國立交通大學

管理科學系

博 士 論 文

No. 046

已實現波動度於動態期貨避險之應用

Applications of Realized Volatility for Futures Hedging

研 究 生：賴雨聖

指導教授：許和鈞　教授

中 華 民 國 九 十 八 年 十 一 月

國立交通大學

管理科學系

博 士 論 文

No. 046

已實現波動度於動態期貨避險之應用

Applications of Realized Volatility for Futures Hedging

研 究 生：賴雨聖

研究指導委員會：沈華榮　教授

謝國文　教授

鍾惠民　教授

指導教授：許和鈞　教授

中 華 民 國 九 十 八 年 十 一 月

已實現波動度於動態期貨避險之應用

Applications of Realized Volatility for Futures Hedging

研 究 生：賴雨聖　　Student：Yu-Sheng Lai

指導教授：許和鈞　　Advisor：Her-Jiun Sheu

國 立 交 通 大 學

管 理 科 學 系

博 士 論 文

A Dissertation

Submitted to Department of Management Science

College of Management

National Chiao Tung University

in Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

in

Management

November 2009

Hsin-Chu, Taiwan, Republic of China

中華民國九十八年十一月

已實現波動度於動態期貨避險之應用

研究生：賴雨聖　　　　　　　　　　　　　　　　　　　　　　　　指導教授：許和鈞

國立交通大學管理科學系博士班

中文摘要

本論文旨在應用已實現波動度(realized volatility)於動態避險議題上。首先我們提出新的以高頻日內資訊為基礎之多變量波動度模式，並將此模式應用至動態避險比率之估計上。在實證分析上，我們以美國S&P 500期貨為研究對象，探討此新的避險模式是否可以改進期貨避險之績效。與傳統以低頻資訊為基礎之避險模式比較後發現，新的避險模式不但可以大幅降低投資組合在樣本外之風險暴露額，並可為避險者產生正面的經濟價值。探究其主因，主要是因為已實現波動度可較報酬率平方(return squares)提供更精確之波動度估計值，以致在波動度(或是避險比率)的預測上將得以較傳統方式有較佳的表現。

此外我們亦探討如何將此方法應用於事後(ex-post)避險績效之評估上。將Andersen et al. (2005, 2006)之研究成果應用至期貨避險議題上，我們建構之已實現避險比率得以用於評估各種事前(ex-ante)避險模式之預測能力，同時漸進理論並提供衡量避險比率精確度之方法。另者，已實現避險效果亦可用於衡量期貨避險之績效。

本論文最後探討已實現避險比率之動態性質。藉由門檻自我迴歸模式之分析，我們得以研究避險比率是否存在門檻效果並探討其在不同狀態下之動態性質。應用Hansen (1996)提出之拔靴檢定方法，我們發現門檻效果確實存在於已實現避險比率中。當避險比率高(低)於門檻變數的情況下，避險比率本身將會有較低(高)的波動度。門檻效果以及正向自我迴歸現象的存在均顯示避險比率應是與時變動(time-varying)的，因此實證結果支持動態避險比率假說。

藉由日內高頻資料的應用，已實現波動度提供我們一個不需模式設定(model-free)的方法來估計不可觀測到的波動度指標。本論文的研究成果指出，高頻日內資料的使用將可為許多財務上的議題，例如資產配置或期貨避險等，開啟另一嶄新的研究題材。

關鍵辭：動態避險比率；多變量波動度模式；已實現波動度；避險績效。

Applications of Realized Volatility for Futures Hedging

Student: Yu-Sheng Lai Advisor: Her-Jiun Sheu

Department of Management Science

National Chiao Tung University

Abstract

The dissertation applies the realized volatility (RV) approach to the futures hedging problem. Firstly, we propose a new class of multivariate volatility models encompassing RV estimates to estimate the risk-minimizing hedge ratio, and compare the performance of the proposed models with those generated by return-based models. In an out-of-sample context with a daily rebalancing approach, the empirical results show that improvement can be substantial when switching from daily to intraday. This essentially comes from the advantage that the intraday-based RV potentially can provide more accurate daily covariance matrix estimates than RV utilizing daily prices.

Next, we describe ex-post measures for assessing ex-ante hedge ratio estimates. Applying the realized beta framework of Andersen et al. (2005, 2006), the realized hedge ratio, realized hedging effectiveness, and the asymptotic confidence interval are constructed. The realized hedge ratio, which is consistent with the integrated hedge ratio, provides a natural benchmark for assessing the forecasting ability of any ex-ante hedge ratio estimates. Meanwhile, the asymptotic distribution provides insights into the precision of the realized hedge ratio. Furthermore, the realized hedging effectiveness provides an ex-post estimate for the integrated hedging effectiveness.

Then, the dynamics of the realized hedge ratio is investigated via a two-regime Self-Exciting Threshold Autoregressive (SETAR) model. The SETAR is tackled with a linear Autoregressive (AR) model if the threshold effect is not significant. Empirical results conclude the realized daily hedge ratio is characterized as regime-dependent dynamics and is likely to be positively autocorrelated so that the usual assumption of constant hedge ratio seems inappropriate.

The RV approach, which utilizes finer information in intraday high-frequency data, provides a direct and consistent technique for estimating the latent volatility without the need for relying on explicit models. Our investigation may provoke further study on the benefits of utilizing intraday information in volatility modeling, which is relevant to asset allocating and futures hedging.

Keywords: Dynamic hedge ratio; Multivariate volatility model; Realized volatility; Hedging performance.

致謝辭

在交大管科系博士班的學習過程，隨著博士論文的付梓即將劃上句點。回想剛進入校園修業時的諸多挑戰，到畢業喜悅收成這段時間的點點滴滴，這一切都要感謝許多人對我的提攜與幫助。

本論文能順利完成，首先最要感謝的就是我的指導老師－許和鈞教授，一路走來，老師總是不厭其煩地叮嚀我論文的進度，並隨時教導我正確的研究態度與方式，我的研究成果能順利發表於學術期刊及研討會上，這都需感謝許老師對於我的辛勤指導跟協助。另外我也要特別感謝交通大學鍾惠民教授、中研院經濟所周雨田研究員及陳宜廷研究員，多次透過課堂以及私下的互動機會，給予我學術上的引導與建議，並讓我了解身為研究者應有之積極態度與堅持。此外我也要感謝我的碩士論文指導老師－盧陽正教授，由於盧老師的鼓勵與推薦，讓我累積更多的本錢與勇氣能繼續攻讀博士學位，我亦永遠銘記在心。當然我的博士論文能夠完成，也要感謝交通大學謝國文教授、中央大學周冠男教授以及聯合大學林美貞教授在百忙中抽空擔任我的論文口試委員，並給予我許多寶貴的建議與方向，讓本篇論文更臻周延。

我也要感謝一群好友：交大財金所陳煒朋、劉炳麟、陳清和；交大管科系徐淑芳、王若蓮、張玲玲、魏裕珍等同窗；交大經管所劉志良；以及暨南大學陳永泓。感謝你們的鼓勵與陪伴，讓我苦悶的研究生活添加了許多樂趣及回憶。同時也要感謝管科系林碧梧小姐與葉秀敏小姐在行政事務上的協助，讓我可以省卻許多繁瑣的業務手續。

最後感謝我的家人，由於你們的支持，使得我在求學的路上，少了許多的擔憂。老婆珮琪是我這一路上最佳的諮詢者，除了擔起經濟重任外並身負小女瀅如的教養全責。感謝岳父岳母接手小孩日間的褓母工作，免去我們夫妻倆許多煩惱。最後將本論文獻給我摯愛的父母親及家人，感謝您無怨無悔的養育與無時無刻的關懷照顧，讓我能專注於課業研究中。從小到大的求學路上，謝謝您們多年來的支持與鼓勵，我要把我所有的驕傲與你們分享。由衷感懷之情，溢於言表！

賴雨聖

民國九十八年十一月中

Table of Contents

List of Tables

List of Figures

Chapter 1. Introduction

1.1. Motivation

Futures contracts are important hedging instruments for hedgers. By taking opposite positions in spot and futures markets, the price risk of a spot position can be reduced. Hence, theoretical and empirical aspects of a futures hedge have been the focus of much academic research. At the theoretical level, the hedging theories have shown that a hedge is the optimal when a hedger uses an optimal hedge ratio, e.g., Johnson (1960). Traditionally, this hedge ratio is derived by the portfolio approach via an expected-utility maximization scheme.[1] With some additional restrictions, the minimum-variance hedge ratio, which is equivalent to the covariance of spot and futures over the variance of futures, is generally the optimal, e.g., Benninga et al. (1983). Due to the simplicity, this preference-free hedge ratio is easily to be implemented so that it has been widely adopted in the empirical studies.

At the empirical level, a considerable amount of studies on futures hedging have focused on modeling the joint distribution of spot and futures prices and applying the results to estimate the optimal hedge ratio. While the early studies assume that the hedge ratio is constant over time (e.g., Ederington, 1979), recent studies have documented that the joint distribution, and hence the hedge ratio, should be time dependent due to the time-varying nature of risks (e.g., Kroner & Sultan, 1993).[2] Since then, varieties of multivariate volatility techniques have been applied. Baillie and Myers (1991), Myers (1991), Kroner and Sultan (1993), Brooks et al. (2002), Lien et al. (2002), and Lien and Yang (2006) are examples of studies that apply gen-

---

[1] On the other hand, the development of stochastic dominance theory has also facilitated the implementation of a futures hedge. An overview of this application could be found in Lien and Tse (2002).

[2] With the aim of reducing risk, this hedge ratio is equivalent to the ratio of the conditional covariance between spot and futures over the conditional variance of futures.

eralized autoregressive conditional heteroscedasticity (GARCH) models.[3] As a result, the central issue in the context of a dynamic hedge is to provide the conditional covariance matrix forecast that can characterize the dynamics of the distribution more realistically.

Conventionally, the standard GARCH conditional covariance matrix forecast is simply specified as some functions of its past values as well as outer product of past daily or weekly returns.[4] In other words, by only utilizing daily or weekly price information, this category of volatility models provides relatively convenient and traceable method to give the forecast. Nevertheless, even with correctly specified models, it has been shown that the GARCH-type forecasts do not fully mimic the properties of integrated covariance matrix, which serves as an ideal theoretical ex-post benchmark for assessing the quality of ex-ante covariance matrix forecasts (see Andersen et al, 2006). One possible reason of this imperfect forecast may come from that the low frequency data do not fully convey all the relevant information so that the outer product of past returns itself is a nosy proxy of the multivariate volatility.

To judge the success of a futures hedge, the hedging effectiveness (HE) of Ederington (1979) has been extensively adopted in the empirical studies as a benchmark for hedging performance and as a measure to select the best hedging method. This traditional measure defines the effectiveness of a hedge by calculating the percentage reduction from the variance of the spot position to the variance of the hedged portfolio. Empirical results on futures hedging using this unconditional HE has shown that the simple ordinary-least-square (OLS) regression method generally has a best performance in sample, and the superiority of this OLS method is also supported by some out-of-sample comparisons.[5] For example, Kroner and Sultan (1993)

---

[3] In addition to the GARCH framework, other studies use stochastic volatility models (e.g., Lien & Wilson, 2001).

[4] The standard multivariate GARCH models specify the conditional covariance matrix by either generalizing the univariate GARCH models of Bollerslev (1986) or by combining some univariate GARCH models. The former includes the VECH model of Bollerslev et al. (1988) and the BEKK (Baba-Engle-Kraft-Kroner) model of Engle and Kroner (1995). The latter includes the constant conditional correlation (CCC) model of Bollerslev (1990), the dynamic conditional correlation (DCC) model of Engle (2002), and the copula-based GARCH model of Patton (2004, 2006). A review of a wide range of multivariate GARCH models is referred to Bauwens et al. (2006).

[5] It is noted that the variance-based hedging effectiveness is the focus of this study although Cotter and Hanly

indicate that the OLS method outperforms all other constant hedge methods in a within-sample context; and, Lien et al. (2002) indicate that the OLS hedge even can beat the constant conditional correlation GARCH (CCC-GARCH) hedge in an out-of-sample context. Lien (2005a,b, 2008, 2009) shows that the OLS hedge tends to outperform others, such as the naïve, error correction (EC), or the GARCH hedges, for within-sample comparisons because the hedging effectiveness of Ederington (1979) only considers the proportional reductions in the unconditional variance. Hence, whether the dynamic hedge can surpass the static OLS is still studied both theoretically and empirically, although the literature has identified with the hedge ratio should be time varying as the new information has arrived to the market (e.g., Kroner & Sultan, 1993).

1.2. Dynamic Futures Hedge and Hedging Effectiveness

Consider a one-period futures hedging problem. Suppose an investor (hedger) who longs a fixed spot portfolio at beginning would like to reduce the price risk of the spot at the end of the period. To achieve this goal, he may go to a futures market to short a proportion of futures contracts. Usually, nearby contracts are used due to liquidity concerns. Let S_t and F_t be the logarithmic prices of spot and futures, respectively, at time t; and, β_t be a hedge ratio, which is defined by the amount of futures per unit spot, at time t. Then the realization on the hedged portfolio return for the hedging period (from time t to $t+1$) is given by

$$r_{p,t+1} = r_{s,t+1} - \beta_t r_{f,t+1} \tag{1.1}$$

where $r_{s,t+1} = S_{t+1} - S_t$ is the return for holding the spot, and $r_{f,t+1} = F_{t+1} - F_t$ is the return for holding the futures. Note that the hedge ratio is an unknown decision variable for the

(2006) argue that the chosen (unconditional) risk measures that are used in calculating the hedging effectiveness has important implications in determining the best hedging method when the hedgers have their specific aims. With the use of an extensive set of risk measures, including variance, semi-variance, lower partial moments, value at risk, and conditional value at risk (or called expected shortfall), they find that the best hedging method is sensitive to the specific risk measures.

hedger so that it needs to be estimated via some models.[6]

Assume that the hedger would like to choose the optimal futures holdings by maximizing the expected (mean-variance) utility function:

$$EU(r_{p,t+1} \mid \Phi_t; \beta_t, \gamma) = E(r_{p,t+1} \mid \Phi_t) - \gamma \operatorname{var}(r_{p,t+1} \mid \Phi_t) \tag{1.2}$$

at time t with the degree of risk aversion $\gamma > 0$, where the expectation and the variance operators are calculated conditional on the set of all available information Φ_t at time t (Kroner & Sultan, 1993). With some mathematical derivations, the optimal hedge ratio equals to

$$\beta_t^* = \frac{E(r_{f,t+1} \mid \Phi_t) + 2\gamma \operatorname{cov}(r_{s,t+1}, r_{f,t+1} \mid \Phi_t)}{2\gamma \operatorname{var}(r_{f,t+1} \mid \Phi_t)} \tag{1.3}$$

When the expected return for holding futures is zero (i.e. $E(r_{f,t+1} \mid \Phi_t) = 0$) or the degree of risk aversion is high ($\gamma \to \infty$), the expected utility-maximizing problem simplifies to the (conditional) variance-minimizing problem:

$$\min_{\beta_t} \operatorname{var}(r_{p,t+1} \mid \Phi_t; \beta_t) \tag{1.4}$$

and the optimal hedge ratio (or minimum-variance hedge ratio) is formulated as

$$\beta_t^* = \frac{\operatorname{cov}(r_{s,t+1}, r_{f,t+1} \mid \Phi_t)}{\operatorname{var}(r_{f,t+1} \mid \Phi_t)} \tag{1.5}$$

The derivation of this hedge ratio is generally valid for von-Neumann Morgenstern utility functions for $U'(\cdot) > 0$ and $U''(\cdot) < 0$ (Benninga et al., 1983). This simplified hedge ratio only depends on conditional second moments of spot and futures, and therefore, one central issue in the context of a dynamic hedge is to provide the conditional covariance matrix fore-

---

[6] Traditional approach to commodity futures hedging adopts the naïve strategy, which suggests a hedger who longs a unit of spot position should sell a unit of futures today and then buy the contracts back when he sells the spot. A perfect hedge is achieved when the spot and futures prices both move by the same amount; however, in practice, it is found that the prices usually do not have identical co-movements.

casts that can characterize the dynamics of the second moments more realistically.

Assume that the 2×1 vector of returns $R_t = (r_{s,t}, r_{f,t})'$ follows the discrete-time process:

$$R_t = M_t + \Omega_t^{1/2} Z_t, \quad t = 1, 2, \ldots, T \tag{1.6}$$

where $M_t \equiv E(R_t \mid \Phi_{t-1})$ and $\Omega_t \equiv \mathrm{var}(R_t \mid \Phi_{t-1})$ represent the 2×1 conditional mean vector and 2×2 conditional covariance matrix of R_t, respectively; $Z_t = (\eta_{s,t}, \eta_{f,t})'$ is an 2×1 vector of serially uncorrelated disturbances with $E(Z_t) = \mathbf{0}$ and $\mathrm{var}(Z_t) = \mathbf{I}$. Note that M_t consists of individual conditional means, and Ω_t consists of individual conditional variance $\sigma_{i,t}^2$ $(i = s, f)$ in the diagonal and conditional covariance $\sigma_{sf,t}$ in the off-diagonal. This decomposition is of much popular with the empirical success of multivariate volatility models, e.g. the multivariate GARCH-type models; and, over years, a large body of studies has applied these models to estimate the dynamic hedge ratio, e.g., Kroner and Sultan (1993), and among many others.

