國立交通大學

電子工程學系 電子研究所碩士班
碩士論文

適用於多輸入多輸出系統之

低複雜度K-Best 球體解碼演算法

Low-complexity Techniques of K-Best Sphere Decoding for MIMO systems

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摘要

這篇論文中，我們在維持和傳統K-Best球體解碼演算法及最大概似偵側(ML detection)相近的效能的前提下提出了兩個化簡K-Best 球體解碼演算法的方法。其中可變動式K-Best 球體解碼演算法提供利用接收訊號來決定K值大小的方式。 而分群式K-Best球體解碼演算法利用接收訊號的統計特性僅僅需要粗略排序的比較器就可以替換運算複雜的排序電路。藉由 4x4 64-QAM的系統模擬，位元錯誤率(BER)訂在 $5x10^{-4}$ 的條件下與傳統的 64-Best 球體解碼演算法做比較，使用可變動式K-Best 球體解碼演算法可以化簡 23.65% 到 52.22% 的計算複雜度，並且僅造成 0.13dB到 1.18dB的效能衰減。使用分群式K-Best球體解碼演算法可以化簡計算複雜度超過 99%，並且僅造成 0.09dB的效能衰減。

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ABSTRACT

In the thesis, two low-complexity techniques of K-best SD algorithm are proposed while remain similar performance to conventional K-best SD algorithm and ML detection. Adaptive K-Best SD algorithm provides a means to determine the value K according to the received signals. Clustered K-Best SD algorithm uses the statistics knowledge of the received signal, and the clustering technique replaces the high complexity of the sorter with a few comparators. As compared with conventional 64-Best SD algorithm for 4x4 64 -QAM system, the adaptive K-Best SD algorithm can reduce complexity ranges from 23.65% to 52.22% within 0.13dB and 1.18dB performance degradation, whereas the clustered K-Best SD algorithm can reduce over 99% complexity within 0.09dB performance degradation.

誌　謝

本論文承蒙吾師 張錫嘉博士指導，適時給予指示與建議，使我研究的過程順利，並於學業、生活上給予關懷，師恩浩瀚，永銘於心。感謝交大電信系的吳文榕老師、王忠炫老師以及交大電子系的桑梓賢老師在口試的時候給我指導以及建議，使本論文更加完善。

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修齊

謹致於　新竹

2007　九月

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Chapter 1

Introduction

1.1 Research Motivation

Recently, multiple-input multiple-out (MIMO) systems are applied in many wireless applications for better transmission efficiency and signal quality due to the inherent diversity gain provided by the multi-path environment. Maximum-likelihood (ML) sequence detection is one of the detection schemes for detecting the received signals in MIMO systems. By searching for the constellation point nearest to the received signal, ML detection is optimized for minimizing the symbol error probabilities, but exhaustive search becomes infeasible since the computation complexity grows as the number of antenna or the constellation points increases. Sphere decoding (SD) algorithm can reduce the computation complexity by confining the number of constellation points to be searched, Fincke-Pohst [1] and Schnorr-Euchner [2] are two of the most common computationally efficient search strategies for realizing the ML detection. Nevertheless, the difficulties in hardware implementation arise because of the non-constant computation complexity and decoding throughput. Alternatively, K-Best SD algorithm [3], [4] simplifies the hardware implementation of SD algorithm by keeping at most K best paths in each layer, leading to fixed-throughput and predictable complexity. Note that the term layer refers to the signal constellations of an transmit antenna. However, K-Best SD algorithm can not guarantee ML performance since the ML path might be eliminated due to the *breadth-first* nature of K-Best SD search approach. Thus the value of K should be large enough, and the value K dominates the performance and computation complexity.

Although K-Best SD algorithm solved the non-constant decoding speed problem of SD algorithm, the sorting computation complexity is still heavy. In this thesis, two modified K-Best SD algorithms are proposed for reducing the sorting computation complexity while remaining the performance similar to ML detection. An adaptive K-best SD algorithm is proposed, providing an adaptive selection of K by observing the ratio of the second minimum and minimum of all paths at the previous decoding layer, and with predicted candidate technique, we can compute only a fraction of the paths before select the K best candidates. During the computing process, sorting becomes a serious problem when the number of K is large. A divided sorting strategy is proposed to achieve the near conventional sorter performance, we divid one global sorter into several local sorters, simulation results show the performance will achieve near one global sorter when the K selection is large enough. Furthermore, because the sorting operations cause the most computation complexity of K-Best SD algorithm, a concept of clustered K-Best SD algorihtm is proposed. Due to the statistcs knowledge of receved signal, we can get the possible candidates by using a few comparators instead of sorter which reduce the computation complexity.

As compared with conventional 64-Best SD aglroithm for 4×4 64-QAM system. The adaptive K-Best SD algorithm can reduce complexity ranges from 23.65% to 52.22% within 0.13dB and 1.18dB performance degradation, whereas the clustered K-Best SD algorihtm can reduce over 99% complexity within 0.09dB performance degradation.

1.2 Thesis Organization

This thesis focuses on reducing the computation complexity of K-Best SD algorithm while achieve similar performance to ML detection. The organizeion is as follows. In chapter 2, basic concept of MIMO system model, traditional detecting techinque and Sphere Decoding(SD) algorihtm are introduced. Adaptive K-Best SD algorithm are described in Chapter 3. In Chapter 4 divided sorting strategy and clustered K-Best SD algorithm are introduced. The simulatioin and comparison results are shown in Chapter 5. At last, we mention the conclusion and give some potential future work in Chapter 6.

Chapter 2

MIMO System Model

Multiple-input-multiple-output(MIMO) communcation systems and spatial multiplexing have recently drawn significant attention. This is a means to achieve gains in system capacity [5] and use spatial diversity to manage multipath fading. The following introduces the concept of diversity and the advantage of using MIMO system and brifely explain linear and non-linear decoding technique. Further, the sphere decoding algorithm is described and the K-Best sphere decoding algorithm is mentioned for easier hardware implementation.

2.1 Diversity gain

Fading, is caused by the random fluctuations in signal level, is a probelm in the wireless communication. Diversity provides multiple path(ideally independent) for the same transmitted signal. The probability that all braches sufferd in deep fade fade is small if the number of brach increases. Thus diveristy technique plays an important role in the wireless communication to handle fading channel. The symbol error rate(SER) for a system employing diversity techniques at high SNR can be approximated by

$$P_e \approx \frac{c}{\rho^M} \tag{2.1}$$

utilizing log-scale

$$log(P_e) \approx -Mlog(\rho) + c' \tag{2.2}$$

where c is a scaling constant to specify the nature of channel and the modulation type of the system and M is the diversity order of the system, and c' is the log term of c. Fig. 2.1

introduces diversity gain. The slope of diversity gain will become sharp in log-scale by increasing M (diversity order) in high SNR region.

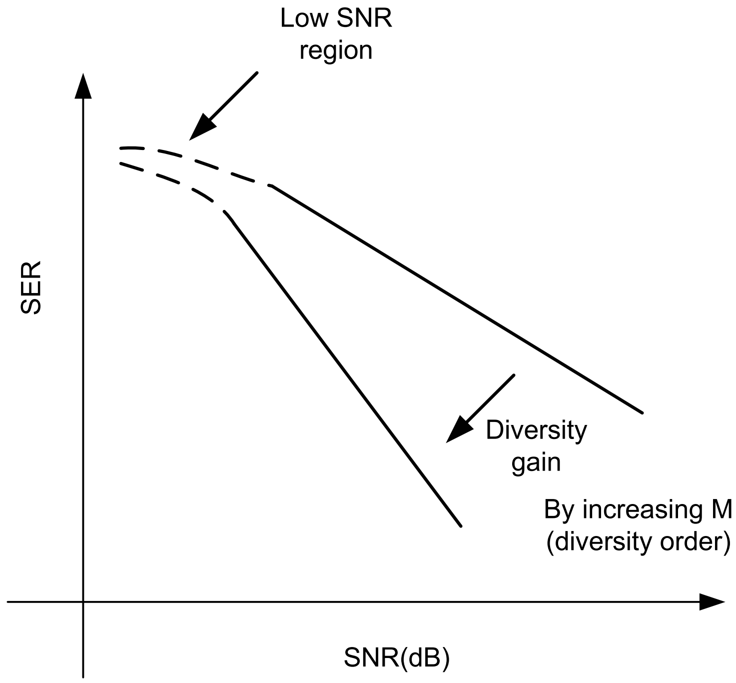

Figure 2.1: Diversity gain increases due to SNR advantage

From the previous disussion, it is obvious that diversity is a powerful technique to manage fading channel in wireless systems. The technique with the highest diversity will be prefered for the MIMO system design.

2.2 Channel model

For a MIMO system with N_T transmit antennas and N_R receive antennas, the transmitted and received signals can be represented by

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}, \tag{2.3}$$

where $\tilde{\mathbf{y}}$ is the $N_R \times 1$ received complex signals, $\tilde{\mathbf{H}}$ is an $N_R \times N_T$ matrix of independent and identical distributed (i.i.d.) circular Gaussian random variables (flat fading is assumed), $\tilde{\mathbf{s}}$ is an $N_T \times 1$ complex vector representing the signals transmitted by each transmit antenna, and $\tilde{\mathbf{n}}$ is the $N_R \times 1$ i.i.d. complex Gaussian noise vector. Moreover, the complex model in Eq.(2.3) is often described by the equivalent real-valued representation, which is

$$\begin{aligned} \mathbf{y} &= \begin{bmatrix} Re\{\tilde{\mathbf{y}}\} \\ Im\{\tilde{\mathbf{y}}\} \end{bmatrix} \\ &= \begin{bmatrix} Re\{\tilde{\mathbf{H}}\} & -Im\{\tilde{\mathbf{H}}\} \\ Im\{\tilde{\mathbf{H}}\} & Re\{\tilde{\mathbf{H}}\} \end{bmatrix} \begin{bmatrix} Re\{\tilde{\mathbf{s}}\} \\ Im\{\tilde{\mathbf{s}}\} \end{bmatrix} + \begin{bmatrix} Re\{\tilde{\mathbf{n}}\} \\ Im\{\tilde{\mathbf{n}}\} \end{bmatrix} \\ &= \mathbf{Hs} + \mathbf{n}. \end{aligned} \tag{2.4}$$

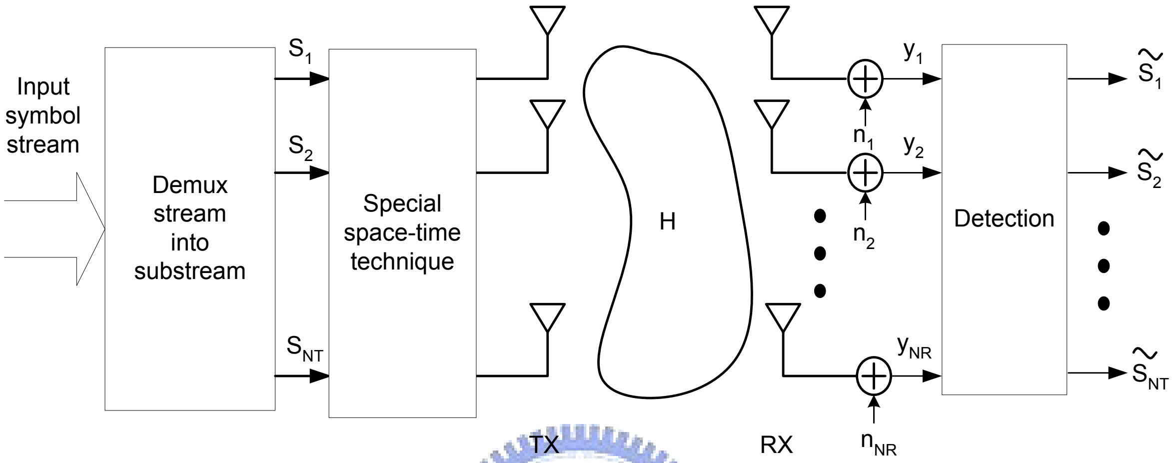

Figure 2.2: Schematic of a linear receiver for separating the transmitted data streams over a MIMO channel

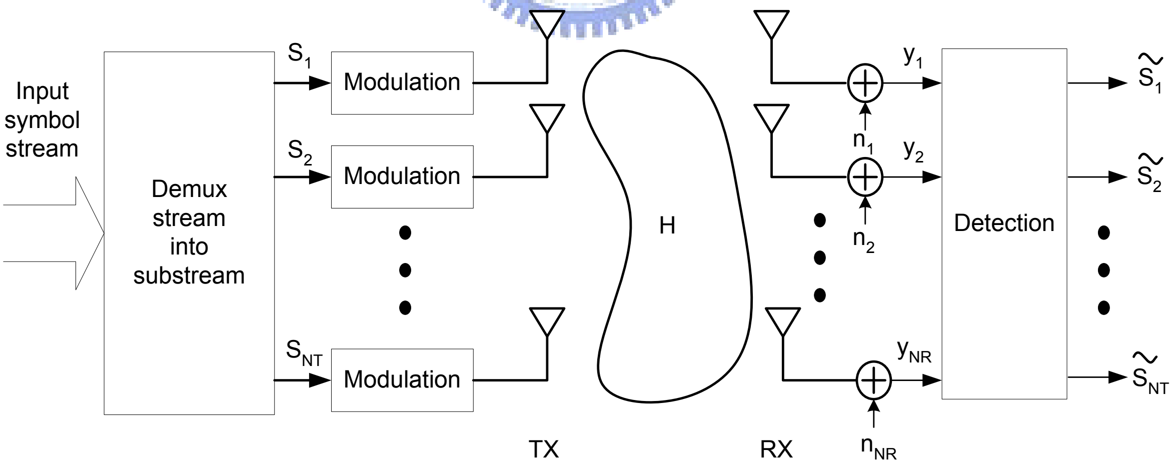

Figure 2.3: Simplified schematic of a linear receiver for separating the transmitted data streams over a MIMO channel

This is also referred to as the *real value decomposition*. For QAM signals, real value decomposition transforms the complex constellation into two real-valued PAM constellations, which can result to fewer computation. Fig.2.2 shows the general block diagram of MIMO system. As the figure shows, a MIMO system takes N_T paralle data streams, and uses special space-time encoding techniques such as spatial interleaving and space-time coding [6] [7]. However, in this thesis to simplify the problem, we use simple block as Fig.2.3. After takes N_T paralle data streams, the system modulates each of them using complex constellations as previous introduced and arrages them through N_T antennas.

