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Stability conditions for the Bianchi type II anisotropically inflating universes

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Abstract. Stability conditions for a class of anisotropically inflating solutions in the Bianchi type II background space are shown explicitly in this paper. These inflating solutions were known to break the cosmic no-hair theorem such that they do not approach the de Sitter universe at large times. It can be shown that unstable modes of the anisotropic perturbations always exist for this class of expanding solutions. As a result, we show that these set of anisotropically expanding solutions are unstable against anisotropic perturbations in the Bianchi type II space.

Keywords: cosmological perturbation theory; physics of the early universe

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1 Introduction

The inflationary universe is a central phenomena derived from the astronomical observations aim to test, and to understand how the universe might have evolved from a general initial condition into its present state of large-scale isotropy and homogeneity together with an almost flat spectrum of near-Gaussian fluctuations. There should be a brief moment of accelerated expansion during the epoch of the early universe [1]. It has been shown that a simple physically-motivated inflationary scenario can be induced by the acceleration driven by a scalar field with a constant potential. It can also be driven by a higher derivative pure gravity models with natural graceful exit. It is therefore important to find out whether universal acceleration and asymptotic approach to the de Sitter metric always occurs in these models. In fact, a series of cosmic no-hair theorems of varying strengths and degrees of applicability have been proved in support of certain constraints on the field parameters for its occurrence [2–8]. The conformal equivalence between these higher-order theories in vacuum and general relativity in the presence of a scalar field has also been shown in ref. [9–11].

In particular, it was shown that when quadratic terms are added to the Lagrangian of general relativity then new types of cosmological solution arise when $\Lambda > 0$ which have no counterparts in general relativity in the Bianchi type II and type VI_h spaces. [12]. Theses solutions inflate anisotropically and do not approach the de Sitter spacetime at large times. They hence provide counter-examples to the expectation that a cosmic no-hair theorem will continue to hold in simple higher-order extensions of general relativity. Other consequences of these higher-order theories have also been studied in [13–17].

A pure gravity theory which is quadratic in the scalar curvature and the Ricci tensor was considered in ref. [12] for the model consists of the 4-dimensional gravitational action

$$S_{\mathrm{BH}} = \frac{1}{2}\int d^4x\sqrt{g}\;\; L = \frac{1}{2}\int d^4x\sqrt{g}\left(R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} - 2\Lambda\right). \tag{1.1}$$

The Einstein equations can be shown to be [12]

$$H_{\mu\nu} \equiv G_{\mu\nu} + \Phi_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \tag{1.2}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - Rg_{\mu\nu}/2$ and

$$\Phi_{\mu\nu} \equiv 2\alpha R \left(R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} \right) + (2\alpha + \beta) \left(g_{\mu\nu} D^2 - D_\mu D_\nu \right) R$$
$$+ \beta D^2 \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + 2\beta \left(R_{\mu\sigma\nu\rho} - \frac{1}{4} g_{\mu\nu} R_{\sigma\rho} \right) R^{\sigma\rho}. \quad (1.3)$$

Here the tensor $\Phi_{\mu\nu}$ incorporates the deviation from regular Einstein gravity related to the coupling constants α and β.

A new classes of exact solutions are found in a spatially homogeneous universes of the Bianchi types II (BII) space given by the metric

$$\mathrm{d}s_{II}^2 = -\mathrm{d}t^2 + e^{2bt} \left[\mathrm{d}x + \frac{a}{2} (z \mathrm{d}y - y \mathrm{d}z) \right]^2 + e^{bt} (\mathrm{d}y^2 + \mathrm{d}z^2), \quad (1.4)$$

where

$$a^2 = \frac{11 + 8\Lambda(11\alpha + 3\beta)}{30\beta}, \quad b^2 = \frac{8\Lambda(\alpha + 3\beta) + 1}{30\beta}. \quad (1.5)$$

Here a and b are some constant functions of α, β and Λ. These solutions are spacetime homogeneous with a 5-dimensional isotropy group. They have a one-parameter family of 4-dimensional Lie groups [18–20]. Interesting discussions related to these solutions can be found in ref. [12].

