國立交通大學
工業工程與管理學系

碩士論文

考慮製程平均發生偏移下之製程能力評估

Process Capability Measures with Control Chart Mean Shift Consideration

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摘要

製程能力指標被用來衡量製程製造產品符合規格的能力，不僅是提供品質保證的工具，也是在品質改善方面一個方針。不過，自從 Motorola 公司在 1980 年代提出 6 個標準差的觀念後，很多統計學家質疑這些提倡 6 個標準差的人，為什麼他們在衡量製程能力時，會先對製程平均做 1.5 個標準差的調整。Bothe(2002) 針對這個問題，利用管制圖的機制來偵測製程平均發生偏移的情況，發現它隨著不同的抽樣個數可以有不同的調整量，可是 Bothe 的研究是在常態的假設下，事實上，非常態的製程在業界是較常發生的。我們針對非常態的情況，探討了三個 Non-normal (Gamma, Weibull and Lognormal distributions)的平均連串長度，發現其對常態假設非常敏感。所以我們針對這三個非常態 Gamma 分配做詳細的分析，導出在不同非常態分配下應調整的偏移量，並針對非常態適用的 C_{pk} 指標做調整。在論文的最後，我們用一個實例來說明如何在非常態(以 Gamma 分配為例)的情況下，在考慮製程平均會發生變動的情況下，如何調整製程能力指標 C_{pk}。

關鍵字：非常態、Gamma 分配、Weibull 分配、Lognormal 分配、製程偏移、製程能力指標

Process Capability Measure with Control Chart Mean Shift Consideration

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Abstract

Process capability indices have been proposed in the manufacturing industry to provide numerical measures on process reproduction capability, which are effective tools for quality assurance and guidance for process improvement. Motorola, Inc. introduced its Six Sigma quality initiative to the world in the 1980s. Some quality practitioners questioned why the Six Sigma advocates claim it is necessary to add a 1.5σ shift to the average when estimating process capability. Bothe (2002) provides a statistical reason for including such a shift in the process average that is based on the chart's subgroup size. Data in Bothe' study was assumed to be approximately normally distributed. What effects on capability estimates when the process output has a non-normal distribution? This paper investigates the average run length of three non-normal distributions and calculate the mean shift adjustments and addresses this problem computing reliable estimates for capability index C_{pk} for non-normal process (particularly for gamma distribution) when the statistically adjustments is considered. For illustration purpose, an application example is presented.

Keywords: Dynamic C_{pk} , Mean shift, Process capability index, Gamma distribution, Weibull distribution, Log-normal distribution.

誌謝

轉眼間研究所兩年的時光很快就結束了，本篇論文可以順利完成，首先要感謝的就是我的指導老師—彭文理博士，彭老師嚴謹又細心的指導，使得我在做論文時有許多的收獲，雅甄學姊不遺餘力的細心指導，讓我從中學習到很多，協助我完成論文，真是辛苦學姊了，還有鍾老師及許老師在口試時對我的指教也讓我獲益不少。

除此之外，研究所兩年的生活裡，我要感謝的人很多，有陪在我身邊兩年的實驗室各個伙伴們，以及民有二街的室友們，大家一起走過許許多多歡樂與煩惱的時光，還要感謝一直很支持我的家人，要感謝的人太多了，套句陳之藩說的，”那就感謝天吧!”。

最後，不能免俗的，祝大家身體健康、萬事如意。謝謝你們!!

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1. Introduction

Recent years, numerous process capability indices (PCIs) have been widely used in the manufacturing industry, to provide a numerical measure on whether a process is capable of re-producing items meeting the quality requirement preset in the factory. Those indices have become popular as unit-less measures on process potential and performance. The most commonly used indices, C_p and C_{pk} discussed in Kane(1986), and the two more-advanced indices C_{pm} and C_{pmk} developed by Chan *et al.* (1988) and Pearn *et al*. (1992). Based on analyzing the PCIs, a production department can trace and improve a poor process so that the quality level can be enhanced and the requirements of the customers can be satisfied. These PCIs have been defined explicitly as:

$$C_p = \frac{USL-LSL}{6\sigma}, \quad C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\}, \quad C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2+(\mu-T)^2}},$$

$$C_{pmk} = \min\left\{\frac{USL-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-LSL}{3\sqrt{\sigma^2+(\mu-T)^2}}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, and T is the target value. The index C_{pk} has been regarded as a yield-based index. For a C_{pk} level of 1, statistically one would expect that the product's fractions of defectives, is no more than 2700 parts per million (PPM) fall outside the specification limits. At a C_{pk} level of 1.33, the defect rate is 66 PPM. To achieve less than 0.544 PPM defect rate, a C_{pk} level of 1.67 is needed. At a C_{pk} level of 2.0, the defective rate reduced to 0.002 PPM (see Appendix: table 5).

Ever since Motorola, Inc. introduced its Six Sigma quality initiative, quality practitioners have questioned why followers of this philosophy add a 1.5 σ when estimating process capability. The reason for such an adjustment, Six Sigma advocates claim that it is necessary, but they offer only personal experiences and three dated empirical studies as justification (see Bender (1975), Evans(1975), Gilson(1951)). By examining the sensitivity of control charts to detect changes of various magnitudes, Bothe (2002) provides a statistically based reason to this issue. The data in Bothe's study is assumed to be approximately normally distributed. Actually, non-normal processes occur frequently in practice. Pyzdek (1992) has mentioned the distributions of certain chemical processes such as zinc plating thickness of a hot-dip galvanizing process are very quite often skewed. Choi (1996) presents an example of a skewed distribution in the "active area" shaping stage of the wafer's production processes. The abundance of outputs from skewed distributions, the censoring effects induced by the finite precision of actual measurements, stratification, etc., makes the normality assumption often unreasonable. Hence, if the capability indices based on the normal assumption concerning the data are used to deal with non-normal observations, the values of the capability indices may, in a majority of situations, be incorrect and quite likely misrepresent the actual product quality.

In this paper, we investigate three non-normal distributions, Weibull、Log-normal and Gamma, to calculate the ARL(average run length) by simulation. We also show that the detection power performance of $\bar{X}$ chart under the Bothe' adjustments when the process is in control is very sensitive to the assumption of normality (for gamma distribution). We calculate the derived adjustments by simulations. (for weibull and lognormal distributions) and provide the statistically derived adjustments based on the chart's subgroup size (for gamma distribution) to calculate the estimator of dynamic C_{pk} when the data is non-normal distribution. A real-world example take from a semiconductor manufacturing process is investigated in the paper to illustrate the applicability.

2. Process Mean Shift Investigation for Non-normal Process

ARL(average run length) of the control chart is the average number of points that must be poltted before a point indicates an out-of-control condition. If the process observations are uncorrelated, then for any Shewhart control chart, the ARL can be calculated easily from

$$ARL = \frac{1}{p},$$

where p is the probability that any point exceeds the control limits. To illustrate, for the $\bar{x}$ chart with three-sigma limits, p=0.0027 is the probability that a single point falls outside the limits when the process is in control. Therefore, the average run length of the $\bar{x}$ chart when the process is in control (called ARL_0) is

$$ARL = \frac{1}{p} = \frac{1}{0.0027} \approx 370 .$$

That is, even if the process remains in control, an out of control signal will be generated every 370 samples, on the average.

For any Shewshart control chart, we have noted that the ARL can also be expressed as

$$ARL_0 = \frac{1}{\alpha},$$

for the in-control ARL and

$$ARL_1 = \frac{1}{1-\beta},$$

for the out-of-control ARL, where α is the probability of indicating a shift when none has occurred, and β is the probability of failing to indicate a real shift in process level.

In this chapter, we will investigate the ARL_1 of three non-normal distributions, weibull, gamma, and log-normal by simulation to find which one is the most sensitive to the normality assumption.

2.1. Weibull Process

Weibull distributions are often used to model the time until a given technical device fails. Examples of situations is which the weibull has been used include electronic devices such as memory element, mechanical components such as bearings, and structural elements in aircraft and automobiles.

If the failure rate of the device decreases over time, one chooses $k < 1$ (resulting in a decreasing density function). If the failure rate of the device is constant over time, one chooses $k = 1$, again resulting in a decreasing function f. If the failure rate of the device increases over time, one chooses $k > 1$ and obtains a density f which increases towards a maximum and then decreases forever. Manufacturers will often supply the shape and scale parameters for the lifetime distribution of a particular device. The weibull distribution can also be used to model the distribution of wind speeds at a given location on Earth. Again, every location is characterized by a particular shape and scale parameter.

2.1.1. The Weibull Distribution

In this section, we investigate the non-normal distributions under study are various Weibull distributions. Observations from the Weibull distribution are non-negative. The Weibull distribution can be denoted as Weibull(λ, k) with the probability density function given by

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{\left[-\left(\frac{x}{\lambda}\right)^{k}\right]}, \quad x \geq 0,$$

where $\lambda > 0$ is the scale parameter, and $k > 0$ is the shape parameter. The mean and variance of the Weibull distribution are

$$\mu = \lambda\Gamma\left(1+\frac{1}{k}\right) \text{ and } \sigma^2 = \lambda^2\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma\left(1+\frac{1}{k}\right)^2\right], \text{ respectively.}$$

The Weibull distribution is very flexible, and by appropriate selection of the parameters λ and k, the distribution can assume a wide variety of shapes.

Several Weibull distribution are shown in Fig 1 for $\lambda = 1$ and $k = 1/2$, 1, 2, 3, 4 and 8. Note that when $k = 1$, the weibull distribution reduces to the exponential distribution with mean $1/\lambda$.

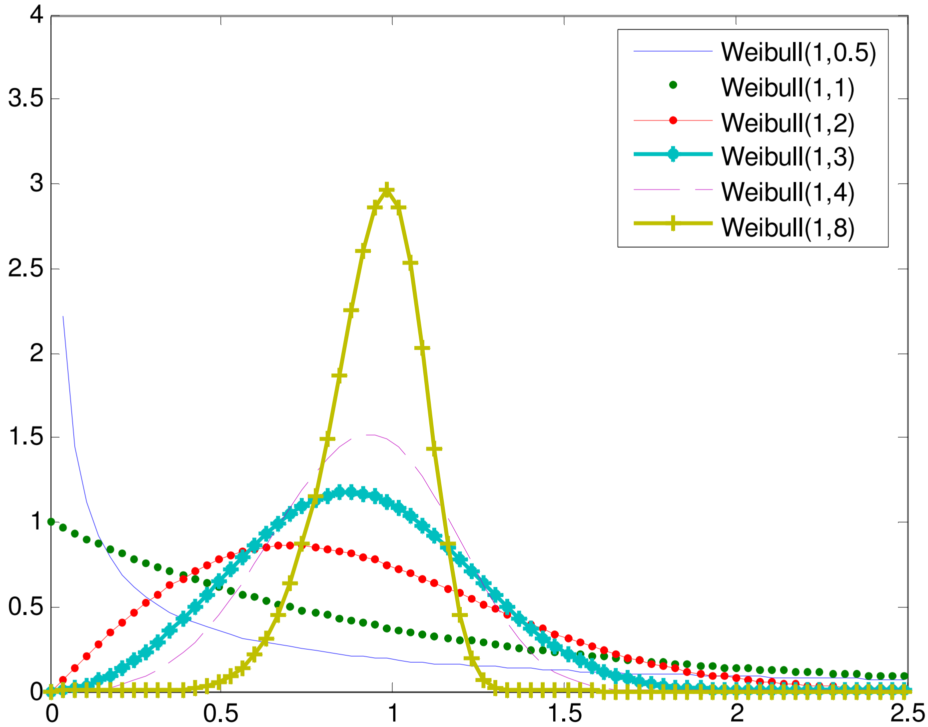

Figure 1. Weibull distributions for selected values of the shape parameter k and scale parameter $\lambda = 1$.

The skewness and kurtosis of weibull are

$$\gamma_1 = \frac{2\Gamma^3\left(1+\frac{1}{k}\right) - 3\Gamma\left(1+\frac{1}{k}\right)\Gamma\left(1+\frac{2}{k}\right)}{\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]^{3/2}} + \frac{\Gamma\left(1+\frac{3}{k}\right)}{\left[\Gamma(1+\frac{2}{k}) - \Gamma^2\left(1+\frac{1}{k}\right)\right]^{3/2}}, \text{ and}$$

$$\gamma_2 = \frac{f(k)}{\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]^2} \text{ ,where}$$

$$f(k) = -6\Gamma^4\left(1+\frac{1}{k}\right) + 12\Gamma^2\left(1+\frac{1}{k}\right)\Gamma\left(1+\frac{2}{k}\right) - 3\Gamma^2\left(1+\frac{2}{k}\right) - 4\Gamma\left(1+\frac{1}{k}\right)\Gamma\left(1+\frac{3}{k}\right) + \Gamma\left(1+\frac{4}{k}\right).$$

Table 1. Values of skewness and kurtosis for various weibull distributions

Distribution	Skewness	Kurtosis
N(0,1)	0.0000	3.0000
W(1,0.5)	6.6188	87.7200
W(1,1)	2.0000	9.0000
W(1,2)	0.6311	3.2451
W(1,3)	0.1681	2.7295
W(1,4)	-0.0872	2.7478
W(1,5)	-0.2541	2.8803
W(1,6)	-0.3733	3.0355
W(1,7)	-0.4632	3.1872
W(1,8)	-0.5337	3.3277
W(1,9)	-0.5907	3.4552
W(1,10)	-0.6376	3.5702

It can be found from Table 1, the weibull distribution is a left-tail distribution for k>3 and is a right-tail distribution for $k \leq 3$. Figure 2 presents several weibull distributions along with a normal distribution for the same mean and variance. In this study, we let k =0.5, 1, 2, 3, 4, and 5, while fixing λ =1.

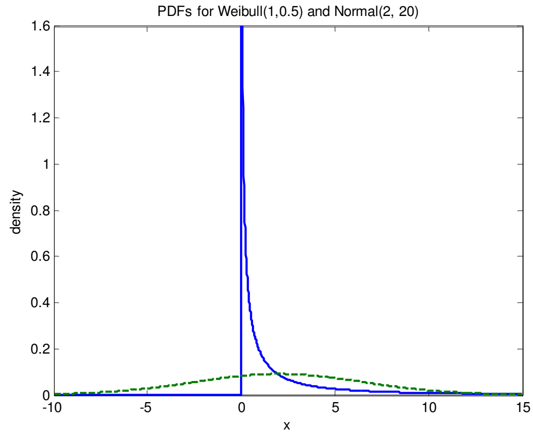

Figure 2(a). Probability density functions for Weibull (1, 0.5) and Normal(2, 20).

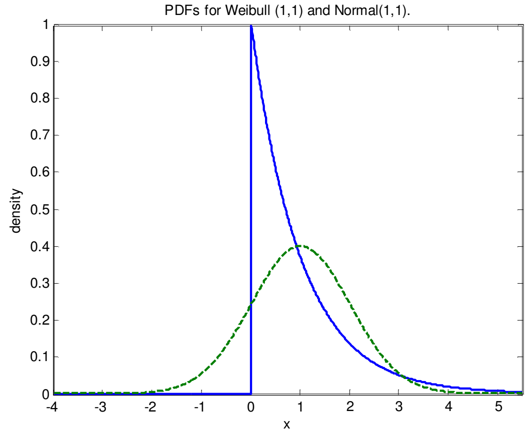

Figure 2(b). Probability density functions for Weibull (1,1) and Normal(1, 1).

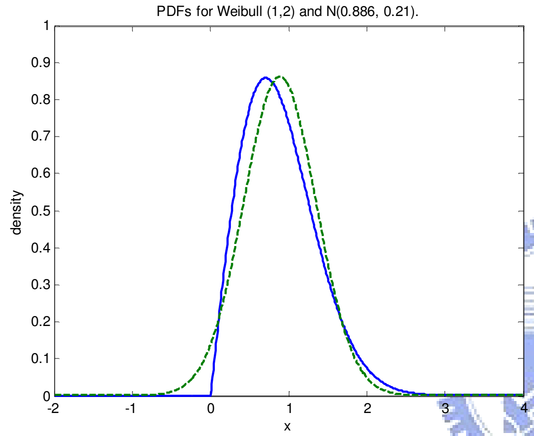

Figure 2(c). Probability density functions for Weibull (1, 2) and Normal(0.886, 0.21).

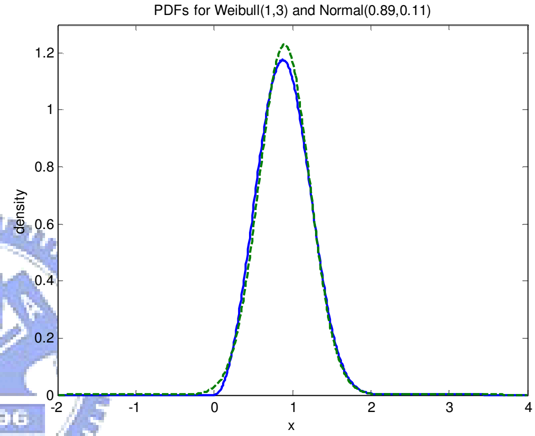

Figure 2(d). Probability density functions for Weibull (1, 3) and Normal(0.89,0.11).

