

國立交通大學

電子工程學系 電子研究所碩士班
碩士論文

適用於多輸入多輸出系統之
低複雜度 K-Best 球體解碼演算法

Low-complexity Techniques of K-Best Sphere
Decoding for MIMO systems

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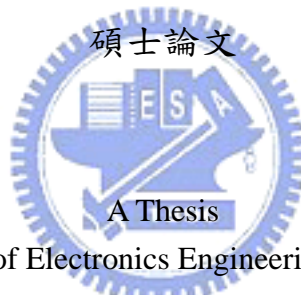
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摘要

這篇論文中，我們在維持和傳統K-Best球體解碼演算法及最大概似偵測(ML detection)相近的效能的前提下提出了兩個化簡K-Best 球體解碼演算法的方法。其中可變動式K-Best 球體解碼演算法提供利用接收訊號來決定K值大小的方式。而分群式K-Best球體解碼演算法利用接收訊號的統計特性僅僅需要粗略排序的比較器就可以替換運算複雜的排序電路。藉由 4x4 64-QAM的系統模擬，位元錯誤率(BER)訂在 5×10^{-4} 的條件下與傳統的 64-Best 球體解碼演算法做比較，使用可變動式K-Best 球體解碼演算法可以化簡 23.65% 到 52.22% 的計算複雜度，並且僅造成 0.13dB到 1.18dB的效能衰減。使用分群式K-Best球體解碼演算法可以化簡計算複雜度超過 99%，並且僅造成 0.09dB的效能衰減。

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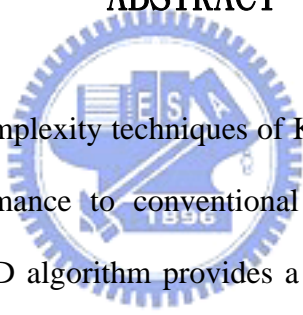
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ABSTRACT



In the thesis, two low-complexity techniques of K-best SD algorithm are proposed while remain similar performance to conventional K-best SD algorithm and ML detection. Adaptive K-Best SD algorithm provides a means to determine the value K according to the received signals. Clustered K-Best SD algorithm uses the statistics knowledge of the received signal, and the clustering technique replaces the high complexity of the sorter with a few comparators. As compared with conventional 64-Best SD algorithm for 4x4 64-QAM system, the adaptive K-Best SD algorithm can reduce complexity ranges from 23.65% to 52.22% within 0.13dB and 1.18dB performance degradation, whereas the clustered K-Best SD algorithm can reduce over 99% complexity within 0.09dB performance degradation.

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修齊

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2007 九月

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Chapter 1

Introduction

1.1 Research Motivation

Recently, multiple-input multiple-out (MIMO) systems are applied in many wireless applications for better transmission efficiency and signal quality due to the inherent diversity gain provided by the multi-path environment. Maximum-likelihood (ML) sequence detection is one of the detection schemes for detecting the received signals in MIMO systems. By searching for the constellation point nearest to the received signal, ML detection is optimized for minimizing the symbol error probabilities, but exhaustive search becomes infeasible since the computation complexity grows as the number of antenna or the constellation points increases. Sphere decoding (SD) algorithm can reduce the computation complexity by confining the number of constellation points to be searched, Fincke-Pohst [1] and Schnorr-Euchner [2] are two of the most common computationally efficient search strategies for realizing the ML detection. Nevertheless, the difficulties in hardware implementation arise because of the non-constant computation complexity and decoding throughput. Alternatively, K-Best SD algorithm [3], [4] simplifies the hardware implementation of SD algorithm by keeping at most K best paths in each layer, leading to fixed-throughput and predictable complexity. Note that the term layer refers to the signal constellations of an transmit antenna. However, K-Best SD algorithm can not guarantee ML performance since the ML path might be eliminated due to the *breadth-first* nature of K-Best SD search approach. Thus the value of K should be large enough, and the value K dominates the performance and computation complexity.