Note that the no-hair theorem for Einstein gravity states that the presence of a positive cosmological constant drives the late-time evolution towards the de Sitter spacetime for Bianchi types $I - VIII$ spaces [6] if the matter sources obey the strong-energy condition. It has also been shown that the cosmic no-hair theorem cannot be proved and counter-examples exist if this condition dose not hold exactly [7, 21–24].

The Bianchi type solutions given above inflate in the presence of a positive cosmological constant Λ. They are, however, neither de Sitter, nor asymptotically de Sitter. Note that it has been shown that this solution is also a solution to a Brans-Dicke type scalar tensor theory in the Bianchi type II background space. It is shown, however, that these anisotropically inflating solutions are not stable under field perturbations [25].

Note that from the point of view of an effective theory of gravity, an action with quadratic curvature terms as the one considered here, should be understood as some perturbative correction to Einstein gravity suitable in some energy scale. Theories quadratic in the curvature give field equations which are higher order than two in time derivatives and generally have run away solutions. The runaway solutions are supposed to be unphysical because they grow with time scales which are beyond the limits of validity of the theory. Thus, in this context not all solutions have physical significance [27]. Note that the BH expanding solution does not have a limit in general relativity (e.g. it is not defined for $\beta \to 0$). An isotropic example of this is the Starobinsky inflation [28]. We will show in this paper that the anisotropically expanding BH solution (1.4) with constraints (1.5) is in fact unstable under field perturbations.

Due to the complexity of the equations of motion it was difficult to extract the stability information for these non-perturbative solutions in the higher derivative models. We will start with a compact and model-independent formula for the field equations [26] in the Bianchi type II background space. Anisotropic perturbations can therefore be performed more easily in this approach. As a result, we will show that the system always admits unstable modes for all $x \equiv a^2/b^2$. In addition, we will also show that unstable mode also exists if $3\alpha + \beta < 0$.

Consequently, we will be able to show that these new classes of anisotropically expanding solutions are not stable solutions in the Bianchi type II space.

This paper will be organized as follows: (i) We will first present a model-independent formulae for the field equations in a BII metric space. These new set of equations can be shown to agree with the $H_{tt} = 0$ and $H_{11} = 0$ components of the eq. (1.2) for the quadratic curvature model (1.1). It can also be verified directly that BH solutions are also solutions to these new equations. (ii) Anisotropically perturbations can be obtained by perturbing these two field equations against any BII background metric. The model-independent, linear order perturbation equations will be presented accordingly. (iii) A complete set of perturbation equations against the BH background metric solutions (1.4) can therefore be obtained directly. (iv) As a result, we end up with a polynomial equation of degree 6 for the perturbation equations. Fortunately, this polynomial can be factorized and some useful information can thus be extracted for the stability analysis. (v) Finally, we can show that unstable modes always exist for these perturbations. Therefore, the BH solution is always unstable against these anisotropic perturbations. Conclusions will also be drawn at the end of this paper.

2 Stability conditions for the higher derivative BH solutions

2.1 The BII Metric and the field equations

The BH solution (1.4) can also be written as, in a more familiar form,

$$ds^2 = -dt^2 + a_1^2(t)dr^2 + g_{mn}dx^m dx^n \tag{2.1}$$

with $(x^0, x^1, x^2, x^3) = (t, r, z, \phi)$, $a_1^2(t) = \exp[bt]/a^2$ and $a_2^2(t) = \exp[2bt]/a^2$ where a and b denote some constant functions of α and β given by the eq. (1.5). Here

$$g_{mn} = \begin{pmatrix} a_2^2(t) & ra_2^2(t) \\ ra_2^2(t) & a_1^2(t) + r^2 a_2^2(t) \end{pmatrix}. \tag{2.2}$$

The Lagrangian of the system $L(\dot{H}_i, H_i, E)$ can be shown to be function of $\dot{H}_i, H_i$and E only in the BII background space. For example, $R = 2(2A + B + C + 2D)$ with $A = \dot{H}_1 + H_1^2$, $B = \dot{H}_2 + H_2^2$, $C = H_1^2 - 3E$and $D = H_1 H_2 + E$ as shorthanded notation. Here $H_i = \dot{a}_i/a_i$ denote the Hubble parameters and $E \equiv a_2^2/(4a_1^4)$ denotes a function of the scale factors a_i. Another example is that the effective Lagrangian of the higher derivative model (1.1) given by

$$L = \frac{1}{2}\left(R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} - 2\Lambda\right) \tag{2.3}$$

can be shown to be:

$$\begin{aligned} L &= (2A + B + C + 2D) + 2\alpha(2A + B + C + 2D)^2 \\ &\quad + \frac{\beta}{2}\left[(2A + B)^2 + 2(A + C + D)^2 + (B + 2D)^2\right] - \Lambda. \end{aligned} \tag{2.4}$$

as functional of the scale factors $\dot{H}_i, H_i$and E.

