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1. Introduction

1.1. Research Background and Motivation

Process capability indices (PCIs) which provide numerical measure of production characteristic to reflect the quality of product have been used in the manufacturing industry. Many authors have promoted the use of various PCIs for evaluating a supplier's process capability. Examples include Boyles (1991), Pearn *et al.* (1992), Kotz and Johnson (1993), Vännman and Kotz (1995), Vännman (1997), Kotz and Lovelace (1998), Pearn *et al* (1998), Kotz and Johnson (2002), Pearn and Shu (2003) and reference therein.

The first process capability index C_p was developed by Kane (1986), which considers the overall process variability relative to the manufacturing tolerance to measure process precision. Due to simplicity of the design, C_p can not reflect the tendency of process centering. The index C_p was defined:

$$C_p = \frac{\text{USL-LSL}}{6\sigma}, \tag{1}$$

where USL is the upper specification limit, LSL is the lower specification limit and σ is the process standard deviation. To measure the degree of process centering, Pearn *et al.* (1998) introduced the following accuracy index C_a:

$$C_a = 1 - \frac{|\mu - \text{m}|}{d}, \tag{2}$$

where μ is the process mean, d= (USL-LSL)/2, and m= (USL+LSL)/2.The index C_a measure the centering tendency, which alerts the user if the process mean deviate from its midpoint. The C_{pk} index considers process variation and the location of process mean

$$C_{pk} = \min\{\text{C}_{\text{pu}}, \text{C}_{\text{pl}}\} = \min\left\{\frac{\text{USL - }\mu}{3\sigma}, \frac{\mu\text{ - LSL}}{3\sigma}\right\} = \frac{d - |\mu - \text{m}|}{3\sigma}. \tag{3}$$

The index C_{pk} does not determine the probability of nonconformance. It does limit nonconformance, and in fact, the probability of nonconformance is never more than $2\Phi(-3C_{pk})$. Taguchi on the other hand, emphasize the product loss when one of its characteristic departs from the target value *T*. Hsiang and Taguchi (1985) introduced the index C_{pm}, which was also proposed independently by Chan *et al* (1988). The index C_{pm} incorporates with the variation of production items with respect to the target value and specification limits preset in the factory. It was defined as

$$C_{pm} = \frac{\text{USL-LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{4}$$

1.2. Research Purpose and Objectives

Ever since Motorola, Inc. introduced its Six Sigma quality initiative, quality practitioners have questioned why followers of this philosophy notion add 1.5σ when estimating process capability. When asked the reason for such an adjustment, six-sigma user claim it is necessary, but offer only personal experiences and three dated empirical literature. Bothe (2002) provides a statistical reason to the issue. The control chart doesn't detect small movement in process mean obviously. Since the practitioners may not detect the process mean shift when it, user estimate process capability optimistically. Bothe (2002) thinks the adjustment of mean shift is under the same detection powerbring 50%. The data of Bothe's study is assumed to be approximately normality distribution. The control chart's subgroup and shift distance could influence the detection power that warn user of the movement of mean. We calculate various adjustments for different subgroup size detection power. However non-normal process occurs frequently in practice. If the process capability indices based on the normal assumption concerning the data are used with non-normal observations, the value of the process capability indices may, in a majority of situation, be incorrect and quite likely misrepresent the actual product quality.

Pyzdek (1995) has mentioned the distributions of certain chemical processes such as zinc plating thickness of a hot-dip galvanizing process are very quite often skewed. Choi (1996) presents an example of a skewed distribution in the 'active area' shaping stage of the wafer's production process. Cygan (1989), Krishnakumer and Laghari have mentioned that the lifetimes of polypropylene films under high ac and dc field stresses be shown as a two-parameter Weibull distribution. The abundance of outputs from skewed distribution, the censoring, etc, makes the normality assumption often being illegitimate. Specifically, we assure the product lifetime which be from skewed distribution by statistic test and historical data. It will lead to underrate the probability of nonconformance that using the adjustment for normal case to adjust the non-normal cases.

In this paper, we show that the detection power performance of $\overline{X}$ control chart under the Bothe' adjustment when the process in control is very sensitive to the assumption of normality. We provide the statistical derived mean shift adjustment based on the chart subgroup size and distribution parameter to calculate the estimator of C_{pk} when the data is non-normal distribution for Weibull distribution, Beta distribution and F distribution. Because the most non-normal distributions do not have reproductive, we use reference to approximate the c.d.f of $\overline{X}$ and distribution ($\overline{X}=\sum_{i=1}^{n} X_i \big/ n$, where $X_i \sim \text{Weibull distribution}$). We use Matlab program simulate the non-normal data for Calculate mean shift adjustment. based on the same detection power with control chart mean shift consideration.

1.3. Research Organization

First, we introduce the research motivation and purpose in Chapter 1. Secondly, a brief introduction of Bothe's study and adjustment reason are included in Chapter 2. In Chapter 3, we introduce the characteristic of Weibull distribution and calculate the c.d.f of $\overline{X}$ and distribution. Then, we define the adjustment and compare the result with normal process. After that we calculate the adjustment for Weibull process and introduce the calculation of dynamic non-normal index C_{pk}. In Chapters 4 and 5, we introduce the characteristic of Beta and F distribution. Then, we use the MATLAB program to simulate the mean shift adjustment for beta and F processes. For illustrative purpose, an application of Printed Circuit Boards (PCBs) weld is presented in Chapter 6. Finally, we give some conclusion in Chapter 7.

2. Process Capability Adjustment for Normal Process

2.1. Detection Power Analysis

Bothe (2002) provides a statistical reason why to add a 1.5σ shift to the process mean. Assuming the process approximately normal distribution, control charts can't reliably detect small movement in process mean. It is hard to detect small movement in process. Table 1 displays the probabilities of detecting changes in μ (process mean) versus subgroup size for shift=0.5(0.5)3σ with n=3, 4 and 5. When μ had a small movement (ex: 0.5σ, 1σ), the detection power is too small. Then, small mean movement affects the PCIs accuracy. However, the probability of nonconformance will increase obviously. For example, when C_{pk} is 1.33, the probability of nonconformance is 64ppm (parts per million). If process mean has 1σ shift, it is difficult to detecte by control chart, the probability of nonconformance becomes 1350ppm. The probability of nonconformance will increase twenty-fold. Bothe (2002) considered the adjustments according with the same detection power(=0.5).

Table 1. Probabilities of detecting changes in μ versus subgroup size.

Shift in μ	Subgroup Size		
	3	4	5
0.5σ	0.0164	0.0228	0.0299
1σ	0.1024	0.1587	0.2225
1.5σ	0.3439	0.5000	0.6384
2σ	0.6787	0.8413	0.9295
2.5σ	0.9083	0.9772	0.9952
3σ	0.9860	0.9986	0.9999

2.2. Bothe's Consideration for 1.5-sigma Adjustment

When subgroup size is four and mean shift is 1.5σ, the detection power will be 0.5. Bothe (2002) considers providing the same detection power in order to define the several adjustments with different subgroup size which were called S_{50}. By this idea, he computed the several adjustments for different subgroup size and showed in Table 2. Table 2 displays shift sizes that have 50 percent chance of remaining undetected for subgroup sizes 1 through 6. Because shifts ranging in size from 0 up to $S_{50}\sigma$ are the ones likely to remain undetected, a conservative approach is to assume that every missed shift is as large as $S_{50}\sigma$. And Bothe (2002) invented dynamic C_{pk} be defined as

$$
\begin{aligned}
Dynamic\ \hat{C}_{pk} &= \min[\frac{(\hat{u} - S_{50}\hat{\sigma}) - LSL}{3\hat{\sigma}}, \frac{USL - (\hat{u} + S_{50}\hat{\sigma})}{3\hat{\sigma}}] \\
&= \min[\frac{(\hat{u} - LSL - S_{50}\hat{\sigma}}{3\hat{\sigma}}, \frac{USL - \hat{u} - S_{50}\hat{\sigma})}{3\hat{\sigma}}].
\end{aligned} \tag{5}
$$

The dynamic C_{pk} could be corrected by subgroup size really not fixed 1.5σ adjustment.

Table 2. Adjustment values for normal distribution with several subgroup size.

Subgroup Size	S_{50}
1	3
2	2.12
3	1.73
4	1.5
5	1.34
6	1.22

3. Process Capability Adjustment for Weibull Process

3.1. The Weibull Distribution

In this section, we investigate Weibull distribution that has been often used in the field of life data analysis due to its flexibility. It can mimic the behavior of other statistical distributions such as the normal and the exponential. The Weibull distributions are also used to model the time until a given technical device fails. If the failurerate of the device decreases over time, one chooses $\beta < 1$ (β is the shape parameter). If the failure rate of the device is constant over time, one choose $\beta = 1$, again resulting in a decreasing function *f*. If the failure rate of the device increases over time, one chooses $\beta > 1$ and obtains a density f which increases towards a maximum and then decreases forever.

The Weibull distribution is non-negative distribution. The Weibull distribution can be denoted as Weibull (α, β) with scale parameter α and shape parameter β. The cumulative density function and the probability density function are defined as

$$F(X) = 1\text{-e}^{-(x/\alpha)^{\beta}}, x > 0, \alpha > 0, \beta > 0, \tag{6}$$

and

$$f(x) = \beta\alpha^{-\beta}x^{\beta-1}e^{-(\frac{x}{\alpha})^{\beta}}, x > 0, \alpha > 0, \beta > 0, \tag{7}$$

and the mean and variance are given respective by

$$E(X) = \alpha[\Gamma(1+\beta^{-1})], \tag{8}$$

and

$$\text{V(X)} = \alpha^{2}[\Gamma(1+2\beta^{-1}) - \Gamma^{2}(1+\beta^{-1})], \tag{9}$$

It should be denote that the Weibull distribution is skewed. To know how this distribution are different from the normal distribution in term of the coefficient of skewness and the kurtosis of coefficient, Table 3 presents the coefficient of skewness and the coefficient of kurtosis of the Weibull distribution under study. The coefficient of skewness Weibull distribution is given by:

$$\gamma_1 = \frac{2\Gamma^{3}(1+\beta^{-1}) - 3\Gamma(1+\beta^{-1})\Gamma(1+2\beta^{-1}) + \Gamma(1+3\beta^{-1})}{[\Gamma(1+2\beta^{-1}) - \Gamma^{2}(1+\beta^{-1})]^{3/2}}, \tag{10}$$

and the kurtosis coefficient of Weibull distribution is given by:

$$\gamma_2 = \frac{f(\beta)}{[\Gamma(1+2\beta^{-1}) - \Gamma^{2}(1+\beta^{-1})]^{2}}, \tag{11}$$

where $\Gamma(x)$ is the gamma function and

$$f(\beta) \equiv -6\Gamma^4(1+\beta^{-1}) + 12\Gamma^2(1+\beta^{-1})\Gamma(1+2\beta^{-1}) - \\ 3\Gamma^2(1+2\beta^{-1}) - 4\Gamma(1+\beta^{-1})\Gamma(1+3\beta^{-1}) + \Gamma(1+4\beta^{-1}). \tag{12}$$

The formula of these modulus let us know that α is scale parameter and β is shape parameter. To make short of the matter, scale parameter can modulate the fold of the mean and the variance. Figure 1 displays Weibull distribution with various values of α. We can find the scale parameter only control the mean and the variance to adjust the distribution size.

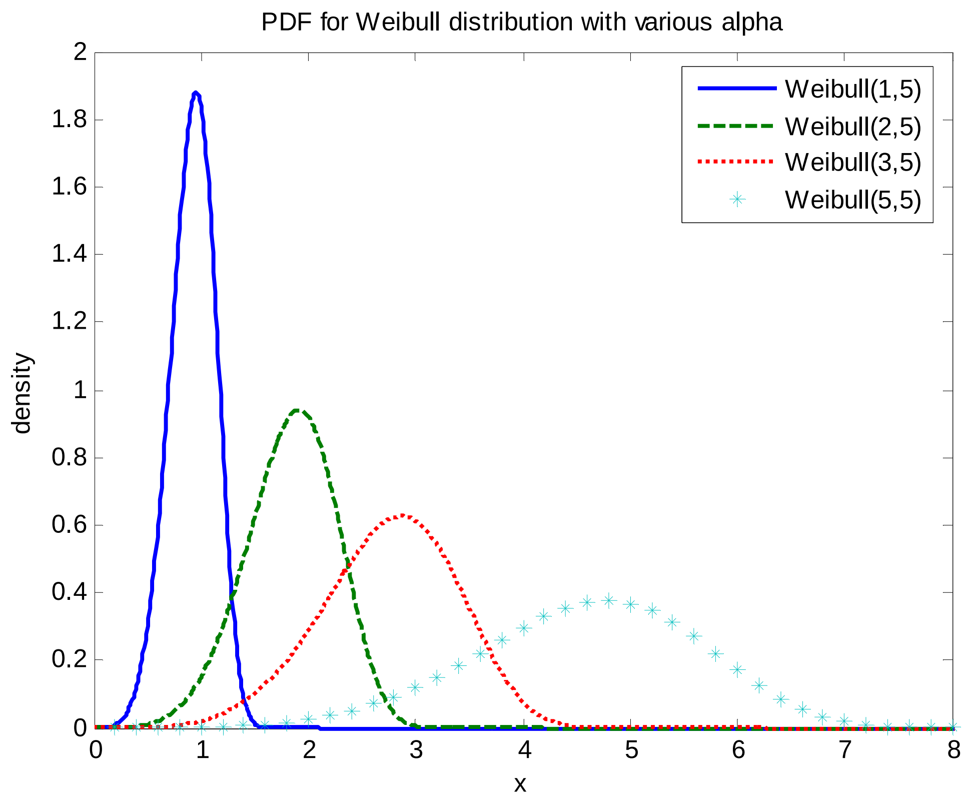

Figure 1. Weibull distribution with various α

The Equations (10), (11) and (12) show that skewness coefficient and the kurtosis coefficient only depend on shape parameter. Figure 2 displays Weibull distribution with various β.

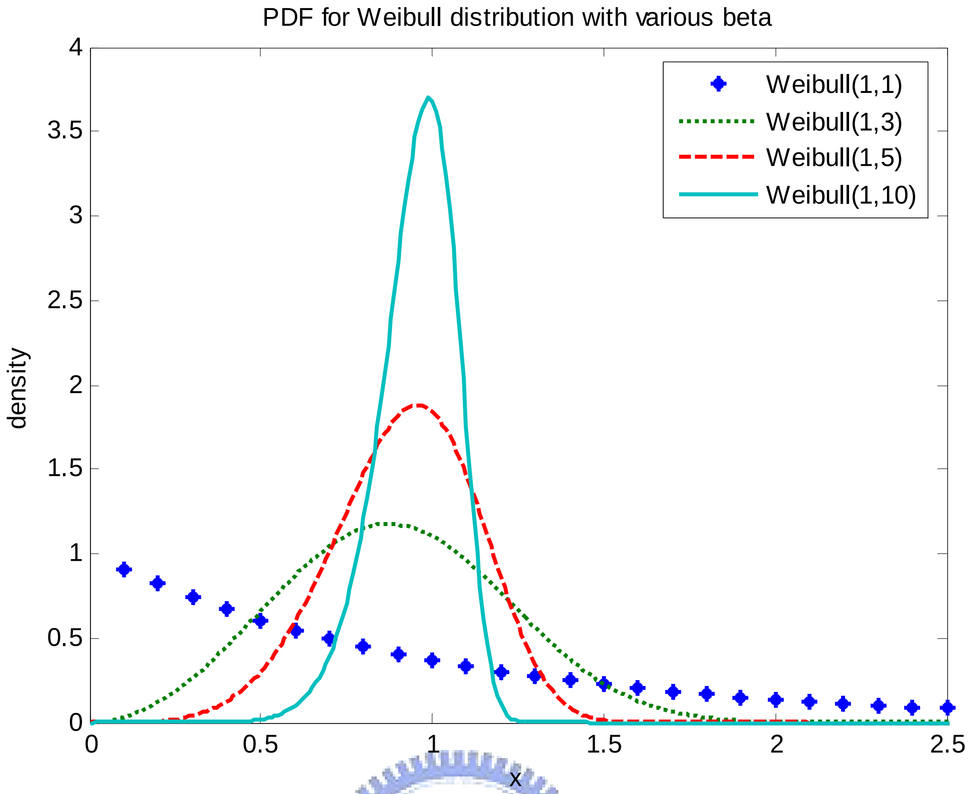

Figure 2 Weibull distribution with various β

Table 3. The M.G.F. of Weibull distribution for several β.

Weibull(α,β)	mean	variance	skewness	Kurtosis
Normal(0,1)	0	1	0	0
Weibull(1,0.5)	2	20	6.61876	84.72
Weibull(1,1)	1	1	2	6
Weibull(1,2)	0.886227	0.214602	0.631111	0.245089
Weibull(1,3)	0.89298	0.105333	0.168103	-0.27054
Weibull(1,3.6)	0.901106	0.27802	0	-0.283255
Weibull(1,4)	0.906402	0.0646615	-0.087237	-0.25217
Weibull(1,5)	0.918169	0.04423	-0.25411	-0.11971
Weibull(1,6)	0.927719	0.0323163	-0.373262	0.035455
Weibull(1,7)	0.935438	0.0247037	-0.46319	0.187183
Weibull(1,8)	0.941743	0.0195232	-0.533726	0.327676
Weibull(1,9)	0.946965	0.0158299	-0.590657	0.455204
Weibull(1,10)	0.951351	0.0131005	-0.637637	0.570166

When the shape parameter increases, the results in Table 3 also indicate that the corresponding values of skewness coefficient becomes small. It is mean the skewness coefficient of the Weibull distribution turn positive skewness into negative skewness when the shape parameter increases. Especially, when shape parameter is 3.6, the skewness coefficient of the Weibull distribution is almost zero. This means distribution is symmetric about median and similar normal distribution. With the shape parameter increasing, the corresponding values of kurtosis coefficient will become large in addition to fall into [0, 3.36].

From Figure 3, we know the Weibull p.d.f. shape is more similar normal distribution while the shape parameter β is more than 2. The kurtosis coefficient of Weibull distribution center with increasing β, the shape is leptokurtic and has a more acute "peak" leptokurtic. By the Equation (10), we would consider the Weibull distribution be most similar to normality distribution when β fall into [3, 4]. The form of Weibull distribution become centralizing and being left skew when the shape parameter exceeds five.

