CHAPTER 2. LITERATURE REVIEW

This chapter overviews the previous researches on chaos testing and prediction. Related works on its applications, including abnormal diagnosis and chaos control, were also reviewed. The chapter is organized as follows. Section 2.1 presents the works on testing for chaos. Section 2.2 reviews the study on the prediction of chaotic time series. Section 2.3 provides an overview of other relevant works. Some comments are concluded in section 2.4.

2.1 Testing for Chaos

1. Ryan, Goldberger, Pincus, Mietus and Lipsitz (1994)

This study aimed to quantify the complex dynamics of beat-to-beat sinus rhythm heart rate fluctuations and to determine their differences as a function of gender and age. The authors analyzed heart rate dynamics during 8-min segments of continuous electrocardiographic recording in healthy young (20 to 39 years old), middle-aged (40 to 64 years old) and elderly (65 to 90 years old) men (n = 40) and women (n = 27) while they performed spontaneous and metronomic (15 breaths/min) breathing. Relatively high (0.15 to 0.40 Hz) and low (0.01 to 0.15 Hz) frequency components of heart rate variability were computed using spectral analysis. The overall "complexity" of each heart rate time series was quantified by its approximate entropy, a measure of regularity derived from nonlinear dynamics ("chaos" theory). The results showed that mean heart rate did not differ between the age groups or genders. High frequency heart rate power and the high/low frequency power ratio decreased with age in both men and women ($p < 0.05$). The high/low frequency power ratio during spontaneous and metronomic breathing was greater in women than men ($p < 0.05$). Heart rate approximate entropy decreased with age and was higher in women than men ($p < 0.05$).

2. Solé and Bascompte (1995)

This study aimed to identify chaos in ecological data. A new class of Lyapunov exponents from a simple numerical approach based on a spatially defined average of local divergence of trajectories was utilized. This was a new way of identifying chaos in ecological data where, because of its limited length and quality, current techniques derived from the dynamical systems theory were not applicable. The results showed that only a very short time series (around 10 time steps) was necessary for obtaining a close estimate of the spatiotemporal Lyapunov exponent if the lattice size was high enough. The method was also applied to three previously studied coupled map lattice models and was proved to be a good measure for spatiotemporal chaos and an alternative way of estimating the underlying dimension.

3. Molnár, Skinner, Csépe, Winkler and Karmos (1995)

This study aimed to apply recently developed mathematical tools of chaos theory to the analysis of event-related potentials (ERPs) recorded in paradigms in which the mismatch negativity (MMN) and the P3 component appeared. A new method, the point correlation dimension (PD2i), was used for data analysis, which was proved to be more accurate than other algorithms for the calculation of the correlation dimension (D2), which was a measure of the complexity of the generator(s) responsible for producing the analyzed time series, i.e., the EEG. ERPs were recorded from Fz, Cz and Pz in 6 subjects. The results showed that the PD2i decreased significantly during the event-related potentials in both situations when the MMN was present and when the P3 was present. However the pattern and magnitude of this decrease were different between these two situations. The pattern of PD2i changed during the occurrence of deviant stimuli eliciting that the MMN suggested the presence of a frontal MMN generator. The conspicuous PD2i decrease during the occurrence of the P3 wave might support the “context closure” hypothesis concerning its functional significance.

4. Snyder and Kurtze (1996)

This study aimed to examine the use of chaos theory in modelling time series data generated by computer mediated communication (CMC). Data generated by CMC bulletin boards was examined to prove the presence of chaotic behavior and to assess the variance which could be accounted for by the deterministic mechanism. The study regarded the time series data generated from a CMC discussion group as the sum of two components. One was a deterministic “signal” which presumably obeyed some unknown chaotic dynamics. The other component was truly random “noise”. The study's overall goal was to assess the relative importance of these two components, using techniques devised by Procaccia and Grassberger for studying chaos in time series data. The results showed that chaotic time series data had increasingly larger fractions of added noise until chaotic behavior was no longer found. Analysis of the data with added noise indicated that from 20 to 30% of the variation in the data was the result of noise. Conversely, 70 to 80% of the variation in the data could be accounted for by deterministic chaos. Implications for future research using chaos and CMC were also discussed.