To judge the success of these methods, the hedging effectiveness (HE) of Ederington (1979) has been extensively adopted as a benchmark for the hedging performance.[7] The measure defines the effectiveness of a futures hedge by calculating the percentage reduction from the variance of the spot position to the variance of the hedged portfolio, or

$$\mathrm{HE} \equiv 1 - \frac{\mathrm{var}(r_{p,t+1})}{\mathrm{var}(r_{s,t+1})} \tag{1.7}$$

where the realization on the hedged portfolio return $r_{p,t+1}$ equals to $r_{s,t+1} - \beta_t^* r_{f,t+1}$; $r_{s,t+1}$ and $r_{f,t+1}$ represent realizations of spot and futures returns, respectively; $\mathrm{var}(\cdot)$ denotes the variance operator. Based on this performance measure, a hedging method is deemed better than others if it can generate a higher HE or equivalently a smaller $\mathrm{var}(r_{p,t+1})$ for the hedged portfolio.

[7] In addition to the statistical measure, alternative economic measures are also discussed, such as the Sharpe-type hedging performance of Howard and D'Antonio (1984) and the economic value of Lence (1995).

1.3. Integrated Covariance Estimation using Intraday Data

The availability of intraday high-frequency data for many financial assets has benefited the measurement of realizations on the unobserved latent volatility process. Without through standard time series techniques, e.g., the GARCH models, the realized volatility (RV) approach has provided a consistent model-free estimate of the price volatility over a given discrete-time interval. To illustrate, suppose the 2×1 vector of (spot-futures) returns is arisen from the continuous-time diffusion process:

$$dP(t) = M(t)dt + \Sigma(t)dW(t),\ \ t \in [0, T] \tag{1.8}$$

where $P(t) = \{S(t), F(t)\}'$ represents the 2×1 vector of logarithmic prices; $M(t)$ represents the 2×1 instantaneous drifts; $\Sigma(t)$ represents the 2×2 instantaneous positively definite diffusion matrix that consists of individual instantaneous variance $\sigma_i^2(t)$ $(i = s, f)$ in the diagonal and instantaneous covariance $\sigma_{sf}(t)$ in the off-diagonal; $W(t)$ represents the 2×1 vector of independent standard Brownian motions. Over the $[t-\triangle, t]$ time interval, the return for the continuous-time model is defined as

$$R(t,\triangle) = P(t) - P(t-\triangle) = \int_{t-\triangle}^{t} M(s)ds + \int_{t-\triangle}^{t} \Sigma(s)dW(s) \tag{1.9}$$

When $\triangle = 1$, it represents the one period return from time $t-1$ to t. As compared with the discrete-time representation, the conditional mean vector and covariance matrix are replaced by the corresponding integrated mean and covariance process with the innovation driven by the continuously evolving standard Brownian motion. As a result, the integrated covariance matrix for the one-period, which is formulated by

$$\mathrm{ICov}(t) = \int_{t-1}^{t} \Sigma(s)\Sigma(s)'ds \tag{1.10}$$

is closely related to the conditional covariance matrix in the discrete-time framework so that the integrated variance (covariance) has been served as an ideal ex-post benchmark for assessing the quality of ex-ante variance (covariance) forecasts (Andersen et al., 2006).

The availability of intraday high-frequency data has provided a way to estimate the integrated covariance matrix. Define the realized covariance matrix for the time interval $[t-1, t]$ as

$$\mathrm{RCov}(t,\triangle) = \sum_{j=1}^{1/\triangle} R(t-1+j\cdot\triangle,\triangle)R(t-1+j\cdot\triangle,\triangle)' \qquad (1.11)$$

As the sampling frequency of returns goes to infinity, the bivariate realized covariation converges to the corresponding bivariate integrated covariation:

$$\mathrm{RCov}(t,\triangle) \to \mathrm{ICov}(t),\ \ \triangle \to 0 \qquad (1.12)$$

Details of the proof are referred to Andersen et al. (2001a, 2001b, 2003), and Barndorff-Nielsen and Shephard (2004). For notation simplicity, hereafter, we denote $\mathrm{RV}_{s,t}$ and $\mathrm{RV}_{f,t}$ the realized variance of the spot and the futures, respectively; and RCov_{t} the realized spot-futures covariance, at time t using the $\triangle$ equally sampled discrete-time returns.

1.4. Research Objectives

The objective of this dissertation is to apply the model-free RV approach to the one-period hedging problem. To do so, there are three topics to be investigated in the following article.

Firstly, a large number of empirical studies on futures hedge have concerned with the conventional multivariate GARCH-type models, which only utilize daily (weekly) price information, in estimating the dynamic hedge ratio. With the use of intraday information, Chapter 2 presents alternative discrete-time multivariate volatility models encompassing the elements of realized covariance in estimating the dynamic hedge ratio. In addition, the benefits on the dynamic hedge over those conventional methods using return-based volatility models are compared based on an extensive set of statistical and economical performance measures.

Next, applying the realized beta framework of Andersen et al. (2005, 2006), Chapter 3 extends the framework to analyze the hedging problem. With the availability of intraday data, it is shown that the realized hedge ratio and realized hedging effectiveness provide alternative ex-post benchmarks for evaluating the performance of ex-ante hedging methods. Furthermore, an empirical example is also exhibited in this chapter.

Finally, with the construction of realized daily hedge ratio series, the dynamics of the hedge ratio is analyzed in Chapter 4. The importance of this analysis is that the insight into the hedge ratio behavior may further the development of dynamic hedge ratio models. Moreover, a two-regime threshold autoregressive model is also applied to detect the regime-switching feather of the ratio. If the feature is appeared in the hedge ratio, the argument of time-varying hedge ratio is further supported by the evidence.

1.5. Organization of the Dissertation

The organization of the dissertation is depicted in Figure 1.1 and briefly introduced as follows. Chapter 1 introduces the dissertation with the organization. Chapter 2 proposes a new class of discrete-time multivariate volatility models encompassing the elements of realized covariance matrix to estimate the risk-minimizing hedge ratio, and compare the performance with those generated by return-based volatility models. Chapter 3 presents a RV-based method for analyzing the one-period hedging problem with an illustrated example. Chapter 4 assesses the dynamics of realized daily hedge ratios. Chapter 5 concludes the dissertation with suggestions for the future research.

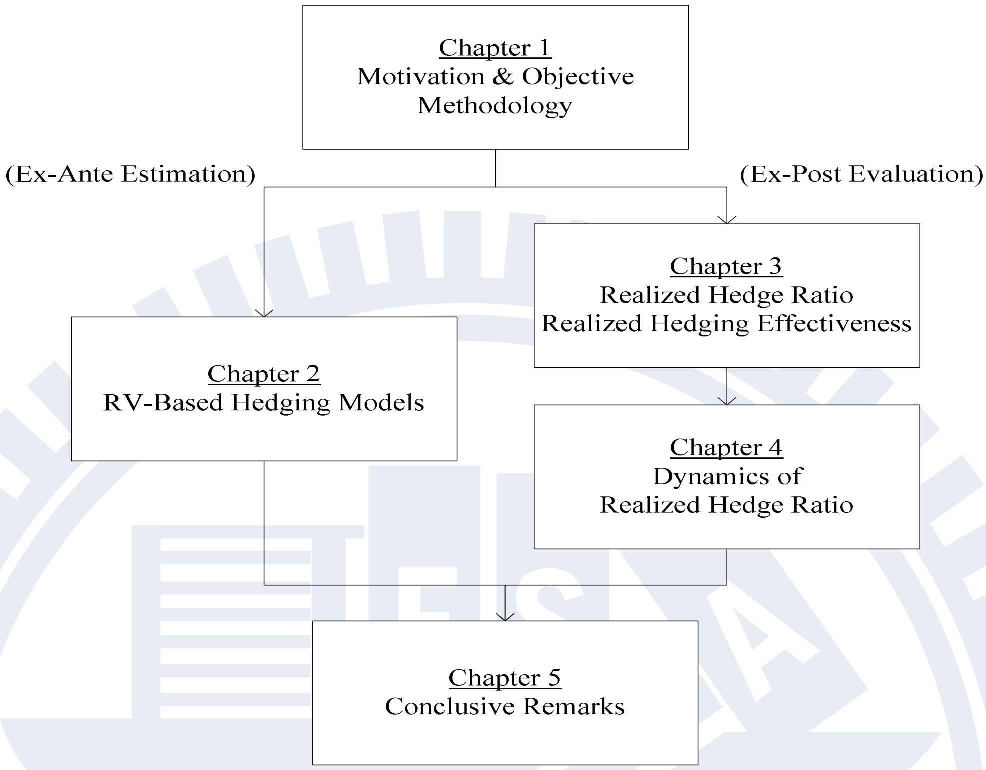

Figure 1.1 Organization of the Dissertation

Chapter 2. The Incremental Value of a Futures Hedge using Realized Volatility

This chapter proposes a new class of multivariate volatility models encompassing RV estimates to estimate the risk-minimizing hedge ratio, and compare the hedging performance of the proposed models with those generated by return-based models. In an out-of-sample context with a daily rebalancing approach, based on an extensive set of statistical and economic performance measures, the empirical results show that improvement can be substantial when switching from daily to intraday. This essentially comes from the advantage that the intraday-based RV potentially can provide more accurate daily covariance matrix estimates than RV utilizing daily prices. Finally, this study also analyzes the effect of hedge horizon on hedge ratio and hedging effectiveness for both the in-sample and the out-of-sample data.

2.1. Research Problem and Objective

It has been shown that most of the latent GARCH models fail to satisfactorily describe the high kurtosis, small first-order autocorrelation of squared returns, and slow decay of the autocorrelation of squared returns toward zero that have been observed in many daily or weekly financial returns (e.g., Carnero et al., 2004).[9] The search for alternative volatility techniques has motivated scholars to exploit information in intraday high-frequency data (e.g., Anderson & Bollerslev, 1998; Andersen et al., 2003; Barndorff-Nielsen & Shephard, 2004; Hayashi & Yoshida, 2005; Voev & Lunde, 2007). This so-called RV approach provides a direct and consistent technique to estimate the latent volatility process without the need for re-

[9] To capture the high kurtosis of returns and low first-order autocorrelation of return squares simultaneously, GARCH models often require a high persistence and/or leptokurtic conditional distributions when they are fitted to the financial time series (Carnero et al., 2004). The restrictions on GARCH then severely restrict the allowed dynamic dependence of the volatility.

lying on explicit models.[10] Therefore, it has served as an ex-post benchmark for assessing the quality of any ex-ante volatility forecasts (e.g., Andersen et al, 2006). In forecasting future volatility, studies have shown that RV can provide more accurate forecasts than methods using daily squared returns (e.g., Blair et al., 2001). Recently, RV has been employed to explore the economic value (EV) of investment by Fleming et al. (2003), Bandi et al. (2008), and De Pooter et al. (2008).[11] They find that the EV is substantial when switching from daily to intraday returns, even without applying bias-correction techniques in constructing realized estimates. As a result, the superiority of RV-based investment may result because it takes into account finer intraday information and thus potentially provides more accurate daily covariance matrix estimates or forecasts than methods utilizing daily prices.

This study attempts to improve the performance of a risk-minimizing futures hedge when the intraday-based RV approach is incorporated into forecasting the relevant covariance matrix. It is not clear, however, whether the accurate RV is used or not may differentiate the performance of a futures hedge. To address this issue, a new RV-based method is demonstrated in this study. The proposed method builds on the bivariate error correction model by employing the flexible CCC-GARCH error structure of Kroner and Sultan (1993). The advantage of extending their model is that it can capture the long-run cointegration relationship and the time-varying second moments simultaneously when the dynamics of the joint distribution is specified. Moreover, when the leverage effect and/or the dynamic correlation are revealed, the asymmetric volatility and/or the dynamic conditional correlation (DCC) models can also be incorporated into this method. These desirable properties thus provide a more

---

[10] Generally, in the absence of market microstructure and non-synchronous trading (Epps effect), a daily measure of variance is computed as the sum of the squared intraday equidistant returns, and a daily measure of covariance is obtained by summing the products of intraday equidistant returns, for the given trading day. For a review of the RV refers to McAleer and Medeiros (2008).

[11] For example, Fleming et al. (2003) show how a risk-averse investor would be willing to pay 50 to 200 basis points per year to capture the multivariate volatility forecasts based on intraday returns instead of daily returns in the context of investment decisions on three actively traded futures contracts (S&P 500 index, Treasury bonds, and gold). It is assumed that the investor follows a volatility-timing strategy, which rebalances his portfolio only when the estimated conditional covariance matrix of the daily returns changes. This case treats the expected daily returns as time-invariant.

convenient way to lodge the realized covariance matrix in the GARCH error structures as compared with the rolling estimators of Fleming et al. (2003), Bandi et al. (2008), and De Pooter et al. (2008). The RV-based method, which uses finer volatility proxies in estimating and forecasting the conditional covariance matrix, is expected to provide better descriptions on the spot-futures dynamics and the resulting hedge ratios than methods using squared return shocks.

In the empirical analyses, the RV-based hedge ratios are calculated in an out-of-sample context spanning the period of December 19, 2003 and March 31, 2009 for the highly traded S&P 500 index futures contracts. we compare the performance of the RV-based hedge with the return-based GARCH and the OLS hedges using an extensive set of statistical and economic measures. The comparisons are conducted for both short and long hedgers. To anticipate the results, this study finds that the RV-based hedge can substantially outperform the return-based GARCH and/or the static OLS hedges especially during the surge in volatility period. Then the RV-based method is applied to examine the effect of hedging horizon (ranges from one week to three month) on hedge ratio and hedging effectiveness. The results show that hedge ratio tends to increase and to approach unity (i.e., naïve hedge ratio) with the length of hedging horizon; and, hedging effectiveness tends to increase as the length of hedging horizon increases. The rest of the chapter is organized as follows. First, we present the conventional hedging method and demonstrate the RV-based method. Next, we present the data and their properties with the empirical results. Finally, the last section concludes the study.

2.2. The Conventional Hedging Models

Kroner and Sultan (1993) have proposed a bivariate GARCH error correction model for modeling the joint distribution of spot and futures. As such, the most prominent application of this model is to estimate time-varying hedge ratios. The econometric model for the daily returns conditioning on the set of all relevant information Φ_{t-1} at time $t-1$ can be described as

$$\begin{aligned} r_{s,t} &= \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{s,t} \\ r_{f,t} &= \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{f,t} \end{aligned} \tag{2.1}$$

$$\begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix} | \, \Phi_{t-1} \sim N(0, H_t) \tag{2.2}$$

where $(S_{t-1} - \delta F_{t-1})$ is the error correction term (ECT). The incorporation of ECT in the conditional mean equations is essential, especially in currency and equity markets (see Park & Switzer, 1995; Brooks et al., 2002; Choudhry, 2003). The residual vector $(\varepsilon_{s,t}, \varepsilon_{f,t})'$ is postulated as a bivariate normal distribution with a 2×1 zero mean vector and a 2×2 time-varying covariance matrix

$$H_t = \begin{bmatrix} h_{s,t} & h_{sf,t} \\ h_{sf,t} & h_{f,t} \end{bmatrix} = \begin{bmatrix} h_{s,t}^{1/2} & 0 \\ 0 & h_{f,t}^{1/2} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t}^{1/2} & 0 \\ 0 & h_{f,t}^{1/2} \end{bmatrix} = D_t R D_t \tag{2.3}$$

where $h_{sf,t}$ is a covariance, and $h_{s,t}$ and $h_{f,t}$ are conditional variances for the spot and futures returns, respectively, and ρ is the time-invariant correlation coefficient between them. That is, the model applies the CCC estimator of Bollerslev (1990) to model and forecast the bivariate conditional covariance matrix.

