

# 國立交通大學

資訊科學與工程研究所

## 碩士論文

在比較模式下對類超立方體局部診斷能力之研究

Local Diagnosability of Hypercube Like Network under  
the Comparison Model

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中華民國九十五年六月

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# 在比較模式下對類超立方體局部診斷能力之研究

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在多處理器系統中，診斷能力是判斷一個系統可靠度的重要依據。以往的文獻都是以全域的角度來看一個系統的診斷能力，本篇論文提出一種局部的診斷方法，並且用局部的診斷方法來證明類超立方體的診斷能力，最後我們提出一個診斷演算法，可以用來快速的診斷一個點的好與壞，其時間複雜度為 $O(n \lg n)$ ，其中 $n$ 為一個系統的点數總和，比Sungupta提出的演算法 $O(n^5)$ 更有效率。

**關鍵字：**診斷能力、局部診斷能力、類超立方體、比較模式、診斷演算法。

# **Local Diagnosability of Hypercube Like Network under the Comparison model.**

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Interconnection networks have been an active research area for parallel and distributed computer system. We usually use a graph  $G=(V,E)$  to represent the topology of a network, where vertices represent processors and edges represent links between processors. The diagnosability has played an important role in the reliability of an interconnection network. In this thesis, we present a novel idea on system diagnosis called local diagnosability. There is a strong relationship between the local diagnosability and the traditional global one. For this local sense, we focus more on a single processor and require only identifying the status of this particular processor correctly. We propose a sufficient condition to determine the local diagnosability of a given processor, and we prove the diagnosability of Hypercube Like network  $HL_n$  is  $n$  for  $n \geq 5$  in this local sense. Moreover, we proposed a diagnosis algorithm to easily compute the local diagnosability of each node based on the comparison model.

**Keywords :** diagnosability,  $t$ -diagnosable, comparison model,  $MM^*$  model, Hypercube Like network, local diagnosability, diagnosis algorithm

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# Chapter 1

## Introduction

With the rapid development of technology, multiprocessor systems are more and more important. The reliability of the processors in multiprocessor systems is therefore becoming an important issue. In order to maintain the reliability of a system, whenever a processor (node) is found faulty, it should be replaced by a fault-free processor (node). The process of identifying all the faulty nodes is called the diagnosis of the system. The maximum number of faulty nodes that the system can guarantee to identify is called the diagnosability of the system.

There are several approaches for interconnected processors to diagnose faulty processors by themselves. One major approach is called the comparison model, first proposed by Malek and Maeng [7], [8]. This approach performs the diagnosis by sending the same input to a pair of adjacent processors and comparing their responses.

In the previous studies on diagnosis, most investigators focused on the global diagnosis ability of a system but ignored some local systematic details. For example, if a system is of diagnosability  $t$ , it is at most  $t$ -diagnosable, i.e. given any syndrome  $\sigma$ , all the faulty nodes in a system  $S$  can be precisely identified if  $S$  is with at most  $t$  faulty nodes. But it

is possible to correctly point out all faulty nodes in some part of the system  $S$  under any given syndrome if  $S$  is with more than  $t$  faulty nodes. Thus, only considering the global status let us lose some local detail of a system.

In this paper, we present a novel idea on system diagnosis which is called local diagnosability. More local information about a system can be retrieved through this concept. In other words, every node in a system has its own local diagnosability which states some kind of connection status around it. Moreover, we proposed a counting algorithm to easily compute the local diagnosability of each node based on the comparison model. Finally, we can get back to the original global diagnosis in the point of view of local diagnosis and prove some existing theorems and other new ones.

The rest of this paper is organized as follows: Chapter 2 provides preliminaries and necessary background for diagnosing a system. Chapter 3 introduces the concepts of local diagnosability and some sufficient condition to check whether it is locally  $t$ -diagnosable around a certain node in a system. In Chapter 4, we discuss the diagnosability of Hypercube like network. Then we propose a counting algorithm to compute the local diagnosability of each node in Chapter 5, and the conclusions are given in Chapter 6.



# Chapter 2

## Preliminaries

### 2.1 Graph definition and notation

In this chapter, we give the basic of graph definition and notation [13].  $G = (V, E)$  is a graph if  $V$  is a finite set and  $E$  is a subset of  $\{ (u, v) \mid (u, v) \text{ is an unordered pair of } V \}$ . The degree of vertex  $v$  in a graph  $G$  is the number of edges incident to  $v$ . A vertex cover of  $G$  is a subset  $K \subseteq V(G)$  such that every edge of  $E(G)$  has at least one end vertex in  $K$ . A vertex cover set with the minimum cardinality is called a minimum vertex cover.

