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Chaos synchronization by variable strength linear coupling and Lyapunov function derivative in series form

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Abstract

A new general strategy to achieve chaos synchronization by variable strength linear coupling without another active control is proposed. They give the criteria of chaos synchronization for two identical chaotic systems and two different chaotic dynamic systems with variable strength linear coupling. In this method, the time derivative of Lyapunov function in series form is firstly used. Lorenz system, Duffing system, Rössler system and Hyper-Rössler system are presented as simulated examples.

Keywords: Chaos; Synchronization; Linear coupling; Coupled chaotic systems

1. Introduction

In recent years, synchronization in chaotic dynamic system has been a very interesting problem and has been widely studied [1–3]. Synchronization means that the state variables of a response system approach eventually to that of a drive system. There are many control techniques to synchronize chaotic systems, such as linear error feedback control, adaptive control, active control [2–17].

In this paper, a new general strategy to achieve chaos synchronization by variable strength linear coupling is proposed. This method, in which the time derivative of Lyapunov function in series form is firstly used, can give either local synchronization which is usually good enough or global synchronization which is usually an unnecessary high demand [18–21].

This paper is organized as follows. In Section 2, synchronization strategy by variable strength linear coupling without another active control is proposed, in which the Lyapunov function derivative in series form is first used. In Section 3, Lorenz system, Duffing system, Rössler system and Hyper-Rössler system are presented as simulated examples. In Section 4, conclusions are given.

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2. Synchronization strategy by variable strength linear coupling and Lyapunov function derivative in series form

(a) Consider the following unidirectional coupled identical chaotic systems

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{f}(\mathbf{x})\\ \dot{\mathbf{y}} &= \mathbf{A}\mathbf{y} + \mathbf{f}(\mathbf{y}) + \boldsymbol{\Gamma}(\mathbf{y} - \mathbf{x}),\end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$, $\mathbf{y} = [y_1, y_2, \ldots, y_n]^{\mathrm{T}} \in R^n$ denote two state vectors, $\mathbf{A}$ is an $n \times n$ constant coefficient matrix, $\mathbf{f}$ is a nonlinear vector function, and $\boldsymbol{\Gamma}$ is an $n \times n$ matrix which gives the variable strength of the linear coupling term $(\mathbf{y} - \mathbf{x})$.

In order to study the synchronization of $\mathbf{x}$ and $\mathbf{y}$, define $\mathbf{e} = \mathbf{y} - \mathbf{x}$ as the state error. Error equation can be written as

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{y} + \mathbf{f}(\mathbf{y}) + \boldsymbol{\Gamma}(\mathbf{y} - \mathbf{x}) - \mathbf{A}\mathbf{x} - \mathbf{f}(\mathbf{x}). \tag{2}$$

By Taylor expansion

$$\begin{aligned}\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}) &= \mathbf{f}(\mathbf{x} + \mathbf{e}) - \mathbf{f}(\mathbf{x}) = \mathbf{f}'(\mathbf{x})\mathbf{e} + \text{HOT of } \mathbf{e}\\ &= \mathbf{F}(\mathbf{x})\mathbf{e} + \text{HOT of } \mathbf{e},\end{aligned} \tag{3}$$

where $\mathbf{f}'(\mathbf{x})$ is the time derivative $\mathbf{f}(\mathbf{x})$, and $\mathbf{F}(\mathbf{x}) = \mathbf{f}'(\mathbf{x})$.

Theorem 1. *The chaotic systems in Eq. (1) can be locally completely synchronized, if $\|\mathbf{e}\|^2$ is smaller than a bounded value and $\boldsymbol{\Gamma}$ is chosen such that $\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{F} = -\mathbf{C}$, where $\mathbf{C}$ is a positive definite diagonal matrix.*