The expressions of the conditional variances $h_{s,t}$ and $h_{f,t}$ in Equation (2.3) are typically thought of as univariate GARCH-type models. For example, the GARCH(1,1) structure introduced by Bollerslev (1986) for the returns can be specified as

$$\begin{aligned} h_{s,t} &= \beta_{0s} + \beta_{1s} h_{s,t-1} + \beta_{2s} \varepsilon_{s,t-1}^2 \\ h_{f,t} &= \beta_{0f} + \beta_{1f} h_{f,t-1} + \beta_{2f} \varepsilon_{f,t-1}^2 \end{aligned} \tag{2.4}$$

When the leverage effect is revealed, asymmetric volatility models are commonly formulized, such as the GJR (Glosten-Jagannathan-Runkle) model of Glosten et al. (1993). This simply modifies the standard GARCH(1,1) model with an additional ARCH (autoregressive conditional heteroscedasticity) term conditional on the sign of the past innovation. The conditional volatilities of Equation (2.4) are then reformulated as

$$\begin{aligned} h_{s,t} &= \beta_{0s} + \beta_{1s} h_{s,t-1} + \beta_{2s}\varepsilon_{s,t-1}^2 + \beta_{3s}\varepsilon_{s,t-1}^2 I(\varepsilon_{s,t-1} < 0) \\ h_{f,t} &= \beta_{0f} + \beta_{1f} h_{f,t-1} + \beta_{2f}\varepsilon_{f,t-1}^2 + \beta_{3f}\varepsilon_{f,t-1}^2 I(\varepsilon_{f,t-1} < 0) \end{aligned} \tag{2.5}$$

where $I(\cdot)$ denotes an indicator function. For estimating the constant correlation coefficient, the ever-popular rolling estimator uses equal weight to all past T return innovations:

$$\hat{\rho} = \frac{\sum_{t=1}^{T} \varepsilon_{s,t}\varepsilon_{f,t}}{\sqrt{(\sum_{t=1}^{T} \varepsilon_{s,t}^2)(\sum_{t=1}^{T} \varepsilon_{f,t}^2)}} \in [-1,1] \tag{2.6}$$

To fit in with reality, however, the conditional correlation coefficient can be relaxed to vary with time (Engle, 2002; Tse & Tsui, 2002). That is, the conditional covariance matrix of Equation (2.3) is generalized to the DCC formulation:

$$H_t = \begin{bmatrix} h_{s,t} & h_{sf,t} \\ h_{sf,t} & h_{f,t} \end{bmatrix} = \begin{bmatrix} h_{s,t}^{1/2} & 0 \\ 0 & h_{f,t}^{1/2} \end{bmatrix} \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix} \begin{bmatrix} h_{s,t}^{1/2} & 0 \\ 0 & h_{f,t}^{1/2} \end{bmatrix} = D_t R_t D_t \tag{2.7}$$

The evolution of ρ_t in this paper is analogous to the univariate GARCH equation:

$$\rho_t = (1-\theta_1-\theta_2)\overline{\rho} + \theta_1\rho_{t-1} + \theta_2 \frac{\sum_{h=1}^{M} \eta_{s,t-h}\eta_{f,t-h}}{\sqrt{(\sum_{h=1}^{M} \eta_{s,t-h}^2)(\sum_{h=1}^{M} \eta_{f,t-h}^2)}} \tag{2.8}$$

where θ_1 and θ_2 are non-negative with $\theta_1 + \theta_2 \leq 1$; the sample size $M = 2$ for estimating the sample correlation coefficient with $\eta_{i,t-h} = \varepsilon_{i,t-h} / h_{i,t-h}^{1/2}$ ($i = s, f$) follows Tse and Tsui (2002); $\overline{\rho}$ is the unconditional correlation between spot and futures.

Given the null hypothesis that the set of all relevant information Φ_{t-1} is observed with correctly specified models, these CCC or DCC models with the ECT describe the dynamic nature of spot-futures distributions. In particular, this GARCH class of volatility (correlation) models provides simple ways to forecast the bivariate volatility by using actual return innovations and the estimated hedge ratio by minimizing the risk of the hedged portfolio return at time t is given by

$$\hat{\beta}_t^* = \hat{h}_{sf,t+1} / \hat{h}_{f,t+1} \tag{2.9}$$

where $\hat{h}_{sf,t+1}$ and $\hat{h}_{f,t+1}$ are the covariance and variance forecasts using Equations (2.3) or (2.7). In brief, Equations (2.1) through (2.9) then construct the conventional hedge ratio models, namely, the ECT-GARCH-CCC model (Equations (2.1)-(2.4), (2.6), and (2.9)), the ECT-GARCH-DCC model (Equations (2.1)-(2.2), (2.4), and (2.7)-(2.9)), the ECT-GJR-CCC model (Equations (2.1)-(2.3), (2.5)-(2.6), and (2.9)), and the ECT-GJR-DCC model (Equations (2.1)-(2.2), (2.5), and (2.7)-(2.9)).

2.3. Alternative Models using Realized Volatility

To explore the incremental value of a RV-based hedge, the realized variance and/or correlation are encompassed within the conventional CCC and DCC models with the ECT specification. Following the previous specifications, the RV-based GARCH(1,1) volatility can be expressed as

$$\begin{aligned} h_{s,t} &= \beta_{0s} + \beta_{1s} h_{s,t-1} + \beta_{2s} \mathrm{RV}_{s,t-1} \\ h_{f,t} &= \beta_{0f} + \beta_{1f} h_{f,t-1} + \beta_{2f} \mathrm{RV}_{f,t-1} \end{aligned} \tag{2.10}$$

where each $\mathrm{RV}_{i,t-1}$ ($i = s, f$) is defined by summing up $\triangle$-minute squared returns at time $t-1$. Analogously, the RV-based GJR(1,1) model can be reformulated as

$$\begin{aligned} h_{s,t} &= \beta_{0s} + \beta_{1s} h_{s,t-1} + \beta_{2s} \mathrm{RV}_{s,t-1} + \beta_{3s} \mathrm{RV}_{s,t-1} I(\varepsilon_{s,t-1} < 0) \\ h_{f,t} &= \beta_{0f} + \beta_{1f} h_{f,t-1} + \beta_{2f} \mathrm{RV}_{f,t-1} + \beta_{3f} \mathrm{RV}_{f,t-1} I(\varepsilon_{f,t-1} < 0) \end{aligned} \tag{2.11}$$

That is, the right-hand side squared residuals in the conventional GARCH or GJR models are replaced by realized variances. To encompass the realized correlation $\mathrm{RCorr}_{t-1} = \mathrm{RCov}_{t-1} \big/ (\mathrm{RV}_{s,t-1} \mathrm{RV}_{f,t-1})^{1/2}$ within the conditional correlation dynamics, Equation (2.8) can be modified as

$$\rho_t = (1 - \theta_1 - \theta_2)\overline{\rho} + \theta_1 \rho_{t-1} + \theta_2 \mathrm{RCorr}_{t-1} \tag{2.12}$$

where the realized covariance RCov_{t-1} is defined by cumulating the cross-products of the intraday spot and futures returns at time $t-1$. For estimating the constant correlation in the RV-based CCC model, however, the sample mean of the realized correlations should be a biased estimate due to non-synchronous trading (Epps effect) and/or market microstructure noise (see, e.g., Hayashi & Yoshida, 2005; Voev & Lunde, 2007). To simplify the estimation process, the rolling estimator of Equation (2.6) is still adopted. Then, these modifications construct a new class of RV-based hedge ratio models, namely, the ECT-RV-GARCH-CCC model (Equations (2.1)-(2.3), (2.6), (2.9), and (2.10)), the ECT-RV-GARCH-DCC model (Equations (2.1)-(2.2), (2.7), (2.9), (2.10), and (2.12)), the ECT-RV-GJR-CCC model (Equations (2.1)-(2.3), (2.6), (2.9), and (2.11)), and the ECT-RV-GJR-DCC model (Equations (2.1)-(2.2), (2.7), (2.9), and (2.11)-(2.12)).

To estimate the parameters in the RV-based or the conventional CCC or DCC models, we follow the two-step estimation procedure of Bollerslev (1990) and Engle (2002). Since this class of multivariate volatility models has separate parameters, it can be estimated easily and consistently in two steps.[12] With the normality assumption of Equation (2.2), we can maximize each volatility term with the conditional mean in the first step:

$$\hat{\vartheta}_i = \arg\max\{L_{V_i}(\vartheta_i)\} \tag{2.13}$$

where $\vartheta_i = (\alpha_i, \beta_i, \delta)$ and L_{V_i} denote the Gaussian quasi-likelihood function for asset i, and then maximize the correlation term:

$$\hat{\theta} = \arg\max\{L_C(\theta \mid \hat{\vartheta})\} \tag{2.14}$$

in the second step, L_C represents the Gaussian quasi-likelihood function of the correlation part, and $\theta = \rho$ for the CCC and $\theta = (\theta_1, \theta_2)$ for the DCC. Without the normality assump-

[12] Engle and Granger (1987) indicate that a cointegration system could consistently be estimated via a two-step estimator, where both steps require only single equation least squares, so that we can estimate the DCC-type models with the ECT by using the two-step procedure of Bollerslev (1990) and Engle (2002).

tion, these estimators still have the quasi-maximum likelihood (QML) interpretation.

2.4. Empirical Analyses

Data Descriptions

The performance of the RV-based hedging method is examined empirically on the S&P 500 index futures contracts traded on Chicago Mercantile Exchange (CME). The sample period is from January 1, 1998 to March 31, 2009, which covers the period of subprime mortgage crisis. We obtain the daily closing (settlement) prices for the spot (futures) from the *Datastream* and the intraday transaction prices for them from *Tick Data Inc*.[13] Note that this study rolls the nearest month contract to the next month when the daily volume of the current contract is exceeded. Specifically, daily and intraday prices of all days corresponding to U.S. public holidays are removed. Hence, there are 2828 trading days for the period examined.

Table 2.1 reports the diagnostic checks on the distribution properties of the daily spot (futures) returns, which are calculated as differenced natural logarithmic daily closing (settlement) prices. The results of unit root and cointegration tests are shown in Panel A, and summary statistics on the returns are reported in Panel B. The augmented Dickey-Fuller tests show that the spot and futures log-prices have a unit root, but their first-differenced series are stationary. The Johansen trace statistic indicates that the spot and futures prices are cointegrated with the cointegrating parameter $\hat{\delta} \approx 1$. The unconditional distributions of the univariate returns reveal non-normality, as evidenced by the non-zero skewness, high kurtosis, and significant Jarque-Bera statistics. Panel C provides the autocorrelation functions (ACF) as well as the Ljung-Box statistics of the squared (cross-product) daily returns. The first-order ACF ranges from 0.1970 to 0.4093 but decays slowly toward zero. Based on the empirical

[13] The intraday transaction observations consist of open, high, low, and close prices at the one-minute sampling frequency.

evidences and the findings of Carnero et al. (2004), the conventional GARCH models may be inadequate to describe the spot-futures volatility dynamics, hence, it is not clear whether the inadequacy may damage the performance of a futures hedge.

TABLE 2.1 Data Description: Daily Price Returns

Indices	Statistics					
Panel A: Unit root and cointegration tests						
	ADF (price)	ADF (return)	Trace	$\hat{\delta}$		
Spot	-1.36	-42.15*	84.97*	0.9841		
Futures	-1.36	-42.40*				
Panel B: Summary statistics of returns						
	Mean	Std. Dev.	Skewness	Kurtosis	Correlation	JB
Spot	-0.0001	0.0138	-0.1280	10.5955	0.9760	6798.74*
Futures	-0.0001	0.0141	0.0369	12.4322		10480.13*
Panel C: Autocorrelation functions of squared (cross-product) returns						
	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)	$Q^2(10)$
Spot	0.3661*	0.1785*	0.2935*	0.3307*	0.2982*	2230.80*
Futures	0.4093*	0.1692*	0.2526*	0.3000*	0.2516*	2032.04*
Spot-Futures	0.1970*	0.4310*	0.1727*	0.3139*	0.3488*	1279.50*

Notes: The daily spot (futures) returns are calculated as differenced natural logarithmic closing (settlement) prices where public holidays are removed. The sample period for the prices runs from January 1, 1998 to March 31, 2009 and the sample size for each is 2828. The values in rows ADF, Trace, JB, ACF(k), and $Q^2(10)$ are statistics of the augmented Dickey-Fuller unit root test, the Johansen cointegration test, the Jarque-Bera normality test, the order k autocorrelation of squared returns, and the Ljung-Box test for the serial correlations in the squared returns. $\hat{\delta}$ is the estimated cointegrating parameter. * indicate significance at the 5% level.

To construct the realized (co-)variance for the alternative method, the intraday futures prices after 3:00 p.m. Chicago time for each day t are dropped since the futures market closes fifteen minutes later than the spot market.[14] we divide the contemporaneous time section across the markets, which runs from 8:30 a.m. until 3:00 p.m. (390 minutes), into m (non-overlapping) intervals of equal lengths $\Delta \equiv 390/m$ such that the times $t_j = t_{j-1} + \Delta$ for $j = 1, \ldots, m$ with $t_0 =$ 8:30 a.m. Chicago time. The log close transaction price at time t_j is denoted as $p(t_j)$, then the equidistant intraday returns on day t, $r_{t_j} \equiv p(t_j) - p(t_{j-1})$,

[14] The (floor) trading section for the S&P 500 index futures on the CME runs from 8:30 a.m. Chicago time until 3:15 p.m. Chicago time.

for $j = 2, \ldots, m$; and, the first period ($j = 1$) intraday return is defined as the difference between the log close and open transaction prices during that time interval. With these mathematical definitions, the realized variance is then defined as $\mathrm{RV}_{i,t}^{(m)} \equiv \sum_{j=1}^{m} r_{i,t_j}^2$ for $i = s, f$, and the realized covariance is defined as $\mathrm{RCov}_t^{(m)} \equiv \sum_{j=1}^{m} r_{s,t_j} r_{f,t_j}$, for each day t.

This study uses fifteen-minute intraday prices ($m = 26$) to construct the realized (co-)variance estimates, and summarizes their descriptive statistics in Table 2.2. Since only observations during the floor trading section are sampled, the average realized variances are smaller than the corresponding unconditional variances obtained from daily returns. For example, the average value of the realized variance estimates for the S&P 500 cash index is 1.11e-4, which is about 58% of the unconditional variance 1.90e-4 calculated from Table 2.1. To measure the realized variance (covariance) for the whole day, the squared (cross-product) overnight returns can further be incorporated into the realized estimators (Martens, 2002; Fleming et al., 2003, Hansen & Lunde, 2005; De Pooter et al., 2008). For the realized correlations, the average level for the S&P 500 (about 0.95) shows slight bias toward zero as compared with the corresponding unconditional correlation estimates (about 0.98) using Equation (2.6). According to Hayashi and Yoshida (2005) and Voev and Lunde (2007), the biasness may come from the non-synchronous trading and/or the market microstructure noise, but it can be corrected using some bias-correction techniques.[15] The ACF of these realized estimates are also reported in Table 2.2. It shows that these realized second moments reveal considerable persistency. However, the ACF of the realized variance (covariance) is higher than the corresponding ACF of squared (cross-product) returns in Table 2.1. For example, in the spot market, the ACF(1) of the realized variance is about 0.66, which is higher than the ACF(1) of the squared returns in Table 2.1 by about 0.37. It is expected that the behavior difference between the realized variance (covariance) and squared (cross-product) returns should

---

[15] We do not adjust for the biases in the empirical analyses because the bias-correction procedures do not guarantee a positive definite realized covariance matrix estimates; see, De Pooter et al. (2008) for discussions.

produce different volatility (covariance) estimates and forecasts based on the alternative and the conventional methods.

TABLE 2.2 Data Description: Realized Variance, Covariance, and Correlation

Realized	Summary Statistics				Autocorrelation Functions				
Estimates	Mean	Median	Min	Max	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)
$\mathrm{RV}_{s,t}$	1.11e-4	5.40e-5	3.00e-6	0.0053	0.6550	0.6632	0.5424	0.5995	0.5357
$\mathrm{RV}_{f,t}$	1.20e-4	5.90e-5	3.00e-6	0.0055	0.6558	0.6677	0.5442	0.6050	0.5467
RCov_t	1.11e-4	5.40e-5	3.00e-6	0.0054	0.6573	0.6710	0.5460	0.6080	0.5451
RCorr_t	0.9506	0.9595	0.2855	0.9986	0.3993	0.3823	0.3999	0.3863	0.4036

Notes: The sample period for these (daily) realized estimates, which are constructed from fifteen-minute equidistant intraday returns, spans the period of January 1, 1998 to March 31, 2009, and the sample size is 2828. It is noted that the futures returns after 3:00 p.m. Chicago time for each day are dropped since the futures markets close fifteen minutes later than the spot markets. The ACF(k) indicates the sample autocorrelation function of the realized estimates corresponding to lags k=1,2,…5. The upper and lower confidence bounds of the ACF with the 5% confidence level are 0.0375 and -0.0375, respectively.

Estimation Results

Table 2.3 presents the estimation results of the return-based and the RV-based GARCH models. Panel A shows the conditional mean and variance estimates, and Panel B shows the conditional correlation estimates. Given the evidence of the cointegration relationship between the spot and futures (in Table 2.1), the restricted ECT, $(S_{t-1} - F_{t-1})$, is parameterized in the conditional mean equations to avoid the loss of long-run information; though, all α_{1f} are insignificant different from zero (Park & Switzer, 1995; Brooks et al., 2002). The insignificance of α_{0f} and α_{1f} coefficients show the expected returns of futures should be zero, meaning the minimum variance hedge is generally the expected utility maximization hedge (Baillie & Myers, 1991; Kroner & Sultan, 1993).

