Chapter 4

STC Combined with FHSS in Quasti-Stastic Fading Channels

In wireless channels, the transmitted signals are usually distorted by some intentional or unintentional jamming noises. From Chapter 2, we know that spread-spectrum systems are the most effective anti-jamming communication techniques, but these techniques cannot resist the fading effects. However, space-time coding combined with error control coding and transmit diversity design are effectively minimizing the effects of multipath fading. Therefore, we propose the design schemes combined with STC and three various types of FHSS systems. One is STC combined with worst-case frequency hopping spread spectrum (WFHSS) which hops the symbols from all transmitter antennas into the same frequency band, another is the joint design of STC and optimum frequency hopping spread spectrum (OFHSS) which avoids any possible collision of the transmitted symbols, and the other is the design of combined STC with uniform frequency hopping spread spectrum (UFHSS) which hops the transmitted symbol randomly over the spread spectrum bandwidth. The three systems we proposed are called STC/WFHSS, STC/OFHSS, and STC/UFHSS systems, respectively.

In this chapter, we give the detailed description of the STC/FHSS system model, and the ML decoding toghther with some suboptimal decoding schemes. We also propose the performance criteria for constructing good space-time codes and present some simulation results.

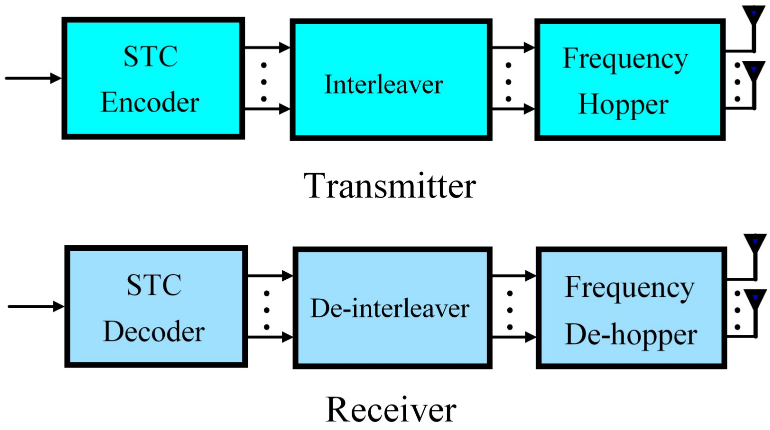

Figure 4.1: The proposed STC/FHSS system.

4.1 STC/FHSS System Model

Consider the proposed systems as shown in Figure 4.1, there are n and m transmitter and receiver antennas, respectively. Interleaver is inserted to break burst channel errors and guarantee memoryless channels, and the MFSK modulation is utilized to be compatible with the FHSS. We also assume slow frequency hopping with one hop per symbol for simplicity, and the hopping patterns generated from the transmitter are available to the receiver.

For the STC/WFHSS system as shown in Figure 4.2, let the receiver signals of the qth receiver antenna be expressed as

$$r_q(t) = \sum_{i=1}^{n} \alpha_{i,q}(t) s_i(t) + \eta_q(t) \tag{4.1}$$

where

$$\begin{aligned} \alpha_{i,q}(t) &= Ae^{j\theta} \\ s_i(t) &= \sqrt{2E_s} sin(\omega_n t) \\ \eta_q(t) &= \eta_I(t)\cos(\omega_n t) - \eta_R(t)\sin(\omega_n t). \end{aligned}$$

A is a Rayleigh random variable, θ is the random phase in $(0, 2\pi]$, E_s is the symbol energy, ω_n is the particular carrier frequency selected by the frequency hopper, $\eta_I(t)$ and $\eta_R(t)$ are statistically independent low-pass white Gaussian noise processes with one-sided noise

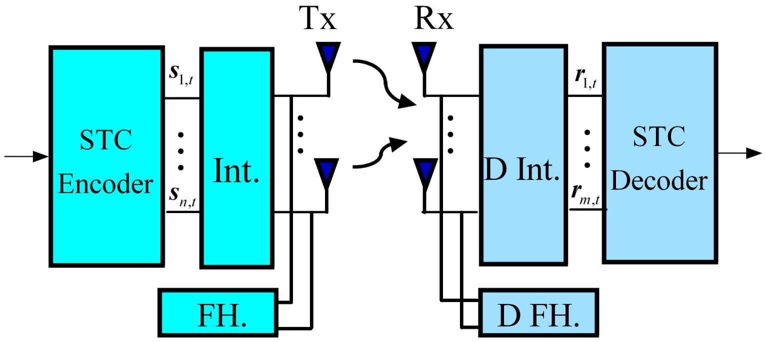

Figure 4.2: STC/WFHSS system model

spectral density N_0. After dehopping and demodulation, the received signal $r^k_{q,t}$ of the qth receiver antenna in the kth frequency slot at time t is given by

$$r^k_{q,t} = \sum_{i=1}^{n} \alpha_{i,q} s^k_{i,t} + \eta^k_{q,t}, \ \forall\ 1 \le q \le m \tag{4.2}$$

where $s^k_{i,t}$ is the symbol transmitted by the ith antenna in the kth frequency slot at time t, for all $1 \le k \le M$ and $1 \le t \le L$, $\alpha_{i,q}$ denotes the equivalent gain of multipath from the ith transmitter antenna to the qth receiver antenna. The transmitted symbols from all transmitter antennas are hopped into the same frequency band. For slow fading, assume that the fading coefficients are constant during a frame L, $L = 128$, and vary from one frame to another. The noise consisting of the AWGN $\eta^k_{W,q,t}$ and the partial-band noise jammer (PBNJ) $\eta^k_{J,q,t}$ is denoted by $\eta^k_{q,t}$, and it can be represented as

$$\eta^k_{q,t} = \begin{cases} \eta^k_{W,q,t} + x_t \eta^k_{J,q,t}, & x_t = 1 \\ \eta^k_{W,q,t}, & x_t = 0 \end{cases} \tag{4.3}$$

where x_t denotes the jamming state indicator (JSI) of the PBNJ taking value from 1 and 0 with probability ρ and $1-\rho$, respectively. Assume $\eta^k_{w,q,t}$ and $\eta^k_{J,q,t}$ are independently Gaussian distributed with zero mean and the variance N_0 and N_J/ρ, respectively [13][30]; $N_J/\rho = J/W_J$, where J is the total jammer power, W_s is the system bandwidth, W_J is the jamming bandwidth, and $\rho = W_J/W_s$ with $0 \le \rho \le 1$. The probability of x_t can be written as

$$P_{X_t}(x_t) = \begin{cases} \rho, & x_t = 1 \\ 1-\rho, & x_t = 0 \end{cases}. \tag{4.4}$$

By (4.2), the probability density function of $\eta_{q,t}^{k}$ is

$$f\left(\eta_{q,t}^{k}\right) = (1-\rho)\, a_1 \exp\left(-\frac{\left|\eta_{q,t}^{k}\right|^2}{N_0}\right) + \rho a_2 \exp\left(-\frac{\left|\eta_{q,t}^{k}\right|^2}{(N_0 + N_J/\rho)}\right) \tag{4.5}$$

where

$$a_1 = \frac{1}{\sqrt{\pi N_0}} \qquad a_2 = \frac{1}{\sqrt{\pi\left(N_0 + N_J/\rho\right)}}.$$

The equation (4.5) can be use to derive the likelihood function of the decoding scheme with respect to the STC/WFHSS system.

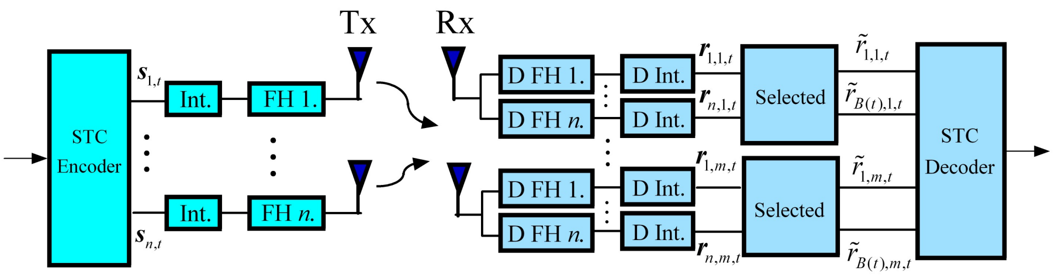

Figure 4.3: STC/UFHSS system model

Figure 4.3 shows the STC/UFHSS system, the transmitter and receiver both have the same frequency hopping patterns, so we could exactly know which transmitted symbols are collided together in the same frequency band. The receiver signal $\boldsymbol{r}_{i',q,t}$ $\forall 1 \leq i' \leq n, 1 \leq q \leq m$, and $1 \leq t \leq L$ can be described by an $M \times 1$ column vector, denoted by

$$\boldsymbol{r}_{i',q,t} = \left(r_{i,q,t}^{1}, r_{i',q,t}^{2}, \ldots, r_{i',q,t}^{M}\right). \tag{4.6}$$