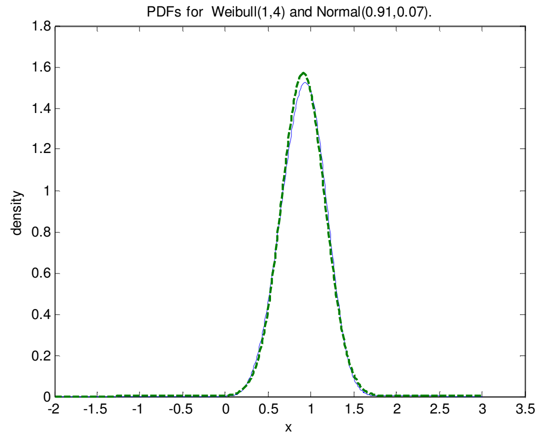

Figure 2(e). Probability density functions for Weibull (1, 4) and Normal(0.91, 0.07).

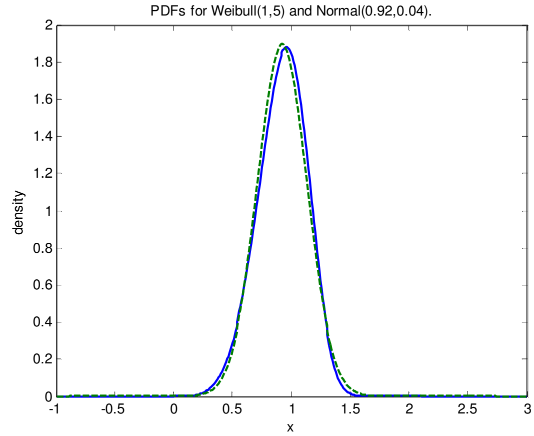

Figure 2(f). Probability density functions for Weibull (1,5) and Normal(0.92, 0.04).

Figure 2. presents several weibull distributions along with a normal distribution for the same mean and variance. Let $k =$ 0.5, 1, 2, 3, 4, and 5, while fixing $\lambda = 1$.

2.1.2. Calculating the Average Run Length of Weibull Process under Different Magnitude of shift by Simulation

In Appendix, tables 8-19 show the ARL of weibull distribution by Simulation with the magnitude of mean shift k= -3(0.5)3.

For different value of k, the shape parameter of weibull, we have various shapes of weibull (λ, k) distribution. From Table 1, we can find that the distribution is a right-tail distribution when $k \leq 3$ and turn to a left-tail distribution when k>3. For different value of k, the positive or negative mean shift, we have different value of ARL_1 by simulation. As we can see from appendix (table 14), obviously, the weibull distribution appears considerably skew to right for small value of k, so that the ARL_1 is smaller than the ARL_1 of N(0,1) when the mean shift is negative. On the other hand, the ARL_1 value increases sharply for the positive mean shift. From Appendix (table 8-13), we found that when the shift of μ is positive, the type II error will decrease sharply and the ARL_1 value of weibull will better then the ARL_1 of N(0,1) for $k > 3$. On the contrary, the ARL_1 is larger than the ARL_1 of N(0,1) for $k \leq 3$. Appendix (table 14-19) show that relative results to above. If the shift of μ is negative, the type II error will decrease sharply and the ARL_1 value of weibull will better then the Normal' for $k \leq 3$.

2.1.3. The Modified Mean Adjustments for Weibull Process

The mean shift adjustment under data comes from Weibull (λ, k) distribution with various values of k (0.5,1(1)10) with n= 2(1)30.in table 2 and table 3 are called as AS_{50} which are the magnitude of positive or negative shift we need to adjust based on required detection power is 0.5. We develop a Matlab program (available on request) to simulate the mean shift adjustment AS_{50}. The program reads the desired detection power (set to be 0.5) and the subgroup sample size n. For example, if k is set to 3, then with n= 5 the mean shift adjustment is AS_{50} = 1.381 for positive mean shift and the mean shift adjustment is AS_{50} = 1.264 for negative mean shift. We conclude that a mean shift adjustment of $AS_{50} = 1.381\sigma$ is required based on the detection power is 0.5 (ARL_1=2) and data comes from weibull (1, 3) distribution. It also shows from table 2 that the adjustment AS_{50} are smaller than Bothe' adjustments as $k \geq 4$ (when n=2(1)6), which is reasonable since the corresponding distributions get closer to the left-tail distribution and it is less affective for positive mean shift. Refer to table 3, it shows the adjustment AS_{50} are smaller than Bothe' adjustments as $k \leq 3$ (when n=2(1)6), which is reasonable since the corresponding distributions.

Table 2. AS_{50} value for several subgroup sizes n and various k values under weibull distribution(shfit to right)

n \ k	0.5	1	2	3	4	5	6	7	8	9	10
2	6.015	3.573	2.541	2.153	2.005	1.923	1.813	1.746	1.714	1.682	1.647
3	4.621	2.716	1.987	1.771	1.647	1.555	1.539	1.503	1.472	1.443	1.436
4	3.744	2.270	1.700	1.541	1.435	1.364	1.327	1.335	1.300	1.283	1.272
5	3.286	1.916	1.488	1.381	1.314	1.243	1.226	1.187	1.181	1.158	1.158
6	2.730	1.723	1.366	1.258	1.183	1.169	1.134	1.102	1.086	1.079	1.075
7	2.516	1.553	1.259	1.141	1.105	1.070	1.052	1.013	1.024	1.006	0.997
8	2.327	1.452	1.149	1.075	1.038	1.006	0.978	0.981	0.966	0.958	0.958
9	2.121	1.341	1.089	1.007	0.978	0.944	0.932	0.928	0.908	0.889	0.893
10	2.022	1.231	1.024	0.949	0.918	0.904	0.888	0.888	0.872	0.861	0.865
11	1.860	1.188	0.985	0.932	0.873	0.866	0.845	0.845	0.836	0.824	0.815
12	1.762	1.111	0.948	0.887	0.854	0.835	0.812	0.807	0.794	0.790	0.792
13	1.599	1.073	0.889	0.847	0.810	0.796	0.791	0.767	0.769	0.751	0.766
14	1.504	1.001	0.865	0.812	0.801	0.785	0.762	0.760	0.750	0.738	0.737
15	1.445	0.963	0.843	0.789	0.758	0.744	0.742	0.714	0.721	0.714	0.712
16	1.406	0.946	0.812	0.769	0.740	0.724	0.713	0.710	0.706	0.686	0.680
17	1.344	0.909	0.776	0.739	0.710	0.703	0.687	0.687	0.681	0.681	0.683
18	1.267	0.864	0.757	0.716	0.691	0.681	0.677	0.666	0.660	0.652	0.655
19	1.253	0.856	0.730	0.706	0.675	0.669	0.654	0.648	0.647	0.640	0.638
20	1.216	0.812	0.709	0.679	0.667	0.660	0.637	0.644	0.624	0.621	0.630
21	1.137	0.803	0.697	0.661	0.646	0.640	0.624	0.629	0.611	0.614	0.595
22	1.101	0.785	0.681	0.655	0.629	0.612	0.609	0.593	0.603	0.607	0.608
23	1.094	0.749	0.665	0.632	0.625	0.601	0.600	0.589	0.586	0.581	0.589
24	1.031	0.739	0.646	0.612	0.607	0.596	0.587	0.582	0.582	0.574	0.572
25	1.038	0.725	0.626	0.611	0.595	0.583	0.571	0.571	0.563	0.565	0.559
26	1.009	0.716	0.629	0.602	0.580	0.570	0.571	0.560	0.555	0.553	0.554
27	0.970	0.702	0.618	0.594	0.565	0.564	0.560	0.552	0.551	0.553	0.538
28	0.951	0.678	0.597	0.587	0.560	0.548	0.552	0.539	0.545	0.536	0.539
29	0.941	0.669	0.597	0.572	0.558	0.544	0.546	0.542	0.537	0.527	0.524
30	0.897	0.653	0.588	0.566	0.544	0.542	0.523	0.526	0.526	0.514	0.527

Table 3. AS_{50} value for several subgroup sizes n and various k values under weibull distribution(shfit to left)

n \ k	0.5	1	2	3	4	5	6	7	8	9	10
2	0.203	0.813	1.530	1.882	2.101	2.247	2.364	2.444	2.481	2.514	2.595
3	0.254	0.820	1.360	1.605	1.714	1.837	1.869	1.955	1.995	2.014	2.034
4	0.285	0.795	1.223	1.400	1.528	1.593	1.638	1.661	1.678	1.723	1.758
5	0.304	0.775	1.116	1.264	1.338	1.400	1.422	1.473	1.495	1.497	1.545
6	0.316	0.751	1.054	1.192	1.233	1.284	1.306	1.327	1.346	1.364	1.397
7	0.324	0.726	0.973	1.076	1.118	1.193	1.218	1.215	1.229	1.271	1.292
8	0.328	0.702	0.926	1.013	1.072	1.099	1.112	1.149	1.154	1.156	1.192
9	0.331	0.677	0.876	0.944	0.997	1.025	1.034	1.077	1.098	1.083	1.112
10	0.334	0.654	0.842	0.912	0.941	0.988	1.008	1.032	1.025	1.041	1.045
11	0.334	0.645	0.816	0.875	0.913	0.941	0.956	0.958	0.967	0.996	0.992
12	0.333	0.620	0.791	0.837	0.875	0.900	0.905	0.911	0.930	0.948	0.966
13	0.331	0.607	0.751	0.815	0.831	0.860	0.884	0.877	0.897	0.898	0.911
14	0.331	0.596	0.725	0.776	0.802	0.824	0.838	0.840	0.864	0.880	0.858
15	0.329	0.579	0.705	0.767	0.779	0.793	0.811	0.826	0.831	0.824	0.825
16	0.329	0.568	0.691	0.742	0.752	0.778	0.784	0.798	0.806	0.803	0.823
17	0.326	0.550	0.674	0.707	0.733	0.749	0.769	0.769	0.778	0.785	0.781
18	0.326	0.539	0.654	0.686	0.716	0.733	0.739	0.743	0.756	0.771	0.756
19	0.322	0.535	0.632	0.666	0.686	0.702	0.708	0.708	0.745	0.740	0.755
20	0.323	0.521	0.616	0.654	0.690	0.683	0.702	0.694	0.714	0.723	0.713
21	0.318	0.517	0.605	0.641	0.664	0.663	0.679	0.703	0.694	0.703	0.699
22	0.318	0.511	0.588	0.627	0.656	0.652	0.663	0.666	0.667	0.691	0.692
23	0.316	0.497	0.579	0.612	0.639	0.643	0.649	0.651	0.654	0.652	0.676
24	0.312	0.490	0.571	0.598	0.621	0.632	0.636	0.641	0.647	0.657	0.643
25	0.309	0.483	0.561	0.581	0.601	0.618	0.614	0.629	0.637	0.631	0.635
26	0.307	0.475	0.549	0.575	0.597	0.594	0.606	0.618	0.616	0.622	0.629
27	0.307	0.468	0.542	0.570	0.587	0.590	0.589	0.612	0.616	0.603	0.617
28	0.304	0.460	0.541	0.556	0.574	0.573	0.584	0.587	0.590	0.603	0.607
29	0.297	0.449	0.519	0.549	0.562	0.566	0.574	0.577	0.581	0.586	0.593
30	0.298	0.450	0.511	0.535	0.552	0.551	0.574	0.565	0.581	0.582	0.579

2.2. Log-normal Process

The log-normal distribution is often assumed to be the distribution of a stock price. A distribution is log-normally distributed when the natural log of the set of the random variables in that distribution is a normally distributed. In plain English, if you take the natural log of each of the random numbers from a log-normal distribution, the new number set will be normally distribution. If x is a random variable with a normal distribution, then exp(x) has a log-normal distribution. Like the normal distribution, log-normal distribution is also defined with mean and standard deviation.

2.2.1. The Log-normal Distribution

In this section, we investigate the non-normal distributions under study are various log-normal distributions. The log-normal distribution can be denoted as log-normal $(\mu, \text{ s})$ with the probability density function given by

$$f(x) = \frac{1}{xs\sqrt{2\pi}} e^{-(\ln x - \mu)^2 / 2s^2}, \quad x \in [0; +\infty)$$

where μ and s are the mean and standard deviation of the variable's logarithm. The expected value and variance of the log-normal distribution are

$$E(x) = e^{\mu + s^2/2} \text{ and } \sigma^2 = \left(e^{s^2} - 1\right) e^{2\mu + s^2} \text{ respectively.}$$

The skewness and kurtosis of log-normal are

$$\gamma_1 = e^{-\mu - s^2/2}\left(e^{s^2} + 2\right)\sqrt{e^{s^2} - 1} \text{ and } \gamma_2 = e^{4s^2} + 2e^{3s^2} + 3e^{2s^2} - 6$$

Table 2 presents the values of skewness and kurtosis (which are defined as the third and fourth moments of the standardized distribution, respectively) of the Log-normal distributions under study.

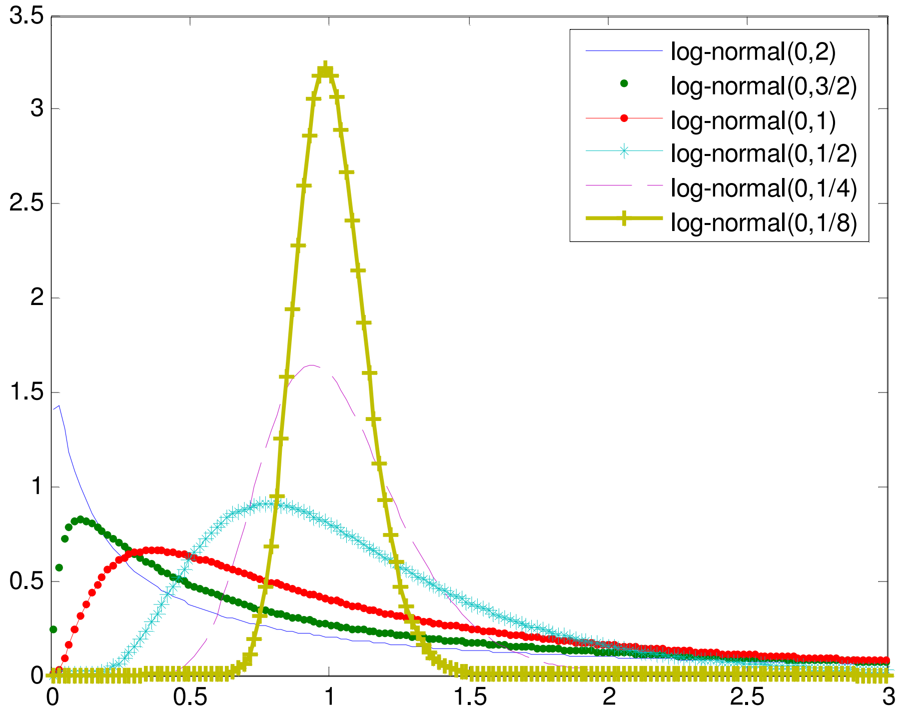

Figure 3. Log-normal distributions for selected values of the shape parameter s and μ=0.

Table 4. Values of skewness and kurtosis for various Log-normal distributions

Distribution	Skewness	Kurtosis
N(0,1)	0.000	3.000
Log(0,2)	414.359	9220563.000
Log(0,1.5)	33.468	10078.300
Log(0,1)	6.185	113.936
Log(0,0.5)	1.750	8.898
Log(0,0.25)	0.778	4.096
Log(0,0.125)	0.378	3.256

As can be seen from Fig 4, as s increases, the log-normal distribution appears more nearly normal.