### 2.2 Hypercube-Like Interconnection Networks

Vaidya et al. [14] introduced a class of hypercube-like interconnection networks, called HL-graphs, which can be defined by applying the  $\oplus$  operation repeatedly as follows:  $HL_0 = \{K_1\}$ ; for  $m \geq 1$ ,  $HL_m = \{G_0 \oplus G_1 \mid G_0, G_1 \in HL_{m-1}\}$ , which has node set  $V(G_0 \oplus G_1) = V(G_0) \cup V(G_1)$  and edge set  $E(G_0 \oplus G_1) = E(G_0) \cup E(G_1) \cup M$ .  $M$  is an arbitrary perfect matching between the nodes of  $G_0$  and  $G_1$ ; i.e.,  $M$  is a set of edges connecting the nodes of  $G_0$  and  $G_1$  in a one to one function. See Fig. 2.1.

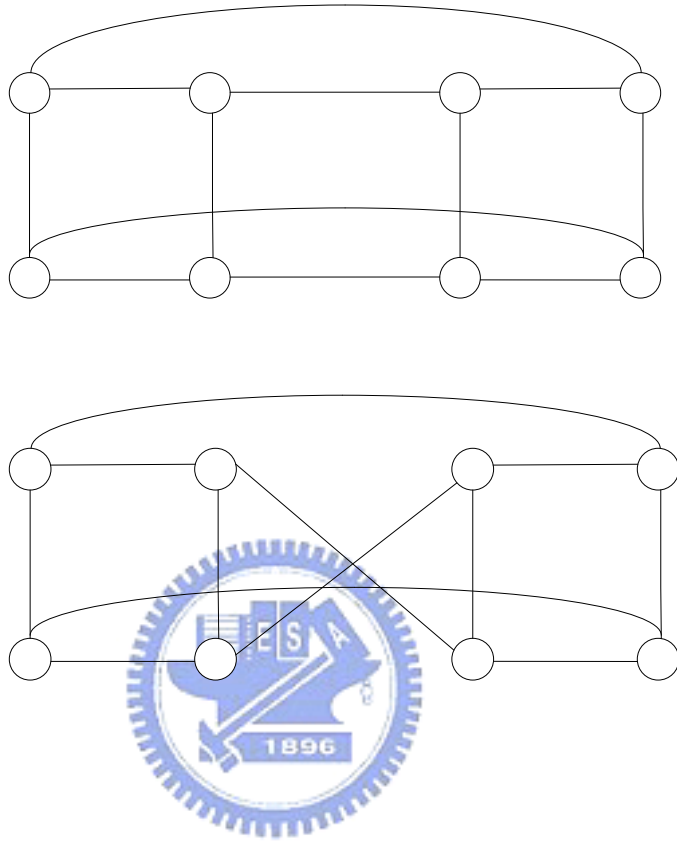


Figure 2.1: Examples of Hypercube Like Network; HL3

### 2.3 The comparison model for diagnosis

For the purpose of self-diagnosis of a given system, several different models have been proposed in the literature [7], [8], [9]. Preparata et al. [9] first introduced a model, the so-called PMC-model, for system level diagnosis in multiprocessor systems. In this model, it is assumed that a processor can test the faulty or fault-free status of another processor.

The comparison model, called the MM model, proposed by Maeng and Malek [8],

[9], is considered to be another practical approach for fault diagnosis in multiprocessor systems. In this approach, the diagnosis is carried out by sending the same testing task to a pair  $\{u,v\}$  of processors and comparing their responses. The comparison is performed by a third processor  $w$  that has direct communication links to both processors  $u$  and  $v$ . The third processor  $w$  is called a comparator of  $u$  and  $v$ .

If the comparator is fault-free, a disagreement between the two responses is an indication of the existence of a faulty processor. To gain as much knowledge as possible about the faulty status of the system, it was assumed that a comparison is performed by each processor for each pair of distinct neighbors with which it can communicate directly. This special case of the MM-model is referred to as the MM\*-model. Sengupta and Dahbura [10] studied the MM model and the MM\*-model, gave a characterization of diagnosable systems under the comparison approach, and proposed a polynomial time algorithm to determine faulty processors under MM\*- model. In this paper, we study the diagnosability of Hypercube Like Network under the MM\*-model.

In the study of multiprocessor systems, the topology of networks is usually represented by a graph  $G=(V,E)$ , where each node  $v \in V$  represents a processor and each edge  $(u,v) \in E$  represents a communication link. The diagnosis by comparison approach can be modeled by a labeled multigraph, called the comparison graph,  $M=(V,C)$  where  $V$  is the set of all processors and  $C$  is the set of labeled edges. A labeled edge  $(u,v)_w \in C$ , with  $w$  being a label on the edge, connects  $u$  and  $v$ , which implies that processors  $u$  and  $v$  are being compared by  $w$ . Under the MM-model, processor  $w$  is a comparator for processors  $u$  and  $v$  only if  $(w,u) \in E$  and  $(w,v) \in E$ . The MM\*-model is a special case of the MM model; it is assumed that each processor  $w$  such that  $(w,u) \in E$  and  $(w,v) \in E$  is a comparator

for the pair of processors  $u$  and  $v$ . The comparison graph  $M = (V, C)$  of a given system can be a multigraph for the same pair of nodes may be compared by several different comparators.