TABLE 2.3 Estimation Results of RV-Based and Return-Based Models

Parameters / Statistics	Return-Based				RV-Based			
	ECT-GARCH		ECT-GJR		ECT-RV-GARCH		ECT-RV-GJR	
	CCC	DCC	CCC	DCC	CCC	DCC	CCC	DCC
Panel A: Estimates of conditional mean and conditional variance equations								
α_{0s}	-1.88e-5		-0.0004		-0.0004		-0.0004	
	(-0.07)		(-1.72)		(-1.64)		(-1.95)	
α_{1s}	-0.1035		-0.0880		-0.0947		-0.1010	
	(-2.54)		(-2.76)		(-2.40)		(-2.58)	
β_{0s}	1.13e-6		1.31e-6		6.76e-6		4.79e-6	
	(5.91)		(7.02)		(7.57)		(7.09)	
β_{1s}	0.9173		0.9258		0.7702		0.8261	
	(121.28)		(113.79)		(33.59)		(45.93)	
β_{2s}	0.0774		0.0001		0.2297		0.0773	
	(10.68)		(0.01)		(9.10)		(4.22)	
β_{3s}	-		0.1311		-		0.1929	
			(10.52)				(7.01)	
α_{0f}	0.0005		1.12e-5		0.0002		0.0001	
	(2.23)		(0.05)		(0.73)		(0.36)	
α_{1f}	0.0440		0.0065		0.0613		0.0486	
	(1.03)		(0.19)		(1.52)		(1.23)	
β_{0f}	1.36e-6		1.45e-6		7.33e-6		5.30e-6	
	(6.99)		(7.67)		(8.24)		(7.65)	
β_{1f}	0.9121		0.9217		0.7325		0.7958	
	(117.37)		(114.13)		(28.70)		(39.87)	
β_{2f}	0.0812		0.0001		0.2674		0.0915	
	(10.98)		(0.01)		(9.44)		(4.29)	
β_{3f}	-		0.1379		-		0.2251	
			(10.86)				(7.51)	
Panel B: Estimates of constant (dynamic) conditional correlation processes								
θ_1	-	0.2537	-	0.1416	-	0.9929	-	0.9932
		(3.24)		(2.50)		(660.87)		(687.19)
θ_2	-	0.0374	-	0.0515	-	0.0018	-	0.0018
		(7.59)		(8.61)		(4.43)		(4.49)
$\overline{\rho}_{sf}$ (ρ_{sf})	0.9732	0.9716	0.9719	0.9700	0.9736	0.9675	0.9728	0.9667
	(2821.10)		(2728.50)		(3036.90)		(2932.00)	

Notes: The entries (in the parentheses) are the Gaussian QML estimates (and their asymptotic t-statistics) of return-based and RV-based models, where parameters are estimated via a two-step estimation method. This method first estimates $\vartheta_i := (\alpha_i, \beta_i)$ for $i = s, f$ by maximizing the Gaussian quasi-likelihood function $L_{V_i}(\vartheta_i) = -\frac{1}{2}\ln 2\pi - \frac{1}{2T}\sum_{t=1}^{T}\ln h_{i,t} - \frac{1}{2T}\sum_{t=1}^{T} h_{i,t}^{-1}[r_{i,t} - \alpha_{0i} - \alpha_{1i}(S_{t-1} - F_{t-1})]^2$, then estimates θ by maximizing the $\hat{\vartheta}_i$ -based quasi-likelihood function $L_C(\theta \mid \hat{\vartheta}) = \frac{1}{T}\sum_{t=1}^{T}\ln f_t(\hat{\varepsilon}_t;\theta)$, where $f_t(\theta)$ represents the conditional probability density function of the standard bivariate normal distribution. * indicates significance at the 5% level.

While the estimates of the conditional mean equations are similar, the results in the conditional variance and/or correlation equations are quite different. Concerning the result of the conventional models, the insignificance of the β_{2i} shows the symmetric GARCH specification seems more suitable for the data. The persistence of the GARCH for the spot and futures are about 0.9947 and 0.9933, respectively, which suggests the conditional volatilities reveal high persistence. For the correlation equations, the inferences conclude that the DCC is held. Hence, the empirical evidence using daily information indicates the ECT-GARCH-DCC model fits reasonably well to the S&P 500 market. Then turn to the estimation results of the RV-based models. The significance of β_{3i} suggests that the asymmetry in volatility are revealed. A positive β_{3i} shows that the impact of a positive return shock on the current volatility is smaller than that of a negative return shock of the same magnitude. Particularly, the persistence of volatility implied by the RV-based models is about 0.9999, which is higher than the persistence implied by the return-based models. Besides the persistence, the weight on the persistence parameters between the two methods also differs. Taking the spot volatility as an example, the return-based GARCH estimates of β_{1i} (β_{2i}) is about 0.91 (0.08), whereas the RV-based GARCH estimates of β_{1i} (β_{2i}) is about 0.77 (0.23). The higher weight on the β_{2i} using RV-based GARCH indicates that past RV may provide more information in predicting the current volatility than those using lagged daily return squares. For the conditional correlation equations, the significance of θ_1 and θ_2 for the S&P 500 indicates the null of mean-reverting DCC hypothesis is held. In addition, the higher persistence of the RV-based correlation than the return-based one is also observed form the empirical evidence. It shows that the persistence of correlation implied by the RV-based models is about 0.99, which is much higher than the persistence (ranges from 0.19 to 0.29) implied by the return-based models. Thus, the empirical evidence using RV indicates the ECT-RV-GJR-DCC model seems suitable to the S&P 500 data. Figure 2.1 compares the conditional volatility and correlation estimates using the two methods. We report the best-fitted models among them to save space. It is evident that the RV-based second moment estimates are not equal to those of the return-based models. Essentially, the difference should come from the dynamics difference between the RV and the return squares.

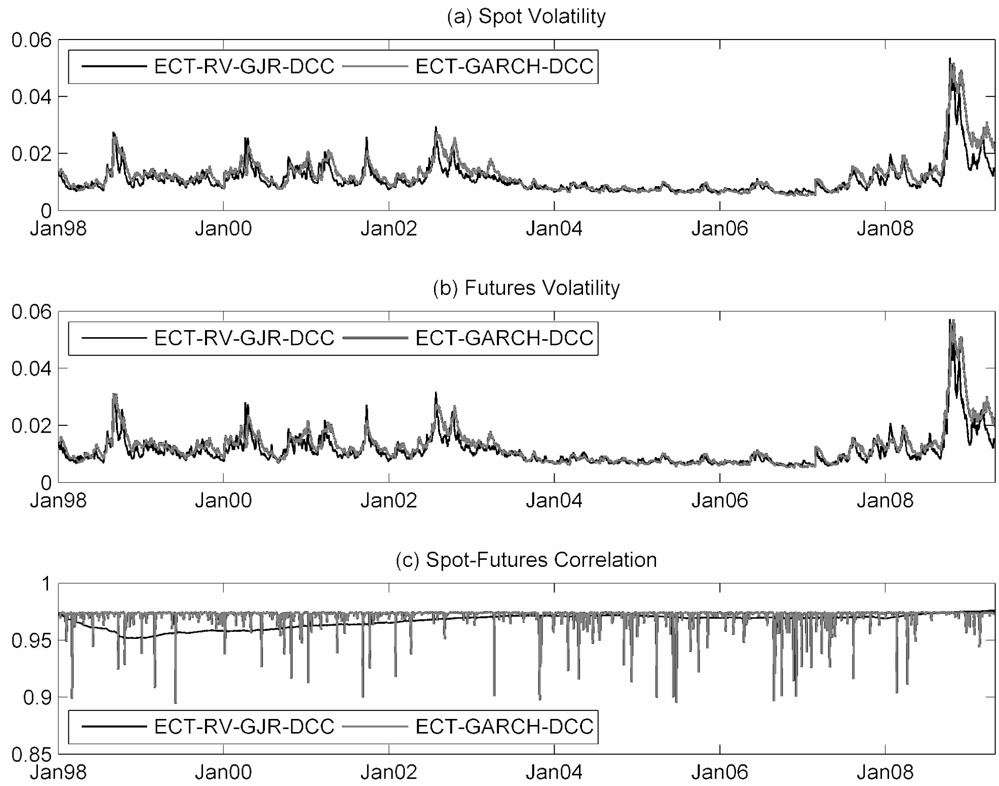

FIGURE 2.1 In-Sample Comparisons on Volatility and Correlation Estimates: RV-Based vs. Return-Based Methods

RV-based Hedge Ratios and their Hedging Performances

We now turn to analyze the performance of a futures hedge using the RV approach. Since the hedging decision has to be made ex-ante, the evaluation is conducted in an out-of-sample context using a rollover method.[16] To do so, this study splits the full sample period into two: the in-sample period (from January 1, 1998 to December 18, 2003; 1500 observations), and the out-of-sample period (from December 19, 2003 to March 31, 2009; 1328 observations). Each model is estimated with the use of the in-sample data and then re-estimated with a daily rollover in the out-of-sample period, keeping the estimation sample size of 1500 (fixed). This

[16] This rollover method is also used in Lien et al. (2002) and Lien and Yang (2006).

rollover method is continued for all the 1328 out-of-samples. The estimated hedge ratios as indicated in Equation (2.9) for each model are subsequently constructed, and the corresponding realized portfolio returns for both the short ($r_{p,t+1} = r_{s,t+1} - \beta_t r_{f,t+1}$) and the long ($r_{p,t+1} = -r_{s,t+1} + \beta_t r_{f,t+1}$) hedges are calculated. In addition to the dynamic models specified in the previous sections, we also evaluate the performance of the static OLS method based on this rollover method. Particularly, to see whether the RV-based models can provide a superior hedging performance during the crisis, the results before (Period I: December 19, 2003 through September 28, 2007, 950 observations) and during (Period II: October 1, 2007 through March 31, 2009, 378 observations) that period are separately reported.

Table 2.4 exhibits some diagnostic statistics of the hedge ratios; and, Figure 2.2 plots the dynamic hedge ratios.[17] On average, the RV-based hedge ratios are larger (smaller) than the conventional return-based hedge ratios in Period I (II); however, they have smaller variation in both the periods. The ADF tests on the hedge ratios illustrate the unit-root hypothesis is rejected at the 5% level except for the results based on the OLS (in both periods) and the ECT-GARCH-CCC (in period II) models.[18] The study also reports the ACF values of these out-of-sample hedge ratios up to lag five. It is apparent that, the ACF of the RV-based hedge ratios is smaller but decays more quickly than the ACF of the return-based hedge ratios.

[17] Since the CCC-based hedge ratios are similar to the DCC-based ones, we do not plot them to save space.
[18] This agrees with the finding of Lien et al. (2002), who report that the out-of-sample GARCH hedge ratios are stationary.

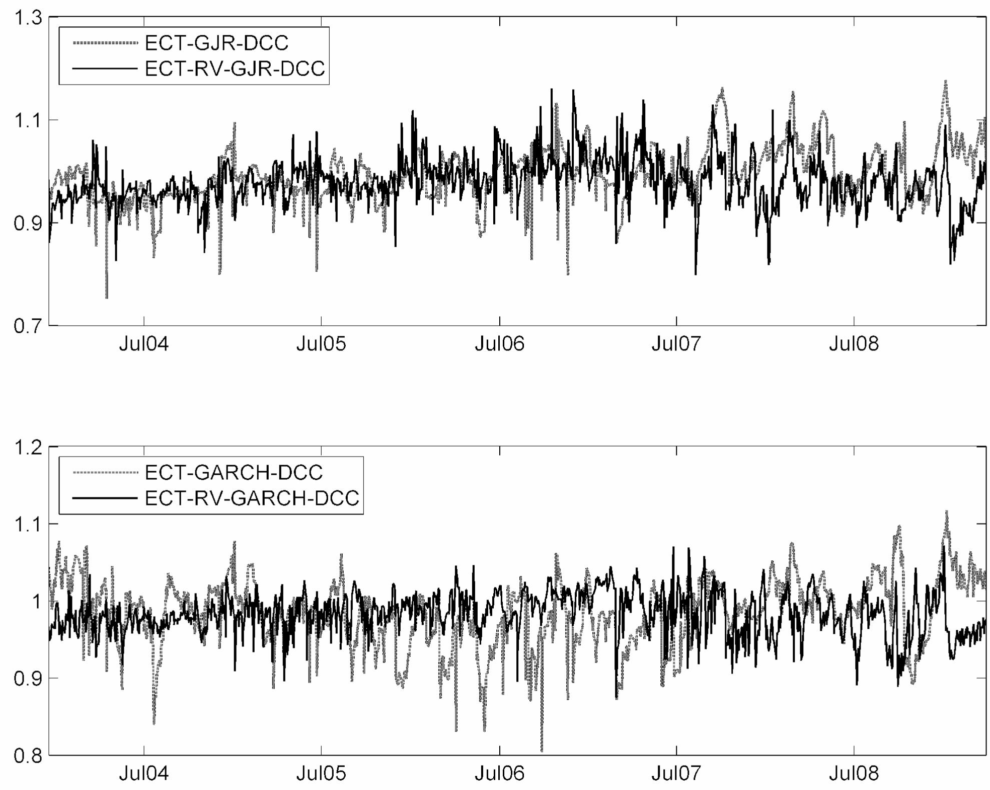

FIGURE 2.2 Out-of-Sample Hedge Ratios: RV-Based vs. Return-Based Methods

Table 2.5 presents several statistics on the realizations of the hedged portfolio returns. Considering the hedging performance using standard deviation (Std. Dev.), the results show that the RV-based method yields an average sample volatility of 0.1730% (0.4284%) for the Period I (II), which is smaller than the 0.1745% (0.4404%) of the conventional method. That is, the improvement of the RV-based method over the return-based method is about 0.90% and 2.74% for the Period I and Period II, respectively. Particularly, the RV-based method even can surpass the simple OLS method during the crisis while the return-based GARCH method does not.[19] Besides the Std. Dev., two alternatives, namely the value-at-risk (VaR) and the ex-

[19] Lien et al. (2002) indicate that the out-of-sample CCC-GARCH hedge does not outperform the OLS hedge in the S&P 500 market, where their data were extracted for the period of January 1988 through June 1998. However,

pected shortfall (ES), are also included in the comparisons since the evaluation results may change if performance criteria other than traditional measures are applied (Cotter & Hanly, 2006).[20] The result shows that using the RV-based method generally can provide a better performance in managing portfolio VaR and ES especially during the crisis period. In addition, in Period II, the improvement of RV-based method over the conventional method tends to enlarge when the percentile has moved toward 99%. For example, on average, the improvement of long hedges in VaR(0.95), ES(0.95), VaR(0.99), and ES(0.99) is about 2.53%, 8.33%, 20.82%, and 11.43% (the percentage change in VaR and ES reduction), respectively; and they all can surpass the simple OLS method whereas the return-based models cannot. For the performance in Period I, it seems that the RV-based method is inferior to the conventional method. In this period, it is observed that the OLS hedge has the best performance in most of the cases. Hence, the empirical evidences conclude that the RV-based hedges are more useful than the return-based hedges in managing portfolio risk especially during the surge of volatility period.

Cotter and Hanly (2006) show that the CCC-GARCH hedge can beat the OLS hedge in the S&P 500 market in terms of variance reduction, where their data were extracted for the period of January 1998 through December 2003.

[20] The VaR is defined by the negative of the α^{th} empirical percentile of the realizations on hedged portfolio returns, i.e., $\mathrm{VaR}(\alpha) = -\hat{F}_n^{-1}(r_{p,t+1};\alpha)$, where $\hat{F}_n$ denotes the empirical distribution of the hedged portfolio returns using the n realized observations. A major shortcoming with the VaR is that it is not a coherent risk measure (Artzner et al., 1999). Hence, the ES measure has received some attention recently. The ES summarizes the negative of the average returns on the portfolio given that the hedged portfolio return has exceeded its α^{th} empirical percentile, or $\mathrm{ES}(\alpha) = -\hat{E}_n(r_{p,t+1} \mid r_{p,t+1} \le -\mathrm{VaR}(r_{p,t+1};\alpha))$, where $\hat{E}_n$ represents the sample average operator. This gives the hedger additional information about both the probability of losses and possible magnitude of losses beyond the α^{th} percentile.