The perturbation equations can be derived from perturbing the field equations $H^\mu_\nu = 0$ given by eq. (1.2) in the presence of the BII background space. The formal derivation is quite complicate. Fortunately, there is a more illuminating and simple method by perturbing the

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field equations derived from varying the effective action given by (2.4). The final expressions for the field equation $H^t_t = 0$ and $H^1_1 = 0$ can be shown to be:

$$D_0 L \equiv L + H_i \left(\frac{\partial}{\partial t} + 3H \right) L^i - H_i L_i - \dot{H}_i L^i = 0, \tag{2.5}$$

$$D_1 L \equiv 2L - 4E\partial_E L + \left(\frac{\partial}{\partial t} + 3H \right)^2 L^i - \left(\frac{\partial}{\partial t} + 3H \right) L_i = 0. \tag{2.6}$$

Here $L_i \equiv \delta L/\delta H_i$, $L^i \equiv \delta L/\delta \dot{H}_i$and $3H \equiv 2H_1 + H_2$. We can also show that the BH solution does solve above equations. Note that we need to restore the lapse function $d(t)$ $(-dt^2 \to -d^2(t)dt^2)$ before varying the g_{tt} component for the $H_{tt} = 0$ field equation [26]. This is because that the g_{tt} information has been removed from the BII metric (2.1) by a time coordinate redefinition for convenience in the literature. We need to restore the lapse function in order to reconstruct the field equations $H_{tt} = 0$ [26]. Once the variation is done, we can set $d(t) = 1$ after replacing the effect of $d(t)$ by H_i through a correspondence relations between the field variables. In addition, the other nonvanishing components of the field equations $H^i_j = 0$ can be shown to be derivable from the field equations (2.5) and (2.6) following the Bianchi identity $D_\mu G^\mu_\nu = 0$ and the generalized energy momentum conservation law. Note also that we can easily show that the field equations (1.2) and (1.3) agree with eqs. (2.5) and (2.6) for the BH model (1.1).

2.2 Anisotropic metric perturbations

For convenience, we will write $a_i(t) = \exp[bA_i(t)]/a$ for the general BII metric and $\delta A_i = k_i \exp[b\nu t]$ for the metric perturbations against the BII background metric with $A_i(t) = A^0_i =$ constant. For example, $(A^0_1, A^0_2) = (1/2, 1)$ stands for the BH background solution of the action (1.1). We will first derive a set of model-independent perturbation equations in an arbitrary background metric space with $A_i(t) = A^0_i$. As a result, the parameter $b\nu$ will stand for the decay (or expansion) constant of the perturbation mode given by $\delta A_i = k_i \exp[b\nu t]$. We will show that positive modes with $\nu > 0$ always exist for the perturbations against the expanding BH background metric solution. Therefore, this will prove that the BH solutions are always unstable.

By writing he perturbation equations $\delta D_0 L$ and $\delta D_1 L$ can be shown to be:

$$\begin{aligned} \mathcal{D}_0 \delta A_0 &= \begin{pmatrix} I_1 + 2J - 2K & I_2 + J + K \\ F_1 + 2G - 2H & F_2 + G + H \end{pmatrix} \begin{pmatrix} \delta A_1 \\ \delta A_2 \end{pmatrix} \\ &\equiv \begin{pmatrix} B_{11} & B_{12} \\ 2b^2 B_{21} & 2b^2 B_{22} \end{pmatrix} \begin{pmatrix} \delta A_1 \\ \delta A_2 \end{pmatrix} = 0, \end{aligned} \tag{2.7}$$

which also defines the matrix components B_{ij}. Here

$$I_j = bH_i L^{ij} \nu^3 + \left(H_i L^i_j + 2bH_i L^{ij} - H_i L^j_i \right) \nu^2 + \left(2L^j + 2H_i L^i_j - \frac{1}{b} H_i L_{ij} \right) \nu \,, \tag{2.8}$$