5. Logan and Mathew (1996)

This paper introduced some of the basic concepts of chaos theory and then detailed a method for quantifying a fractal dimension from a time series, the correlation dimension. Some of the practical difficulties encountered in measuring the correlation dimension from the correlation integral algorithm were also outlined. Finally, some experimental results from a rolling element bearing test rig were presented.

6. Yoshida, Sato, Yamamoto, and Yokota (1997)

This paper aimed to study a method that enabled us to get information on how a chaotic system behaved as its parameters were changed. This method was an application of theory based on statistical mechanics developed by Tomita and others. In a previous paper, in order to apply the theory to time series analysis, they proposed a method that can calculate quantities based on statistical mechanics, rather than on system equations. The results showed that their method was effective for characterization of non-linear systems whose equations were not known. And the authors also focused on the effect of instrumental noise on their method because there was a significant problem when their method was used for time series analysis. The results showed that a qβ-phase transition might become invalid as an identifier of chaos and disappear altogether.

7. Blanco, Figliola, Kochen, Rosso, and Salgado (1997)

This paper aimed to report a new attempt to characterize the global brain dynamics through the electrical activity using these nonlinear dynamical invariants. The methods employed in this work were independent of any modeling of brain activity. It relied solely on the analysis of data obtained from a single variable time series. They analyzed the EEG signals from deep electrodes that intersect different anatomical structures, in a patient with refractory epilepsy that is prone to surgical treatment. The electrical signal provided by deep electrodes guaranteed low noise signals. The methods usually employed in nonlinear dynamics analysis are based on distances, and assume the stationarity of the data sets. Distances between points in appropriated embeddings of the data were used to compute a set of metric parameters, like correlation dimension and Lyapunov exponents, etc. They emphasized that these dynamical invariants were defined only if the series were stationary or weakly stationary. The insight gained by the concept of deterministic chaos for the EEG was that a seemingly disordered process might be governed by a relatively few simple laws, which could be determined. The main results showed that from a systematic and self consistent protocol we could conclude that: a) The brain's spontaneous activity was not a simple noise, but an active signal probably reflecting causal responses from hidden events and sources during sensory and cognitive processing in the brain. b) The time evolution of the system could be described by a low dimensional deterministic dynamic. c) The nonlinear dynamical invariants (embedding dimension, correlation dimension and Lyapunov exponent) were dependent on brain structure as well as brain activity.

8. Schittenkopf and Deco (1997)

This paper aimed to present a practicable procedure which allowed us to decide if a given time series was pure noise, chaotic but distorted by noise, purely chaotic, or a Markov process. Furthermore, the method gave an estimate of the Kolmogorov-Sinai (KS) entropy and the noise level. The procedure was based on a measure of the sensitive dependence on the initial condition called ε-information flow. This measure generalized the concept of KS entropy and characterized the underlying dynamics. The ε-information flow was approximated by the calculation of various correlation integrals.

9. Kawamura, McKerchar, Spigel, and Jinno (1998)

This paper aimed to analyse the chaotic characteristics of the monthly time series of the Southern Oscillation Index (SOI). Three schemes, moving average, low-pass filter and nonlinear smoothing, were used to reduce noise and enhance chaotic properties. Autocorrelation and spectral characteristics, as well as three chaos-oriented properties -- phase space trajectory, the largest Lyapunov exponent and correlation dimension -- were then examined. The results showed that no significant signs of chaotic behavior were found for either the noise-reduced SOI time series or the raw one. Although it contained long-term periodicity, the SOI time series was considered to be stochastic rather than chaotic from the viewpoint of dynamical systems theory.