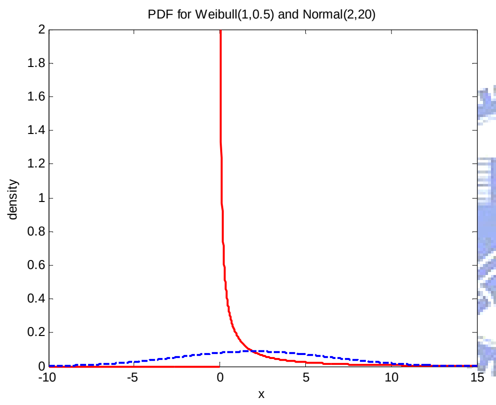

Figure 3(a). Probability density function for Weibull(1,0.5) and N(2,20)

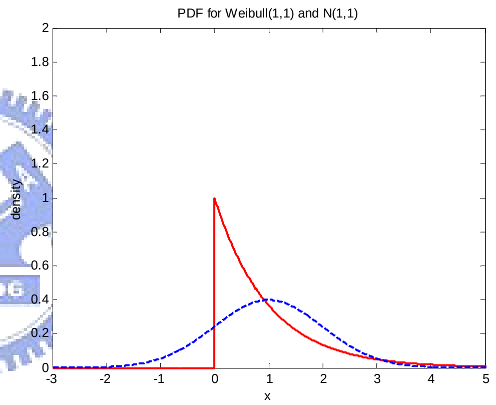

Figure 3(b). Probability density function for Weibull(1,1) and N(1,1)

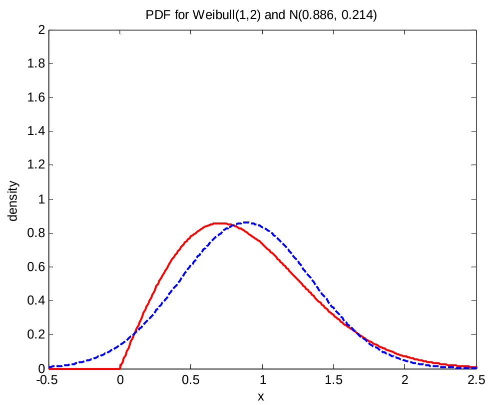

Figure 3(c). Probability density function for Weibull(1,2) and N(0.8862, 0.2146)

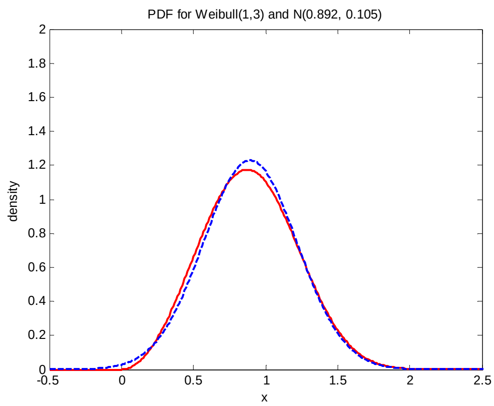

Figure 3(d). Probability density function for Weibull(1,3) and N(0.892, 0.1053)

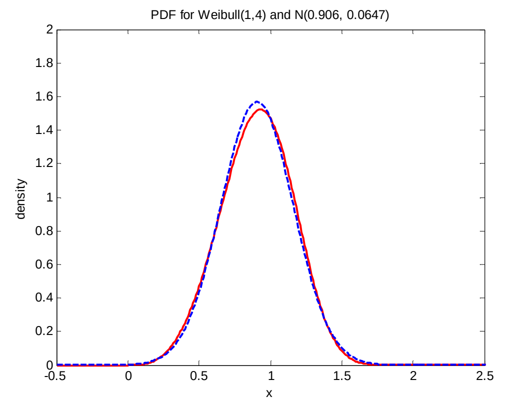

Figure 3(e). Probability density function for Weibull(1,4) and N(0.9064, 0.0646)

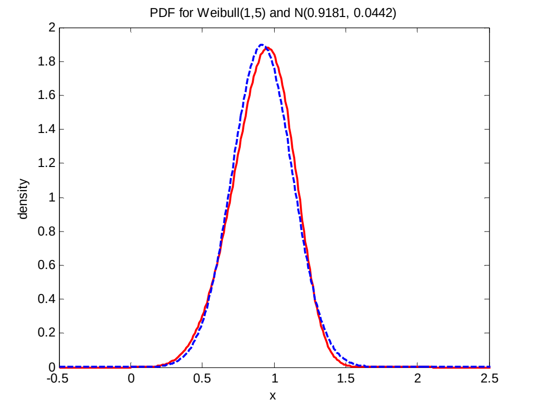

Figure 3(f). Probability density function for Weibull(1,5) and N(0.9181, 0.0442)

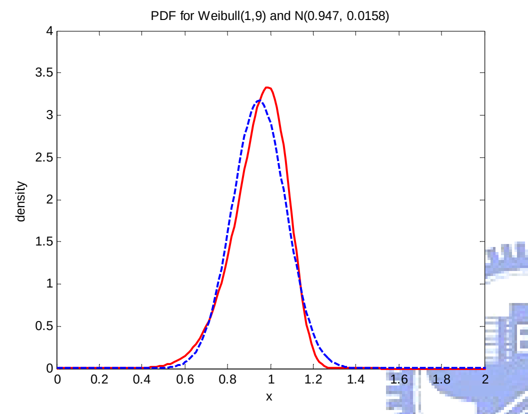

Figure 3(g). Probability density function for Weibull(1,9) and N(0.9469, 0.0158)

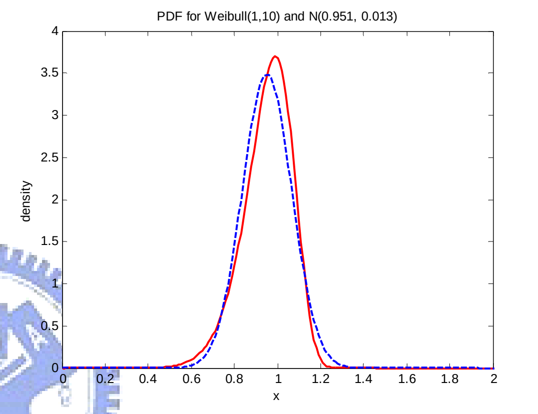

Figure 3(h). Probability density function for Weibull(1,10) and N(0.9513, 0.0131)

3.2. The Sampling Distribution of Weibull Process

Because the Weibull distribution don't have reproductive, we can't know the c.d.f. of $\overline{X}$ distribution by the parameter. We use a reference to approximate the $\overline{X}$ of Weibull distribution. By the reference Lu (2003), we can use three quantile to computation $\overline{X}$ of Weibull distribution which is continuous and interests us. While X_i is a random variable from Weibull distribution ($\alpha = 1,\ \beta$), by transformation variable setting $Y_i = X_i^{\beta}$, then Y_i will be a random variable from exponential distribution (1). We use $Y_1 + Y_2 + \cdots + Y_n$ distribution that we realized to approximate $X_1 + X_2 + \cdots + X_n$ which we interests. We define as

$$P(X_1 + \cdots + X_n \le t) = P(Y_1 + \cdots + Y_n \le w(t)), \qquad (13)$$

where

$$w(t) = a * t^{3\beta} + b * t^{2\beta} + c * t^{3\beta}. \qquad (14)$$

As a result of $w(t)$ having three unknown parameter a b c, we use three quantile (0.1, 0.5 and 0.9) to calculate a, b, c.

$$P(X_1+\cdots+X_n \le t_p)=p \tag{15}$$

$$P(Y_1+\cdots+Y_n \le \theta_p)=p \tag{16}$$

Then, t_p is the quantile of $(Y_1+Y_2+\cdots+Y_n)$. θ_p is the quantile of $X_1+X_2+\cdots+X_n$ equally. We use equation $a*t_p^{\;3\beta}+b*t_p^{\;2\beta}+c*t_p^{\;\beta}=\theta_p$, and setting quantile =0.1, 0.5 and 0.9 to get three linear equations. We can solve the three linear equation to receive the a,b,c parameter and find $w(t)$. Then we calculate the cumulative distribution function of $\overline{X}$

STEP 1: When Y_i is a random variable from exponential ($\alpha=1$) we can use the Poisson distribution concept to find the equation as:

$$P(Y_1+\cdots+Y_n \le \theta_p)=1-\sum_{i=0}^{n-1} e^{-\theta_p} * \theta_p^i / i!. \tag{17}$$

To set p=0.1 0.5 and 0.9, substitution the Equation (18). Then we will receive $\theta_{0.1}, \theta_{0.5}$ and $\theta_{0.9}$

$$1-\sum_{i=0}^{n-1} e^{-\theta p} * \theta_p^i / i! = p. \tag{18}$$

STEP 2: We simulate the large sample, let $S_1, S_2 ....., S_N$ be identical and independent sample from $F_n(x)=P(X_1+\cdots+X_n \le x)$ and to sort S_i, we obtain $S_{(1)}, S_{(2)}, \ldots, S_{(N)}$, so $\hat{t}_p = S_{(N*p)}$, where n is the group size, X_i be identical and independent from Weibull $(1,\beta)$.For example $N=10^7$, $\hat{t}_{0.1}=S_{(10^6)}$, $\hat{t}_{0.5}=S_{(5*10^6)}$, $\hat{t}_{0.9}=S_{(9*10^6)}$

STEP 3: We get $\theta_{0.1}$, $\theta_{0.5}$, $\theta_{0.9}$, $\hat{t}_{0.1}$, $\hat{t}_{0.5}$ and $\hat{t}_{0.9}$ to solve these Equations mentioned above $\hat{a}, \hat{b}$ and $\hat{c}$. Then we can estimate $w(t)$ with given n and β by the Equation (20).

$$\begin{cases} a\hat{t}_{0.1}^{3\beta}+b\hat{t}_{0.1}^{2\beta}+c\hat{t}_{0.1}^{\beta}=\theta_{0.1} \\ a\hat{t}_{0.5}^{3\beta}+b\hat{t}_{0.5}^{2\beta}+c\hat{t}_{0.5}^{\beta}=\theta_{0.5} \\ a\hat{t}_{0.9}^{3\beta}+b\hat{t}_{0.9}^{2\beta}+c\hat{t}_{0.9}^{\beta}=\theta_{0.9} \end{cases} \tag{19}$$

$$\hat{w}(t)=\hat{a}*t^{3\beta}+\hat{b}*t^{2\beta}+\hat{c}*t^{3\beta} \tag{20}$$

STEP 4: The interested distribution of $X_1 + X_2 + \cdots + X_n$ could calculate by the Equation (21). Then the $\overline{X}$ of Weibull distribution could approximate as Equation (22) the same.

$$\begin{aligned} P(X_1 + \cdots + X_n \le t) &= P((X_1 + X_2 + \cdots + X_n)/\alpha) \le t/\alpha) \\ &= P(Y_1 + Y_2 + \cdots + Y_n \le t/\alpha) \end{aligned} \tag{21}$$

$$P(\overline{X} \le t/n) = P(X_1/n + \cdots + X_n/n \le \hat{w}(t/n)) \tag{22}$$

STEP 5: How to do the Weibull distribution when $\alpha \ne 1$ .When $X_1, X_2 \ldots, X_n$ are random variable from Weibull distribution (α, β). The cumulative distribution Equation of X_i be given as

By transformation set $Y = X/\alpha$, Y_i is a random variable from Weibull distribution $(1, \beta)$.

$$\begin{aligned} P(X_1 + \cdots + X_n \le t) &= P((X_1 + X_2 + \cdots + X_n)/\alpha) \le t/\alpha) \\ &= P(Y_1 + Y_2 + \cdots + Y_n \le t/\alpha) \end{aligned} \tag{23}$$

and

$$\begin{aligned} P(\overline{\mathrm{X}}_{\mathrm{n}} \le t) &= P((X_1 + X_2 + \cdots + X_n)/(\alpha * \mathrm{n})) \le t/(\alpha * \mathrm{n})) \\ &= P(Y_1 + Y_2 + \cdots + Y_n \le t/(\alpha * \mathrm{n})) \end{aligned} \tag{24}$$

By the Equation (24), we can find the cumulative distribution Equation of $\overline{X}_n$ when X_i is a random variable from Weibull distribution (α, β).

3.3. Estimating the Parameters of Weibull Process

If the random variable $X_1, X_2 \ldots, X_n$ are independently and identical Weibull distribution with unknown α and β. We will estimate α and β by MLE (Maximum likelihood function)

$$\hat{\alpha} = [\frac{1}{n}\sum_{i=1}^{n} \mathrm{X}_i^{\hat{\beta}}]^{1/\hat{\beta}}, \tag{25}$$

and

$$\frac{1}{\hat{\beta}} = \frac{\sum_{i=1}^{n} \mathrm{X}_i^{\hat{\beta}} \ln \mathrm{X}_i}{\sum_{i=1}^{n} \mathrm{X}_i^{\hat{\beta}}} - \frac{1}{n}\sum_{i=1}^{n} \ln \mathrm{X}_i. \tag{26}$$

Using Equation (25) and (26), we can estimate β and α parameter when data be from Weibull distribution with unknown parameter.

3.4. The Detection Power for Weibull Process Under 1.5-sigma Adjustments

The major purpose of individual control chart is assisting on identifying shifts and drifts in processes it's easily to be implemented. We use the non-normal control chart to detect the power of Weibull distribution under Bothe' adjustments. In the non-normal process control chart, the UCL (upper control limits) is the 99.865 percentile of $\overline{X}_n$ distribution when X_i is random variable form non-normal distribution. The LCL (lower control limits) is the 0.135 percentile of $\overline{X}_n$ distribution when X_i be random variable form non-normal distribution.

Because the shape of the Weibull distribution changing from positive skewness to negative skewness with increasing the shape parameter, we will discuss two different case. Process occurs right shifting and left shifting in control chart. The detective power be defined the probability of outline control chart under the mean being shifted. Its mean 1-(type II error) the same. We suppose μ occurring right movement and left movement. When k>0, it is mean μ occurring right movement; and μ occurring left movement when k<0.

$$\begin{aligned}\text{Detection power} &= 1\text{-}P(\text{LCL} \le \overline{X}_n \le \text{UCL} \mid u_1 = u_0 + k\sigma_x) \\ &= 1\text{-}P(\overline{X}_{(0.00135)} \le \overline{X}_n \le \overline{X}_{(0.99865)} \mid u_1 = u_0 + k\sigma_x).\end{aligned} \quad (27)$$

Table 4 and 5 display the detection power with right shift and left shift when data come from Weibull distribution with $\alpha = 1$ and $\beta = 1\ (1)\ 10$. The shifted distance in the second column is the Bothe' adjustments determined when data come from normal distribution and the detection power was fixed to 50 percent. From Table 4 and 5, we observe the detection power closed to 0.5 under sample size (n) increasing. This means that the more the shape is similar to normal distribution, the more the result is close to normal distribution. The Weibull distribution is most similar to the normality distribution when shape parameter close to 3.6. When the shape of Weibull distribution is positive skewness with right shift, detection powers are smaller than normal distribution. it causes detection power under 50 percent that using Bothe' adjustments substituted for Weibull distribution.

Table 4. Detection power of various Weibull distribution for k>0.

n	Shift σ	Weibull distribution(1, β) for right shift										
		N(0,1)	$\beta=1$	$\beta=2$	$\beta=3$	$\beta=4$	$\beta=5$	$\beta=6$	$\beta=7$	$\beta=8$	$\beta=9$	$\beta=10$
2	2.12	0.5	0.054	0.309	0.525	0.687	0.747	0.785	0.807	0.822	0.833	0.841
3	1.73	0.5	0.091	0.347	0.524	0.664	0.726	0.760	0.782	0.796	0.809	0.815
4	1.5	0.5	0.099	0.375	0.516	0.646	0.699	0.735	0.756	0.775	0.784	0.793
5	1.34	0.5	0.119	0.378	0.514	0.626	0.681	0.712	0.738	0.752	0.764	0.775
6	1.22	0.5	0.141	0.389	0.509	0.614	0.668	0.696	0.719	0.734	0.747	0.755

Table 5. Detection power of various Weibull distribution for k<0.

n	Shift	Weibull distribution(1, β) for left shift										
	σ	N(0,1)	$\beta=1$	$\beta=2$	$\beta=3$	$\beta=4$	$\beta=5$	$\beta=6$	$\beta=7$	$\beta=8$	$\beta=9$	$\beta=10$
2	2.12	0.5	0.928	0.782	0.550	0.513	0.439	0.387	0.350	0.323	0.304	0.288
3	1.73	0.5	0.906	0.733	0.537	0.506	0.449	0.411	0.384	0.364	0.348	0.337
4	1.5	0.5	0.886	0.702	0.532	0.505	0.458	0.426	0.404	0.385	0.375	0.365
5	1.34	0.5	0.868	0.680	0.527	0.504	0.464	0.436	0.416	0.401	0.390	0.381
6	1.22	0.5	0.852	0.664	0.525	0.504	0.467	0.441	0.424	0.411	0.401	0.393

3.5. The Modified Mean Adjustments for Weibull Process

We set a given sample size (n) $\alpha = 1$ and given β, then sampling large data (10^7) which are from Weibull distribution to estimate the $\hat{t}_{0.1}$, $\hat{t}_{0.5}$ and $\hat{t}_{0.9}$. After that we find solution for $\hat{a}, \hat{b}, \hat{c}$ by Equation (19) and compute the cumulative density function of $\overline{X}$ with the given n and β.