The received signals $r_{i',q,t}^{k}$ of the qth receiver antenna transmitted from the i'th antenna in the kth frequency slot at time t can be expressed as

$$\begin{aligned} r_{i',q,t}^{k} &= \alpha_{i',q}\hat{s}_{i',t}^{k} + \sum_{\substack{i=1 \\ i\neq i'}}^{n} \beta_{i',i,t}\alpha_{i,q}\hat{s}_{i,t}^{k} + \eta_{i',q,t}^{k} \\ &= \sum_{\substack{i=1 \\ i\neq i'}}^{n} \beta_{i',i,t}\alpha_{i,q}\hat{s}_{i,t}^{k} + \eta_{i',q,t}^{k} \end{aligned} \tag{4.7}$$

for all $1 \le q \le m, 1 \le i' \le n, 1 \le k \le M,$ and $1 \le t \le L$, where $\eta^k_{i',q,t}$ denotes the composite noise consisting of the AWGN $\eta^k_{W,i',q,t}$ and PBJN $\eta^k_{J,i',q,t}$, and $\beta_{i',i,t}$ stands for the indicator of collided transmitted symbols taking value from 1 and 0. The probability of $\beta_{i',i,t}$ can be written as

$$P(\beta_{i',i,t}, i \neq i') = \begin{cases} \frac{1}{\mu} & , \ \beta_{i',i,t} = 1 \\ 1 - \frac{1}{\mu} & , \ \beta_{i',i,t} = 0 \end{cases} \tag{4.8}$$

and $\beta_{i',i,t} = 1$ for $i = i'$. where μ stands for the number of total frequency hopping bands, and $1/\mu$ is the probability of any two symbols hopped in the same frequency band. If the transmitted symbols are collided together, the corresponding received signals $\boldsymbol{r}_{i',q,t}$ have the same band information. Then, we choose one of the receiver signals and denoted it by

$$\tilde{\boldsymbol{r}}_{q,t,b} = \left(\tilde{r}^1_{q,t,b}, \tilde{r}^2_{q,t,b}, \ldots, \tilde{r}^M_{q,t,b}\right). \tag{4.9}$$

The selected signals $\tilde{r}^k_{q,t,b}$ of the qth receiver antenna in the kth frequency slot at time t at frequency hopping band b can be represented as

$$\tilde{r}^k_{q,t,b} = \sum_{\substack{i=1 \\ i \neq i'}}^{n} \tilde{\beta}_{i,t,b}\alpha_{i,q}s^k_{i,t} + \tilde{\eta}^k_{q,t,b} \tag{4.10}$$

for all $1 \le q \le m, 1 \le k \le M, 1 \le b \le B(t)$, and $1 \le t \le L$, where $B(t)$ denotes the number of total frequency bands which transmitted symbols occupyed at time instant t, $\tilde{\beta}_{i,t,b}$ is the indicator of collided transmitted symbols in the frequency band b taking values from 1 and 0, and $\tilde{\eta}^k_{q,t,b}$ stands for the composite noise consisting of the AWGN $\tilde{\eta}^k_{J,q,t,b}$ and PBNJ $\tilde{\eta}^k_{J,q,t,b}$ after dehopping with respect to the signals in the frequency band b. Let $\eta^k_{q,t,b} = \eta^k_{W,q,t,b} + x_{t,b}\eta^k_{J,q,t,b}$, where $x_{t,b}$ is the indicator of the PBNJ taking value from 1 and 0 with probability ρ and $1-\rho$, respectively. For example, assume that there are three transmitter antennas ($n = 3$), and the signals $\boldsymbol{s}_{1,t}, \boldsymbol{s}_{2,t}$, and $\boldsymbol{s}_{3,t}$ are transmitted at time instant t. As shown in Figure 4.4, we can find that the data $\boldsymbol{s}_{2,t}$ and $\boldsymbol{s}_{3,t}$ are hopped into the same frequency band, and then the receiver signals at qth receiver antenna after selected are denoted by $\tilde{\boldsymbol{r}}_{1,q,t}$ and $\tilde{\boldsymbol{r}}_{2,q,t}$. The selected signals denote the total transmitted signals are hopped into two frequency bands at time instant t. By (4.10), the probability density

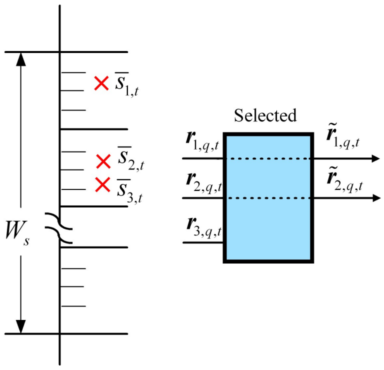

Figure 4.4: Example of the STC/UFHSS system for selected receiver signals

function of $\tilde{\eta}^k_{q,t,b}$ is

$$f\left(\tilde{\eta}^k_{q,t,b}\right) = (1-\rho)\, a_1 \exp\left(-\frac{\left|\tilde{\eta}^k_{q,t,b}\right|^2}{N_0}\right) + \rho a_2 \exp\left(-\frac{\left|\tilde{\eta}^k_{q,t,b}\right|^2}{(N_0 + N_J/\rho)}\right). \tag{4.11}$$

The above equation can be used to derive the likelihood function of the decoding scheme with respect to the STC/UFHSS system.

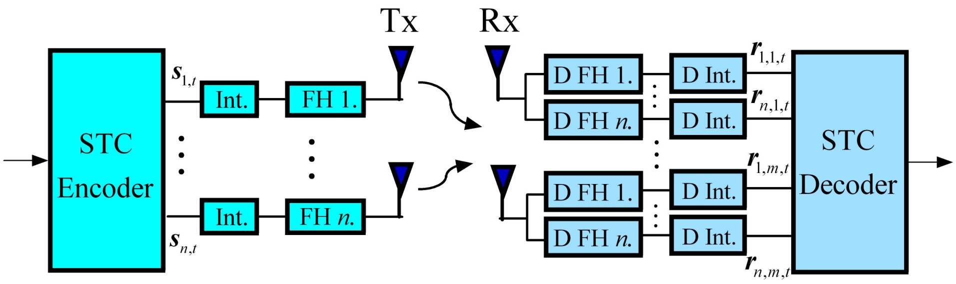

Figure 4.5: STC/OFHSS system model

For the STC/OFHSS system as shown in Figure 4.5, the received signal $r^k_{i,q,t}$ of the qth receiver antenna transmitted from the ith antenna in the kth frequency slot at time t is given by

$$r^k_{i,q,t} = \alpha_{i,q} s^k_{i,t} + \eta^k_{i,q,t} \tag{4.12}$$

for all $1 \le i \le n, 1 \le q \le m, 1 \le k \le M$ and $1 \le t \le L$, where $\eta^k_{i,q,t} = \eta^k_{W,i,i,t} + x_{i,t}\eta^k_{J,i,i,t}$, and all these values are defined the same as that in the STC/UFHSS system. By (4.12), the probability density function of $\eta_{i,q,t}^{k}$ conditioned on $x_{i,t}$ is

$$f\left(\eta_{i,q,t}^{k}\right) = (1-\rho)\, a_1 \exp\left(-\frac{\left|\eta_{i,q,t}^{k}\right|^2}{N_0}\right) + \rho a_2 \exp\left(-\frac{\left|\eta_{i,q,t}^{k}\right|^2}{(N_0 + N_J/\rho)}\right). \tag{4.13}$$

The likelihood function of the decoding schemes can be derived by equation (4.13) in the STC/OFHSS system.

Space-time codes can achieve transmit diversity as well as a coding gain. In additional, the signal transmitted by frequency hopping can avoid PBNJ effectively. Therefore, the STC/FHSS system is combined with temporal, frequency, and spatial domain to against multipath fading and jamming interferences. With respect to these three types of STC/FHSS systems, the performance variation could also be observed for comparison.

4.2 STC Combined with Worst-Case FH

Consider the proposed STC/WFHSS system as shown in Figure 4.2. At time instant t, the encoded codewords from all transmitter antennas are hopped into the same frequency band. Hence, the received symbols from each receiver antenna are dehopped with the same hopping pattern. We assume that the channel is a slowly flat fading channel, and the fading coefficients $\alpha_{i,q}$ are independent complex Gaussian random variable with zero mean and variance $\sigma_{i,q}^2$.

4.2.1 Decoding with CSI Available

In this section, the ML and soboptimal decoding schemes of this proposed system have been derived in [31]. We show the derived results here for discussion and comparison with respect to our proposed system. Assume perfect estimation of $\alpha_{i,q}'s$ is available at the receiver. Let the received signals $\boldsymbol{r} = (r_{q,t}^{k}\ \forall\ q,t,k)$, the jamming indicator $\boldsymbol{x} = (x_t\ \forall\ t)$, path gains $\boldsymbol{\alpha} = (\alpha_{i,q}\ \forall\ i,q)$, and the estimated symbols $\hat{\boldsymbol{s}} = \left(\hat{s}_{i,t}^{k}\ \forall\ i,k,t\right)$. The optimal decoding metrics with respect to CSI available are presented in the following section.