For  $(u, v)_w \in C$ , the output of comparator  $w$  of  $u$  and  $v$  is denoted by  $r((u, v)_w)$ , a disagreement of the outputs is denoted by the comparison results  $r((u, v)_w) = 1$ , whereas an agreement is denoted by  $r((u, v)_w) = 0$ . We list all of the possible result in Table 2.1.

Other node Comparator	Test Result	
	Fault free	At least one is faulty
Fault free	0	1
Fault	0 or 1	0 or 1

Table 2.1: The possible result in Comparison

In this paper, in order to be consistent with the MM model, we have the following assumptions [10]:

1. All faults are permanent;
2. A faulty processor produces incorrect outputs for each of its given testing tasks;

3. The output of a comparison performed by a faulty processor is unreliable; and
4. Two faulty processors with the same input do not produce the same output.

## 2.4 Preliminaries

**Theorem 1** [10] *For any  $F_1, F_2$  where  $F_1, F_2 \subset V$  and  $F_1 \neq F_2$ ,  $(F_1, F_2)$  is a distinguishable pair if and only if at least one of the following conditions is satisfied: (Fig. 2.2)*

1.  $\exists i, k \in V - F_1 - F_2$  and  $\exists j \in (F_1 - F_2) \cup (F_2 - F_1)$  such that  $(i, j)_k \in C$ .
2.  $\exists i, j \in F_1 - F_2$  and  $\exists k \in V - F_1 - F_2$  such that  $(i, j)_k \in C$ .
3.  $\exists i, j \in F_2 - F_1$  and  $\exists k \in V - F_1 - F_2$  such that  $(i, j)_k \in C$ .

Theorem 1 gives a necessary and sufficient condition to ensure distinguishability of a pair of set of vertices  $(F_1, F_2)$ .

**Definition 1** *A system  $G=(V,E)$  is  $t$ -diagnosable if and only if  $\forall F_1 \neq F_2, |F_1| \leq t, |F_2| \leq t, F_1$  and  $F_2$  are distinguishable.*

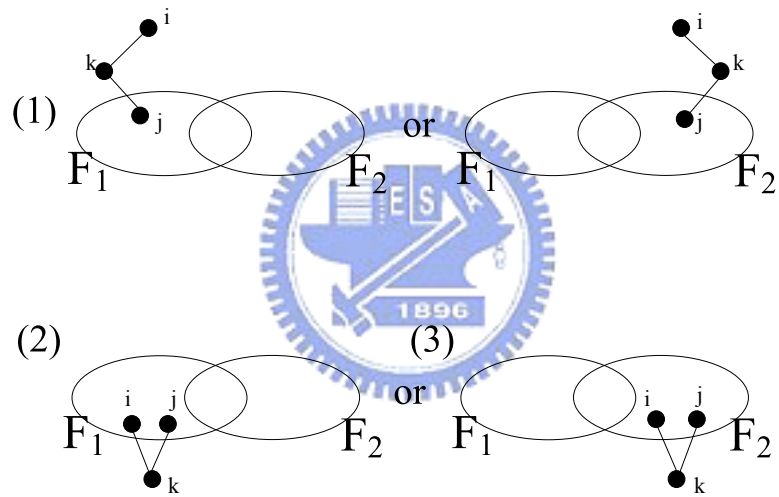


Figure 2.2: Illustrations of a distinguishable pair  $(F_1, F_2)$

# Chapter 3

## Local Diagnosability

In this chapter, we will define the definition of local diagnosability. And we will provide some practical theorem about local diagnosability. By these theorem, we can easily check the diagnosability of a system.

**Definition 2** *A system  $G=(V,E)$  is locally  $t$ -diagnosable if and only if  $\forall F_1 \neq F_2, |F_1| \leq t, |F_2| \leq t, x \in F_1 \Delta F_2, F_1$  and  $F_2$  are distinguishable.*

**Theorem 2** *Let  $G(V,E)$  be the graph of a system  $G$ . Then  $G$  is  $t$ -diagnosable if and only if  $\forall x \in V(G), G$  is locally  $t$ -diagnosable at node  $x$ .*

**Proof.** We prove the necessary condition by contradiction first. By Definition 2, there exists an indistinguishable pair  $(F_1, F_2)$  with  $|F_1| \leq t, |F_2| \leq t$ , and  $x \in F_1 \Delta F_2$ . This contradicts that  $G$  is  $t$ -diagnosable. Then we prove the sufficiency. Suppose  $G$  is not  $t$ -diagnosable. Then there exists an indistinguishable pair  $(F_1, F_2)$  with  $|F_1| \leq t, |F_2| \leq t$ . Pick any node  $y$  in  $F_1 \Delta F_2$ , the system is not locally  $t$ -diagnosable at  $y$ . It contracts with Definition 2.