We can use this c.d.f to compute the relationship between the mean shift and Type Ⅱ error. The mean shift adjustment AS_{50} means that the detection power is fifty percent when process mean shift AS_{50} sigma. AS_{50} is adjustment in C_{pk}. AS_{50} was calculated by how distance shift the detection power reach 50 percent. Therefore, Table 6 and Table 7 display the magnitude of mean shift adjustments AS_{50} based on the detection power is 50 percent and data from Weibull (1, β) distribution for various value of β = (0.5 and 1(1)10) and n=2(1)15 with right shift and left shift. Although we consider process mean shift with right shift and left shift, we will use worse case. For example, if β is 3 with n=3, the mean right shift adjustment AS_{50} is 1.642. In other words, we use adjustment in Table 6 when $\beta \leqq 3$; there against using adjustment in Table 7 when $\beta \geqq 4$. We conclude that a mean shift adjustment of $AS_{50} = 1.642\sigma$ is required based on the detection power is 50 percent and data come from Weibull (1, 3) distribution. When β=0.5, AS_{50} is larger than Bothe' adjustment apparently. Because the shape is extraordinarily unlike the normal distribution when β=0.5. In the same way, AS_{50} would be near to Bothe' adjustment with β=3.6.

Why not discuss the relationship between AS_{50} and scale parameter? To view the formula of skewness coefficient and kurtosis coefficient for Weibull distribution, we know scale parameter unable to affect these. So the fixed β and subgroup size can look for the AS_{50}.

Table 6. AS_{50} values for several n and various β values when k>0.

AS_{50}	Weibull distribution(1, β) for right shift										
n	0.5	1	2	3	4	5	6	7	8	9	10
2	6.167	3.611	2.492	2.009	1.767	1.632	1.536	1.470	1.424	1.387	1.359
3	4.634	2.735	1.967	1.642	1.482	1.373	1.307	1.261	1.228	1.197	1.182
4	3.755	2.250	1.663	1.448	1.309	1.232	1.175	1.138	1.103	1.087	1.071
5	3.165	1.944	1.484	1.301	1.196	1.127	1.084	1.047	1.025	1.006	0.988
6	2.869	1.716	1.343	1.201	1.104	1.043	1.009	0.981	0.960	0.942	0.932
7	2.572	1.569	1.239	1.119	1.037	0.990	0.954	0.928	0.907	0.892	0.881
8	2.317	1.440	1.159	1.051	0.984	0.939	0.905	0.883	0.864	0.852	0.839
9	2.143	1.340	1.086	0.991	0.930	0.891	0.865	0.845	0.828	0.814	0.805
10	1.970	1.251	1.031	0.943	0.889	0.853	0.828	0.811	0.797	0.784	0.773
11	1.846	1.185	0.975	0.899	0.854	0.816	0.799	0.777	0.768	0.756	0.748
12	1.720	1.110	0.932	0.858	0.820	0.787	0.767	0.752	0.741	0.729	0.722
13	1.651	1.066	0.893	0.828	0.788	0.763	0.746	0.728	0.721	0.708	0.701
14	1.539	1.021	0.861	0.801	0.762	0.737	0.723	0.709	0.696	0.688	0.684
15	1.468	0.974	0.829	0.772	0.745	0.717	0.701	0.689	0.675	0.669	0.660

Table 7. AS_{50} values for several n and various β values when k<0.

AS_{50}	Weibull distribution(1, β) for left shift										
n	0.5	1	2	3	4	5	6	7	8	9	10
2	0.203	0.820	1.532	1.888	2.098	2.236	2.333	2.405	2.461	2.504	2.540
3	0.255	0.813	1.356	1.591	1.723	1.808	1.866	1.909	1.941	1.967	1.987
4	0.285	0.802	1.225	1.399	1.494	1.554	1.596	1.626	1.649	1.667	1.681
5	0.304	0.776	1.125	1.263	1.337	1.384	1.416	1.439	1.456	1.470	1.481
6	0.316	0.749	1.047	1.160	1.221	1.259	1.285	1.304	1.318	1.329	1.338
7	0.324	0.724	0.983	1.079	1.131	1.163	1.185	1.201	1.213	1.222	1.230
8	0.329	0.700	0.929	1.013	1.058	1.086	1.105	1.118	1.129	1.137	1.144
9	0.331	0.678	0.884	0.958	0.998	1.022	1.039	1.051	1.060	1.067	1.073
10	0.333	0.658	0.844	0.911	0.947	0.969	0.984	0.994	1.003	1.009	1.014
11	0.333	0.640	0.810	0.871	0.903	0.923	0.936	0.946	0.954	0.959	0.964
12	0.333	0.623	0.780	0.835	0.865	0.883	0.895	0.904	0.911	0.916	0.921
13	0.332	0.607	0.753	0.804	0.831	0.848	0.859	0.867	0.873	0.879	0.882
14	0.331	0.593	0.728	0.776	0.801	0.816	0.827	0.834	0.840	0.845	0.848
15	0.330	0.579	0.706	0.751	0.774	0.788	0.798	0.805	0.811	0.815	0.818

Figure 4 displays power curve for various sample size. The mean of power curve is detection power with various shift sigma units for mean. For small shift in μ all curves are close to zero. As the distance of shift creasing, so does the power of chart to detect it. Three detect power are equalto1 for excess of 3.5σ.

The horizontal line drawn on this graph shows that is a 50% chance of missing a 1.94σ shifts in μ when n is 5, whereas μ must move by 2.735σ to have this same probability when n is 3.

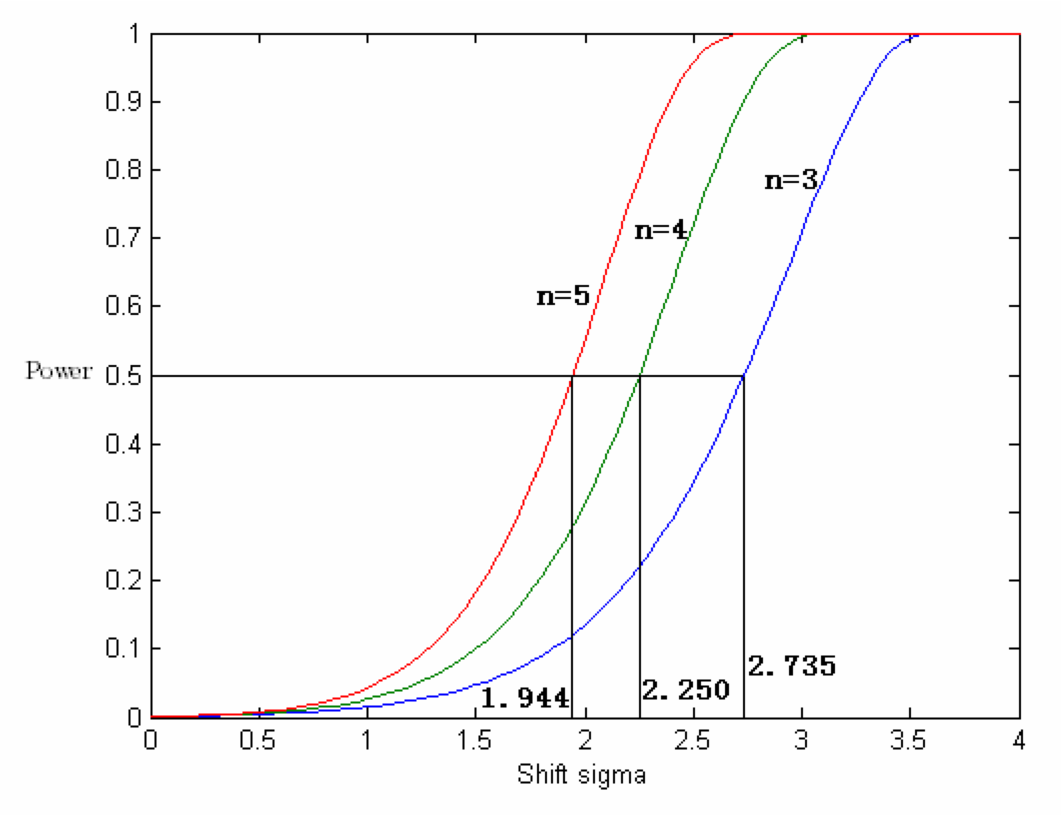

Figure 4. Power curve for subgroup size 3, 4 and 5 when $(\alpha, \beta) = (1,1)$.

3.6. The Capability Adjustment for Weibull Process

The index C_{pk} has been viewed as a yield-based index since it provides bound on the process on the process yield for a normality distribution process with a fixed value of C_{pk} which is defined as Equation (3). The purpose of process capability indices, which are statistical measures of process capability, is based on several assumptions. Two of the most important assumption is that the process monitored is supposed to be stable and the output is approximately normal distribution. When the distribution of a process characteristic is non normal, PCIs could often lead to erroneous and misleading interpretation of the process capability.

In the recent years, several approaches the problems of PCIs for the non-normal populations have been suggested. Chen and Pearn (1997) consider come generalizations of these basic capability indices to cover non-normal distribution. Where $X_{0.135}$ is the 0.135th percentile, $X_{99.865}$ is the 99.865th percentile. Since the median is usually the preferable central value for a skewed distribution, the corresponding C_{pu} and C_{pl} are defined as:

$$C_{pl} = \frac{median - \mathrm{LSL}}{median - (lower\ 0.135\%\ point)} = \frac{median - \mathrm{LSL}}{median - X_{0.135}}, \tag{29}$$

and

$$C_{pu} = \frac{\mathrm{USL} - median}{(upper\ 99.865\%\ point) - median} = \frac{\mathrm{USL} - median}{X_{99.865} - median}. \tag{30}$$

The index C_{pk} will then be calculated as the minimum of C_{pu} and C_{pl}, namely:

$$\begin{aligned} C_{pk} &= \min\{C_{pl}, C_{pu}\} \\ &= \min\{\frac{median - \mathrm{LSL}}{median - X_{0.135}}, \frac{\mathrm{USL} - median}{X_{99.865} - median}\}. \end{aligned} \tag{31}$$

Acknowledging that a process will experience shifts in $X_{0.50}$ (median) of various magnitudes and knowing that not all of these will be discovered, some allowance for them must be made when estimating outgoing quality so customers are not disappointed. Because shifts ranging in size from 0 up to $AS_{50}\sigma$ are the likely to main undetected, a conservative approach it to assume that every missed shift it as large as AS_{50}. When estimating capability, $X_{0.5}$ minus $AS_{50}\sigma$ is used to evaluate how well the process output meets the LSL and $X_{0.5}$ plus $AS_{50}\sigma$ is used for determining conformance to the USL. Both of these adjustments are incorporated into the C_{pk} formula, now called the "dynamic" C_{pk} index, by making the following modifications:

$$\begin{aligned} C_{pk} &= \min\{\frac{(X_{0.5} - AS_{50}\sigma) - \mathrm{LSL}}{X_{0.5} - X_{0.135}}, \frac{\mathrm{USL} - (X_{0.5} + AS_{50}\sigma)}{X_{99.865} - X_{0.5}}\} \\ &= \min\{\frac{X_{0.5} - AS_{50}\sigma - \mathrm{LSL}}{X_{0.5} - X_{0.135}}, \frac{\mathrm{USL} - X_{0.5} - AS_{50}\sigma}{X_{99.865} - X_{0.5}}\} \end{aligned} \tag{32}$$

By including an adjustment in this assessment for undetected shifts in median, the estimate of capability with decrease and the expected total number nonconforming parts will increase. From Table 6, AS_{50} is 1.663 when n = 4 and data comes from Weibull (1, 2) distribution.

3.7. Comparing with Simulate

We don't know the $\overline{X}$ of the most non-normal distribution (except gamma distribution) by the parameter because the most non-normal distributions have no reproductive. We use other way to know the $\overline{X}$ of the non-normal distribution like approximate method and simulate method. The approximate method would get the cumulative distribution function of $\overline{X}$. By the reference Lu (2003), we can use three quantile to computation the c.d.f of $\overline{X}$ distribution. The c.d.f of $\overline{X}$ distribution could approximate as Equation (22).Using this c.d.f is to compute the relationship between the mean shift and Type Ⅱ error. The simulate method is using MATLAB program to sample repeatedly and loop function to estimation the influence of mean shift on Type Ⅱ error.

We give some instance to compare. We take four examples of different Weibull process cases and repeat ten times. We show difference between approximate method and simulate method. We take Weibull (1, 5,) n=2, Weibull (1, 5,) n=10, Weibull (1, 5,) n=15 and Weibull (1, 10,) n=10 for examples. Table 8 displays four different cases for example. We compare the standard deviation of outcome and spent mean time. We find that approximate method had computed fast and stable. Figure 5 displays twenty computations for approximate mothed and simulate mothed. We find that approximate method is more stable than simulate method.

Table 8. Four cases for example.

	approximate method	simulate method
	Weibull(α=1, β=5), n=2	
Mean Standard deviation Spent mean time	2.236 3.96E-04 50 second	2.2379 0.0167 542 second
	Weibull(α=1, β=5), n=10	
Mean Standard deviation Spent mean time	0.9687 2.21E-04 240 second	0.9837 0.011 450 second
	Weibull(α=1, β=5), n=15	
Mean Standard deviation Spent mean time	0.7882 2.04E-04 320 second	0.7976 0.0074 423 second
	Weibull(α=1, β=10), n=10	
Mean Standard deviation Spent mean time	1.0142 4.63E-04 240 second	1.0377 0.0122 469 second

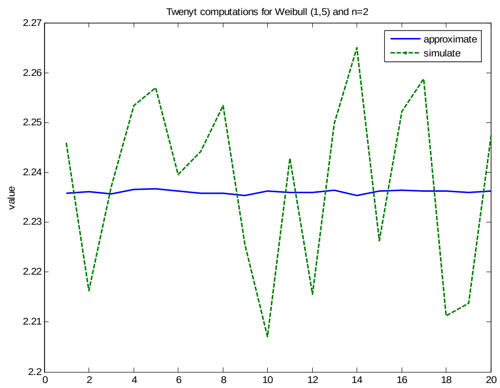

Figure 5(a).Twenty computations for Weibull(1, 5) and n=2

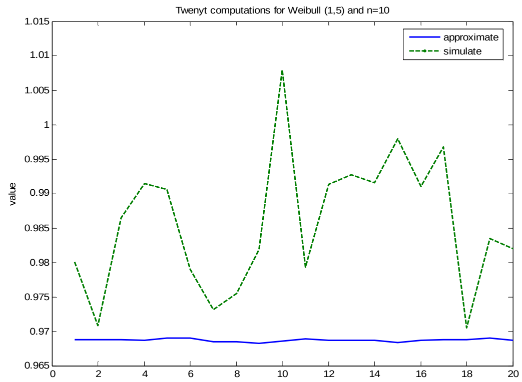

Figure 5(b).Twenty computations for Weibull(1, 5) and n=10

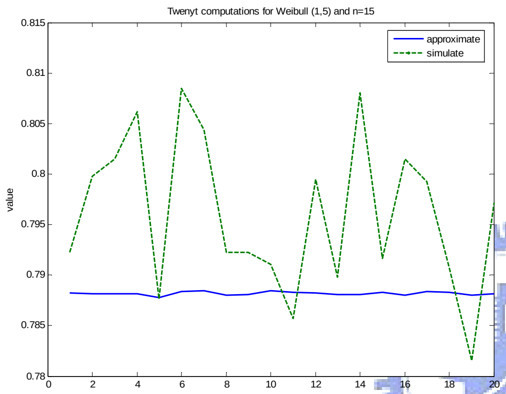

Figure 5(c).Twenty computations for Weibull(1, 5) and n=15

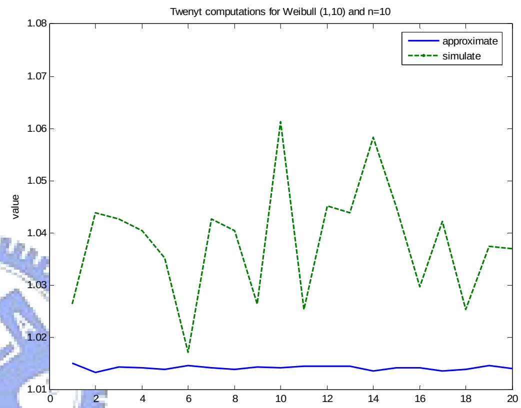

Figure 5(d).Twenty computations for Weibull(1, 10) and n=10

The approximate method has the advantage of spend time and convenient. The simulate method must spend lashings time to estimate AS_{50}. The mean shift adjustment AS_{50} means that the detection power is fifty percent when process mean shift AS_{50} sigma. The engineer can regulate detection power according to personal experience. The approximate method can compute speedily by the cumulative distribution function of $\overline{X}$ distribution. The simulate method must repeat to simulate again. The precise is the weakness of approximate method. The approximate method can not compute real the c.d.f of $\overline{X}$ distribution precisely.

Table 9. Approximate method compare with simulate method.

	approximate method	simulate method
Spent time	less	more
Convenience	better	worse
Promote accurate	hard	spent more time

4. Process Capability Adjustment for Beta Process

4.1. The Beta Distribution

Beta distributions are very versatile and a variety of uncertainties can be usefully modeled by them. This flexibility encourages its empirical use in a wide range of applications. The beta distributions are among the most frequently employed to model theoretical distribution. An example of some importance is the use of beta distributions of fit the distribution of certain criteria used in statistical likelihood ratio test.

In this section, we investigate the non-normal distributions under study are various beta distributions. The beta distribution with parameters p and q can be denoted as beta(p,q) and the probability density function given by

$$f(x)=\frac{\Gamma(p+q)}{\Gamma(p)+\Gamma(q)}x^{p-1}(1-x)^{q-1},\ \ 0\le x\le 1, q>0, q>0, \tag{33}$$

where $\Gamma(x)$ is the gamma function. The mean and variance of the Beta distribution are

$$E(X)=\frac{p}{p+q}, \tag{34}$$

and

$$V(X)=\frac{pq}{(p+q)^2(p+q+1)}. \tag{35}$$

The coefficient of skewness and the coefficient of kurtosis of beta distribution are

$$\gamma_1=\frac{2(q-p)\sqrt{(p+q+1)}}{(p+q+2)\sqrt{pq}}, \tag{36}$$

and

$$\gamma_2=\frac{6\left[p^3-p^2(2q-1)+q^2(q+1)-2pq(q+2)\right]}{pq(p+q+2)(p+q+3)}. \tag{37}$$

Figure 5 displays Beta distributions for selected values of the ratio p/q. Table 8 presents the coefficient of skewness and the coefficient of kurtosis of the beta distribution under study. When p=q, the beta density is symmetric about 0.5, giving more and more weight to regions about 0.5 as the common value p (=q) increases. When q>p, the density is skewed to the left and belong right-tail distribution; and it is skewed to the right when p>q.