4.2.1.1 Maximum Likelihood Decoding with JSI Available

By (4.5), assume perfect estimation of JSI x_t's are available at the receiver. The likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$ and $\boldsymbol{\alpha}$ is given by

$$\begin{aligned} f\left\{\boldsymbol{r}\left|\hat{\boldsymbol{s}},\boldsymbol{\alpha},\boldsymbol{x}\right.\right\} &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m} f\left\{\eta_{q,t}^{k}=r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\left|\hat{s}_{i,t}^{k},\alpha_{i,q},x_t\ \forall\ i,q,t\right.\right\} \\ &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\frac{1}{\sqrt{\pi\left(N_0+x_t^2N_J/\rho\right)}}\exp\left(-\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{\left(N_0+x_t^2N_J/\rho\right)}\right) \end{aligned} \quad (4.14)$$

By taking logarithm on the likehood function, $\hat{\boldsymbol{s}}_{i,t}^{k}$'s can be decoded in the ML sense by maximizing the following metric:

$$\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\ln\left\{\frac{1}{\sqrt{\pi\left(N_0+x_t^2N_J/\rho\right)}}\exp\left(-\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{\left(N_0+x_t^2N_J/\rho\right)}\right)\right\}. \quad (4.15)$$

Suppose x_t's are not available at the receiver. The likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$, and $\boldsymbol{\alpha}$, i.e., $f\left\{\boldsymbol{r}\left|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\right.\right\}$, can be obtained by averaging (4.14) with respect to x_t's. A closed-form expression of $\ln f\left\{\boldsymbol{r}\left|\hat{\boldsymbol{s}},\ \boldsymbol{\alpha}\right.\right\}$ is then derived as

$$\begin{aligned} &\prod_{t=1}^{L}\ln\left\{\rho\prod_{q=1}^{m}\prod_{k=1}^{M}\frac{1}{\sqrt{\pi\left(N_0+x_t^2N_J/\rho\right)}}\exp\left(-\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{\left(N_0+x_t^2N_J/\rho\right)}\right)\right. \\ &\left.+(1-\rho)\prod_{q=1}^{m}\prod_{k=1}^{M}\frac{1}{\sqrt{\pi\left(N_0+x_t^2N_J/\rho\right)}}\exp\left(-\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{\left(N_0+x_t^2N_J/\rho\right)}\right)\right\}. \end{aligned} \quad (4.16)$$

$\hat{s}_{i,t}^{k}$'s can hence be choosen in the ML sense by maximizing the decoding metric in (4.16).

4.4.1.2 Suboptimal Decoding Schemes

Although the ML decoding can provide the optimum error correcting performance, the computation required to execute (4.16) might limit its feasibility in practical applications. Besides, the side informations of channel and jamming, i.e., $E_b/N_0, E_b/N_J,$ and ρ are also required to be estimated [32][33]. Hence, by ignoring all coefficients in (4.16) and using the approximation of

$$\exp(x) \approx 1 + x. \quad (4.17)$$

A suboptimal scheme SUB1.1 is proposed with the following decoding metric:

$$\min_{\hat{\boldsymbol{s}}} \sum_{t=1}^{L} \ln \left(1 + \sum_{q=1}^{m} \sum_{k=1}^{M} \left| r_{q,t}^{k} - \sum_{i=1}^{n} \alpha_{i,q} \hat{s}_{i,t}^{k} \right|^{2} \right). \tag{4.18}$$

4.2.2 Decoding without CSI

Suppose $\alpha_{i,q}$'s are not available at the receiver, and the path gains are modeled as independent complex Gaussian random variables with zero mean and variance $\sigma_{i,q}^2$ per dimension with respect to Rayleigh fading channels. The optimal decoding metrics of the STC/OFHSS system with respect to CSI unknown are presented in the following section.

4.2.1.1 Maximum Likelihood Decoding with JSI Available

Let $a_{1,x_t} = \frac{1}{\sqrt{\pi\left(N_0 + x_t^2 N_J/\rho\right)}}$, $a_{2,x_t} = \frac{1}{\left(N_0 + x_t^2 N_J/\rho\right)}$, and then $f\left(\boldsymbol{r} \left| \hat{\boldsymbol{s}}, \boldsymbol{\alpha}, \boldsymbol{x} \right.\right)$ can be written as

$$\begin{aligned}
f\left\{\boldsymbol{r} \left| \hat{\boldsymbol{s}}, \boldsymbol{\alpha}, \boldsymbol{x} \right.\right\} &= \prod_{t=1}^{L} \prod_{k=1}^{M} \prod_{q=1}^{m} f\left\{ r_{q,t}^{k} = \sum_{i=1}^{n} \alpha_{i,q} \hat{s}_{i,t}^{k} + \eta_{q,t}^{k} \left| \hat{s}_{i,t}^{k}, \alpha_{i,q}, x_t \;\forall\; i, q, t \right. \right\} \\
&= \prod_{t=1}^{L} \prod_{k=1}^{M} \prod_{q=1}^{m} \frac{1}{\sqrt{\pi\left(N_0 + x_t^2 N_J/\rho\right)}} \exp\left\{ -\frac{\left| r_{q,t}^{k} - \sum_{i=1}^{n} \alpha_{i,q} \hat{s}_{i,t}^{k} \right|}{N_0 + x_t^2 N_J/\rho} \right\} \\
&= \left\{ \prod_{t=1}^{L} \prod_{k=1}^{M} \prod_{q=1}^{m} a_{1,x_t} \right\} \cdot \exp\left(-\sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} a_{2,x_t} \left[\left| r_{q,t}^{k} \right|^2 \right.\right. \\
&\quad \left.\left. -2\mathrm{Re}\left(r_{q,t}^{k} \sum_{i=1}^{n} \alpha_{i,q}^{*} \hat{s}_{i,t}^{k*} \right) + \sum_{i=1}^{n} \alpha_{i,q} \hat{s}_{i,t}^{k} \sum_{l=1}^{n} \alpha_{i,q}^{*} \hat{s}_{i,t}^{k*} \right) .
\end{aligned} \tag{4.19}$$

The fading gain $\alpha_{i,q}$ can be presented as

$$\alpha_{i,q} = \alpha_{R,i,q} + j\alpha_{I,i,q} \tag{4.20}$$

where $\alpha_{R,i,q}$ and $\alpha_{I,i,q}$ are statistically independent Gaussian random variable with zero mean and variance $\sigma_{i,q}^2 = 1/2$. Then, the $\mathrm{Re}\left(r_{q,t}^{k} \sum_{i=1}^{n} \alpha_{i,q}^{*} \hat{s}_{i,t}^{k*} \right)$ and $\left(\sum_{i=1}^{n} \alpha_{i,q} \hat{s}_{i,t}^{k} \sum_{l=1}^{n} \alpha_{i,q}^{*} \hat{s}_{i,t}^{k*} \right)$ of the exponent can be rewritten as

$$\begin{aligned}
\mathrm{Re}\left(r_{q,t}^k \sum_{i=1}^n \alpha_{i,q}^* \hat{s}_{i,t}^{k*}\right) &= \mathrm{Re}\left(r_{q,t}^k \sum_{i=1}^n (\alpha_{R,i,q} - j\alpha_{I,i,q})\hat{s}_{i,t}^k\right) \\
&= \mathrm{Re}\left(r_{q,t}^k \sum_{i=1}^n \alpha_{R,i,q}\hat{s}_{i,t}^k\right) - \mathrm{Re}\left(r_{q,t}^k \sum_{i=1}^n j\alpha_{I,i,q}\hat{s}_{i,t}^k\right) \\
&= \mathrm{Re}\left(r_{q,t}^k \sum_{i=1}^n \alpha_{R,i,q}\hat{s}_{i,t}^k\right) + \mathrm{Im}\left(r_{q,t}^k \sum_{i=1}^n \alpha_{I,i,q}\hat{s}_{i,t}^k\right), \qquad (4.21)
\end{aligned}$$

and

$$\begin{aligned}
\sum_{i=1}^n \alpha_{i,q}\hat{s}_{i,t}^k \sum_{l=1}^n \alpha_{i,q}^* \hat{s}_{i,t}^{k*} &= \sum_{i=1}^n \sum_{l=1}^n (\alpha_{R,i,q} + j\alpha_{I,i,q})(\alpha_{R,l,q} - j\alpha_{I,l,q})\hat{s}_{i,t}^k \hat{s}_{l,t}^k \\
&= \sum_{i=1}^n \sum_{l=1}^n (\alpha_{R,i,q}\alpha_{R,l,q} + \alpha_{I,i,q}\alpha_{I,l,q})\hat{s}_{i,t}^k \hat{s}_{l,t}^k \\
&= \sum_{i=1}^n \alpha_{R,i,q}^2 |\hat{s}_{i,t}^k|^2 + \sum_{i=1}^n \alpha_{I,i,q}^2 |\hat{s}_{i,t}^k|^2 \\
&\quad + \sum_{\substack{i=1\\ i\neq l}}^n \sum_{\substack{l=1\\ l\neq i}}^n \alpha_{R,i,q}\alpha_{R,l,q}\hat{s}_{i,t}^k \hat{s}_{l,t}^k + \sum_{\substack{i=1\\ i\neq l}}^n \sum_{\substack{l=1\\ l\neq i}}^n \alpha_{I,i,q}\alpha_{I,l,q}\hat{s}_{i,t}^k \hat{s}_{l,t}^k. \qquad (4.22)
\end{aligned}$$

