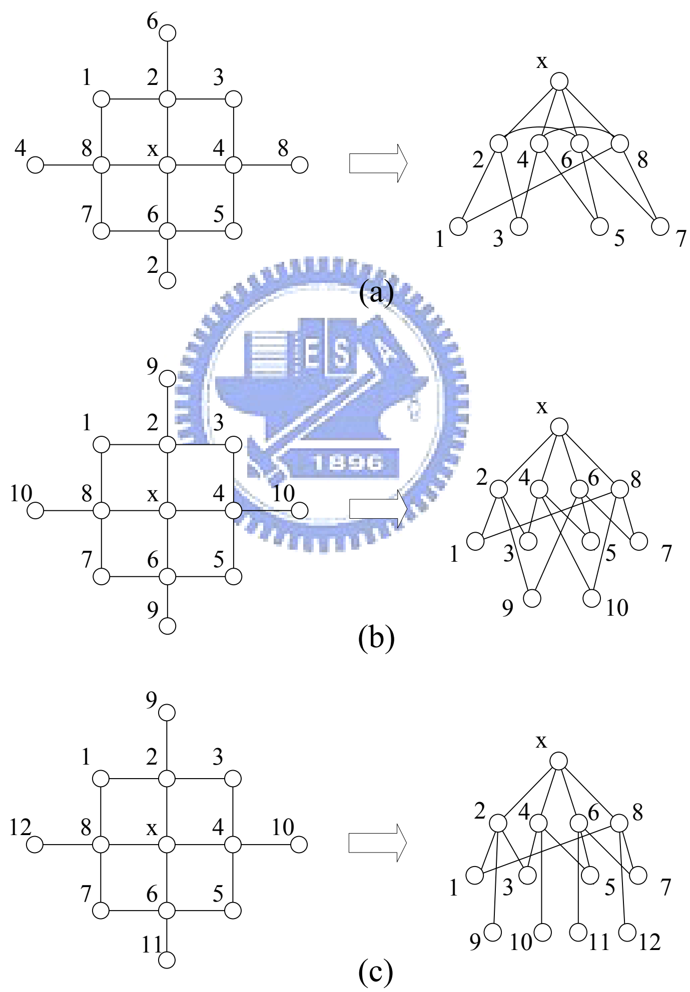


Figure 3.5: Illustrations of three cases (of the proof of Lemma 6). (a) Case 1, (b) Case 2, and (c) Case 3.

For all the cases prove above, we observe that there exists a Type I at any vertex x $\in Q_2^k$, for all $k \geq 3$.

Theorem 4 *There exists a Type I at any vertex* $x \in Q_n^k$*, for some* $n \geq 2$ *and for all* $k \geq 3$.

Proof. We prove this theorem by induction on n. Clearly by Lemma 6, this theorem is true for Q_2^k, for all $k \geq 3$. Assume it holds for some $n \geq 2$, for all $k \geq 3$. We now show that it holds for $n+1$.

Let Q_{n+1}^k be obtained from k copies of subgraph Q_n^k, denoted by $Q_n^k[0]$, $Q_n^k[1]$, ..., and $Q_n^k[k-1]$, by adding dimension edges between them.

Without loss of generality, we take node x to be one node of the subgraph Q_n^k. Let y and y' be the neighboring nodes of x exclusive those in the subgraph Q_n^k which contains the node x. For the degree of each node is two, therefore there are one more node (not the node x) links to y and y', y and y' are in different subgraph. Also, by induction hypothesis, there exists a Type I at node x $\in Q_n^k$ (see Figure 3.6).

Consequently, this theorem holds.

3.3 Local Diagnosability of Q_n^k with Faulty Edges

Commonly, there are faults occur in a multi-processor system. In this thesis, we discuss the issue when the linking edges are broken or missing, called faulty edges.