10. Jayanthi and Sinha (1998)

This paper aimed to conceptualize the process of innovation implementation in high technology manufacturing, a natural setting of multiple and ongoing innovation implementation. Building on the developments in organizational learning theory, they framed the process of innovation implementation in high technology manufacturing as a problem of balancing between exploitation and exploration. Through the application of a logistic difference equation, they provided insights into the dynamics of balancing between exploitation and exploration, and showed that innovation implementation in high technology manufacturing could be conceptualized as a chaotic process, in a scientific sense. Using time series data from a wafer fabrication plant, the high technology manufacturing plant that served as our research site over a period of 125 weeks, they tested this conceptualization. The results showed that empirical support for the conceptualization of innovation implementation in high technology manufacturing was a chaotic process.

11. Longstaff and Heath (1999)

This paper considered the problem of how the psychomotor system translated a stable motor memory into an invariant spatial output within an infinitely variable biomechanical and environmental context. Initially the validity of a novel methodology, based on the concatenation of handwriting velocity data over several trials to form long time series, combined with singular value decomposition to reduce noise, was confirmed. The data analyzed were the horizontal and vertical velocity of the stylus as eight participants wrote the pseudo-word madronal on a computer graphics tablet. Nonlinear dynamic analysis techniques such as examination of delay portraits, as well as calculation of the correlation dimension and Lyapunov spectra were applied to test the hypothesis that handwriting velocity profiles were chaotic. The results showed that the largest Lyapunov exponents were positive, the sums of Lyapunov spectra components were negative and the correlation dimensions were low and fractional, supporting this hypothesis. They concluded by proposing that the psychomotor actions found in handwriting were a product of a chaotic dynamic process whose initial conditions depended on the environmental and biomechanical context.

12. Sivakumar (2000)

This paper was designed: (1) to address some of the important issues in the application of chaos theory in hydrology; and (2) to provide possible interpretations to the results reported by past studies reporting chaos in hydrological processes. A brief review of some of the past studies investigating chaos in hydrological processes was presented. An insight into the studies revealed that most of the problems, such as data size, noise, delay time, in the application of chaos theory have been addressed by past studies, and caution has been taken in the application of the methods and interpretation of the results. The study also revealed that the problem of data size was not as severe as it was assumed to be, whereas the presence of noise seemed to have much more influence on the nonlinear prediction method than the correlation dimension method. The study indicated that the presence of noise in the data could be an important reason for the low-prediction accuracy estimates achieved in some of the past studies. These observations, with the fact that most of the past studies used the correlation dimension either as a proof or as a preliminary evidence of chaos, suggested that the hypothesis of deterministic chaos, as the basis in those studies, for hydrological processes was valid and had great practical potential.

13. Radhakrishna, Narayana, Dutt, and Yeragani (2000)

This paper aimed to investigate measures of nonlinear dynamics and chaos theory in relation to heart rate variability in 27 normally controlled subjects in supine and standing postures, and 14 subjects in spontaneous and controlled breathing conditions. They examined minimum embedding dimension (MED), largest Lyapunov exponent (LLE) and measures of nonlinearity (NL) of heart rate time series. MED quantifies the system's complexity, LLE predictability and NL, a measure of deviation from linear processes. There was a significant decrease in complexity ($P<0.00001$), a decrease in predictability ($P<0.00001$) and an increase in nonlinearity ($P=0.00001$) during the change from supine to standing posture. Decrease in MED, and increase in NL score and LLE in standing posture appeared to be partly due to an increase in sympathetic activity of the autonomous nervous system in standing posture. An improvement in predictability during controlled breathing appeared to be due to the introduction of a periodic component.