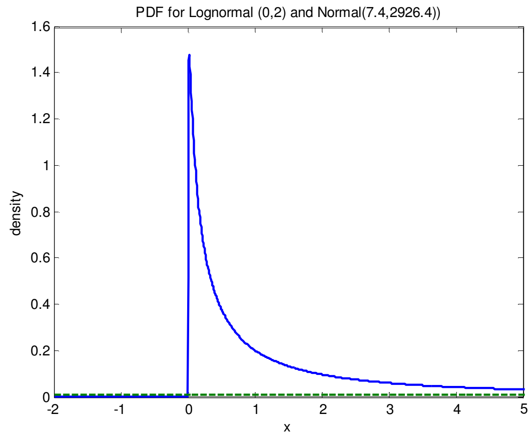

Figure 4(a).Probability density functions for Lognormal (0,2) and Normal(7.4,2926.4)

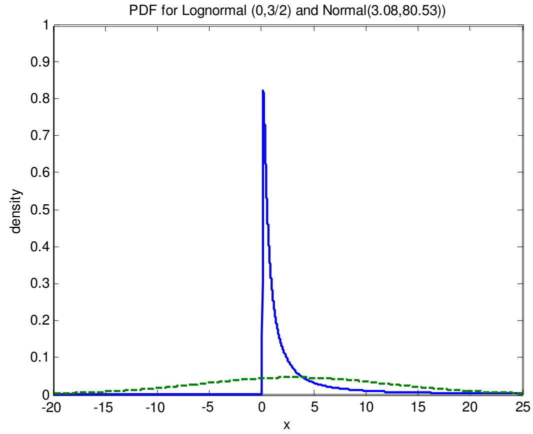

Figure 4(b). Probability density functions for Lognormal (0,3/2) and Normal(3.08,80.53)

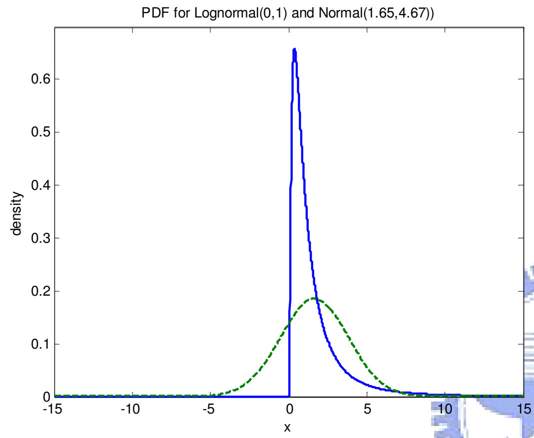

Figure 4(c). Probability density functions for Lognormal (0,1) and Normal(1.65,4.67)

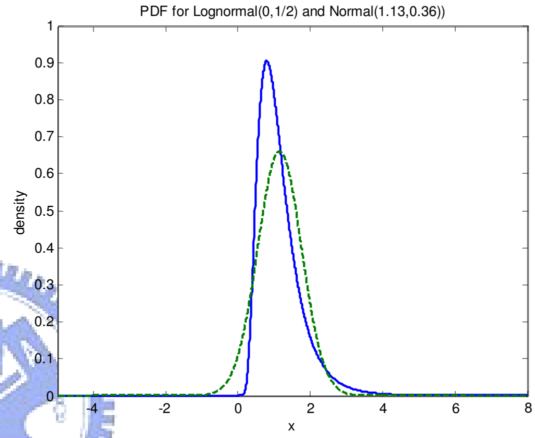

Figure 4(d). Probability density functions for Lognormal (0,1/2) and Normal(1.13,0.36)

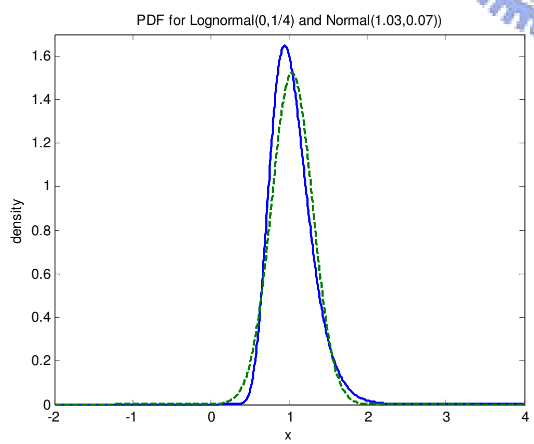

Figure 4(e). Probability density functions for Lognormal (0,1/4) and Normal(1.03,0.07)

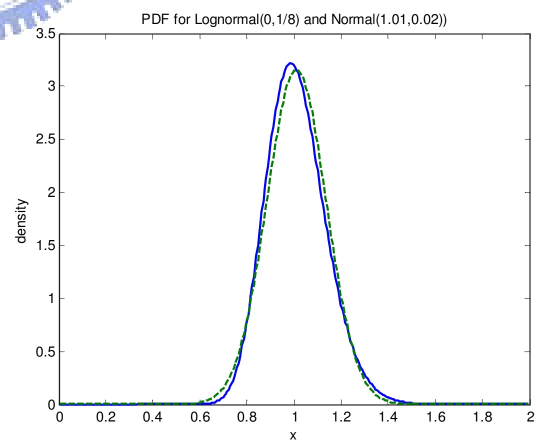

Figure 4(f).Probability density functions for Lognormal (0,1/8) and Normal(1.01,0.02)

Figure 4 presents several log-normal distributions along with a normal distribution for the same mean and variance. Let $s = 0.5$, 1, 2, 3, 4, and 5, while fixing $\mu = 0$.

2.2.2. Calculating the Average Run Length of Log-normal Process under Different Magnitude of shift by Simulation

In Appendix (table 20-31) show the ARL_1 of log-normal distribution by simulation with the magnitude of mean shift k= -3(0.5)3.

We can find in table 2 when s decreases, the corresponding values of skewness will become less and close to the values of the standard normal distribution. The result through these distributions, we can get some insights of the effects of non-normality in terms of skewness and kurtosis.

Since the log-normal distribution is a right-tail distribution, we can find in Appendix (table 26-31), the effect of the ARL_1 when the positive mean shift (k>0) is more remarkable when the negative mean shift (k<0), especially in the small shape parameter of log-normal, s,. Even when the sample size, n, and the magnitude of mean shift to left k is small, the performance of detecting power is still better specially as the magnitude of mean shift is 2σ.

What we need to pay attention is the positive mean shift of log-normal. We all know when s is large, the skewness and kurtosis are comparatively large. So the type II error will increase sharply if the positive mean shift move and the ARL_1 will be large. Comparatively, the effect of the mean shift to right needs to be paid attention much more.

In Appendix (table 20-31) show that the ARL_1 gets closer to the ARL_1 value of N(0,1) as s decreases, which is reasonable since the corresponding distributions get closer to the standard normal distribution.

2.2.3. The Modified Mean Adjustments for Lognormal Process

The mean shift adjustment under data comes from lognormal (μ , s) distribution with various values of s (2, 3/2, 1, 1/2, 1/4, 1/8) with n= 2(1)30.in table 4 is called as AS_{50} which is the magnitude of shift we need to adjust based on required detection power is 0.5 We develop a Matlab program (available on request) to simulate the mean shift adjustment AS_{50}. The program reads the desired detection power (set to be 0.5) and the subgroup sample size n. For example, if s is set to 1/2, then with n= 5 the mean shift adjustment is AS_{50} = 1.896. We conclude that a mean shift adjustment of AS_{50} = 1.896σ is required based on the detection power is 0.5 (ARL_1=2) and data comes from lognormal (0, 1/2) distribution. It also shows from table 4 that the adjustment AS_{50} gets closer to Bothe' adjustments as s decreases (when n=2(1)6), which is reasonable since the corresponding distributions get closer to the standard normal distribution. We should notice that when s is large, the AS_{50} value of lognormal is large different from the S_{50} value of Normal.

Table 5. AS_{50} value for several subgroup sizes n and various s values under log-normal distribution

n \ s	2	3/2	1	1/2	1/4	1/8
2	5.970	6.548	5.538	3.622	2.764	2.426
3	4.987	5.073	4.359	2.703	2.161	1.961
4	4.268	4.417	3.657	2.205	1.792	1.641
5	3.833	4.076	3.019	1.896	1.592	1.472
6	3.529	3.639	2.666	1.691	1.422	1.320
7	3.293	3.251	2.460	1.536	1.333	1.219
8	3.035	3.078	2.161	1.414	1.220	1.137
9	3.119	2.919	2.023	1.327	1.116	1.066
10	2.850	2.753	1.853	1.228	1.075	0.992
11	2.498	2.547	1.753	1.173	1.031	0.948
12	2.624	2.374	1.715	1.104	0.957	0.912
13	2.410	2.373	1.551	1.029	0.929	0.873
14	2.264	2.146	1.511	0.996	0.878	0.848
15	2.173	2.096	1.427	0.974	0.846	0.815
16	2.252	2.171	1.376	0.915	0.826	0.775
17	1.976	1.991	1.315	0.878	0.792	0.762
18	2.133	1.909	1.269	0.866	0.757	0.735
19	2.219	1.857	1.169	0.843	0.756	0.707
20	2.000	1.774	1.177	0.822	0.734	0.699
21	1.919	1.743	1.142	0.792	0.714	0.680
22	1.912	1.623	1.095	0.767	0.685	0.676
23	1.796	1.604	1.058	0.748	0.667	0.651
24	1.952	1.558	1.072	0.723	0.666	0.635
25	1.736	1.592	1.009	0.716	0.650	0.616
26	1.595	1.527	0.977	0.695	0.637	0.610
27	1.737	1.449	0.991	0.675	0.619	0.596
28	1.585	1.461	0.936	0.676	0.616	0.593
29	1.677	1.356	0.914	0.653	0.594	0.580
30	1.734	1.405	0.917	0.640	0.583	0.558

2.3. Gamma Process

All of us know that the case of non-normal processes occurs frequently in practice, for example, in the semiconductor industry. Pyzdek(1992) points out the skewed distributions that are bounded on one side occur frequently in industry and gives several examples, such as a shearing process and a chemical dip process. The abundance of outputs from skewed distributions makes the normality assumption often unreasonable. In the recent years, several approaches to the problems of PCIs for the non-normal populations have been suggested. For the skewed processes, the proportion of nonconforming items for fixed values of standard PCIs tends to increase as skewness increases, i.e. the standard PCIs simply ignore the skewness of the underlying population. We would compare the different between normal and gamma distributions in Section 2.3.1 The skewed gamma distribution can represent different process for various values of N and θ.And the statistical property of gamma distribution is discussed in Section 3.1.

2.3.1. The Gamma Distribution

In this section, we investigate the non-normal distributions under study are various gamma distributions. Observations from the Gamma distribution are non-negative. The Gamma distribution can be denoted as Gamma$(N,\ \theta)$ with the probability density function given by

$$f(x)=\frac{1}{\theta^{N}\Gamma(N)}x^{N-1}\exp\{-x/\theta\},\quad x>0,N>0,\theta>0$$

and the mean and variance given, respectively, by

$$\mu=N\theta \ \text{ and } \ \sigma^{2}=N\theta^{2}$$

Denote the family of gamma distributions with mean $N\theta$ by Gamma $(N,\ \theta)$. The Gamma distributions are skewed. To see how this distribution are different from the standard normal distribution in terms of skewness and kurtosis, table 5 presents the values of skewness and kurtosis (which are defined as the third and fourth moments of the standardized distribution, respectively) of the gamma distributions under study. Note that the case $N=1$ corresponds to the exponential distribution and the skewness of Gamma$(N,\ \theta)$ is $2/\sqrt{N}$ and kurtosis is $\frac{6}{N}+3$. We can find in table 5 when the N decreases, the corresponding values of skewness will become large and far away from the values of the standard normal distribution. The result through these distributions, we can get some insights of the effects of non-normality in terms of skewness and kurtosis.

Table 6. Values of skewness and kurtosis for various gamma distributions.

Distribution	Skewness	Kurtosis
N(0,1)	0.000	3.000
Gamma(5,1)	0.894	4.200
Gamma(4,1)	1.000	4.500
Gamma(3,1)	1.155	5.000
Gamma(2,1)	1.414	6.000
Gamma(1,1)	2.000	9.000
Gamma(0.5,1)	2.828	15.000

Figure 5 presents several gamma distributions along with a normal distribution for the same mean and variance. In this study, we let $N =$ 0.5, 1, 2, 3, 4, and 5, while (without loss of generality) fixing $\theta = 1$. These values of N and θ correspond to the values used by Schiling and Nelson (1976). As can be seen from Figure 5(a)-(f), as N increases, the gamma distribution appears more nearly normal. Through these distributions, we wish to get some insights of the effects of non-normality on the detection power in terms of skewness and kurtosis in Section 3.2.

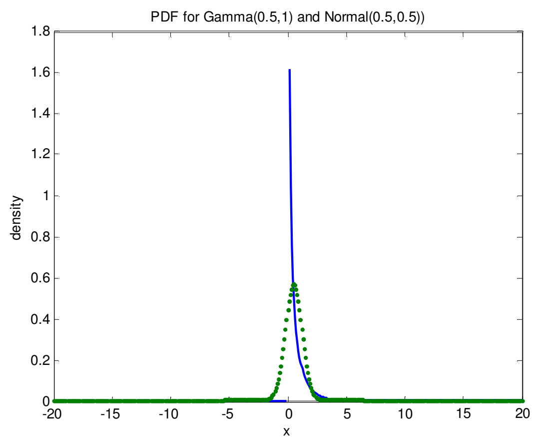

Figure 5(a). Probability density functions for Gamma (0.5, 1) and N(0.5, 0.5).

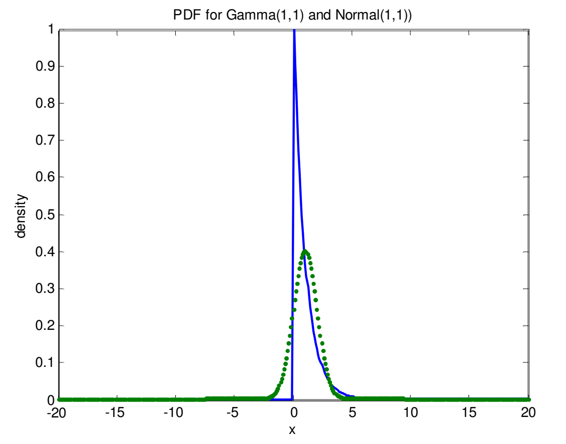

Figure 5(b). Probability density functions for Gamma (1, 1) and N(1, 1).

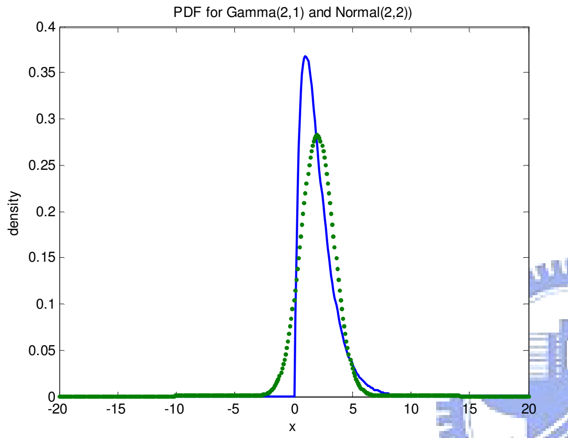

Figure 5(c). Probability density functions for Gamma (2, 1) and N(2, 2).

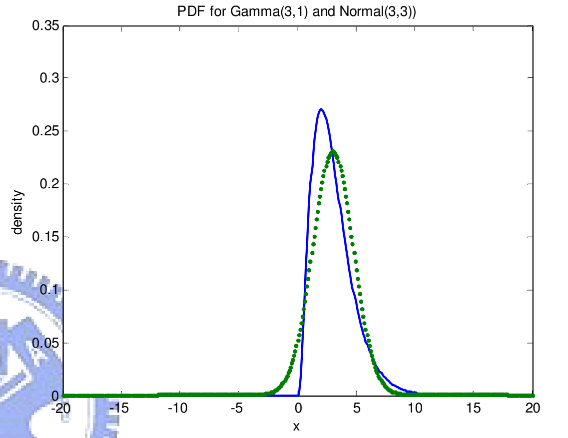

Figure 5(d). Probability density functions for Gamma (3, 1) and N(3, 3).

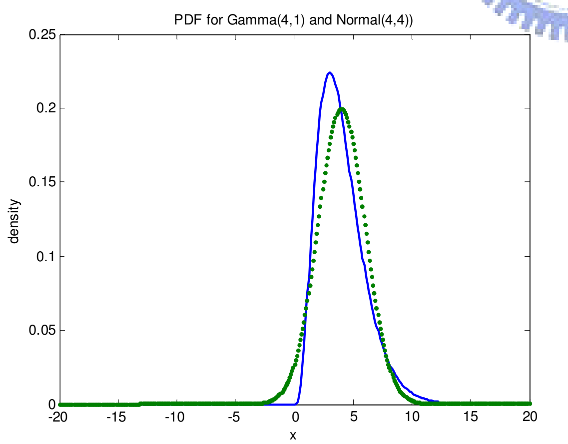

Figure 5(e). Probability density functions for Gamma (4, 1) and N(4, 4).

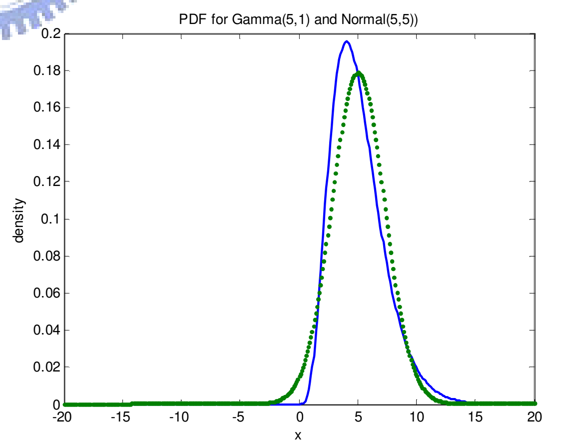

Figure 5(f). Probability density functions for Gamma (5, 1) and N(5, 5).

Figure 5. presents several gamma distributions along with a normal distribution for the same mean and variance. Let N= 0.5, 1, 2, 3, 4, and 5, while fixing $\theta = 1$.