2.3 MIMO detection methods

Several detection methods are introduced in the following

2.3.1 Linear Detection Methods

Assume a channel has a response, linear detction methods try to estimate that match the inverse of the channel. This is done by multiply a compensate matrix to orignal channel matrix which is usually based on Zero Forcing(ZF) the SER of ZF is defined by a upper bound by

$$P_e \leq N_e \left(\frac{\rho d_{min}^2}{2N_T} \right)^{-(N_R - N_T + 1)} \tag{2.5}$$

where N_e is the number of the neighbors of the constellation, d_{min} is the minimum distance of two constelation, and ρ represents the SNR, N_T, N_R are transmit antennas and receive antennas respectively. where Eq.(2.5) demonstrates the diveristy oreder of each stream is $N_R - N_T + 1$. ZF receiver has low complexity but it suffers from noise enhancement. Minimum Mean Square Error(MMSE) is another linear detection methods. It concerns the noise enhancement problem and minimizes the total error rate. The MMSE receiver can also acheive $N_R - N_T + 1$ [8] diversity order of ZF receiver. Though the linear detection methods requires low computational complexity, but the performance degradtation is significant.

2.3.2 Suppression and Successive Cancellation

Successive cancellation(SC) decodes the transmission signal by iterative decoding a up-triangular matrix using bottom up method. It decodes new data stream iteratively until all the transmitted streams are solved it provides only $N_R - N_T + 1$ diversity order but the performance is only slightly better than MMSE. Ordered successive cancellation (OSC) receiver or called V-BLAST [9] is the improved method. It sorts the decoding order from the highest SNR to the lowest SNR. The OSC may have diversity more than $N_R - N_T + 1$ [8] and have a better performance than SC, but it suffers from error propagation problem, and the performance is still suboptimal.

2.3.3 Maximum-likelihood detection

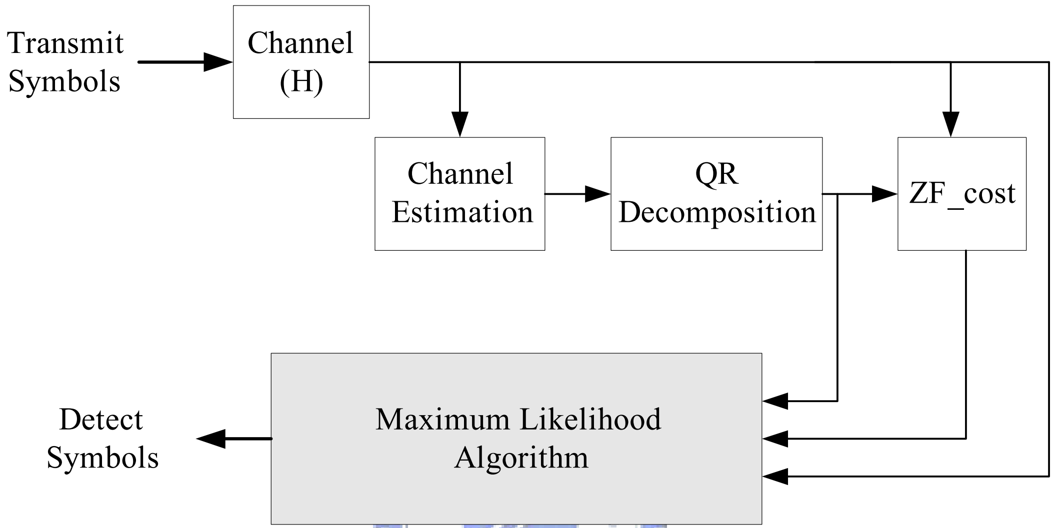

Figure 2.4: Block diagram of MIMO detection

For detecting the received signals, maximum likelihood (ML) sequence detection is one of the MIMO system detection technique that optimizes the symbol error probability [10]. According to the system model described in, Fig.2.4 ML detection is equivalent to searching for the vector $\hat{s}$ that minimizes $\|\mathbf{y} - \mathbf{H}s\|^2$. That is,

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathbf{\Omega}} \|\mathbf{y} - \mathbf{H}s\|^2 , \tag{2.6}$$

where Ω is the set consisting of all possible $2N_t$-dimensional signal constellation points. Fig.2.4 shows the simplified block diagram of a MIMO receiver. The channel estimator provides the required channel state information $\mathbf{H}$. By QR decomposition, the channel matrix $\mathbf{H}$ is decomposed by $\mathbf{H} = \mathbf{QR}$, and Eq.(2.6) can be rewritten as

$$\begin{aligned} \|\mathbf{y} - \mathbf{Hs}\|^2 &= (\mathbf{s} - \mathbf{s_{zf}})^H \mathbf{H}^H \mathbf{H} (\mathbf{s} - \mathbf{s_{zf}}) \\ &+ \mathbf{y}^H (I - \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^T)\mathbf{y} \end{aligned}$$

and

$$\begin{aligned}\hat{\mathbf{s}} &= \arg\min_{\mathbf{s}\in\mathbf{\Omega}}(\mathbf{s}-\mathbf{s_{zf}})^H\mathbf{H}^T\mathbf{H}(\mathbf{s}-\mathbf{s_{zf}}) \\ &= \arg\min_{\mathbf{s}\in\mathbf{\Omega}}\bar{\mathbf{s}}^H\mathbf{R}^H\mathbf{R}\bar{\mathbf{s}}. \end{aligned} \tag{2.7}$$

Note that the matrix $\mathbf{R}$ derived from QR decomposition is an upper triangular matrix with non-negative diagonal elements, and $H^HH=R^HR$. Moreover, $\mathbf{s_{zf}}$ is the zero-forcing (ZF) solution that can be derived by $\mathbf{s_{zf}} = \mathbf{H}^+\mathbf{y}$ for $\mathbf{H}^+$ is the pseudo-inverse of $\mathbf{H}$. It is perceived that $\bar{\mathbf{s}} = \mathbf{s} - \mathbf{s_{zf}}$ is the distance from the candidates of signal to the ZF solution.

Due to the triangular form of $\mathbf{R}$, we can rewrite Eq.(2.7) as

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathbf{\Omega}}\sum_{i=1}^{N_R}\left\|y_i - \sum_{j=i}^{N_T}R_{ij}s_j^{(i)}\right\|^2, \tag{2.8}$$

where R_{ij} and s_j denote the i-th row, j-th column of $\mathbf{R}$ and the j-th element of $\mathbf{s}$. Moreover, we can define $e(\mathbf{s}^{(i)})$, the partial square Euclidean distance(PED) of the i-th layer, by

$$e(\mathbf{s}^{(i)}) = \left\|y_i - \sum_{j=i}^{N_T}R_{ij}s_j^{(i)}\right\|^2, \tag{2.9}$$

where $\mathbf{s}^{(i)} = [s_i^{(i)}s_{i+1}^{(i)}\cdots s_{N_T}^{(i)}]^T$ and $s_j^{(i)}$ is the j-th element of $\mathbf{s}^{(i)}$. Then the accumulated Euclidean distance corresponding to the candidate $\mathbf{s}^{(i)}$ can be derived recursively from the PED and the accumulated Euclidean distance corresponding to $\mathbf{s}^{(i+1)}$, denoted by $T(\mathbf{s}^{i+1)})$, that is

$$T(\mathbf{s}^{(i)}) = T(\mathbf{s}^{(i+1)}) + e(\mathbf{s}^{(i)}). \tag{2.10}$$

The detection process starts from i=N_T, resulting to a tree-structure, or called depth-first, search strategy. However, exhaustively searching for the ML solution becomes infeasible [11] since the computation complexity grows exponentially with N_t or the number of constellation points.

The ML pefromance can be defined by pairwise error probability(PEP) [8], which determines the probability when the input vector symbol $s^{(i)}$ is transmitted while detected as $s^{(j)}$ with $i \neq j$. The average PEP is upper-bounded at high SNR by

$$P(s^{(i)} \to s^{(j)}) \leq \left(\frac{\rho}{4N_T}\left\|d_{i,j}\right\|^2\right)^{-N_R} \tag{2.11}$$

where $d_{i,j} = s^{(i)} - s^{(j)}$. From Eq.(2.11) N_R order of diversity is achieved. For uncoded system, ML detection techinques outperfroms other in diversity, however the computation

Table 2.1: Summary of comparative performance and computation complexity of receivers for different detection techniques

Receiver	Diversity order	Performance	Complexity
ZF	$N_R - N_T + 1$	Poor	Low
MMSE	$\approx N_R - N_T + 1$	Poor	Low
SC	$\approx N_R - N_T + 1$	Medium	Medium
OSC	$N_R - N_T + 1 \leq, \leq N_R$	Medium	Medium
ML	N_R	Good	High

comlexity is increased in an exponetial form, thus next section will introduce Sphere Decoding algorihtm to maintain the diversity of ML, while decrease the computation complexity.

Tabel 2.1 is the summary of diversity order and SNR loss for different detection techinues with spatial multiplexing. The ML receiver has zero SNR loss and achiver N_R order of diversity.

2.4 Sphere Decoding

As the previous section mentioned, ML detection can achieve full diversity and good performance for MIMO systems. However, in order to achive more diversity gain, increasing number of antennas is necessary, which will cause the computation complexity of ML detection higher than linear detection and successive cancellation method. Thus, there should be some techniques to simplify the ML detection method. The following of this section introduced Sphere Decoding(SD) algorithm and K-Best SD algorithm to achieve the goal.

2.4.1 Concept of Sphere Decoding (SD) algorithm

Sphere decoding (SD) algorithm has been proposed and recognized as a powerful means to solve the ML detection problems [4] [12] [13]. SD algorithm reduces the computation by restricting the search range. Instead of searching all candidates in $\mathbf{\Omega}$ Eq.(2.6), SD algorithm constrains a much smaller search range $\mathbf{\Omega_{SD}} = \{\mathbf{s} : \bar{\mathbf{s}}^H \mathbf{R}^H \mathbf{R} \bar{\mathbf{s}} \leq d^2\}$; only the candidates in $\mathbf{\Omega_{SD}}$ will be compared. By the aforementioned procedure, the candidate of the smallest $T(\mathbf{s}^{(1)})$ in Eq.(2.10) is always the ML solution as long as d is properly defined. The problem can be illustrated as a two-dimensional problem in Fig. 2.5, the solution can be obtained by drawing a circle around the received signal, and chosen proper radius to discard the points outside the radius.

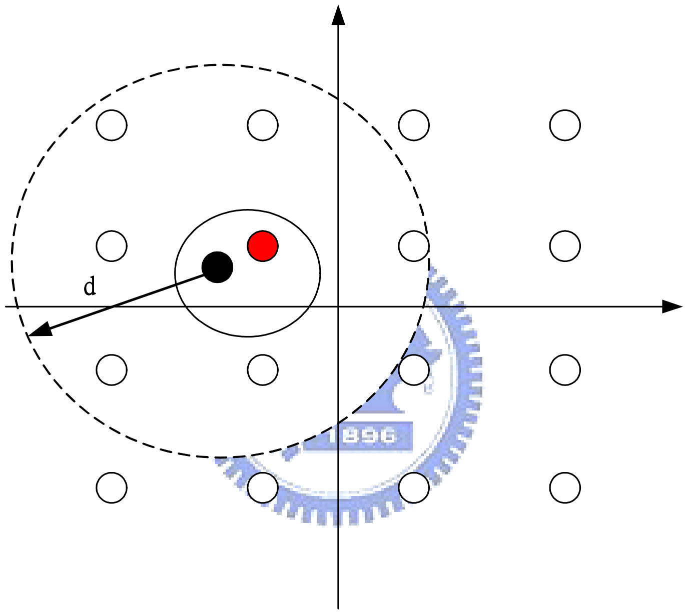

Figure 2.5: Geometrical representation of the sphere decoding algorithm

Fig. 2.6 is an illustrative concept of Sphere Decoding algorithm. We can map the two dimensional problem into a tree search problem. For a N_T=2 antenna system. The possible singal candidates are on the green path and passed the initail radius constraint. The rest of the path are pruned during the process.

However, not only the value d, but the computation varies with SNR, leading to a non-constant decoding throughput. Hardware implementation of SD algorithm becomes complicated.