Although K-Best SD algorithm solved the non-constant decoding speed problem of SD algorithm, the sorting computation complexity is still heavy. In this thesis, two modified K-Best SD algorithms are proposed for reducing the sorting computation complexity while remaining the performance similar to ML detection. An adaptive K -best SD algorithm is proposed, providing an adaptive selection of K by observing the ratio of the second minimum and minimum of all paths at the previous decoding layer, and with predicted candidate technique, we can compute only a fraction of the paths before select the K best candidates. During the computing process, sorting becomes a serious problem when the number of K is large. A divided sorting strategy is proposed to achieve the near conventional sorter performance, we divid one global sorter into several local sorters, simulation results show the performance will achieve near one global sorter when the K selection is large enough. Furthermore, because the sorting operations cause the most computation complexity of K-Best SD algorithm, a concept of clustered K-Best SD algorihtm is proposed. Due to the statistics knowledge of received signal, we can get the possible candidates by using a few comparators instead of sorter which reduce the computation complexity.

As compared with conventional 64-Best SD aglroithm for 4×4 64-QAM system. The adaptive K-Best SD algorithm can reduce complexity ranges from 23.65% to 52.22% within 0.13dB and 1.18dB performance degradation, whereas the clustered K-Best SD algorihtm can reduce over 99% complexity within 0.09dB performance degradation.

1.2 Thesis Organization

This thesis focuses on reducing the computation complexity of K-Best SD algorithm while achieve similar performance to ML detection. The organizeion is as follows. In chapter 2, basic concept of MIMO system model, traditional detecting techinque and Sphere Decoding(SD) algorihtm are introduced. Adaptive K-Best SD algorithm are described in Chapter 3. In Chapter 4 divided sorting strategy and clustered K-Best SD algorithm are introduced. The simulatioin and comparison results are shown in Chapter 5. At last, we mention the conclusion and give some potential future work in Chapter 6.

Chapter 2

MIMO System Model

Multiple-input-multiple-output(MIMO) communication systems and spatial multiplexing have recently drawn significant attention. This is a means to achieve gains in system capacity [5] and use spatial diversity to manage multipath fading. The following introduces the concept of diversity and the advantage of using MIMO system and briefly explain linear and non-linear decoding technique. Further, the sphere decoding algorithm is described and the K-Best sphere decoding algorithm is mentioned for easier hardware implementation.



2.1 Diversity gain

Fading, is caused by the random fluctuations in signal level, is a problem in the wireless communication. Diversity provides multiple path(ideally independent) for the same transmitted signal. The probability that all branches suffer in deep fade is small if the number of branch increases. Thus diversity technique plays an important role in the wireless communication to handle fading channel. The symbol error rate(SER) for a system employing diversity techniques at high SNR can be approximated by

$$P_e \approx \frac{c}{\rho^M} \quad (2.1)$$

utilizing log-scale

$$\log(P_e) \approx -M\log(\rho) + c' \quad (2.2)$$

where c is a scaling constant to specify the nature of channel and the modulation type of the system and M is the diversity order of the system, and c' is the log term of c . Fig. 2.1

introduces diversity gain. The slope of diversity gain will become sharp in log-scale by increasing M (diversity order) in high SNR region.