**Theorem 3** *The diagnosability of a system  $G(V,E)$  is  $t(G)$  if and only if  $t(G)=\min \{t(x)| \forall x \in V(G)\}$ , where  $t(x)$  is the local diagnosability of the node  $x$ .*

**Proof.** The theorem holds from Theorem 2.

**Theorem 4** *Let  $G(V,E)$  be the graph of a system  $G$ . Then  $G$  is locally  $t$ -diagnosable at node  $x$  if, for each vertex set  $S \subset V(G)$  with  $|S|=p$ ,  $0 \leq p \leq t-1$  let the connected component which  $x$  belongs to in  $G-S$  be denoted by  $C_x$ , the cardinality of the vertex cover of  $C_x$  including  $x$  is at least  $2(t-p)+1$ .*

**Proof.** We prove this by contradiction. Suppose  $G$  is not locally  $t$ -diagnosable at node  $x$ . By Definition 2,  $\exists F_1 \neq F_2 \subseteq V$  with  $|F_1| \leq t, |F_2| \leq t$ , and  $x \in F_1 \Delta F_2$ ,  $(F_1, F_2)$  is an indistinguishable pair. Let  $S = F_1 \cap F_2$  with  $|S|=p$ , and let the component which  $x$  belongs to in  $G-S$  is  $C_x$ . Assume that the cardinality of the vertex cover of  $C_x$  is at least  $2(t-p)+1$ . As we know,  $|F_1 \Delta F_2| = 2(t-p)$ . Then the maximum overlapping of  $C_x$  and  $F_1 \Delta F_2$  is  $F_1 \Delta F_2$  itself. In the worst case, the vertex cover in  $C_x$  all fall into  $C_x \cap (F_1 \Delta F_2)$  where  $|C_x \cap (F_1 \Delta F_2)| = 2(t-p)$ . There is still one vertex which is a member of the whole vertex cover of  $C_x$  in  $C_x - (F_1 \Delta F_2)$ . Consequently, there is an edge lying in  $C_x - (F_1 \Delta F_2)$ . Then  $(F_1, F_2)$  is a distinguishable pair since it satisfies the condition 1 of Theorem 1. Therefore  $G$  is locally  $t$ -diagnosable at node  $x$  By Definition 2 which is a contradiction.

**Definition 3** *Let  $G(V,E)$  be a graph, for any  $x \in V$ , a subgraph  $ES(x;n)$  is defined as  $ES(x;n)=[V(x;n),E(x;n)]$ , for some  $n \in N$ , where  $V(x;n)=\{x\} \cup \{v_{ij}|1 \leq i \leq n, 1 \leq j \leq 4\}$   
 $E(x;n)=\{(x,v_{k1}), (v_{k1},v_{k2}), (v_{k2},v_{k3}), (v_{k3},v_{k4})|1 \leq k \leq n\}$*



**Theorem 5** *If there is subgraph  $ES(x;n)$  around  $x$  in a system  $G(V,E)$  and  $\deg(x) \geq n$ , then  $t(x) \geq n$ .*

**Proof.** We prove this theorem by following the condition of the Theorem 4. First we let  $l_k=(v_{k1}, v_{k2}, v_{k3}, v_{k4})$  for some  $k, 1 \leq k \leq n$ , in  $ES(x;n)$ . Then consider the vertex cover on each  $l_k$ . The cardinality of vertex on each  $l_k$  is 2. After deleting a set of nodes  $S$  in  $ES(x;n)$  where  $|S|= p, 0 \leq p \leq n-1$ , there are at least  $(n-p)$  complete  $l_k$  quadruples where the word "complete" means that all  $v_{k1}, v_{k2}, v_{k3}, v_{k4}$  on a  $l_k$  have not been deleted in  $G-S$ . Then the cardinality of vertex cover including  $x$  on the connected component  $C_x$  which contains  $x$  is more than  $1+ 2(n-p)$ . Therefore, the system  $G$  with a subgraph  $ES(x;n)$  is locally  $n$ -diagnosable at  $x$  by Theorem 4.

**Theorem 6** *In a system  $G(V,E)$ , for some  $x \in V(G)$ , if  $\deg(x)=n$ , then  $t(x) \leq n$ .*

**Proof.** We prove this theorem by contradiction. Suppose on the contrary that  $t(x) > n$ , say  $t(x)= n+1$ . Then the system is locally  $(n+1)$ -diagnosable on  $x$  and  $\deg(x)= n$ . Assume there is a subgraph  $ES(x;n)$  around  $x$ . Let  $F_1$  and  $F_2$  be the set of  $\{x\} \cup \{v_{k3}\}$  and  $\{v_{k1}$  for all  $k, 1 \leq k \leq n$ , in  $ES(x;n)$ , respectively. Then,  $(F_1, F_2)$  is not a distinguishable pair according to Theorem 1, which is a contradiction. Then the proof is completed.