$$J = \frac{1}{b} H_i L^i \nu \,, \tag{2.9}$$

$$K = \frac{x}{2} (bH_i L^i_E \nu + L_E + 2bH_i L^i_E - H_i L_{iE}) \,, \tag{2.10}$$

$$\begin{aligned} F_i &= L^{1i} \nu^4 + \frac{1}{b} (L^1_i + 4bL^{1i} - L^i_1) \nu^3 + \frac{1}{b^2} (2L^i + 4b^2 L^{1i} + 4bL^1_i - a^2 L^i_E - L_{1i} - 2bL^i_1) \nu^2 \\ &\quad + \frac{1}{b^3} \left(2L_i + 4b^2 L^1_i - a^2 L_{iE} - 2bL_{1i} \right) \nu \,, \end{aligned} \tag{2.11}$$

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$$G = \frac{1}{b^2}\left[L^1\nu^2 + \left(4L^1 - \frac{1}{b}L_1\right)\nu\right], \tag{2.12}$$

$$H = \frac{x}{2}\left[L_E^1\nu^2 + \left(4L_E^1 - \frac{1}{b}L_{1E}\right)\nu + 4L_E^1 - \frac{2}{b^2}L_E - xL_{EE} - \frac{2}{b}L_{1E}\right] \tag{2.13}$$

are functions of the field parameters. Moreover, we have also defined $L^{ij} = \delta^2 L/\delta\dot{H}_i\delta\dot{H}_j\,|_{A_i(t)=A_i^0}$, $L_j^i = \delta^2 L/\delta\dot{H}_i\delta H_j\,|_{A_i(t)=A_i^0}$, $L_{ij} = \delta^2 L/\delta H_i\delta H_j\,|_{A_i(t)=A_i^0}$, $L_E^i = \delta^2 L/\delta\dot{H}_i\delta E\,|_{A_i(t)=A_i^0}$, $L_{iE} = \delta^2 L/\delta H_i\delta E\,|_{A_i(t)=A_i^0}$, $L_E = \delta L/\delta E\,|_{A_i(t)=A_i^0}$ and $L_{EE} = \delta^2 L/(\delta E)^2\,|_{A_i(t)=A_i^0}$ for convenience. The differential equations can be restored by replacing each ν with ∂_t/b. In addition, we have extracted the scale parameter $2b^2$ from the matrix components in eq. (2.7) when we define B_{21} and B_{22}. Note also that the matrix eq. (2.7) is a model-independent close form of the stability equations for any anisotropically evolving solutions in the BII background space.

2.3 Perturbation equations in the BH background metric space

By identifying the background metric $(A_1^0(t), A_2^0(t)) = (1/2, 1)$ we can compute the parameters $I_j, F_j, J, K, G$and H of the BH model with L given by (2.3). In addition, we note that there is an identity relating the parameters $\alpha_1 \equiv \alpha b^2$, $\beta_1 \equiv \beta b^2$and $x \equiv a^2/b^2$:

$$1 = [x - 11, 3x - 3]. \tag{2.14}$$

Here $[A, B] \equiv A\alpha_1 + B\beta_1$. This follows directly from the constraints in eq. (1.5). This formula is useful in cancelling all the α and β independent terms in the perturbation equations.

We can also write the perturbation equation (2.7) as

$$\begin{aligned}\mathcal{D}_1\delta A_1 &\equiv \begin{pmatrix} B_{11}/2 & B_{12} \\ B_{21}/2 & B_{22}\end{pmatrix}\begin{pmatrix} 2\delta A_1 \\ \delta A_2\end{pmatrix} \\ &= \begin{pmatrix} I_1/2 + J - K & I_2 + J + K \\ (F_1/2 + G - H)/2b^2 & (F_2 + G + H)/2b^2\end{pmatrix}\begin{pmatrix} 2\delta A_1 \\ \delta A_2\end{pmatrix} = 0,\end{aligned} \tag{2.15}$$

after identifying the BH solutions with $a_1^2(t) = \exp[bt]/a^2$ and $a_2^2(t) = \exp[2bt]/a^2$. Note that nontrivial solution δA_i exists, from the eq. (2.15), only when

$$\begin{aligned}\det\mathcal{D}_1 &= \det\begin{pmatrix} B_{11}/2 & B_{12} \\ B_{21}/2 & B_{22}\end{pmatrix} = \det\begin{pmatrix} B_{11}/2 & B_{12} + B_{11}/2 \\ B_{21}/2 & B_{22} + B_{21}/2\end{pmatrix} \\ &\equiv \det\begin{pmatrix} A_{11} & A_{12}\nu \\ A_{21} & A_{22}\nu\end{pmatrix} = 0.\end{aligned} \tag{2.16}$$