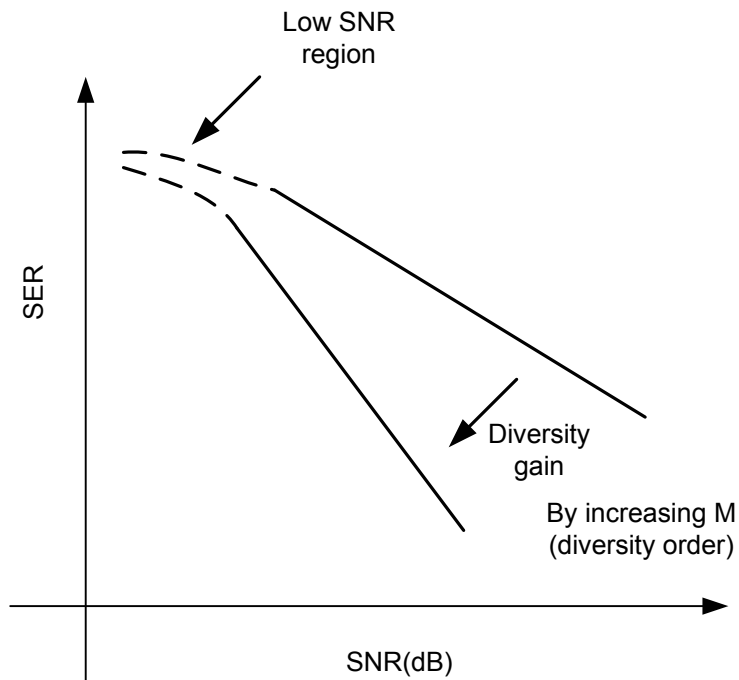


Figure 2.1: Diversity gain increases due to SNR advantage

From the previous discussion, it is obvious that diversity is a powerful technique to manage fading channel in wireless systems. The technique with the highest diversity will be preferred for the MIMO system design.

2.2 Channel model

For a MIMO system with N_T transmit antennas and N_R receive antennas, the transmitted and received signals can be represented by

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}, \quad (2.3)$$

where $\tilde{\mathbf{y}}$ is the $N_R \times 1$ received complex signals, $\tilde{\mathbf{H}}$ is an $N_R \times N_T$ matrix of independent and identical distributed (i.i.d.) circular Gaussian random variables (flat fading is assumed), $\tilde{\mathbf{s}}$ is an $N_T \times 1$ complex vector representing the signals transmitted by each transmit antenna, and $\tilde{\mathbf{n}}$ is the $N_R \times 1$ i.i.d. complex Gaussian noise vector. Moreover, the complex model

in Eq.(2.3) is often described by the equivalent real-valued representation, which is

$$\begin{aligned}
 \mathbf{y} &= \begin{bmatrix} \text{Re}\{\tilde{\mathbf{y}}\} \\ \text{Im}\{\tilde{\mathbf{y}}\} \end{bmatrix} \\
 &= \begin{bmatrix} \text{Re}\{\tilde{\mathbf{H}}\} & -\text{Im}\{\tilde{\mathbf{H}}\} \\ \text{Im}\{\tilde{\mathbf{H}}\} & \text{Re}\{\tilde{\mathbf{H}}\} \end{bmatrix} \begin{bmatrix} \text{Re}\{\tilde{\mathbf{s}}\} \\ \text{Im}\{\tilde{\mathbf{s}}\} \end{bmatrix} + \begin{bmatrix} \text{Re}\{\tilde{\mathbf{n}}\} \\ \text{Im}\{\tilde{\mathbf{n}}\} \end{bmatrix} \\
 &= \mathbf{H}\mathbf{s} + \mathbf{n}.
 \end{aligned} \tag{2.4}$$

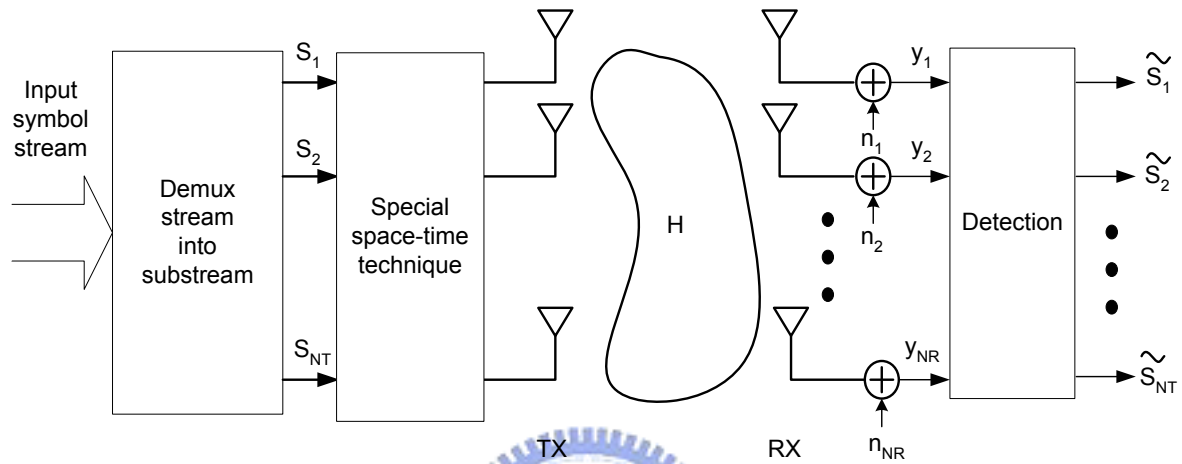


Figure 2.2: Schematic of a linear receiver for separating the transmitted data streams over a MIMO channel