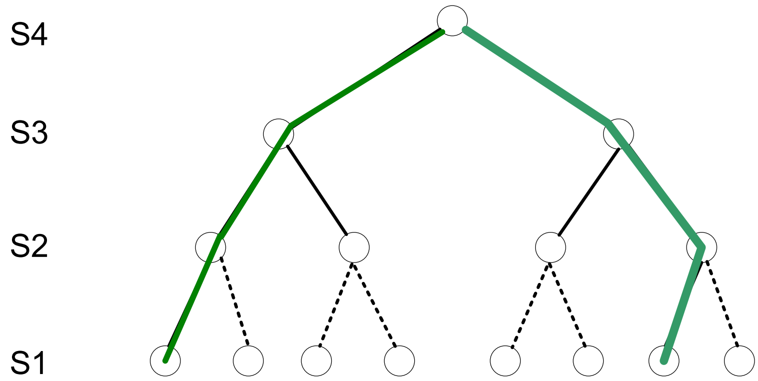

Figure 2.6: Extend sphere decoding to tree search representation

2.4.2 Sphere Decoding algorithm

The sphere decoding can be ragarded as finding s [14]

$$y = Hs + n \tag{2.12}$$

where $\mathbf{s} = [s_1 s_2 \cdots s_{N_T}]^T$ The maximum likelihood decoding algorihtm can be rewritten as from in Eq.(2.4) the

$$\begin{aligned} \|n\|^2 = \|\mathbf{y} - \mathbf{Hs}\|^2 &= (\mathbf{s} - \mathbf{s_{zf}})^H \mathbf{H}^H \mathbf{H} (\mathbf{s} - \mathbf{s_{zf}}) \\ &+ \|\mathbf{y}\|^2 - \|\mathbf{Hs_{zf}}\|^2 . \end{aligned} \tag{2.13}$$

Based on Fincke Pohst method in [1], the lattice point $(H)\mathbf{s_{zf}}$ lies inside the sphere of radius d

$$\begin{aligned} d^2 \geq \|\mathbf{y} - \mathbf{Hs}\|^2 &= (\mathbf{s} - \mathbf{s_{zf}})^H \mathbf{H}^H \mathbf{H} (\mathbf{s} - \mathbf{s_{zf}}) \\ &+ \|\mathbf{y}\|^2 - \|\mathbf{Hs_{zf}}\|^2 . \end{aligned} \tag{2.14}$$

By using the transform of Eq.(2.4)(2.5), Eq.(2.14) can be rewritten as

$$d'^2 \geq \sum_{i=1}^{N_R} \left\| y_i - \sum_{j=i}^{N_T} R_{ij} s_j^{(i)} \right\|^2 , \tag{2.15}$$

Futher simplification of Eq.(2.15) and assume $N_T = N_R = M$ and $s_{zf} = \tilde{\mathbf{s}}$ gives

$$
\begin{aligned}
d'^2 &\geq (\mathbf{s}-\tilde{\mathbf{s}})^H \mathbf{H}^H \mathbf{H} (\mathbf{s}-\tilde{\mathbf{s}}) \\
&= (\mathbf{s}-\tilde{\mathbf{s}})^H \mathbf{R}^H \mathbf{R} (\mathbf{s}-\tilde{\mathbf{s}}) \\
&= \sum_{i=1}^{M} R_{i,i}^2 \left((\mathbf{s}_i - \tilde{\mathbf{s}}) + \sum_{j=i+1}^{M} \frac{R_{i,j}}{R_{i,i}} (\mathbf{s}_i - \tilde{\mathbf{s}}) \right)^2 \\
&= R_{M,M}^2 (\mathbf{s}_M - \tilde{\mathbf{s}}_M)^2 \\
&+ R_{M-1,M-1}^2 \left(\mathbf{s}_{M-1} - \tilde{\mathbf{s}}_{M-1} + \frac{R_{M-1,M}}{R_{M-1,M-1}} (\mathbf{s}_M - \tilde{\mathbf{s}_M}) \right)^2 + \ldots.
\end{aligned} \tag{2.16}
$$

The condition leads to s_M falls in the interval

$$
\left\lceil \tilde{\mathbf{s}}_M - \frac{d'}{R_{M,M}} \right\rceil \leq \mathbf{s}_M \leq \left\lfloor \tilde{\mathbf{s}}_M + \frac{d'}{R_{M,M}} \right\rfloor \tag{2.17}
$$

For every s_M satisfy Eq.(2.17) we defined a new constant

$$
d_{M-1}'^2 = d'^2 - R_{M,M}^2 (\mathbf{s}_M - \tilde{\mathbf{s}}_M)^2 \tag{2.18}
$$

and a new condition can be modified as

$$
{d'_{M-1}}^2 \geq R_{M-1,M-1}^2 \left(\mathbf{s}_{M-1} - \tilde{\mathbf{s}}_{M-1} + \frac{R_{M-1,M}}{R_{M-1,M-1}} (\mathbf{s}_M - \tilde{\mathbf{s}_M}) \right)^2 \tag{2.19}
$$

Defined $\mathbf{s}_{M-1|M} = \tilde{\mathbf{s}}_{M-1} + \frac{R_{M-1,M}}{R_{M-1,M-1}} (\mathbf{s}_M - \tilde{\mathbf{s}_M})$ and Eq.(2.19) is equuvalent to

$$
\left\lceil \tilde{\mathbf{s}}_{M-1|M} - \frac{d'_{M-1}}{R_{M-1,M-1}} \right\rceil \leq \mathbf{s}_{M-1} \leq \left\lfloor \tilde{\mathbf{s}}_{M-1|M} + \frac{d'_{M-1}}{R_{M-1,M-1}} \right\rfloor \tag{2.20}
$$

In a similar process, one can find possible $\mathbf{s}_{M-2}$ and so on, starting nested condition until possible $\mathbf{s}_1$ is found.

2.5 K-Best Sphere Decoding algorithm

K-best SD algorithm is an alternative method that improves the decoding throughput. It simplified the original SD algorithm and maintains a constant throughput by keeping only the K smallest accumulated PED at each layer. However, K-best SD algorithm can not guarantee the performance of ML detection since the ML solution may be eliminated when it is not of the K best accumulated PEDs. Thus, larger K is required and the value K becomes a tradeoff between complexity and error performance.

Fig. 2.8 illustrates the bit error rate of a 4×4 MIMO detector of different values of K, Experimental results show that for 64-QAM, if K is equal to 64, there is practically no performance degradation to the ML perfromance. However, there is performance degradation when K is chosen too small.

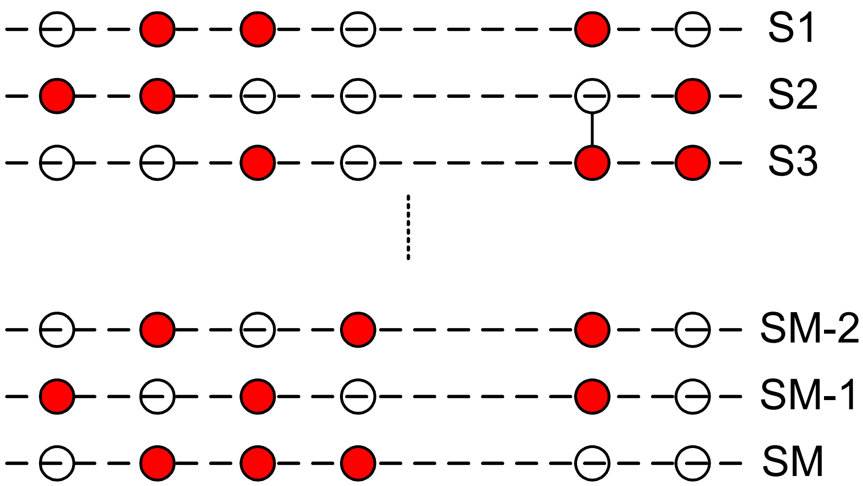

Figure 2.7: Geometrical representation for K-Best SD algorithm in each decoding layer

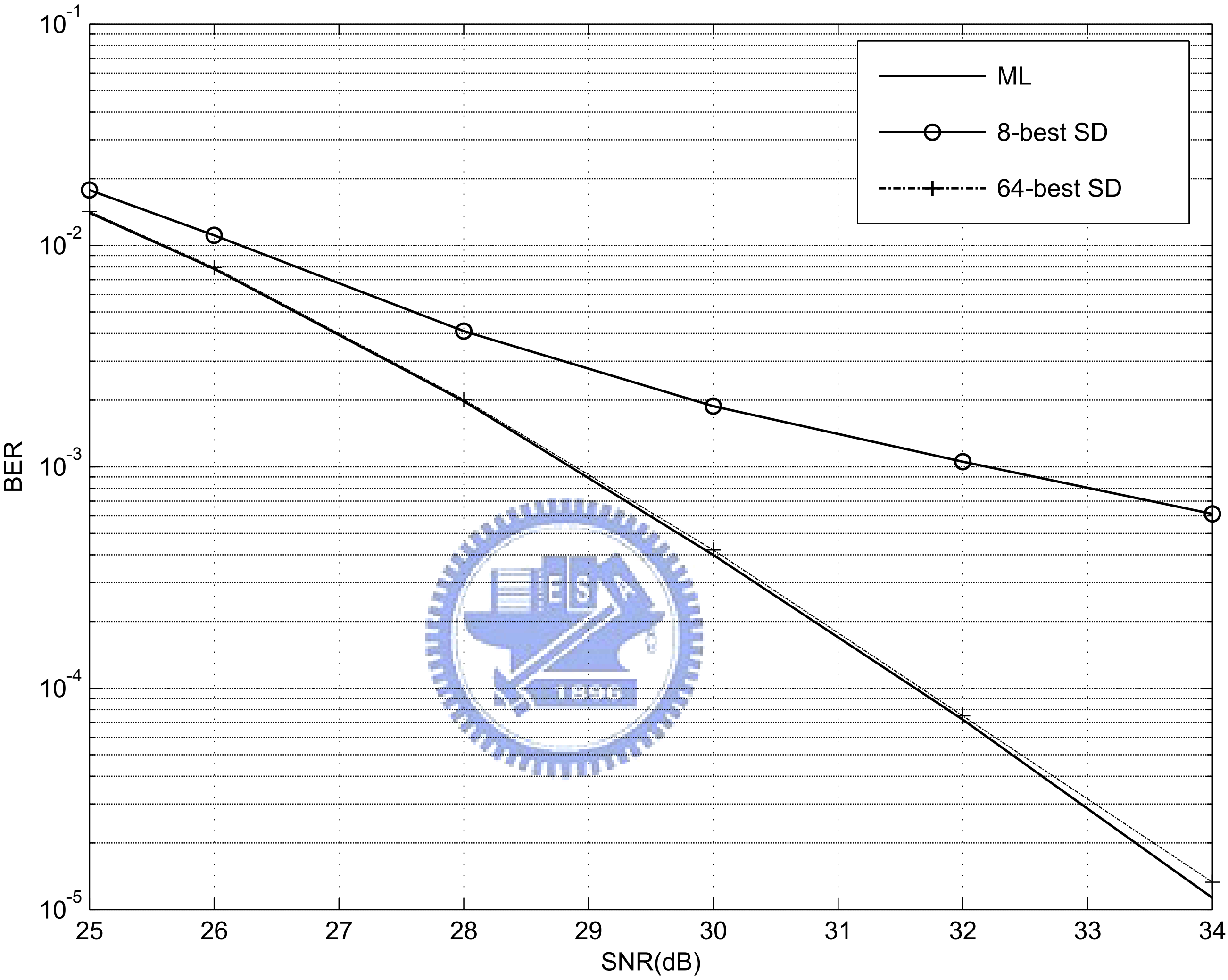

Figure 2.8: Comparison of ML and K-Best SD algorithm for 4×4 64-QAM MIMO system with K=8 and 64

Chapter 3

Proposed Adaptive K-Best SD Algorihtm

To keep the performance similar to ML detection, we should take advantage of both Sphere Decoding algorithm and K-Best SD algorithm. Where one used specific radius to choose possible candidates and the other kept constant K to achieve constant decoding throughput and easier for parrallel and pipeline hardware implemtation. The adaptive K-Best SD algorithm are introduced which use signal indicator to relfect the signal condition to choose different K to achieve low computation complexity. Predicted candidates kept the paths with smaller Partial Euclidean Distance(PED), and pruned those improper ones. The other method is introduced in the next Chapter.

3.1 Adaptive K-Best SD algorithm

Determining a proper K value is a way to reduce complexity and error probability. Due to fading, the signals suffer from low SNR when they are in deep fades, and K should be chosen larger. Contrarily, smaller K is sufficient when the signal strength is high. Dynamic K implies an signal quality indicator is required.

A technique for supporting dynamic K which is referred as adaptive K-best SD algorithm, provides a means to observe the required signal quality. For a MIMO system of N_T transmit antennas, this indicator can be acquired by the ratio

$$R = \frac{M_2}{M_1}, \tag{3.1}$$

where M_2 and M_1 are the second minimum and minimum of the N_t-th decoding layer, respectively. It can be observed that when the value R is below some threshold, the probability of the ML path being eliminated during the K-best SD processing increases.

Fig.3.1 is an illustrative example of a 4×4 64-QAM system, which shows the relation between T and the symbol error probability conditioned on the value T. The curve stands for the probability $Pr(R < T)$, and the histogram shows the the conditional symbol error probability. It is perceived that symbol error probability is small as T increases. Thus, the value K can be determined by first computing R in Eq.(3.1) , then

$$K = \begin{cases} K_1 & \text{if } R \leq T; \\ K_2 & \text{otherwise.} \end{cases} \tag{3.2}$$

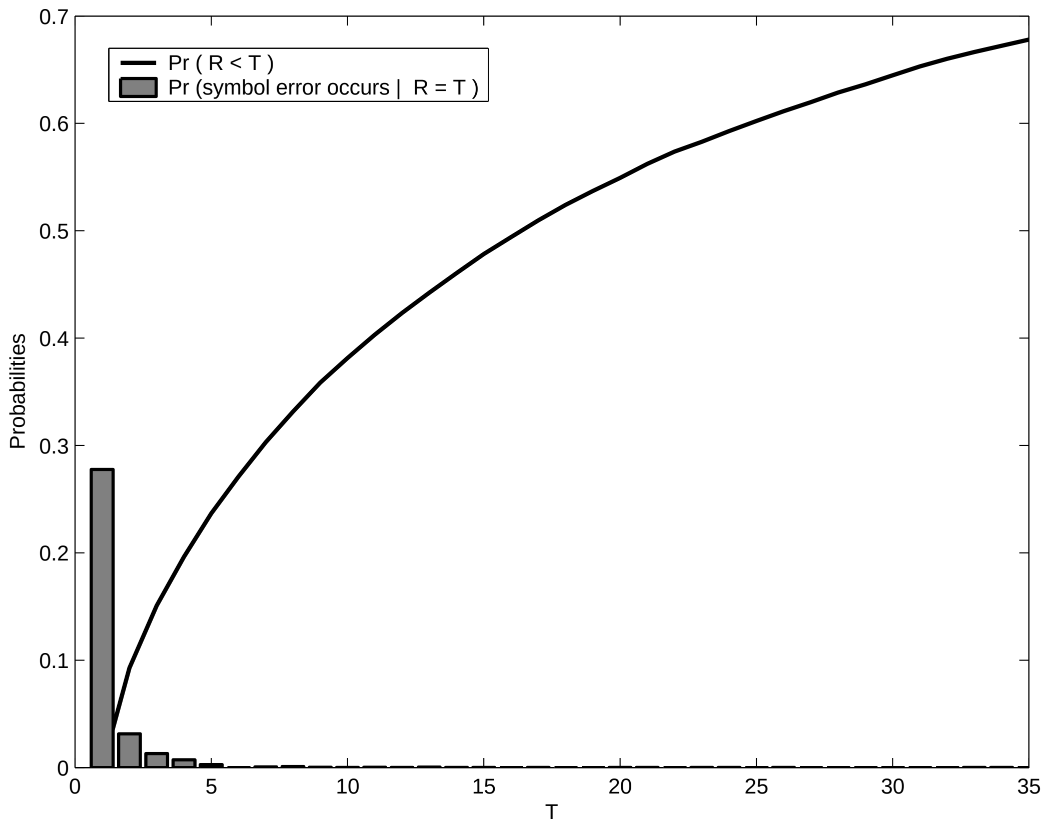

Figure 3.1: The probability of $R < T$ and the conditional symbol error probability.