2.3.2. Calculating the Average Run Length of Gamma Process under Different Magnitude of shift by Simulation

Appendix-table 32-43 shows the ARL_1 of Gamma distribution by Simulation with the magnitude of mean shift from -3σ to 3σ.

Refer to table 3, we know when N is larger, the Gamma distribution is closer to Normal distribution and Gamma is a right-tail distribution. From Appendix-table 38-43, because gamma distribution is a right-tail distribution, the effect of the negative mean shift is far less than the positive mean shift. What we need to pay attention is that when N is bigger, the effect of the negative mean shift is more serious. Comparatively, the effect of the positive mean shift needs to be paid attention to even more.

Appendix-table 34 show that arl1 value of Gamma distribution is similar to the value of Normal when N is bigger. Appendix-table 43 appears that the magnitude of mean shift is -3σ, the performance of gamma distribution is better than N(0,1), even N and n are both small. But small shift to right, like 0.5 or 1, will far worse than the performance of N(0,1) for any shape parameter of gamma and small sample sizes. Compare to the performance of weibull distribution, it's also poor than the performance of weibull, especially when shift is small. We find that some values of weibull are still better than N(0,1) when n, N and mean shift are both small.

3. Process Mean Shift Investigation for Gamma Process

3.1. Statistical Properties of Gamma Distribution

The Gamma distribution has a reproductive property: If X_1 and X_2 are independent random variables and each has a Gamma distribution with possible different values of N_1, N_2 of N, but with common values of θ, then $X_1 + X_2$ also has a Gamma distribution, with the same value of θ, and with $N = N_1 + N_2$ (refer to Norman(1994)). Applying this property, let $X_1, X_2, \ldots, X_n$ be a sequence of independent distribution of Gamma(N , θ) and then the distribution of $X_1 + X_2 + \cdots + X_n$ is Gamma (nN, θ). Using simply statistical technique, we can conclude that $Y_n = (X_1 + X_2 + \cdots + X_n)/n$ ~Gamma $(nN, \theta/n)$. (see Figure 6)

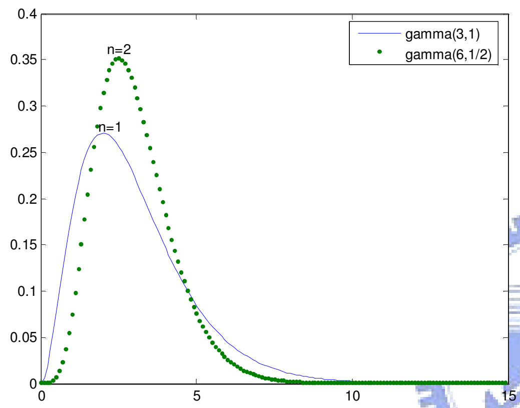

Figure 6(a). Probability density functions for Gamma (3, 1) and Gamma (6, 1/2).

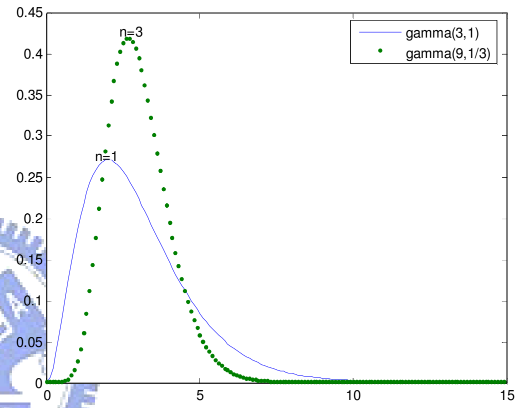

Figure 6(b). Probability density functions for Gamma (3, 1) and Gamma (9, 1/3)

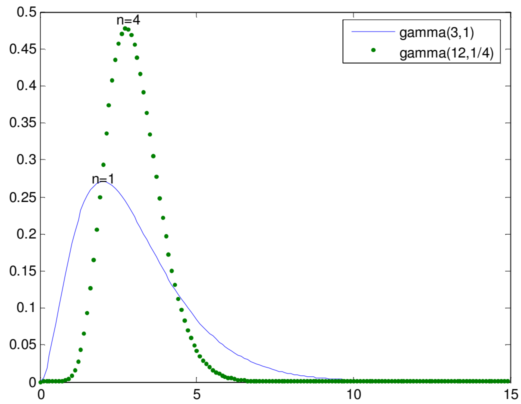

Figure 6(c). Probability density function for Gamma (3, 1) and Gamma (12, 1/4)

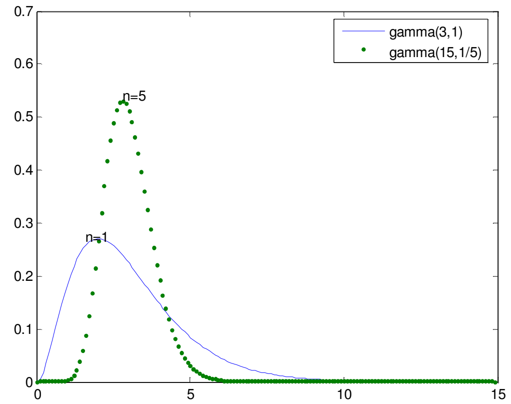

Figure 6(d). Probability density function for Gamma (3, 1) and Gamma(15, 1/5)

Figure 6. Probability density functions for Gamma distributions with different sample sizes.

3.2. The Detection Power of Gamma Process under the Bothe' Adjustments

The major purpose of individuals control chart is assisting on identifying shifts and drifts in processes and it's easily to be implemented. But, some assumptions should be satisfied before control charts are used. The assumptions include that the process characteristics must follow normal distributions. Due to above-mentioned statements, we replace the traditional , $\mu \pm 3\sigma$, to be the upper or lower control limits by the quantile of cumulatived distribution function for different parameters of gamma$(N,\ \theta)$ and detect the power of Gamma process under Bothe' adjustments.

Let X_1 , X_2 ,….., X_n be a sequence observations of independent and identically distributed in gamma$(N,\ \theta)$. Using the reproductive property of gamma distribution, the mean of the observations is $\bar{X}_n$ ($\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$) which is distributed in Gamma (nN , θ / n). X_i and $\bar{X}_n$ are distributed as gamma distribution, we can obtain the following notations:

$\mu_{X_i} = N \times \theta$, $\sigma_{X_i} = \sqrt{N} \times \theta$, and $\sigma_{\bar{X}} = \left(\sqrt{N} \times \theta\right) / \sqrt{n}$. Consequently, we derived the power of gamma process as follows. Since the type II error β is

$$\begin{aligned} \beta &= P\left(LCL \le \bar{X}_n \le UCL \mid \mu_1 = \mu_0 + k\sigma_{X_i}\right) \\ &= P\left(F_{0.00135} \le \bar{X}_n \le F_{0.99865} \mid \mu_1 = \mu_0 + k\sigma_{X_i}\right) \\ &= G_{\bar{x}}\left(F_{0.99865}\right) - G_{\bar{x}}\left(F_{0.00135}\right) \end{aligned}$$

where $1-\beta$ is the detection power of gamma process. $G_{\bar{X}_n}(\cdot)$ is the cumulative distribution function of gamma distribution with that mean has shifted and μ_1 is the mean after process shift (μ_0 is the mean of the original process). The control limits LCL and UCL are calculated as $F_{0.00135}$ and $F_{0.99865}$ respectively.

Table 2 presents the detecting power when data come from gamma distribution with N=0.5,1(1)10 and θ=1. The magnitude of shift in the second column on the left is the Bothe' adjustments determined when data come from normal distribution and the detecting power was fixed to 0.5.

Table 7. Detection power of various gamma distributions

Subgroup size n	Shift δ	Distribution Gamma $(N,1)$											
		$N=0.5$	$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$	$N=7$	$N=8$	$N=9$	$N=10$	N(0,1)
2	2.12	0.027	0.054	0.100	0.136	0.164	0.187	0.206	0.222	0.235	0.247	0.257	0.5
3	1.73	0.040	0.078	0.136	0.176	0.205	0.228	0.246	0.262	0.274	0.285	0.294	0.5
4	1.50	0.054	0.100	0.164	0.206	0.236	0.258	0.275	0.289	0.301	0.311	0.320	0.5
5	1.34	0.066	0.119	0.187	0.228	0.257	0.278	0.294	0.308	0.319	0.328	0.336	0.5
6	1.22	0.077	0.134	0.203	0.244	0.272	0.292	0.307	0.320	0.330	0.339	0.346	0.5

From table 6, we can find that the detection power is less than 0.5 when data come from gamma distribution under the Bothe' mean adjustments. Since the Bothe'(2002) approach is based on the normality assumption of the data. And our study shows that the detection power gets larger as N increases, which is reasonable since the corresponding distributions get closer to the standard normal distribution. But it's still far away from the results of Normal. The skewness of $\text{Gamma}(N,\ 1)$ is $2/\sqrt{N}$. Therefore, as N decreases the gamma distribution is more skewed and the detection power is poorer. For example, when $N=10$ and the subgroup size $n=2$, the detection power is 0.257. It implies the Bothe' adjustments are quite sensitive to the skewed processes. Hence in our study, we should modify the mean adjustments when data come from gamma distribution.

3.3. The Modified Mean Adjustments for Gamma Process

The mean shift adjustment under data comes from Gamma (N , 1) distribution with various values of N (=0.5 and 1(1)10) with n= 2(1)30 in table 7 is called as AS_{50} which is the magnitude of shift we need to adjust based on required detection power is 0.5 We develop a Matlab program (available on request) to compute the mean shift adjustment AS_{50}. The program reads the desired detection power (set to be 0.5) and the subgroup sample size n. For example, if N is set to 3, then with n= 5 the mean shift adjustment is AS_{50} = 1.69. We conclude that a mean shift adjustment of AS_{50} = 1.69σ is required based on the detection power is 0.5 and data comes from Gamma(3,1) distribution. It also shows from table 7 that the adjustment AS_{50} gets closer to Bothe' adjustments as N increases (when n=2(1)6), which is reasonable since the corresponding distributions get closer to the standard normal distribution. We should notice that when N is small, the AS_{50} value of Gamma is large different from the S_{50} value of Normal.

Table 8. AS_{50} value for several subgroup sizes n and various N values

n \ N	0.5	1	2	3	4	5	6	7	8	9	10
2	4.182	3.611	3.185	2.992	2.876	2.797	2.738	2.692	2.655	2.625	2.599
3	3.127	2.732	2.443	2.313	2.236	2.182	2.143	2.113	2.088	2.067	2.050
4	2.553	2.252	2.034	1.936	1.878	1.838	1.808	1.785	1.767	1.752	1.738
5	2.188	1.944	1.769	1.690	1.644	1.612	1.588	1.570	1.555	1.543	1.532
6	1.932	1.727	1.581	1.515	1.476	1.450	1.430	1.415	1.403	1.392	1.384
7	1.741	1.565	1.439	1.383	1.350	1.327	1.310	1.297	1.286	1.278	1.270
8	1.592	1.438	1.328	1.279	1.249	1.229	1.215	1.203	1.194	1.186	1.180
9	1.473	1.336	1.237	1.194	1.168	1.150	1.137	1.127	1.118	1.112	1.106
10	1.375	1.251	1.162	1.123	1.100	1.084	1.072	1.063	1.055	1.049	1.044
11	1.292	1.179	1.099	1.063	1.042	1.027	1.016	1.008	1.001	0.996	0.991
12	1.222	1.118	1.044	1.011	0.992	0.978	0.969	0.961	0.955	0.950	0.945
13	1.160	1.064	0.996	0.966	0.948	0.936	0.927	0.920	0.914	0.909	0.905
14	1.107	1.018	0.954	0.926	0.910	0.898	0.890	0.883	0.878	0.874	0.870
15	1.059	0.976	0.917	0.891	0.875	0.864	0.857	0.850	0.846	0.842	0.838
16	1.017	0.939	0.883	0.859	0.844	0.834	0.827	0.821	0.817	0.813	0.810
17	0.979	0.905	0.853	0.830	0.816	0.807	0.800	0.795	0.790	0.787	0.784
18	0.944	0.875	0.826	0.804	0.791	0.782	0.775	0.770	0.766	0.763	0.760
19	0.913	0.847	0.801	0.780	0.768	0.759	0.753	0.748	0.744	0.741	0.738
20	0.884	0.822	0.778	0.758	0.746	0.738	0.732	0.728	0.724	0.721	0.718
21	0.858	0.798	0.756	0.738	0.726	0.719	0.713	0.709	0.705	0.702	0.700
22	0.834	0.777	0.737	0.719	0.708	0.701	0.695	0.691	0.688	0.685	0.683
23	0.811	0.757	0.718	0.701	0.691	0.684	0.679	0.675	0.672	0.669	0.667
24	0.790	0.738	0.701	0.685	0.675	0.669	0.664	0.660	0.657	0.654	0.652
25	0.771	0.721	0.685	0.670	0.660	0.654	0.649	0.646	0.643	0.640	0.638
26	0.753	0.704	0.670	0.655	0.646	0.640	0.636	0.632	0.629	0.627	0.625
27	0.736	0.689	0.656	0.642	0.633	0.627	0.623	0.619	0.617	0.615	0.613
28	0.720	0.675	0.643	0.629	0.621	0.615	0.611	0.608	0.605	0.603	0.601
29	0.704	0.661	0.631	0.617	0.609	0.604	0.599	0.596	0.594	0.592	0.590
30	0.690	0.648	0.619	0.606	0.598	0.593	0.589	0.586	0.583	0.581	0.579

Figure 7 presents the AS_{50} curves of the gamma process with θ=0.5 and $\theta=1$ for subgroup sizes $n=1(1)5$, (expressed in θ values on the horizontal axis). Figure 7 depict that the magnitude of shift detected by $\overline{X}$ chart under Gamma process would not change for any θ values. It would be affected only with the N value.

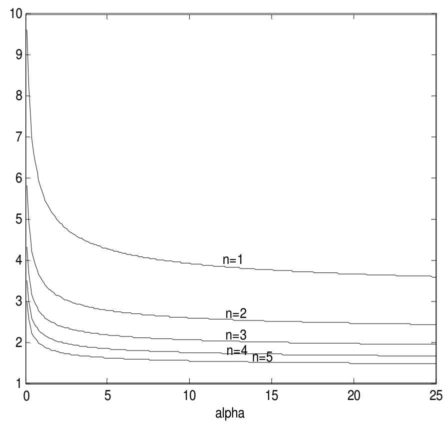

Figure 7(a). The AS_{50} curves of the gamma process with $\theta = 0.5$ for different N values on the horizontal axis.

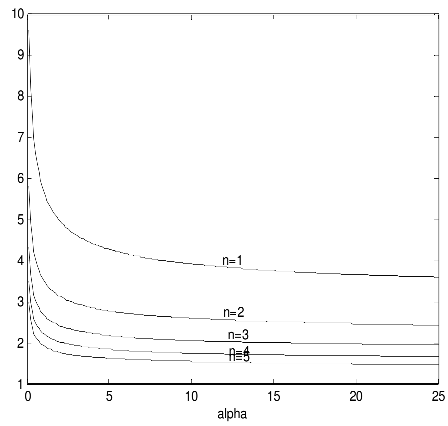

Figure 7(b). The AS_{50} curves of the gamma process with $\theta = 1$ for different N values on the horizontal axis.

Figure 7. show that AS_{50} not be affected by changing θ values

Speak of the figure 8, we called it as power curves, these lines on figure 8 portray the chances of detecting a shift in μ of a given size (expressed in σ units on the horizontal axis). For small shifts in μ, all three curves are close to zero. As the size of the shift increases, so does the power of the chart to detect it, with all three curves eventually leveling off close to 100% for shifts in excess of 3.5σ.

The dashed horizontal line drawn on this graph shows that there is a 50% chance of missing a 1.69σ shift in μ when *n* is 5, whereas μ must move by 2.313σ to have this same probability when *n* is only 3. Temporary movements in μ smaller than $AS_{50}\sigma$ are more than likely to be missed by a control chart.

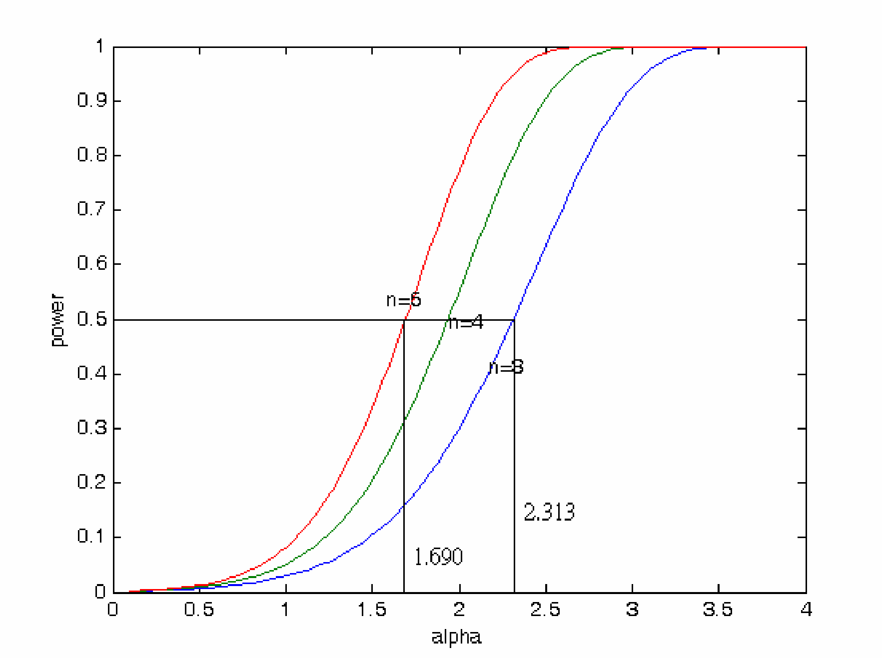

Figure 8. Power curve for subgroup sizes 3,4 and 5 when N =3.