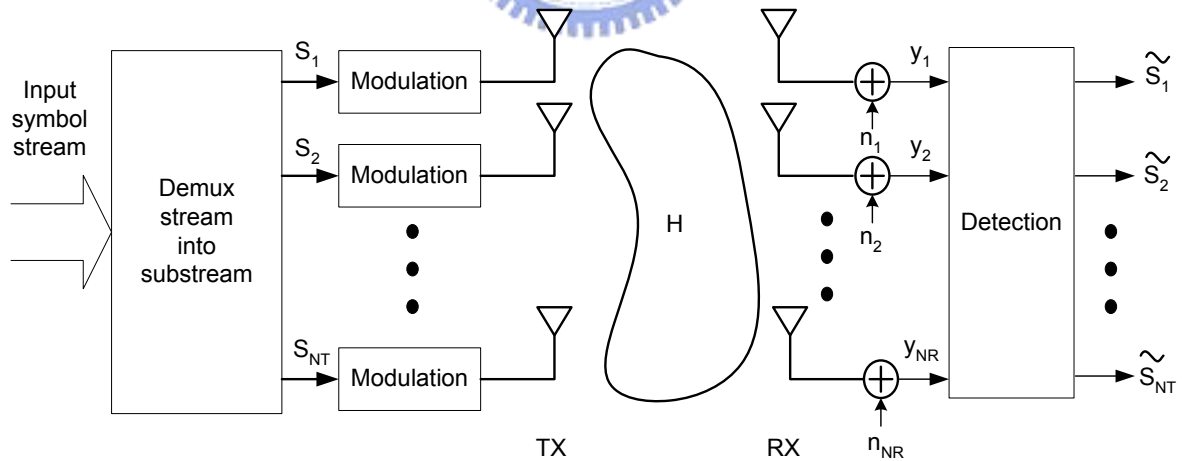


Figure 2.3: Simplified schematic of a linear receiver for separating the transmitted data streams over a MIMO channel

This is also referred to as the *real value decomposition*. For QAM signals, real value

decomposition transforms the complex constellation into two real-valued PAM constellations, which can result to fewer computation. Fig.2.2 shows the general block diagram of MIMO system. As the figure shows, a MIMO system takes N_T parallel data streams, and uses special space-time encoding techniques such as spatial interleaving and space-time coding [6] [7]. However, in this thesis to simplify the problem, we use simple block as Fig.2.3. After takes N_T parallel data streams, the system modulates each of them using complex constellations as previously introduced and arranges them through N_T antennas.

2.3 MIMO detection methods

Several detection methods are introduced in the following

2.3.1 Linear Detection Methods

Assume a channel has a response, linear detection methods try to estimate that match the inverse of the channel. This is done by multiplying a compensate matrix to original channel matrix which is usually based on Zero Forcing(ZF) the SER of ZF is defined by an upper bound by

$$P_e \leq N_e \left(\frac{\rho d_{min}^2}{2N_T} \right)^{-(N_R - N_T + 1)} \quad (2.5)$$

where N_e is the number of the neighbors of the constellation, d_{min} is the minimum distance of two constellations, and ρ represents the SNR, N_T , N_R are transmit antennas and receive antennas respectively. where Eq.(2.5) demonstrates the diversity order of each stream is $N_R - N_T + 1$. ZF receiver has low complexity but it suffers from noise enhancement. Minimum Mean Square Error(MMSE) is another linear detection method. It concerns the noise enhancement problem and minimizes the total error rate. The MMSE receiver can also achieve $N_R - N_T + 1$ [8] diversity order of ZF receiver. Though the linear detection methods require low computational complexity, but the performance degradation is significant.