Proof. Choose a positive definite function as

$$V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{e}. \tag{4}$$

Then

$$\begin{aligned}\dot{V}(\mathbf{e}) &= \mathbf{e}^{\mathrm{T}}\dot{\mathbf{e}}\\ &= \mathbf{e}^{\mathrm{T}}(\mathbf{A}\mathbf{y} + \mathbf{f}(\mathbf{y}) + \boldsymbol{\Gamma}(\mathbf{y} - \mathbf{x}) - \mathbf{A}\mathbf{x} - \mathbf{f}(\mathbf{x}))\\ &= \mathbf{e}^{\mathrm{T}}(\mathbf{A}\mathbf{e} + \boldsymbol{\Gamma}\mathbf{e} + \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}))\\ &= \mathbf{e}^{\mathrm{T}}(\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{F})\mathbf{e} + \text{HOT of } \mathbf{e}.\end{aligned} \tag{5}$$

Since $\|\mathbf{e}\|^2$ is smaller than a bounded value and $\boldsymbol{\Gamma}$ is chosen such that $\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{F} = -\mathbf{C}$, Eq. (5) becomes $\dot{V}(\mathbf{e}) = -\mathbf{e}^{\mathrm{T}}\mathbf{C}\mathbf{e} + \text{HOT of } \mathbf{e} < 0$, since $-\mathbf{e}^{\mathrm{T}}\mathbf{C}\mathbf{e}$ is a definite form, the higher-order terms of $\mathbf{e}$ have no influence on the definiteness of $\dot{V}$, provided that $\|\mathbf{e}\|^2$ is smaller than a bounded value. The proof of this theorem can be found in [22,23], which is used extensively in the theory of stability of motion. By the Lyapunov asymptotical stability theorem, the origin of error equation (2) is locally asymptotically stable and the chaotic systems in Eq. (1) are locally completely synchronized. □

Corollary 1. *If $\mathbf{f}(\mathbf{x} + \mathbf{e}) - \mathbf{f}(\mathbf{x})$ is a linear function of $\mathbf{e}, \mathbf{D}\mathbf{e}$, Eq. (5) becomes $\dot{V}(\mathbf{e}) = \mathbf{e}^{\mathrm{T}}(\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{D})\mathbf{e}$. Let $\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{D} = -\mathbf{C}$, then $\dot{V}(\mathbf{e}) = -\mathbf{e}^{\mathrm{T}}\mathbf{C}\mathbf{e} < 0$. By the Lyapunov asymptotical stability theorem, the origin of error equation (2) is globally asymptotically stable. Hence, the chaotic systems in Eq. (1) are globally completely synchronized.* □

(b) Consider the following two unidirectional coupled different chaotic systems

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{f}(\mathbf{x})\\ \dot{\mathbf{y}} &= \hat{\mathbf{A}}\mathbf{y} + \mathbf{f}(\mathbf{y}) + \mathbf{u},\end{aligned} \tag{6}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$, $\mathbf{y} = [y_1, y_2, \ldots, y_n]^{\mathrm{T}} \in R^n$ denote two state vectors, $\mathbf{A}$ and $\hat{\mathbf{A}}$ are two different $n \times n$ constant coefficient matrices, $\mathbf{f}$ is a nonlinear vector function, and $\mathbf{u}$ is the coupling vector of which the elements are functions of $\mathbf{x}$ and $\mathbf{y}$.

In order to study the synchronization of $\mathbf{x}$ and $\mathbf{y}$, define $\mathbf{e} = \mathbf{y} - \mathbf{x}$ as the state error. Error equation can be written as

$$\dot{\mathbf{e}} = \hat{\mathbf{A}}\mathbf{y} + \mathbf{f}(\mathbf{y}) + \mathbf{u} - \mathbf{A}\mathbf{x} - \mathbf{f}(\mathbf{x}). \tag{7}$$

By Taylor expansion

$$\begin{aligned} \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}) &= \mathbf{f}(\mathbf{x}+\mathbf{e}) - \mathbf{f}(\mathbf{x}) = \mathbf{f}'(\mathbf{x})\mathbf{e} + \text{HOT of } \mathbf{e} \\ &= \mathbf{F}(\mathbf{x})\mathbf{e} + \text{HOT of } \mathbf{e}. \end{aligned} \tag{8}$$