But exactly, how many missing links or broken edges will make the system not locally t-diagnosable for any vertex x? Let t be the degree of each node, that is $2n$. When there

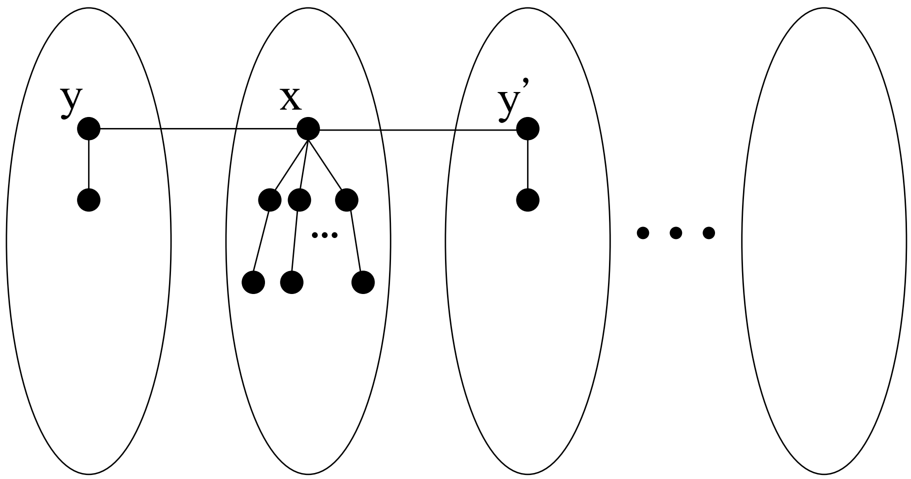


Figure 3.6: An illustration of the proof of Theorem 4.

are $2n$ broken edges then we learn that, if all these $2n$ broken edges gather at one specific node, then the system is not locally t-diagnosable at this node. Because this node is totally isolated, there are no any edges linking to this node. Further more, if there are $2n - 1$ broken edges, simply as above, let all these $2n - 1$ broken edges gather at one specific node x, and let node y be the only node connected to x. Then the system is not locally t-diagnosable at node y. By lemma 4, there exists a indistinguishable pair, $S_1 = N(y)$ and $S_2 = y \bigcup N(y) - x$, $|S_1| = t$ and $S_2 = t$ (see Figure 3.7). As results show above, we then investigate when there are $2n - 2$ broken edges.

Q_2^3 with two broken edges is not locally t-diagnosable at vertex x, $x \in Q_2^3$, because of there exists an indistinguishable pair $S_1 \in \{x, a, b, c\}$ and $S_2 \in \{y, a, b, c\}$, by lemma 4 (see Figure 3.8).

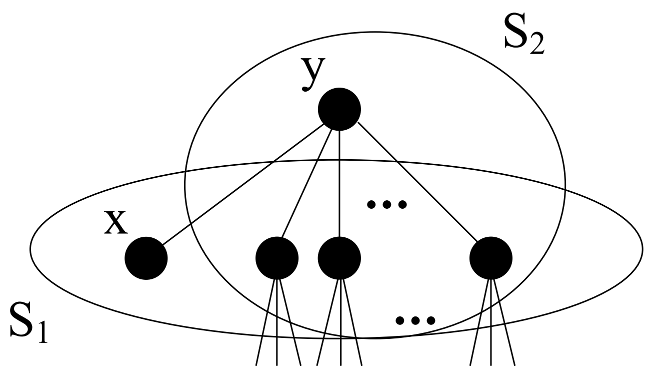


Figure 3.7: An illustration of indistinguishable pair S_1 and S_2 (of Q_n^k with 2n broken edges).

Lemma 7 *Let Q_2^k, for all $k \geq 4$, has no more than two faulty edges. Then there exists a Type I at any vertex $x \in Q_2^k$, for all $k \geq 4$.*

Proof. Let node x be any node in Q_2^k, we discuss the number faulty edges around node x. We have three cases: 1) There are two faulty edges on the neighboring edges of x, 2) There is exactly one faulty edge on the neighboring edges of x, and 3) No faulty edges are the neighboring edges of x.

Case 1: There are two faulty edges on the neighboring edges of x.

Case one can be further discussed into two subcases, for how many faulty edges are there on the dimension edges.

Case 1.1: In case one, there are two faulty edges on the dimension edges.

Case 1.2: In case one, there are only one faulty edge on the dimension edges.