By averaging $\alpha_{i,q}$'s with respect to the probability density function in (4.19), we can get

$$\begin{aligned}
& f(\boldsymbol{r}|\hat{\boldsymbol{s}}, \boldsymbol{x}) \\
&= \int_{-\infty}^{\infty} P_r(\boldsymbol{r}|\hat{\boldsymbol{s}}, \boldsymbol{x}, \boldsymbol{\alpha}) f(\boldsymbol{\alpha}) d\boldsymbol{\alpha} \\
&= \left\{\prod_{t=1}^L \prod_{k=1}^M \prod_{m=1}^q \left[a_{1,x_t} \cdot \exp\left(-a_{2,x_t}|r_{q,t}^k|^2\right)\right]\right\} \cdot \left\{\prod_{q=1}^m \prod_{i=1}^n \left[\left(\sum_{t=1}^L \sum_{k=1}^M a_{2,x_t}\lambda_{i,t}^k + 1\right)^{-1}\right.\right. \\
&\quad \left.\left. \cdot \exp\left(\frac{\left|\left(\sum_{t=1}^L \sum_{k=1}^M a_{2,x_t} u_{i,q,t}^k\right)\right|^2}{\sum_{t=1}^L \sum_{k=1}^M a_{2,x_t}\lambda_{i,t}^k + 1}\right)\right]\right\} \qquad (4.23)
\end{aligned}$$

where

$$a_{1,x_t} = \frac{1}{\sqrt{\pi(N_0 + x_t^2 N_J/\rho)}} \quad a_{2,x_t} = \frac{1}{N_0 + x_t^2 N_J/\rho}$$
$$u_{i,q,t}^k = \left[\left(r_{q,t}^k \hat{s}_{1,t}^k\right)\ \left(r_{q,t}^k \hat{s}_{1,t}^k\right) \cdots \left(r_{q,t}^k \hat{s}_{1,t}^k\right)\right] \boldsymbol{v}_{i,t}^k.$$

$\boldsymbol{v}_{i,t}^k$'s and $\lambda_{i,t}^k$'s stand for the eigenvectors and eigenvalues of the following matrix, respectively:

$$\begin{pmatrix} |\hat{s}_{1,t}^k|^2 & \hat{s}_{1,t}^k \hat{s}_{2,t}^{k*} & \cdots & \hat{s}_{1,t}^k \hat{s}_{n,t}^{k*} \\ \hat{s}_{2,t}^k \hat{s}_{1,t}^{k*} & |\hat{s}_{2,t}^k|^2 & \cdots & \hat{s}_{2,t}^k \hat{s}_{n,t}^{k*} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{s}_{n,t}^k \hat{s}_{1,t}^{k*} & \hat{s}_{n,t}^k \hat{s}_{2,t}^{k*} & \cdots & |\hat{s}_{n,t}^k|^2 \end{pmatrix}. \tag{4.24}$$

The ML decoding then chooses $\hat{s}_{i,t}^k$'s by maximizing (4.23).

4.2.1.2 Suboptimal Decoding Schemes

Although the ML decoding can provide the optimum error-correcting performance, the complicated arithmetic not only requires high computational complexity but also excludes the use of the efficient Viterbi algorithm as the decoding metric in (4.23) cannot not be separated in time domain. To optimize the trade-off between decoding complexity and performance, we first present the suboptimal scheme SUB1.2 and SUB1.3 with JSI available, and the values of a_{1,x_t}, a_{2,x_t}, $\lambda_{i,t}^k$, and $u_{i,q,t}^k$ in the following suboptimal schemes are the same as that in (4.23). The suboptimal scheme SUB1.2 which chooses $\hat{s}_{i,t}^k$'s by maximizing

$$\sum_{t=1}^{L} \ln \left\{ \left[\prod_{k=1}^{M} \prod_{q=1}^{m} K_{i,t,x_t}^{k,q} \right] \cdot \left[\prod_{i=1}^{n} \prod_{q=1}^{m} \left[\left(a_{2,x_t} \sum_{k=1}^{M} \lambda_{i,t}^k + 1 \right) \exp \left(a_{2,x_t} \frac{\left| u_{i,q,t}^k \right|^2}{\sum_{k=1}^{M} \lambda_{i,t}^k + 1} \right) \right] \right] \right\} \tag{4.25}$$

where

$$K_{i,t,x_t}^{k,q} = a_{1,x_t} \exp \left(-a_{2,x_t} \left| r_{q,t}^k \right|^2 \right).$$

The other suboptimal scheme is SUB1.3 denoted by

$$\text{SUB1.3} : \max_{\hat{\boldsymbol{s}}} \sum_{q=1}^{m} \sum_{i=1}^{n} \left(\frac{\left| \sum_{t=1}^{L} \sum_{k=1}^{M} a_{2,x_t} u_{i,q,t}^k \right|^2}{\sum_{t=1}^{L} \sum_{k=1}^{M} a_{2,x_t} \lambda_{i,t}^k + 1} \right). \tag{4.26}$$

Suppose $\alpha_{i,q}$'s and x_t's are both not available at the receiver, and the closed-form of $f(\boldsymbol{r}|\hat{\boldsymbol{s}})$ is too complexity to be written. Therefore, we propose three suboptimal decoding schemes to reduce the decoding complexity. The first one suboptimal scheme is SUB1.4 which is represented as

$$\left\{ \left[\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m} K_{i,t,x_t}^{k,q}\right] \cdot \left[\prod_{i=1}^{n}\prod_{q=1}^{m}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t=1}\lambda_{i,t}^{k}+1\right)\exp\left(\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t=1}u_{i,q,t}^{k}\right|^2}{\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M} a_{2,x_t=1}\lambda_{i,t}^{k}+1}\right)\right]\right]\right\}$$

$$+\sum_{t'=1}^{L}\left[\frac{\rho^{(L-1)}}{(1-\rho)^{-1}}\cdot\frac{\left(a_{1,x_t=1}\right)^{mM(L-1)}}{\left(a_{1,x_t=0}\right)^{-mM}}\cdot\exp\left(-a_{2,x_{t=1}}\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\left(\left|r_{q,t}^k\right|^2+a_3\left|r_{q,t'}^k\right|^2\right)\right)\right]$$

$$\cdot\prod_{q=1}^{m}\prod_{i=1}^{n}\left\{X_{i,q}\cdot\exp\left\{\frac{\left|a_{2,x_{t=1}}\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M} u^k_{i,q,t}+a_{2,x_{t=1}}a_3\sum\limits_{k=1}^{M} u^k_{i,q,t'}\right|^2}{a_{2,x_{t=1}}\sum\limits_{k=1}^{M}\lambda^k_{i,t}+a_{2,x_{t=1}}a_3\sum\limits_{k=1}^{M}\lambda^k_{i,t'}+1}\right\}\right\} \tag{4.27}$$

where

$$a_3=\frac{N_J}{N_0\rho} \quad a_{2,x_t=1}=\frac{1}{N_0+N_J/\rho} \quad a_{1,x_t=0}=\frac{1}{\sqrt{\pi N_0}} \quad a_{1,x_t=1}=\frac{1}{\sqrt{\pi\left(N_0+N_J/\rho\right)}}$$

$$X_{i,q}=\left[a_{2,x_t=1}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\lambda^k_{i,t}+a_3\sum_{k=1}^{M}\lambda^k_{i,t}\right)+1\right]^{-1}. \tag{4.28}$$