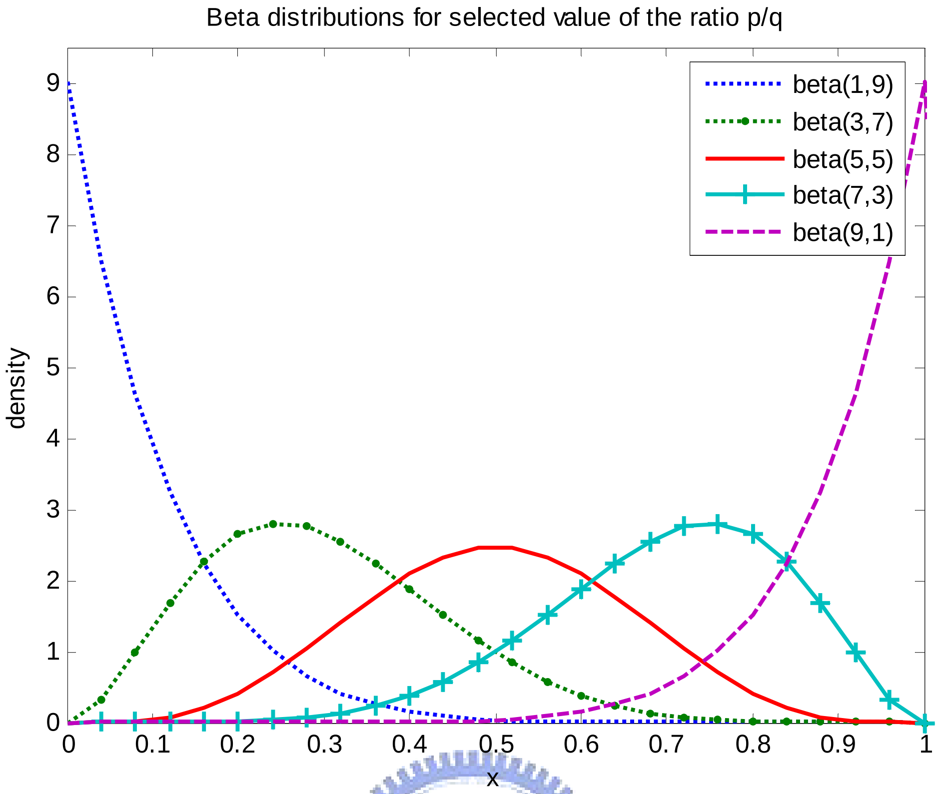

Figure 6. Beta distributions for selected values of the ratio p/q.

Table 10. The M.G.F. of beta distribution for several ratio p/q.

beta(p,q)	mean	variance	skewness	kurtosis
Normal(0,1)	0	1	0	0
beta(1,9)	0.1	0.008182	-1.474055	2.547009
beta(2,8)	0.2	0.014545	-0.829156	0.490385
beta(3,7)	0.3	0.019091	-0.482498	-0.139194
beta(4,6)	0.4	0.021818	-0.225668	-0.391026
Beta(5,5)	0.5	0.022727	0	-0.461538
Beta(6,4)	0.6	0.021818	0.225668	-0.391026
Beta(7,3)	0.7	0.019091	0.482498	-0.139194
Beta(8,2)	0.8	0.014545	0.829156	0.490385
Beta(9,1)	0.9	0.008182	1.474055	2.547009

Figure 6 displays several beta distributions along with a normal distribution for the same mean and variance. From Figure 6, we know the beta p.d.f. shape is more similar normal distribution while p is close to q.

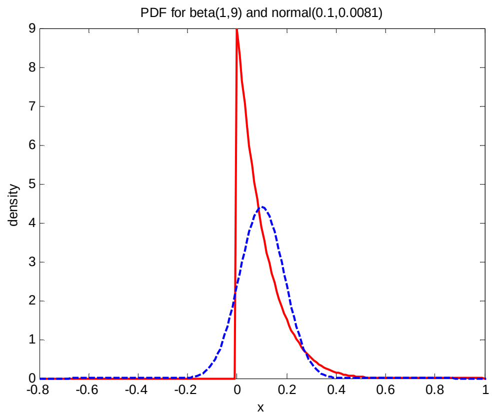

Figure 7(a).Probability density functions for beta(1,9) and normal(0.1, 0.00818)

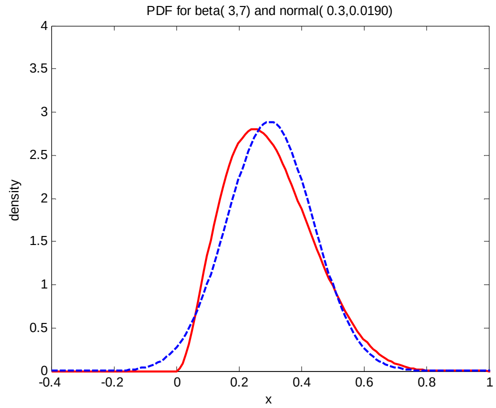

Figure 7(b). Probability density functions for beta(3,7) and normal(0.3, 0.01909)

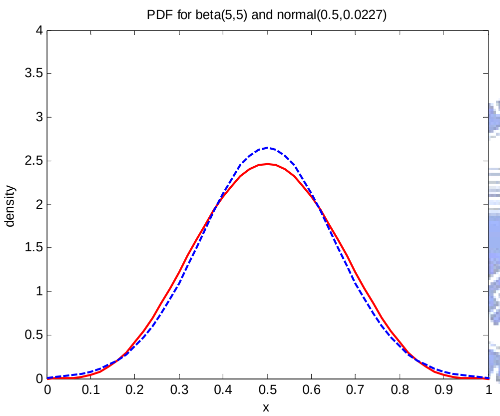

Figure 7(c). Probability density functions for beta(5,5) and normal(0.5, 0.02272)

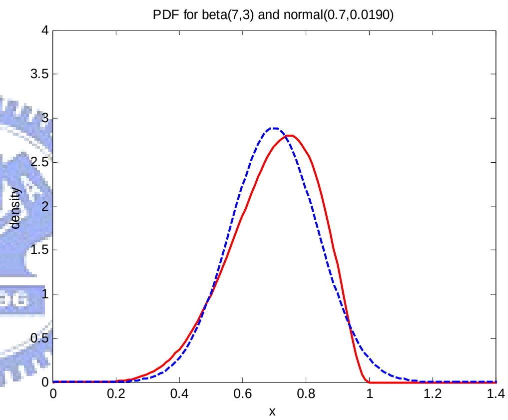

Figure 7(d). Probability density functions for beta(7,3) and normal(0.7, 0.01909)

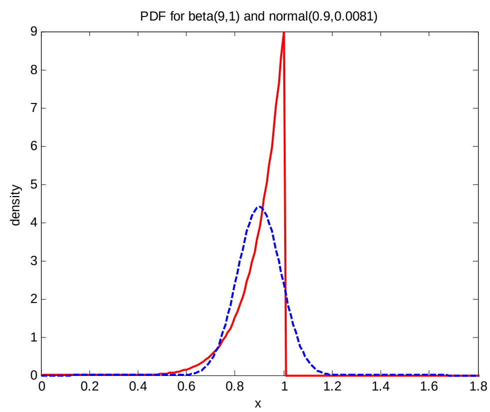

Figure 7(e). Probability density functions for beta(9,1) and normal(0.9, 0.0081)

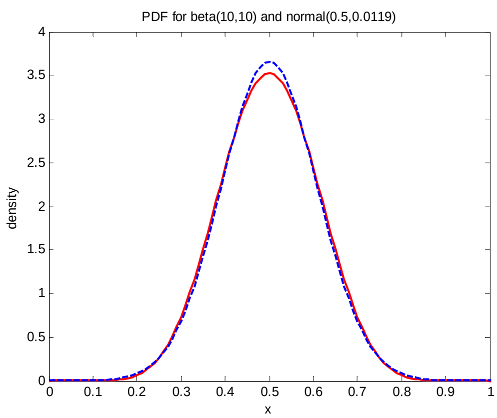

Figure 7(f). Probability density functions for beta(10,10) and normal(0.5, 0.0119)

4.2. The Modified Mean Adjustments for Beta Process

When q>p, the density is skewed to the left and belong right-tail distribution; and it is skewed to the right when p>q. We will also discuss two different cases. We discuss μ occurring right movement and left movement. We develop a Matlab program to simulate the mean shift adjustment AS_{50}. To simulate how distance mean shift the detection power is fifty percent.

Table 9 and Table 10 display the magnitude of mean shift adjustments AS_{50} based on the detection power is 50 percent and data from beta distribution(p,q) with various value of ratio 0.1 (0.1) 0.9 and n=2 (1) 25. For example, if beta (p, q) is (3, 7) and with n=5, the mean right shift adjustment AS_{50} is 1.456. The AS_{50} of beta distributions are usually larger than Bothe' adjustment apparently. When p=q, the AS_{50} of beta distributions are slightly smaller than Bothe' adjustment. When p=q, the shape of beta distributions belong to platykurtic and symmetrical distribution. The shape of platykurtic can decrease the AS_{50} .In Appendix, Tables 14-23 show the AS_{50} of beta distribution by Simulation with the various p=1(1)10 and q=1(1)10.

Table 11. AS_{50} values for several subgroup sizes n,p and q when k>0.

AS_{50}	beta(p, q) for right shift								
n	(1,9)	(2,8)	(3,7)	(4,6)	(5,5)	(6,4)	(7,3)	(8,2)	(9,1)
2	3.106	2.627	2.358	2.164	1.987	1.812	1.615	1.359	0.937
3	2.405	2.086	1.907	1.778	1.662	1.544	1.413	1.238	0.932
4	2.006	1.764	1.638	1.539	1.457	1.369	1.270	1.136	0.899
5	1.751	1.559	1.456	1.378	1.310	1.240	1.161	1.056	0.863
6	1.571	1.409	1.325	1.258	1.198	1.143	1.079	0.990	0.827
7	1.430	1.294	1.218	1.164	1.115	1.065	1.011	0.934	0.794
8	1.321	1.200	1.136	1.088	1.043	1.002	0.953	0.887	0.765
9	1.232	1.123	1.067	1.023	0.986	0.950	0.905	0.848	0.738
10	1.158	1.063	1.009	0.972	0.937	0.903	0.865	0.811	0.714
11	1.095	1.008	0.959	0.925	0.895	0.862	0.827	0.780	0.691
12	1.039	0.962	0.917	0.886	0.858	0.829	0.798	0.753	0.671
13	0.995	0.920	0.879	0.851	0.825	0.798	0.769	0.727	0.652
14	0.953	0.883	0.847	0.820	0.795	0.771	0.743	0.705	0.635
15	0.915	0.852	0.817	0.791	0.768	0.745	0.720	0.685	0.619
16	0.883	0.823	0.790	0.767	0.744	0.723	0.698	0.666	0.605
17	0.853	0.796	0.765	0.742	0.722	0.702	0.679	0.649	0.590
18	0.823	0.771	0.742	0.723	0.702	0.684	0.663	0.633	0.578
19	0.800	0.749	0.723	0.701	0.684	0.665	0.645	0.618	0.566
20	0.778	0.729	0.702	0.685	0.666	0.650	0.630	0.604	0.554

Table 12. AS_{50} values for several subgroup sizes n,p and q when k<0.

AS_{50}	beta(p, q) for left shift								
n	(1,9)	(2,8)	(3,7)	(4,6)	(5,5)	(6,4)	(7,3)	(8,2)	(9,1)
2	0.937	1.358	1.616	1.812	1.984	2.163	2.360	2.629	3.106
3	0.931	1.238	1.413	1.546	1.659	1.777	1.907	2.080	2.405
4	0.899	1.137	1.269	1.368	1.455	1.540	1.638	1.771	2.008
5	0.862	1.055	1.161	1.240	1.309	1.378	1.457	1.561	1.753
6	0.827	0.989	1.077	1.143	1.201	1.259	1.324	1.410	1.565
7	0.794	0.934	1.010	1.064	1.114	1.164	1.218	1.292	1.428
8	0.765	0.888	0.954	1.003	1.044	1.087	1.135	1.203	1.322
9	0.738	0.847	0.906	0.948	0.986	1.023	1.067	1.127	1.231
10	0.714	0.810	0.865	0.904	0.937	0.972	1.010	1.061	1.157
11	0.691	0.780	0.829	0.863	0.895	0.925	0.961	1.007	1.094
12	0.670	0.753	0.796	0.829	0.858	0.887	0.917	0.961	1.041
13	0.652	0.728	0.768	0.798	0.825	0.852	0.879	0.921	0.994
14	0.635	0.706	0.743	0.770	0.794	0.820	0.847	0.884	0.953
15	0.619	0.684	0.719	0.746	0.768	0.791	0.817	0.852	0.914
16	0.605	0.665	0.698	0.723	0.744	0.765	0.790	0.823	0.883
17	0.592	0.649	0.679	0.703	0.722	0.742	0.765	0.796	0.852
18	0.577	0.632	0.662	0.683	0.703	0.722	0.742	0.772	0.823
19	0.566	0.618	0.645	0.666	0.684	0.703	0.722	0.749	0.800
20	0.555	0.603	0.630	0.650	0.667	0.684	0.704	0.729	0.777

5. Process Capability Adjustment for F Process

5.1. The F Distribution

In this section we introduce the F distribution which is closely related to the normal and χ^2 distributions and to each other, and they have important statistical applications. The F distribution arise frequently as the null distribution of a test statistic, especially in likelihood-ratio tests, perhaps most notably in the analysis of variance.

Suppose that U and V are independent variates with $U \sim \chi^2_{(v_1)}$ and $V \sim \chi^2_{(v_2)}$. The ratio of a χ^2 variate to its degrees of freedom is called a mean square. Thus $U \div v_1$ and $V \div v_2$ are mean squares, and they are independent because U and V are independent. Let X is the ratio of independent mean squares:

$$X \equiv \frac{U \div v_1}{V \div v_2}. \tag{38}$$

Then X is a continuous variate which can take any positive real value. Its distribution is called the F Distribution with v_1 numerator and v_2 denominator degrees of freedom, and we write $X \sim F_{v_1,v_2}$ for short. The probability density function of F_{v_1,v_3} is defined as

$$f_{v_1,v_2}(x) = \frac{\Gamma(\frac{v_1+v_2}{2}) v_1^{v_1/2} v_2^{v_2/2}}{\Gamma(\frac{v_1}{2})\Gamma(\frac{v_2}{2})} \frac{x^{v_1/2-1}}{(v_2+v_1x)^{(v_1+v_2)/2}}, \text{for } x > 0, \tag{39}$$

and the mean and variance are given respective by

$$E(X) = \frac{v_2}{v_2-2}, \ for\ \mathrm{v}_2 > 0, \tag{40}$$

and

$$V(X) = \frac{2v_2^2(v_2+v_1-2)}{v_1(v_2-2)^2(v_2-4)}, \ for\ v_2 > 4, \tag{41}$$

where v_1 and v_2 are degrees of freedom for F distribution.

The skewness coefficient and kurtosis coefficient of F distribution are

$$\gamma_1 = \frac{2(v_2+2v_1-2)}{v_2-6}\sqrt{\frac{2(v_2-4)}{v_1(v_2+v_1-2)}}, \ for\ \mathrm{v}_2 > 6, \tag{42}$$

and

$$\gamma_2 = \frac{12(-16+20v_2-8v_2^2+v_2^3+44v_1-32v_1v_2+5v_2^2v_1-22v_1^2+5v_2v_1^2}{v_1(v_2-6)(v_2-8)(v_1+v_2-2)}, \quad for\ \mathrm{v}_2 > 8 \tag{43}$$

Figure 7 displays F distributions for selected values of the v_1 and v_2. Table 11 presents the coefficient of skewness and the coefficient of kurtosis of the beta distribution. By the Equation (42) and (43), we know that F distributions are positively skewed and leptokurtic for any v_1 and v_2. The parameter v_2 is more powerful to affect γ_1 and γ_2.

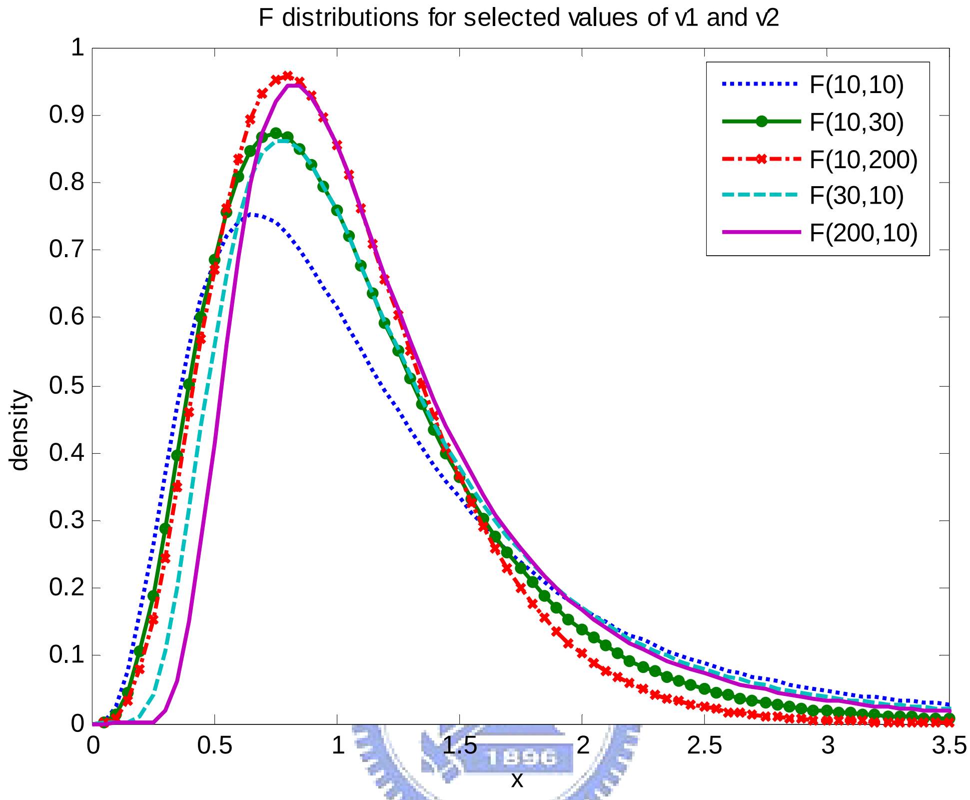

Figure 8. F distributions for selected values degree of freedom v_1 and v_2.