14. Skjeltorp (2000)

This paper aimed to investigate the validity of the much-used assumption that the stock market returns followed a random walk and were normally distributed. For this purpose the concepts of chaos theory and fractals were applied. Two independent models were used to examine price variations in the Norwegian and US stock markets. The first model used was the range over standard deviation or R/S statistic which tested for persistence or antipersistence in the time series. Both the Norwegian and US stock markets showed significant persistence caused by long-run "memory" components in the series. In addition, an average non-periodic cycle of four years was found for the US stock market. These results were not consistent with the random walk assumption. The second model investigated the distributional scaling behavior of the high-frequency price variations in the Norwegian stock market. The results showed a remarkable constant scaling behavior between different time intervals. This means that there is no intrinsic time scale for the dynamics of stock price variations. The relationship can be expressed through a scaling exponent, describing the development of the distributions as the time scale changes. This description may be important when constructing or improving pricing models in such a way that they coincide more closely with the observed market behaviour. The empirical distributions of high-frequency price variations for the Norwegian stock market were then compared to the Lévy stable distribution with the relevant scaling exponent found by using the R/S- and distributional scaling analysis. Good agreement was found between the Lévy profile and the empirical distribution for price variations less than ±6 standard deviations, covering almost three orders of magnitude in the data. For probabilities larger than ±6 standard deviations, there seemed to be an exponential fall-off from the Lévy profile in the tails which indicated that the second-moment might be finite.

15. Bogaert, Beckers, Ramaekers,and Aubert (2001)

In this study, the correlation dimension (CD) method was related to chaos theory and was used to quantify heart rate variability (HRV). The CD was a measure for the amount of correlations present in the signal. The algorithm used to calculate the CD was based on the method of Grassberger and Proccacia. The method was first validated on signals with known CD and then applied to HRV-signals of heart transplants and an age-matched control group of healthy subjects. The CD of the corresponding surrogate time series was calculated to investigate non-linear correlations in the HRV-signal. Circadian variations of the CD were studied in 20 healthy subjects, including men and women. The results showed that the value of the CD for healthy subjects ranged from 2.12 to 5.53 with a mean value of 4.32. For heart transplants, only a few time series showed a finite value of the CD that varied between 2.10 and 5.60. Also, a significant difference was found between the CD of the original and the surrogate time series in healthy subjects. The CD of women was higher than that of men, and this difference was more pronounced during the night than during the day. This limited study showed that the CD alone could not be used to make a distinction between HRV-signals of healthy subjects and heart transplants. However, the evidence showed that there were non-linear correlations present in the HRV-signal and significant gender and circadian differences in the CD.

16. Panas (2001)

This paper aimed to investigate price behavior in the London Metal Exchange market. Thus, this study tested the two most attractive nonlinear models—long memory and chaos—on six metal commodities to ascertain which model was consistent with the observed metal price nonlinear dynamics. Application of long memory and chaos analysis provided new approaches for assessing the behavior of metal prices. They identified, in tin, a case of chaos. The empirical results in the case of aluminium supported the long memory hypothesis. A short memory model explained the underlying processes of the nickel and lead returns series, while zinc returns reflected an anti-persistent process. To our knowledge, this was one of the first attempts to apply long memory and chaos analysis in the evaluation of the behavior of metal prices.

17. Islam and Sivakumar (2002)

This paper aimed to characterize and predict runoff dynamics, using ideas gained from nonlinear dynamical theory. Daily runoff data observed over a period of 19 years (January 1, 1975–December 31, 1993) at the Lindenborg catchment in Denmark was studied using a variety of techniques. First, the autocorrelation function and the Fourier power spectrum were used as indicators to obtain some preliminary information regarding the runoff behavior. A comprehensive characterization was done next through the correlation integral analysis, the false nearest neighbor algorithm, and the nonlinear prediction method, all using the concept of phase-space reconstruction, i.e., reconstruction of the single-dimensional (or variable) runoff series in a multi-dimensional phase-space to represent its dynamics. The average mutual information method was used to estimate the delay time for the phase-space reconstruction. The exponential decay in the autocorrelation function plot and the sharp spectral lines in the Fourier power spectrum seemed to provide some preliminary indication regarding the possible presence of chaos in the runoff dynamics. The results showed that the (low) correlation dimension (of about 3.76) obtained from the correlation integral analysis, the (low) global dimension (of 4 or 5) obtained from the false nearest neighbor algorithm, and the (low) optimal embedding dimension (of 3) from the nonlinear prediction method were in close agreement with each other, providing convincing evidence regarding the presence of low-dimensional chaotic behavior in the runoff dynamics. The near-accurate predictions achieved for the runoff series (correlation coefficient of about 0.99 and coefficient of efficiency of about 0.98) indicated the appropriateness of the chaotic dynamical approach for characterizing and predicting the runoff dynamics at the Lindenborg catchment.