**Theorem 7** *If there is a subgraph  $ES(x;n)$  around  $x$  in a system  $G(V,E)$  and  $\deg(x)=n$ , then  $t(x)=n$ .*

**Proof.** This theorem holds from the combination of Theorem 5 and Theorem 6, therefore this theorem is also completed.

By Theorem 7, we can check the local diagnosability of any node in a system. It's more convenient than global view.



## Chapter 4

# Diagnosability of Hypercube Like Network

**Theorem 8** *The diagnosability of Hypercube Like Network  $HL_n$  is  $n$  for  $n \geq 5$ .*

**Proof.** We prove the theorem by induction on  $n$ , the dimension of Hypercube Like Network  $HL_n$ .

Basis: We will prove that  $HL_5$  is 5-diagnosable. Consider any node  $x$  in  $HL_5$ , we find that there is a subgraph  $ES(x,5)$  around  $x$  (See Fig. 4.1). Hence node  $x$  in  $HL_5$  is locally 5-diagnosable by Theorem 7. Because  $HL_5$  is node symmetric, therefore every node in  $HL_5$  is locally 5-diagnosable. Hence  $HL_5$  is 5-diagnosable by Theorem 2.

Hypothesis: The claim holds for  $HL_{n-1}$ .

Induction: Consider an  $n$ -cube,  $HL_n$ . We want to show that each node of  $HL_n$  all have the subgraph  $ES(x,n)$  around it. Consider any node  $x$  in  $HL_n$ , we can separate  $HL_n$  into two  $HL_{n-1}$  which called  $G$  and  $H$ . Without loss of generality, we may assume that  $x$  is in  $G$ . By hypothesis, there is a subgraph  $ES(x,n-1)$  in  $G$ . Consider  $x'$  in  $H$ , there is a subgraph  $ES(x',n-1)$  in  $H$ . Hence there is a subgraph  $ES(x,n)$  in  $HL_n$ . Therefore  $x$  is

locally  $n$ -diagnosable by Theorem 7. And  $HL_n$  is node symmetric, each node in  $HL_n$  is locally  $n$ -diagnosable, hence  $HL_n$  is  $n$  diagnosable Theorem 2.

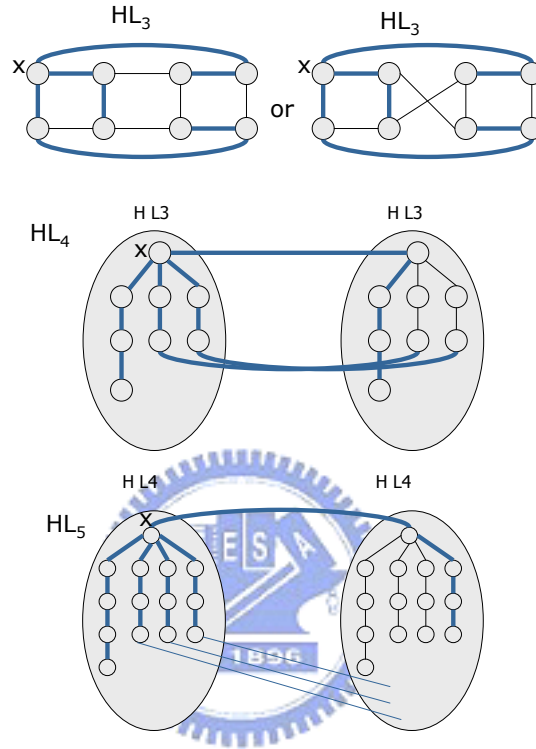


Figure 4.1: It shows  $HL_5$  has a subgraph  $ES(x,5)$  around  $x$ .

**Theorem 9** *If the local diagnosability of any node in  $HL_{n-1}$  equals to its degree with  $n-3$  edge faults, then the local diagnosability of any node in  $HL_n$  equals to its degree with  $n-2$  edge faults.*

**Proof.** First we explain why not the edge fault in  $HL_n$  equals to  $n-1$ . We give a counterexample in the Fig. 5.2. We can see that  $F_1, F_2$  are indistinguishable pair by Theorem 1. See Fig. 4.2.

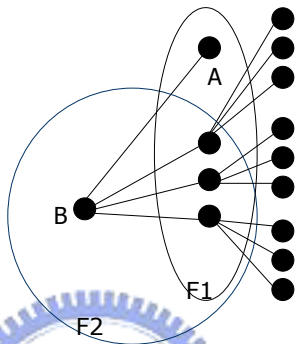


Figure 4.2: The illustration of indistinguishable pair.

Now we are going to prove the theorem. Suppose the local diagnosability of any node in  $HL_{n-1}$  with  $n-3$  edge faults equals to its degree. Consider  $HL_n$  which has  $n-2$  faulty edges is constructed with two copies of  $HL_{n-1}$ , one is  $G$  and the other is  $H$ . Without loss of generality, we may assume that  $x$  is in  $G$  and  $\deg(x)=m$ . And the degree of node  $x'$  in  $H$  correspond to  $x$  is  $m'$ . We prove it in three cases.

case 1.1: There are  $k$  faulty edges in the crossed edge, where  $1 \leq k \leq n-2$ , and  $(x, x')$  is faulty. See Fig. 4.3.