The derivation of the metric in (4.27) is similar to that in Appendix B and Appendix C. Another suboptimal scheme is SUB1.5 with the following decoding metric

$$\sum_{t=1}^{L}\ln\left\{(1-\rho)\left(K_{i,t,x_t=0}^{k,q}\right)^{Mm}\cdot\left[\prod_{i=1}^{n}\prod_{q=1}^{m}\left[\left(a_{2,x_t=0}\sum_{k=1}^{M}\lambda^k_{i,t}+1\right)\exp\left(\frac{\left|a_{2,x_t=0}u^k_{i,q,t}\right|^2}{a_{2,x_t=0}\sum\limits_{k=1}^{M}\lambda^k_{i,t}+1}\right)\right]\right]\right.$$

$$\left.+\rho\left(K_{i,t,x_t=0}^{k,q}\right)^{Mm}\cdot\left[\prod_{i=1}^{n}\prod_{q=1}^{m}\left[\left(a_{2,x_t}\sum_{k=1}^{M}\lambda^k_{i,t}+1\right)\exp\left(\frac{\left|a_{2,x_t=1}u^k_{i,q,t}\right|^2}{a_{2,x_t=1}\sum\limits_{k=1}^{M}\lambda^k_{i,t}+1}\right)\right]\right]\right\} \tag{4.29}$$

where

$$a_{2,x_t=0}=\frac{1}{N_0} \quad a_{2,x_t=1}=\frac{1}{N_0+N_J/\rho} \quad a_{1,x_t=0}=\frac{1}{\sqrt{\pi N_0}} \quad a_{1,x_t=1}=\frac{1}{\sqrt{\pi\left(N_0+N_J/\rho\right)}}$$

$$K_{i,t,x_t=0}^{k,q}=a_{1,x_t=0}\exp\left(-a_{2,x_t=0}\left|r^k_{q,t}\right|^2\right) \quad K_{i,t,x_t=1}^{k,q}=a_{1,x_t=1}\exp\left(-a_{2,x_t=1}\left|r^k_{q,t}\right|^2\right)$$

The other suboptimal scheme SUB1.6 is

$$\text{SUB1.6} : \max_{\hat{\boldsymbol{s}}} \sum_{q=1}^{m} \sum_{i=1}^{n} \left(\frac{\left| \sum_{t=1}^{L} \sum_{k=1}^{M} u_{i,q,t}^{k} \right|^2}{\sum_{t=1}^{L} \sum_{k=1}^{M} \lambda_{i,t,x_t}^{k,q} + 1} \right). \tag{4.30}$$

4.2.3 Design Criteria for Constructing Good Space-Time Codes

We propose two design criteria for constructing good space-time codes of the STC/WFHSS systems with respect to the wireless jamming channels. The first design criterion is provided for the case of low diversity. That is, the number of independent subchannels is small. On the other hands, the second design criterion is proposed for the case of high diversity with respect to large number of independent subchannels.

First of all, we derive the design criterion for the case of low diversity. To evaluate the performance of the ML decoding, consider two transmitted sequences $\boldsymbol{s} = (s_{i,t}^k \ \forall\ i,t,k)$ and $\tilde{\boldsymbol{s}} = (\tilde{s}_{i,t}^k \ \forall\ i,t,k)$. Assume perfect estimation of $\alpha_{i,q}$'s and x_t's are both available at the receiver. The conditional pairwise error probability that the decoder decides in favor of $\tilde{\boldsymbol{s}}$ than $\boldsymbol{s}$ is given by

$$\begin{aligned}
&\mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \alpha_{i,q}, x_t \ \forall\ i,q,t\right) \\
&= \mathrm{P_r}\left\{ \sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M} \ln\left[\frac{1}{\sqrt{\pi(N_0 + x_t^2 N_J/\rho)}} \exp\left(-\frac{|r_{q,t}^k - \sum_{i=1}^n \alpha_{i,q} s_{i,t}^k|^2}{N_0 + x_t^2 N_J/\rho}\right)\right]\right. \\
&\left. \leq \sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M} \ln\left[\frac{1}{\sqrt{\pi(N_0 + x_t^2 N_J/\rho)}} \exp\left(-\frac{|r_{q,t}^k - \sum_{i=1}^n \alpha_{i,q} \tilde{s}_{i,t}^k|^2}{N_0 + x_t^2 N_J/\rho}\right)\right]\right\} \\
&= Q\left(\sqrt{\frac{\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M} \left|\sum_{i=1}^{n} \alpha_{i,q}(s_{i,t}^k - \tilde{s}_{i,t}^k)\right|^2}{2(N_0 + x_t^2 N_J/\rho)}}\right)
\end{aligned} \tag{4.31}$$

where $Q(a)$ is the complementary error function defined by

$$Q\left(a\right) \doteq \frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} e^{-x^2/2} dx. \tag{4.32}$$

Using the inequality $Q(a) \leq \frac{1}{2}\exp(-a^2/2)\ \forall\ a \geq 0$, the conditional pairwise error probability (4.31) can be upper bounded by

$$\mathrm{P_r}\left(\boldsymbol{s} \to \tilde{\boldsymbol{s}} | \alpha_{i,q}, x_t\ \forall\ i, q, t\right) \leq \frac{1}{2}\exp\left(-\sum_{t=1}^{L} a_{x_t} d_t^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})\right) \tag{4.33}$$

where

$$d_t^2(\boldsymbol{s}, \tilde{\boldsymbol{s}}) = \sum_{q=1}^{m}\sum_{k=1}^{M}\left|\sum_{i=1}^{n}\alpha_{i,q}(s_{i,t}^k - \tilde{s}_{i,t}^k)\right|^2 \text{ and } a_{x_t} = \frac{1}{4(N_0 + x_t^2 N_J/\rho)}.$$

By averaging (4.33) with respect to x_t's and $\alpha_{i,q}$'s, the pairwise error probability is approximated as

$$P_r\left(\boldsymbol{s} \to \tilde{\boldsymbol{s}}\right) \cong \frac{\rho^{\mathrm{L}}}{2}\left(\frac{1}{4(N_0 + N_J/\rho)}\right)^{-\mathrm{mn}} \cdot \mathrm{W}_1 \tag{4.34}$$

where

$$W_1 = \left[\prod_{q=1}^{m}\prod_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}\right)^{-1} + \sum_{t'=1}^{L}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(\left(\sum_{t=1}^{L}\lambda_{i,t} + \lambda_{i,t'}\right)\right)^{-1}\right].$$

To minimize decoding error probability, good codes should hence be constructed by maximizing W_1 for all possible $\boldsymbol{s}$ and $\tilde{\boldsymbol{s}}$. According to the above design criterion, space-time codes of good performance with respcet to the small values of rm are given in Table 4.1 by a computer search. In Table 4.1, (a_t, b_t) denote the binary inputs at time t and the transmitted symbol $s_{i,t}^k$ is determined from the encoder output x_i^t by

$$s_{i,t}^k = \begin{cases} \sqrt{E_s}, & \text{if } k = x_i^t \\ 0, & \text{otherwise} \end{cases} \tag{4.35}$$

for all i and t.

Then, we discuss the design criterion for the case of high diversity. The pairwise error probability conditioned on $\alpha_{i,q}$'s is upper bounded by

$$\mathrm{P_r}\left(\boldsymbol{s} \to \tilde{\boldsymbol{s}}\,|\boldsymbol{\alpha_{i,q}}\right) \leq \frac{1}{2}\prod_{t=1}^{L}\left\{(1-\rho)\exp\left(-\frac{d_t^2\left(\boldsymbol{s}, \tilde{\boldsymbol{s}}\right)}{4N_0}\right) + \rho\exp\left(-\frac{d_t^2\left(\boldsymbol{s}, \tilde{\boldsymbol{s}}\right)}{4\left(N_0 + N_J/\rho\right)}\right)\right\}. \tag{4.36}$$

The pairwise error probability can be further approximated as

$$\mathrm{P_r}\left(\boldsymbol{s} \to \tilde{\boldsymbol{s}}\right) \cong \frac{1}{2}\rho^L \cdot W_2 \tag{4.37}$$

Table 4.1: Optimal Space-time codes of the STC/WFHSS system with 4FSK and 2 transmitter antennas for wireless jamming channels with respect to the case of low diversity.

Memory	Generator Sequences
2	$(x_1^t, x_2^t) = b_{t-1}(3,2) \oplus_4 a_{t-1}(2,1) \oplus_4 b_t(0,2) \oplus_4 a_t(0,3)$
3	$(x_1^t, x_2^t) = a_{t-2}(2,2) \oplus_4 b_{t-1}(3,1) \oplus_4 a_{t-1}(3,3) \oplus_4 b_t(2,2)$ $\oplus_4 a_t(2,1)$
4	$(x_1^t, x_2^t) = b_{t-2}(0,2) \oplus_4 a_{t-2}(0,3) \oplus_4 b_{t-1}(1,2) \oplus_4 a_{t-1}(2,2)$ $\oplus_4 b_t(3,3) \oplus_4 a_t(2,2)$

where

$$W_2 = \exp\left(-\frac{\left(\sum_{t=1}^{L}\sum_{i=1}^{r}\sum_{q=1}^{m}\lambda_{i,t}\right)^2}{\sum_{q=1}^{m}\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}\right)}\right) + \sum_{t'=1}^{L}\exp\left(-\frac{\left(\sum_{q=1}^{m}\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}+\lambda_{i,t'}\right)\right)^2}{\sum_{q=1}^{m}\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}+\lambda_{i,t'}\right)}\right).$$

Good codes should hence be constructed by maximizing W_2 for all possible $\boldsymbol{s}$ and $\tilde{\boldsymbol{s}}$. According to the design criterion, space-time codes of good performance with respcet to the large values of rm are given in Table 4.2 by a computer search.

Table 4.2: Optimal Space-time codes of the STC/WFHSS system with 4FSK and 2 transmitter antennas for wireless jamming channels with respect to the case of high diversity.