Note that the ν-independent terms only appear in the functions K and H. Therefore the ν-independent terms are eliminated by writing $A_{12}\nu = B_{12} + B_{11}/2$ and $A_{22}\nu = B_{22} + B_{21}/2$. As a result, the determinant $\det\mathcal{D}_1$ can be derived from the definition

$$\begin{pmatrix} A_{11} & A_{12}\nu \\ A_{21} & A_{22}\nu\end{pmatrix} = \begin{pmatrix} I_1/2 + J - K & I_1/2 + I_2 + 2J \\ (F_1/2 + G - H)/2b^2 & (F_1/2 + F_2 + 2G)/2b^2\end{pmatrix}. \tag{2.17}$$

Therefore, nontrivial solutions exist only when $\det\mathcal{D}_1 = (A_{11}A_{22} - A_{12}A_{21})\nu = 0$.

The parameters I_i, F_i, J, K, G and H of the BH model can be shown to be:

$$I_1 = 2[16,5]\nu^3 + 2[28,9]\nu^2 - 2[30,3x+8]\nu \,, \tag{2.18}$$
$$I_2 = 2[8,3]\nu^3 + [36,13]\nu^2 - 2[18,3x+5]\nu \,, \tag{2.19}$$
$$J = 12[0,x]\nu \,, \tag{2.20}$$
$$K = [-8x,0]\nu + 3x[2,1] \,, \tag{2.21}$$
$$b^2F_1 = 4[8,3]\nu^4 + 16[8,3]\nu^3 + 8[2x+11,5]\nu^2 + 4[10x-20,3x-4]\nu \,, \tag{2.22}$$
$$b^2F_2 = 4[4,1]\nu^4 + 18[4,1]\nu^3 + 2[4x+28,8-x]\nu^2 + 8[3x-6,x-1]\nu \,, \tag{2.23}$$
$$b^2G = 2[0,5x-1]\nu^2 + 2[0,7x-2]\nu \,, \tag{2.24}$$
$$b^2H = -4x[2,1]\nu^2 - 4x[3,2]\nu + 2x[4-2x,-3-6x] \,. \tag{2.25}$$

Note that we have extracted the scale parameters $1/b^2$ from the functions F_i, G and H for convenience in writing above list. We can therefore derive the following matrix components for the model (1.1):

$$A_{11} = [16,5]\nu^3 + [28,9]\nu^2 + [8x-30,9x-8]\nu - 3x[2,1] \,, \tag{2.26}$$
$$A_{12} = [32,11]\nu^2 + [64,22]\nu + [-66,15x-18] \,, \tag{2.27}$$
$$A_{21} = [8,3]\nu^4 + [32,12]\nu^3 + [8x+22,7x+9]\nu^2 + 2[8x-10,7x-3]\nu + x[2x-4,6x+3] \,, \tag{2.28}$$
$$A_{22} = [16,5]\nu^3 + [68,21]\nu^2 + [8x+50,9x+16]\nu + [22x-44,21x-12]. \tag{2.29}$$

2.4 Stability conditions

Note that $\det \mathcal{D} \equiv \det \mathcal{D}_1/\nu = A_{11}A_{22} - A_{12}A_{21} = 0$ is a polynomial equation of degree 6 in ν. The coefficients of this polynomial equation also depends on the choices of α and β. In fact the polynomial equation $A_{11}A_{22} - A_{12}A_{21} = 0$ can be factorized as

$$\det \mathcal{D} = -2F(\nu)\beta_1 \left[6\nu^2 + 12\nu + x - 11, \;\; 2\nu^2 + 4\nu + 3x - 3\right] = 0 \tag{2.30}$$