2.3.2 Suppression and Successive Cancellation

Successive cancellation(SC) decodes the transmission signal by iterative decoding a upper-triangular matrix using bottom up method. It decodes new data stream iteratively until

all the transmitted streams are solved it provides only $N_R - N_T + 1$ diversity order but the performance is only slightly better than MMSE. Ordered successive cancellation (OSC) receiver or called V-BLAST [9] is the improved method. It sorts the decoding order from the highest SNR to the lowest SNR. The OSC may have diversity more than $N_R - N_T + 1$ [8] and have a better performance than SC, but it suffers from error propagation problem, and the performance is still suboptimal.

2.3.3 Maximum-likelihood detection

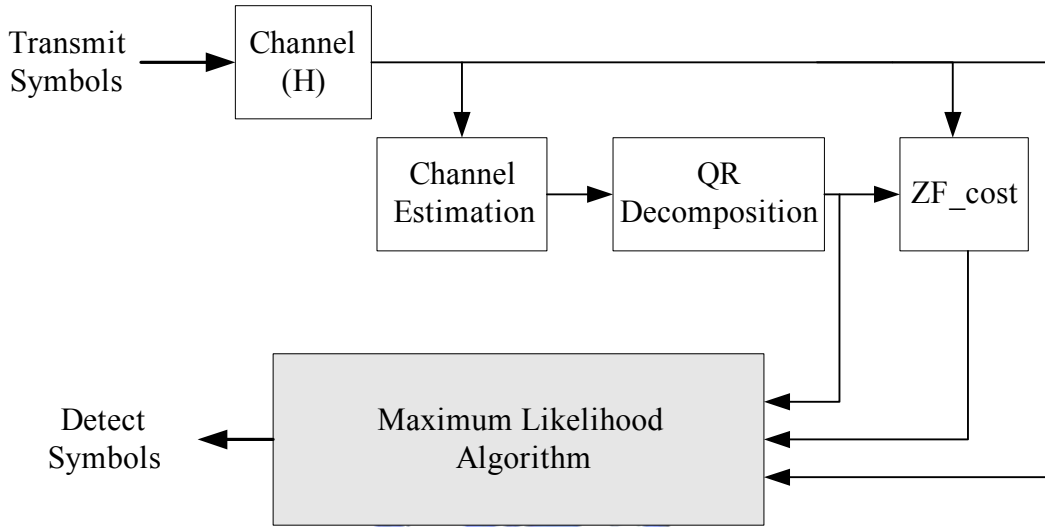


Figure 2.4: Block diagram of MIMO detection

For detecting the received signals, maximum likelihood (ML) sequence detection is one of the MIMO system detection technique that optimizes the symbol error probability [10]. According to the system model described in, Fig.2.4 ML detection is equivalent to searching for the vector \hat{s} that minimizes $\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2$. That is,

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \Omega} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2, \quad (2.6)$$

where Ω is the set consisting of all possible $2N_t$ -dimensional signal constellation points. Fig.2.4 shows the simplified block diagram of a MIMO receiver. The channel estimator provides the required channel state information \mathbf{H} . By QR decomposition, the channel matrix \mathbf{H} is decomposed by $\mathbf{H} = \mathbf{Q}\mathbf{R}$, and Eq.(2.6) can be rewritten as

$$\begin{aligned} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 &= (\mathbf{s} - \mathbf{s}_{zf})^H \mathbf{H}^H \mathbf{H} (\mathbf{s} - \mathbf{s}_{zf}) \\ &+ \mathbf{y}^H (\mathbf{I} - \mathbf{H}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^T) \mathbf{y} \end{aligned}$$

and

$$\begin{aligned}\hat{\mathbf{s}} &= \arg \min_{\mathbf{s} \in \Omega} (\mathbf{s} - \mathbf{s}_{\text{zf}})^H \mathbf{H}^T \mathbf{H} (\mathbf{s} - \mathbf{s}_{\text{zf}}) \\ &= \arg \min_{\mathbf{s} \in \Omega} \bar{\mathbf{s}}^H \mathbf{R}^H \mathbf{R} \bar{\mathbf{s}}.\end{aligned}\quad (2.7)$$

Note that the matrix \mathbf{R} derived from QR decomposition is an upper triangular matrix with non-negative diagonal elements, and $\mathbf{H}^H \mathbf{H} = \mathbf{R}^H \mathbf{R}$. Moreover, \mathbf{s}_{zf} is the zero-forcing (ZF) solution that can be derived by $\mathbf{s}_{\text{zf}} = \mathbf{H}^+ \mathbf{y}$ for \mathbf{H}^+ is the pseudo-inverse of \mathbf{H} . It is perceived that $\bar{\mathbf{s}} = \mathbf{s} - \mathbf{s}_{\text{zf}}$ is the distance from the candidates of signal to the ZF solution.