Since there are  $k$  faulty edges in the crossed edge, where  $1 \leq k \leq n-2$ , the faulty edges in  $G$  are at most  $n-3$ . Since the faulty edges in  $G$  are at most  $n-3$ , the local diagnosability

of  $x$  in  $G$  equals to its degree  $m$ . Hence there is a subgraph  $ES(x,m)$  around  $x$  in  $HL_n$  because  $(x,x')$  is faulty. By theorem 7, the local diagnosability of  $x$  in  $HL_n$  equals to its degree  $m$  with  $n-2$  faulty edges.

case 1.2: There are  $k$  faulty edges in the crossed edge, where  $1 \leq k \leq n-2$ , and  $(x,x')$  is fault-free. See Fig. 4.3.

Since there are  $k$  faulty edges in the crossed edge, where  $1 \leq k \leq n-2$ , the faulty edges in  $G,H$  are at most  $n-3$ . Since the faulty edges in  $G,H$  are at most  $n-3$ , the local diagnosability of  $x$  in  $G$  equal to its degree  $m-1$  and the local diagnosability of  $x'$  in  $H$  equal to its degree  $m'-1$ . Hence there is a subgraph  $ES(x,m-1)$  around  $x$  in  $G$  and  $ES(x',m'-1)$  in  $H$ . So there is a subgraph  $ES(x,m)$  around  $x$  in  $HL_n$ . By theorem 7, the local diagnosability of  $x$  in  $HL_n$  equals to its degree  $m$  with  $n-2$  faulty edges.

case 2: All faulty edges are in  $G$ . (i.e there are  $n-2$  faulty edges in  $G$ .) See Fig. 4.4.

If there is a faulty edge  $S$  belongs to  $\{(x,v11),(x,v21),(x,v31),(x,v41)\}$ , we assume that  $S$  is fault-free. Hence there are  $n-3$  faulty edges in  $G$ . By assumption, the local diagnosability of  $x$  in  $G$  equals to its degree, So we can find  $ES(x,m-1)$  in  $G$ . Consider  $x'$  in  $H$ , we can also find  $ES(x',m'-1)$  in  $H$  by assumption. Therefore, we can easily find  $ES(x,m)$  in  $HL_n$ . Then the local diagnosability of  $x$  in  $HL_n$  equals to its degree with  $n-2$  faulty edges by Theorem 7.

If there is a faulty edge  $S=(x,y)$  belongs to  $\{(v11,v12),(v21,v22),(v31,v32),(v41,v42)\}$ , we assume that  $S$  is fault free. Hence there are  $n-3$  faulty edges in  $G$ . By assumption, the local diagnosability of  $x$  in  $G$  equals to its degree, So we can find  $ES(x,m-1)$  in  $G$ . Consider  $x'$  in  $H$ , we can also find  $ES(x',m'-1)$  in  $H$  by assumption. Consider node  $y'$  in  $H$ , we can

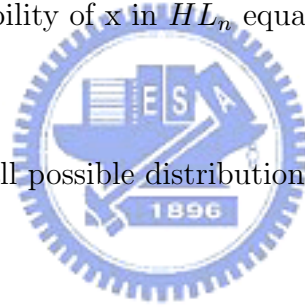
find  $ES(y', \deg(y')-1)$  in  $H$ . Therefore, we can easily find  $ES(x, m)$  in  $HL_n$ . Hence the local diagnosability of  $x$  in  $HL_n$  equals to it's degree with  $n-2$  faulty edges by Theorem 7.

If there is a faulty edge  $S$  belongs to  $\{(v_{12}, v_{13}), (v_{22}, v_{23}), (v_{32}, v_{33}), (v_{42}, v_{43})\}$  or  $\{(v_{13}, v_{14}), (v_{23}, v_{24}), (v_{33}, v_{34}), (v_{43}, v_{44})\}$ , it can be proved using by the same way.

case 3: There are  $x$  faulty edges in  $G$ , where  $1 \leq x \leq n-2$ . And there are  $y$  faulty edges in  $H$ , where  $1 \leq y \leq n-2$ . See Fig. 4.5.

Because faulty edges in  $H$  and  $G$  are at most  $n-2$ . By the assumption, the local diagnosability of  $x$  in  $G$  equals to it's degree  $m-1$ . Hence we can find  $ES(x, m-1)$  in  $G$ . We can find  $ES(x', m'-1)$  in  $H$  by the same way. Hence we can find  $ES(x, m)$  in  $HL_n$ . Therefore the local diagnosability of  $x$  in  $HL_n$  equals to it's degree with  $n-2$  faulty edges by Theorem 7.

In case 1,2,3, we proved all possible distribution of faulty edges. Therefore, the proof is complete.