18. Sakakura (2003)

This paper aimed to evaluate the chaotic nature of snoring sound. Phase space, Poincare section, and time evolutions of Lyapunov exponents were calculated. At the same time, the snoring sound was analyzed by the conventional FFT method. Analytical results were as follows. An attractor of snoring sound was drawn in a four-dimensional space using the embedding theorem of Takens. The results showed that the values of Lyapunov exponent (one of the indices of chaotic nature) were positive, suggesting that snoring sound demonstrates the characteristics of chaos. These nonlinear dynamic methods were expected to provide useful information for better understanding of irregular snoring sounds.

19. Ferreira, Francisco, Machado, and Muruganandam (2003)

In this paper they performed a time series analysis of the model employing tools from statistics, dynamical systems theory and stochastic processes. Using benchmark systems and a financial index for comparison, several conclusions were obtained about the generating mechanism for this kind of evolution. The results showed that the motion was deterministic, driven by occasional random external perturbation. When the interval between two successive perturbations was sufficiently large, one could find low dimensional chaos in this regime. However, the full motion of the MG model was found to be similar to that of the first differences of the SP500 index: stochastic, nonlinear and (unit root) stationary.

20. Sprott (2003)

In this book, Sprott (2003) proposed a general strategy to determine the data to see whether the underlying mechanism was chaotic or random:

(1) Verifying the integrity of the data – Examine a printout or graph to identify outrageously silly mistakes, bad or missing points, and formatting errors to determine the precision (number of significant digits) of the data.

(2) Testing for stationary – Look for obvious trends or low-frequency structure and compare the first half of the data with the second half. Try detrending if necessary, for example by taking log first differences or fitting to a low-order polynomial or superposition of sine waves.

(3) Plotting the data in various ways – Plot X_n versus X_{n-1}, return maps (such as each local maximum versus the previous maximum) and Poincare' sections.

(4) Determining the correlation time or minimum of the mutual information to see if the sampling rate is adequate but not excessive.

(5) If the autocorrelation function oscillates and decays slowly, look for periodicities. Examine the power spectrum for evidence of discrete, perhaps harmonically-related, frequencies. With a broadband spectrum, see if it is a straight line on a log-log or log-linear plot.

(6) Making a space-time separation plot to see if the data record is too short for meaningful analysis.

(7) See if there is a low-dimensional embedding using the method of false nearest neighbors or saturation in the correlation dimension.

(8) If the embedding is sufficiently low, determine the correlation dimension.

Make sure log C(r) versus log$r$ has a scaling (linear) region by plotting its derivative v(r) versus log$r$. Make sure the calculated dimension is insensitive to the embedding dimension and time step. Make sure you have sufficiently many data points for the measured dimension, for example using the Tsonis criterion.

(9) If you have evidence of a low-dimensional attractor, you can then try to calculate at least the largest Lyapunov exponent, entropy, and growth rate of unpredictability. If the dimension is too high, you can try re-predictor or use principal component analysis, keeping only a few dominant components.

(10) If you think you have found chaos in your data, construct some appropriately randomized surrogate data sets and subject them to the same analysis to verify your conclusion and test the statistical significance of your estimates of dimension, entropy, predictability, Lyapunov exponents, and so forth.

(11) If you find evidence of low-dimensional chaos, you can construct model equations and attempt short-term predictions. However, if the dimension is unmeasurably high, you might have correlated noise for which some predictability is possible, and whose power spectrum and probability distribution allow comparison with theoretical models. In real experiments, the latter case is overwhelmingly more common, apart from specially-constructed laboratory experiments governed by simple physical laws, such as the driven pendulums and other nonlinear mechanical or electrical oscillators.