Theorem 2. *Choose* $\boldsymbol{\Gamma} = -\mathbf{C} - \mathbf{A} - \mathbf{F}$ *and* $\mathbf{B} = -\tilde{\mathbf{A}}$, *where* $\mathbf{C}$ *is positive definite diagonal matrix and* $\tilde{\mathbf{A}} = \hat{\mathbf{A}} - \mathbf{A}$. *The chaotic systems in Eq. (6) can be locally completely synchronized, if* $\|\mathbf{e}\|^2$ *is smaller than a bounded value and* $\mathbf{u} = \boldsymbol{\Gamma}\mathbf{e} + \mathbf{B}\mathbf{y}$.

Proof. Choose a positive definite function as

$$V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{e}. \tag{9}$$

Then

$$\begin{aligned} \dot{V}(\mathbf{e}) &= \mathbf{e}^{\mathrm{T}}\dot{\mathbf{e}} \\ &= \mathbf{e}^{\mathrm{T}}(\hat{\mathbf{A}}y + \mathbf{f}(\mathbf{y}) + \mathbf{u} - \mathbf{A}\mathbf{x} - \mathbf{f}(\mathbf{x})) \\ &= \mathbf{e}^{\mathrm{T}}(\tilde{\mathbf{A}}y + \mathbf{A}\mathbf{e} + \mathbf{u} + \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x})). \end{aligned} \tag{10}$$

Let $\mathbf{u} = \boldsymbol{\Gamma}\mathbf{e} + \mathbf{B}\mathbf{y}$, Eq. (10) becomes

$$\begin{aligned} \dot{V}(\mathbf{e}) &= \mathbf{e}^{\mathrm{T}}(\tilde{\mathbf{A}}\mathbf{y} + \mathbf{A}\mathbf{e} + \boldsymbol{\Gamma}\mathbf{e} + \mathbf{B}\mathbf{y} + \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x})) \\ &= \mathbf{e}^{\mathrm{T}}(\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{F})\mathbf{e} + \mathbf{e}^{\mathrm{T}}(\tilde{\mathbf{A}} + \mathbf{B})\mathbf{y} + \text{HOT of } \mathbf{e}. \end{aligned} \tag{11}$$

Since $\|\mathbf{e}\|^2$ is smaller than a bounded value, $\boldsymbol{\Gamma}$ and $\mathbf{B}$ are chosen such that $\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{F} = -\mathbf{C}$ and $\mathbf{B} = -\tilde{\mathbf{A}}$, Eq. (10) becomes $\dot{V}(\mathbf{e}) = -\mathbf{e}^{\mathrm{T}}\mathbf{C}\mathbf{e}$+HOT of $\mathbf{e} < 0$. By the Lyapunov asymptotical stability theorem, the origin of error equation (7) is locally asymptotically stable and the chaotic systems in Eq. (6) are locally completely synchronized. □

Corollary 2. *If* $\mathbf{f}(\mathbf{x}+\mathbf{e}) - \mathbf{f}(\mathbf{x})$ *is a linear function of* $\mathbf{e}$, $\mathbf{D}\mathbf{e}$, *Eq. (11) becomes* $\dot{V}(\mathbf{e}) = \mathbf{e}^{\mathrm{T}}(\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{D})\mathbf{e} + \mathbf{e}^{\mathrm{T}}(\tilde{\mathbf{A}} + \mathbf{B})\mathbf{y}$. *Let* $\mathbf{A} + \boldsymbol{\Gamma} + \mathbf{D} = -\mathbf{C}$ *and* $\mathbf{B} = -\tilde{\mathbf{A}}$, *then* $\dot{V}(\mathbf{e}) = -\mathbf{e}^{\mathrm{T}}\mathbf{C}\mathbf{e} < 0$. *By the Lyapunov asymptotical stability theorem, the origin of error equation (7) is globally asymptotically stable, and the chaotic systems in Eq. (6) are globally completely synchronized.* □