TABLE 2.4 Summary Statistics of Out-of-Sample Hedge Ratios

Statistics	ECT-GARCH		ECT-RV-GARCH		ECT-GJR		ECT-RV-GJR		OLS
	CCC	DCC	CCC	DCC	CCC	DCC	CCC	DCC	
Panel A: Period I (December 19, 2003 ~ September 28, 2007, 950 observations)									
Mean	0.9820	0.9761	0.9908	0.9864	0.9906	0.9828	0.9890	0.9840	0.9602
Std. Dev.	0.0415	0.0427	0.0246	0.0242	0.0438	0.0474	0.0442	0.0438	0.0108
Min	0.8314	0.8037	0.8795	0.8749	0.8321	0.7523	0.8028	0.7979	0.9393
Max	1.0859	1.0778	1.0771	1.0707	1.1562	1.1494	1.1692	1.1613	0.9789
ADF	-7.1189	-9.5666	-15.5897	-15.6399	-7.1890	-12.7961	-16.1559	-16.1489	-1.8493
ACF(1)	0.9036	0.8353	0.6509	0.6453	0.8990	0.7332	0.6529	0.6503	0.9962
ACF(2)	0.8272	0.7486	0.4817	0.4735	0.8457	0.6382	0.5309	0.5287	0.9924
ACF(3)	0.7585	0.6850	0.3343	0.3264	0.7859	0.6064	0.4783	0.4756	0.9986
ACF(4)	0.6996	0.6335	0.2570	0.2487	0.7344	0.6017	0.4358	0.4333	0.9848
ACF(5)	0.6500	0.5886	0.2206	0.2104	0.6798	0.5334	0.4030	0.3985	0.9810
Panel B: Period II (October 1, 2007 ~ March 31, 2009, 378 observations)									
Mean	1.0123	1.0065	0.9761	0.9714	1.0329	1.0264	0.9704	0.9651	0.9807
Std. Dev.	0.0405	0.0401	0.0319	0.0317	0.0542	0.0532	0.0480	0.0477	0.0081
Min	0.8965	0.8923	0.8923	0.8888	0.9168	0.9120	0.8228	0.8181	0.9648
Max	1.1236	1.1176	1.0793	1.0723	1.1992	1.1785	1.1308	1.1204	1.0077
ADF	-3.1989	-3.4508	-6.4422	-6.4631	-3.9150	-4.0537	-7.4427	-7.4055	-1.8333
ACF(1)	0.9493	0.9411	0.8037	0.8026	0.9263	0.9211	0.7518	0.7536	0.9858
ACF(2)	0.8988	0.8878	0.6263	0.6239	0.8623	0.8549	0.6081	0.6090	0.9704
ACF(3)	0.8464	0.8347	0.4938	0.4908	0.8067	0.7973	0.4836	0.4839	0.9510
ACF(4)	0.8004	0.7897	0.3729	0.3690	0.7504	0.7419	0.4093	0.4091	0.9303
ACF(5)	0.7619	0.7509	0.2658	0.2617	0.6902	0.6789	0.2968	0.2963	0.9123

Notes: The ADF indicates the augmented Dickey-Fuller unit root test based on trend stationary AR model, where the 5% critical values for Period I and II are -3.4150 and -3.4234, respectively. The ACF(k) indicates the sample autocorrelation function of the hedge ratios corresponding to lags k. Their 95% confidence bonds for Period I and II are [-0.0649, 0.0649] and [-0.1029, 0.1029], respectively.

TABLE 2.5 Out-of-Sample Comparisons of Hedging Performance: Statistical Evaluations

Models	Std. Dev. ($\times 10^{-2}$)	Short Hedge VaR(.95) ($\times 10^{-2}$)	ES(.95) ($\times 10^{-2}$)	VaR(.99) ($\times 10^{-2}$)	ES(.99) ($\times 10^{-2}$)	Long Hedge VaR(.95) ($\times 10^{-2}$)	ES(.95) ($\times 10^{-2}$)	VaR(.99) ($\times 10^{-2}$)	ES(.99) ($\times 10^{-2}$)
Panel A: Period I (December 19, 2003 ~ September 28, 2007, 950 observations)									
ECT-GARCH-CCC	0.1753	0.2917	0.4305	0.4859	0.7269	0.2493	0.3592	0.4476	0.5557
ECT-RV-GARCH-CCC	0.1729	0.2719	0.4220	0.5037	0.7291	**0.2444**	0.3677	0.4781	0.6005
	(-0.0024)	(-0.0198)	(-0.0085)	(+0.0178)	(+0.0022)	(-0.0049)	(+0.0085)	(+0.0305)	(+0.0448)
ECT-GARCH-DCC	0.1750	0.2888	0.4280	0.4848	0.7143	0.2495	0.3591	0.4323	0.5570
ECT-RV-GARCH-DCC	0.1727	0.2719	0.4206	0.5022	0.7238	0.2461	0.3668	0.4767	0.5990
	(-0.0023)	(-0.0169)	(-0.0074)	(+0.0174)	(+0.0095)	(-0.0034)	(+0.0077)	(+0.0444)	(+0.0420)
ECT-GJR-CCC	0.1740	0.2830	0.4251	**0.4760**	0.7247	0.2498	0.3577	0.4165	0.5560
ECT-RV-GJR-CCC	0.1732	**0.2699**	0.4087	0.4891	0.7266	0.2563	0.3754	0.4635	0.5852
	(-0.0008)	(-0.0131)	(-0.0164)	(+0.0131)	(+0.0019)	(+0.0065)	(+0.0177)	(+0.0470)	(+0.0292)
ECT-GJR-DCC	0.1738	0.2860	0.4208	0.4987	0.7064	0.2494	0.3606	0.4656	0.5663
ECT-RV-GJR-DCC	0.1730	0.2700	**0.4074**	0.4875	0.7224	0.2553	0.3746	0.4635	0.5831
	(-0.0008)	(-0.0160)	(-0.0134)	(-0.0112)	(+0.0160)	(+0.0059)	(+0.0140)	(-0.0021)	(+0.0168)
OLS	**0.1719**	0.2719	0.4213	0.4845	**0.7059**	0.2493	**0.3533**	**0.4142**	**0.5526**
Panel B: Period II (October 1, 2007 ~ March 31, 2009, 378 observations)									
ECT-GARCH-CCC	0.4378	**0.6735**	1.0276	**1.1741**	1.6902	0.7315	1.0899	1.5817	1.6778
ECT-RV-GARCH-CCC	**0.4241**	0.7007	**1.0166**	1.2711	1.4693	0.7125	1.0128	1.2146	1.5001
	(-0.0137)	(+0.0272)	(-0.0110)	(+0.0970)	(-0.2209)	(-0.0190)	(-0.0771)	(-0.3671)	(-0.1777)
ECT-GARCH-DCC	0.4367	0.6807	1.0274	1.1885	1.6829	0.7213	1.0853	1.5714	1.6599
ECT-RV-GARCH-DCC	0.4250	0.6991	1.0178	1.2924	1.4838	0.7222	**1.0091**	1.2231	1.4931
	(-0.0117)	(+0.0184)	(-0.0096)	(+0.1039)	(-0.1991)	(+0.0009)	(-0.0762)	(-0.3483)	(-0.1668)
ECT-GJR-CCC	0.4447	0.6908	1.0240	1.2684	1.6324	0.7365	1.1332	1.4526	1.6762
ECT-RV-GJR-CCC	0.4316	0.7013	1.0646	1.2761	**1.4530**	**0.7055**	1.0194	1.1723	1.4649
	(-0.0131)	(+0.0105)	(+0.0406)	(+0.0077)	(-0.1794)	(-0.0310)	(-0.1138)	(-0.2803)	(-0.2113)
ECT-GJR-DCC	0.4425	0.6799	1.0225	1.2839	1.6275	0.7553	1.1207	1.4315	1.6574
ECT-RV-GJR-DCC	0.4328	0.7143	1.0677	1.2908	1.4654	0.7299	1.0189	**1.1703**	**1.4510**
	(-0.0097)	(+0.0344)	(+0.0452)	(+0.0069)	(-0.1621)	(-0.0254)	(-0.1018)	(-0.2612)	(-0.2064)
OLS	0.4357	0.6855	1.0752	1.2734	1.5828	0.7193	1.0316	1.3290	1.6004

Notes: The values in columns Std. Dev., VaR, and ES are standard deviation, value-at-risk, and expected shortfall of the realized hedged portfolio returns. The entries in the parentheses are the relative values of the improvement of the alternative over the conventional hedge ratio models. The bold values are the best hedging model based on each risk measure.

TABLE 2.6 Out-of-Sample Comparisons of Hedging Performance: EV Gains

Benchmark	Alternative	Short Hedge				Long Hedge			
		$\gamma = 1$	$\gamma = 3$	$\gamma = 7$	$\gamma = 10$	$\gamma = 1$	$\gamma = 3$	$\gamma = 7$	$\gamma = 10$
Panel A: Period I (December 19, 2003 ~ September 28, 2007, 950 observations)									
ECT-GARCH-CCC	ECT-RV-GARCH-CCC	0.1556	0.1572	0.1604	0.1629	-0.1539	-0.1522	-0.1490	-0.1465
ECT-GARCH-DCC	ECT-RV-GARCH-DCC	0.1250	0.1267	0.1299	0.1324	-0.1234	-0.1217	-0.1184	-0.1159
ECT-GJR-CCC	ECT-RV-GJR-CCC	0.2061	0.2067	0.1926	0.1946	-0.2056	-0.2051	-0.2041	-0.2033
ECT-GJR-DCC	ECT-RV-GJR-DCC	0.1754	0.1758	0.1769	0.1777	-0.1748	-0.1743	-0.1732	-0.1725
OLS	ECT-RV-GARCH-CCC	0.0230	0.0223	0.0209	0.0199	-0.0237	-0.0243	-0.0257	-0.0267
OLS	ECT-RV-GARCH-DCC	0.0371	0.0366	0.0356	0.0348	-0.0376	-0.0381	-0.0391	-0.0398
OLS	ECT-RV-GJR-CCC	0.1009	0.1001	0.0983	0.0970	-0.1018	-0.1027	-0.1045	-0.1058
OLS	ECT-RV-GJR-DCC	0.1156	0.1148	0.1134	0.1123	-0.1163	-0.1170	-0.1185	-0.1196
OLS	ECT-GARCH-CCC	-0.1326	-0.1349	-0.1395	-0.1430	0.1302	0.1279	0.1233	0.1198
OLS	ECT-GARCH-DCC	-0.0879	-0.0901	-0.0943	-0.0976	0.0858	0.0836	0.0793	0.0761
OLS	ECT-GJR-CCC	-0.1052	-0.1066	-0.0943	-0.0976	0.1038	0.1024	0.0996	0.0975
OLS	ECT-GJR-DCC	-0.0598	-0.0610	-0.0635	-0.0654	0.0585	0.0573	0.0547	0.0529
Panel B: Period II (October 1, 2007 ~ March 31, 2009, 378 observations)									
ECT-GARCH-CCC	ECT-RV-GARCH-CCC	-1.0951	-1.0714	-1.0241	-0.9887	1.1187	1.1423	1.1896	1.2251
ECT-GARCH-DCC	ECT-RV-GARCH-DCC	-1.0760	-1.0558	-1.0152	-0.9849	1.0963	1.1166	1.1571	1.1875
ECT-GJR-CCC	ECT-RV-GJR-CCC	-1.5027	-1.4799	-1.4340	-1.3995	1.5257	1.5486	1.5946	1.6289
ECT-GJR-DCC	ECT-RV-GJR-DCC	-1.4798	-1.4625	-1.4283	-1.4025	1.4969	1.5141	1.5483	1.5740
OLS	ECT-RV-GARCH-CCC	-0.2879	-0.2678	-0.2277	-0.1977	0.3079	0.3279	0.3680	0.3981
OLS	ECT-RV-GARCH-DCC	-0.3628	-0.3443	-0.3072	-0.2795	0.3813	0.3998	0.4368	0.4646
OLS	ECT-RV-GJR-CCC	0.1916	0.1987	0.2131	0.2239	-0.1844	-0.1772	-0.1628	-0.1521
OLS	ECT-RV-GJR-DCC	0.1068	0.1120	0.1222	0.1300	-0.1017	-0.0965	-0.0863	-0.0786
OLS	ECT-GARCH-CCC	0.8072	0.8036	0.7964	0.7910	-0.8108	-0.8144	-0.8216	-0.8270
OLS	ECT-GARCH-DCC	0.7132	0.7115	0.7080	0.7054	-0.7150	-0.7168	-0.7203	-0.7229
OLS	ECT-GJR-CCC	1.6943	1.6786	1.6471	1.6234	-1.7101	-1.7258	-1.7574	-1.7810
OLS	ECT-GJR-DCC	1.5866	1.5745	1.5505	1.5325	-1.5986	-1.6106	-1.6346	-1.6526

Notes: The table shows the basis point fees per day that an hedger with the quadratic utility and the constant relative risk aversion of γ would willing to pay to switch the benchmark to the alternative strategies. Note that the effect of transaction costs on the hedging performance is excluded.

In addition to statistical evaluations, hedgers may wish to understand the economic gains of a futures hedge using the intraday-based RV approach. As a result, the EV, $\hat{E}_n U(r^b_{p,t+1};\gamma) = \hat{E}_n U(r^a_{p,t+1} - \mathrm{EV};\gamma)$, gives an amount that the hedger would be willing to sacrifice each day to switch from the benchmark strategy to the alternative strategy, where $\hat{E}_n$ represents the sample average operator (Lence, 1995). Note that the mean-variance utility function $EU(\gamma) = E(r_{p,t+1}) - \gamma\,\mathrm{var}(r_{p,t+1})$ is specified with the degree of risk aversion $\gamma > 0$.[21] The advantage of using EV is that it further accounts for the hedger's risk preference. This study considers four different levels of risk aversion $\gamma = 1,3,7,10$ to assess the performance gains across hedgers.[22] The results for each model and each period examined are summarized in Table 2.6.[23]

Table 2.6 illustrates that the RV-based hedges can substantially outperform the static OLS and/or the return-based dynamic GARCH hedges in terms of EV gains. For short hedges in Period I, the RV-based method generates positive EV gains over the conventional and the OLS methods, while the EV of the return-based GARCH method over the OLS is negative. Taking $\gamma = 1$ as an example, the average EV of the RV-based hedges surpasses the return-based GARCH (OLS) hedges by about 0.17 (0.07) basis points per day, while the average EV gain of the return-based GARCH method over the OLS is about -0.10 basis points per day. For long hedges, however, it is observed that the RV-based method not only underperforms the return-based GARCH method but also loses the static OLS method in Period I. Since the expected hedged portfolio returns $E(r_{p,t+1})$ for short and long hedges have the same magnitude but with opposite sign, the benefits for short hedging using RV should be harmful to the long hedge when the hedgers have the quadratic utility form. Then examining

---

[21] The utility function is also considered in Kroner and Sultan (1993) and Lien and Yang (2006).

[22] This specification on risk aversion follows Patton (2004), who considers $\gamma = 1,3,7,10,20$ for asset allocating. In addition, Lien and Yang (2006) assume that the risk aversion to be 4 in measuring the expected utility of a futures hedge.

[23] Given risk aversion γ, the EV of the alternative over the benchmark model is obtained by solving the nonlinear equation: $E(r^b_{p,t+1}) - \gamma\,\mathrm{var}(r^b_{p,t+1}) = E(r^a_{p,t+1} - \mathrm{EV}) - \gamma\,\mathrm{var}(r^a_{p,t+1} - \mathrm{EV})$ with the use of the out-of-sample hedged portfolio returns.

the performance in Period II, the average EV gain over the conventional method by the RV-based method for long (short) hedges is about 1.35 (-1.24) basis points per day. As compared with the OLS method, it is observed that the ECT-RV-GARCH-based (ECT-RV-GJR-based) models for long (short) hedges can generate positive EV gains in the period. Clearly, a short (long) hedger with the mean-variance utility would prefer using the RV-based method for his/her hedging activity in Period I (II). Moreover, it is evident that the EV gains of the RV-based models differentiate across hedgers with different risk aversions. The EV of the alternative method surpasses the conventional method because it increases as the risk aversion of the hedger increases. That is, a hedger with a higher risk aversion in the S&P 500 market can benefit more when he/she uses the RV-based models in hedging.

Hedge Horizon, Hedge Ratio, and Hedging Effectiveness

As we have discussed the results based on a daily hedging horizon, since individuals and institutions may not have the same hedging horizon for their specific purposes, the RV-based method is further applied to study the effect of hedging horizon on hedge ratio and hedging effectiveness. A few studies have found that hedge ratio depends on the hedging horizon and approaches unity (i.e., naïve hedge ratio) for a longer horizon; and, hedging effectiveness tends to increase as the length of hedging horizon increases (see, for example, Ederington, 1979; Geppert, 1995; Chen et al., 2004; Lien and Shrestha, 2007).[24] It is noted that, however, this result has never been examined empirically using the RV-based and the return-based GARCH methods. Thus, this study analyzes the relationships of the dynamic methods for six hedging horizons: 1-week (1W), 2-week (2W), 3-week (3W), 1-month (1M), 2-month (2M), and 3-month (3M), and the in-sample and out-of-sample results are plotted in Panel (a)-(b)

[24] For example, the findings are supported by Ederington (1979), Geppert (1995), and Lien and Shrestha (2007). These studies use the OLS technique, the cointegration method, and the wavelet analysis, respectively, to study the effect of hedging horizon length on the minimum-variance hedge ratio and the hedging effectiveness of Ederington (1979).

and (c)-(d) of Figure 2.3, respectively.[25] Panel (a) and (c) show the hedge ratio tends to increase and to approach unity with increase of the length of hedging horizon. For the in-sample case, the conventional hedge ratio is likely to exceed the alternative hedge ratio except the 3W and the 1M horizons. For the out-of-sample study, however, the alternative generally has higher hedge ratio estimates but not for the 3W and the 3M cases. In terms of hedging performance, Panel (b) and (d) plot the effect of hedging horizon on the hedging effectiveness as shown by Ederington (1979), which is estimated by calculating the percentage reduction in the variance of the naked spot portfolio. It is apparent that the hedging effectiveness increases with the length of hedging horizon, but the degree of hedging effectiveness does not approach one. For the out-of-sample analysis, the RV-based method generally outperforms the conventional method in terms of the hedging effectiveness for shorter horizons (within 2 weeks), but fail to outperform the conventional method for longer horizons.[26] As a result, to achieve a better hedging performance, it is suggested that hedgers should use more futures contracts for hedging when their hedging horizon is no longer than 2 weeks.