The value R can be regarded as a signal quality indicator of the visited signals. In fact, at each decoding layer, there is always a corresponding R, and the layer number in which

R is determined becomes a tradeoff between computation complexity and performance. If R is determined at the first few decoding layers, the computation of the rest of the decoding layers can be reduced if $K = K_2$ is chosen. However, if R is determined earlier, there are chances that R cannnot provide sufficient information to report the signal quality and the performance will degrade.

The following analysis will show that why R can be regarded as a signal quality indicator. We defined $\tilde{S}_{2N}$ as the received signal of the decoding layer and $\hat{S}_{2N}$ is the signal received that makes Eq.(2.10) smallest. $\tilde{S}_{2N}$ may have a distance $K \times \Delta$ to $\hat{S}_{2N}$. $C_{min}^{(2N)}$ is the min cost in one decoding layer and $C_{2nd-min}^{(2N)}$ is the second min cost of the same decoding layer. The ratio of $C_{2nd-min}^{(2N)}$ and $C_{min}^{(2N)}$ can be the same as previous mentioned R. There are $k = 0$ and $k \neq 0$ case. For $k = 0$ case, it illustrates that the decoding signal is the transmitted signal , which implies the noise is samll. From the equation when the SNR becomes larger, the ratio increased. Non-zero k implies large noise. Larger K indicates the $2N$ layer signal suffer from server noise, therefore the ratio becomes smaller.

$$\hat{s}_{2N} = \tilde{s}_{2N} + k\Delta \tag{3.3}$$

$$\begin{aligned} C_{min}^{(2N)} &= (y_{2N} - R_{2N}\hat{s}_{2N})^2 \\ &= (y_{2N} - R_{2N}\tilde{s}_{2N} - R_{2N}k\Delta)^2 \\ &= (n_{2N} - R_{2N}k\Delta)^2. \end{aligned} \tag{3.4}$$

$$\begin{aligned} C_{2nd-min}^{(2N)} &= (y_{2N} - R_{2N}(\hat{s}_2N + u\Delta))^2, u \in \{+1, -1\} \\ &= (n_{2N} - R_{2N}(k+u)\Delta)^2. \end{aligned} \tag{3.5}$$

$$\frac{C_{2nd-min}^{(2N)}}{C_{min}^{(2N)}} = \begin{cases} 1 + \Delta^2 SNR_{2N} - 2u\Delta\sqrt{SNR_{2N}} & \text{if } k = 0; \\ 1 + \dfrac{(2ku+1)\Delta^2 SNR_{2N} - 2u\Delta(\sqrt{SNR_{2N}})}{1 + SNR_{2N}k^2(\Delta^2) - 2\sqrt{SNR_{2N}k\Delta}} & \text{if } k \neq 0. \end{cases} \tag{3.6}$$

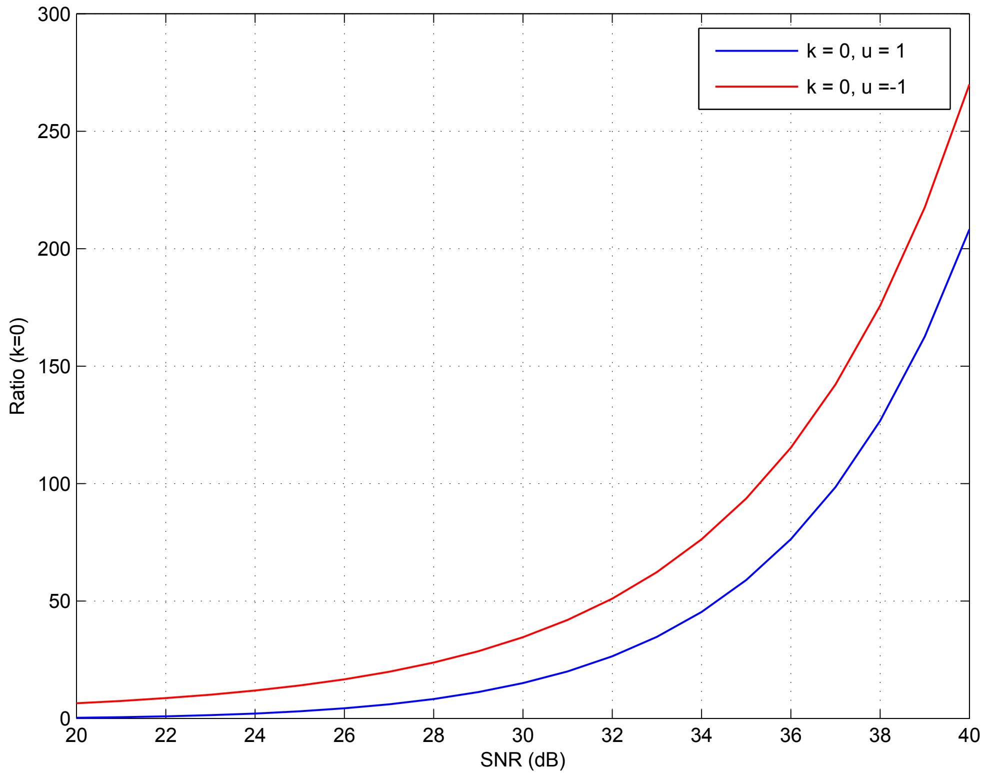

Figure 3.2: The ratio of second minimum over mimimum for k=0 figure

3.2 Predicted candidates

Although the computation complexity of adaptive K-best SD algorihtm is necessarily lower than the conventional SD algorithm since we use different K to choose the PEDs of each decoding process need to be calculated. However, in some case (low SNR region) only the K PEDs resulting to the K best accumulated PEDs can affect the PED calculation in the next decoding layer. That is, part of computations of the PEDs are unnecessary. A method to predict the more likely PEDs is presented in the following. Only a fraction of the PEDs are computed, and thus, the computation can be greatly reduced.

At decoding layer i, the point $\hat{s}_i$ results in the smallest PED for a given $\mathbf{s}^{(i+1)}$ can be derived by

$$\hat{s}_i{}^{(i+1)} = Q\left[\frac{y_i - \sum_{j=i+1}^{N_T} R_{ij} s_j^{(i+1)}}{R_{ii}}\right], \tag{3.7}$$

where $Q[*]$ represents for quntization value and only the $L-1$ points nearest to $\hat{s}_i{}^{(i+1)}$ will be computed for $e(\mathbf{s}^{(i)})$. That is, the $s_i^{(i)}$ of the vector $\mathbf{s}^{(i)}$ will be $\hat{s}_i{}^{(i+1)}$ and its $L-1$ nearest constellation points. Only L PEDs from $e(\mathbf{s}^{(i+1)})$ should be calculated instead.

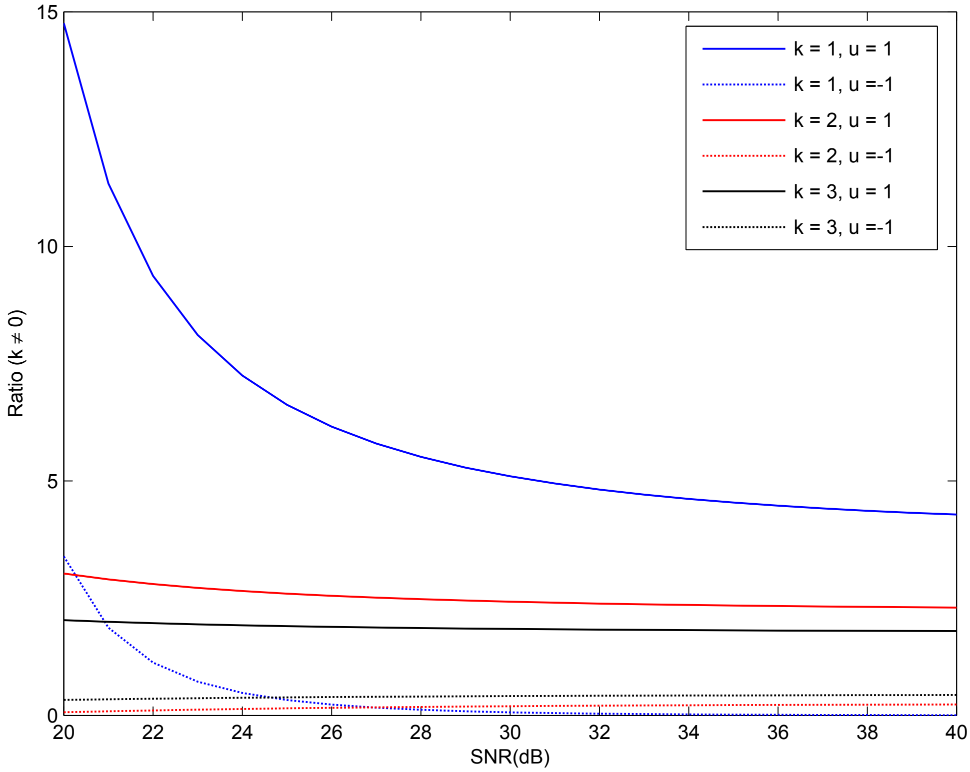

Figure 3.3: The ratio of second minimum over mimimum for $k \neq 0$ figure

Accordingly, we can always have the PED values computed in an ascending order, and the first L smallest PEDs will contribute to more likely candidates.

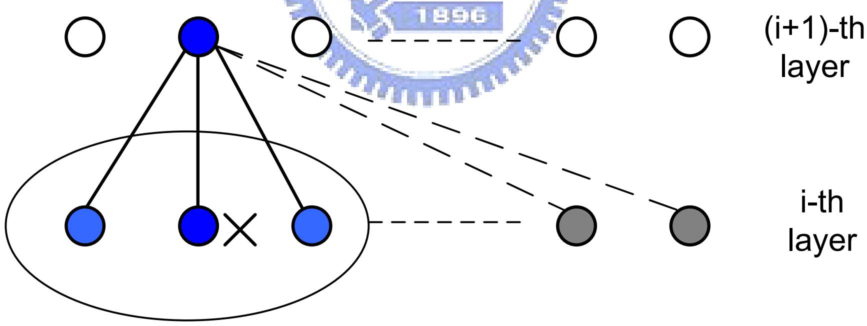

Figure 3.4: Adaptive K-Best SD algorithm by using predicted candidates

Fig.3.4 is a 64-QAM example with $L = 3$. The constellation corresponds to the i-th layer is denoted as s_j(blue ball), as the figure shows, the points with cross mark will be quantized to the $\hat{s}^{(i+1)}$, and only the three constellation points (linked by solid lines) will be computed. Thus, the computation complexity can be reduced, especially when N_T is large.

Chapter 4

Proposed Divided Sorting Strategy and Clustered K-Best SD algorithm

During the decoding, sorter is a bottle neck in the computing process, thus divdided sorting strategy is proposed to use several local sorter instead of one global one. Reducing the sorter size will reduce the computation complexity. Further, we enhance the idea of not using any sorter to keep the candidates by utilizing the concept to set specific radius of Sphere Decoding, constant candidates of K-best SD algorithm, and replace sorter with a few comparators. Without sorter, we loosely keep the possible candidates instead of precisely order all the candidates. The algorihtms are introduced as follows.

4.1 Divided Sorting Strategy

Though we have used adaptvie K-Best SD algorithm to support dynamic K value according to the signal quality, and predicted candidates method to keep the nearest node, the heavy computation complexity of sorting is still a unsolved problem. A technique to reduce the number of sorting operation is the divided sorter technique. The concept is based on divides the original one global sorter into several local sorters. Arithmetic complexity is generally written in a form known as Big-O notation, where the O represents the complexity of the algorithm and a value n represents the size of the set the algorithm is run against. The two classes of sorting algorithms are $O(n^2)$, which includes the bubble, insertion, selection, and shell sorts; and $O(n \log n)$ [15] which includes the heap, merge, and quick sorts. for $O(n^2)$ algorithms, the divided sorting strategy for four local sorter can be expressed as

$$(n^2) > (n/4)^2 \times 4 \tag{4.1}$$

and for $O(n \log n)$ algorihtms, the divided sorting strategy for four local sorter can be rewritten as

$$(nlogn) > (n/4)log(n/4) \times 4 \tag{4.2}$$

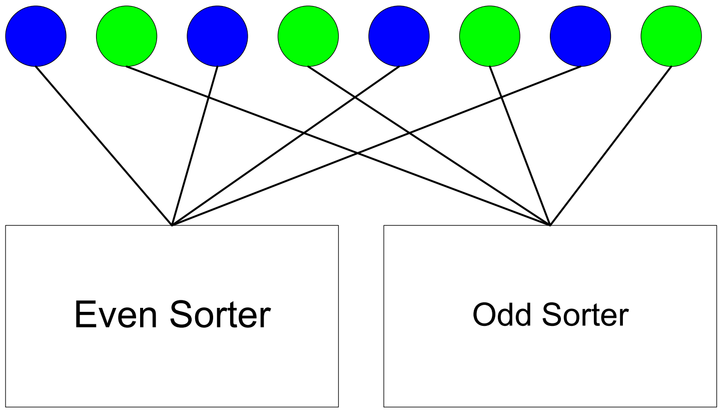

Figure 4.1: divided sorter

Fig. 4.1 is a example. The computation effort is reduced to 25% for $O(n^2)$ algorithms, and the reduction of computation complexity for $O(n \log n)$ will be small when n is large. As long as K is sufficiently large, each local sorter will still have similar input distribution as the orignal global sorter. Thus using divided sorter strategy will keep similar peformance to conventional K-Best SD algorithm The simulation results show in chapter 5.