3. Numerical results for typical chaotic systems

First example for Theorem 1 is the Rössler system. Consider the following two unidirectional coupled chaotic Rössler systems:

$$\begin{aligned} \dot{x}_1 &= -y_1 - z_1 \\ \dot{y}_1 &= x_1 + ay_1 \\ \dot{z}_1 &= b + z_1(x_1 - c) \\ \dot{x}_2 &= -y_2 - z_2 + \Gamma_{11}e_1 + \Gamma_{12}e_2 + \Gamma_{13}e_3 \\ \dot{y}_2 &= x_2 + ay_2 + \Gamma_{21}e_1 + \Gamma_{22}e_2 + \Gamma_{23}e_3 \\ \dot{z}_2 &= b + z_2(x_2 - c) + \Gamma_{31}e_1 + \Gamma_{32}e_2 + \Gamma_{33}e_3, \end{aligned} \tag{12}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ 0 & 0 & -c \end{bmatrix}. \tag{13}$$

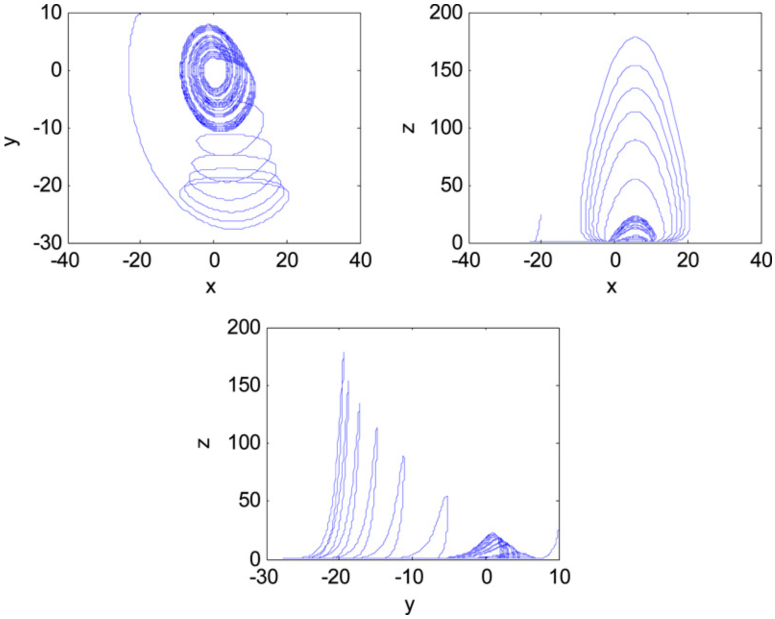

Fig. 1. Chaotic phase portraits for the Rössler system.

Choose a Lyapunov function in the form of a positive definite function:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{14}$$

by Taylor Formula

$$\begin{aligned}\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}) &= \begin{bmatrix} 0 \\ 0 \\ z_1 e_1 + x_1 e_3 + e_1 e_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ z_1 & 0 & x_1 \end{bmatrix} \mathbf{e} + \begin{bmatrix} 0 \\ 0 \\ e_1 e_3 \end{bmatrix} \\ &= \mathbf{Fe} + \cdots . \end{aligned} \tag{15}$$

Let

$$\mathbf{\Gamma} = -\mathbf{I} - \mathbf{A} - \mathbf{F} = \begin{bmatrix} -1 & 1 & 1 \\ -1 & -1-a & 0 \\ -z_1 & 0 & -1+c-x_1 \end{bmatrix}. \tag{16}$$

According to Theorem 1, we obtain that

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 + \text{HOT of } \mathbf{e} < 0 \tag{17}$$

is negative definite when $\|\mathbf{e}\|^2$ is smaller than a bounded value. The Rössler systems in Eq. (12) are locally synchronized. For the initial states $(-20, 10, 25)$, $(-21, 10.5, 25)$ and system parameters $a = 0.2$, $b = 0.2$, $c = 5.7$, the chaotic phase portraits and state errors versus time are shown in Figs. 1 and 2.