3.4. The Modified Estimator of Process Capability C_{pk}

3.4.1. C_{pk} in the Non-Normal Case

The index C_{pk} has been viewed as an yield-based index since it provides bounds on the process yield for a normally distributed process with a fixed value of C_{pk}. This index C_{pk} is defined as:

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\},$$

where as above USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean and σ is the process standard deviation. The proper use of process capability indices, which are statistical measures of process capability, is based on several assumptions. One of the most essential is that the process monitored is supposed to be stable and the output is approximately normally distributed. When the distribution of a process characteristic is non-normal, PCIs calculated using conventional methods could often lead to erroneous and misleading interpretation of the process's capability.

In the recent years, several approaches to the problems of PCIs for the non-normal populations have been suggested. A widely accepted approach for PCI computation is to use the popular normal probability plot so that the normality assumption can be verified simultaneously. Analogous to the normal probability plot, where the "natural" process width is between the 0.135 percentile and the 99.865 percentile, surrogate PCI values may be obtained via appropriately selected probability plots. Since the median is usually the preferable central value for a skewed distribution, the corresponding C_{pu} and C_{pl} are defined as:

$$C_{pu} = \frac{USL - \text{median}}{(\text{upper } 0.135\% \text{ point}) - \text{median}} = \frac{USL - \text{median}}{X_{0.99865} - \text{median}},$$

$$\text{and } \; C_{pl} = \frac{\text{median} - USL}{\text{median} - (\text{lower } 0.135\% \text{ point})} = \frac{\text{median} - LSL}{\text{median} - X_{0.00135}}.$$

The index C_{pk} will then be calculated as the minimum of C_{pu} and C_{pl}, namely:

$$C_{pk} = \min\{C_{pu}, C_{pl}\}$$

$$= \min\left\{\frac{USL - \text{median}}{X_{0.99865} - \text{median}}, \frac{\text{median} - LSL}{\text{median} - X_{0.00135}}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

where these percentile points can easily be obtained from a simple computer code performing probability plots.

In the non-normal case, if we are able to find a better distributional form for the data, which provides a very satisfactory fit (this can be tested by means of goodness-of-fit tests), we can obtain more accurate measures of the three quantiles ($X_{0.00135}$, $X_{0.50}$ (= median) and $X_{0.99865}$) under consideration. This involves modeling the process data with alternative probability models, such as the Weibull or gamma ones (see e.g. Dudewicz and Mishra (1998), Kotz and Lovelace (1998)). However, an obvious disadvantage of probability plotting is that it is not a truly objective procedure. It is quite possible for two analysts to arrive at different conclusions using the same data. For this reason, it is often desirable to supplement probability plots with goodness-of-fit tests, which possess more formal statistical foundations (see, e.g., Shapiro (1980)). Choosing the distribution to fit the data is also an important step in probability plotting. Sometimes one can use the available knowledge of the physical phenomenon or the past experience to suggest a choice of the distribution.

3.4.2. Adjustment of C_{pk}

Acknowledging that a process will experience shifts in $X_{0.50}$ (= median) of various magnitudes and knowing that not all of these will be discovered, some allowance for them must be made when estimating outgoing quality so customers are not disappointed. Because shifts ranging in size from 0 up to $AS_{50}\sigma$ are the ones likely to remain undetected (larger moves should be caught by the chart), a conservative approach is to assume that every missed shift is as large as AS_{50} .

When estimating capability, $X_{0.50}$ minus $AS_{50}\sigma$ is used to evaluate how well the process output meets the LSL and $X_{0.50}$ plus $AS_{50}\sigma$ is used for determining conformance to the USL. Both of these adjustments are incorporated into the C_{pk} formula (See (1)), now called the “dynamic” C_{pk} index, by making the following modifications:

$$C_{pk} = \min\left\{\frac{USL-(X_{0.50}+AS_{50}\sigma)}{X_{0.99865}-X_{0.50}}, \frac{(X_{0.50}-AS_{50}\sigma)-LSL}{X_{0.50}-X_{0.00135}}\right\}$$

$$= \min\left\{\frac{USL-X_{0.50}-AS_{50}\sigma}{X_{0.99865}-X_{0.50}}, \frac{X_{0.50}-AS_{50}\sigma-LSL}{X_{0.50}-X_{0.00135}}\right\}$$

By including an adjustment in this assessment for undetected shifts in median, the estimate of capability will decrease and the expected total number nonconforming parts will increase. From table 5, AS_{50} is 1.936 when n equals 4 and data comes from Gamma(3,1) distribution. Factoring in the possibility of missing shifts in median of up to 1.936σ drops the C_{pk} index.

4. Application

The manufacturing of ICs includes the front end process of wafer and the back end process of integrated circuit packaging. In the integrated circuit packaging factory, the manufacturing process generally contains the following main steps, die sawing, die mounting, wire bonding, molding, trimming and forming, marking, plating and testing. Wire bonding are the most common means of providing an electrical connection from the IC device to the lead-frame, the process uses ultra-thin gold or aluminum wire to make the electrical inter-connect between the chip and the package leads. High-speed wire bond equipment consists of a handling system to feed the lead-frame into the work area. Image recognition systems ensure the die is orientated to match the bonding diagram for a particular device. Wires are bonded one wire at a time each interconnection two wire bonds are formed, one at the die (first bond) and the other at the lead-frame(second bond). The first bond involves the formation of a ball. The ball is placed in direct contact within the bond pad opening on the die, under load and ultrasonic energy within a few milliseconds and forms a ball bond at the bond pad metal as shown in figure 9 and figure 10 shows that the completion of wire bonding.

At wire bonding process, one of the most important factors to affect the quality of wire bonding is the ball size. Because we know that it may easily shut down when the width between two bond balls is too small, the size of the ball bond is one of the most important factors to be considered. We let the USL and LSL of ball size are 8mil and 0.5mil (1 mil = 1/1000 inch= 0.0254 mm)

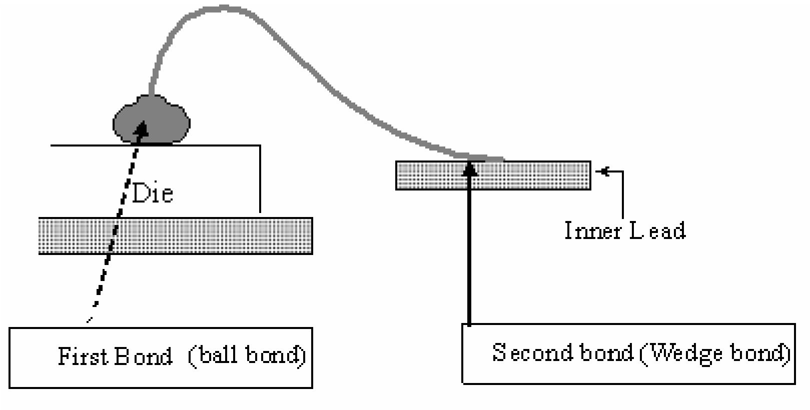

Figure 9. wire bonding process

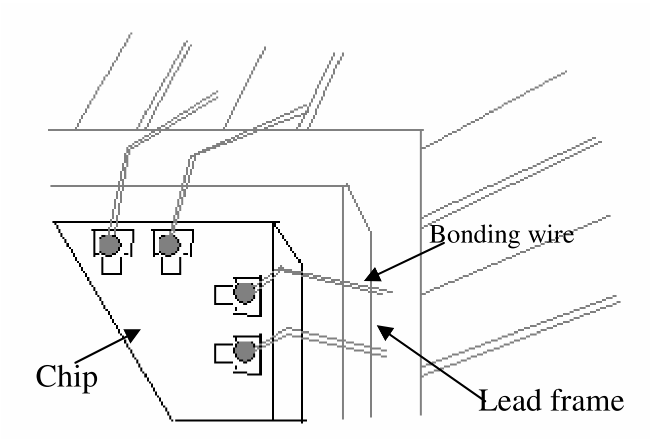

Figure 10. The chip at the wire bonding process

A part of historical data is display in table 8. The historical data indicates that the process pretty approximate to a gamma distribution (this can be tested by means of goodness-of-fit tests). The parameters N and θ of gamma distribution could be calculated from historical data giving $N=3$ and $\theta=1$. Therefore, it is appropriate to use this approach and we can obtain more accurate measures of the three quantiles : $X_{0.00135}$, $X_{0.50}$ (= median), and $X_{0.99865}$ for $\sigma=\sqrt{\frac{N}{n}}\times\theta=\sqrt{\frac{3}{10}}=0.547$ under consideration. Then "dynamic" C_{pk} index can be calculated as follows:

$$
\begin{aligned}
C_{pk} &= \min\left\{\frac{USL - X_{0.50} - AS_{50}\sigma}{X_{0.99865} - X_{0.50}}, \frac{X_{0.50} - AS_{50}\sigma - LSL}{X_{0.50} - X_{0.00135}}\right\} \\
&= \min\left\{\frac{8-2.67-1.123(0.547)}{10.87-2.67}, \frac{2.67-1.123(0.547)-0.5}{2.67-0.211}\right\} \\
&= \min\{0.58, 0.63\} \\
&= 0.58
\end{aligned}
$$

with AS_{50} =1.123 for n=10 from table 5. Compared it to the value of the following index：

$$C_{pk} = \left\{ \frac{USL - X_{0.5}}{X_{0.99865} - X_{0.5}}, \frac{X_{0.5} - LSL}{X_{0.5} - X_{0.00135}} \right\}$$
$$=\{0.65, 0.88\}$$
$$=0.65$$

that we don't consider the shift in μ ，we can find that the value of dynamic C_{pk} much smaller. By increasing n, shift in μ have a higher probability of detection. For example, if n=15, the AS_{50} would be 0.891 for Gamma distribution (3, 1) then

$$C_{pk} = \min\left\{ \frac{USL - X_{0.50} - AS_{50}\sigma}{X_{0.99865} - X_{0.50}}, \frac{X_{0.50} - AS_{50}\sigma - LSL}{X_{0.50} - X_{0.00135}} \right\}$$
$$=\min\left\{ \frac{8 - 2.67 - 0.891(0.547)}{10.87 - 2.67}, \frac{2.67 - 0.891(0.547) - 0.5}{2.67 - 0.211} \right\}$$
$$=\min\{0.6, 0.68\}$$
$$=0.6$$

Enlarging n by 2 increases the dynamic C_{pk} index from 0.58 to 0.6.

Table 9. The 100 observations are collected of the historical data.

1.399	1.215	1.841	2.079	2.497	2.955	3.290	2.886	1.235	2.044
1.435	1.235	1.855	2.100	2.507	3.002	2.782	2.955	1.251	2.050
1.709	1.251	1.867	2.169	2.513	3.018	2.814	3.002	1.313	2.096
1.731	1.313	1.928	2.173	2.572	3.020	2.886	3.018	1.399	2.212
1.748	1.399	1.998	2.202	2.639	3.024	2.955	3.020	1.435	2.363
1.782	1.435	2.079	2.238	2.653	3.037	3.002	3.024	1.512	2.402
1.808	1.512	2.100	2.323	2.700	3.060	3.018	1.808	1.562	2.413
1.841	1.562	2.169	2.374	2.782	3.066	3.020	1.841	1.602	2.422
1.251	1.602	2.173	2.440	2.814	3.135	3.024	1.748	1.609	2.494
1.011	1.609	2.202	2.473	2.886	3.259	2.782	1.782	1.709	2.512

5. Conclusion

This paper has considered the problem for adjusting estimates of process capability by including a mean shift when data is from non-normal distribution (particularly for gamma distribution). In the Bothe' study, statistically derived adjustments are proposed under the data assumed to be approximately normally distributed. But the case of non-normal processes occurs frequently in practice, for example, in the semiconductor industry. We develop a Matlab program (available on request) to simulate the average run length of non-normal distributions and compute the mean shift adjustment AS_{50} based on the detection power is 0.5 ($ARL_1 = 2$) for data comes from weibull and lognormal distribution and Gamma distribution with various values of N (=0.5 and 1(1)10) with n= 2(1)30. We also provided tables for the engineers/ practitioners to use for their in-plant applications. A real-world example taken from manufacturing process is investigated to illustrate the applicability of our approach.

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Appendix A. The Average Run Length of Three Non-normal Distributions

Table 10. Index values and the corresponding bounds on NCPPM for a Normal process

C_{pk}	Lower bound	Upper bound	C_{pk}	Lower bound	Upper bound
0.6	35930	71861	1.33	33	66
0.7	17864	35729	1.4	13	27
0.8	8198	16395	1.45	6.807	13.614
0.9	3467	6934	1.5	3.398	6.795
1.0	1350	2700	1.6	0.793	1.587
1.1	483	967	1.67	0.272	0.544
1.2	159	318	1.7	0.170	0.340
1.24	100	200	1.8	0.033	0.067
1.25	88	177	1.9	0.006	0.012
1.3	48	96	2.0	0.001	0.002

Table 11. Average run length of weibull with $+0.5\sigma$ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	529.52	305.95	136.90	91.16	71.80	55.99	53.17	46.38	43.30	40.69	37.11	90.64
3	428.45	223.75	86.77	73.08	51.91	41.32	39.40	33.76	30.94	27.93	30.19	60.68
4	453.35	156.02	68.90	43.41	38.11	29.82	27.67	26.34	24.75	24.26	21.73	43.89
5	325.26	126.74	50.24	32.74	26.40	27.52	22.64	19.65	19.99	17.39	17.17	33.40
6	294.21	90.32	41.00	26.73	22.30	20.48	17.38	17.60	17.19	15.30	14.99	26.36
7	291.06	75.24	31.89	22.88	18.12	16.02	15.68	12.32	13.51	12.16	12.20	21.38
8	274.15	58.39	27.07	18.85	14.97	13.67	12.74	11.78	10.98	11.25	9.77	17.73
9	212.59	44.13	23.89	17.16	12.31	11.49	11.23	10.45	9.85	9.34	9.08	14.97
10	193.57	40.32	16.99	13.03	12.21	10.24	8.84	9.21	8.86	8.56	7.39	12.82
11	185.19	38.94	16.12	11.99	10.36	9.56	7.78	7.64	7.47	7.41	7.02	11.13
12	155.52	28.74	13.11	10.60	8.71	8.33	7.57	6.73	6.82	6.90	6.62	9.76
13	152.89	24.39	11.30	9.04	8.05	6.91	6.94	5.89	6.45	6.32	6.47	8.65
14	129.30	23.41	10.18	8.11	7.03	6.61	6.16	5.72	5.86	5.68	5.18	7.73
15	112.20	18.93	9.00	7.27	6.53	6.27	5.68	5.41	5.17	4.85	5.03	6.96
16	110.19	16.17	9.16	6.49	5.86	5.46	5.14	4.67	4.69	4.63	4.45	6.30
17	103.46	15.82	7.34	6.75	5.24	4.88	4.55	4.32	4.26	4.08	4.35	5.75
18	72.92	15.05	6.62	5.98	4.81	4.28	4.78	4.38	3.99	4.04	3.93	5.27
19	80.79	11.90	6.31	5.17	4.37	4.34	3.95	3.91	3.95	3.78	3.56	4.86
20	67.94	10.90	5.80	4.77	4.17	3.81	3.93	3.77	3.54	3.51	3.41	4.50
21	64.90	9.88	5.60	4.49	4.03	3.68	3.65	3.35	3.41	3.42	3.49	4.18
22	50.44	8.79	4.64	3.94	3.83	3.51	3.37	3.24	3.13	3.19	3.06	3.90
23	52.61	7.90	4.60	3.92	3.54	3.21	3.26	3.07	3.01	2.95	2.77	3.66
24	37.69	7.03	4.42	3.40	3.19	3.15	3.03	2.88	2.79	2.83	2.79	3.44
25	42.50	6.81	4.05	3.20	3.05	2.98	2.76	2.95	2.69	2.61	2.61	3.24
26	30.01	6.09	3.67	3.22	3.07	2.79	2.67	2.63	2.63	2.55	2.64	3.07
27	36.45	5.58	3.43	3.02	2.76	2.62	2.58	2.56	2.50	2.43	2.42	2.91
28	26.42	5.76	3.33	2.89	2.92	2.46	2.40	2.53	2.45	2.50	2.29	2.77
29	32.36	5.10	3.20	2.80	2.61	2.50	2.37	2.32	2.37	2.36	2.32	2.64
30	25.78	4.73	3.00	2.68	2.54	2.30	2.23	2.33	2.22	2.29	2.21	2.52