(see Figure 3.9).

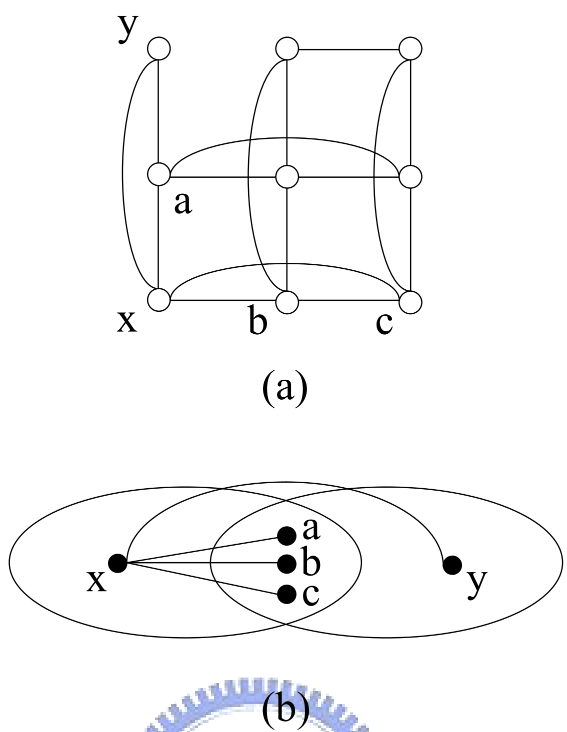


Figure 3.8: Illustrations of indistinguishable pair S_1 and S_2 (of Q_2^3 with two broken edges).

Clearly, we learn that each neighboring node of x in the Case 1.1 and Case 1.2, connects at leat two different nodes which are not including node x.

Case 2: There is exactly one faulty edge on the neighboring edges of x.

Case two can be further discussed into two cases, with $k = 4$ and $k > 4$.

Case 2.1: k = 4.

Because there are six different nodes which are adjacent to the neighboring nodes of x, and there are only one more faulty edge may happened, by *Pigeonhole Principle*, each node of the $N(x)$ has at least two links to test two different nodes.

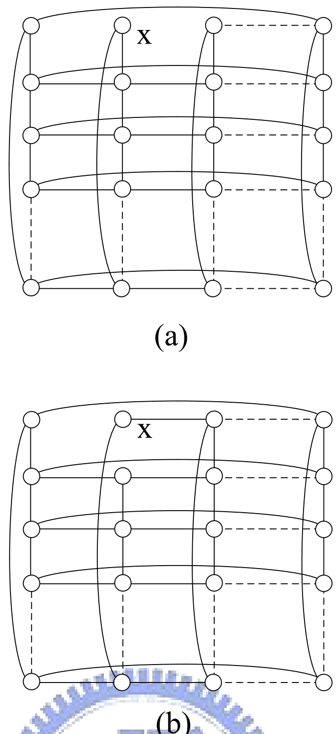


Figure 3.9: Illustrations of case 1 (of the proof of Lemma 7), (a) Case 1.1 and (b) Case 1.2 .

Case 2.2: k $>$ 4.

Because there are seven different nodes which are adjacent to the neighboring nodes of x, and there are only one more faulty edge may happened, by *Pigeonhole Principle*, each node of the $N(x)$ has at least two links to test two different nodes.

(see Figure 3.10).

Case 3: No faulty edges are the neighboring edges of x.

Case three can be further discussed into two subcases, with $k = 4$ and $k > 4$.

Case 3.1: $\mathbf{k = 4}$.

Because there are six different nodes which are adjacent to the neighboring nodes of x, and there are two faulty edges may happened, by *Pigeonhole Principle*, each node of the $N(x)$ has at least one link to test a different nodes.

Case 3.2: $\mathbf{k > 4}$.

Because there are seven different nodes which are adjacent to the neighboring nodes of x, and there are two faulty edges may happened, by *Pigeonhole Principle*, each node of the $N(x)$ has at least one link to test a different nodes.

(see Figure 3.11).

For all the cases prove above, we observe that there exists a Type I at any vertex x $\in Q_2^k$, with no more than two faulty edges, for all $k \geq 4$.