4.2 Clustered K-Best SD algorithm

In [16] the paper choose an upper bound C and discard $e(s_i) > C\text{-}\alpha \times i \times C/8$ where $i = 9 - k$, and k is the detection layer. But this bound is set by simulation result and this bound may vary with different channel nature. Our method use statstics characteristcs [17] thus the bound will not vary, and is suitalbe for hardware implementation. As the equation model in Chap 2, a MIMO system with N_T transmit antennas and N_R receive antennas, the transmitted and received signals can be represented by

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}, \tag{4.3}$$

the assumption of each parameter is specified in Chap.2 channel model.

The equation is equivalent to minimize the vector $\hat{n}$. To make the $\|\mathbf{y} - \mathbf{H}s\|$ part as small as possible. for

$$\|\tilde{\mathbf{n}}\| = \|\mathbf{y} - \mathbf{H}s\|, \tag{4.4}$$

we can rewrite Eq.(4.4) as follows

$$\sum\nolimits_{i=1}^{N_R} \left\|n^{(i)}\right\|^2 = \sum_{i=1}^{N_R} \left\| y_i - \sum_{j=i}^{N_T} R_{ij} s_j^{(i)} \right\|^2, \tag{4.5}$$

where $\mathbf{s}^{(i)} = [s_i^{(i)} s_{i+1}^{(i)} \cdots s_{N_T}^{(i)}]^T$ and $s_j^{(i)}$ is the j-th element of $\mathbf{s}^{(i)}$. Then the $\left\|\mathbf{n}^{(\mathbf{i})}\right\|^2$ is the square term of the i-th layer. Since we assume the noise is a i.i.d Gaussian random varialbe with variance σ_n^2, the $\sum_{i=1}^{N_R} \left\|n^{(i)}\right\|^2$ term can be viewed as a Chi-square with n degrees of freedom. To take the advantage of Chi-sqaure for using the confidence interval of probability, we use the inverse of the χ^2 Chi-square cumulative distribution which is a sumation of n i.i.d $N(0,1)$ function. The inverse of the χ^2 cdf for a given probability p and v degrees of freedom is

$$x = F^{-1}(p|v) = \left\{ x : F(x|v) = p \right\} \tag{4.6}$$

where

$$p = F(x|v) = \int_0^x \frac{t^{(v-2)/2} e^{-t/2}}{2^{v/2}\Gamma(v/2)} dt \tag{4.7}$$

and $\Gamma(*)$ is the Gamma function [18]. Each element of output x is the value whose cumulative probability under the $\chi^2(F)$ cdf defined by the corresponding degrees of freedom v (which means the N_R-th receiver antenna) is specified the corresponding probability p.

Fig. 4.2 shows that there is always a minimum working BER corresponding to a minimum working SNR for a system to operate properly. By using this SNR, we could calculate the σ_n^2 of noise and use this σ_n to decide the invese Chi-square cdf mentioned above. On the receiver side, it is not possible to derive the true variance of the transmitted signals, thus using variance of noise is an alternative way to be a criterion. And the value x will need to multiply σ_n^2 to satisfy the Chi-square condition. We defined

$$A_{ml} = x \times \sigma_n^2 \tag{4.8}$$

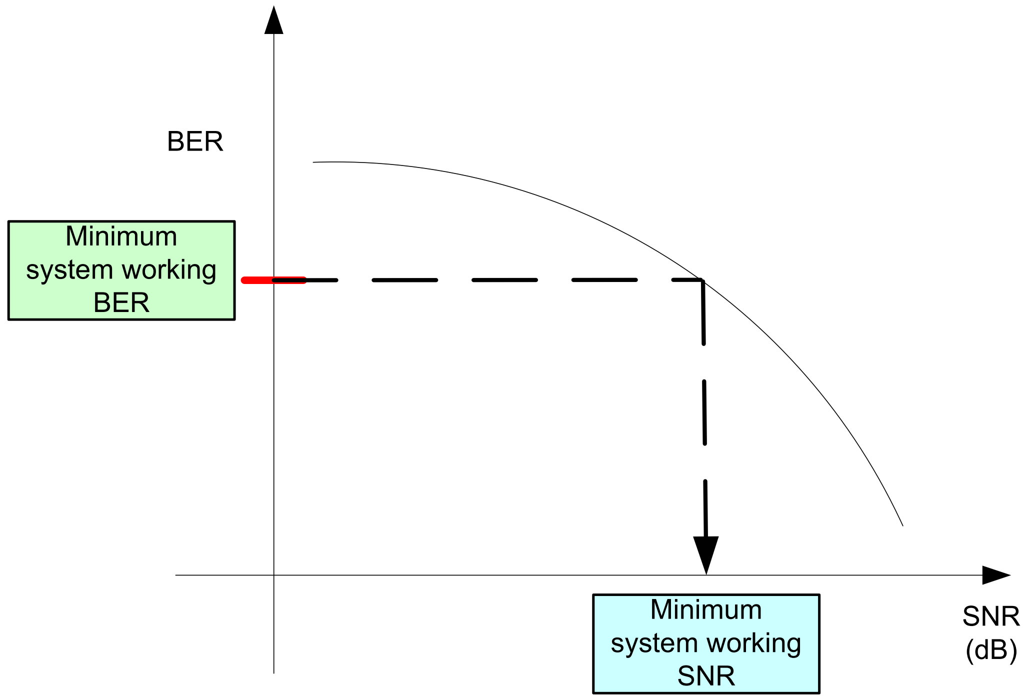

Figure 4.2: Representation of minimum working SNR and corresponding mimimum working BER

That is , there is very large probability that the value $\sum_{k=i}^{N_R} \|n\|^2$ will fall in the region $\left[0, A_{ml}^{(i)}\right]$. Different layers will have their own $A_{ml}^{(i)}$ constraint and has been decided before the system start to operate. From previous, one can obtain a typical value of r, for a true transmitted signal s

$$\|y - Hs\|^2 = \|N\|^2 \propto \sigma^2 \cdot \chi_{2N}^2 \tag{4.9}$$

where χ_{2N}^2 is a Chi-square random variable with $2N$ degrees of freedom. We can derived this expacted random varialbe by $\sigma^2 E\chi_{2N}^2 = 2\sigma^2 N$. In [5] the paper counts in the channel effect to choose a proper radius. Thus from Eq.(2.7) one possible choice of radius is

$$r^2 = 2\sigma^2 KN - y^*(I - H(H^*H)^{-1}H^*)y \tag{4.10}$$

where $K \geq 1$ is chosen, and a confidance interval is set up for the χ_{2N}^2 random variable, then one can capture the true s. However, this method have to choose proper K and a confidance interval of χ_{2N}^2 for the radius by try and error. In fact, the channel gain should be estimated correctly or the choice of radius will not keep proper candidates. It is diffucult to choose a proper radius since the channel estimation is never easy to be estimated precisely. Also, for MIMO system, the probability for all the channel gain increased or decreased simultaneously is very low. Thus in this thesis, we will not consider channel gain, we only use statistcis of chi-square and choose a loosely bound based on minimum working BER to keep K candidates in each decoding layer.

Fig. 4.3 shows the clustered K-Best SD algorithm of keep all the survival paths under the constraint of $A_{ml}^{(i)}$. After load the initial data, we calculate the path cost of each survival path. Compare with minimum cost criterion, if there is no path pass the A_{ml} criterion, load minimum cost and index back to the initial state. In the decoding process, there may have large candidates pass the A_{ml} criterion, thus the system have to keep all the survival candidates, this becomes a problem for hardware implementation since memory will be concerned in design a system. Thus we have to take the concept similar to the K-Best SD algorithm, to keep certain K candidates in each decoding iteration instead of keep all the survival paths. Also, how to keep the survival paths without sorter is a problem. Since the size of storage is only K, we should put those path's cost smallest in to the memory. However, we didn't preciesly have the order of all survival paths, we should prune some possible path in the decoding process. To provide the solution to this problem we divided $A_{ml}^{(i)}$ into C block. By this arrangement, we could arrange the smaller paths in the former then we could prevent the pruning smaller path's cost event.

Fig. 4.4 is the statsitcs cumulative probability of Aml constraint, by using minimum working SNR to get σ_n we can derive Aml criterion as Eq.(4.8). Fig. 4.5 is an illustrative example of how Aml consrtaint works. There is a specific Aml distribution for each layer. In the i-th layer compare the present PED with $A_{ml}^{(i)}$ constraint. Those path below the constraint which is on the left side of the red line, will be kept. Fig. 4.6 shows that divided the original Aml constraint into C block. For the original Aml constraint, there may be more candidates than K will survive under the constraint. However, there are only K candidates can be kept during the decoding process. One can keep the smaller path in the former to prevent pruing the possible candidates by this method. Though because we didn't prercisely sort the order of each path's cost in the storage blocks. Those paths in the last block may slightly cause calculation error. But as the simulation shows this can be sovled by increasing C and have sufficinet number of K. Chapping criterion into more piceses takes the advantage of get the more possible candidates in the former.

Fig. 4.7 shows the block diagram. The decoding process is as follows. At each detection

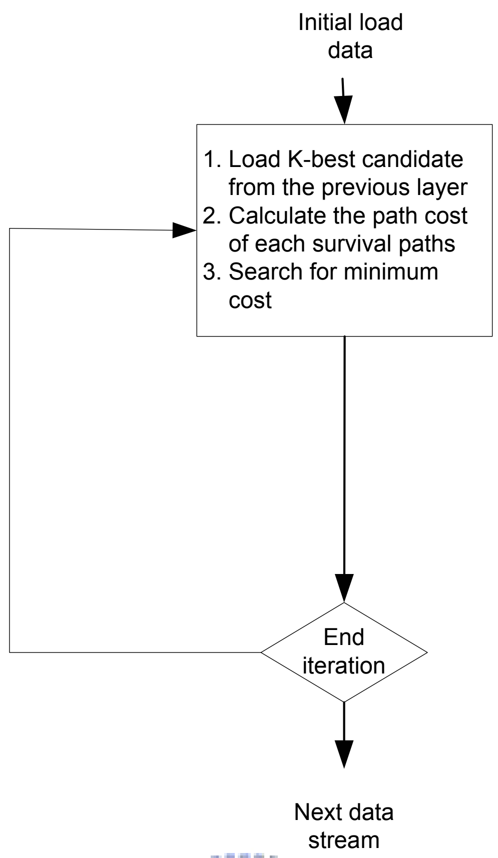

Figure 4.3: Clustered K-Best SD algorithm diagram

layer we refresh the candidate lists from the previous layer. There are memory blocks contain cnadidates of each possible path with their cost at present. We calculate the n-th child index from their m-th parent and its path's cost. To prevent the case that no path survived under the $A_{ml}^{(i)}$ constraint when the system will lose the possible candidates, the path with minimum cost should always be recorded and kept.