Table 13. The M.G.F. of F distribution for several values degree of freedom.

$F(v_1,v_2)$	mean	variance	skewness	kurtosis
Normal (0,1)	0	1	0	0
F(10,10)	1.25	0.9375	3.614784	45.2
F(10,30)	1.071429	0.335557	1.479687	4.12823
F(10,50)	1.041667	0.273626	1.231023	2.667085
F(10,100)	1.020408	0.234277	1.058578	1.847857
F(10,200)	1.010101	0.216554	0.975655	1.505227
F(30,10)	1.25	0.659722	3.488326	42.505263
F(50,10)	1.25	0.604167	3.473645	42.198621
F(100,10)	1.25	0.5625	3.466667	42.053333
F(200,10)	1.25	0.541667	3.464768	42.013846

Figure 8 display several F distributions along with a normal distribution for the same mean and variance. From Figure 8, we know the F p.d.f. shape is more similar normal distribution while v_1 and v_2 increase.

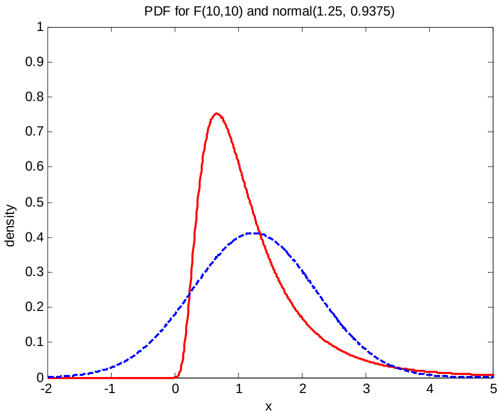

Figure 9(a). Probability density functions for F(10,10) and normal(1.25, 0.9375)

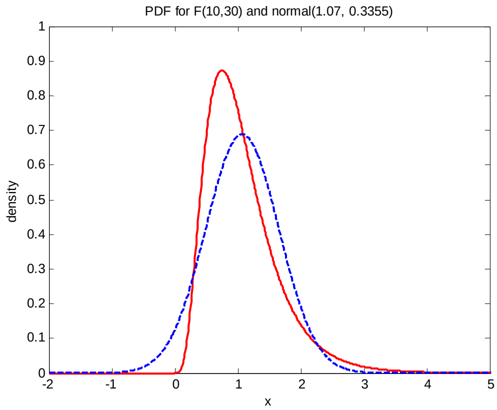

Figure 9(b). Probability density functions for F(10,30) and normal(1.07, 0.3355)

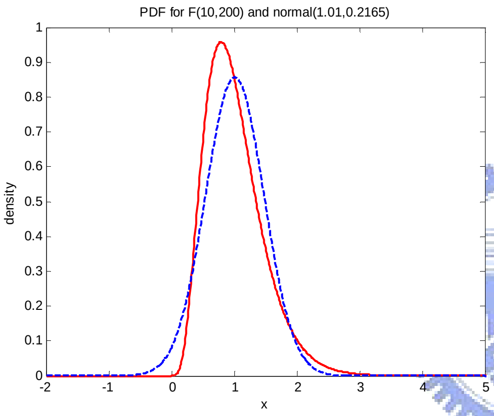

Figure 9(c). Probability density functions for F(10,200) and normal(1.01, 0.2165)

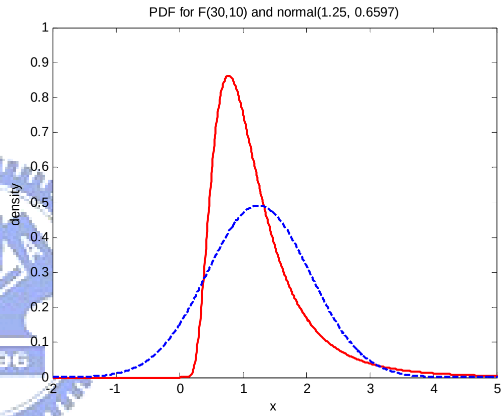

Figure 9(d). Probability density functions for F(30,10) and normal(1.25, 0.65972)

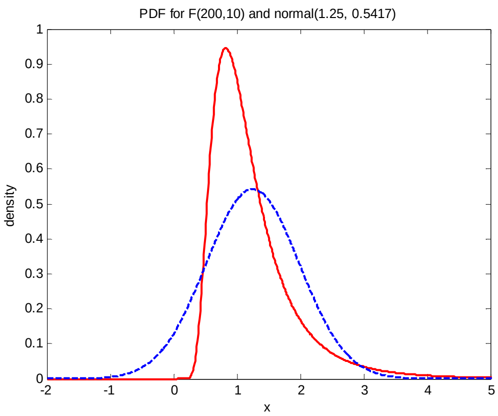

Figure 9(e). Probability density functions for F(200,10) and normal(1.25, 0.5416)

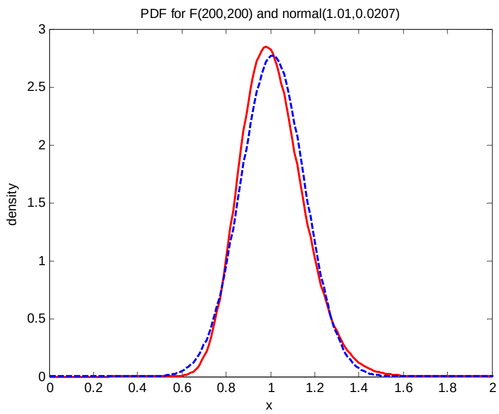

Figure 9(f). Probability density functions for F(200,200) and normal(1.01, 0.0207)

5.2. The Modified Mean Adjustments of F Process

Because the parameter v_2 is more powerful to effect the coefficient of skewness, we fix the parameter v_1 and compute various subgroup sizes n, and the parameter v_2. F distributions are positive skewness, so we only discuss μ occurring right movement. Table 12 displays AS_{50} based on the detection power is 50 percent and data from F (5, v_2) distribution with various value of v_2 =5, 10, 15, 20, 25, 30, 40, 50, 60, 120, ∞ and n=2 (1) 25. For example, if v_2 is 30 with n=2, the mean shift adjustment AS_{50} is 3.541. We conclude that a mean shift adjustment of $AS_{50} = 3.541\sigma$ is required based on the detection power is 50 percent and data come from F (5, 30) distribution. AS_{50} of F distributions are larger than Bothe' adjustment obviously. That is because the skewness and the kurtosis of F distributions are bigger than normal distribution.

In Appendix, Tables 24-33 show the AS_{50} of F distribution by Simulation with the various v_1=10,15,20,25,30,40,50,60,120, ∞ and v_2 =5,10,15,20,25,30, 40,50,60,120, ∞.

Table 14. AS_{50} values for several subgroup sizes n and various v_2 values.

AS_{50}	v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.807	4.727	4.098	3.810	3.651	3.541	3.416	3.343	3.297	3.184	3.088
3	4.613	3.588	3.101	2.881	2.763	2.686	2.601	2.548	2.515	2.440	2.378
4	3.918	2.948	2.543	2.371	2.276	2.221	2.151	2.114	2.090	2.029	1.986
5	3.458	2.532	2.185	2.040	1.964	1.918	1.862	1.832	1.812	1.767	1.729
6	3.120	2.238	1.933	1.810	1.745	1.703	1.658	1.632	1.617	1.579	1.546
7	2.863	2.014	1.744	1.634	1.581	1.544	1.504	1.484	1.471	1.438	1.412
8	2.645	1.842	1.597	1.499	1.450	1.420	1.384	1.366	1.354	1.326	1.303
9	2.473	1.706	1.476	1.390	1.345	1.318	1.288	1.271	1.259	1.235	1.215
10	2.335	1.588	1.378	1.299	1.259	1.236	1.208	1.192	1.182	1.160	1.141
11	2.208	1.489	1.295	1.223	1.186	1.164	1.139	1.125	1.118	1.096	1.080
12	2.099	1.404	1.224	1.157	1.124	1.103	1.082	1.069	1.061	1.042	1.027
13	2.010	1.332	1.163	1.101	1.069	1.051	1.030	1.019	1.012	0.994	0.981
14	1.925	1.270	1.110	1.051	1.023	1.005	0.985	0.975	0.968	0.952	0.940
15	1.846	1.213	1.061	1.008	0.979	0.965	0.946	0.936	0.930	0.916	0.904
16	1.779	1.163	1.018	0.967	0.942	0.927	0.911	0.901	0.896	0.882	0.871
17	1.717	1.116	0.981	0.932	0.909	0.894	0.879	0.870	0.865	0.851	0.841
18	1.663	1.074	0.944	0.900	0.878	0.865	0.849	0.841	0.836	0.824	0.815
19	1.612	1.038	0.914	0.871	0.849	0.836	0.823	0.815	0.810	0.799	0.790
20	1.563	1.004	0.885	0.844	0.824	0.812	0.799	0.792	0.787	0.776	0.767

6. An Application Example

PCB mean s "Printed Circuit Board" literally means the products are made by pattern printing technology. PCB replaces conventional copper wiring methodology and enables mass production process to duplicate the products. Therefore, PCB can increase the throughput and the mobility, also reducing the size and the cost. As the industry transferred from analog to digital, PCB manufacturing was stimulated to provide high volume production. Walk-man/Calculator/ computer /fax machine /TV/Cell phone/ PDA. Factories producing various PCB and related products generally are classified as "the PCB industry" because the core components inside those products are the PCBs.

The PCB manufacturing process mainly consists of a series of chemical related operations and the chemical operations determine the functions of the PCB. PCB is laminate and be made from two or more sheets of material stuck together; often copper and fiberglass is laminate. The makers have to create patterns required on the laminate. For multi layer PCBs, the patterns will include inner layer and outer layer. The PCBs pass through a chain of inner layer pattererns coating, mass lamination press, drilling, outer layer pattern creation, solder resist coating. For electronic devices minimization sizing trend semiconductor package gradually transfer to like BGA/CSP type of high-density packaging. The dimension shrinkage is really obviously. The high-density PCBs have to progress faster. High I/O BGA as the diagram show that rounting density and connecting density increasing that fine line capability will be a really sever challenge. For electronic device packaging shrinking size then bonding pad design on PCB change also. The exact solder become more and more important.

Because we know that PCBs may shut down when the weld is too small or large, the size of the weld ball is one of the most important factors to be considered. We let the LSL and USL of weld ball size are 0.5mil and 4mil. A part of historical data is displayed in Table 14. The historical data indicates that the process pretty approximate to a Weibull distribution (this can be tested by means of goodness-of-fit tests). The parameters α and β of Weibull distribution could be calculated from historical data giving $\alpha = 2.5$ and $\beta = 5$.

Therefore, it is appropriate to use this approach and we can obtain more accurate measures of the three quantile : $X_{0.00135} = 0.667$, $X_{0.5} = 2.323$, and

$X_{0.99865} = 3.647$ for $\sigma = \alpha^2[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})] = 0.5258$ under consideration.

Then "dynamic" C_{pk} index can be calculated as follows:

$$\text{dynamic } C_{pk} = \min\left\{\frac{\text{USL} - X_{0.5} - AS_{50}\sigma}{X_{0.99865} - X_{0.5}}, \frac{X_{0.5} - \text{LSL} - AS_{50}\sigma}{X_{0.5} - X_{0.00135}}\right\}$$
$$= \min\left\{\frac{4\text{-}2.323\text{-}0.969{*}0.5258}{3.647\text{-}2.32}, \frac{2.323\text{-}0.5\text{-}0.969{*}0.5258}{2.323\text{-}0.667}\right\}$$
$$= \min\{0.882, 0.793\}$$
$$= 0.793,$$

with AS_{50} =0.969 for n=10 from Table 7. Compared it to the value of the following index：

$$C_{pk} = \min\left\{\frac{\text{USL} - X_{0.5}}{X_{0.99865} - X_{0.5}}, \frac{X_{0.5} - \text{LSL}}{X_{0.5} - X_{0.00135}}\right\}$$
$$= \min\{1.27, 1.10\}$$
$$= 1.1.$$

That we don't consider the shift in μ, we can find that the value of dynamic C_{pk} much smaller. By increasing n, shift in μ have a higher probability of detection. For example, if n=15, the AS_{50} would be 0.788 for Weibull distribution ($\alpha = 2.5, \beta = 5$) then

$$\text{dynamic } C_{pk} = \min\left\{\frac{\text{USL} - X_{0.5} - AS_{50}\sigma}{X_{0.99865} - X_{0.5}}, \frac{X_{0.5} - \text{LSL} - AS_{50}\sigma}{X_{0.5} - X_{0.00135}}\right\}$$
$$= \min\left\{\frac{4\text{-}2.323\text{-}0.788{*}0.5258}{3.647\text{-}2.32}, \frac{2.323\text{-}0.5\text{-}0.788{*}0.5258}{2.323\text{-}0.667}\right\}$$
$$= \min\{0.953, 0.851\}$$
$$= 0.851$$

Enlarging n by 2 increases the dynamic C_{pk} index from 0.793 to 0.851.

Table 15. The 100 observations are collected of the historical data.

2.346	2.188	1.860	2.164	2.799	2.211	2.549	2.801	2.627	2.895
2.200	2.286	2.175	3.077	2.543	2.905	2.747	2.747	2.814	1.575
3.117	2.619	1.484	1.552	2.804	1.845	1.847	1.128	2.370	2.138
2.839	2.141	2.945	3.243	1.429	2.529	2.957	1.507	1.952	1.329
2.680	2.687	2.998	2.544	1.736	1.950	2.119	2.542	2.280	1.620
2.614	2.274	2.598	2.611	2.410	2.872	2.371	1.186	2.496	2.774
2.312	2.736	3.015	1.884	1.858	2.472	2.329	2.073	2.728	1.940
1.684	2.740	1.247	2.874	2.247	1.891	1.518	3.126	1.672	2.364
2.125	2.894	0.961	1.663	2.609	2.309	1.771	3.348	2.285	2.266
3.185	1.615	2.486	2.304	2.035	1.371	1.647	1.832	2.371	3.141

7. Conclusion

This paper has considered the problem for adjusting estimates of process capability by including a mean shift when data is from non-normal distribution. In the Bothe' studies, statistically derived adjustments are proposed under the data assumed to be approximately normally distributed. But the case of non-normal processes occurs frequently in practice. We use a Matlab program to compute the mean shift adjustment AS_{50} based on the detection power is 50% for data comes from Weibull distribution and Beta distribution and F distribution with various parameters. In this paper, we know the skewness and kurtosis effecting the AS_{50}. The more skewness, the lager AS_{50} than normal distribution. Especially, when the skewness approximate 0 and the kurtosis <0, the AS_{50} will be better than normal distributions slightly.

References

1. Boyles, R. A. (1991). The Taguchi capability index. *Journal of Quality Technology,* 23, 17-26.
2. Chen, L. K., Cheng, S. W. and Spiring, F. A. (1988). A new measure of process capability: C_{pm} . *Journal of Quality Technology*, 20, 162-173.
3. Cygan P., Krishnakumar, B., and Laghari, J. R. (1989). Lifetimes of polypropylene films under combined high electric field and thermal stresses, *IEEE Transactions on Electrical Insulation*, 24, 619-625.
4. Bothe, D. R. (2002).Statistical Reason for the 1.5σ shift. *Quality Engineering*, 14(3), 479-487.
5. Kalbfleisch, J.G. (1985). *Probability and Statistical Inference,* 2nd edition. Springer-Verlag, New York.
6. Kane, V. E. (1986). Process capability indices. *Journal of Quality Technology*, 18, 41-52.
7. Kotz, S. and Johnson, N. L. (1993).*Process Capability Indices*. Chapman & Hall, London, U.K.
8. Kotz, S. and Lovelace, C. (1998). *Process Capability Indices in Theory and Practice*. Arnold, London, U.K.
9. Kotz, S. and Johnson, N. L. (2002). Process Capability indices – a review, 1992-2000. *Journal of Quality Technology,* 34(1), 1-19.
10. Kotz, S. and Johnson, N. L. and N.Balakrishnam (1995). *Continuous Univariate Distributions*. A Wiley-Interscience Publication, New York.
11. Chen, K. S. and Pearn, W. L. (1997). An application of non-normal process capability indices, *Quality and Reliability Engineering International*, 13, 355-360.
12. Pearn, W. L., Kotz, S. and Johnson, N. L. (1992). Distributional and inferential properties of process capability indices. *Journal of Quality Technology,* 24(4), 216-233.
13. Pearn, W. L., Lin, G. H. and Chen, K. S. (1998). Distribution and inferential properties of process accuracy and process precision indices. *Communications in Statistics: Theory & Method*, 27(4), 985-1000.
14. Lu, X. M. and Peng N. F. (2003).*The Approximation of the Distribution Function of Sum of Independent and Identical Weibull distributions*. National Chiao Tung University, Taiwan.
15. Pyzdek, T. (1995). Process Capability Analysis Using Personal Computers, *Quality. Engineering*, 4(3), 419-440.
16. Ross, S. (1992). *A First Course in Probability*, fifth edition. Prentice-Hall International.

17. Taguchi, G. and Hsiang, T. C. (1985). A tutorial on quality control and assurance –the Taguchi methods. *ASA Annual Meeting*, Las Vegas, Nevada.
18. Vännman, K. (1997). Distribution and moments in simplified from for a general class of capability indices. *Communications in Statistics: Theory & Methods*, 26, 159-179.

Appendix

Table 16. AS_{50} values for q=1, n=2(1)25 and various p values.