Memory	Generator Sequences
2	$(x_1^t, x_2^t) = b_{t-1}(3,0) \oplus_4 a_{t-1}(0,3) \oplus_4 b_t(2,2) \oplus_4 a_t(3,3)$
3	$(x_1^t, x_2^t) = a_{t-2}(2,2) \oplus_4 b_{t-1}(1,1) \oplus_4 a_{t-1}(3,1) \oplus_4 b_t(2,3)$ $\oplus_4 a_t(1,2)$
4	$(x_1^t, x_2^t) = b_{t-2}(1,1) \oplus_4 a_{t-2}(2,2) \oplus_4 b_{t-1}(1,0) \oplus_4 a_{t-1}(1,3)$ $\oplus_4 b_t(1,1) \oplus_4 a_t(2,2)$

4.3 STC Combined with Optimum FH

Another system we proposed is STC/OFHSS system as shown in Figure 4.5. The encoded codewords from all transmitter antennas are hopped into distinct frequency bands to avoid any possible collision of the transmitted symbols. In this section, we derive the ML decoding together with some suboptimal decoding schemes with respect to the STC/OFHSS system, and we also provide the performance criteria for constructing good space-time codes.

4.3.1 Decoding with CSI Available

Assume perfect estimation of $\alpha_{i,q}$'s is available at the receiver. Let the selected received signals $\boldsymbol{r} = (r^k_{i,q,t} \ \forall \ i, q, t, k)$, the jamming indicator $\boldsymbol{x} = (x_{i,t} \ \forall \ i, t)$, path gains $\boldsymbol{\alpha} = (\alpha_{i,q} \ \forall \ i, q)$, and the estimated symbols $\hat{\boldsymbol{s}} = \left(\hat{s}^k_{i,t} \ \forall \ i, k, t\right)$. The optimal decoding metrics with respect to CSI available are presented in the following section.

4.4.1.1 Maximum Likelihood Decoding with JSI Available

To evaluate the system performance, we derive the ML decoding of space-time codes as follows. By (4.13), assume perfect estimation of JSI $x_{i,t}$'s are available at the receiver. The likelihood function is given by

$$\begin{aligned} f\left\{\boldsymbol{r} \left| \hat{\boldsymbol{s}}, \boldsymbol{\alpha}, \boldsymbol{x} \right.\right\} &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n} f\left\{r^k_{i,q,t} = \alpha_{i,q}\hat{s}^k_{i,t} + \eta^k_{i,q,t} \left| \hat{s}^k_{i,t}, \alpha_{i,q}, x_{i,t}, \ \forall \ i, q, t, k \right.\right\} \\ &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n} \frac{1}{\sqrt{\pi\left(N_0 + x^2_{i,t}N_J/\rho\right)}} \exp\left\{-\frac{\left|r^k_{i,q,t} - \alpha_{i,q}\hat{s}^k_{i,t}\right|}{N_0 + x^2_{i,t}N_J/\rho}\right\}. \end{aligned} \tag{4.38}$$

By taking logarithm on the likehood function, $\hat{\boldsymbol{s}}^k_{i,t}$'s can be decoded in the ML sense by maximizing the following metric:

$$\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{i=1}^{n} \ln\left\{\frac{1}{\sqrt{\pi\left(N_0 + x^2_{i,t}N_J/\rho\right)}} \exp\left\{-\frac{\left|r^k_{i,q,t} - \alpha_{i,q}\hat{s}^k_{i,t}\right|}{N_0 + x^2_{i,t}N_J/\rho}\right\}\right\}. \tag{4.39}$$

Suppose $x_{i,t}$'s are not available at the receiver. The likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$, and $\hat{\boldsymbol{\alpha}}$, i.e., $f\left\{\boldsymbol{r} \left| \hat{\boldsymbol{s}}, \boldsymbol{\alpha} \right.\right\}$, can be obtained by averaging (4.38) with respect to $x_{i,t}$'s. A closed-form expression of $\ln f\{\boldsymbol{r}\,|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\}$ with respect to two transmitter antennas is then derived as

$$\ln\Bigg\{(\phi\rho)\,a_{1,x_{i,t}=1}^{2mM}\exp\left(-\sum_{q=1}^{m}\sum_{k=1}^{M}\left(\frac{\left|r_{q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^{2}}{N_0+N_J/\rho}+\frac{\left|r_{q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^{2}}{N_0+N_J/\rho}\right)\right)$$
$$+(1-\phi)\rho a_{1,x_{i,t}=1}^{mM}a_{1,x_{i,t}=0}^{mM}\exp\left(-\sum_{q=1}^{m}\sum_{k=1}^{M}\left(\frac{\left|r_{q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^{2}}{N_0+N_J/\rho}+\frac{\left|r_{q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^{2}}{N_0}\right)\right)$$
$$+\beta(1-\rho)a_{1,x_{i,t}=0}^{mM}a_{1,x_{i,t}=1}^{mM}\exp\left(-\sum_{q=1}^{m}\sum_{k=1}^{M}\left(\frac{\left|r_{q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^{2}}{N_0}+\frac{\left|r_{q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^{2}}{N_0+N_J/\rho}\right)\right)$$
$$+(1-\beta)(1-\rho)a_{1,x_{i,t}=0}^{2mM}\exp\left(-\sum_{q=1}^{m}\sum_{k=1}^{M}\left(\frac{\left|r_{q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^{2}}{N_0}+\frac{\left|r_{q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^{2}}{N_0}\right)\right)\Bigg\} \quad (4.40)$$

where

$$a_{1,x_{i,t}=0}=\frac{1}{\sqrt{\pi N_0}} \qquad a_{1,x_{i,t}=1}=\frac{1}{\sqrt{\pi(N_0+N_J/\rho)}} \quad (4.41)$$

$$P(x_{1,t}=1)=\frac{Q}{N_t}=\rho \qquad P(x_{1,t}=0)=1-\frac{Q}{N_t}=1-\rho \quad (4.42)$$

$$P(x_{2,t}=1|x_{1,t}=1)=\frac{Q-1}{N_t-1}=\phi \qquad P(x_{2,t}=0|x_{1,t}=1)=1-\frac{Q-1}{N_t-1}=1-\phi \quad (4.43)$$

$$P(x_{2,t}=1|x_{1,t}=0)=\frac{Q}{N_t-1}=\beta \qquad P(x_{2,t}=0|x_{1,t}=0)=1-\frac{Q}{N_t-1}=1-\beta \quad (4.44)$$

N_t is the total frequency hopping bands and Q is the occupied jamming bands of the total frequency hopping bands.

4.4.1.2 Suboptimal Decoding Schemes

Althought the ML decoding can provide the optimum error correcting performance, the computation required to execute (4.40) might limit its feasibility in practical applications. Besides, the side informations of channel and jamming, i.e., $E_b/N_0, E_b/N_J,$ and ρ are also required to be estimated. Hence, by ignoring all coefficients in (4.40) and using the approximation of $\exp(x)\approx 1+x$, we proposed a suboptimal scheme SUB2.1 with the following decoding metric:

$$\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M}\sum_{i=1}^{n}\ln\left[1+\left|r_{i,q,t}^{k}-\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^{2}\right]. \quad (4.45)$$

4.3.2 Decoding without CSI

Suppose $\alpha_{i,q}$'s are not available at the receiver, the optimal decoding metrics of the STC/OFHSS system with respect to CSI unknown are presented in the following section.

4.4.2.1 Maximum Likelihood Decoding with JSI Available

Let $a_{1,x_{i,t}} = \frac{1}{\sqrt{\pi\left(N_0+x_{i,t}^2 N_J/\rho\right)}}, a_{2,x_{i,t}} = \frac{1}{\left(N_0+x_{i,t}^2 N_J/\rho\right)}$, and then (4.38) can be rewritten as

$$\begin{aligned} f\left\{\boldsymbol{r}\left|\hat{\boldsymbol{s}}, \boldsymbol{\alpha}, \boldsymbol{x}\right.\right\} &= \left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n} a_{1,x_{i,t}}\right\}\cdot\exp\left(-\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{i=1}^{n} a_{2,x_{i,t}}\left[\left|r_{i,q,t}^{k}\right|^2\right.\right. \\ &\quad \left.\left. -2\mathrm{Re}\left(r_{i,q,t}^{k}\alpha_{i,q}^{*}\hat{s}_{i,t}^{k*}\right)\right] + \left|\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2\right). \end{aligned} \tag{4.46}$$

The $\mathrm{Re}\left(r_{i,q,t}^{k}\alpha_{i,q}^{*}\hat{s}_{i,t}^{k*}\right)$ and $|\alpha_{i,q}\hat{s}_{i,t}^{k}|^2$ of the exponent can be rewritten as

$$\begin{aligned} \mathrm{Re}\left(r_{i,q,t}^{k}\alpha_{i,q}^{*}\hat{s}_{i,t}^{k*}\right) &= \mathrm{Re}\left(r_{i,q,t}^{k}\left(\alpha_{R,i,q} - j\alpha_{I,i,q}\right)\hat{s}_{i,t}^{k}\right) \\ &= \mathrm{Re}\left(r_{i,q,t}^{k}\alpha_{R,i,q}\hat{s}_{i,t}^{k}\right) + \mathrm{Im}\left(r_{i,q,t}^{k}\alpha_{I,i,q}\hat{s}_{i,t}^{k}\right) \end{aligned} \tag{4.47}$$

and

$$\left|\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2 = \left(\alpha_{R,i,q}^2 + \alpha_{I,i,q}^2\right)\left|\hat{s}_{i,t}^{k}\right|^2. \tag{4.48}$$