Theorem 5 *Let $|F|$ be the number of faulty edges in Q_n^k, for some $n \geq 2$ and for all $k \geq 4$. Then there exists a Type I in any vertex $x \in Q_n^k$, for some $n \geq 2$, for all $k \geq 4$ and $|F| \leq 2n - 2$.*

Proof. We prove this theorem by induction on n. Clearly by Lemma 7, this theorem is true for Q_2^k, for all $k \geq 4$. Assume it holds for some $n \geq 2$, for all $k \geq 4$. We now show that it holds for $n + 1$.

Let Q_{n+1}^k be obtained from k copies of subgraph Q_n^k, denoted by $Q_n^k[0]$, $Q_n^k[1]$, ..., and $Q_n^k[k-1]$, by adding dimension edges between them. Let x be any node in Q_{n+1}^k. We investigate two cases: 1) There are not more than two faulty edges and at least one faulty edges on the dimension edges along some dimension, 2)There are not more than one faulty edges on the dimension edges.

Case 1: There are not more than two faulty edges and at least one faulty edges on the dimension edges along some dimension.

We discuss the case, when $n + 2 \leq |F| \leq 2n$. By Pigeonhole Principle, there are at least two faulty edges on the dimension edges along some dimension d, $0 \leq d \leq n$.

Thus there are at most two dimensional edges that connect to each node, we then have three more subcases:

Case 1.1: There are two faulty edges on the dimension edges and this two faulty edges are adjacent to the node x.

By induction hypotheses, there exists a Type I at node x.

Case 1.2: There are one faulty edge on the dimension edges and the faulty edge is adjacent to the node x.

Let y be the neighboring node of x, because each node has at least two good neighbors and by induction hypotheses, there exists a Type I at node x.

Case 1.3: There are no faulty edges on the dimension edges which is adjacent to the node x.

Let y and y' be the neighboring nodes of x, because each node has at least two good neighbor and by induction hypotheses, there exists a Type I at node x.

(see Figure 3.12).

Case 2: There are not more than one faulty edges on the dimension edges.

We discuss the case, when $1 \leq |F| \leq n+1$. There are not more than one faulty edge on the dimension edges and there are at most $n+1$ faulty edges along dimension $n+1$. For each subgraph there are at most n broken edges, we have $2n-2 \geq n$, for some $n \geq 2$.

There are at most two dimension edges that connect to each node. We have two more subcases:

Case 2.1: There are one faulty edge on the dimension edges, and the faulty edge is connected to the node x.

Let y be the neighboring node of x, because each node has at least two good neighbor and by induction hypotheses, there exists a Type I at node x.

Case 2.2: There are no faulty edges on the dimension edges which is connected to the node x.

Let y and y' be the neighboring node of x, Similarly above, each node has at least two good neighbor and by induction hypotheses, there exists a Type I at node x.

(see Figure 3.13).

3.4 Diagnosis Algorithm (Counting Algorithm)

Base on Type I, we propose an algorithm to diagnose a system and the time-complexity is $O(n \log n)$. It is more efficient than the fault identification algorithm which the time-complexity is $O(n^{2.5})$ [7].

Let node p be the node which adjacent to node x, and node q be the node which adjacent to node p, both p and q are in the Type I of node x. A vote is said to be positive, negative or spoilt is defined as follows:

Positive vote, if $\sigma(q,p)=0$ and $\sigma(p,x)=0$.

Negative vote, if $\sigma(q,p)=0$ and $\sigma(p,x)=1$.

Spoilt vote, if $\sigma(q,p)=1$ and $\sigma(p,x)=0$, or $\sigma(q,p)=1$ and $\sigma(p,x)=1$

A local syndrome is the set of syndrome which are tested in Type I of node x, showed as above.

(see Figure 3.14).

In a locally t-diagnosable system, the number of faulty nodes are under t. First, we have a set of local syndrome from the Type I of node x, then we count the positive votes and negative votes to tell if node x is faulty or fault free. Third, provided that the positive votes are greater than or equal to the negative votes, we say the node x is fault free, vice versa.