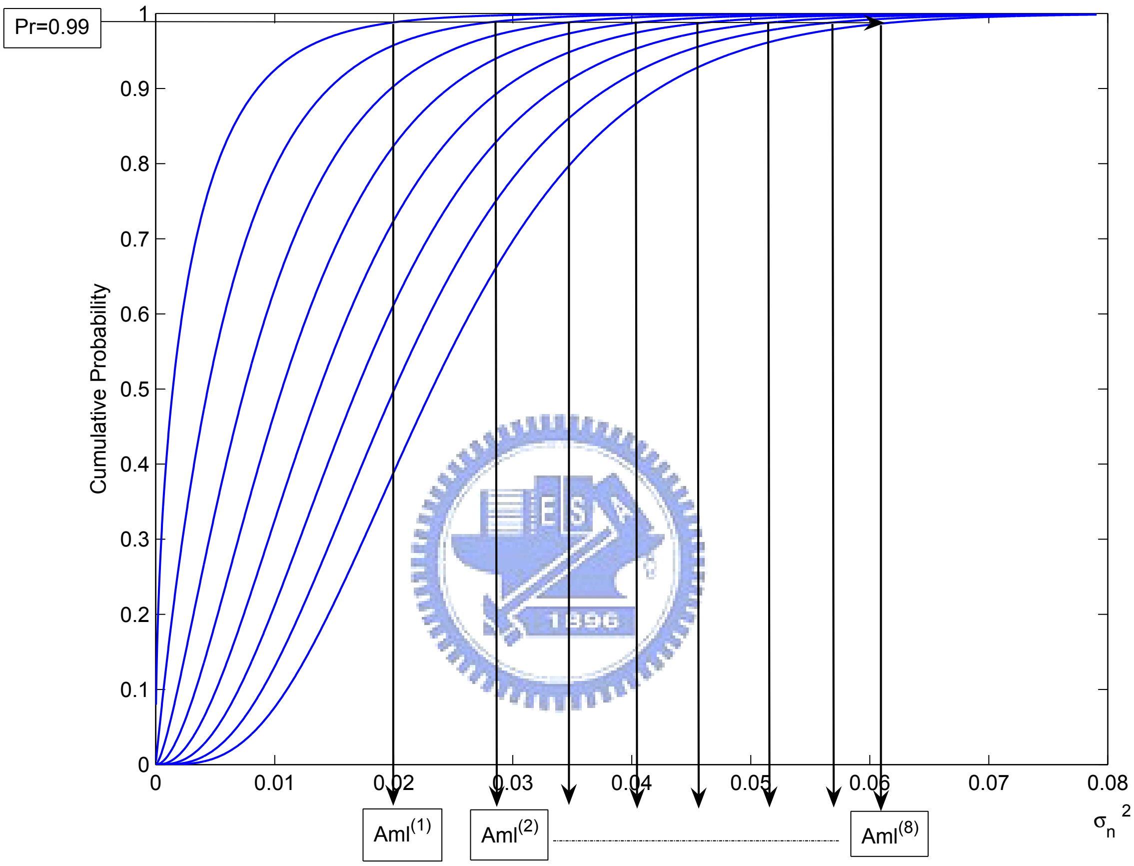

Figure 4.4: Cumulative probability of Aml constraint

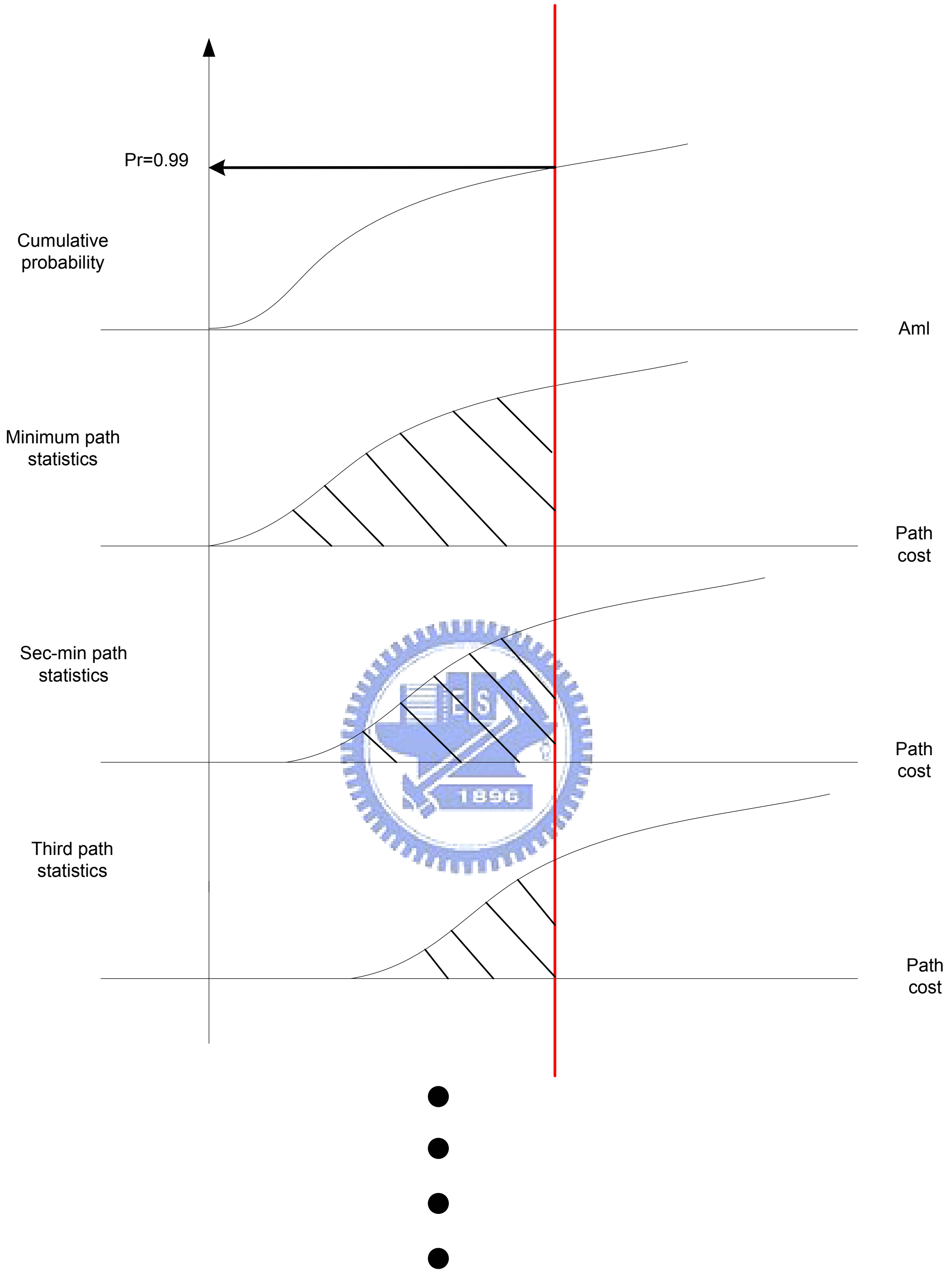

Figure 4.5: Concept of Aml constaint and ordered statistics of each path

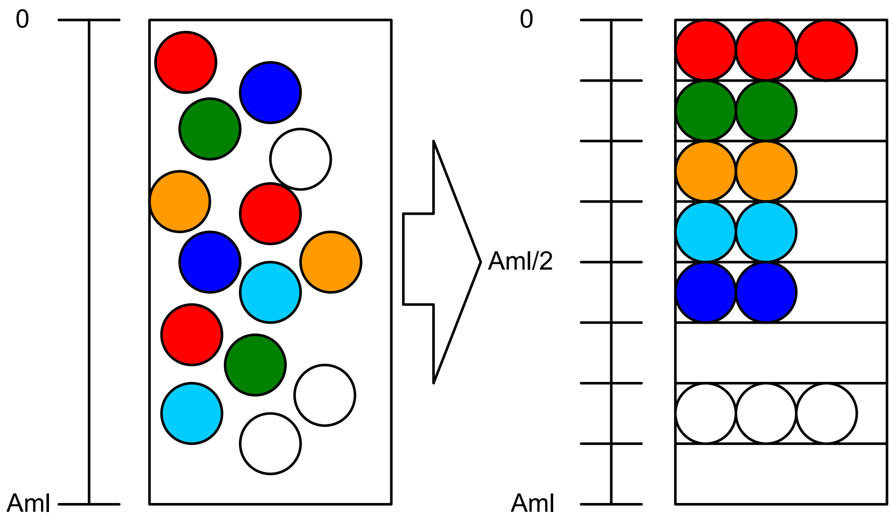

Figure 4.6: Group the Aml criterion into C blocks

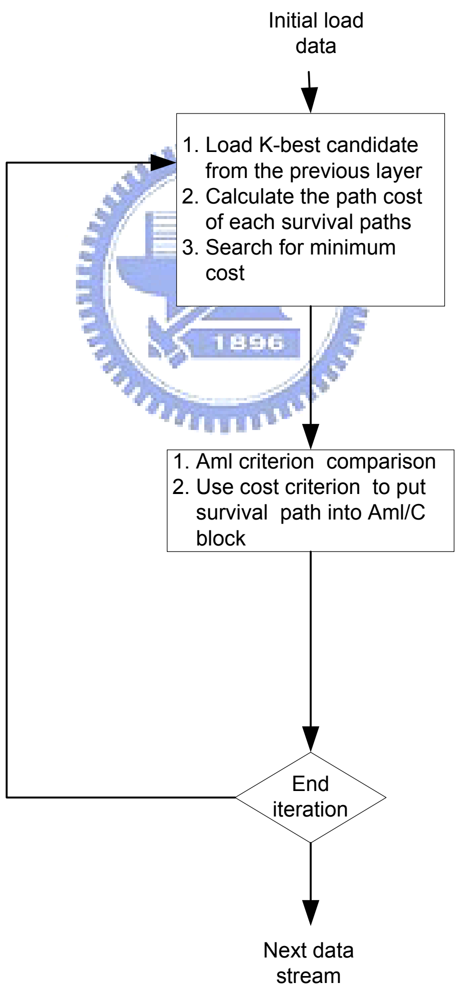

Figure 4.7: Improved clustered K-Best SD algorithm diagram

Chapter 5

Simulation Results and Comparison

In this section, a 4×4 MIMO system is simulated for comparing the proposed schemes and the conventional SD and K-best SD algorithms ($K = 64$), whereas the ML detection provides a performance baseline. The signal is modulated by 64-QAM and the MIMO channel is assumed to fade uncorrelatedly and independently. Totally 10^6 bits are simulated when the SNR is below 30dB, and 10^7 bits are simulated for SNR $\geq 30dB$.

5.1 Proposed Adpative K-Best SD Algorithm

The proposed adaptive K-best SD algorithm can be applied with the above mentioned candidate prediction technique, whereas the K_1 and K_2 can have distinct L_1 and L_2 values, respectively. Fig.5.1 presents the error probabilities versus SNR for different detection methods. It is perceived that for SNR lower or equal to 30 dB, all the proposed schemes can provide performance very close to that of the ML detection. When SNR is greater than 30dB, a slight degradation is shown, and the value L dominates the degradation. As shown in Fig. 5.1, for $K_1 = K_2 = 64$, the one with $L_1 = L_2 = 8$ outperforms the one with $L_1 = L_2 = 3$.

The value T provides a tradeoff between the complexity and error probability. Since smaller K_2 may lead to performance degradation in high SNR, a larger T will be required. On the other hand, Fig. 3.1 shows that symbol error probability drops when $T > 10$. Accordingly, we first compare two cases $K_1 = 64, K_2 = 32, T = 30$ with $L_1 = 8, L_2 = 8$ and $K_1 = 64, K_2 = 32, T = 15$ with $L_1 = L_2 = 8$, As Fig. 5.1 shows, the former

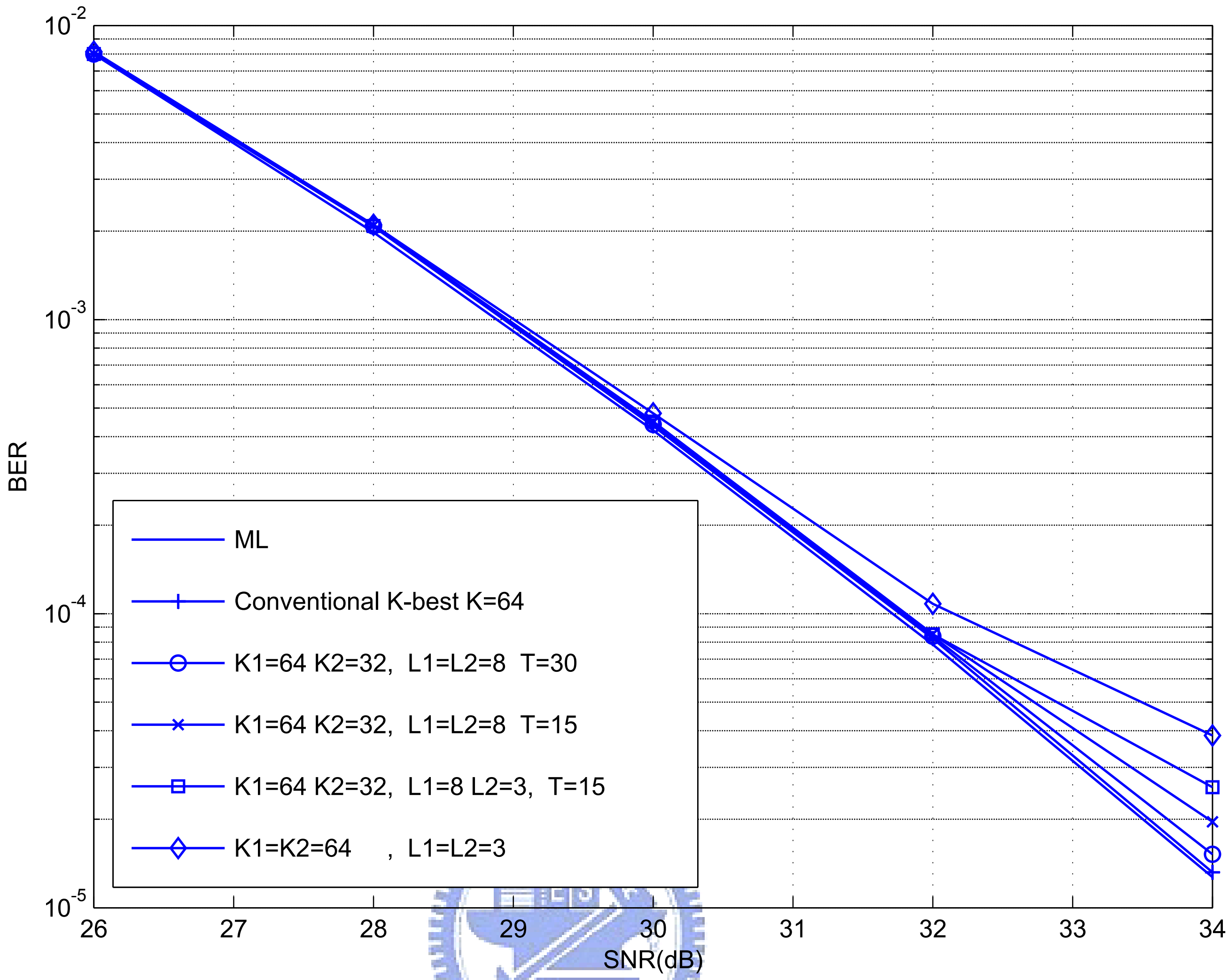

Figure 5.1: BER comparisons of different detection schemes for 4×4 64-QAM MIMO system

results to slightly smaller error probabilities, because $K_1 = 64$ is used more often than $K_2 = 32$, thus, the former case will gain some performance advantage. Next, we compare $K_1 = 64, K_2 = 32, T = 15$ with $L_1 = 8, L_2 = 8$ and $K_1 = 64, K_2 = 32, T = 15$ with $L_1 = 8, L_2 = 3$ cases, whereas the parameters chosen will result to similar computation complexities. It can be observed that the value L affect error probability. The maximum value of L is the dimension of the PAM constellation. Smaller L will reduce computation effort, however, the performance will also degrade since some computation is ignored.