By averaging $\alpha_{i,q}$'s with respect to the conditional probability density function in (4.46), we can get

$$\begin{aligned} &\int_{-\infty}^{\infty} f(\boldsymbol{r}|\hat{\boldsymbol{s}}, \boldsymbol{x}, \boldsymbol{\alpha}) f(\boldsymbol{\alpha}) d\boldsymbol{\alpha} \\ &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n}\left(a_{1,x_{i,t}}\cdot\exp\left(-a_{2,x_{i,t}}\left|r_{i,q,t}^{k}\right|^2\right)\right)\cdot\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^2 + 1\right)^{-1}\right. \\ &\quad \left.\cdot\exp\left(\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_{i,t}} r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right|^2}{\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^2 + 1}\right)\right). \end{aligned} \tag{4.49}$$

The ML decoding then chooses $\hat{s}_{i,t}^{k}$'s by maximizing (4.49).

4.4.2.2 Suboptimal Decoding Schemes

To optimize the trade-off between decoding complexity and performance, we first present the suboptimal scheme SUB2.2 and SUB2.3 with JSI available, and the values of $a_{1,x_{i,t}}$ and $a_{2,x_{i,t}}$ of the following decoding metrics are the same as that in (4.49). The suboptimal scheme SUB2.2 which chooses $\hat{s}_{i,t}^k$'s by maximizing

$$\sum_{t=1}^{L} \ln \left\{ \prod_{k=1}^{M} \prod_{q=1}^{m} \prod_{i=1}^{n} \left(a_{1,x_{i,t}} \cdot \exp\left(-a_{2,x_{i,t}} \left| r_{i,q,t}^k \right|^2 \right) \right) \cdot \prod_{q=1}^{m} \prod_{i=1}^{n} \left(\left(\sum_{k=1}^{M} a_{2,x_{i,t}} \left| \hat{s}_{i,t}^k \right|^2 + 1 \right)^{-1} \cdot \exp\left(\frac{\left| \sum_{k=1}^{M} a_{2,x_{i,t}} r_{i,q,t}^k \hat{s}_{i,t}^k \right|^2}{\sum_{k=1}^{M} a_{2,x_{i,t}} \left| \hat{s}_{i,t}^k \right|^2 + 1} \right) \right) \right\}. \tag{4.50}$$

The other suboptimal scheme SUB2.3 which uses the approximation of $\exp(x) \approx 1 + x$ is denoted by

$$\text{SUB2.3} : \max_{\hat{\boldsymbol{s}}} \sum_{t=1}^{L} \sum_{q=1}^{m} \sum_{i=1}^{n} \ln \left[1 + \frac{\left| \sum_{k=1}^{M} a_{2,x_{i,t}} r_{i,q,t}^k \hat{s}_{i,t}^k \right|^2}{\sum_{k=1}^{M} a_{2,x_{i,t}} \left| \hat{s}_{i,t}^k \right| + 1} \right]. \tag{4.51}$$

Suppose $\alpha_{i,q}$'s and $x_{i,t}$'s are both not available at the receiver, and the closed-form of $f(\boldsymbol{r}|\hat{\boldsymbol{s}})$ is too complexity to be written. Therefore, we propose three suboptimal decoding schemes to reduce the decoding complexity. We also assume that the number of total frequency hopping bans is very large, and there are few transmitter antennas we used. That is, we assume the transmitted signals from each transmitted antennas are jammed independently. The first one suboptimal scheme is SUB2.4 which is represented as

$$
\begin{aligned}
&\left[\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n} a_{1,x_{i,t}=1}\exp\left(-a_{2,x_{i,t}=1}\left|r_{i,q,t}^{k}\right|^{2}\right)\right]\Bigg[\prod_{i=1}^{n}\prod_{q=1}^{m}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}=1}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)^{-1}\\
&\cdot\exp\left(\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}=1}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right|^{2}}{\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}=1}\left|\hat{s}_{i,t}^{k}\right|^{2}+1}\right)+\sum_{t'=1}^{L}\sum_{i'=1}^{n}\Bigg[\frac{\rho^{nL-1}}{(1-\rho)^{-1}}\cdot\frac{a_{1,x_{i,t}=1}^{Mm(Ln-1)}}{a_{1,x_{i,t}=0}^{Mm}}\cdot\\
&\exp\left(-a_{2,x_{i,t}=1}\sum_{q=1}^{m}\sum_{k=1}^{M}\left(\sum_{t=1}^{L}\sum_{i=1}^{n}\left|r_{i,q,t}^{k}\right|^{2}+a_{3}\left|r_{i',q,t'}^{k}\right|^{2}\right)\right)\cdot\prod_{q=1}^{m}\Bigg[\prod_{\substack{i=1\\ i\neq i'}}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}=1}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)^{-1}\\
&\cdot\exp\left(\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}=1}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right|^{2}}{\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}=1}\left|\hat{s}_{i,t}^{k}\right|^{2}+1}\right)\cdot\left(a_{2,x_{i,t}=1}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|\hat{s}_{i',t}^{k}\right|^{2}+a_{3}\sum_{k=1}^{M}\left|\hat{s}_{i',t'}^{k}\right|^{2}\right)+1\right)^{-1}\\
&\cdot\exp\left(\frac{\left|a_{2,x_{i,t}=1}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}r_{i,q,t}^{k}\hat{s}_{i',t}^{k}+a_{3}\sum_{k=1}^{M}r_{i',q,t'}^{k}\hat{s}_{i',t'}^{k}\right)\right|^{2}}{a_{2,x_{i,t}=1}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|\hat{s}_{i',t}^{k}\right|^{2}+\sum_{k=1}^{M}\left|\hat{s}_{i',t'}^{k}\right|^{2}\right)+1}\right)\Bigg]\Bigg]
\end{aligned}
\tag{4.52}
$$

where

$$a_{3}=\frac{N_{J}}{N_{0}\rho}\qquad a_{2,x_{i,t}=1}=\frac{1}{N_{0}+N_{J}/\rho}\qquad a_{1,x_{i,t}=0}=\frac{1}{\sqrt{\pi N_{0}}}\qquad a_{1,x_{i,t}=1}=\frac{1}{\sqrt{\pi\left(N_{0}+N_{J}/\rho\right)}}.$$

The derivation of the metric in (4.52) is similar to that in Appendix D and Appendix E. Another suboptimal scheme is SUB2.5 with the following decoding metric

$$
\begin{aligned}
&\sum_{t=1}^{L}\sum_{i=1}^{n}\ln\Bigg\{(1-\rho)\prod_{k=1}^{M}\prod_{q=1}^{m}\left(a_{1,x_{i,t}=0}\cdot\exp\left(-a_{2,x_{i,t}=0}\left|r_{i,q,t}^{k}\right|^{2}\right)\right)\cdot\prod_{q=1}^{m}\left(\left(\sum_{k=1}^{M}a_{2,x_{i,t}=0}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)^{-1}\right.\\
&\left.\cdot\exp\left(\frac{\left|\sum_{k=1}^{M}a_{2,x_{i,t}=0}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right|^{2}}{\sum_{k=1}^{M}a_{2,x_{i,t}=0}\left|\hat{s}_{i,t}^{k}\right|^{2}+1}\right)\right)+\rho\prod_{k=1}^{M}\prod_{q=1}^{m}\left(a_{1,x_{i,t}=1}\cdot\exp\left(-a_{2,x_{i,t}=1}\left|r_{i,q,t}^{k}\right|^{2}\right)\right)\\
&\cdot\prod_{q=1}^{m}\left(\left(\sum_{k=1}^{M}a_{2,x_{i,t}=1}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)^{-1}\cdot\exp\left(\frac{\left|\sum_{k=1}^{M}a_{2,x_{i,t}=1}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right|^{2}}{\sum_{k=1}^{M}a_{2,x_{i,t}=1}\left|\hat{s}_{i,t}^{k}\right|^{2}+1}\right)\right)\Bigg\}
\end{aligned}
\tag{4.53}
$$

where

$$a_{2,x_{i,t}=0}=\frac{1}{N_{0}}\qquad a_{2,x_{i,t}=1}=\frac{1}{N_{0}+N_{J}/\rho}\qquad a_{1,x_{i,t}=0}=\frac{1}{\sqrt{\pi N_{0}}}\qquad a_{1,x_{i,t}=1}=\frac{1}{\sqrt{\pi\left(N_{0}+N_{J}/\rho\right)}}$$

The other suboptimal scheme SUB2.6 is

$$\max_{s} \sum_{q=1}^{m} \sum_{i=1}^{n} \frac{\left| \sum_{t=1}^{L} \sum_{k=1}^{M} r_{i,q,t}^{k} \hat{s}_{i,t}^{k} \right|^2}{\sum_{t=1}^{L} \sum_{k=1}^{M} \left| \hat{s}_{i,t}^{k} \right|^2 + 1}. \tag{4.54}$$

4.3.3 Design Criteria for Constructing Good Space-Time Codes

We propose two design criteria for constructing good space-time codes of the STC/OFHSS systems with respect to the wireless jamming channels. The first design criterion is provided for the case of low diversity. That is, the number of independent subchannels is small. On the other hands, the second design criterion is proposed for the case of high diversity with respect to large number of independent subchannels. We also assume that the number of total frequency hopping bans is very large, and there are few transmitter antennas we used. That is, we assume the transmitted signals from each transmitted antennas are jammed independently.