---

[25] Lien and Shrestha (2007) indicate that there are two ways to incorporate hedging horizon in estimating hedge ratio. One way is to derive an optimal hedge ratio that explicitly depends on hedging horizon based on some models, such as the model of Geppert (1995). The other way is to estimate the hedge ratio by matching the data frequency with the hedging horizon, such as the approach used by Chen et al. (2004). In this study, we use the non-overlapping approach of Chen et al. (2004) that matches the data frequency to estimate the hedge ratio and the resulting hedging effectiveness. The figures plot the average values of the empirical results based on the conventional and the alternative models mentioned in the previous sections.

[26] It should be noted that, however, the sample size used to estimate hedge ratio and hedging effectiveness decreases quickly with hedging horizon length (e.g., there are only 20 out-of-sample estimates for 3M hedging horizon). Hence, the results for longer horizons, such as 2M and 3M, may not be reliable because a lower frequency results in a substantial reduction in the sample size (see, e.g., Lien & Shrestha, 2007).

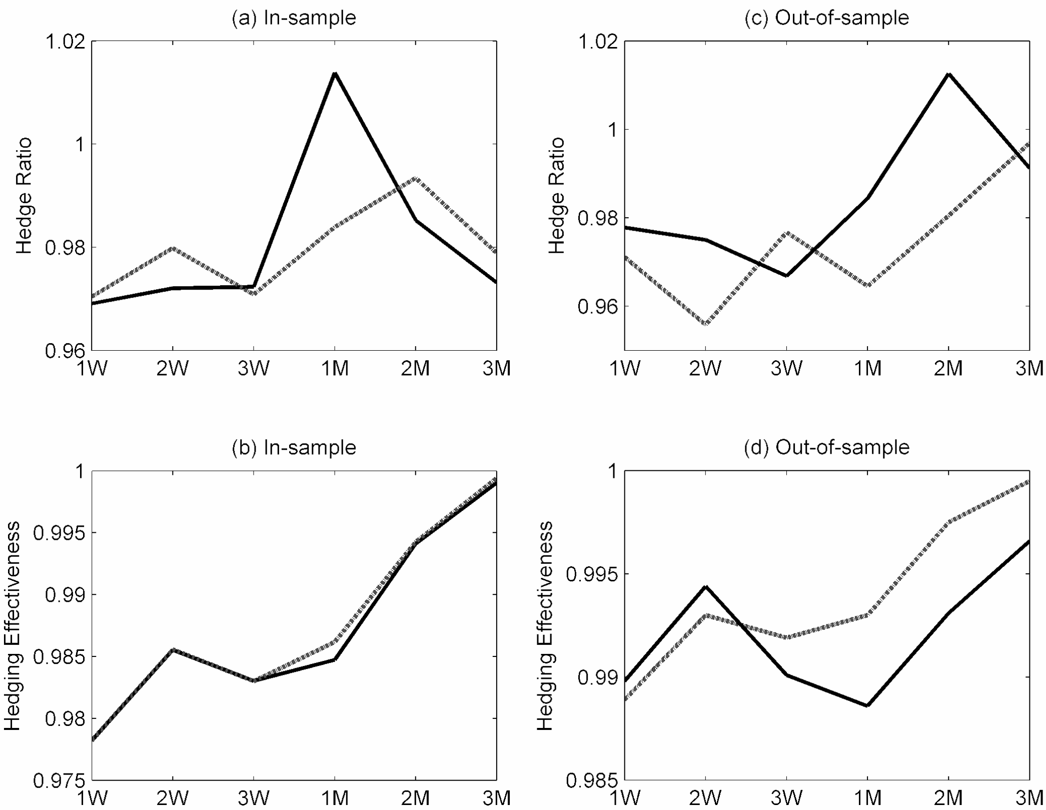

FIGURE 2.3 The Effect of Hedge Horizon on Hedge Ratio and Hedging Effectiveness: RV-Based (Solid) vs. Return-Based (Dash) Methods

Chapter 3. An Application of Realized Regression to the Hedging Problem

This chapter presents a high-frequency based method for analyzing the one-period futures hedging problem. Applying the realized regression, the realized hedge ratio is constructed; meanwhile, the R-squared coefficient provides an ex-post performance measure. The asymptotic theory enables us to assess the parameter estimation risk of the hedge ratios. An empirical study is conducted on the S&P 500 index and their hedging performance is compared to the conventional hedge ratios. Moreover, impacts of the market microstructure effect on the realized hedge ratio and the effect of the parameter estimation risk on the corresponding hedging performance are discussed.

3.1. Research Problem and Objective

Under the specification of one-factor capital asset pricing model, Andersen et al. (2005, 2006) gave the concept of integrated beta, which is defined as the ratio between the integrated covariance of the market and the i^{th} individual stocks and the integrated market variance, and further studied the macroeconomic determinants and dynamics of the systematic risk via the realized beta approach. Applying the results to the one-period hedging problem, the aim of this chapter attempts to show that the usefulness of the realized beta approach for analyzing the problem.

With some assumptions in the underlying price processes, the integrated hedge ratio is defined firstly. The estimator, realized hedge ratio, is consistent with the integrated hedge ratio, and the estimate is achieved by applying the realized regression of Barndorff-Nielsen and Shephard (2004), using larger data sets of intraday returns. Meanwhile, the asymptotic distribution provides insights into the precision of the realized hedge ratio. Furthermore, the realized hedging effectiveness provides an ex-post estimate for the integrated hedging effectiveness. This realized method is then applied to the S&P 500 market spanning the period of January 2000 and December 2004 by assuming a weekly (Wednesday-based) hedging horizon. Moreover, its performance is compared to the conventional hedging models.

Analyzing the hedging problem using the proposed method has the following advantages. First, the realized hedge ratio provides explicitly continuous evaluation in the integrated hedge ratio without assuming discrete-time multivariate volatility models or time variations in the hedge ratio dynamics. Meanwhile, the ex-post realizations further enable us to study the properties within the hedge ratio. Second, the conventional hedge ratio is estimated by the parameter certainty equivalent (PCE) procedure. Substituting sample estimates of models' parameters, the estimated hedge ratio neglects the issue of parameter estimation risk (Lence & Hayes, 1994a,b; Shi & Irwin, 2005). In contrast, the asymptotic confidence interval allows exploring the estimation risk at the empirical level. Third, the integrated volatility corresponds closely to the conditional volatility for discrete-time returns. Thus, the realized hedging effectiveness provides an ex-post measure for conditional hedging effectiveness, as compared with the unconditional version of Ederington (1979).

The empirical findings are summarized as follows. First, 15-minute returns may be suitable for constructing realized weekly hedge ratios for the S&P 500 index. The market microstructure effect leads the realized hedge ratio downward-biased when the sampling frequency chosen is high, such as 5-minute returns. Second, the realized weekly hedge ratio is generally superior to other conventional hedge ratios; meanwhile, the GARCH hedge ratios do not perform better than the OLS estimate, based on realized hedging effectiveness measures. They are evaluated for the in-sample and out-of-sample period. Third, a regression analysis indicates that the realized hedging effectiveness is lower if the realized weekly hedge ratio becomes imprecise. The implication is that the parameter estimation risk is important when hedgers make hedging decisions. Therefore, the estimation risk should be incorporated into the hedging models (Lence and Hayes, 1994a, 1994b; Shi and Irwin, 2005). The rest of this chapter is organized as follows. Next, we present the concept of integrated hedge ratio and integrated hedging effectiveness, and further apply the realized regression of Barndorff-Nielsen and Shephard (2004) to have the estimates. This is followed by illustrating an empirical example; and finally, the chapter is concluded in the last section.

3.2. Realized Hedge Ratio and Hedging Effectiveness

To illustrate, assume the 2×1 vector of spot-futures returns follows the continuous-time diffusion process as mentioned before. Based on the derivation of an optimal (risk-minimizing) futures hedge, the integrated hedge ratio over the time interval $[t, t+1]$ is formally defined as

$$\beta_{t+1} \equiv \int_t^{t+1} \sigma_{sf}(s)ds \Big/ \int_t^{t+1} \sigma_f^2(s)ds \tag{3.1}$$

Note this definition is also found in Harris et al. (2007). Extending the specification to the hedging effectiveness of Ederington (1979), the integrated hedging effectiveness is defined:

$$\mathrm{IHE}_{t+1}^{\beta_t^*} \equiv 1 - \int_t^{t+1} \sigma_p^2(s;\beta_t^*)ds \Big/ \int_t^{t+1} \sigma_s^2(s)ds \tag{3.2}$$

It measures the percentage reduction in the integrated hedged portfolio variance for a particular (ex-ante) hedge ratio, β_t^*, with the integrated variance of the spot position. Since the integrated (co-)variance is closely related to the conditional (co-)variance, the realized hedging effectiveness may be regarded as a measure for calculating the percentage reduction from the conditional variance of the spot position to the conditional variance of the hedged portfolio.

Andersen et al. (2005, 2006) studied the macroeconomic determinants and dynamics of the systematic risk in the one-factor capital asset pricing model by using the realized beta approach. The approach then provides an empirical framework for analyzing the hedging problem. Based on their results, the realized hedge ratio for the integrated hedge ratio could be estimated by using intraday high-frequency data as follows:

$$\hat{\beta}_{t+1} = \mathrm{RCov}_{sf,t+1} / \mathrm{RV}_{f,t+1} \rightarrow \beta_{t+1}, \quad \triangle \rightarrow 0 \tag{3.3}$$

and it provides a natural benchmark for assessing the forecasting ability of any ex-ante (risk-minimizing) hedge ratios. For example, the hedger may consider the loss functions, such as $\mathrm{MSE}(\beta_t^*) \equiv \hat{E}_n(\beta_{t+1} - \beta_t^*)^2$ or $\mathrm{MAE}(\beta_t^*) \equiv \hat{E}_n \mid \beta_{t+1} - \beta_t^* \mid$, for the evaluation process. With the truth underlying integrated hedge ratio, the hedging methods are directly assessed by comparing relative size of the MSE and/or MAE without computing the realization on the hedged portfolio returns or gained utilities. In addition, for estimating the integrated hedging effectiveness, the realized hedging effectiveness for any particular β_t^* gives the result:

$$\mathrm{RHE}_{t+1}^{\beta_t^*} = 1 - \mathrm{RV}_{p,t+1}^{\beta_t^*} / \mathrm{RV}_{s,t+1} \rightarrow \mathrm{IHE}_{t+1}^{\beta_t^*}, \quad \triangle \rightarrow 0 \tag{3.4}$$

where

$$\mathrm{RV}_{p,t+1}^{\beta_t^*} = \mathrm{RV}_{s,t+1} + \mathrm{RV}_{f,t+1} - 2\beta_t^* \mathrm{RCov}_{sf,t+1} \tag{3.5}$$

Barndorff-Nielsen and Shephard (2004) have discussed some issues in the realized measures. Focusing on the realized hedge ratio (beta) in the analysis, the realized regression method:

$$r_{s,t_j} = \beta_{t+1} \cdot r_{f,t_j} + \varepsilon_{t_j}, \quad j = 1, \ldots, 1/\triangle \tag{3.6}$$

could be applied to estimate the integrated hedge ratio (beta). Note that the upper bound of the realized hedging effectiveness is equivalent to R-squared coefficient in the realized regression (Harris et al., 2007). In addition, the asymptotic property for the realized regression is also presented in Barndorff-Nielsen and Shephard (2004). They have shown that the asymptotic distribution of the integrated hedge ratio follows

$$\frac{\hat{\beta}_{t+1} - \beta_{t+1}}{\sqrt{\mathrm{RV}_{f,t+1}^{-2}\tilde{g}_t}} \sim N(0,1) \tag{3.7}$$

where

$$\tilde{g}_t = \sum_{j=1}^{1/\Delta} a_j^2 - \sum_{j=1}^{1/\Delta-1} a_j a_{j-1} \quad \text{and} \tag{3.8}$$

$$a_j = r_{s,t_j} r_{f,t_j} - \hat{\beta}_{t+1} r_{f,t_j}^2 \tag{3.9}$$

With the asymptotic distribution, the $\alpha\%$ asymptotic confidence interval for the integrated hedge ratio is constructed:

$$\beta_{t+1} \in \hat{\beta}_{t+1} \pm z_{\alpha/2}\sqrt{\mathrm{RV}_{f,t+1}^{-2}\tilde{g}_t} \tag{3.10}$$

With the interval, interval forecast ability of ex-ante hedge ratios could also be assessed via some designed loss functions.

3.3. An Illustrated Example

An example on the mid-week (Wednesday-base) futures hedge is presented in this section. The underlying assets contain the S&P 500 index and the S&P 500 futures traded on the CME. Data sets contain weekly and high frequency intraday prices spanning the period of January 1, 2000 and December 31, 2004, provided by the *Datastream* and *Tick Data Inc*. The intraday price data are equidistant in time. The continuous price series for the futures are created when the 2nd month future volume exceeds the first future month volume. Moreover, the futures returns after 15:00 for each day are dropped because the futures market closes fifteen minutes later than the spot market. Then the realized weekly hedge ratio series is constructed using the transactions prices based on the previous tick method.

Sampling Frequencies and Realized Hedge Ratios

The issue of sampling frequency chosen should be important for calculating the realized weekly hedge ratios. For example, due to the market microstructure noises, Barndorff-Nielsen and Shephard (2002) showed that the realized daily volatility is biased when the sampling frequency is high. Thus, we study the properties of realized weekly hedge ratios against various sampling frequencies. Table 3.1 summarizes the statistics of realized weekly hedge ratios against five sampling frequencies, including 5-, 10-, 15-, 30-, and 60-minute. Applying these data sets to Equation (3.3), we have 261 realized weekly hedge ratios for each sampling method. The statistics in Table 3.1 conclude that all the hedge ratios exhibit time varying, left-skewed and leptokurtic. Moreover, the means of the realized hedge ratios are about 0.90 except the result obtained from the 5-minute data. For example, a two-tail t statistic (-22.17) indicates that the mean of the 5-minute and the 15-minute realized hedge ratios are different at the 1% significance level. The preliminary analysis shows that the realized weekly hedge ratios may be biased estimates for the integrated weekly hedge ratios if 5-minute returns are used.

Table 3.1 Summary Statistics of Realized Hedge Ratios against Sampling Frequencies

Δ (Minute)	5	10	15	30	60
Min.	0.5552	0.6779	0.7554	0.4107	0.3273
25^{th}	0.7972	0.8749	0.8925	0.8678	0.8717
50^{th}	0.8332	0.9014	0.9219	0.9052	0.9286
75^{th}	0.8565	0.9206	0.9413	0.9419	0.9816
Max	0.9319	0.9779	1.1448	1.0374	1.1261
Mean	0.8229	0.8959	0.9176	0.8989	0.9218
Std. Dev.	0.0535	0.0394	0.0409	0.0659	0.0920
Skewness	-1.3426	-1.1587	-0.0592	-2.0097	-1.5209
Kurtosis	6.3308	6.6692	7.5263	14.4019	10.1414
Obs.	261	261	261	261	261

A proper sampling frequency for realized weekly hedge ratios may be addressed by plotting the hedge ratio signature plot. This provides a visual trade-off between the accuracy and the precision of the realized weekly hedge ratio, and the spirit is similar to the volatility signature plot of Andersen et al. (2000). In this study, the accuracy is measured by the mean of the realized weekly hedge ratios, and the precision is measured by the mean of the 95% asymptotic confidence intervals. Figure 3.1 shows the plot. It can be observed that the 95% asymptotic confidence interval narrows when the sampling frequency increases. That is, the realized hedge ratio becomes precise if a larger number of observations are used. The smallest two confidence intervals, 0.1147 and 0.1184, occur in the 5-minute and the 15-minute, respectively, and the highest, 0.2401, occurs in the 60-minute. In balancing the accuracy and the precision of the realized weekly hedge ratio, it seems that the 15-minute hedge ratio is more reliable.