AS_{50}	beta(p,1) for mean shift									
n	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)	(7,1)	(8,1)	(9,1)	(10,1)
2	1.642	2.245	2.544	2.722	2.845	2.935	3.007	3.064	3.110	3.148
3	1.500	1.868	2.042	2.150	2.230	2.288	2.334	2.373	2.403	2.429
4	1.365	1.617	1.742	1.824	1.880	1.925	1.957	1.986	2.008	2.028
5	1.250	1.444	1.543	1.605	1.651	1.683	1.711	1.734	1.750	1.766
6	1.157	1.315	1.396	1.447	1.485	1.513	1.535	1.553	1.568	1.581
7	1.081	1.215	1.283	1.327	1.358	1.382	1.402	1.417	1.429	1.439
8	1.017	1.134	1.193	1.232	1.259	1.280	1.296	1.310	1.321	1.330
9	0.964	1.067	1.120	1.153	1.178	1.196	1.210	1.222	1.231	1.239
10	0.918	1.009	1.057	1.087	1.109	1.126	1.139	1.149	1.159	1.165
11	0.879	0.962	1.004	1.031	1.051	1.067	1.078	1.087	1.094	1.101
12	0.844	0.919	0.957	0.982	1.001	1.014	1.025	1.034	1.040	1.046
13	0.812	0.881	0.918	0.940	0.957	0.969	0.979	0.986	0.994	1.001
14	0.784	0.847	0.881	0.902	0.918	0.931	0.939	0.946	0.952	0.957
15	0.758	0.818	0.849	0.870	0.884	0.894	0.903	0.909	0.915	0.920
16	0.736	0.791	0.820	0.839	0.853	0.863	0.870	0.877	0.881	0.888
17	0.714	0.766	0.794	0.811	0.824	0.834	0.841	0.848	0.852	0.856
18	0.695	0.744	0.770	0.786	0.798	0.807	0.814	0.820	0.825	0.828
19	0.677	0.724	0.748	0.764	0.775	0.783	0.790	0.795	0.799	0.803
20	0.660	0.705	0.728	0.743	0.754	0.761	0.768	0.773	0.777	0.780
21	0.645	0.687	0.709	0.724	0.734	0.741	0.747	0.752	0.756	0.759
22	0.631	0.671	0.692	0.705	0.715	0.722	0.728	0.732	0.736	0.739
23	0.617	0.656	0.676	0.688	0.697	0.705	0.710	0.714	0.718	0.721
24	0.605	0.641	0.661	0.672	0.682	0.688	0.693	0.698	0.701	0.704
25	0.593	0.628	0.646	0.658	0.667	0.673	0.678	0.682	0.685	0.688

Table 17. AS_{50} values for q=2, n=2(1)25 and various p values.

AS_{50}	beta(p, 2) for mean shift									
n	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)	(7,2)	(8,2)	(9,2)	(10,2)
2	2.242	1.833	2.115	2.288	2.408	2.502	2.571	2.627	2.676	2.713
3	1.868	1.587	1.760	1.869	1.943	2.001	2.047	2.084	2.112	2.137
4	1.617	1.410	1.535	1.613	1.669	1.710	1.741	1.769	1.789	1.808
5	1.443	1.279	1.376	1.437	1.482	1.514	1.540	1.559	1.577	1.591
6	1.315	1.178	1.258	1.308	1.344	1.370	1.391	1.409	1.423	1.434
7	1.214	1.097	1.164	1.207	1.239	1.261	1.280	1.293	1.304	1.315
8	1.134	1.031	1.089	1.125	1.153	1.173	1.189	1.201	1.211	1.221
9	1.067	0.975	1.027	1.059	1.082	1.100	1.115	1.126	1.133	1.141
10	1.010	0.928	0.973	1.002	1.024	1.040	1.052	1.062	1.070	1.079
11	0.961	0.886	0.928	0.954	0.974	0.988	0.999	1.007	1.016	1.021
12	0.919	0.850	0.888	0.913	0.930	0.943	0.954	0.961	0.968	0.975
13	0.881	0.818	0.853	0.876	0.891	0.904	0.913	0.920	0.926	0.931
14	0.848	0.789	0.822	0.843	0.857	0.868	0.877	0.884	0.890	0.894
15	0.818	0.763	0.793	0.812	0.827	0.837	0.845	0.851	0.857	0.861
16	0.791	0.740	0.768	0.787	0.799	0.809	0.815	0.822	0.827	0.833
17	0.766	0.719	0.745	0.761	0.774	0.782	0.790	0.796	0.800	0.805
18	0.744	0.699	0.723	0.739	0.751	0.760	0.766	0.772	0.776	0.779
19	0.723	0.681	0.704	0.720	0.730	0.738	0.744	0.750	0.754	0.757
20	0.705	0.663	0.686	0.700	0.710	0.718	0.724	0.729	0.733	0.736
21	0.687	0.648	0.669	0.683	0.692	0.700	0.705	0.710	0.714	0.717
22	0.670	0.633	0.654	0.667	0.676	0.683	0.688	0.693	0.696	0.699
23	0.656	0.620	0.639	0.652	0.661	0.667	0.672	0.677	0.681	0.682
24	0.641	0.607	0.625	0.638	0.645	0.652	0.657	0.661	0.665	0.668
25	0.628	0.595	0.613	0.625	0.632	0.639	0.643	0.647	0.650	0.653

Table 18. AS_{50} values for q=3, n=2(1)25 and various p values.

AS_{50}	beta(p, 3) for mean shift									
n	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)	(7,3)	(8,3)	(9,3)	(10,3)
2	2.544	2.113	1.914	2.084	2.203	2.290	2.364	2.419	2.467	2.506
3	2.040	1.760	1.626	1.733	1.807	1.864	1.909	1.944	1.973	1.999
4	1.744	1.535	1.433	1.510	1.566	1.607	1.638	1.665	1.688	1.706
5	1.543	1.374	1.294	1.355	1.399	1.431	1.455	1.479	1.494	1.509
6	1.396	1.258	1.189	1.239	1.273	1.301	1.323	1.340	1.353	1.366
7	1.282	1.165	1.106	1.148	1.179	1.200	1.219	1.233	1.244	1.256
8	1.193	1.090	1.038	1.074	1.101	1.120	1.136	1.148	1.161	1.169
9	1.118	1.026	0.980	1.013	1.036	1.054	1.067	1.079	1.088	1.096
10	1.057	0.974	0.933	0.961	0.982	0.999	1.009	1.020	1.029	1.036
11	1.003	0.928	0.890	0.917	0.936	0.950	0.962	0.970	0.977	0.984
12	0.958	0.888	0.855	0.878	0.895	0.908	0.918	0.926	0.932	0.939
13	0.917	0.853	0.821	0.843	0.860	0.871	0.880	0.887	0.894	0.899
14	0.881	0.822	0.792	0.812	0.827	0.838	0.847	0.854	0.861	0.864
15	0.849	0.794	0.766	0.785	0.799	0.809	0.817	0.824	0.829	0.833
16	0.820	0.769	0.742	0.760	0.773	0.782	0.790	0.796	0.801	0.806
17	0.794	0.745	0.720	0.737	0.749	0.758	0.765	0.771	0.777	0.780
18	0.770	0.724	0.701	0.716	0.727	0.736	0.742	0.748	0.753	0.756
19	0.748	0.704	0.682	0.697	0.708	0.715	0.723	0.727	0.732	0.735
20	0.727	0.686	0.665	0.680	0.690	0.697	0.703	0.708	0.712	0.716
21	0.709	0.669	0.649	0.662	0.673	0.679	0.686	0.690	0.694	0.697
22	0.692	0.654	0.635	0.648	0.657	0.664	0.669	0.674	0.677	0.680
23	0.675	0.639	0.621	0.633	0.642	0.649	0.654	0.658	0.662	0.664
24	0.661	0.626	0.608	0.620	0.629	0.634	0.640	0.643	0.647	0.651
25	0.646	0.613	0.596	0.607	0.615	0.622	0.627	0.630	0.634	0.637

Table 19. AS_{50} values for q=4, n=2(1)25 and various p values.

AS_{50}	beta(p, 4) for mean shift									
n	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)	(7,4)	(8,4)	(9,4)	(10,4)
2	2.724	2.287	2.082	1.958	2.074	2.163	2.231	2.288	2.333	2.377
3	2.150	1.868	1.732	1.647	1.721	1.776	1.821	1.858	1.888	1.912
4	1.822	1.613	1.510	1.445	1.501	1.543	1.571	1.600	1.621	1.638
5	1.605	1.438	1.355	1.304	1.348	1.378	1.405	1.425	1.441	1.456
6	1.448	1.308	1.239	1.196	1.231	1.258	1.279	1.298	1.310	1.322
7	1.328	1.206	1.149	1.112	1.141	1.164	1.181	1.197	1.208	1.218
8	1.233	1.126	1.074	1.041	1.068	1.088	1.104	1.116	1.126	1.135
9	1.152	1.059	1.014	0.985	1.008	1.025	1.039	1.049	1.059	1.068
10	1.087	1.003	0.961	0.936	0.957	0.970	0.985	0.994	1.003	1.009
11	1.032	0.955	0.917	0.893	0.911	0.925	0.937	0.947	0.954	0.960
12	0.983	0.912	0.878	0.856	0.873	0.885	0.895	0.905	0.912	0.917
13	0.940	0.875	0.843	0.824	0.840	0.850	0.860	0.868	0.874	0.880
14	0.904	0.842	0.813	0.794	0.809	0.820	0.829	0.835	0.841	0.846
15	0.869	0.813	0.785	0.768	0.781	0.791	0.799	0.806	0.811	0.816
16	0.839	0.786	0.760	0.743	0.756	0.766	0.775	0.780	0.784	0.789
17	0.811	0.761	0.736	0.721	0.734	0.743	0.749	0.755	0.761	0.765
18	0.786	0.740	0.716	0.702	0.713	0.722	0.728	0.734	0.738	0.741
19	0.764	0.720	0.697	0.683	0.694	0.702	0.708	0.713	0.718	0.721
20	0.743	0.700	0.679	0.666	0.676	0.684	0.690	0.694	0.699	0.702
21	0.722	0.682	0.663	0.650	0.660	0.667	0.673	0.677	0.681	0.685
22	0.705	0.666	0.648	0.635	0.645	0.651	0.658	0.661	0.666	0.668
23	0.688	0.652	0.634	0.621	0.631	0.636	0.643	0.646	0.650	0.653
24	0.672	0.637	0.619	0.609	0.617	0.623	0.629	0.632	0.636	0.639
25	0.657	0.624	0.607	0.597	0.605	0.611	0.616	0.619	0.623	0.626

Table 20. AS_{50} values for q=5, n=2(1)25 and various p values.

AS_{50}	beta(p, 5) for mean shift									
n	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)	(7,5)	(8,5)	(9,5)	(10,5)
2	2.847	2.413	2.202	2.074	1.986	2.071	2.138	2.197	2.244	2.284
3	2.229	1.943	1.806	1.719	1.660	1.717	1.759	1.795	1.825	1.851
4	1.880	1.667	1.566	1.500	1.454	1.496	1.527	1.555	1.577	1.595
5	1.651	1.480	1.399	1.345	1.310	1.341	1.367	1.387	1.405	1.420
6	1.482	1.343	1.274	1.231	1.201	1.228	1.248	1.265	1.279	1.291
7	1.360	1.238	1.178	1.141	1.114	1.137	1.155	1.169	1.181	1.191
8	1.258	1.152	1.101	1.067	1.045	1.065	1.080	1.092	1.104	1.113
9	1.178	1.084	1.037	1.007	0.986	1.005	1.018	1.028	1.038	1.047
10	1.109	1.025	0.982	0.956	0.937	0.953	0.965	0.976	0.984	0.991
11	1.051	0.974	0.936	0.911	0.895	0.909	0.920	0.929	0.936	0.943
12	1.002	0.930	0.895	0.873	0.858	0.871	0.880	0.888	0.895	0.902
13	0.956	0.892	0.859	0.839	0.825	0.837	0.846	0.853	0.860	0.865
14	0.917	0.858	0.827	0.808	0.795	0.807	0.815	0.822	0.827	0.833
15	0.882	0.826	0.798	0.781	0.769	0.779	0.787	0.794	0.799	0.804
16	0.853	0.798	0.773	0.756	0.744	0.753	0.762	0.767	0.774	0.778
17	0.824	0.773	0.749	0.733	0.722	0.731	0.739	0.745	0.749	0.754
18	0.799	0.751	0.728	0.712	0.703	0.711	0.718	0.723	0.727	0.731
19	0.775	0.730	0.708	0.695	0.684	0.692	0.698	0.703	0.708	0.712
20	0.753	0.709	0.689	0.676	0.667	0.674	0.680	0.685	0.690	0.694
21	0.733	0.692	0.673	0.660	0.651	0.658	0.664	0.669	0.673	0.676
22	0.714	0.676	0.657	0.645	0.636	0.643	0.649	0.653	0.657	0.660
23	0.698	0.661	0.642	0.630	0.622	0.629	0.634	0.638	0.642	0.645
24	0.682	0.646	0.628	0.617	0.610	0.616	0.620	0.625	0.628	0.631
25	0.667	0.632	0.615	0.605	0.597	0.603	0.608	0.612	0.616	0.618

Table 21. AS_{50} values for q=6, n=2(1)25 and various p values.

AS_{50}	beta(p, 6) for mean shift									
n	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)	(7,6)	(8,6)	(9,6)	(10,6)
2	2.934	2.499	2.292	2.162	2.072	2.006	2.073	2.128	2.172	2.215
3	2.287	2.002	1.864	1.776	1.716	1.672	1.715	1.751	1.781	1.805
4	1.924	1.712	1.605	1.540	1.495	1.461	1.492	1.519	1.540	1.558
5	1.684	1.513	1.430	1.377	1.342	1.314	1.339	1.359	1.377	1.392
6	1.512	1.370	1.300	1.257	1.228	1.204	1.224	1.243	1.256	1.269
7	1.382	1.261	1.201	1.163	1.138	1.117	1.135	1.150	1.161	1.173
8	1.277	1.172	1.121	1.088	1.064	1.047	1.062	1.075	1.086	1.093
9	1.196	1.100	1.054	1.025	1.004	0.989	1.002	1.013	1.023	1.031
10	1.126	1.040	0.998	0.971	0.953	0.939	0.952	0.961	0.970	0.978
11	1.065	0.987	0.950	0.926	0.908	0.897	0.908	0.916	0.924	0.931
12	1.014	0.943	0.907	0.886	0.870	0.859	0.868	0.877	0.884	0.890
13	0.970	0.904	0.870	0.851	0.836	0.825	0.835	0.842	0.849	0.855
14	0.930	0.868	0.837	0.819	0.805	0.796	0.804	0.811	0.817	0.823
15	0.894	0.837	0.809	0.791	0.779	0.769	0.778	0.784	0.789	0.795
16	0.863	0.809	0.782	0.765	0.754	0.745	0.753	0.759	0.764	0.768
17	0.833	0.783	0.758	0.742	0.731	0.723	0.731	0.736	0.741	0.745
18	0.807	0.759	0.736	0.722	0.712	0.703	0.709	0.716	0.720	0.724
19	0.784	0.738	0.716	0.702	0.692	0.684	0.691	0.696	0.700	0.704
20	0.761	0.718	0.698	0.684	0.674	0.668	0.673	0.678	0.683	0.686
21	0.741	0.700	0.680	0.668	0.658	0.652	0.658	0.662	0.665	0.669
22	0.722	0.682	0.664	0.652	0.643	0.636	0.643	0.646	0.650	0.654
23	0.704	0.666	0.649	0.637	0.629	0.623	0.628	0.632	0.635	0.638
24	0.688	0.652	0.635	0.624	0.616	0.610	0.614	0.619	0.622	0.625
25	0.672	0.638	0.621	0.611	0.603	0.598	0.603	0.607	0.609	0.612

Table 22. AS_{50} values for q=7, n=2(1)25 and various p values.

AS_{50}	beta(p, 7) for mean shift									
n	(1,7)	(2,7)	(3,7)	(4,7)	(5,7)	(6,7)	(7,7)	(8,7)	(9,7)	(10,7)
2	3.007	2.574	2.363	2.231	2.142	2.072	2.021	2.076	2.120	2.157
3	2.332	2.046	1.910	1.820	1.759	1.713	1.678	1.714	1.743	1.768
4	1.959	1.742	1.638	1.572	1.528	1.493	1.465	1.492	1.514	1.532
5	1.712	1.539	1.456	1.403	1.368	1.341	1.318	1.338	1.355	1.370
6	1.534	1.390	1.322	1.278	1.248	1.225	1.207	1.224	1.237	1.250
7	1.402	1.278	1.219	1.181	1.155	1.134	1.119	1.134	1.146	1.157
8	1.296	1.189	1.135	1.103	1.079	1.063	1.049	1.062	1.072	1.080
9	1.209	1.114	1.068	1.039	1.017	1.002	0.990	1.001	1.010	1.017
10	1.138	1.052	1.011	0.983	0.965	0.950	0.940	0.951	0.958	0.966
11	1.077	1.000	0.961	0.936	0.919	0.908	0.897	0.906	0.914	0.921
12	1.025	0.953	0.918	0.896	0.880	0.869	0.861	0.869	0.874	0.881
13	0.978	0.913	0.880	0.860	0.846	0.835	0.826	0.834	0.841	0.846
14	0.939	0.877	0.847	0.828	0.815	0.804	0.797	0.803	0.809	0.815
15	0.903	0.845	0.817	0.799	0.787	0.778	0.770	0.776	0.782	0.787
16	0.870	0.816	0.789	0.774	0.762	0.752	0.746	0.752	0.757	0.761
17	0.842	0.789	0.765	0.750	0.738	0.730	0.724	0.730	0.734	0.739
18	0.813	0.766	0.743	0.728	0.717	0.710	0.703	0.709	0.714	0.718
19	0.791	0.745	0.722	0.708	0.698	0.690	0.685	0.690	0.695	0.698
20	0.768	0.724	0.703	0.690	0.681	0.673	0.668	0.673	0.677	0.680
21	0.747	0.705	0.685	0.673	0.664	0.657	0.652	0.656	0.661	0.664
22	0.727	0.687	0.670	0.657	0.648	0.643	0.637	0.642	0.646	0.648
23	0.710	0.672	0.654	0.643	0.634	0.628	0.624	0.627	0.631	0.634
24	0.693	0.657	0.639	0.628	0.621	0.615	0.610	0.614	0.617	0.620
25	0.678	0.642	0.627	0.615	0.608	0.602	0.598	0.601	0.605	0.608

Table 23. AS_{50} values for q=8, n=2(1)25 and various p values.