Table 12. Average run length of weibull with $+1\sigma$ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	382.64	110.02	34.46	19.33	13.42	11.01	8.85	7.95	7.78	7.10	6.62	17.73
3	302.59	64.18	18.22	10.85	8.10	6.52	5.49	5.35	4.76	4.77	4.64	9.76
4	252.39	37.29	10.34	6.51	5.25	4.45	4.15	3.99	3.71	3.61	3.39	6.30
5	176.44	22.34	7.52	5.05	3.91	3.52	3.31	2.96	2.93	2.77	2.70	4.50
6	145.11	14.75	5.11	3.46	2.95	2.84	2.75	2.50	2.40	2.31	2.30	3.44
7	85.95	10.88	3.86	2.97	2.50	2.28	2.27	2.14	2.02	2.00	2.02	2.77
8	87.12	7.10	3.15	2.43	2.22	2.00	1.94	1.90	1.79	1.82	1.79	2.32
9	60.60	5.85	2.63	2.08	1.91	1.79	1.71	1.69	1.62	1.66	1.64	2.00
10	44.89	4.29	2.21	1.87	1.71	1.65	1.57	1.55	1.53	1.48	1.48	1.77
11	41.04	3.64	2.00	1.70	1.54	1.47	1.41	1.42	1.44	1.38	1.38	1.60
12	27.94	2.81	1.69	1.50	1.41	1.42	1.34	1.32	1.36	1.31	1.30	1.47
13	23.13	2.38	1.60	1.41	1.35	1.32	1.29	1.28	1.27	1.26	1.24	1.37
14	15.93	2.05	1.45	1.30	1.27	1.23	1.23	1.22	1.21	1.22	1.20	1.30
15	13.02	1.88	1.33	1.25	1.23	1.20	1.20	1.18	1.17	1.15	1.15	1.24
16	12.96	1.73	1.24	1.19	1.18	1.14	1.14	1.14	1.14	1.13	1.13	1.19
17	9.56	1.52	1.22	1.17	1.13	1.13	1.12	1.12	1.11	1.11	1.10	1.15
18	9.00	1.38	1.17	1.13	1.10	1.10	1.11	1.10	1.09	1.09	1.08	1.12
19	5.43	1.31	1.14	1.10	1.09	1.08	1.08	1.08	1.07	1.07	1.06	1.10
20	5.73	1.24	1.09	1.09	1.07	1.07	1.06	1.06	1.06	1.06	1.05	1.08
21	4.50	1.20	1.08	1.06	1.06	1.05	1.05	1.05	1.05	1.05	1.04	1.06
22	3.81	1.14	1.06	1.05	1.05	1.04	1.04	1.04	1.04	1.04	1.04	1.05
23	3.44	1.10	1.06	1.04	1.04	1.03	1.03	1.03	1.03	1.03	1.03	1.04
24	2.37	1.08	1.04	1.03	1.03	1.03	1.03	1.02	1.03	1.02	1.03	1.03
25	2.02	1.06	1.03	1.03	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
26	1.91	1.04	1.03	1.02	1.02	1.02	1.02	1.01	1.01	1.01	1.02	1.02
27	1.87	1.03	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
28	1.50	1.03	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
29	1.49	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
30	1.32	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01

Table 13. Average run length of weibull with $+1.5\sigma$ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	272.40	57.00	10.10	5.28	4.17	3.38	3.06	2.85	2.66	2.64	2.45	5.27
3	180.20	20.54	4.84	3.05	2.62	2.28	2.02	1.98	1.91	1.88	1.79	2.91
4	110.11	10.69	3.08	2.15	1.85	1.72	1.63	1.56	1.53	1.50	1.48	2.00
5	83.98	4.77	2.00	1.60	1.48	1.40	1.36	1.31	1.31	1.31	1.28	1.57
6	61.30	3.47	1.57	1.38	1.27	1.23	1.22	1.21	1.19	1.19	1.19	1.33
7	35.94	2.36	1.38	1.23	1.16	1.15	1.14	1.13	1.11	1.12	1.12	1.20
8	24.23	1.69	1.18	1.13	1.11	1.09	1.08	1.09	1.08	1.07	1.07	1.12
9	17.31	1.41	1.10	1.08	1.07	1.06	1.06	1.05	1.05	1.05	1.05	1.07
10	9.73	1.21	1.07	1.04	1.04	1.03	1.03	1.03	1.03	1.03	1.03	1.04
11	5.70	1.14	1.03	1.03	1.03	1.02	1.02	1.02	1.02	1.02	1.02	1.03
12	5.09	1.06	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
13	3.63	1.03	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
14	2.67	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
15	1.84	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.52	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.32	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 14. Average run length of weibull with $+2\sigma$ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	164.04	21.72	4.28	2.42	1.98	1.75	1.65	1.59	1.56	1.49	1.49	2.32
3	95.42	7.23	1.98	1.60	1.39	1.33	1.26	1.25	1.23	1.22	1.20	1.47
4	62.91	3.14	1.40	1.21	1.16	1.13	1.12	1.11	1.11	1.09	1.09	1.19
5	29.92	1.73	1.14	1.08	1.06	1.05	1.05	1.05	1.05	1.04	1.04	1.08
6	15.40	1.27	1.05	1.03	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.03
7	10.52	1.09	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
8	4.95	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	2.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.42	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 15. Average run length of weibull with +2.5σ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	122.53	10.09	2.12	1.49	1.35	1.25	1.22	1.19	1.18	1.17	1.17	1.42
3	56.97	3.05	1.20	1.12	1.09	1.07	1.05	1.06	1.05	1.05	1.05	1.10
4	26.44	1.39	1.05	1.02	1.02	1.02	1.02	1.01	1.01	1.01	1.01	1.02
5	9.61	1.03	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	5.69	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 16. Average run length of weibull with $+3\sigma$ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	87.05	4.46	1.32	1.14	1.09	1.07	1.06	1.06	1.05	1.05	1.05	1.12
3	32.79	1.45	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
4	14.66	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	3.71	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	1.23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 17. Average run length of weibull with -0.5σ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.36	3.53	22.60	47.62	73.25	92.35	119.53	109.86	140.19	151.85	140.33	90.64
3	1.39	4.04	20.66	40.77	63.53	81.37	85.66	98.87	82.80	96.66	100.57	60.68
4	1.42	4.25	17.21	30.78	45.67	51.07	58.52	61.80	64.52	68.99	79.26	43.89
5	1.42	4.18	15.16	26.28	33.73	35.37	45.27	49.83	49.66	52.08	51.88	33.40
6	1.43	4.19	12.56	21.08	25.96	30.72	33.56	38.56	44.74	40.79	40.05	26.36
7	1.42	3.94	11.35	17.39	21.11	25.80	26.92	29.64	30.39	31.55	32.95	21.38
8	1.42	3.79	9.95	14.48	18.06	21.67	22.62	27.52	25.60	24.88	28.86	17.73
9	1.41	3.56	8.45	11.06	16.05	18.09	19.40	22.02	21.10	24.23	24.38	14.97
10	1.40	3.37	7.80	12.03	13.71	14.16	16.18	18.11	17.21	19.67	17.83	12.82
11	1.39	3.19	6.95	9.34	11.15	12.50	14.71	14.35	14.26	14.63	16.62	11.13
12	1.38	3.04	6.49	8.39	9.36	11.94	12.59	13.20	12.46	13.92	13.18	9.76
13	1.36	2.93	5.52	7.27	9.00	9.18	10.98	11.97	11.05	11.38	12.13	8.65
14	1.35	2.84	5.22	7.20	7.70	8.35	9.20	9.26	9.65	10.16	10.72	7.73
15	1.34	2.65	4.87	6.18	7.27	7.91	8.27	9.18	9.05	9.04	9.86	6.96
16	1.32	2.62	4.55	5.51	6.57	7.78	7.30	7.83	8.09	8.35	8.26	6.30
17	1.31	2.42	4.14	4.98	5.94	6.36	6.73	7.08	7.64	7.75	7.26	5.75
18	1.30	2.35	3.75	4.80	5.42	6.13	6.23	6.60	7.09	6.80	7.19	5.27
19	1.29	2.32	3.59	4.48	5.20	5.52	6.02	5.76	6.25	5.92	6.40	4.86
20	1.28	2.16	3.20	4.23	4.66	4.64	5.30	5.32	5.99	5.64	5.59	4.50
21	1.27	2.10	3.20	3.74	4.23	4.55	4.90	4.86	5.06	5.46	5.38	4.18
22	1.26	2.03	3.03	3.52	3.90	4.02	4.49	4.77	4.70	4.99	5.38	3.90
23	1.25	2.00	2.79	3.35	3.47	3.87	4.12	4.37	4.54	4.69	4.54	3.66
24	1.24	1.95	2.83	3.20	3.57	3.65	3.94	4.09	4.38	4.06	4.36	3.44
25	1.23	1.84	2.59	3.09	3.32	3.57	3.83	3.85	4.06	4.17	4.05	3.24
26	1.22	1.82	2.43	2.91	3.02	3.48	3.55	3.74	3.62	3.86	3.78	3.07
27	1.21	1.78	2.34	2.68	2.90	3.07	3.29	3.57	3.54	3.69	3.54	2.91
28	1.20	1.71	2.32	2.65	2.67	3.07	2.96	3.11	3.14	3.24	3.60	2.77
29	1.19	1.67	2.18	2.48	2.67	2.74	2.88	3.09	3.01	3.19	3.05	2.64
30	1.19	1.64	2.20	2.44	2.63	2.75	2.81	2.85	2.87	2.87	3.02	2.52

Table 18. Average run length of weibull with -1 σ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.13	1.64	4.65	10.11	15.78	22.72	26.33	26.04	34.57	36.10	40.43	17.73
3	1.12	1.61	3.76	6.88	9.39	10.63	15.48	16.02	17.57	19.19	20.85	9.76
4	1.11	1.53	3.01	4.82	6.66	7.46	8.13	9.53	10.96	11.76	12.42	6.30
5	1.10	1.46	2.60	3.70	4.51	5.09	5.70	6.78	7.19	8.02	8.46	4.50
6	1.09	1.38	2.19	2.91	3.61	3.96	4.39	5.00	4.74	5.02	5.41	3.44
7	1.08	1.33	1.95	2.39	2.75	3.09	3.27	3.50	3.85	3.84	3.91	2.77
8	1.07	1.28	1.74	2.04	2.36	2.54	2.74	2.94	3.05	3.06	3.18	2.32
9	1.06	1.23	1.60	1.94	2.01	2.18	2.31	2.56	2.54	2.55	2.50	2.00
10	1.06	1.19	1.50	1.67	1.76	1.87	2.07	2.11	2.17	2.18	2.40	1.77
11	1.05	1.16	1.40	1.54	1.65	1.70	1.79	1.86	1.85	1.99	1.97	1.60
12	1.04	1.14	1.28	1.41	1.50	1.53	1.62	1.58	1.71	1.68	1.75	1.47
13	1.04	1.11	1.25	1.35	1.38	1.44	1.47	1.45	1.58	1.51	1.62	1.37
14	1.04	1.09	1.18	1.28	1.31	1.35	1.37	1.42	1.45	1.43	1.42	1.30
15	1.03	1.08	1.16	1.22	1.25	1.27	1.31	1.32	1.31	1.33	1.37	1.24
16	1.03	1.07	1.13	1.17	1.18	1.21	1.24	1.26	1.27	1.27	1.27	1.19
17	1.02	1.06	1.10	1.13	1.17	1.16	1.18	1.21	1.23	1.21	1.21	1.15
18	1.02	1.05	1.08	1.11	1.11	1.15	1.14	1.18	1.16	1.17	1.17	1.12
19	1.02	1.04	1.07	1.09	1.09	1.12	1.12	1.12	1.12	1.12	1.14	1.10
20	1.02	1.03	1.05	1.07	1.08	1.09	1.10	1.10	1.10	1.10	1.12	1.08
21	1.02	1.03	1.05	1.06	1.07	1.06	1.07	1.08	1.08	1.08	1.09	1.06
22	1.01	1.02	1.04	1.05	1.05	1.06	1.06	1.06	1.06	1.07	1.07	1.05
23	1.01	1.02	1.03	1.04	1.04	1.04	1.06	1.05	1.04	1.05	1.05	1.04
24	1.01	1.02	1.02	1.03	1.03	1.04	1.04	1.03	1.04	1.04	1.04	1.03
25	1.01	1.01	1.02	1.02	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.02
26	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.03	1.02
27	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.01
28	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
29	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
30	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01

Table 19. Average run length of weibull with -1.5σ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.07	1.24	2.05	3.49	4.74	6.10	7.52	9.62	9.74	11.75	12.01	5.27
3	1.05	1.18	1.67	2.31	2.94	3.36	3.83	4.23	5.03	5.39	5.15	2.91
4	1.04	1.13	1.42	1.73	1.97	2.27	2.34	2.63	2.85	2.91	3.08	2.00
5	1.03	1.09	1.27	1.44	1.59	1.66	1.79	1.87	2.04	2.09	2.17	1.57
6	1.03	1.07	1.16	1.26	1.36	1.41	1.46	1.48	1.55	1.60	1.64	1.33
7	1.02	1.05	1.12	1.16	1.22	1.25	1.27	1.31	1.30	1.32	1.32	1.20
8	1.02	1.03	1.07	1.10	1.13	1.14	1.16	1.17	1.20	1.21	1.20	1.12
9	1.01	1.02	1.04	1.06	1.07	1.09	1.09	1.10	1.11	1.10	1.11	1.07
10	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.05	1.07	1.06	1.07	1.04
11	1.01	1.01	1.02	1.02	1.03	1.03	1.03	1.03	1.03	1.04	1.04	1.02
12	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.01
13	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
14	1.01	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 20. Average run length of weibull with -2σ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.04	1.10	1.37	1.80	2.17	2.71	3.16	3.73	3.66	3.97	4.48	2.32
3	1.03	1.06	1.17	1.33	1.49	1.59	1.70	1.79	1.95	2.20	2.16	1.47
4	1.02	1.03	1.08	1.15	1.19	1.23	1.30	1.33	1.38	1.38	1.44	1.19
5	1.01	1.02	1.04	1.06	1.08	1.09	1.09	1.13	1.14	1.15	1.17	1.08
6	1.01	1.01	1.02	1.02	1.03	1.04	1.04	1.04	1.04	1.05	1.05	1.03
7	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.01
8	1.01	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.00	1.01	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 21. Average run length of weibull with -2.5σ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.02	1.04	1.12	1.27	1.42	1.53	1.71	1.85	1.92	2.02	2.17	1.42
3	1.01	1.02	1.04	1.07	1.10	1.13	1.17	1.20	1.19	1.22	1.25	1.10
4	1.01	1.01	1.01	1.02	1.02	1.03	1.03	1.03	1.04	1.04	1.06	1.02
5	1.01	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 22. Average run length of weibull with -3σ mean shift

n \ k	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.01	1.02	1.04	1.07	1.11	1.15	1.20	1.21	1.28	1.31	1.37	1.12
3	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.03	1.03	1.03	1.01
4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 23. Average run length of Lognormal with +0.5σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	722.91	692.28	535.80	333.40	169.77	135.27	90.64
3	478.31	560.76	446.17	248.97	132.12	79.32	60.68
4	513.76	540.47	432.15	175.85	88.23	59.12	43.89
5	504.35	572.39	443.00	155.09	60.98	54.55	33.40
6	485.37	427.96	369.61	94.75	43.95	37.79	26.36
7	536.90	485.29	278.55	79.22	35.25	27.68	21.38
8	509.88	422.36	245.53	65.76	29.33	24.62	17.73
9	426.20	391.85	210.10	47.87	27.47	17.89	14.97
10	499.28	400.38	214.46	36.84	21.34	15.54	12.82
11	502.22	371.49	185.80	35.42	19.50	15.42	11.13
12	482.61	385.00	189.60	29.82	14.41	12.89	9.76
13	472.04	482.94	158.93	25.64	13.80	11.37	8.65
14	471.73	408.93	141.57	20.96	11.12	10.00	7.73
15	426.65	385.68	124.87	17.64	10.35	9.29	6.96
16	436.54	328.91	122.67	18.59	9.64	7.45	6.30
17	403.63	312.28	103.36	14.09	7.82	7.05	5.75
18	430.10	314.97	84.33	13.02	7.14	5.97	5.27
19	415.26	265.38	88.66	11.10	7.24	5.69	4.86
20	439.86	279.23	85.16	10.94	6.32	5.32	4.50
21	403.41	289.42	60.25	8.68	5.86	4.82	4.18
22	410.08	249.76	57.75	8.52	5.32	4.39	3.90
23	289.71	267.64	57.30	7.48	5.03	4.30	3.66
24	474.62	238.28	53.70	7.18	4.45	3.86	3.44
25	449.56	244.76	49.61	6.94	4.42	3.78	3.24
26	369.49	199.88	40.84	5.90	4.16	3.27	3.07
27	430.59	194.93	30.74	5.82	3.75	3.48	2.91
28	331.94	279.38	36.08	5.23	3.39	3.08	2.77
29	373.80	177.13	25.43	4.89	3.26	3.05	2.64
30	320.34	209.77	23.94	4.44	3.19	2.76	2.52