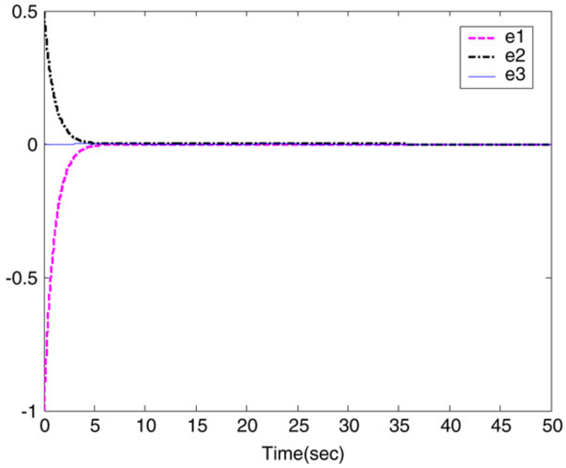

Fig. 2. Time histories of errors for two Rössler systems.

Second example for Corollary 1 is the Hyper-Rössler system. Consider the following two unidirectional coupled chaotic Hyper-Rössler systems:

$$\begin{aligned}
&\dot{x}_1 = -x_2 - x_3\\
&\dot{x}_2 = x_1 + ax_2 + x_4\\
&\dot{x}_3 = b + x_1x_3\\
&\dot{x}_4 = cx_4 - dx_3\\
&\dot{y}_1 = -y_2 - y_3 + \Gamma_{11}e_1 + \Gamma_{12}e_2 + \Gamma_{13}e_3 + \Gamma_{14}e_4\\
&\dot{y}_2 = y_1 + ay_2 + y_4 + \Gamma_{21}e_1 + \Gamma_{22}e_2 + \Gamma_{23}e_3 + \Gamma_{24}e_4\\
&\dot{y}_3 = b + y_1y_3 + \Gamma_{31}e_1 + \Gamma_{32}e_2 + \Gamma_{33}e_3 + \Gamma_{34}e_4\\
&\dot{y}_4 = cy_4 - dy_3 + \Gamma_{41}e_1 + \Gamma_{42}e_2 + \Gamma_{43}e_3 + \Gamma_{44}e_4,
\end{aligned} \tag{18}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & a & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -d & c \end{bmatrix}. \tag{19}$$

Choose a Lyapunov function in the form of a positive definite function:

$$V(e_1, e_2, e_3, e_4) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \tag{20}$$

$$\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ y_1y_3 - x_1x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} y_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & x_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{e} = \mathbf{De}. \tag{21}$$

Let

$$\mathbf{\Gamma} = -\mathbf{C} - \mathbf{A} - \mathbf{D} = \begin{bmatrix} -1 - y_3 & 1 & 1 & 0 \\ -1 & -1 - a & 0 & -1 \\ 0 & 0 & -1 - x_1 & 0 \\ 0 & 0 & d & -1 - c \end{bmatrix}. \tag{22}$$

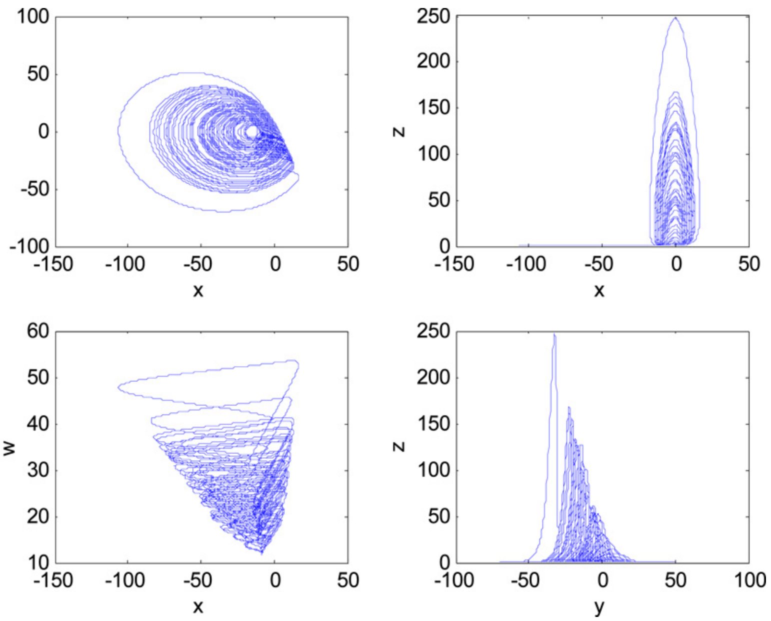

Fig. 3. Chaotic phase portraits for the Hyper-Rössler system.