Theorem 6 *For the local syndrome from Type I of node x of a locally t-diagnosable system, the node x is fault-free if the positive votes are greater than or equal to the negative votes.*

Proof. By contradiction, let node x be the faulty node. Without loss of generality, we have A positive votes, B negative votes and C spoilt votes in the set of local syndrome of node x, for $A+B+C=t$ and $deg(x)=t$. As the node x is faulty, the minimum number of faulty nodes of Type I is $2A+C+1$, for a positive vote have two faulty nodes, a spoilt vote have at least one faulty node and the node x is one faulty node.

As a result, $2A+C+1 \geq A+B+C+1$, for $A-B \geq 0$. Because $A+B+C+1=t+1$, the total faulty nodes are greater than or equal to $t+1$. This contradicts the assumption that the node x is fault-free.

Theorem 7 *For the local syndrome from Type I of node x of a locally t-diagnosable system, the node x is faulty if the positive votes are less than the negative votes.*

Proof. By contradiction, let node x be the faulty-free node. Without loss of generality, we have A positive votes, B negative votes and C spoilt votes in the set of local syndrome of node x, for $A+B+C=t$ and $deg(x)=t$. The minimum number of faulty nodes of Type I is $2B+C$, for a negative vote have two faulty nodes and a spoilt vote have at least one faulty node.

As a result, $2B+C > A+B+C$, for $A-B<0$. Because $A+B+C=t$, the total faulty nodes is greater than t. This contradicts the assumption that the node x is faulty.

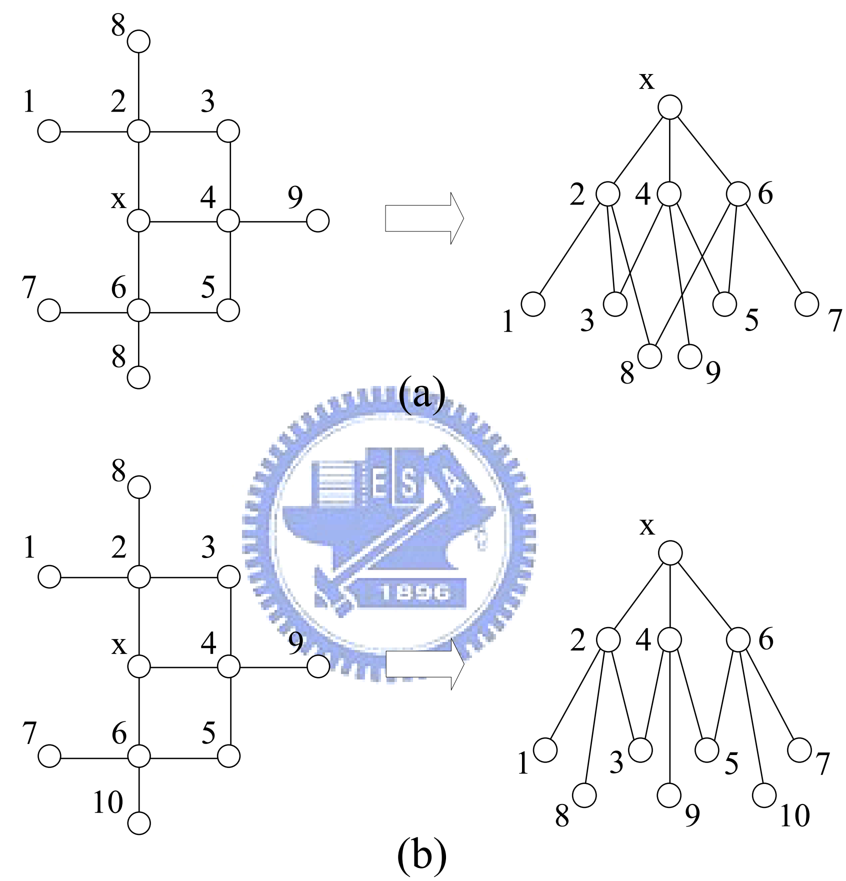


Figure 3.10: Illustrations of case 2 (of the proof of Lemma 7), (a) Case 2.1 and (b) Case 2.2 .