Fig.5.3 and Fig.5.4 shows the percentage of K_1 and K_2 are selected for SNR = 30, 32, and 34 dB. As the SNR increases, the percentage of K_2 being selected also increases, and more computation complexity can be reduced. For all detection schemes, sorting

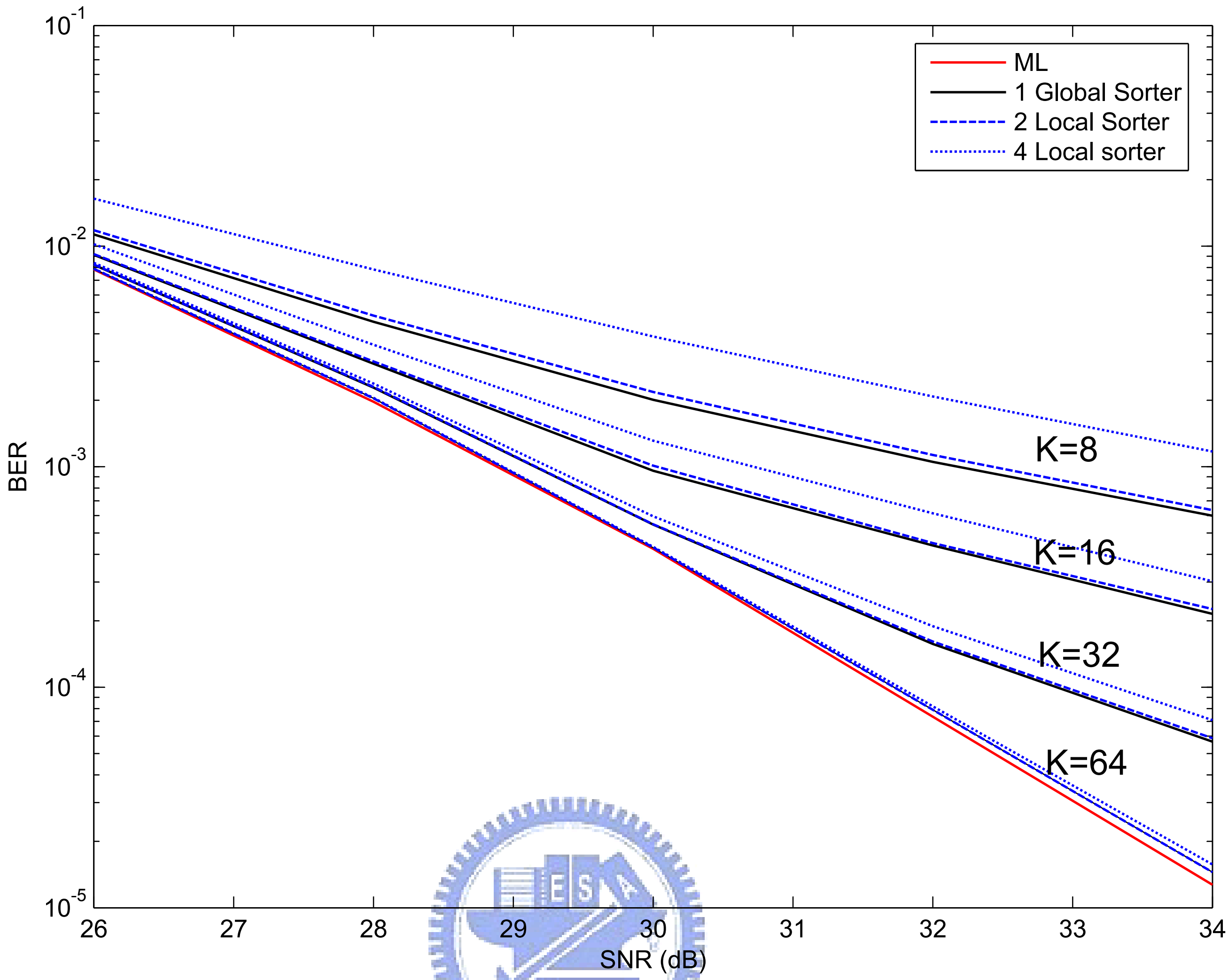

Figure 5.2: Performance comparison of different K for different size of sorting group

always contributes the most to the overall computation complexity. Thus, the number of sorting operations are recorded and shown in Table 5.1 and Table 5.2 for comparing the complexities. As compared with conventional 64-Best SD algorithm in a 64-QAM 4×4 MIMO system, The table shows that the reduction in the comparing complexity ranges from 23.65% to 52.22%, whereas the corresponding SNR degradation is maintained within 0.13dB and 1.1dB for a . Also, the reduction of addition and multiplication operation ranges from 18.59% to 61.66%.

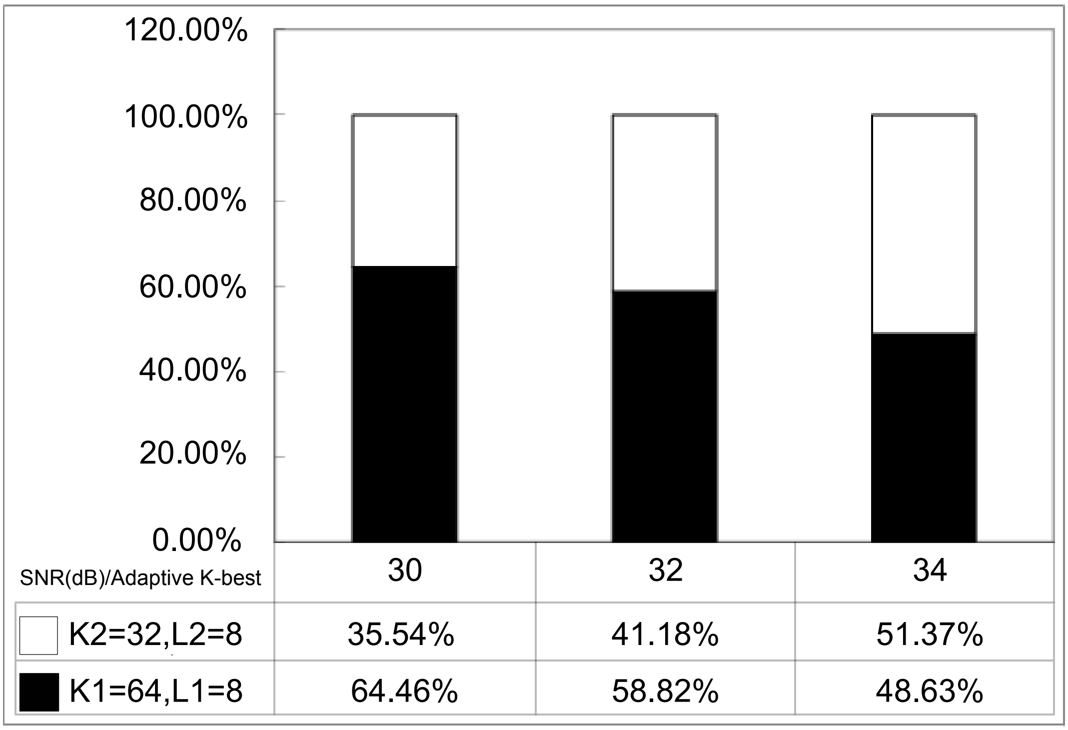

Figure 5.3: Reduce computation effort of adpative K-Best SD algorithm in SNR = 30, 32, and 34dB for T = 30.

5.2 Divided Sorting Strategy

Divided sorting strategy is based on the knowledge as the kept path increased, the performance degradation with ML detection is samll. Fig. 5.2 shows that when $K = 64$, the performance of dividing local sorter into 4 groups is close to 2 local sorter and global sorter. This provides the information that we can use smaller size local sorter in hardware consideration to achieve similar performance of global sorter technique and ML detection.

5.3 Clustered K-Best SD algorithm

Fig. 5.5 shows the perfornace comparison of using A_{ml} constraint with conventional K-Best SD algorihtm and ML detection. From the figure we can observe that when we choose the same candidates as 64-best SD algorithm divided Aml criterion into 16 block, their is only slightly performance degradation with conventional 64-Best SD algorihtm and ML detection. When the number of blocks is decreased, the performance degradation is significant. When one takes Aml constraint with $K = 64$ candidates and $C = 4$ block

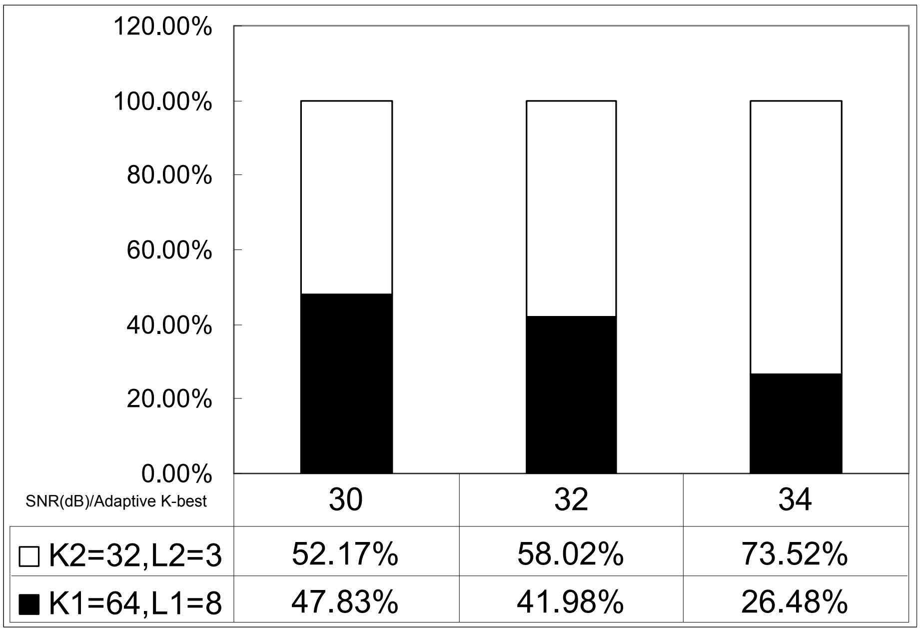

Figure 5.4: Reduce computation effort of adpative K-Best SD algorithm in SNR = 30, 32, and 34dB for T = 15.

the performance degradation is 2dB compared to the same candidates but with $C = 16$ block at $BER = 10^{-4}$. And when the number of candidates is decreased to 32, the performance degradation is not endured. From the simulation shows, we should choose $K = 64$ candidates and divided the block into 16, to maintain similar pefromance of conventional 64-Best SD algorihtm and ML detection.

Table 5.3 for comparing the complexities. The normalized comparing complexity refers to the number of compare operation of all methods normalized to that of the conventional 64-Best SD algorithm. As compared to 64-Best SD algorithm in a a 64-QAM 4×4 MIMO system. The reduction in the comparing complexity is over 99%, whereas the corresponding SNR degradation is maintained within 0.09dB. Also, the reduction copmlexity in addition and multiplication operation is over 98%.

Table 5.4 shows the average path using clustered K-Best SD algorithm in each decoding layer. The average path in each layer is 4.56 per layer. In consequence, the path needed to be calculated is in a small proportion in the decoding process when it compared to the conventional 64-Best SD algorithm. Besides we normalized addtion and multiplication, the reduction is 98.83% when compared to conventional 64-best algorithm, this benifit comes from the average path derived by clustered K-best SD algorithm is very

Table 5.1: Comparison of ML and conventional K-Best SD algorithm and adaptive K-Best SD algorithm

Method	ML	Conventional 64-best	$K_1 = K_2 = 64$ $L_1 = L_2 = 3$
Normalized add/mulp operations	$1.21 \times 10^5\%$	100%	38.34%
Comparing Operations	3.45×10^9	2.03×10^6	7.02×10^5
Normalized comparing Complexity	1.67×10^5 %	100%	34.58%
SNR (dB) for BER $= 5 \times 10^{-4}$	32.64	32.72	33.82

small thus the number of addition and multiplication is decreased. From this result, we can show that the clustered K-Best SD algorithm can reduce most of the addition and multiplication, and comparing operations to keep performance near conventional K-Best SD algorithm and ML detection. Fig 5.6 shows the comparing complexity normalized to conventional 64-Best SD algorithm with performance at BER=5×10^{-4} and Fig 5.7 shows the normalized add/mutiply operation to convnetional 64-Best SD algortithm with performance at BER=5×10^{-4}. They are the position of complexity with the corresponding performance.