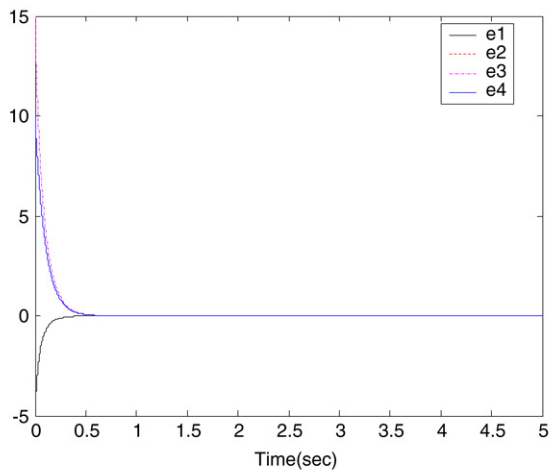

Fig. 4. Time histories of errors for two synchronized Hyper-Rössler systems.

According to Corollary 1, we obtain

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0. \tag{23}$$

The Hyper-Rössler systems in Eq. (18) are globally synchronized. For the initial states $(-20, 0, 0, 15)$, $(-20, 10.15, 15)$ and system parameters $a = 0.25$, $b = 3$, $c = 0.05$, $d = 0.5$, the chaotic phase portraits and state errors versus time are shown in Figs. 3 and 4.

Third example for Theorem 2 is the Duffing system. Consider the following two unidirectional coupled chaotic Duffing systems:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\delta x_2 + \alpha x_1 - \beta x_1^3 + a \cos \omega t \end{aligned} \tag{24}$$

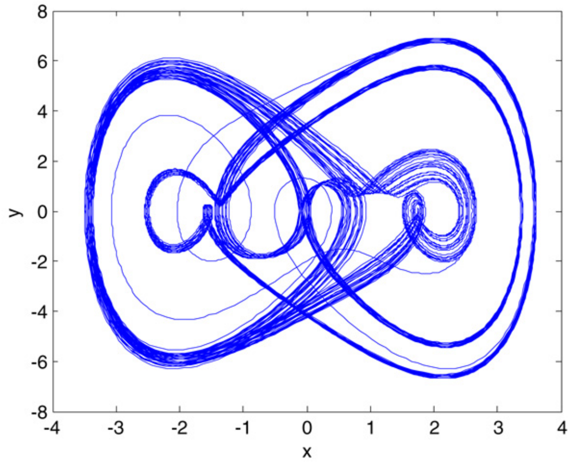

Fig. 5. Chaotic phase portrait for the Duffing system.

$$\dot{y}_1 = y_2 + u_1$$
$$\dot{y}_2 = -\hat{\delta} y_2 + \hat{\alpha} y_1 - \beta y_1^3 + a \cos \omega t + u_2,$$

where $\mathbf{u} = [u_1, u_2]^{\mathrm{T}}$ is the coupling term.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ \alpha & -\delta \end{bmatrix}. \tag{25}$$

Choose a Lyapunov function in the form of a positive definite function:

$$V(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2). \tag{26}$$

By Taylor expansion

$$\begin{aligned} \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}) &= \begin{bmatrix} 0 \\ -\beta y_1^3 + \beta x_1^3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -3\beta x_1^2 & 0 \end{bmatrix} \mathbf{e} + \begin{bmatrix} 0 \\ -6\beta x_1 e_1^2 + \cdots \end{bmatrix} \\ &= \mathbf{Fe} + \text{H.O.T. of } \mathbf{e}. \end{aligned} \tag{27}$$