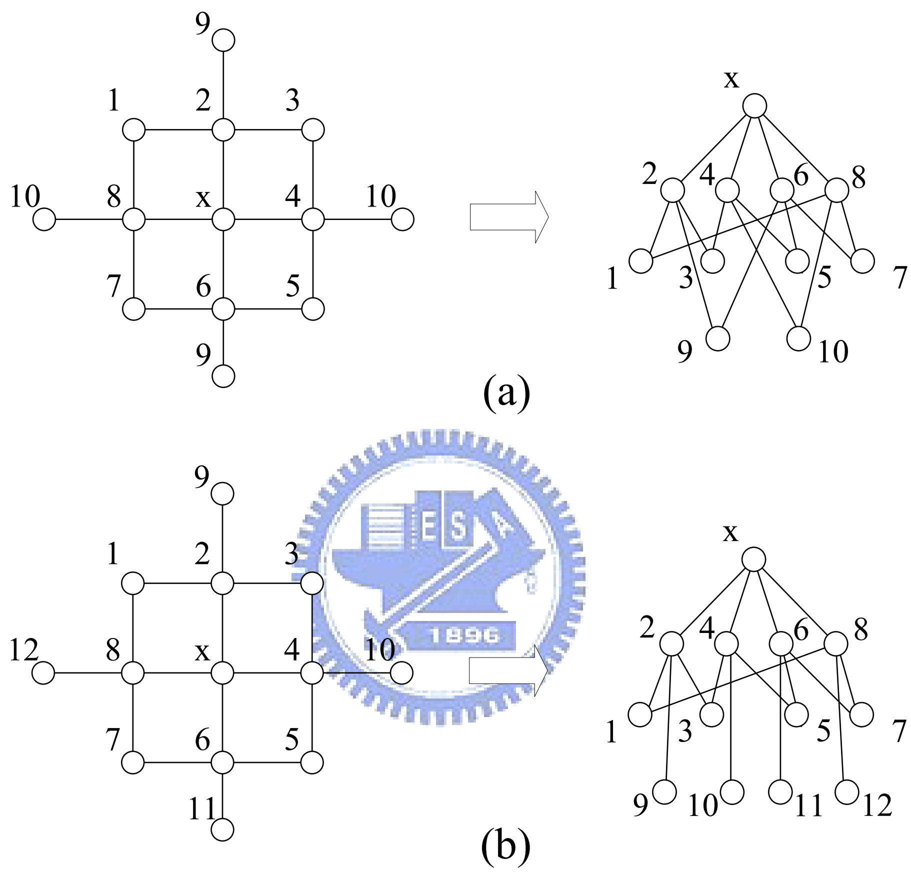


Figure 3.11: Illustrations of case 3 (of the proof of Lemma 7), (a) Case 3.1 and (b) Case 3.2 .