2.2 Prediction of the Chaotic Time Series

2.2.1 State-space prediction

1. Lorenz (1969)

Lorenz (1969) proposed a new prediction method for predicting the weather. The main concept was that if we observe the system for a long time there would be states in the past which were arbitrarily close to the present state, and our prediction, $\hat{X}_{N+1} = X_{n_0+1}$, would be arbitrarily close to the truth, i.e., looking for a sequence of points in the series that most closely resembles the current sequence. Thus, in theory we have a very nice prediction algorithm. It is usually referred to as the "Lorenz method of analogues", because it was proposed as a forecasting method by Lorenz (1969).

2. Kennel and Isabelle (1992)

Kennel and Isabelle (1992) proposed a new prediction method, which faced reality one step at a time. Even if the assumption of an underlying deterministic system is correct, they usually do not measure the actual states X_n but one (or few) quantities, which functionally depend on these states. Most commonly we have scalar measurements

$S_n = S(X_n)$, n=1,….,N.

where, more often than not, the measurement function *S* is as unknow as *F*. Obviously we cannot invert *S* but we can use a delay reconstruction to obtain vectors equivalent to the original ones.

This procedure introduced two adjustable parameters into the prediction method: the delay time τ and the embedding dimension *m*.

2.2.2 State-space averaging

1. Eckmann and Ruelle (1985); Sano and Sawada (1985)

Eckmann and Ruelle (1985) proposed a new prediction method, which worked even better than estimating the local Jacobian matrix at each point along the orbit by least-squares fitting its elements to a collection of nearby points.

2. Farmer and Sidorowich (1987)

Farmer and Sidorowich (1987) proposed a new state-space averaging model, which improved by including a cluster of nearby points rather than just the nearest one and either taking their average (zero-order approximation) or fitting them to a locally linear model (first-order approximation).

3. Sugihara and May (1990)

Sugihara and May (1990) proposed a different approach, which abandoned model equations altogether and looked for points in the time-delay state space that were close to the present point to see how they evolved, and used a weighted average of the images of exactly m+1 neighbors which form a simplex containing the current point.

4. Kantz (1994)

Kantz (1994) proposed a new prediction method, which suggested using not just the nearest neighbor of each point but all points within some small neighborhood.

5. Paparella (1997)

Paparella (1997) proposed a new prediction method, called random analog prediction (RAP), which selects a neighbor randomly with a probability to decrease with its state-space distance.

6. Hegger (1999)

Hegger (1999) proposed a new prediction method, which averaged over a cluster of neighbors and, used every point in the data set, but weighted by e^{-d^2/σ^2_N}, where d was the distance of each point in the m-dimensional embedding from the last known value X_N, and σ_N was the standard deviation of the noise. The prediction was

$$X_{N+k} = \frac{\sum_{n=m}^{N-k} w_n X_{n+k}}{\sum_{n=m}^{N-k} w_n}$$

where

$$w_n = \exp\left[-\frac{1}{\sigma^2_N}\sum_{j=0}^{m-1}(X_{N-j} - X_{n-j})^2\right]$$

Other *weighting functions* (or *kernels*) that fell to zero for large separation such as $w_n = 1/d^{\alpha}$ gave similar results. A "*kernels*" was a weighting function that depended only on the distance to the point.