AS_{50}	beta(p, 8) for mean shift									
n	(1,8)	(2,8)	(3,8)	(4,8)	(5,8)	(6,8)	(7,8)	(8,8)	(9,8)	(10,8)
2	3.063	2.628	2.419	2.287	2.196	2.129	2.075	2.031	2.078	2.116
3	2.372	2.082	1.944	1.856	1.796	1.749	1.713	1.684	1.713	1.739
4	1.984	1.769	1.664	1.598	1.551	1.520	1.490	1.468	1.491	1.511
5	1.734	1.561	1.476	1.424	1.388	1.360	1.338	1.321	1.337	1.353
6	1.553	1.409	1.339	1.296	1.264	1.241	1.223	1.210	1.222	1.235
7	1.418	1.293	1.235	1.196	1.169	1.149	1.133	1.120	1.134	1.143
8	1.309	1.200	1.149	1.115	1.092	1.074	1.061	1.050	1.060	1.070
9	1.221	1.124	1.079	1.050	1.029	1.013	1.002	0.992	1.000	1.009
10	1.148	1.062	1.021	0.994	0.975	0.962	0.950	0.941	0.949	0.957
11	1.087	1.008	0.970	0.946	0.930	0.916	0.906	0.898	0.906	0.912
12	1.033	0.961	0.927	0.904	0.889	0.877	0.868	0.860	0.867	0.872
13	0.987	0.920	0.888	0.868	0.853	0.842	0.834	0.827	0.833	0.839
14	0.946	0.883	0.854	0.835	0.821	0.811	0.804	0.797	0.803	0.808
15	0.909	0.851	0.823	0.806	0.794	0.783	0.776	0.770	0.776	0.781
16	0.877	0.823	0.797	0.779	0.767	0.759	0.752	0.747	0.752	0.756
17	0.847	0.796	0.770	0.756	0.744	0.736	0.730	0.724	0.729	0.734
18	0.821	0.771	0.748	0.734	0.723	0.715	0.709	0.704	0.709	0.712
19	0.795	0.749	0.727	0.713	0.704	0.697	0.690	0.685	0.690	0.694
20	0.773	0.729	0.708	0.694	0.685	0.678	0.672	0.668	0.672	0.676
21	0.752	0.710	0.690	0.677	0.669	0.662	0.657	0.653	0.656	0.660
22	0.733	0.693	0.673	0.662	0.653	0.646	0.642	0.637	0.641	0.644
23	0.714	0.676	0.658	0.646	0.638	0.633	0.627	0.624	0.627	0.630
24	0.698	0.661	0.644	0.633	0.625	0.619	0.615	0.610	0.614	0.617
25	0.681	0.647	0.630	0.619	0.612	0.606	0.602	0.598	0.601	0.604

Table 24. AS_{50} values for q=9, n=2(1)25 and various p values.

AS_{50}	beta(p, 9) for mean shift									
n	(1,9)	(2,9)	(3,9)	(4,9)	(5,9)	(6,9)	(7,9)	(8,9)	(9,9)	(10,9)
2	3.106	2.675	2.463	2.334	2.243	2.175	2.119	2.076	2.042	2.080
3	2.402	2.114	1.973	1.886	1.825	1.778	1.744	1.713	1.690	1.715
4	2.008	1.790	1.686	1.622	1.576	1.541	1.513	1.492	1.471	1.491
5	1.750	1.578	1.494	1.441	1.405	1.377	1.355	1.337	1.322	1.337
6	1.568	1.423	1.354	1.309	1.280	1.256	1.239	1.222	1.210	1.222
7	1.430	1.303	1.245	1.207	1.181	1.163	1.147	1.133	1.123	1.132
8	1.321	1.211	1.157	1.127	1.103	1.085	1.072	1.059	1.051	1.060
9	1.230	1.135	1.088	1.059	1.039	1.023	1.011	1.000	0.992	1.001
10	1.157	1.070	1.029	1.002	0.983	0.970	0.958	0.949	0.942	0.949
11	1.094	1.015	0.977	0.953	0.937	0.924	0.914	0.905	0.899	0.905
12	1.042	0.967	0.933	0.911	0.896	0.884	0.875	0.867	0.861	0.866
13	0.993	0.926	0.895	0.874	0.860	0.849	0.839	0.834	0.828	0.833
14	0.951	0.890	0.860	0.841	0.828	0.818	0.809	0.803	0.798	0.803
15	0.915	0.857	0.829	0.812	0.799	0.790	0.782	0.776	0.771	0.775
16	0.880	0.826	0.801	0.784	0.773	0.764	0.757	0.751	0.747	0.751
17	0.852	0.800	0.776	0.760	0.749	0.741	0.735	0.729	0.725	0.729
18	0.826	0.776	0.752	0.738	0.728	0.720	0.714	0.709	0.705	0.708
19	0.800	0.754	0.731	0.718	0.707	0.700	0.694	0.690	0.686	0.689
20	0.777	0.733	0.712	0.698	0.689	0.682	0.677	0.673	0.669	0.672
21	0.756	0.714	0.694	0.681	0.673	0.665	0.660	0.656	0.652	0.656
22	0.737	0.696	0.677	0.665	0.656	0.650	0.645	0.641	0.638	0.640
23	0.717	0.680	0.662	0.650	0.642	0.635	0.631	0.627	0.624	0.627
24	0.700	0.664	0.647	0.636	0.629	0.622	0.618	0.613	0.611	0.613
25	0.685	0.650	0.633	0.622	0.615	0.609	0.605	0.601	0.598	0.601

Table 25. AS_{50} values for q=10, n=2(1)25 and various p values.

AS_{50}	beta(p, 10) for mean shift									
n	(1,10)	(2,10)	(3,10)	(4,10)	(5,10)	(6,10)	(7,10)	(8,10)	(9,10)	(10,10)
2	3.150	2.715	2.504	2.375	2.284	2.216	2.161	2.114	2.078	2.048
3	2.431	2.138	1.997	1.912	1.853	1.805	1.768	1.739	1.714	1.694
4	2.027	1.809	1.706	1.641	1.594	1.559	1.530	1.509	1.490	1.475
5	1.765	1.593	1.510	1.456	1.420	1.392	1.369	1.352	1.336	1.324
6	1.579	1.434	1.365	1.323	1.291	1.268	1.249	1.235	1.221	1.212
7	1.440	1.317	1.256	1.219	1.191	1.172	1.156	1.143	1.131	1.124
8	1.328	1.221	1.168	1.135	1.113	1.095	1.082	1.068	1.060	1.052
9	1.239	1.142	1.096	1.067	1.046	1.031	1.019	1.008	0.999	0.993
10	1.163	1.077	1.034	1.008	0.990	0.978	0.965	0.957	0.949	0.941
11	1.100	1.021	0.984	0.960	0.943	0.930	0.920	0.912	0.905	0.899
12	1.046	0.974	0.940	0.917	0.902	0.890	0.881	0.872	0.866	0.861
13	1.000	0.932	0.899	0.880	0.865	0.854	0.845	0.839	0.833	0.827
14	0.957	0.895	0.864	0.845	0.832	0.822	0.815	0.808	0.803	0.798
15	0.920	0.862	0.833	0.815	0.803	0.794	0.787	0.780	0.776	0.771
16	0.886	0.831	0.806	0.788	0.777	0.768	0.761	0.756	0.750	0.747
17	0.855	0.805	0.780	0.765	0.753	0.744	0.739	0.733	0.729	0.725
18	0.828	0.781	0.756	0.742	0.732	0.723	0.717	0.713	0.708	0.704
19	0.804	0.757	0.735	0.721	0.712	0.704	0.697	0.693	0.689	0.686
20	0.780	0.736	0.715	0.702	0.693	0.686	0.680	0.676	0.672	0.669
21	0.759	0.717	0.698	0.685	0.675	0.669	0.663	0.659	0.656	0.652
22	0.739	0.699	0.680	0.668	0.660	0.654	0.648	0.644	0.641	0.638
23	0.721	0.682	0.665	0.653	0.645	0.639	0.634	0.630	0.626	0.624
24	0.704	0.667	0.650	0.639	0.631	0.625	0.621	0.617	0.614	0.611
25	0.688	0.653	0.636	0.625	0.618	0.612	0.607	0.604	0.602	0.598

Table 26. AS_{50} values for v_1=10, n=2(1)25 and various v_2 values.

AS_{50}	v_1=10 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.777	4.618	3.946	3.632	3.448	3.338	3.192	3.109	3.057	2.923	2.811
3	4.606	3.501	2.987	2.754	2.622	2.545	2.443	2.389	2.352	2.265	2.190
4	3.931	2.874	2.454	2.270	2.170	2.108	2.033	1.992	1.965	1.900	1.842
5	3.452	2.471	2.111	1.960	1.878	1.828	1.767	1.734	1.712	1.659	1.617
6	3.106	2.190	1.871	1.739	1.670	1.629	1.579	1.550	1.533	1.490	1.454
7	2.847	1.973	1.687	1.576	1.516	1.479	1.438	1.412	1.398	1.360	1.331
8	2.638	1.803	1.546	1.446	1.394	1.361	1.324	1.304	1.291	1.259	1.232
9	2.469	1.665	1.433	1.342	1.295	1.266	1.234	1.215	1.205	1.176	1.152
10	2.323	1.552	1.338	1.256	1.214	1.188	1.160	1.143	1.132	1.107	1.085
11	2.197	1.458	1.259	1.183	1.145	1.122	1.096	1.080	1.072	1.049	1.030
12	2.095	1.375	1.191	1.121	1.085	1.065	1.040	1.027	1.018	0.998	0.980
13	1.994	1.305	1.131	1.068	1.034	1.016	0.993	0.981	0.972	0.954	0.938
14	1.915	1.244	1.079	1.019	0.990	0.971	0.951	0.940	0.931	0.914	0.900
15	1.839	1.188	1.033	0.978	0.949	0.933	0.913	0.902	0.896	0.879	0.865
16	1.775	1.139	0.992	0.941	0.915	0.898	0.881	0.870	0.864	0.849	0.835
17	1.713	1.096	0.956	0.906	0.882	0.867	0.851	0.841	0.835	0.821	0.808
18	1.654	1.057	0.922	0.876	0.852	0.839	0.823	0.814	0.808	0.795	0.783
19	1.610	1.018	0.892	0.847	0.826	0.813	0.798	0.789	0.784	0.771	0.760
20	1.556	0.985	0.864	0.823	0.801	0.789	0.774	0.767	0.762	0.750	0.739
21	1.515	0.954	0.839	0.798	0.778	0.766	0.753	0.746	0.741	0.730	0.720
22	1.475	0.925	0.815	0.776	0.757	0.747	0.734	0.727	0.721	0.711	0.701
23	1.437	0.901	0.793	0.757	0.738	0.728	0.715	0.708	0.704	0.694	0.685
24	1.400	0.876	0.772	0.737	0.721	0.710	0.698	0.692	0.687	0.677	0.669
25	1.366	0.853	0.754	0.720	0.703	0.694	0.682	0.676	0.672	0.663	0.655

Table 27. AS_{50} values for v_1=15, n=2(1)25 and various v_2 values.

AS_{50}	v_1=15 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.771	4.575	3.894	3.569	3.379	3.256	3.109	3.015	2.956	2.815	2.689
3	4.588	3.476	2.950	2.712	2.568	2.490	2.388	2.326	2.286	2.190	2.109
4	3.905	2.854	2.426	2.238	2.131	2.065	1.991	1.943	1.917	1.843	1.781
5	3.436	2.455	2.087	1.933	1.847	1.794	1.731	1.697	1.671	1.616	1.567
6	3.107	2.171	1.852	1.714	1.643	1.599	1.548	1.518	1.498	1.453	1.410
7	2.852	1.957	1.670	1.554	1.494	1.455	1.411	1.384	1.368	1.330	1.296
8	2.636	1.792	1.531	1.427	1.374	1.340	1.301	1.279	1.265	1.230	1.201
9	2.455	1.654	1.417	1.325	1.277	1.249	1.213	1.195	1.182	1.150	1.125
10	2.320	1.543	1.325	1.242	1.198	1.173	1.141	1.123	1.112	1.084	1.061
11	2.207	1.449	1.247	1.169	1.131	1.108	1.077	1.061	1.052	1.028	1.006
12	2.087	1.368	1.179	1.108	1.073	1.051	1.026	1.010	1.002	0.979	0.959
13	1.994	1.296	1.121	1.056	1.022	1.002	0.979	0.965	0.957	0.936	0.918
14	1.914	1.235	1.069	1.009	0.977	0.959	0.938	0.925	0.918	0.898	0.882
15	1.836	1.181	1.024	0.968	0.939	0.921	0.901	0.889	0.883	0.864	0.849
16	1.768	1.133	0.985	0.931	0.903	0.887	0.869	0.858	0.851	0.834	0.820
17	1.715	1.090	0.948	0.897	0.872	0.857	0.840	0.830	0.822	0.808	0.793
18	1.657	1.049	0.915	0.867	0.844	0.829	0.812	0.803	0.796	0.782	0.770
19	1.605	1.012	0.885	0.840	0.817	0.804	0.788	0.778	0.773	0.759	0.748
20	1.552	0.980	0.858	0.814	0.794	0.781	0.765	0.757	0.751	0.738	0.727
21	1.510	0.949	0.833	0.791	0.771	0.759	0.745	0.737	0.731	0.719	0.708
22	1.473	0.921	0.809	0.770	0.750	0.738	0.726	0.718	0.712	0.701	0.690
23	1.432	0.896	0.787	0.750	0.731	0.720	0.708	0.700	0.695	0.684	0.674
24	1.397	0.872	0.768	0.732	0.714	0.703	0.691	0.684	0.679	0.669	0.659
25	1.369	0.848	0.748	0.714	0.697	0.687	0.675	0.668	0.664	0.654	0.644

Table 28. AS_{50} values for v_1=20, n=2(1)25 and various v_2 values.

AS_{50}	v_1=20 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.773	4.558	3.876	3.543	3.342	3.218	3.059	2.966	2.905	2.753	2.614
3	4.604	3.466	2.929	2.690	2.553	2.464	2.353	2.292	2.251	2.152	2.059
4	3.917	2.853	2.414	2.219	2.113	2.048	1.964	1.918	1.887	1.813	1.746
5	3.440	2.452	2.078	1.918	1.830	1.774	1.711	1.675	1.650	1.592	1.540
6	3.117	2.167	1.840	1.705	1.631	1.588	1.534	1.502	1.482	1.434	1.387
7	2.853	1.952	1.660	1.544	1.482	1.444	1.397	1.368	1.354	1.310	1.275
8	2.639	1.787	1.524	1.418	1.365	1.330	1.290	1.268	1.252	1.217	1.182
9	2.464	1.649	1.412	1.318	1.270	1.238	1.205	1.182	1.170	1.137	1.109
10	2.320	1.537	1.319	1.235	1.188	1.163	1.132	1.112	1.101	1.071	1.046
11	2.196	1.448	1.243	1.163	1.124	1.099	1.070	1.054	1.042	1.017	0.994
12	2.088	1.365	1.175	1.104	1.066	1.044	1.018	1.003	0.994	0.969	0.948
13	2.001	1.294	1.116	1.051	1.015	0.996	0.971	0.959	0.950	0.926	0.907
14	1.912	1.234	1.068	1.004	0.972	0.953	0.932	0.919	0.910	0.889	0.872
15	1.837	1.177	1.021	0.962	0.934	0.916	0.895	0.883	0.876	0.856	0.840
16	1.769	1.129	0.981	0.926	0.899	0.882	0.863	0.852	0.844	0.827	0.811
17	1.711	1.085	0.944	0.894	0.868	0.852	0.834	0.822	0.817	0.800	0.786
18	1.649	1.048	0.911	0.864	0.838	0.824	0.807	0.797	0.791	0.775	0.761
19	1.604	1.012	0.883	0.836	0.813	0.798	0.782	0.773	0.768	0.753	0.739
20	1.555	0.978	0.854	0.810	0.789	0.777	0.760	0.752	0.746	0.732	0.720
21	1.512	0.947	0.829	0.788	0.767	0.755	0.739	0.732	0.726	0.713	0.700
22	1.474	0.920	0.807	0.767	0.746	0.735	0.721	0.713	0.708	0.696	0.684
23	1.432	0.893	0.785	0.747	0.729	0.717	0.704	0.696	0.691	0.679	0.668
24	1.398	0.869	0.765	0.728	0.711	0.700	0.687	0.680	0.675	0.663	0.653
25	1.369	0.847	0.747	0.711	0.694	0.684	0.671	0.665	0.660	0.649	0.638

Table 29. AS_{50} values for v_1 =25, n=2(1)25 and various v_2 values.

AS_{50}	v_1 =25 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.760	4.563	3.859	3.523	3.329	3.197	3.036	2.941	2.874	2.713	2.562
3	4.590	3.466	2.921	2.674	2.536	2.447	2.336	2.273	2.229	2.123	2.024
4	3.918	2.852	2.403	2.210	2.103	2.035	1.953	1.903	1.870	1.794	1.720
5	3.450	2.449	2.070	1.909	1.822	1.768	1.703	1.663	1.639	1.576	1.519
6	3.117	2.162	1.834	1.700	1.622	1.579	1.522	1.494	1.471	1.418	1.372
7	2.843	1.948	1.657	1.539	1.476	1.437	1.388	1.362	1.344	1.300	1.260
8	2.642	1.782	1.521	1.415	1.359	1.324	1.283	1.260	1.245	1.206	1.171
9	2.461	1.650	1.409	1.314	1.264	1.233	1.196	1.176	1.162	1.128	1.098
10	2.319	1.539	1.316	1.232	1.186	1.156	1.125	1.108	1.095	1.063	1.038
11	2.198	1.444	1.238	1.161	1.120	1.094	1.063	1.050	1.037	1.010	0.983
12	2.095	1.364	1.172	1.100	1.063	1.039	1.013	0.998	0.986	0.962	0.939
13	1.994	1.295	1.114	1.047	1.013	0.992	0.967	0.954	0.943	0.921	0.900
14	1.902	1.230	1.064	1.001	0.971	0.950	0.926	0.915	0.905	0.884	0.864
15	1.838	1.175	1.019	0.960	0.930	0.914	0.890	0.879	0.872	0.852	0.832
16	1.763	1.128	0.979	0.925	0.896	0.879	0.860	0.848	0.840	0.821	0.805
17	1.708	1.082	0.944	0.890	0.866	0.849	0.829	0.820	0.812	0.795	0.780
18	1.655	1.044	0.910	0.861	0.837	0.822	0.804	0.793	0.787	0.770	0.755
19	1.604	1.009	0.880	0.834	0.810	0.796	0.779	0.770	0.764	0.748	0.734
20	1.554	0.977	0.853	0.808	0.787	0.774	0.758	0.749	0.743	0.728	0.715
21	1.512	0.945	0.828	0.786	0.765	0.753	0.737	0.729	0.723	0.709	0.696
22	1.475	0.918	0.805	0.765	0.745	0.732	0.718	0.710	0.705	0.692	0.680
23	1.434	0.892	0.784	0.745	0.726	0.715	0.700	0.693	0.688	0.676	0.664
24	1.397	0.867	0.762	0.727	0.709	0.698	0.684	0.676	0.673	0.660	0.650
25	1.367	0.846	0.745	0.709	0.692	0.681	0.668	0.662	0.658	0.646	0.635

Table 30. AS_{50} values for v_1 =30, n=2(1)25 and various v_2 values.