Due to the triangular form of \mathbf{R} , we can rewrite Eq.(2.7) as

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \Omega} \sum_{i=1}^{N_R} \left\| y_i - \sum_{j=i}^{N_T} R_{ij} s_j^{(i)} \right\|^2, \quad (2.8)$$

where R_{ij} and s_j denote the i -th row, j -th column of \mathbf{R} and the j -th element of \mathbf{s} . Moreover, we can define $e(\mathbf{s}^{(i)})$, the partial square Euclidean distance (PED) of the i -th layer, by

$$e(\mathbf{s}^{(i)}) = \left\| y_i - \sum_{j=i}^{N_T} R_{ij} s_j^{(i)} \right\|^2, \quad (2.9)$$

where $\mathbf{s}^{(i)} = [s_i^{(i)} s_{i+1}^{(i)} \cdots s_{N_T}^{(i)}]^T$ and $s_j^{(i)}$ is the j -th element of $\mathbf{s}^{(i)}$. Then the accumulated Euclidean distance corresponding to the candidate $\mathbf{s}^{(i)}$ can be derived recursively from the PED and the accumulated Euclidean distance corresponding to $\mathbf{s}^{(i+1)}$, denoted by $T(\mathbf{s}^{(i+1)})$, that is

$$T(\mathbf{s}^{(i)}) = T(\mathbf{s}^{(i+1)}) + e(\mathbf{s}^{(i)}). \quad (2.10)$$

The detection process starts from $i=N_T$, resulting to a tree-structure, or called depth-first, search strategy. However, exhaustively searching for the ML solution becomes infeasible [11] since the computation complexity grows exponentially with N_i or the number of constellation points.

The ML performance can be defined by pairwise error probability (PEP) [8], which determines the probability when the input vector symbol $s^{(i)}$ is transmitted while detected as $s^{(j)}$ with $i \neq j$. The average PEP is upper-bounded at high SNR by

$$P(s^{(i)} \rightarrow s^{(j)}) \leq \left(\frac{\rho}{4N_T} \|d_{i,j}\|^2 \right)^{-N_R} \quad (2.11)$$

where $d_{i,j} = s^{(i)} - s^{(j)}$. From Eq.(2.11) N_R order of diversity is achieved. For uncoded system, ML detection techniques outperforms other in diversity, however the computation

Table 2.1: Summary of comparative performance and computation complexity of receivers for different detection techniques

Receiver	Diversity order	Performance	Complexity
ZF	$N_R - N_T + 1$	Poor	Low
MMSE	$\approx N_R - N_T + 1$	Poor	Low
SC	$\approx N_R - N_T + 1$	Medium	Medium
OSC	$N_R - N_T + 1 \leq, \leq N_R$	Medium	Medium
ML	N_R	Good	High

complexity is increased in an exponential form, thus next section will introduce Sphere Decoding algorithm to maintain the diversity of ML, while decrease the computation complexity.

Table 2.1 is the summary of diversity order and SNR loss for different detection techniques with spatial multiplexing. The ML receiver has zero SNR loss and achieves N_R order of diversity.



2.4 Sphere Decoding

As the previous section mentioned, ML detection can achieve full diversity and good performance for MIMO systems. However, in order to achieve more diversity gain, increasing number of antennas is necessary, which will cause the computation complexity of ML detection higher than linear detection and successive cancellation method. Thus, there should be some techniques to simplify the ML detection method. The following of this section introduced Sphere Decoding(SD) algorithm and K-Best SD algorithm to achieve the goal.