2.3 Other Relevant Works

2.3.1 Applications on Abnormal Diagnosis

1. Rajkovic, Radivojevic and Timotijevic (1995)

This paper aimed to study the relationship between the qualitative change in the behavior on one side, and the electric (or) magnetic response of the brain itself on the other side. A new method was presented that combined nonlinear dynamics theory, signal processing and information theory for the analysis of electric and magnetic activity of the human brain. Of particular interest was the application of this method for the analysis of event related desynchronization of EEG activity, imaging of brain functions, diagnosis of generalized and local brain disfunctions. Since the brain is a pattern forming, self-organizing dynamical system, the method analyzed spatial and temporal patterns of electric and magnetic fields and compared information contents of each self-organized state as well as the information transfer between them. Each state of brain activity was characterized by its complexity represented by entropy and dimension, which might vary in space, time or simultaneously in space and time. The results showed how patterns in normal subjects differed from the pathological cases representative of generalized and focal dysfunctions and how changes in sensori-motor coordination induced changes in the brain activity as reflected in entropies and dimension. The method might be applied to a local spatial region of the brain as well as for the analysis of transients. Under certain symmetry conditions the method was equivalent to the spatio-temporal wavelet transform. Use of cellular automata in this approach was also presented.

2. Iokibe, et al. (1996, 1997)

This paper aimed to employ the change in chaotic parameters to diagnose the abnormality of automobile engines as well as to discriminate such diseases as diabetes, arrhythmia, and ventricular fibrillatory. The results showed the chaotic approach suitable for real testing.

3. Logan and Mathew (1996)

This paper introduced some of the basic concepts of chaos theory, and then detailed a method for quantifying a fractal dimension from a time series - the correlation dimension. Some of the practical difficulties encountered in measuring the correlation dimension from the correlation integral algorithm were also outlined.

Finally, some experimental results from a rolling element bearing test rig were presented.

2.3.2 Applications on Chaos Control

1.Geldof (1995)

This paper aimed to give form and content to integrated water management. They applied an internal approach, based on the component parts. With this approach they encountered at least three problems: the problem of scale, the problem of level and the problem of assessment. They solved these problems by applying an external approach to integrated water management, in addition to the internal approach. This is possible by application of the theory of complexity. The results showed that they could describe the sum of the processes within integrated water management as a complex adaptive system and a learning and evolving system. If they use this system as a model, they come to the insight that they must not try to achieve as much order as possible, but that they have to look for a good balance between order and chaos. If they find that balance, they are able to adapt water management to the ever changing surroundings. The usefulness of the approach becomes clear if they look at the discussion in urban water management about the applicability of source control and end-of-pipe techniques. The approach provided a strategy for handling uncertainties.

2. Murphy (1996)

This paper used chaos theory to model public relations situations whose salient feature was the volatility of public perceptions. After discussing the central premises of the theory itself, it applied chaos theory to issues management, the evolution of interest groups, crises, and rumors. The results showed that chaos theory was most useful as an analogy to structure persistent image problems and to raise questions about organizational control of public perceptions. Because it emphasized uncertainty, open-endedness, plurality, and change, chaos theory set limits on the purposeful management of volatile issues.

3.Garfinkel and Alan (1997)

This paper presented some applications in the social sciences. They first discussed chaotic behavior in a supply-and-demand order and delivery chain. The focus was not just on documenting the fact of chaotic behavior, but also on understanding the specific mechanisms that caused chaos in such systems, and on the possibilities for ‘chaos control’. They also discussed recent applications to the dynamics of economic markets and game-theoretic models of social processes. The emphasis was on new concepts that had been developed within chaos theory, such as the role of spatial pattern formation, ‘self-organized criticality’ and the idea saying that social systems naturally evolved to ‘the edge of chaos.’

4.Tao, Yang, and Yang (1998)

In this paper they presented the theory of control of chaotic systems using sampled data. Firstly the output of the chaotic system was sampled at a given sampling rate. Then the samples of the output were used to construct control signals. Finally, the control signals were held by holding blocks as the control input of the chaotic system during each sampling duration. During each iteration, the control input kept unchanged. The theoretical results on the asymptotic stability of the controlled chaotic system and the controller were presented. The numerical experimental result on the Lorenz system was used to verify the theoretical results.

5.Oshima and Kosuda (1998)

In this study they developed a method for making hourly forecasts of water demand, using the theory of chaos control and evaluated its usefulness in solving distribution reservoir problems through simulation. The future water level of a reservoir was predicted based on the data obtained by demand forecasting and water pumps were operated to keep the reservoir water level in the goal range. As a result, an adequate balance could be maintained between the two reservoir functions so that the frequency of pump operation could be reduced to improve energy efficiency.