AS_{50}	v_1 =30 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.768	4.564	3.850	3.512	3.312	3.182	3.018	2.916	2.853	2.686	2.533
3	4.590	3.458	2.917	2.671	2.526	2.438	2.323	2.258	2.214	2.106	2.005
4	3.919	2.839	2.400	2.205	2.094	2.024	1.940	1.892	1.860	1.779	1.704
5	3.441	2.442	2.067	1.904	1.820	1.761	1.694	1.654	1.628	1.562	1.502
6	3.106	2.163	1.829	1.694	1.619	1.571	1.515	1.484	1.462	1.410	1.360
7	2.838	1.944	1.657	1.536	1.470	1.431	1.384	1.355	1.337	1.292	1.250
8	2.639	1.785	1.520	1.410	1.356	1.322	1.279	1.255	1.238	1.197	1.162
9	2.459	1.651	1.405	1.311	1.261	1.230	1.193	1.172	1.158	1.122	1.089
10	2.321	1.537	1.313	1.228	1.183	1.156	1.121	1.101	1.090	1.058	1.029
11	2.189	1.444	1.237	1.157	1.117	1.092	1.061	1.044	1.033	1.004	0.978
12	2.089	1.361	1.171	1.098	1.059	1.036	1.009	0.994	0.983	0.958	0.933
13	1.999	1.292	1.114	1.046	1.011	0.989	0.963	0.950	0.941	0.917	0.894
14	1.903	1.230	1.063	0.999	0.968	0.949	0.925	0.910	0.902	0.880	0.858
15	1.841	1.175	1.018	0.959	0.930	0.910	0.888	0.876	0.868	0.847	0.829
16	1.765	1.125	0.978	0.923	0.896	0.877	0.857	0.846	0.838	0.818	0.800
17	1.705	1.084	0.941	0.890	0.864	0.848	0.828	0.816	0.809	0.792	0.776
18	1.646	1.044	0.909	0.861	0.835	0.819	0.802	0.791	0.785	0.767	0.752
19	1.604	1.007	0.878	0.833	0.808	0.795	0.777	0.767	0.761	0.746	0.731
20	1.551	0.977	0.854	0.808	0.785	0.772	0.756	0.747	0.740	0.726	0.711
21	1.513	0.946	0.827	0.784	0.765	0.750	0.736	0.728	0.721	0.707	0.693
22	1.471	0.918	0.803	0.764	0.744	0.731	0.717	0.709	0.703	0.690	0.676
23	1.436	0.891	0.783	0.744	0.725	0.713	0.700	0.691	0.686	0.673	0.660
24	1.403	0.868	0.762	0.726	0.708	0.697	0.684	0.675	0.670	0.658	0.646
25	1.364	0.845	0.744	0.709	0.691	0.680	0.668	0.661	0.655	0.644	0.632

Table 31. AS_{50} values for v_1=40, n=2(1)25 and various v_2 values.

AS_{50}	v_1=40 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.746	4.544	3.845	3.503	3.300	3.160	2.995	2.896	2.825	2.647	2.480
3	4.600	3.449	2.909	2.660	2.517	2.426	2.311	2.242	2.195	2.079	1.968
4	3.908	2.838	2.396	2.197	2.090	2.019	1.933	1.881	1.846	1.759	1.676
5	3.438	2.445	2.067	1.901	1.810	1.754	1.687	1.644	1.618	1.549	1.483
6	3.098	2.162	1.831	1.691	1.616	1.567	1.508	1.476	1.455	1.396	1.342
7	2.847	1.951	1.653	1.532	1.468	1.425	1.377	1.347	1.329	1.280	1.234
8	2.639	1.783	1.516	1.409	1.350	1.315	1.273	1.249	1.233	1.190	1.149
9	2.460	1.646	1.404	1.308	1.259	1.226	1.188	1.166	1.152	1.115	1.079
10	2.316	1.539	1.311	1.224	1.180	1.152	1.118	1.098	1.084	1.051	1.020
11	2.189	1.442	1.235	1.156	1.115	1.090	1.058	1.039	1.029	0.996	0.970
12	2.092	1.363	1.169	1.095	1.057	1.034	1.006	0.990	0.978	0.951	0.925
13	1.989	1.292	1.110	1.044	1.009	0.987	0.961	0.945	0.936	0.911	0.886
14	1.908	1.229	1.060	0.996	0.965	0.946	0.922	0.907	0.898	0.875	0.851
15	1.838	1.174	1.016	0.956	0.928	0.907	0.885	0.874	0.864	0.842	0.822
16	1.773	1.126	0.976	0.920	0.892	0.875	0.855	0.842	0.833	0.814	0.794
17	1.709	1.083	0.939	0.889	0.861	0.846	0.826	0.815	0.807	0.787	0.770
18	1.655	1.042	0.908	0.858	0.833	0.818	0.799	0.788	0.782	0.764	0.746
19	1.600	1.009	0.877	0.832	0.809	0.793	0.776	0.765	0.759	0.742	0.725
20	1.549	0.977	0.851	0.807	0.783	0.770	0.754	0.744	0.738	0.722	0.706
21	1.513	0.945	0.826	0.783	0.762	0.750	0.734	0.724	0.719	0.703	0.688
22	1.469	0.919	0.804	0.762	0.742	0.730	0.715	0.706	0.700	0.686	0.672
23	1.436	0.891	0.781	0.742	0.724	0.711	0.697	0.689	0.683	0.670	0.656
24	1.397	0.868	0.762	0.725	0.706	0.695	0.681	0.673	0.668	0.654	0.641
25	1.369	0.846	0.743	0.707	0.690	0.679	0.666	0.659	0.653	0.640	0.628

Table 32. AS_{50} values for v_1=50, n=2(1)25 and various v_2 values.

AS_{50}	v_1=50 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞										
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.762	4.536	3.851	3.488	3.290	3.157	2.986	2.875	2.808	2.624	2.443
3	4.590	3.447	2.912	2.658	2.514	2.421	2.304	2.232	2.185	2.066	1.946
4	3.907	2.840	2.396	2.199	2.085	2.012	1.925	1.875	1.840	1.749	1.662
5	3.452	2.441	2.063	1.897	1.810	1.752	1.680	1.638	1.612	1.540	1.468
6	3.106	2.154	1.828	1.687	1.612	1.563	1.506	1.472	1.448	1.392	1.330
7	2.844	1.950	1.654	1.530	1.466	1.423	1.373	1.344	1.324	1.275	1.226
8	2.632	1.780	1.514	1.408	1.348	1.311	1.270	1.244	1.227	1.182	1.140
9	2.465	1.649	1.401	1.306	1.256	1.224	1.185	1.161	1.148	1.109	1.070
10	2.322	1.535	1.311	1.223	1.178	1.149	1.114	1.095	1.081	1.047	1.013
11	2.191	1.438	1.232	1.154	1.112	1.087	1.055	1.036	1.025	0.993	0.962
12	2.084	1.359	1.168	1.093	1.055	1.031	1.004	0.987	0.976	0.946	0.918
13	1.990	1.289	1.109	1.042	1.008	0.984	0.959	0.943	0.934	0.907	0.882
14	1.912	1.228	1.059	0.997	0.964	0.944	0.919	0.905	0.896	0.871	0.848
15	1.835	1.173	1.015	0.957	0.925	0.906	0.884	0.870	0.862	0.839	0.816
16	1.766	1.126	0.977	0.921	0.891	0.874	0.854	0.840	0.832	0.810	0.790
17	1.711	1.084	0.939	0.887	0.861	0.844	0.824	0.812	0.805	0.785	0.765
18	1.649	1.043	0.908	0.857	0.832	0.817	0.798	0.787	0.780	0.761	0.743
19	1.610	1.008	0.878	0.831	0.807	0.791	0.775	0.764	0.757	0.739	0.722
20	1.552	0.973	0.850	0.806	0.783	0.770	0.752	0.743	0.737	0.720	0.702
21	1.515	0.945	0.826	0.783	0.761	0.749	0.732	0.723	0.717	0.701	0.685
22	1.471	0.917	0.803	0.762	0.741	0.729	0.713	0.704	0.698	0.684	0.669
23	1.434	0.890	0.781	0.743	0.722	0.711	0.696	0.688	0.682	0.668	0.653
24	1.397	0.867	0.761	0.724	0.706	0.694	0.681	0.673	0.667	0.653	0.639
25	1.361	0.844	0.743	0.708	0.689	0.678	0.665	0.657	0.652	0.639	0.626

Table 33. AS_{50} values for v_1=60, n=2(1)25 and various v_2 values.

AS_{50}	v_1=60 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞										
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.745	4.542	3.839	3.495	3.284	3.150	2.975	2.873	2.801	2.610	2.420
3	4.597	3.455	2.904	2.652	2.514	2.414	2.302	2.228	2.178	2.058	1.930
4	3.921	2.843	2.391	2.193	2.082	2.011	1.920	1.868	1.831	1.742	1.650
5	3.439	2.445	2.059	1.895	1.807	1.750	1.678	1.637	1.607	1.535	1.461
6	3.099	2.157	1.825	1.686	1.609	1.563	1.502	1.469	1.443	1.386	1.323
7	2.843	1.945	1.652	1.530	1.464	1.426	1.371	1.344	1.322	1.270	1.219
8	2.628	1.778	1.514	1.404	1.348	1.311	1.268	1.241	1.224	1.180	1.135
9	2.472	1.646	1.402	1.305	1.257	1.222	1.182	1.161	1.145	1.106	1.065
10	2.317	1.533	1.310	1.223	1.178	1.148	1.114	1.092	1.078	1.045	1.008
11	2.187	1.439	1.234	1.155	1.112	1.086	1.054	1.035	1.023	0.991	0.958
12	2.081	1.358	1.168	1.093	1.055	1.032	1.003	0.985	0.974	0.944	0.916
13	1.998	1.292	1.110	1.043	1.007	0.984	0.958	0.942	0.930	0.904	0.877
14	1.904	1.228	1.060	0.997	0.963	0.944	0.918	0.904	0.895	0.869	0.843
15	1.840	1.173	1.016	0.957	0.926	0.906	0.883	0.870	0.861	0.838	0.814
16	1.770	1.127	0.975	0.918	0.890	0.873	0.853	0.840	0.831	0.809	0.786
17	1.712	1.081	0.939	0.887	0.860	0.843	0.824	0.811	0.803	0.783	0.763
18	1.658	1.042	0.906	0.858	0.831	0.816	0.797	0.785	0.779	0.759	0.739
19	1.602	1.006	0.876	0.830	0.806	0.791	0.774	0.763	0.755	0.738	0.719
20	1.551	0.975	0.850	0.805	0.783	0.768	0.752	0.742	0.736	0.717	0.700
21	1.515	0.943	0.825	0.783	0.762	0.747	0.732	0.722	0.716	0.699	0.682
22	1.472	0.916	0.802	0.761	0.741	0.728	0.713	0.704	0.698	0.683	0.666
23	1.430	0.891	0.782	0.743	0.723	0.710	0.696	0.687	0.681	0.666	0.651
24	1.400	0.865	0.761	0.724	0.704	0.693	0.679	0.671	0.666	0.652	0.637
25	1.364	0.844	0.742	0.707	0.690	0.678	0.664	0.657	0.651	0.637	0.624

Table 34. AS_{50} values for v_1=120, n=2(1)25 and various v_2 values.

AS_{50}	v_1=120 and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞										
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.760	4.554	3.837	3.480	3.279	3.141	2.958	2.850	2.779	2.572	2.347
3	4.574	3.450	2.902	2.647	2.504	2.409	2.289	2.217	2.167	2.030	1.883
4	3.897	2.837	2.389	2.191	2.077	2.006	1.912	1.860	1.823	1.722	1.612
5	3.457	2.440	2.056	1.895	1.800	1.745	1.670	1.627	1.596	1.519	1.430
6	3.102	2.157	1.823	1.683	1.606	1.557	1.498	1.460	1.437	1.372	1.299
7	2.832	1.942	1.652	1.526	1.461	1.419	1.368	1.337	1.314	1.259	1.197
8	2.635	1.778	1.511	1.404	1.345	1.309	1.264	1.237	1.220	1.172	1.117
9	2.461	1.647	1.401	1.303	1.251	1.219	1.180	1.156	1.140	1.098	1.050
10	2.327	1.535	1.309	1.221	1.176	1.145	1.111	1.090	1.075	1.036	0.993
11	2.191	1.440	1.233	1.153	1.110	1.083	1.051	1.033	1.018	0.983	0.945
12	2.092	1.360	1.165	1.093	1.054	1.029	0.999	0.982	0.971	0.938	0.903
13	1.990	1.289	1.109	1.041	1.005	0.983	0.955	0.939	0.928	0.900	0.866
14	1.915	1.229	1.059	0.995	0.962	0.941	0.916	0.902	0.891	0.864	0.834
15	1.832	1.173	1.016	0.954	0.923	0.905	0.881	0.866	0.857	0.833	0.804
16	1.773	1.124	0.973	0.919	0.890	0.871	0.850	0.838	0.828	0.804	0.778
17	1.708	1.082	0.939	0.886	0.858	0.842	0.822	0.809	0.801	0.779	0.754
18	1.652	1.045	0.906	0.857	0.831	0.815	0.796	0.783	0.777	0.755	0.731
19	1.601	1.006	0.876	0.831	0.805	0.789	0.771	0.761	0.754	0.734	0.711
20	1.554	0.974	0.849	0.805	0.782	0.768	0.751	0.740	0.733	0.715	0.693
21	1.513	0.945	0.824	0.782	0.760	0.746	0.730	0.720	0.714	0.696	0.676
22	1.476	0.916	0.801	0.761	0.740	0.727	0.712	0.703	0.696	0.679	0.659
23	1.432	0.891	0.781	0.742	0.722	0.710	0.694	0.685	0.679	0.663	0.645
24	1.401	0.868	0.761	0.723	0.705	0.692	0.678	0.670	0.665	0.648	0.629
25	1.366	0.844	0.742	0.707	0.688	0.676	0.664	0.655	0.650	0.634	0.617

Table 35. AS_{50} values for $v_1=\infty$, n=2(1)25 and various v_2 values.

AS_{50}	$v_1=\infty$ and v_2 for 5 10 15 20 25 30 40 50 60 120 ∞									
n	5	10	15	20	25	30	40	50	60	120	∞
2	5.761	4.545	3.834	3.481	3.271	3.136	2.953	2.840	2.766	2.551	2.253
3	4.573	3.445	2.906	2.646	2.504	2.404	2.283	2.208	2.157	2.016	1.821
4	3.912	2.842	2.389	2.188	2.080	2.004	1.909	1.852	1.818	1.709	1.566
5	3.444	2.443	2.056	1.893	1.802	1.742	1.669	1.626	1.595	1.510	1.393
6	3.105	2.156	1.827	1.681	1.607	1.556	1.495	1.459	1.433	1.364	1.268
7	2.844	1.942	1.647	1.527	1.459	1.417	1.366	1.334	1.314	1.254	1.171
8	2.629	1.779	1.512	1.403	1.345	1.306	1.262	1.235	1.215	1.164	1.093
9	2.458	1.646	1.402	1.303	1.252	1.221	1.178	1.153	1.138	1.092	1.028
10	2.317	1.532	1.311	1.222	1.175	1.147	1.109	1.087	1.073	1.031	0.974
11	2.190	1.439	1.231	1.153	1.109	1.082	1.048	1.030	1.019	0.981	0.928
12	2.092	1.358	1.166	1.091	1.053	1.028	0.998	0.981	0.968	0.935	0.888
13	1.992	1.289	1.109	1.040	1.005	0.982	0.955	0.938	0.926	0.895	0.852
14	1.911	1.228	1.058	0.994	0.961	0.941	0.916	0.899	0.888	0.860	0.820
15	1.835	1.173	1.015	0.954	0.923	0.906	0.881	0.866	0.856	0.829	0.792
16	1.769	1.126	0.975	0.918	0.889	0.872	0.849	0.836	0.827	0.801	0.766
17	1.710	1.082	0.939	0.886	0.859	0.841	0.821	0.809	0.799	0.776	0.742
18	1.648	1.042	0.905	0.857	0.831	0.814	0.795	0.782	0.776	0.753	0.721
19	1.599	1.006	0.877	0.829	0.805	0.789	0.771	0.761	0.752	0.732	0.701
20	1.554	0.974	0.849	0.804	0.782	0.767	0.749	0.739	0.732	0.711	0.684
21	1.513	0.943	0.825	0.782	0.760	0.746	0.730	0.720	0.713	0.693	0.667
22	1.476	0.916	0.801	0.760	0.741	0.726	0.711	0.702	0.694	0.676	0.651
23	1.438	0.889	0.780	0.741	0.722	0.708	0.693	0.684	0.678	0.661	0.637
24	1.404	0.866	0.759	0.723	0.704	0.692	0.678	0.669	0.662	0.647	0.623
25	1.364	0.844	0.742	0.706	0.688	0.676	0.662	0.654	0.649	0.632	0.610