with

$$F(\nu) = 2\nu^4 + 8\nu^3 + (5x+7)\nu^2 + 2(5x-1)\nu + 15x(x+1) \tag{2.31}$$

as a polynomial of degree 4. $F(\nu) = 0$ can be solved by noting that

$$8F(\nu) = [4\Delta_1 + (5x-1)]^2 + (95x^2 + 130x - 1) \tag{2.32}$$

with $\Delta_1 = \nu^2 + 2\nu$. Therefore the polynomial equation $\det \mathcal{D} = 0$ has 4 different solutions

$$\nu = \nu_\pm \equiv -1 + \frac{1}{2}\left[5 - 5x \pm \sqrt{1 - 130x - 95x^2}\right]^{1/2} \,, \tag{2.33}$$
$$\nu = \tilde{\nu}_\pm \equiv -1 - \frac{1}{2}\left[5 - 5x \pm \sqrt{1 - 130x - 95x^2}\right]^{1/2} \tag{2.34}$$

from the equation $F(\nu) = 0$. There are also two additional solutions

$$\nu = \nu_5 \equiv -1 + \left[\frac{2(3\alpha_1 + \beta_1) - 1}{2(3\alpha_1 + \beta_1)}\right]^{1/2} \,, \tag{2.35}$$
$$\nu = \nu_6 \equiv -1 - \left[\frac{2(3\alpha_1 + \beta_1) - 1}{2(3\alpha_1 + \beta_1)}\right]^{1/2} \tag{2.36}$$

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from the equation

$$\left[6\nu^2 + 12\nu + x - 11,\ \ 2\nu^2 + 4\nu + 3x - 3\right] = 1 + 2\Delta_1(3\alpha_1 + \beta_1) = 0. \tag{2.37}$$

Note that there is a trivial solution $\nu = 0$ and another un-physical solution $\beta_1 = 0$.

3 Conclusions

The stability properties of these solutions can be shown to be helpful in determining whether or not the BH solution is stable against small perturbations. Note that the system is stable against any particular mode of the perturbations only when these perturbation solutions do not obey the inequality $\nu > 0$. It is apparent that unstable mode exists only when $\nu > 0$. In particular, it can be shown that $\nu = \tilde{\nu}_\pm$ and $\nu = \nu_6$ are all stable models.

In addition, we can also show that the reality condition $\sqrt{1 - 130x - 95x^2} \in \mathbf{R}$ remains valid only when $a^2/b^2 = x \leq x_+ \equiv (12\sqrt{6} - 13\sqrt{5})/19\sqrt{5} \sim 0.0076495$. Therefore, we can show that

$$\nu_\pm = -1 + \frac{1}{2}\left[5 - 5x \pm \sqrt{1 - 130x - 95x^2}\right]^{1/2} > 0 \tag{3.1}$$

provided that $x < 1/5$. Hence we conclude that $\nu_\pm > 0$ if $0 < a^2/b^2 = x < x_+$ because that $x_+ < 1/5$. In addition, we can also show that Re $[\nu_\pm] > 0$ if $x > x_+$. Since Re $[\nu_\pm] > 0$ implies that $\nu_\pm$ are oscillating divergent modes. Therefore the system is always unstable for all x in the presence of the BII background solutions found in ref. [12].

Moreover, we can also show that

$$\nu_5 = -1 + \left[\frac{2(3\alpha_1 + \beta_1) - 1}{2(3\alpha_1 + \beta_1)}\right]^{1/2} > 0 \tag{3.2}$$

if $3\alpha_1 + \beta_1 < 0$. Therefore, an additional unstable mode $\nu_5 > 0$ exists if $3\alpha_1 + \beta_1 < 0$.

In conclusion, the whole classes of BH solutions admit three stable modes $\tilde{\nu}_\pm$, ν_6 and ν_5 if $3\alpha_1 + \beta_1 > 0$. There are also two unstable modes $\nu_\pm$ when we perturb the BH solutions anisotropically in the BII background space. In addition, the ν_5 mode becomes unstable mode if $3\alpha_1 + \beta_1 < 0$. Therefore the system can only remain stable for a brief moment before the unstable modes $\nu = \nu_\pm$ dominant the expanding process even if the energy conditions are violated. In contrast to the stability conditions for the same anisotropically expanding solutions found in the Brans-Dicke model [25], stable modes do exist in the higher derivative models.

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