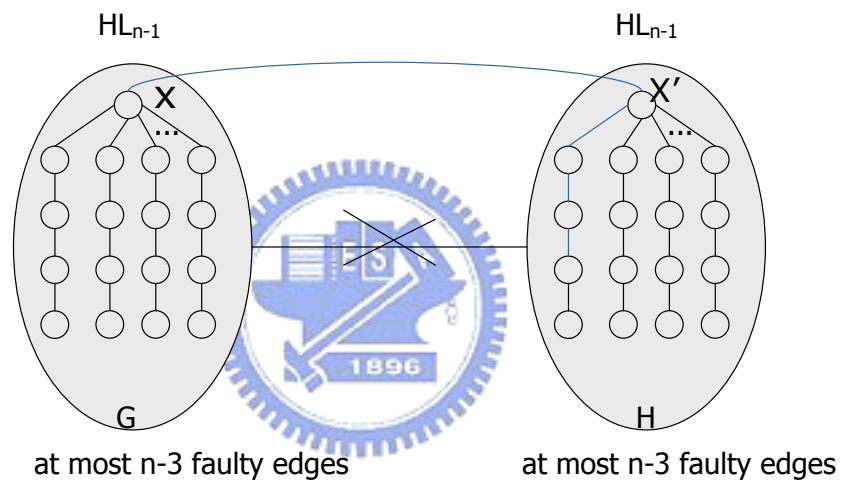


Figure 4.3: Case 1: There are  $x$  faulty edges in the crossed edge, where  $1 \leq x \leq n-2$ .



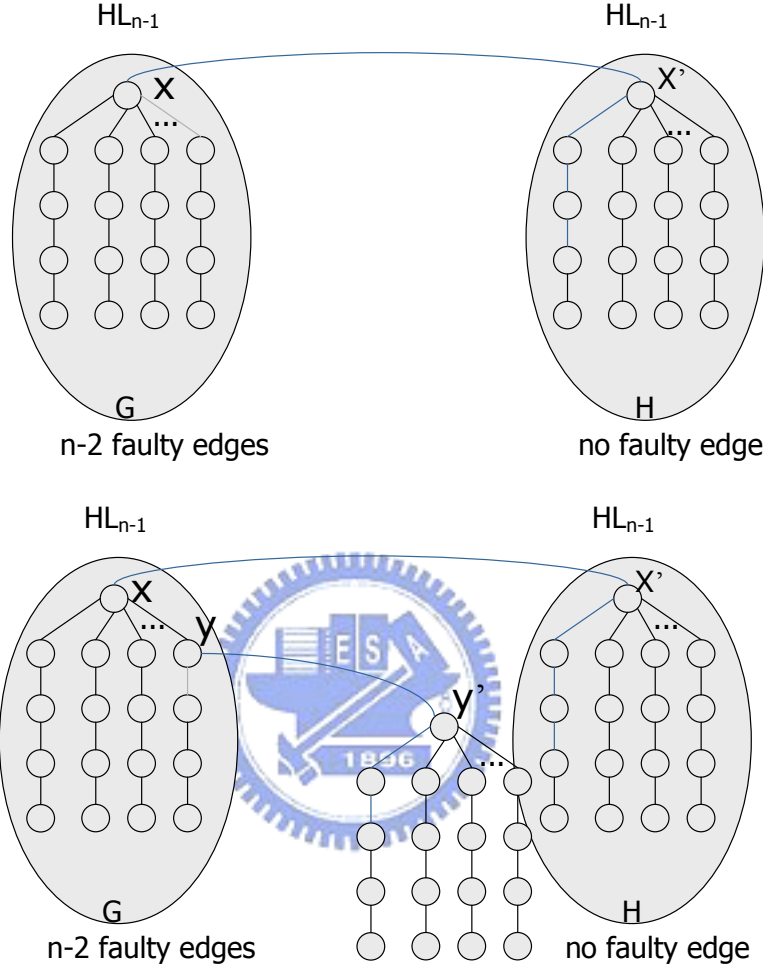


Figure 4.4: Case 2: All faulty edges are in G.(i.e there are n-2 fault edges in G.)