2.4.1 Concept of Sphere Decoding (SD) algorithm

Sphere decoding (SD) algorithm has been proposed and recognized as a powerful means to solve the ML detection problems [4] [12] [13]. SD algorithm reduces the computation by restricting the search range. Instead of searching all candidates in Ω Eq.(2.6), SD algorithm constrains a much smaller search range $\Omega_{\text{SD}} = \{\mathbf{s} : \bar{\mathbf{s}}^H \mathbf{R}^H \mathbf{R} \bar{\mathbf{s}} \leq d^2\}$; only the candidates in Ω_{SD} will be compared. By the aforementioned procedure, the candidate of the smallest $T(\mathbf{s}^{(1)})$ in Eq.(2.10) is always the ML solution as long as d is properly defined. The problem can be illustrated as a two-dimensional problem in Fig. 2.5, the solution can be obtained by drawing a circle around the received signal, and chosen proper radius to discard the points outside the radius.

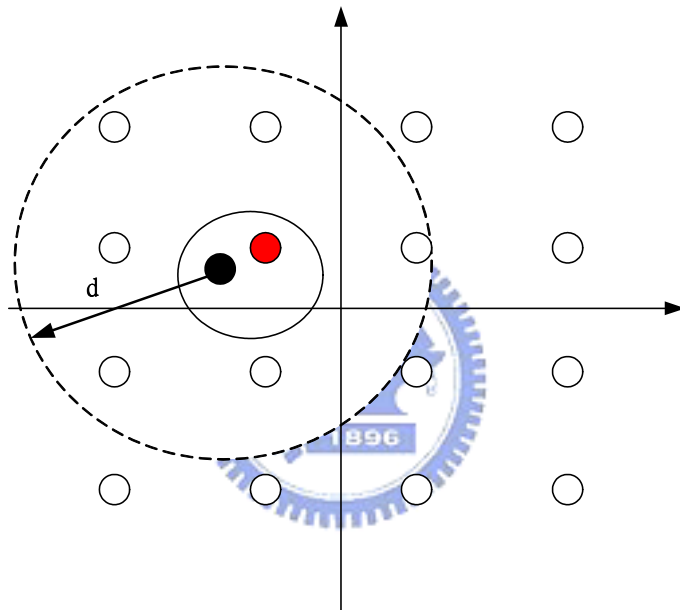


Figure 2.5: Geometrical representation of the sphere decoding algorithm

Fig. 2.6 is an illustrative concept of Sphere Decoding algorithm. We can map the two dimensional problem into a tree search problem. For a $N_T=2$ antenna system. The possible signal candidates are on the green path and passed the initial radius constraint. The rest of the path are pruned during the process.

However, not only the value d , but the computation varies with SNR, leading to a non-constant decoding throughput. Hardware implementation of SD algorithm becomes complicated.

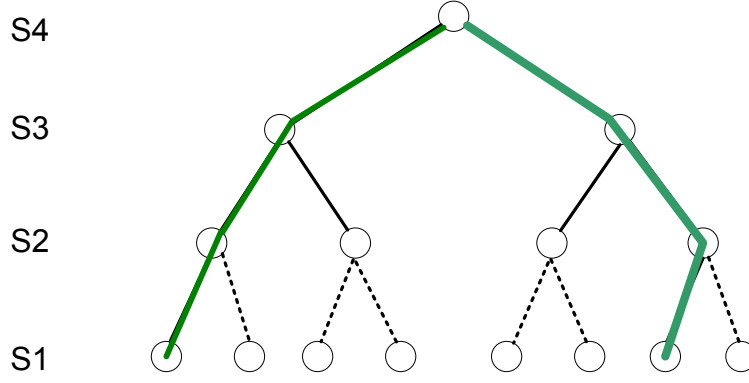


Figure 2.6: Extend sphere decoding to tree search representation

2.4.2 Sphere Decoding algorithm

The sphere decoding can be regarded as finding s [14]

$$y = Hs + n \quad (2.12)$$

where $\mathbf{s} = [s_1 s_2 \cdots s_{N_T}]^T$. The maximum likelihood decoding algorithm can be rewritten as from in Eq.(2.4) the

$$\|n\|^2 = \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = (\mathbf{s} - \mathbf{s}_{zf})^H \mathbf{H}^H \mathbf{H} (\mathbf{s} - \mathbf{s}_{zf}) + \|\mathbf{y}\|^2 - \|\mathbf{H}\mathbf{s}_{zf}\|^2. \quad (2.13)$$