Table 24. Average run length of Lognormal with $+1\sigma$ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	482.41	494.46	340.25	147.19	52.50	29.76	17.73
3	435.80	492.81	312.01	67.64	26.65	17.12	9.76
4	460.26	327.55	232.96	45.21	14.85	10.31	6.30
5	405.13	416.57	156.41	21.27	10.69	6.14	4.50
6	368.91	324.26	150.42	14.50	6.51	4.78	3.44
7	352.43	294.69	86.88	11.00	4.62	3.58	2.77
8	332.60	305.38	88.11	6.49	3.65	2.93	2.32
9	271.05	266.86	44.13	5.48	2.84	2.36	2.00
10	304.88	187.04	50.03	4.08	2.57	1.98	1.77
11	263.02	226.04	40.51	3.29	2.16	1.86	1.60
12	238.11	155.76	27.03	2.55	1.80	1.61	1.47
13	260.64	191.99	21.16	2.44	1.61	1.53	1.37
14	244.66	155.67	17.69	1.98	1.50	1.37	1.30
15	238.91	136.09	12.91	1.84	1.38	1.31	1.24
16	209.51	169.01	9.62	1.66	1.31	1.23	1.19
17	215.59	146.22	8.60	1.46	1.24	1.20	1.15
18	207.47	75.02	8.51	1.39	1.18	1.17	1.12
19	162.89	100.24	6.07	1.27	1.16	1.12	1.10
20	196.30	92.69	5.23	1.24	1.11	1.09	1.08
21	175.90	65.58	3.67	1.19	1.10	1.08	1.06
22	155.15	49.86	3.56	1.13	1.08	1.05	1.05
23	147.57	46.62	3.20	1.09	1.06	1.05	1.04
24	93.93	51.07	2.43	1.09	1.04	1.04	1.03
25	136.65	48.69	2.47	1.06	1.04	1.03	1.02
26	108.08	41.21	1.90	1.04	1.02	1.02	1.02
27	131.62	34.43	1.63	1.03	1.02	1.02	1.01
28	110.64	30.17	1.72	1.02	1.01	1.01	1.01
29	90.70	32.33	1.33	1.02	1.01	1.01	1.01
30	108.18	22.09	1.40	1.01	1.01	1.01	1.01

Table 25. Average run length of Lognormal with $+1.5\sigma$ mean shift

s / n	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	443.89	383.94	235.70	55.84	17.42	9.26	5.27
3	311.98	359.64	181.82	19.44	6.86	4.18	2.91
4	327.92	296.28	133.35	9.83	3.96	2.68	2.00
5	293.65	262.97	83.79	5.30	2.33	1.83	1.57
6	335.72	219.03	44.12	3.06	1.68	1.51	1.33
7	231.68	167.58	32.29	2.10	1.42	1.27	1.20
8	185.80	187.48	17.67	1.63	1.22	1.17	1.12
9	145.55	121.83	13.48	1.34	1.13	1.11	1.07
10	205.92	115.37	9.47	1.19	1.08	1.06	1.04
11	181.25	94.61	5.86	1.12	1.05	1.04	1.03
12	104.64	72.94	3.71	1.07	1.02	1.02	1.01
13	103.54	59.38	2.43	1.02	1.01	1.01	1.01
14	97.27	43.94	2.01	1.01	1.01	1.01	1.01
15	98.86	39.37	1.69	1.01	1.00	1.00	1.00
16	106.94	28.99	1.34	1.00	1.00	1.00	1.00
17	68.70	30.69	1.21	1.00	1.00	1.00	1.00
18	75.61	18.67	1.04	1.00	1.00	1.00	1.00
19	51.64	11.55	1.03	1.00	1.00	1.00	1.00
20	42.92	7.66	1.01	1.00	1.00	1.00	1.00
21	57.41	5.54	1.00	1.00	1.00	1.00	1.00
22	72.56	5.36	1.00	1.00	1.00	1.00	1.00
23	34.57	7.83	1.00	1.00	1.00	1.00	1.00
24	38.83	4.30	1.00	1.00	1.00	1.00	1.00
25	13.66	2.19	1.00	1.00	1.00	1.00	1.00
26	15.62	4.36	1.00	1.00	1.00	1.00	1.00
27	3.82	1.26	1.00	1.00	1.00	1.00	1.00
28	18.07	1.29	1.00	1.00	1.00	1.00	1.00
29	8.75	1.47	1.00	1.00	1.00	1.00	1.00
30	3.01	1.09	1.00	1.00	1.00	1.00	1.00

Table 26. Average run length of Lognormal with +2σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	422.90	323.09	168.37	26.70	5.99	3.56	2.32
3	277.98	172.90	101.10	6.91	2.36	1.78	1.47
4	270.75	263.54	58.71	2.89	1.55	1.29	1.19
5	188.97	125.30	24.41	1.67	1.23	1.12	1.08
6	142.32	85.23	15.35	1.28	1.07	1.04	1.03
7	146.74	90.86	6.58	1.08	1.02	1.01	1.01
8	138.11	53.95	3.46	1.02	1.01	1.01	1.00
9	70.67	42.13	2.09	1.01	1.00	1.00	1.00
10	57.84	37.02	1.34	1.00	1.00	1.00	1.00
11	54.84	25.04	1.10	1.00	1.00	1.00	1.00
12	44.92	10.98	1.01	1.00	1.00	1.00	1.00
13	72.26	8.75	1.01	1.00	1.00	1.00	1.00
14	36.54	5.01	1.00	1.00	1.00	1.00	1.00
15	17.69	4.21	1.00	1.00	1.00	1.00	1.00
16	8.05	4.90	1.00	1.00	1.00	1.00	1.00
17	4.39	2.21	1.00	1.00	1.00	1.00	1.00
18	7.23	1.01	1.00	1.00	1.00	1.00	1.00
19	1.00	1.34	1.00	1.00	1.00	1.00	1.00
20	2.38	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.06	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 27. Average run length of lognormal with $+2.5\sigma$ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	340.91	309.89	122.03	10.32	2.75	1.82	1.42
3	253.83	199.03	48.75	2.66	1.32	1.19	1.10
4	126.41	107.07	20.18	1.37	1.07	1.04	1.02
5	95.26	83.46	5.96	1.05	1.01	1.01	1.00
6	58.55	50.46	2.98	1.00	1.00	1.00	1.00
7	47.29	37.49	1.68	1.00	1.00	1.00	1.00
8	33.08	29.72	1.07	1.00	1.00	1.00	1.00
9	17.92	5.70	1.00	1.00	1.00	1.00	1.00
10	17.66	9.21	1.00	1.00	1.00	1.00	1.00
11	4.63	2.41	1.00	1.00	1.00	1.00	1.00
12	3.23	1.12	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.16	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 28. Average run length of Lognormal with +3σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	267.232	224.385	55.728	4.674	1.524	1.240	1.12
3	166.265	101.691	20.746	1.353	1.046	1.025	1.01
4	80.574	74.157	7.293	1.009	1.003	1.002	1.00
5	46.503	49.044	2.085	1.000	1.000	1.000	1.00
6	12.028	15.476	1.147	1.000	1.000	1.000	1.00
7	32.340	7.052	1.000	1.000	1.000	1.000	1.00
8	1.000	1.467	1.000	1.000	1.000	1.000	1.00
9	1.000	1.956	1.000	1.000	1.000	1.000	1.00
10	1.000	1.496	1.000	1.000	1.000	1.000	1.00
11	1.000	1.000	1.000	1.000	1.000	1.000	1.00
12	1.000	1.000	1.000	1.000	1.000	1.000	1.00
13	1.000	1.000	1.000	1.000	1.000	1.000	1.00
14	1.000	1.000	1.000	1.000	1.000	1.000	1.00
15	1.000	1.000	1.000	1.000	1.000	1.000	1.00
16	1.000	1.000	1.000	1.000	1.000	1.000	1.00
17	1.000	1.000	1.000	1.000	1.000	1.000	1.00
18	1.000	1.000	1.000	1.000	1.000	1.000	1.00
19	1.000	1.000	1.000	1.000	1.000	1.000	1.00
20	1.000	1.000	1.000	1.000	1.000	1.000	1.00
21	1.000	1.000	1.000	1.000	1.000	1.000	1.00
22	1.000	1.000	1.000	1.000	1.000	1.000	1.00
23	1.000	1.000	1.000	1.000	1.000	1.000	1.00
24	1.000	1.000	1.000	1.000	1.000	1.000	1.00
25	1.000	1.000	1.000	1.000	1.000	1.000	1.00
26	1.000	1.000	1.000	1.000	1.000	1.000	1.00
27	1.000	1.000	1.000	1.000	1.000	1.000	1.00
28	1.000	1.000	1.000	1.000	1.000	1.000	1.00
29	1.000	1.000	1.000	1.000	1.000	1.000	1.00
30	1.000	1.000	1.000	1.000	1.000	1.000	1.00

Table 29. Average run length of Log-normal with -0.5σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	1.05	1.20	2.01	10.16	29.68	58.99	90.64
3	1.05	1.20	2.05	9.22	24.41	39.12	60.68
4	1.05	1.20	2.04	8.58	19.80	29.01	43.89
5	1.04	1.19	2.03	7.12	16.76	24.86	33.40
6	1.04	1.19	1.97	6.44	13.50	18.39	26.36
7	1.04	1.18	1.91	5.82	12.06	14.96	21.38
8	1.04	1.17	1.87	5.42	9.69	13.11	17.73
9	1.04	1.16	1.84	4.89	9.08	11.97	14.97
10	1.04	1.15	1.77	4.40	7.84	9.77	12.82
11	1.03	1.15	1.72	4.05	7.50	8.73	11.13
12	1.03	1.14	1.70	3.97	6.23	7.81	9.76
13	1.03	1.14	1.66	3.62	5.53	7.18	8.65
14	1.03	1.13	1.62	3.35	5.21	6.17	7.73
15	1.03	1.12	1.57	3.18	4.56	5.75	6.96
16	1.03	1.12	1.55	3.05	4.29	5.28	6.30
17	1.03	1.11	1.53	2.78	3.97	4.51	5.75
18	1.03	1.11	1.49	2.70	3.68	4.47	5.27
19	1.03	1.10	1.45	2.63	3.44	4.09	4.86
20	1.02	1.10	1.44	2.43	3.30	3.89	4.50
21	1.02	1.10	1.43	2.41	3.16	3.53	4.18
22	1.02	1.09	1.41	2.28	2.93	3.32	3.90
23	1.02	1.09	1.36	2.18	2.76	3.23	3.66
24	1.02	1.09	1.35	2.09	2.68	2.81	3.44
25	1.02	1.08	1.33	2.06	2.58	2.88	3.24
26	1.02	1.08	1.33	1.99	2.37	2.71	3.07
27	1.02	1.08	1.31	1.91	2.24	2.66	2.91
28	1.02	1.07	1.29	1.85	2.34	2.54	2.77
29	1.02	1.07	1.28	1.79	2.20	2.50	2.64
30	1.02	1.07	1.27	1.78	2.15	2.28	2.52

Table 30. Average run length of Log-normal with -1σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	1.02	1.07	1.25	2.53	5.91	9.47	5.27
3	1.02	1.06	1.22	2.21	4.31	6.31	2.91
4	1.02	1.05	1.19	1.91	3.09	4.35	2.00
5	1.01	1.05	1.16	1.69	2.52	3.42	1.57
6	1.01	1.04	1.14	1.57	2.20	2.61	1.33
7	1.01	1.04	1.12	1.44	1.91	2.29	1.20
8	1.01	1.03	1.10	1.37	1.69	1.91	1.12
9	1.01	1.03	1.09	1.30	1.59	1.73	1.07
10	1.01	1.03	1.08	1.26	1.47	1.59	1.04
11	1.01	1.02	1.07	1.21	1.36	1.42	1.03
12	1.01	1.02	1.06	1.17	1.27	1.34	1.01
13	1.01	1.02	1.05	1.14	1.22	1.29	1.01
14	1.01	1.02	1.04	1.11	1.19	1.24	1.01
15	1.01	1.02	1.04	1.09	1.15	1.19	1.00
16	1.01	1.02	1.03	1.08	1.12	1.14	1.00
17	1.01	1.02	1.03	1.06	1.10	1.12	1.00
18	1.01	1.01	1.03	1.06	1.08	1.10	1.00
19	1.01	1.01	1.02	1.05	1.07	1.08	1.00
20	1.01	1.01	1.02	1.04	1.05	1.07	1.00
21	1.01	1.01	1.02	1.03	1.04	1.05	1.00
22	1.01	1.01	1.02	1.02	1.04	1.04	1.00
23	1.01	1.01	1.01	1.02	1.03	1.03	1.00
24	1.01	1.01	1.01	1.02	1.02	1.03	1.00
25	1.01	1.01	1.01	1.02	1.02	1.02	1.00
26	1.00	1.01	1.01	1.01	1.01	1.02	1.00
27	1.00	1.01	1.01	1.01	1.01	1.01	1.00
28	1.00	1.01	1.01	1.01	1.01	1.01	1.00
29	1.00	1.01	1.01	1.01	1.01	1.01	1.00
30	1.00	1.01	1.01	1.01	1.01	1.01	1.00

Table 31. Average run length of Log-normal with -1.5σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	1.01	1.03	1.10	1.45	2.27	3.19	5.27
3	1.01	1.03	1.08	1.29	1.70	2.10	2.91
4	1.01	1.02	1.06	1.20	1.40	1.64	2.00
5	1.01	1.02	1.04	1.13	1.25	1.38	1.57
6	1.01	1.02	1.03	1.09	1.16	1.23	1.33
7	1.01	1.01	1.03	1.06	1.10	1.14	1.20
8	1.01	1.01	1.02	1.04	1.07	1.09	1.12
9	1.00	1.01	1.02	1.03	1.04	1.06	1.07
10	1.00	1.01	1.01	1.02	1.03	1.04	1.04
11	1.00	1.01	1.01	1.01	1.02	1.02	1.02
12	1.00	1.01	1.01	1.01	1.01	1.01	1.01
13	1.00	1.01	1.01	1.01	1.01	1.01	1.01
14	1.00	1.01	1.01	1.00	1.00	1.00	1.00
15	1.00	1.01	1.01	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 32. Average run length of Log-normal with -2σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	1.01	1.02	1.05	1.16	1.44	1.72	2.32
3	1.01	1.01	1.03	1.09	1.19	1.29	1.47
4	1.00	1.01	1.02	1.04	1.09	1.13	1.19
5	1.00	1.01	1.02	1.03	1.04	1.05	1.08
6	1.00	1.01	1.01	1.01	1.02	1.02	1.03
7	1.00	1.01	1.01	1.01	1.01	1.01	1.01
8	1.00	1.01	1.01	1.00	1.00	1.00	1.00
9	1.00	1.01	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 33. Average run length of Log-normal with -2.5σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	1.01	1.01	1.03	1.06	1.14	1.21	1.42
3	1.00	1.01	1.02	1.03	1.04	1.07	1.10
4	1.00	1.01	1.01	1.01	1.01	1.02	1.02
5	1.00	1.01	1.01	1.00	1.00	1.00	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 34. Average run length of Log-normal with -3σ mean shift

n \ s	2	3/2	1	1/2	1/4	1/8	N(0,1)
2	1.00	1.01	1.02	1.03	1.05	1.07	1.12
3	1.00	1.01	1.01	1.01	1.01	1.01	1.01
4	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 35. Average run length of Gamma with +0.5σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	382.69	307.59	236.12	249.44	194.06	201.78	187.38	152.53	182.08	153.96	161.67	90.64
3	270.43	251.79	174.44	146.76	132.60	130.67	132.18	114.57	112.95	113.75	121.56	60.68
4	231.71	166.85	120.07	104.25	104.67	91.17	83.79	78.82	85.57	72.39	76.20	43.89
5	177.25	123.87	91.70	69.72	73.52	70.26	61.88	58.95	54.08	66.26	60.15	33.40
6	113.86	92.55	68.24	58.33	57.36	56.86	45.13	46.45	43.69	39.94	44.12	26.36
7	111.35	70.04	60.58	43.21	42.09	47.91	38.13	37.54	36.83	38.17	35.37	21.38
8	79.22	70.14	45.87	47.96	34.06	34.16	30.29	29.54	27.03	30.38	27.14	17.73
9	71.58	50.66	35.47	34.14	27.73	23.61	26.95	23.91	24.83	22.46	22.13	14.97
10	52.44	38.03	30.33	25.50	24.33	22.46	21.66	21.80	21.09	19.60	18.62	12.82
11	48.88	32.82	26.91	25.32	22.54	20.14	18.47	16.31	18.42	15.84	15.92	11.13
12	48.18	31.23	20.94	19.71	17.43	16.56	16.26	16.06	17.18	14.92	12.72	9.76
13	38.68	25.72	19.38	17.18	15.92	15.50	13.60	13.68	11.50	12.91	12.44	8.65
14	34.40	20.89	17.57	15.01	13.24	13.14	12.63	11.99	11.67	11.48	11.07	7.73
15	24.66	19.21	14.63	11.85	12.00	11.66	11.43	9.82	9.91	9.83	10.86	6.96
16	24.14	18.02	12.88	11.07	10.26	10.48	10.21	8.87	8.72	8.36	8.82	6.30
17	21.00	15.69	11.38	9.88	9.85	8.56	8.57	8.14	8.12	8.75	7.60	5.75
18	16.69	13.69	10.09	8.67	8.37	7.99	7.25	6.95	7.10	7.06	7.16	5.27
19	18.08	11.79	9.51	7.72	8.27	7.64	6.44	7.02	6.70	6.27	5.97	4.86
20	16.26	11.92	8.77	7.89	7.44	7.09	6.37	5.99	5.72	6.30	5.70	4.50
21	12.77	9.63	7.48	7.12	6.78	6.47	5.97	5.98	5.66	5.40	5.42	4.18
22	12.10	9.09	7.49	7.02	5.66	5.76	5.18	5.59	5.31	4.96	5.28	3.90
23	10.52	8.08	6.66	5.52	5.40	4.75	5.00	4.97	4.71	4.74	4.75	3.66
24	10.10	7.00	5.90	4.92	5.59	4.66	4.78	4.31	4.29	4.36	4.37	3.44
25	9.96	6.58	5.00	4.86	4.52	4.25	4.35	4.29	4.02	3.78	3.92	3.24
26	8.56	5.99	4.86	4.59	4.29	3.99	4.09	3.99	3.80	3.86	4.20	3.07
27	7.51	5.89	4.59	4.26	4.21	3.81	3.70	3.75	3.70	3.90	3.54	2.91
28	7.63	5.07	4.09	3.81	3.70	3.95	3.61	3.46	3.60	3.31	3.32	2.77
29	6.63	4.90	4.14	4.01	3.79	3.40	3.40	3.24	3.18	3.15	3.36	2.64
30	5.88	4.63	3.73	3.33	3.22	3.34	3.26	3.09	3.18	3.04	3.10	2.52