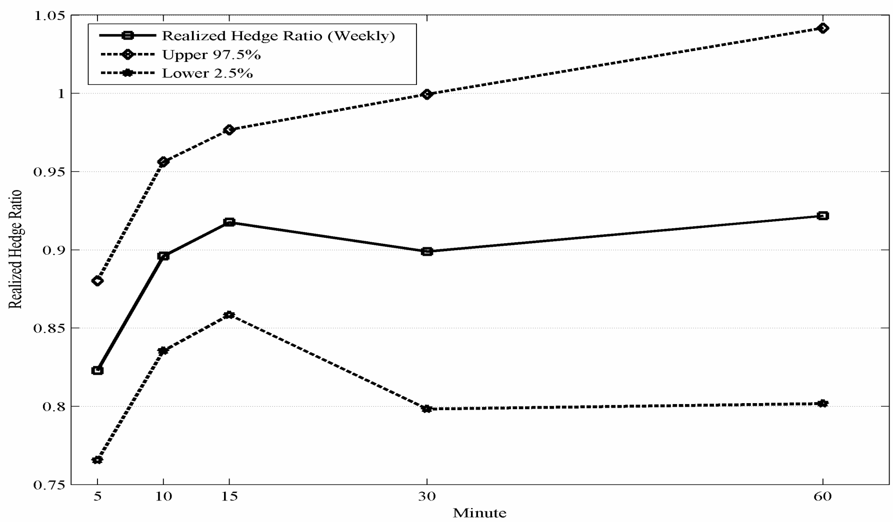

Figure 3.1 Average Values of Realized Weekly Hedge Ratios and 95% Confidence Intervals drawn against Sampling Frequencies.

The Impact of Parameter Estimation Risk

Figure 3.2 plots (15-min) realized weekly hedge ratio and the corresponding upper bound of realized hedging effectiveness in panel (a) and panel (b), respectively. Note that the solid line in panel (a) represents the realized weekly hedge ratio and the dotted lines represent its 95% asymptotic confidence intervals. It is observed that the realized hedge ratio, the confidence interval, and the realized hedging effectiveness are fluctuated with time as the market environment changes. Their relationship is further built by the time-series regression:

$$\underset{}{\mathrm{RHE}_t} = \underset{(18.77)}{0.7536} + \underset{(5.76)}{0.2477}\hat{\beta}_t - \underset{(-14.06)}{0.8424}\mathrm{ACI}_t + u_t, \quad R^2 = 0.50$$

The result shows that the realized (upper bound) hedging effectiveness is positively related to the level of the realized hedge ratio and negatively related to the asymptotic confidence interval.[27] The implication of the results is that the issue of parameter estimation risk for the hedge ratio should be important so that the risk should be incorporated into the hedging models (Lence & Hayes, 1994a,b; Shi & Irwin, 2005).

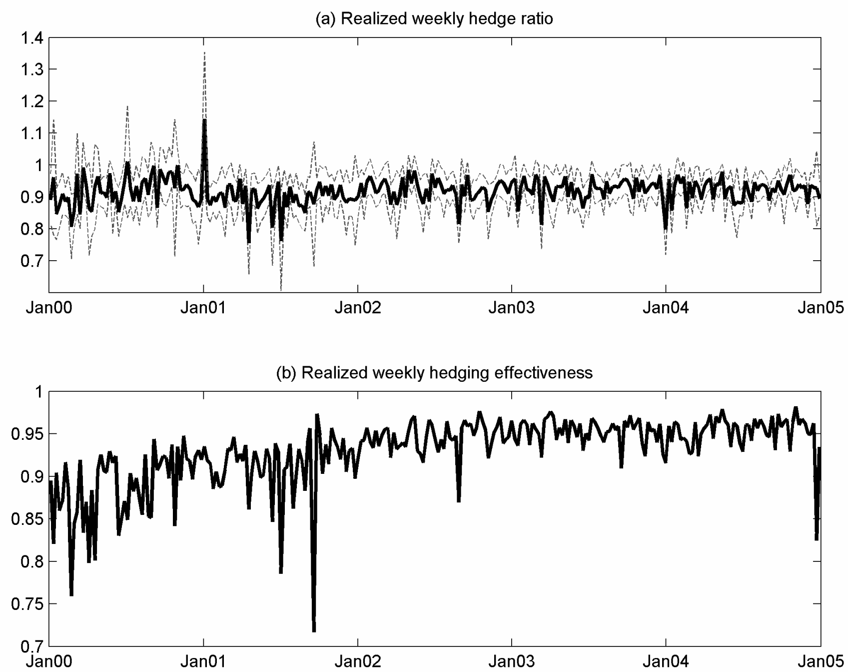

Figure 3.2 (a) Realized Weekly Hedge Ratios with 95% Asymptotic Confidence Intervals; (b) Upper Bound of Realized Weekly Hedging Effectiveness

[27] As suggested by an anonymous referee, this positively relates to the realized hedge ratio follows from the definition of regression R-squared so that it should be regarded as a control variable in the analysis.

Performance Evaluation Results

The benefit of using larger intraday data sets in estimating weekly hedge ratios is compared with conventional approaches, including the Naïve, the OLS, and the CCC-GARCH estimates. They are evaluated for both in-sample and out-of-sample, based on the proposed realized hedging effectiveness of Equation (3.4). The sample period, January 2000 to December 2003, is used for the in-sample estimation (209 weeks), which leaves one year for the out-of-sample comparisons (52 weeks). The comparison results are shown in Table 3.2. Since the estimation of the Naive, the OLS, and the CCC-GARCH hedge ratios are quite standard so that the result is not reported.[28] The corresponding realized hedging effectiveness is then computed using five sampling frequencies, say, 5-, 10-, 15-, 30-, and 60-minute returns, to achieve a robust result.

Table 3.2 Unconditional Sample Means of Realized Weekly Hedging Effectiveness

△ (Minute)	Naive	OLS	CCC-GARCH	Realized
		In-sample (209 Obs.)		
5	0.7708	0.7830	0.7828	0.7984
10	0.8828	0.8885	0.8877	0.8924
15	0.9169	0.9210	0.9198	0.9245
30	0.8857	0.8918	0.8911	0.8982
60	0.9028	0.9074	0.9070	0.9111
		Out-of-sample (52 Obs.)		
5	0.7881	0.7995	0.7967	0.8135
10	0.8920	0.8974	0.8954	0.9008
15	0.9227	0.9267	0.9245	0.9297
30	0.8887	0.8947	0.8922	0.9007
60	0.9058	0.9103	0.9083	0.9137

The result concludes that the realized hedge ratio yields the best performance for the in-sample; and, the superior OLS than the CCC-GARCH model agrees with previous researches, see, e.g., Lien et al. (2002). For the out-of-sample evaluations, the one-step-ahead

28 In the analysis, the OLS gives static estimates, 0.96, while the CCC-GARCH gives time-varying estimates ranging from 0.86 to 1.06.

forecasts for the Naïve, OLS, and CCC-GARCH methods are estimated using a rollover method. We keep the fixed estimation sample size 209 (observation for time $T+1$ is incorporated and the observation for 1 is deleted) and three series of hedge ratio forecasts are then obtained. For the realized hedge ratios, the lagged one period realized hedge ratio is applied as the proxy for the one-step-ahead forecast. The average realized hedging effectiveness for each model is compared. It is concluded that the realized hedge ratio yields the best performance.

Chapter 4. Regime-Dependent Dynamics of a Futures Hedge

A large number of prior studies have developed models for futures hedging. Constructed from high-frequency data and realized beta framework of Andersen et al. (2005, 2006), this chapter has analyzed the dynamics in the realized daily hedge ratio, which serves as an accurate estimate for the integrated daily hedge ratio. Moreover, a two-regime threshold autoregressive model is applied to detect the regime-switching feather of the ratio. Empirical studies on two equity index futures show that the hedge ratio behaves in a regime-dependent dynamics and tends to be more volatile in the low regime than in the high regime. The result then supports the argument of time-varying hedge ratio.

4.1. Research Problem and Objective

A large number of studies have documented that the conventional regression-based static approach is inappropriate for futures hedge because the joint distribution of spot and futures prices is not constant through time.[30] Henceforth, models for time-varying (or conditional) minimum-variance hedge ratios have emerged and been discussed. For example, the bivariate GARCH model of Baillie and Myers (1991) or the random coefficient autoregressive model of Bera et al. (1997) both lead estimates of dynamic hedge ratios. The latter directly estimates time-varying coefficients instead of conditional second moments so that different behavior may be appeared in the estimated hedge ratio. The insight into the hedge ratio behavior may further the development of dynamic hedge ratio models. Set against this background, we analysis the dynamic property in integrated daily hedge ratio via the realized beta framework of Andersen et al. (2005, 2006), which allows explicitly for approximating the integrated

[30] See, for example, Baillie and Myers (1991), Kroner and Sultan (1993), and among many others for evidences of this.

hedge ratio from underlying covariance and variance components.

Empirical studies are conducted on the S&P 500 and the NASDAQ 100 futures contracts. Using the intraday data for the spot and the futures over a six-year period from January 1, 1999 to December 31, 2004, their realized daily hedge ratios are firstly constructed. To explore the possibility of nonlinear dynamics in the realized hedge ratio series, a two-regime Self-Exciting Threshold Autoregressive (SETAR) model is then considered in this paper. The two-regime SETAR is tackled with a linear Autoregressive (AR) model if the threshold effect is not significant. In testing the linear AR against the nonlinear SETAR models, a heteroskedasticity-consistent LM-based statistic of Hansen (1996) is applied via a bootstrap procedure. Empirical results conclude the realized daily hedge ratio is characterized as regime-dependent dynamics and is likely to be positively autocorrelated so that the usual assumption of constant hedge ratio seems inappropriate. In addition, the main effect of the regime demonstrates different variation in the daily-realized hedge ratio. That's, the hedge ratio tends to be more volatile in the low regime than in the high regime. The rest of the article is organized as follows. Next, we demonstrate the SETAR model and then elaborate on the realized daily hedge ratio. This is followed by providing the empirical evidence, and finally, the conclusion is presented in the last section.

4.2. A Two-Regime SETAR Model

To explore the dynamics in the integrated daily hedge ratio, consider the two-regime SETAR model with the form:

$$
\begin{aligned}
y_t = {} & (\alpha_0 + \alpha_1 y_{t-1} + \cdots + \alpha_m y_{t-m}) 1(y_{t-d} \le \gamma) \\
& + (\beta_0 + \beta_1 y_{t-1} + \cdots + \beta_m y_{t-m}) 1(y_{t-d} > \gamma) + e_t
\end{aligned}
\tag{4.1}
$$

where $1(\cdot)$ is an indicator function; y_{t-d} is the threshold variable; γ is the threshold parameter. The autoregressive order is $m \geq 1$, and the parameters α_j and β_j are autoregressive slopes. The SETAR is composed of two regime-dependent piecewise linear models for which the regime-switching dynamics is controlled by a lagged dependent variable. Hansen (1996) provided a heteroskedasticity-consistent LM-based statistic for testing the linearity against the nonlinear SETAR. As the threshold parameter is not identified under the null, the asymptotic distribution of the statistic is not standard and may be approximated by using a bootstrap procedure. In estimating the SETAR model, the sequential conditional least squares or the sequential conditional quasi-maximum likelihood method may be applied. The threshold variable of Equation (4.1) is set as $y_{t-d} \equiv y_{t-1}$ because it should provide relevant information for hedgers when making hedging decisions.

4.3. Realized Daily Hedge Ratios

We investigate the dynamics of the integrated daily hedge ratio in the context of hedging with two stock index futures, namely the S&P 500 and the NASDAQ 100 (traded in CME), over a six-year period from January 1, 1999 to December 31, 2004. Applying the realized beta framework of Andersen et al. (2005, 2006) to this one-period hedging problem, the estimates of the integrated daily hedge ratios for each of the two index pairs are given by

$$\hat{y}_t = \mathrm{RCov}_{sf,t} / \mathrm{RV}_{f,t} \rightarrow y_t \tag{4.2}$$

almost surely for all t as the time between sampling observations $\Delta \rightarrow 0$. In this chapter, both the realized daily hedge ratio series are constructed from the transactions prices based on the previous tick method.[31] The price data sets from the *Tick Data Inc.* are equidistant in time, and 5-minute sampling is often used in practice, see, Andersen et al. (2001a) for discussion.

[31] Several sampling methods have been proposed for constructing the intraday returns, such as the linear interpolation method of Andersen and Bollerslev (1997). One of the discussions of these methods can be referred to Hansen and Lunde (2006).

Specifically, for the futures contracts, the nearest month contract is rolled to the next month when the daily volume of the current contract is exceeded. Moreover, to construct the realized covariance, the futures returns after 15:00 for each day are dropped because the futures markets close fifteen minutes later than the spot markets. The 5-minute returns of the futures and the spot are used to calculate the realized daily hedge ratio for each product. Table 4.1 reports their descriptive statistics and dynamic dependence. The Phillips-Perron unit root tests conclude that the two realized daily hedge ratios are stationary, and the Ljung-Box Q statistics conclude that they are strongly autocorrelated. The preliminary analysis suggests that the realized daily hedge ratio may be modeled as stationary $I(0)$ processes although the realized variance and covariance may be well approximated by a nonlinear fractionally cointegration (Andersen et al., 2006).

Table 4.1 Statistics and Dynamic Dependences of Realized Daily Hedge Ratios

Index	S&P 500	NASDAQ 100
Mean	0.801	0.804
Std. Dev.	0.094	0.105
Skewness	-1.045	-1.268
Kurtosis	4.878	5.367
Q(5)	**978.094**	**1525.645**
PP-AR test	**-2.466**	**-2.564**
PP-ARD test	**-25.470**	**-22.845**
PP-TS test	**-32.298**	**-26.208**
Observations	1508	1508

Notes: The Q(k) statistic (Ljung and Box, 1978) tests the null hypothesis of no autocorrelation up to order k. PP test is a nonparametric unit root test proposed by Phillips and Perron (1988); PP-AR test is based on zero drift AR(1) process; PP-ARD test is based on AR(1) model with drift; and PP-TS test is based on trend stationary AR(1) model. Statistics in bold indicate significance at the 5% level.

4.4. Empirical Results

The bootstrap-calculated asymptotic p-value for the S&P 500 ($p = 0.05$) and the NASDAQ 100 ($p = 0.02$) concludes that the null hypothesis of a single regime (no threshold effect) is rejected at the 5% significance level.[32] The least squares estimates of the threshold are $\hat{\gamma} = 0.82$ and $\hat{\gamma} = 0.79$ for the S&P 500 and the NASDAQ 100, respectively, where the optimal autoregressive orders for them is set by the Bayesian information criteria. The optimal order for our data is 4 and 8 for the S&P 500 and the NASDAQ 100, respectively. The threshold principle divides the linear regression into two regimes by piecewise linear AR function depending on whether the previous realized daily hedge ratio has been exceeding the threshold estimate. Table 4.2 reports the parameter estimates for the two-regime SETAR models with heteroskedasticity consistent standard errors. Although the realized daily hedge ratio is likely to be positively autocorrelated, its variation in the two regimes behaves differently. It is observed that the realized hedge ratio tends to be more volatile in the low regime than in the high regime. As a result, the empirical results show the null of time-invariant hedge ratio hypothesis seems inappropriate and thus support the argument of time-varying hedge ratios.

Table 4.2 Two-regime SETAR estimates

Index	S&P 500		NASDAQ 100	
Regime	$y_{t-1} \leq 0.821$ (794 Obs.)	$y_{t-1} > 0.821$ (710 Obs.)	$y_{t-1} \leq 0.790$ (519 Obs.)	$y_{t-1} > 0.790$ (981 Obs.)
Intercept	**0.248 [0.035]**	**0.507 [0.074]**	**0.131 [0.040]**	**0.345 [0.056]**
y_{t-1}	**0.184 [0.047]**	0.005 [0.071]	**0.155 [0.062]**	0.071 [0.051]
y_{t-2}	**0.135 [0.033]**	**0.130 [0.041]**	**0.127 [0.048]**	**0.139 [0.030]**
y_{t-3}	**0.202 [0.036]**	**0.081 [0.036]**	0.053 [0.046]	**0.114 [0.034]**
y_{t-4}	**0.164 [0.037]**	**0.178 [0.045]**	**0.154 [0.052]**	**0.107 [0.036]**
y_{t-5}	-	-	**0.158 [0.047]**	0.060 [0.031]
y_{t-6}	-	-	0.017 [0.054]	**0.079 [0.033]**
y_{t-7}	-	-	0.025 [0.051]	0.027 [0.032]
y_{t-8}	-	-	**0.140 [0.043]**	-0.010 [0.035]
$\mathrm{std}(e_t)$	0.087	0.071	0.099	0.070
R-squared	0.222	0.074	0.308	0.127
Joint R-squared	0.265		0.376	
LM test	**17.019**		**25.677**	
(p-value)	**(0.050)**		**(0.021)**	

Notes: Standard errors are in brackets. Statistics in bold indicate significance at the 5% level.

[32] The bootstrap replications 1,000 and the trimming percentage 15% are used, see Hansen (1996) for details.