Based on Fincke Pohst method in [1], the lattice point $(\mathbf{H})\mathbf{s}_{zf}$ lies inside the sphere of radius d

$$d^2 \geq \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = (\mathbf{s} - \mathbf{s}_{zf})^H \mathbf{H}^H \mathbf{H} (\mathbf{s} - \mathbf{s}_{zf}) + \|\mathbf{y}\|^2 - \|\mathbf{H}\mathbf{s}_{zf}\|^2. \quad (2.14)$$

By using the transform of Eq.(2.4)(2.5), Eq.(2.14) can be rewritten as

$$d^2 \geq \sum_{i=1}^{N_R} \left\| y_i - \sum_{j=i}^{N_T} R_{ij} s_j^{(i)} \right\|^2, \quad (2.15)$$

Further simplification of Eq.(2.15) and assume $N_T = N_R = M$ and $s_{zf} = \tilde{\mathbf{s}}$ gives

$$\begin{aligned}
d'^2 &\geq (\mathbf{s} - \tilde{\mathbf{s}})^H \mathbf{H}^H \mathbf{H} (\mathbf{s} - \tilde{\mathbf{s}}) \\
&= (\mathbf{s} - \tilde{\mathbf{s}})^H \mathbf{R}^H \mathbf{R} (\mathbf{s} - \tilde{\mathbf{s}}) \\
&= \sum_{i=1}^M R_{i,i}^2 \left((\mathbf{s}_i - \tilde{\mathbf{s}}_i) + \sum_{j=i+1}^M \frac{R_{i,j}}{R_{i,i}} (\mathbf{s}_i - \tilde{\mathbf{s}}_i) \right)^2 \\
&= R_{M,M}^2 (\mathbf{s}_M - \tilde{\mathbf{s}}_M)^2 \\
&+ R_{M-1,M-1}^2 \left(\mathbf{s}_{M-1} - \tilde{\mathbf{s}}_{M-1} + \frac{R_{M-1,M}}{R_{M-1,M-1}} (\mathbf{s}_M - \tilde{\mathbf{s}}_M) \right)^2 + \dots \quad (2.16)
\end{aligned}$$

The condition leads to s_M falls in the interval

$$\left[\tilde{\mathbf{s}}_M - \frac{d'}{R_{M,M}} \right] \leq \mathbf{s}_M \leq \left[\tilde{\mathbf{s}}_M + \frac{d'}{R_{M,M}} \right] \quad (2.17)$$

For every s_M satisfy Eq.(2.17) we defined a new constant

$$d'_{M-1} = d'^2 - R_{M,M}^2 (\mathbf{s}_M - \tilde{\mathbf{s}}_M)^2 \quad (2.18)$$

and a new condition can be modified as

$$d'_{M-1}{}^2 \geq R_{M-1,M-1}^2 \left(\mathbf{s}_{M-1} - \tilde{\mathbf{s}}_{M-1} + \frac{R_{M-1,M}}{R_{M-1,M-1}} (\mathbf{s}_M - \tilde{\mathbf{s}}_M) \right)^2 \quad (2.19)$$

Defined $\mathbf{s}_{M-1|M} = \tilde{\mathbf{s}}_{M-1} + \frac{R_{M-1,M}}{R_{M-1,M-1}} (\mathbf{s}_M - \tilde{\mathbf{s}}_M)$ and Eq.(2.19) is equivalent to

$$\left[\tilde{\mathbf{s}}_{M-1|M} - \frac{d'_{M-1}}{R_{M-1,M-1}} \right] \leq \mathbf{s}_{M-1} \leq \left[\tilde{\mathbf{s}}_{M-1|M} + \frac{d'_{M-1}}{R_{M-1,M-1}} \right] \quad (2.20)$$

In a similar process, one can find possible \mathbf{s}_{M-2} and so on, starting nested condition until possible \mathbf{s}_1 is found.

2.5 K-Best Sphere Decoding algorithm

K -best SD algorithm is an alternative method that improves the decoding throughput. It simplified the original SD algorithm and maintains a constant throughput by keeping only the K smallest accumulated PED at each layer. However, K -best SD algorithm can not guarantee the performance of ML detection since the ML solution may be eliminated