Table 36. Average run length of Gamma with $+1\sigma$ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	167.62	125.57	84.68	64.87	57.68	57.47	56.23	41.80	41.98	40.05	38.31	17.73
3	91.76	65.58	38.11	33.33	29.07	26.36	22.83	21.30	21.85	20.65	21.30	9.76
4	63.70	44.36	24.90	19.58	17.47	16.89	14.48	14.47	14.60	12.13	11.19	6.30
5	43.30	22.50	15.21	11.61	11.55	9.59	9.27	8.74	8.37	7.20	7.55	4.50
6	20.97	16.10	10.14	8.52	7.29	6.56	5.70	5.73	5.65	5.50	4.77	3.44
7	14.96	10.33	7.16	5.67	5.22	4.74	4.52	4.74	4.54	4.00	3.86	2.77
8	11.70	7.75	5.20	4.58	3.80	4.10	3.75	3.51	3.39	3.60	3.39	2.32
9	8.45	5.49	4.11	3.56	3.26	3.11	2.84	2.74	2.77	2.74	2.69	2.00
10	7.03	4.29	3.10	2.94	2.58	2.63	2.35	2.29	2.29	2.33	2.20	1.77
11	5.64	3.62	2.62	2.37	2.20	2.15	2.08	2.16	2.15	1.96	2.01	1.60
12	4.34	2.90	2.33	2.11	2.01	1.95	1.85	1.81	1.75	1.75	1.74	1.47
13	3.46	2.53	1.94	1.79	1.80	1.65	1.64	1.59	1.58	1.54	1.56	1.37
14	2.92	2.23	1.75	1.59	1.62	1.58	1.49	1.47	1.44	1.46	1.44	1.30
15	2.38	1.70	1.65	1.51	1.46	1.45	1.39	1.35	1.37	1.36	1.36	1.24
16	2.10	1.64	1.47	1.46	1.34	1.34	1.31	1.33	1.27	1.30	1.29	1.19
17	1.93	1.47	1.37	1.32	1.26	1.26	1.23	1.23	1.23	1.23	1.22	1.15
18	1.67	1.41	1.29	1.22	1.21	1.22	1.22	1.18	1.17	1.16	1.16	1.12
19	1.50	1.35	1.19	1.20	1.18	1.15	1.18	1.13	1.14	1.14	1.14	1.10
20	1.38	1.27	1.17	1.16	1.15	1.13	1.12	1.11	1.12	1.11	1.12	1.08
21	1.31	1.17	1.13	1.11	1.10	1.11	1.09	1.08	1.09	1.09	1.09	1.06
22	1.25	1.14	1.09	1.08	1.08	1.08	1.07	1.07	1.08	1.07	1.06	1.05
23	1.21	1.10	1.08	1.07	1.06	1.06	1.06	1.05	1.05	1.05	1.05	1.04
24	1.14	1.07	1.07	1.05	1.05	1.04	1.05	1.04	1.04	1.04	1.04	1.03
25	1.11	1.07	1.05	1.04	1.04	1.03	1.03	1.03	1.03	1.03	1.03	1.02
26	1.08	1.05	1.04	1.03	1.03	1.03	1.03	1.02	1.02	1.02	1.02	1.02
27	1.05	1.04	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.01
28	1.03	1.03	1.02	1.02	1.02	1.01	1.01	1.02	1.01	1.02	1.01	1.01
29	1.03	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
30	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01

Table 37. Average run length of Gamma with +1.5 σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	82.27	52.57	34.69	22.53	22.99	19.42	16.38	17.12	12.46	14.79	12.27	5.27
3	33.68	20.24	11.91	9.83	7.77	6.82	6.50	7.15	5.84	5.60	5.13	2.91
4	19.14	9.17	5.97	5.03	4.45	4.37	3.67	3.32	3.66	3.05	3.06	2.00
5	9.04	5.40	3.55	3.04	2.67	2.50	2.41	2.11	2.09	2.18	2.10	1.57
6	5.65	3.51	2.36	2.06	1.98	1.83	1.70	1.67	1.67	1.70	1.61	1.33
7	3.73	2.56	1.85	1.57	1.53	1.47	1.45	1.39	1.38	1.39	1.36	1.20
8	2.42	1.72	1.42	1.35	1.30	1.26	1.25	1.22	1.20	1.22	1.21	1.12
9	1.97	1.48	1.23	1.19	1.15	1.16	1.15	1.13	1.13	1.11	1.11	1.07
10	1.45	1.24	1.14	1.11	1.09	1.09	1.08	1.07	1.07	1.07	1.07	1.04
11	1.28	1.13	1.07	1.07	1.06	1.04	1.04	1.04	1.04	1.04	1.04	1.03
12	1.16	1.05	1.04	1.03	1.03	1.03	1.02	1.02	1.02	1.02	1.02	1.01
13	1.08	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
14	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
15	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 38. Average run length of Gamma with $+2\sigma$ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	42.63	22.19	11.94	9.05	6.93	6.20	6.16	5.46	5.24	4.97	4.80	2.32
3	17.65	7.88	4.09	3.30	2.70	2.70	2.44	2.26	2.21	2.19	2.16	1.47
4	6.45	3.10	2.08	1.76	1.65	1.61	1.54	1.49	1.45	1.42	1.37	1.19
5	2.87	1.72	1.38	1.28	1.23	1.23	1.20	1.20	1.16	1.17	1.18	1.08
6	1.92	1.31	1.15	1.09	1.09	1.08	1.07	1.07	1.06	1.06	1.05	1.03
7	1.23	1.07	1.04	1.03	1.02	1.03	1.02	1.02	1.02	1.02	1.02	1.01
8	1.06	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 39. Average run length of Gamma with +2.5σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	20.41	9.65	5.66	3.98	2.93	2.86	3.09	2.51	2.38	2.28	2.18	1.42
3	5.71	3.06	1.77	1.63	1.40	1.38	1.33	1.28	1.29	1.30	1.23	1.10
4	2.07	1.35	1.16	1.12	1.08	1.08	1.07	1.06	1.05	1.05	1.06	1.02
5	1.20	1.06	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 40. Average run length of Gamma with +3σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	10.29	5.13	2.78	1.95	1.78	1.72	1.54	1.44	1.42	1.41	1.42	1.12
3	2.35	1.39	1.13	1.08	1.06	1.05	1.04	1.05	1.04	1.03	1.04	1.01
4	1.07	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 41. Average run length of Gamma with -0.5σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.97	3.51	7.96	12.94	17.94	21.60	24.20	30.03	29.15	31.85	37.52	90.64
3	2.18	4.07	8.61	12.66	15.56	19.04	21.09	23.07	27.60	26.49	27.18	60.68
4	2.33	4.27	8.22	11.27	13.84	14.55	18.21	18.14	20.20	21.40	21.54	43.89
5	2.40	4.29	7.43	10.17	11.67	12.99	14.68	14.87	15.82	16.91	18.52	33.40
6	2.42	4.06	7.05	8.80	9.60	11.18	11.93	14.02	13.47	15.12	14.12	26.36
7	2.43	4.03	6.53	7.79	8.89	9.25	11.29	11.30	12.12	12.19	12.15	21.38
8	2.42	3.81	5.65	6.90	8.02	8.38	9.56	9.15	10.28	11.05	11.21	17.73
9	2.37	3.58	5.20	6.37	7.26	7.76	8.22	8.26	8.30	8.96	9.92	14.97
10	2.31	3.38	5.06	5.56	6.18	7.10	6.79	7.61	7.69	7.65	7.70	12.82
11	2.24	3.25	4.60	5.13	5.75	6.00	6.25	6.88	7.36	6.66	7.21	11.13
12	2.18	3.12	4.27	5.12	5.02	5.92	5.99	6.25	6.37	6.58	6.62	9.76
13	2.16	2.94	3.98	4.50	5.01	5.39	5.34	5.71	5.69	5.79	6.30	8.65
14	2.06	2.75	3.73	4.12	4.53	5.01	5.14	5.44	5.46	5.41	5.63	7.73
15	2.02	2.69	3.43	3.99	4.09	4.45	4.62	4.70	4.55	4.75	5.02	6.96
16	1.94	2.51	3.23	3.70	3.91	4.08	4.22	4.47	4.58	4.52	4.73	6.30
17	1.94	2.44	2.99	3.50	3.62	3.88	3.91	3.87	4.18	4.32	4.37	5.75
18	1.90	2.32	2.89	3.13	3.39	3.61	3.71	3.82	3.87	3.99	3.95	5.27
19	1.81	2.25	2.80	2.99	3.11	3.24	3.55	3.62	3.44	3.48	3.76	4.86
20	1.78	2.19	2.65	2.82	2.97	3.07	3.31	3.41	3.45	3.45	3.37	4.50
21	1.73	2.07	2.53	2.73	2.69	3.06	3.03	3.14	3.33	3.10	3.11	4.18
22	1.70	2.02	2.43	2.65	2.64	2.83	2.83	2.95	2.98	3.21	2.98	3.90
23	1.65	1.97	2.28	2.53	2.67	2.70	2.70	2.85	2.79	2.95	3.06	3.66
24	1.63	1.89	2.24	2.41	2.49	2.64	2.59	2.68	2.88	2.67	2.71	3.44
25	1.61	1.86	2.18	2.29	2.38	2.49	2.58	2.55	2.69	2.62	2.68	3.24
26	1.57	1.81	2.07	2.20	2.36	2.29	2.33	2.44	2.51	2.58	2.53	3.07
27	1.54	1.75	1.96	2.13	2.17	2.26	2.32	2.40	2.34	2.45	2.41	2.91
28	1.51	1.72	1.92	2.12	2.14	2.18	2.26	2.42	2.31	2.29	2.36	2.77
29	1.48	1.71	1.88	1.97	2.08	2.10	2.08	2.19	2.29	2.23	2.24	2.64
30	1.45	1.65	1.82	1.87	1.95	2.02	2.09	2.08	2.06	2.15	2.26	2.52

Table 42. Average run length of Gamma with -1σ mean shift to left

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.32	1.64	2.42	3.11	3.81	4.32	4.74	5.34	5.44	6.17	6.13	17.73
3	1.30	1.61	2.16	2.67	3.05	3.39	3.86	4.04	4.04	4.37	4.37	9.76
4	1.28	1.54	1.96	2.31	2.51	2.80	2.88	3.02	3.21	3.37	3.38	6.30
5	1.25	1.45	1.77	2.00	2.24	2.31	2.44	2.53	2.77	2.75	2.81	4.50
6	1.22	1.39	1.61	1.78	1.92	1.96	2.07	2.15	2.18	2.23	2.31	3.44
7	1.19	1.33	1.51	1.64	1.71	1.83	1.83	1.92	1.93	2.00	2.04	2.77
8	1.16	1.28	1.41	1.50	1.55	1.64	1.68	1.69	1.73	1.79	1.79	2.32
9	1.14	1.23	1.33	1.40	1.46	1.53	1.52	1.56	1.57	1.59	1.59	2.00
10	1.12	1.20	1.28	1.34	1.36	1.40	1.42	1.45	1.44	1.46	1.51	1.77
11	1.10	1.16	1.22	1.27	1.30	1.32	1.34	1.36	1.35	1.39	1.39	1.60
12	1.09	1.13	1.19	1.22	1.24	1.26	1.28	1.27	1.31	1.30	1.31	1.47
13	1.08	1.11	1.16	1.18	1.20	1.22	1.21	1.23	1.22	1.24	1.26	1.37
14	1.06	1.09	1.13	1.15	1.16	1.18	1.18	1.18	1.19	1.20	1.21	1.30
15	1.05	1.08	1.11	1.12	1.14	1.13	1.15	1.16	1.16	1.16	1.16	1.24
16	1.05	1.07	1.09	1.10	1.11	1.12	1.12	1.13	1.12	1.13	1.13	1.19
17	1.04	1.06	1.07	1.08	1.08	1.09	1.10	1.10	1.10	1.10	1.10	1.15
18	1.03	1.05	1.06	1.07	1.07	1.08	1.08	1.08	1.08	1.09	1.09	1.12
19	1.03	1.04	1.05	1.06	1.06	1.06	1.07	1.07	1.07	1.07	1.07	1.10
20	1.02	1.03	1.04	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.06	1.08
21	1.02	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.05	1.05	1.05	1.06
22	1.02	1.02	1.03	1.03	1.03	1.04	1.03	1.04	1.04	1.04	1.03	1.05
23	1.01	1.02	1.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.04
24	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.03
25	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
26	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.01	1.02	1.02	1.01	1.02
27	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
28	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
29	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
30	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01

Table 43. Average run length of Gamma with -1.5σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.14	1.23	1.45	1.63	1.80	1.92	2.07	2.16	2.26	2.39	2.44	5.27
3	1.10	1.18	1.31	1.42	1.50	1.55	1.63	1.69	1.71	1.78	1.80	2.91
4	1.08	1.13	1.21	1.28	1.31	1.35	1.39	1.42	1.44	1.45	1.47	2.00
5	1.06	1.09	1.14	1.17	1.21	1.22	1.26	1.25	1.27	1.28	1.29	1.57
6	1.04	1.07	1.10	1.12	1.13	1.14	1.16	1.16	1.18	1.18	1.19	1.33
7	1.03	1.05	1.07	1.08	1.09	1.09	1.10	1.10	1.11	1.10	1.13	1.20
8	1.02	1.03	1.04	1.05	1.05	1.06	1.06	1.07	1.07	1.07	1.08	1.12
9	1.02	1.02	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.05	1.05	1.07
10	1.01	1.02	1.02	1.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.04
11	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.03
12	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
13	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 44. Average run length of Gamma with -2σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.06	1.10	1.17	1.23	1.26	1.31	1.35	1.38	1.41	1.45	1.47	2.32
3	1.04	1.06	1.09	1.12	1.14	1.15	1.16	1.19	1.19	1.20	1.21	1.47
4	1.02	1.03	1.05	1.06	1.06	1.07	1.08	1.08	1.09	1.09	1.09	1.19
5	1.01	1.02	1.02	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.08
6	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.03
7	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 45. Average run length of Gamma with -2.5σ mean shiftt

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.03	1.04	1.06	1.08	1.10	1.11	1.12	1.14	1.14	1.16	1.15	1.42
3	1.01	1.02	1.02	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.05	1.10
4	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.02
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 46. Average run length of Gamma with -3σ mean shift

n \ N	0.5	1	2	3	4	5	6	7	8	9	10	N(0,1)
2	1.02	1.02	1.02	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.05	1.12
3	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00