Table 5.2: Comparison of conventional K-Best SD algorithm and adaptive K-Best SD algorithm with predicted candidates

Method	Conventional 64-best	$K_1 = 64, K_2 = 32$ $L_1 = 8, L_2 = 8$ T=30	$K_1 = 64, K_2 = 32$ $L_1 = 8, L_2 = 8$ T=15	$K_1 = 64, K_2 = 32$ $L_1 = 8, L_2 = 3$ T=15
Normalized add/mulp operations	100%	81.41%	63.72%	54.65%
Comparing Operations	2.03×10^6	1.55×10^6	1.28×10^6	9.7×10^5
Normalized Comparing Complexity	100%	76.35%	63.05%	47.78%
SNR (dB) for BER $= 5 \times 10^{-4}$	32.72	32.85	33.05	33.24

Table 5.3: Comparison of conventional K-Best SD algorithm and clustered K-Best SD algorithm

Method	Conventional 64-best	A_{ml} criterion $K = 64$ block $C = 16$
Normalized add/mulp operations	100%	1.17%
Comparing Operations	2.03×10^6	156
Normalized Comparing Complexity	100%	7.68×10^{-3}%
SNR (dB) for BER $= 5 \times 10^{-4}$	32.72	32.81

Table 5.4: Average path number of clustered K-Best SD algorithm for K=64, C=16

decoding layer	average path
8	2.91
7	3.86
6	4.91
5	5.79
4	6.09
3	6.01
2	4.70
1	2.22

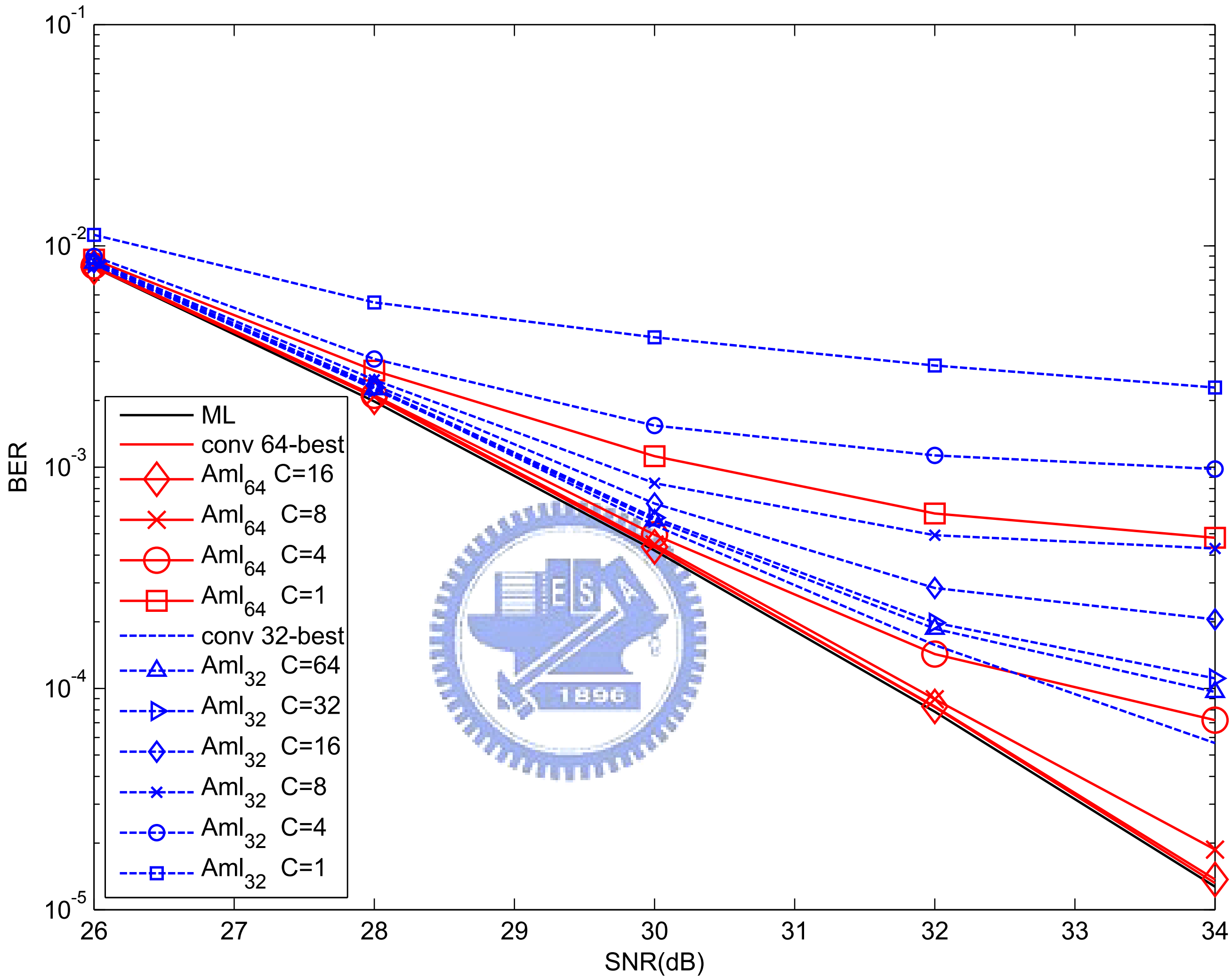

Figure 5.5: Comparison of ML and conventional K-Best SD algorithm and clustered K-Best SD algorithm performance

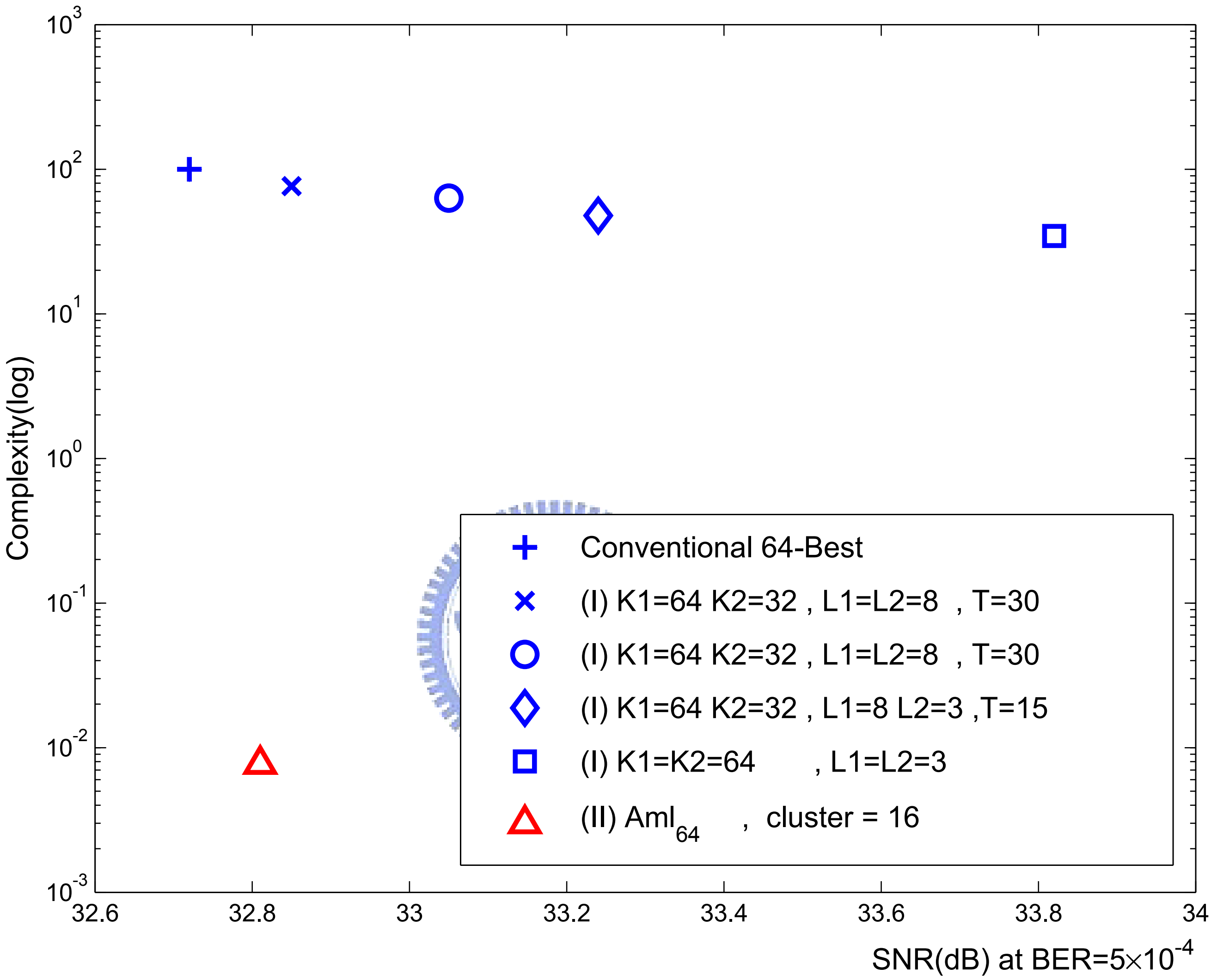

Figure 5.6: Comparison of comparing complexity for conventional 64-Best SD algorithm and adpative K-Best SD algorithm and clustered K-Best SD algorithm

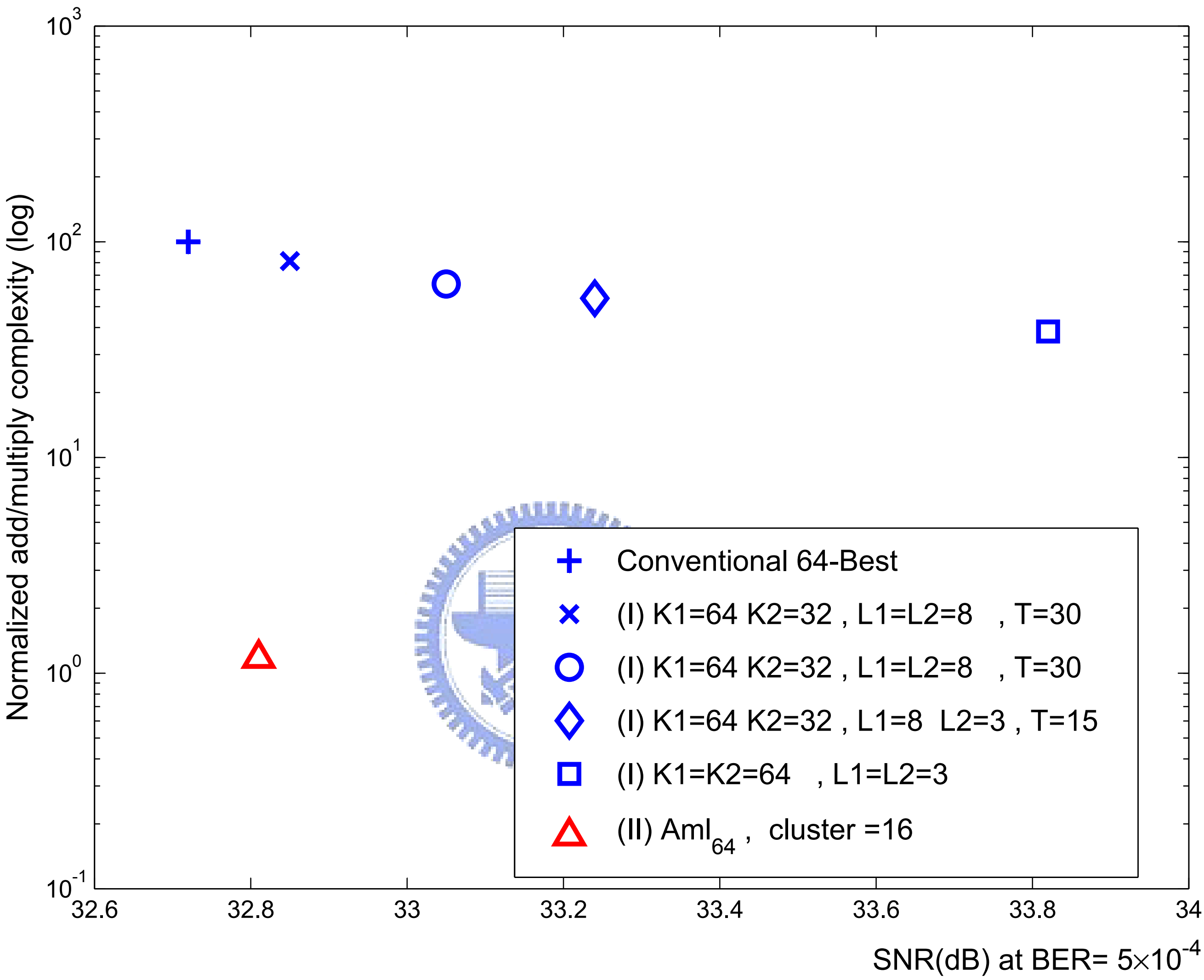

Figure 5.7: Normalized add/multiply operation complexity of conventional 64-Best SD algorithm and adpative K-Best SD algorithm and clustered K-Best SD algorithm

Chapter 6

Conclusion

In this thesis, low-complexity prediction techniques of K-Best SD algorithm are proposed. Adaptive K-Best SD algorithm, based on the signal indicator R derived in the computation process, the approach can reflect the channel nature and noise interfernce to choose proper K for different singal level. Predicted candidates technique can compute only a fraction of the PEDs and thus the computation can be greatly reduced. During the decoding process, sorter is always a computation bottleneck when K is large. A divdied sorting strategy is introduced to reduce the compuatation complexity. Sorting arithmetic complexity is refer to the input set size of the sorter. Several local sorters take advantage of having smaller input set size than the original global sorter. In order to reduce the sorting operation and keep similar performance with conventional K-Best SD algorihtm. The idea of clustered K-Best SD algorithm is introduced. Due to the statistcs knowledge of the received signal, we can replace high complexity sorter with a few comparators. One can use the Chi-square statistics, which provides a lower bound with high probability for possible candidates to survive under the constraint. Instead of sorting precisely, comparators loosely delivering the candidates into their specific cluster. Based on the concept of K-Best SD algorithm, keeping constant candidates in each decoding layer, we can use pipeline architecture for hardware implementation to achive constant decoding speed.

In conclusion, the thsis is focused on how to solve the computation complexity problem of K-Best SD algorithm. As compared with conventional 64-Best SD aglroithm for 4×4 64-QAM system. The adaptive K-Best SD algorithm can reduce complexity ranges from 23.65% to 52.22% within 0.13dB and 1.18dB performance degradation, whereas the clustered K-Best SD algorihtm can reduce over 99% complexity within 0.09dB performance degradation.

The proposed methods including adaptive K-Best SD algorithm and clustered K-Best SD algorithm can apply not only to the described MIMO signal detection probelms, but also the detection of non-orthogonal Space-Time-Block-Code (STBC) signals. Further, our proposed methods can reduce computation complexity and peform easier hardware implementation for those nearest lattice point problem utilizing SD algorithm.

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