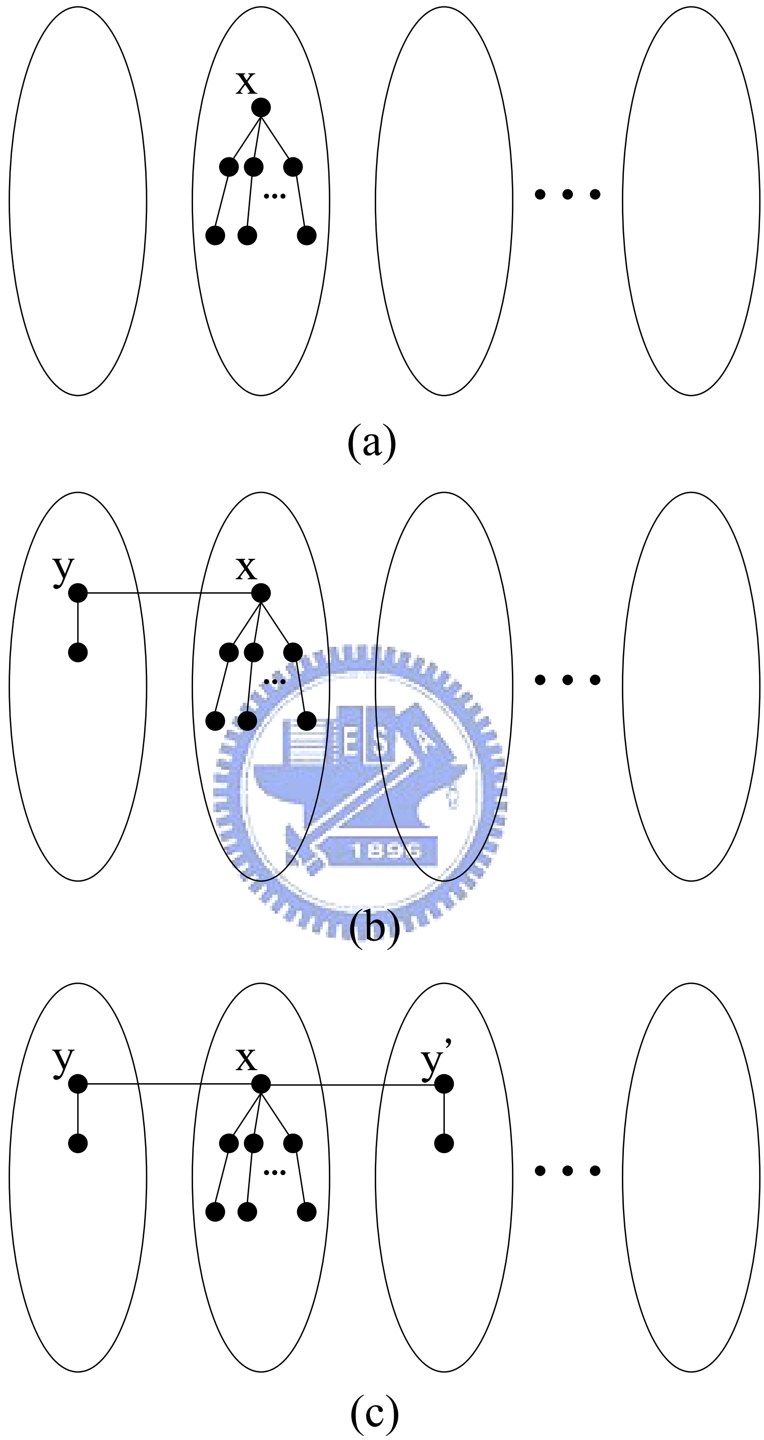


Figure 3.12: Illustrations of case 1 (of the proof of Theorem 5), (a) Case 1.1 and (b) Case 1.2 (c) Case 1.3.