6.Boccaletti, Grebogi, Lai, Mancini, and Maza (2000)

In this paper, they reviewed the major ideas involved in the control of chaos, and presented in detail two methods: the Ott–Grebogi–Yorke (OGY) method and the adaptive method. They also discussed a series of relevant issues connected with chaos control, such as the targeting problem, i.e., how to bring a trajectory to a small neighborhood of a desired location in the chaotic attractor in both low and high dimensions, and pointed out applications for controlling fractal basin boundaries. In short, they described procedures for stabilizing desired chaotic orbits embedded in a chaotic attractor and discussed the issues of communicating with chaos by controlling symbolic sequences and synchronizing chaotic systems. Finally, they gave a review of relevant experimental applications of these ideas and techniques.

7.Harb (2004)

This paper aimed to design a nonlinear controller based on the back stepping nonlinear control theory. The objective of the designed control was to stabilize the output chaotic trajectory by forcing it to the nearest constant solution in the basin of attraction. The author analyzed the dynamics of a permanent magnet synchronous machine (PMSM). The study showed that under certain conditions the PMSM was experiencing chaotic behavior. The result was compared with a nonlinear sliding mode controller. The designed controller based on backstepping nonlinear control was able to eliminate the chaotic oscillations. Also the results showed that the designed controller was mush better than the sliding mode control.

8.Harb and Harb (2004)

The main goal of this paper was to control this chaotic behavior. A nonlinear controller based on the theory of back stepping was designed. The results showed the effectiveness of the designed nonlinear controller in controlling the undesirable and unstable behavior and pulling the PLL back to the in-lock state.

20.Ruan, Gu, and Zhao (2004)

In this paper, they introduced the nonlinear measure of time-continuous system into the control of chaos, and verified that nonlinear measure could characterize the exponential stability and the size of stable basin. They also, based on it and the polar coordinates transformation, derived a general and precise algorithm for determining the radius of stable basin in controlling time-continuous chaotic dynamical system and for estimating the exponential decay of the controlled system converge to the desired goal dynamics. Furthermore, they took the well-known Lorenz, Rösslor system and Chua system as examples to illustrate the implementation of their theory.

2.4 Some Comments

Based on the aforementioned review, some comments are described as follows:

1. It is easy to make mistake to think that a random system is chaotic, and the literature is full of false claims. And the question "Is it chaos?" may be too simplistic since there is a hierarchy of dynamical behaviors with varying amounts of determinism. New analysis techniques have been developed specifically to characterize chaotic systems, and the availability of powerful computers has brought these methods to the desktops of most scientists and researchers. However, chaotic time-series analysis is still more art than science, and there are few sure-fire methods. We need a battery of tests and the conclusions are seldom definitive. New tests are constantly being developed. In traffic flow study, whether there exist chaotic phenomena for the traffic dynamics is rarely studied and it is still not generally clear. If we can demonstrate it with enough evidences then we can utilize chaos theory to elucidate the traffic flow phenomena and to apply it to traffic prediction, incident detection and chaos control.

2. The Lorenz method (1969) looked for a sequence of points in the time series that most closely resembled the current sequence, which was proposed for predicting the weather, but it failed for high-dimensional systems because of the paucity of close analogs. For a deterministic system with sufficient analogs, predictability can be good. With the weighting functions (or kernels), the prediction is always bounded even when predicting many time steps ahead, although the predicted orbit for chaotic data may be periodic. Alternately, use the nearest points in state space to predict the change $\triangle X$ rather than X. The near-term prediction may be improved, but there is then no guarantee that the orbit is bounded (Sprott, 2003).

3. Many previous studies have shown that the chaos approach by means of state space reconstruction has the advantages of prediction. However, different prediction reasoning rationales can come up with various prediction models that might perform quite differently. Therefore, seeking for appropriate chaos prediction reasoning to predict traffic flow dynamics is a challenging issue worthy of exploration.