Let $\mathbf{u} = \mathbf{\Gamma e} + \mathbf{By}$

$$\mathbf{\Gamma} = -\mathbf{I} - \mathbf{A} - \mathbf{F} = \begin{bmatrix} -1 & -1 \\ -\alpha + 3\beta x_1^2 & -1 + \delta \end{bmatrix} \tag{28}$$

$$\mathbf{B} = -\tilde{\mathbf{A}} = \begin{bmatrix} 0 & 0 \\ \hat{\alpha} - \alpha & -\hat{\delta} + \delta \end{bmatrix}. \tag{29}$$

According to Theorem 2, we obtain that

$$\dot{V} = -e_1^2 - e_2^2 + \text{HOT of } \mathbf{e} < 0 \tag{30}$$

is negative definite when $\|\mathbf{e}\|^2$ is smaller than a bounded value. The Duffing systems (24) are locally synchronized. For the initial states (2, 2), (5, 5) and system parameters $\alpha = -0.01$, $\delta = 0.1$, $\beta = \omega = 1$, $a = 10$, $\hat{\alpha} = 1$ and $\hat{\delta} = 0.15$, the chaotic phase portrait and state errors versus time are shown in Figs. 5 and 6.

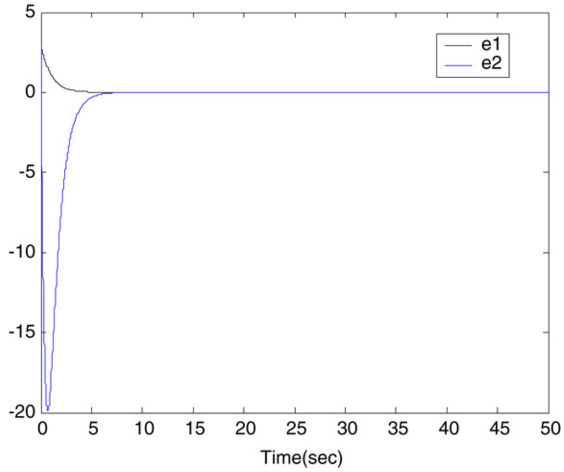

Fig. 6. Time histories of errors for two synchronized Duffing systems.

Last example for Corollary 2 is the Lorenz system. Consider the following two unidirectional coupled chaotic Lorenz systems:

$$\begin{aligned}
\dot{x}_1 &= \sigma(y_1 - x_1) \\
\dot{y}_1 &= \gamma x_1 - x_1 z_1 - y_1 \\
\dot{z}_1 &= x_1 y_1 - \beta z_1 \\
\dot{x}_2 &= \hat{\sigma}(y_2 - x_2) + u_1 \\
\dot{y}_2 &= \hat{\gamma} x_2 - x_2 z_2 - y_2 + u_2 \\
\dot{z}_2 &= x_2 y_2 - \hat{\beta} z_2 + u_3,
\end{aligned} \tag{31}$$

where $\mathbf{u} = [u_1, u_2, u_3]^{\mathrm{T}}$ is the coupling term.

$$\mathbf{A} = \begin{bmatrix} -\sigma & \sigma & 0 \\ \gamma & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}. \tag{32}$$

Choose a Lyapunov function in the form of a positive definite function:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{33}$$

$$\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}) = \begin{bmatrix} 0 \\ -x_2 z_2 + x_1 z_1 \\ x_2 y_2 - x_1 y_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -z_2 & 0 & -x_1 \\ y_2 & x_1 & 0 \end{bmatrix} \mathbf{e} = \mathbf{De}. \tag{34}$$

Let $\mathbf{u} = \mathbf{\Gamma e} + \mathbf{By}$

$$\mathbf{\Gamma} = -\mathbf{I} - \mathbf{A} - \mathbf{D} = \begin{bmatrix} \sigma - 1 & -\sigma & 0 \\ -\gamma + z_2 & 0 & x_1 \\ -y_2 & -x_1 & \beta - 1 \end{bmatrix} \tag{35}$$

$$\boldsymbol{B} = -\tilde{\mathbf{A}} = \begin{bmatrix} 6 & -6 & 0 \\ -17.92 & 0 & 0 \\ 0 & 0 & 4/3 \end{bmatrix}. \tag{36}$$