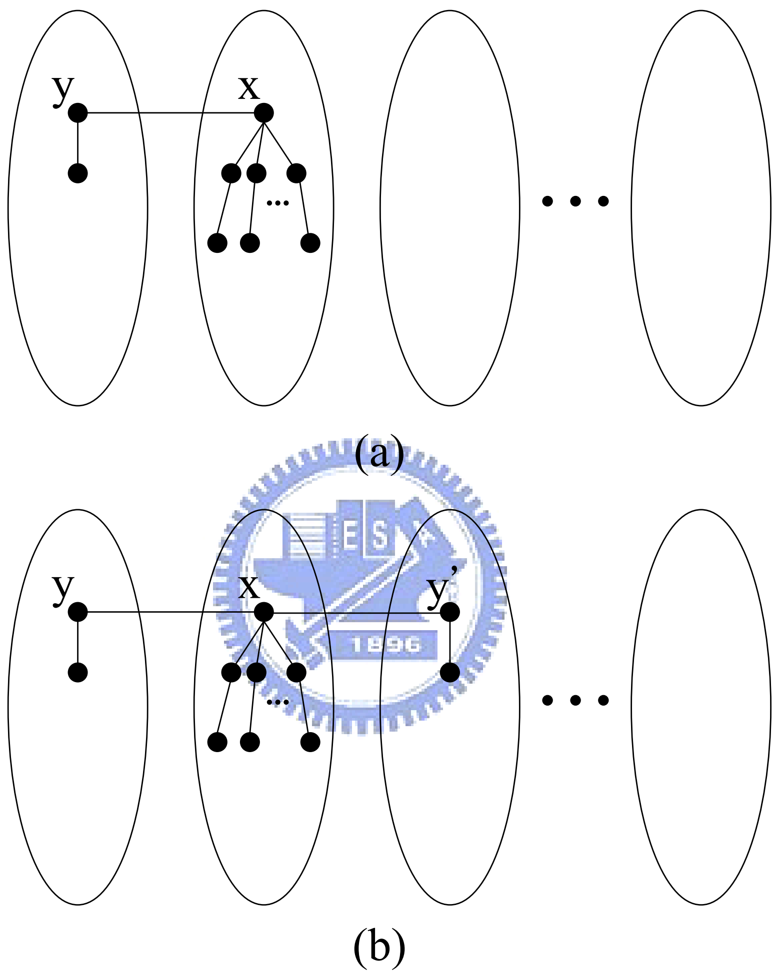


Figure 3.13: Illustrations of case 2 (of the proof of Theorem 5), (a) Case 2.1 and (b) Case 2.2.

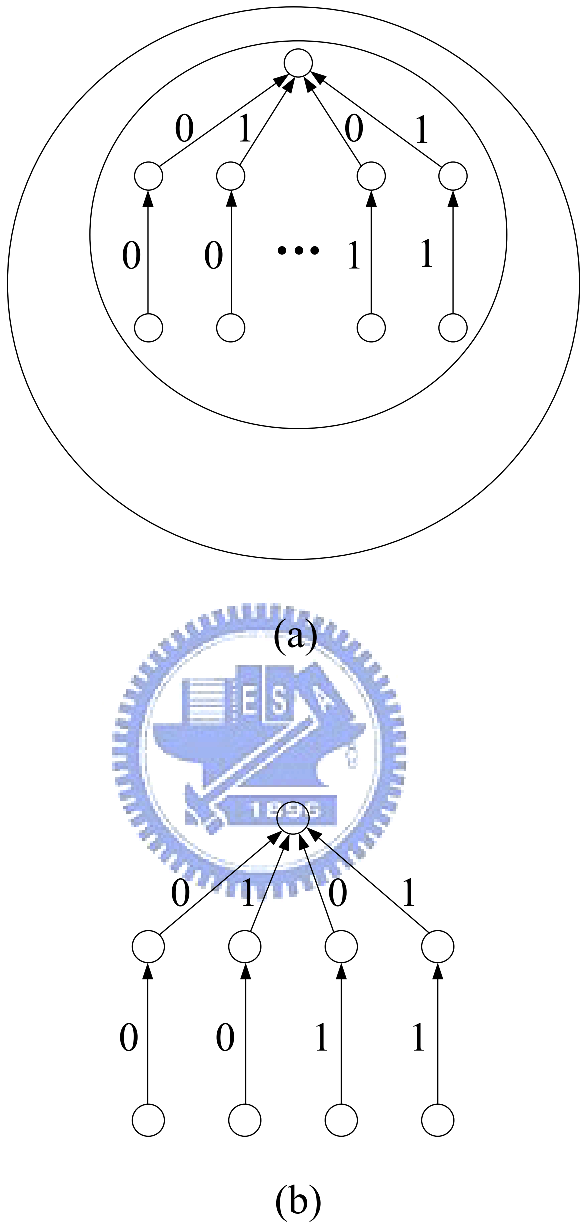


Figure 3.14: Illustrations of (a) Local syndrome and (b)From left to right, there are positive, negative, spoilt, and spoilt votes.

Chapter 4

Conclusion

In this thesis we introduce a new concept, called local diagnosability, and we studied the local diagnosability of *k-ary n-cube* both with no missing links and with missing links. We then proved that the local diagnosability of each node of *k-ary n-cube* equals to the degree of each node of *k-ary n-cube*, with the missing links limited to the number of $2n - 2$. Moreover, by observing whether there exists a Type I at a node of *k-ary n-cube*, we can determine whether a node of *k-ary n-cube* is locally t-diagnosable. By the method described in this thesis, we represented an efficient algorithm to diagnose a multiprocessor system.

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