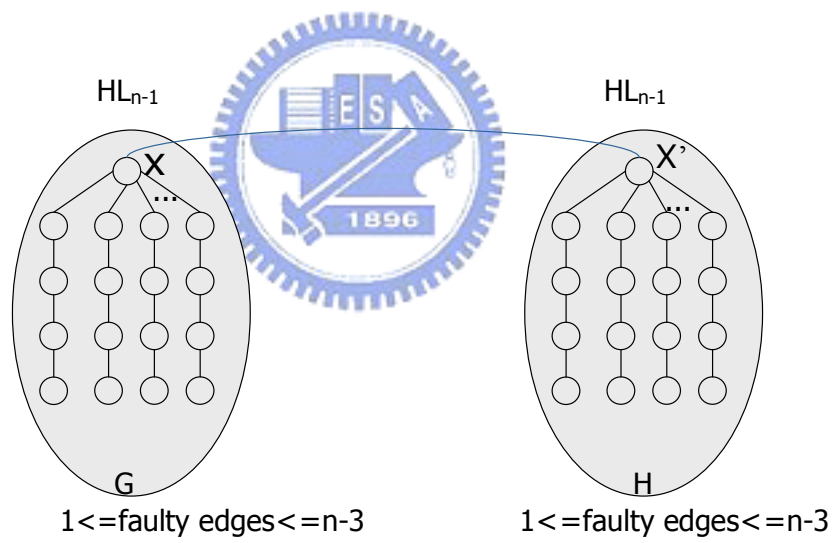


Figure 4.5: Case 3: There are  $x$  faulty edges in  $G$ , where  $1 \leq x \leq n-2$ . And there are  $y$  faulty edges in  $H$ , where  $1 \leq y \leq n-2$

# Chapter 5

## Counting Algorithm

We propose an algorithm in Theorem 10 to compute the local diagnosability of each node based on the comparison model, which is called Counting algorithm. This algorithm is very efficient, the time complexity is  $O(n \lg n)$  where  $n$  is the number of nodes in a system. It is more efficient than the Sengupta's algorithm [10], which the time complexity is  $O(n^5)$ .

**Theorem 10** *For any node  $x$  in a  $t$ -diagnosable system  $S$ , the number of faulty nodes in system  $S$  is  $\leq t$ . First use the Comparison Model to test the nodes in  $V(x;t)$ , it will output some syndrome. Now we will diagnose node  $x$  by the syndrome which output from  $V(x;t)$ . Case 1: If  $A_0(x) \geq A_4(x)$ ,  $x$  is fault-free node, Case 2: If  $A_0(x) < A_4(x)$ ,  $x$  is faulty, where  $A_i(x)$  is the number of output  $G_i$  around  $x$  where  $1 \leq i \leq 7$ . See Fig. 5.1.*

**Proof.** We prove the Case I by contradiction. Suppose  $x$  is faulty, then the number of faulty node is at least  $n = 1 + 3A_0(x) + 2A_1(x) + A_2(x) + 2A_3(x) + A_5(x) + A_6(x) + A_7(x)$  but  $t(x) = A_0(x) + A_1(x) + A_2(x) + A_3(x) + A_4(x) + A_5(x) + A_6(x) + A_7(x)$  it contradict with  $n > t(x)$ , hence  $x$  is fault-free.

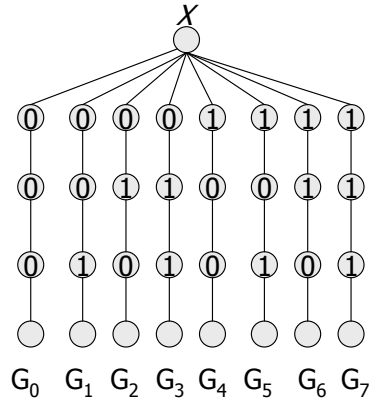


Figure 5.1: 8 types of output around  $x$

We prove the Case II by contradiction. Suppose  $x$  is fault-free, then the number of faulty node is at least  $m=A_1+A_2+A_3+2A_4+A_5+A_6+A_7$ , but  $t(x)=A_0(x)+A_1(x)+A_2(x)+A_3(x)+A_4(x)+A_5(x)+A_6(x)+A_7(x)$  it contradict with  $m>t(x)$ , hence  $x$  is faulty.

There is an example of counting algorithm. Suppose the system is 7-diagnosable, we test  $ES(x;7)$  under the Comparison Model. The result is in Fig. 5.2. Hence  $A_0=3, A_4=2, A_5=1, A_6=1$ . Because  $A_0>A_4$ , by Theorem 10 node  $x$  is fault-free.

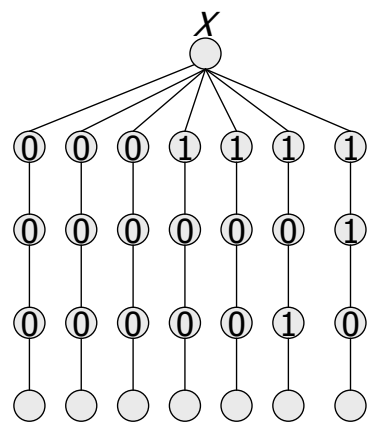


Figure 5.2: The test result in  $ES(x;7)$

# Chapter 6

## Conclusions

The reliability of an interconnection network is an important issue. The diagnosability is also an important factor in measuring the reliability of an interconnection network. In this paper, we propose a new point of view which is called the local diagnosability, and a theorem to verify the diagnosability of multiprocessor systems under the comparison-based model. Then we prove the diagnosability of Hyper-cube Like network under the comparison-based model more easily in our theorem. Finally we propose a counting algorithm, it can diagnose a  $t$ -diagnosable system efficiently.

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