According to Corollary 2, we obtain that

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 < 0 \tag{37}$$

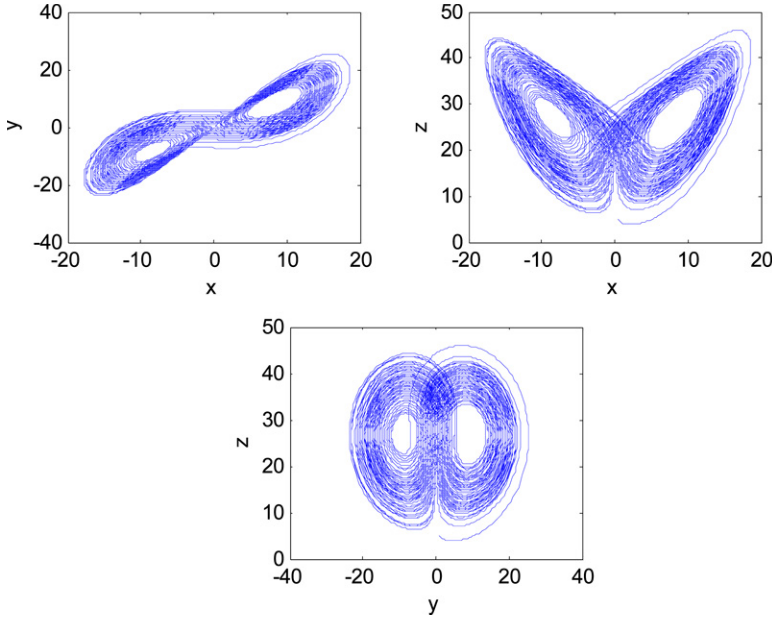

Fig. 7. Chaotic phase portraits for the Lorenz system.

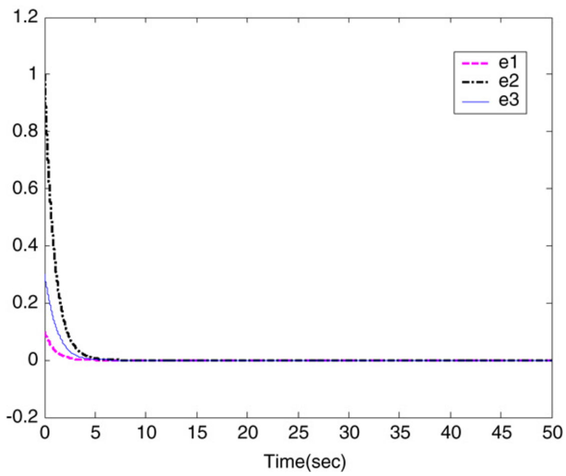

Fig. 8. Time histories of errors for two synchronized Lorenz systems.

is negative definite. The Lorenz systems (31) are global synchronized. For the initial states (0.5, 1, 5), (0.6, 2, 5.3) and system parameters $\sigma = 10$, $\gamma = 28$, $\beta = 8/3$, $\hat{\sigma} = 16$, $\hat{\gamma} = 45.92$ and $\hat{\beta} = 4$, the chaotic phase portraits and state errors versus time are shown in Figs. 7 and 8.

4. Conclusions

In this paper, two theorems for chaos synchronization are proposed by using variable strength linear coupling without another active control, while the time derivative of the Lyapunov function in series form is firstly used, which makes the demand for the Lyapunov function derivative as negative sum of the square of state variables, lower. They give the criteria of chaos synchronization for two identical chaotic systems and for two different chaotic dynamic systems. Either local synchronization which is mostly good enough or global synchronization which is mostly an unnecessary high demand, can be obtained. Lorenz system, Duffing system, Rössler system and Hyper-Rössler system are used as simulation examples which effectively confirm the scheme.

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