Chapter 5. Conclusive Remarks

The dissertation has presented the usefulness of intraday-based RV approach for estimating, evaluating, and investigating the one-period futures hedging problem. Firstly, Chapter 2 starts discussing the incremental value of a futures hedge using the RV approach. To do so, a new class of discrete-time multivariate volatility models encompassing the elements of realized covariance matrix for estimating the risk-minimizing hedge ratio is proposed. Then, the performance of the RV-based models is compared with those generated by return-based models under an out-of-sample context with daily rebalancing approach. The empirical results have indicated that the improvement of switching from daily to intraday can be substantial, based on an extensive set of statistical and economical performance measures.

Next, Chapter 3 has turn to the issue of performance evaluation on futures hedging. By applying of the RV approach, it has presented alternative ex-post measures for assessing the forecasting ability of any ex-ante hedge ratio estimates. With some assumptions in the underlying price processes, the realized (risk-minimizing) hedge ratio is shown to be consistent with the integrated (risk-minimizing) hedge ratio, and it can be estimated consistently using the realized regression of Barndorff-Nielsen and Shephard (2004). Meanwhile, the asymptotic distribution further provides insights into the precision of the hedge ratio. On the other hand, the realized hedging effectiveness offers an alternative ex-post estimate for the integrated hedging effectiveness. With these measures, hedgers may evaluate the performance of their hedging methods via some designed loss functions without only through the hedging effectiveness of Ederington (1979).

At last, but not the last, Chapter 4 has assessed the dynamics of the realized daily hedge ratio. A two-regime SETAR model is applied to detect the regime-switching feather of the hedge ratio. Empirical evidences on the two equity markets have shown that the threshold effect does exist, and the ratio is likely to be positively autocorrelated and tends to be more volatile in the low regime than in the high regime. The empirical finding then supports the regime-switching dynamic hedge; see, for example, Lee et al. (2006).

The availability of intraday high-frequency data for many financial assets has motivated the literature to develop methods for measuring, modeling, and forecasting daily volatility. Besides the well-known RV estimator, the realized range estimator has also been considered in the literature for this purpose. Building on the high-low range (RR) estimator of Parkinson (1980), Martens and Van Dijk (2007) and Christensen and Podolskij (2007) have developed alternative intraday-based realized range estimator for estimating daily volatility, as follows:

$$\mathrm{RR}_t^{(m)} = \frac{1}{4\log 2}\sum_{j=1}^{m}(\log H_{t_j} - \log L_{t_j})^2 \tag{5.1}$$

where the high price $H_{t_j} \equiv \sup_{(j-1)\Delta<i<j\Delta} P_{t-1+i}$ and the low price $L_{t_j} \equiv \inf_{(j-1)\Delta<i<j\Delta} P_{t-1+i}$. By means of sum of squared intraday price ranges, it has been shown that this RR estimator could provide more efficient estimate of daily volatility than the RV that only utilizes intraday price returns. Extending to the multivariate case, the estimate of covariance between assets using intraday data has been suggested by Bannouh et al. (2009), who combines the idea of Brandt and Diebold (2006) as well as the intraday high-low ranges to have the estimate. This so-called realized co-range estimator is formulated as follows:

$$\mathrm{RRCov}_t^{(m)} = \frac{1}{2\lambda_i\lambda_j}(\mathrm{RR}_{p,t}^{(m)} - \lambda_i^2\mathrm{RR}_{i,t}^{(m)} - \lambda_j^2\mathrm{RR}_{j,t}^{(m)}) \tag{5.2}$$

where λ_i and $\lambda_j = 1-\lambda_i$ are the weights of a portfolio that contains assets i and j; $\mathrm{RR}_{p,t}^{(m)}$ is the realized range of the portfolio, and $\mathrm{RR}_{i,t}^{(m)}$ and $\mathrm{RR}_{j,t}^{(m)}$ are the realized ranges of the individual assets.

With the realized (co-)range estimators, there are further extensions for forming the dynamic futures hedge. For example, based on the empirical study of volatility-timing strategy using the realized (co-)ranges, Bannouh et al. (2009) have shown that the covariance prediction using realized (co-)ranges outperforms the realized (co-)variances by about 60 basis points per annum after the transaction cost is taken into accounted. While utilizing realized (co-)variances for predicting the relevant (co-)variances in Chapter 2, the performance of a futures hedge that utilizes realized (co-)ranges is of much interest for further studies. The flexible CCC-GARCH error structure of Kroner and Sultan (1993) provides more flexible and easily ways to estimate the daily (co-)variances as compared with the rolling estimators of Fleming et al. (2003), Bandi et al. (2008), and De Pooter et al. (2008). Besides, the realized (co-)range estimators should provide alternative methods to evaluate the performance of a futures hedge. It is not clear whether the evaluation results using realized (co-)ranges is consistent with the results that use realized (co-)variances as shown in Chapter 3.

Bibliography

Andersen, T. G., & Bollerslev, T. (1997). Intraday periodicity and volatility persistence in financial markets. Journal of Empirical Finance, 4, 115-158.

Andersen, T. G., & Bollerslev, T. (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International Economic Review, 39, 885-905.

Andersen, T. G., Bollerslev, T., Christoffersen, P. F., & Diebold, F. X. (2006). Volatility and correlation forecasting. In G. Elliot, C.W.J. Granger & A. Timmerman (Eds.), Handbook of economic forecasting (pp. 778-878). Amsterdam: North-Holland.

Andersen, T. G., Bollerslev, T., Diebold, F.X., & Ebens, H. (2001a). The distribution of realized stock return volatility. Journal of Financial Economics, 61, 43-76.

Andersen, T.G., Bollerslev, T., Diebold, F.X., & Labys, P. (2000) Great realizations. Risk, 13, 105-108.

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2001b). The distribution of realized exchange rate volatility. Journal of the American Statistical Association, 96, 42-55.

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003). Modeling and forecasting realized volatility. Econometrica, 71, 579-625.

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Wu, J. (2005). A framework for exploring the macroeconomic determinants of systematic risk. American Economic Review, 95, 398-404.

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Wu, J. (2006). Realized beta: Persistence and predictability. Advances in Econometrics, 20, 1-39.

Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1999). Coherent measures of risk. Mathematical Finance, 9, 203-228.

Baillie, R. T., & Myers, R. J. (1991). Bivariate GARCH estimation of the optimal commodity futures hedge. Journal of Applied Econometrics, 6, 109-124.

Bandi, F. M., Russell, J. R., & Zhu, Y. (2008). Using high-frequency data in dynamic portfolio choice. Econometric Reviews, 27, 163-198.

Bannouh, K., Van Dijk, D., & Martens, M. (2009). Range-based covariance estimation using high-frequency data: The realized co-range. Journal of Financial Econometrics, forthcoming.

Barndorff-Nielsen, O.E., & Shephard, N. (2002) Estimating quadratic variation using realized variance. Journal of Applied Econometrics, 17, 457-477.

Barndorff-Nielsen, O. E., & Shephard, N. (2004). Econometric analysis of realized covariation: High frequency based covariance, regression, and correlation in financial economics. Econometrica, 72, 885-925.

Bauwens, L., Laurent, S., & Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey. Journal of Applied Econometrics, 21, 79-109.

Benninga, S., Eldor, R., & Zilcha, I. (1983). Optimal hedging in the futures market under price uncertainty. Economic Letters, 13, 141-145.

Bera, A. K., Garcia, P., & Roh, J. S. (1997). Estimation of time-varying hedge ratios for corn and soybean: BGARCH and random coefficient approaches. Indian Journal of Statistics, 59, 346-68.

Blair, B. J., Poon, S.-H., & Taylor, S. J. (2001). Forecasting S&P 100 volatility: The incremental information content of implied volatilities and high-frequency index returns. Journal of Econometrics, 105, 5-26.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31, 307-327.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized arch model. Review of Economics and Statistics, 72, 498-505.

Bollerslev, T., Engle, R. F., & Wooldridge, J. M. (1988). A capital-asset pricing model with time-varying covariances. Journal of Political Economy, 96, 116-131.

Brandt, M. W., & Diebold, F. X. (2006). A no-arbitrage approach to range-based estimation of return covariances and correlations. Journal of Business 79, 61-74.

Brooks, C., Henry, O. T., & Persand, G. (2002). The effect of asymmetries on optimal hedge ratios. Journal of Business, 75, 333-352.

Carnero, M. A., Peña, D., & Ruiz, E. (2004). Persistence and kurtosis in GARCH and stochastic volatility models. Journal of Financial Econometrics, 2, 319-342.

Chen, S.-S., Lee, C.-F., & Shrestha, K. (2004). An empirical analysis of the relationship between the hedge ratio and hedging horizon: A simultaneous estimation of the short- and long-run hedge ratios, The Journal of Futures Markets, 24, 359-386.

Choudhry, T. (2003). Short run deviations and optimal hedge ratios: Evidence from stock futures. Journal of Multinational Financial Management, 13, 171-192.

Christensen, K., & Podolskij, M. (2007). Realized Range-Based Estimation of Integrated Variance. Journal of Econometrics, 141, 323-349.

Cotter, J., & Hanly, J. (2006). Reevaluating hedging performance. Journal of Futures Markets, 26, 677-702.

De Pooter, M., Martens, M., & van Dijk, D. (2008). Predicting the daily covariance matrix for S&P 100 stocks using intraday data - But which frequency to use?. Econometric Reviews, 27, 199-229.

Ederington, L. H. (1979). The hedging performance of the new futures markets. Journal of Finance, 34, 157-170.

Engle, R. F. (2002). Dynamic conditional correlation: A simple class of multivariate GARCH models. Journal of Business and Economic Statistics, 20, 339-350.

Engle, R. F., & Granger, C. W. J. (1987). Co-integration and error correction: Representation, estimation, and testing. Econometrica, 55, 251-276.

Engle, R. F., & Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. Econometric Theory, 11, 122-150.

Fleming, J., Kirby, C., & Ostdiek, B. (2003). The economic value of volatility timing using "realized" volatility. Journal of Financial Economics, 67, 473-509.

Geppert, J. M. (1995). A statistical model for the relationship between futures contracts hedging effectiveness and investment horizon length. The Journal of Futures Markets, 15, 507-536.

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance, 48, 1779-1801.

Hansen, B. E. (1996). Inference when a nuisance parameter is not identified under the null hypothesis. Econometrica, 64, 413-30.

Hansen, P. R., & Lunde, A. (2005). A realized variance for the whole day based on intermittent high-frequency data. Journal of Financial Econometrics, 3, 525-554.

Hansen, P. R., & Lunde, A. (2006). Realized variance and market microstructure noise. Journal of Business and Economic Statistics, 24, 127-161.

Harris, R. D. F., Shen, J., & Stoja, E. (2007). The limits to minimum-variance hedging. Working Paper, University of Exeter, UK.

Hayashi, T., & Yoshida, N. (2005). On covariance estimation of non-synchronously observed diffusion processes. Bernoulli, 11, 359-379.

Howard, C. T., & D'Antonio, L. J. (1984). A risk-return measure of hedging effectiveness. Journal of Financial and Quantitative Analysis, 19, 101-112.

Johnson, L. (1960). The theory of hedging and speculation in commodity futures. Review of Economic Studies, 27, 139-151.

Kroner, K. F., & Sultan, J. (1993). Time-varying distributions and dynamic hedging with foreign currency futures. Journal of Financial and Quantitative Analysis, 28, 535-551.

Lee, H.-T., Yoder, J. K., Mittelhammer, R. C., & McCluskey, J. J. (2006). A random coefficient autoregressive Markov regime switching model for dynamic futures hedging. Journal of Futures Markets, 26, 103-29.

Lence, S. H. (1995). The economic value of minimum-variance hedges. American Journal of Agricultural Economics, 77, 353-364.

Lence, S. H., & Hayes, D. J. (1994a). The empirical minimum-variance hedge. American Journal of Agricultural Economics, 76, 94-104.

Lence, S. H., & Hayes, D. J. (1994b). Parameter-based decision making under estimation risk: An application to futures trading. Journal of Finance, 49, 345-357.

Lien, D. (2005a). The use and abuse of the hedging effectiveness measure. International Review of Financial Analysis, 14, 277-282.

Lien, D. (2005b). A note on the superiority of the OLS hedge ratio. Journal of Futures Markets, 25, 1121-1126.

Lien, D. (2008). A further note on the optimality of the OLS hedge strategy. Journal of Futures Markets, 28, 308-311.

Lien, D. (2009). A note on the hedging effectiveness of GARCH models. International Review of Economics and Finance, 18, 110-112.

Lien, D., & Shrestha, K. (2007). An empirical analysis of the relationship between hedge ratio and hedging horizon using wavelet analysis, The Journal of Futures Markets, 27, 127-150.

Lien, D., & Tse, Y. K. (2002). Some recent developments in futures hedging. Journal of Economic Surveys, 16, 357-396.

Lien, D., & Wilson, B. K. (2001). Multiperiod hedging in the presence of stochastic volatility. International Review of Financial Analysis, 10, 395-406.

Lien, D., & Yang, L. (2006). Spot-futures spread, time-varying correlation, and hedging with currency futures. Journal of Futures Markets, 26, 1019-1038.

Lien, D., Tse, Y. K., & Tsui, A. K. C. (2002). Evaluating the hedging performance of the constant-correlation GARCH model. Applied Financial Economics, 12, 791-798.

Ljung, G. M., & Box, G. E. P. (1978). On a measure of lack of fit in time series models. Biometrika, 65, 297-303.

Martens, M. (2002). Measuring and forecasting S & P 500 index-futures volatility using high-frequency data. Journal of Futures Markets, 22, 497-518.

Martens, M., & Van Dijk, D. (2007). Measuring volatility with the realized range. Journal of Econometrics, 138, 181-207.

McAleer, M., & Medeiros, M. C. (2008). Realized volatility: A review. Econometric Reviews, 27, 10-45.

Myers, R. J. (1991). Estimating time-varying optimal hedge ratios on futures markets. Journal of Futures Markets, 11, 39-53.

Park, T. H., & Switzer, L. N. (1995). Bivariate GARCH estimation of the optimal hedge ratios for stock index futures. Journal of Futures Markets, 15, 61-67.

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. Journal of Business, 53, 61-65.

Patton, A. J. (2004). On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. Journal of Financial Econometrics, 2, 130-168.

Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. International Economic Review, 47, 527-556.

Phillips, P. C. B., & Perron, P. (1988). Testing for a unit root in time series regression. Biometrika, 75, 335-346.

Shi, W., & Irwin, S. H. (2005). Optimal hedging with a subjective view: An empirical Bayesian approach. American Journal of Agricultural Economics, 87, 918-930.

Tse, Y. K., & Tsui, A. K. C. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. Journal of Business and Economic Statistics, 20, 351-362.

Voev, V., & Lunde, A. (2007). Integrated covariance estimation using high-frequency data in the presence of noise. Journal of Financial Econometrics, 5, 68-104.

CURRICULUM VITAE

基本資料 / Personal Information

姓名：賴雨聖 Yu-Sheng Lai
出生日期：1974/06/02
E-mail：alexlai.ms93g@nctu.edu.tw

主要學歷 / Education

國立交通大學 管理科學系 博士	（2004~2009）
私立銘傳大學 財務金融所 管理碩士	（1998~2000）
國立交通大學 應用數學系 理學學士	（1992~1996）

期刊論文 / Referred Journal Articles

1. Yu-Sheng Lai and Her-Jiun Sheu, 2008, "An Application of Realized Regression to the Futures Hedging Problem", *Asia Pacific Management Review*, Vol. 13, No. 4, 655-666.
2. Yu-Sheng Lai and Her-Jiun Sheu, "The Incremental Value of a Futures Hedge Using Realized Volatility", *Journal of Futures Markets*, forthcoming.

研討會論文 / Conference Presentations

1. Yu-Sheng Lai and Her-Jiun Sheu, "Threshold Effects and Realized Hedge Ratios: Evidence from the S&P 500 and the NASDAQ 100 Markets", Dec. 12-14, 2007, *20th Australasian Finance & Banking Conference*, Shangri-La Hotel, Sydney, Australia.
2. Yu-Sheng Lai and Her-Jiun Sheu, "Performance Evaluation of a Futures Hedge: A Generalized Approach", Jun. 20-21, 2008, *2008 Annual Conference of Taiwan Finance Association*, National Dong Hwa University, Hualien, Taiwan.
3. Yu-Sheng Lai and Her-Jiun Sheu, "Re-Evaluating Futures Hedge under Time-Varying Risks", Dec. 16-18, 2008, *21st Australasian Finance & Banking Conference*, Shangri-La Hotel, Sydney, Australia.
4. Yu-Sheng Lai and Her-Jiun Sheu, "Regime-Dependent Dynamics of the Optimal Futures Hedge: Evidence from the S&P 500 and the NASDAQ 100 Indices", Jul. 6-8, 2009, *International Symposium on Finance and Accounting 2009*, Parkroyal Hotel, Kuala Lumpur, Malaysia.