First of all, we derive the design criterion for the case of low diversity. To evaluate the performance of the ML decoding, consider two transmitted sequences $\boldsymbol{s} = (s_{i,t}^{k} \ \forall \ i, t, k)$ and $\tilde{\boldsymbol{s}} = (\tilde{s}_{i,t}^{k} \ \forall \ i, t, k)$. Assume perfect estimation of $\alpha_{i,q}$'s and $x_{i,t}$'s are both available at the receiver. The conditional pairwise error probability that the decoder decides in favor of $\tilde{\boldsymbol{s}}$ than $\boldsymbol{s}$ is given by

$$\begin{aligned}
&\mathrm{P_r}\left(\boldsymbol{s} \to \tilde{\boldsymbol{s}} | \alpha_{i,q}, x_{i,t} \ \forall \ i, q, t\right) \\
&= P\left\{ \sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \sum_{i=1}^{n} \ln \left[\frac{1}{\sqrt{\pi \left(N_0 + x_{i,t}^2 N_J/\rho\right)}} \exp\left(-\frac{|r_{i,q,t}^{k} - \alpha_{i,q} s_{i,t}^{k}|^2}{N_0 + x_{i,t}^2 N_J/\rho} \right) \right] \right. \\
&\left. \leq \sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \sum_{i=1}^{n} \ln \left[\frac{1}{\sqrt{\pi \left(N_0 + x_{i,t}^2 N_J/\rho\right)}} \exp\left(-\frac{|r_{i,q,t}^{k} - \alpha_{i,q} \tilde{s}_{i,t}^{k}|^2}{N_0 + x_{i,t}^2 N_J/\rho} \right) \right] \right\} \\
&= Q\left(\sqrt{\frac{\sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \sum_{i=1}^{n} \left| \alpha_{i,q} \left(s_{i,t}^{k} - \tilde{s}_{i,t}^{k} \right) \right|^2}{2\left(N_0 + x_{i,t}^2 N_J/\rho\right)}} \right).
\end{aligned} \tag{4.55}$$

Using the inequality $Q(a) \leq \frac{1}{2}\exp(-a^2/2)\ \forall\ a \geq 0$, the conditional pairwise error probability (4.55) can be upper bounded by

$$\mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \alpha_{i,q}, x_t\ \forall\ i,q,t\right) \leq \frac{1}{2}\exp\left(-\sum_{t=1}^{L}\sum_{i=1}^{n} a_{x_{i,t}} d_{i,t}^2(s,\tilde{s})\right) \tag{4.56}$$

where

$$d_{i,t}^2(s,\tilde{s}) = \sum_{q=1}^{m}\sum_{k=1}^{M}\left|\alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2 \text{ and } a_{x_{i,t}} = \frac{1}{4\left(N_0 + x_{i,t}^2 N_J/\rho\right)}.$$

By averaging (4.56) with respect to $x_{i,t}$'s and $\alpha_{i,q}$'s, the pairwise error probability is approximated as

$$\mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}}\right) \cong \frac{\rho^L}{2}\left(\frac{1}{4(N_0 + N_J/\rho)}\right)^{-mn} \cdot W_3 \tag{4.57}$$

where

$$\begin{aligned} W_3 \;=\; & \prod_{q=1}^{m}\prod_{i=1}^{r}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^k - \tilde{s}_{i,t}^k\right|^2\right)^{-1} + \sum_{t'=1}^{L}\sum_{i'=1}^{n}\left[\prod_{q=1}^{m}\left(\prod_{\substack{i=1\\ i\neq i'}}^{r}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^k - \tilde{s}_{i,t}^k\right|^2\right)^{-1}\right.\right. \\ & \left.\left. \cdot\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^k - \tilde{s}_{i,t}^k\right|^2 + \sum_{k=1}^{M}\left|s_{i',t'}^k - \tilde{s}_{i',t'}^k\right|^2\right)\right)\right]. \end{aligned}$$

To minimize decoding error probability, good codes should hence be constructed by maximizing W_3 for all possible $\boldsymbol{s}$ and $\tilde{\boldsymbol{s}}$. According to the above design criterion, space-time codes of good performance with respcet to the small values of rm are given in Table 4.3 by a computer search.

Table 4.3: Optimal Space-time codes of the STC/OFHSS system with 4FSK and 2 transmitter antennas for wireless jamming channels with respect to the case of low diversity.

Memory	**Generator Sequences**
2	$(x_1^t, x_2^t) = b_{t-1}(2,2) \oplus_4 a_{t-1}(1,3) \oplus_4 b_t(1,1) \oplus_4 a_t(2,2)$
3	$(x_1^t, x_2^t) = a_{t-2}(2,3) \oplus_4 b_{t-1}(3,2) \oplus_4 a_{t-1}(3,1) \oplus_4 b_t(2,2)$ $\oplus_4 a_t(1,1)$
4	$(x_1^t, x_2^t) = b_{t-2}(1,1) \oplus_4 a_{t-2}(2,2) \oplus_4 b_{t-1}(3,0) \oplus_4 a_{t-1}(2,0)$ $\oplus_4 b_t(2,2) \oplus_4 a_t(3,3)$

Then, we discuss the design criterion for the case of high diversity. The pairwise error probability conditioned on $\boldsymbol{\alpha}$ is upper bounded by

$$\begin{aligned} & \mathrm{P_r}(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \alpha_{i,q} \ \forall \ i, q) \\ & \leq \frac{1}{2} \prod_{t=1}^{L} \prod_{i=1}^{n} \left\{ (1-\rho) \exp\left(-\frac{d_{i,t}^2(s,\tilde{s})}{4N_0} \right) + \rho \exp\left(-\frac{d_{i,t}^2(s,\tilde{s})}{4\left(N_0 + N_J/\rho\right)} \right) \right\}. \end{aligned} \quad (4.58)$$

The pairwise error probability can be further approximated as

$$\mathrm{P_r}\left(\boldsymbol{s} \longrightarrow \tilde{\boldsymbol{s}}\right) \cong \frac{1}{2} \rho^L \exp\left(-W_4\right) \quad (4.59)$$

where

$$W_4 = \frac{\left(\sum_{q=1}^{m} \sum_{i=1}^{r} \left(\sum_{t=1}^{L} \sum_{k=1}^{M} \left| s_{i,t}^k - \tilde{s}_{i,t}^k \right|^2 \right) \right)^2}{\sum_{q=1}^{m} \sum_{i=1}^{r} \left(\sum_{t=1}^{L} \sum_{k=1}^{M} \left| s_{i,t}^k - \tilde{s}_{i,t}^k \right|^2 \right)^2}.$$

Good codes should hence be constructed by maximizing W_4 for all possible $\boldsymbol{s}$ and $\tilde{\boldsymbol{s}}$. According to the design criterion, space-time codes of good performance with respcet to the large values of rm are given in Table 4.4 by a computer search.

Table 4.4: Optimal Space-time codes of the STC/OFHSS system with 4FSK and 2 transmitter antennas for wireless jamming channels with respect to the case of high diversity.

Memory	Generator Sequences
2	$(x_1^t, x_2^t) = b_{t-1}(2,2) \oplus_4 a_{t-1}(3,3) \oplus_4 b_t(3,1) \oplus_4 a_t(2,2)$
3	$(x_1^t, x_2^t) = a_{t-2}(1,2) \oplus_4 b_{t-1}(2,1) \oplus_4 a_{t-1}(1,3) \oplus_4 b_t(3,3)$ $\oplus_4 a_t(2,2)$
4	$(x_1^t, x_2^t) = b_{t-2}(1,1) \oplus_4 a_{t-2}(2,2) \oplus_4 b_{t-1}(2,0) \oplus_4 a_{t-1}(3,0)$ $\oplus_4 b_t(2,1) \oplus_4 a_t(3,2)$

4.4 STC Combined with Uniform FH

The other system we proposed is STC/UFHSS system as shown in Figure 4.3. At time instant t, the encoded codewords from all transmitter antennas are hopped into the frequency bands randomly over the total spread spectrum bandwidth. In this section, we derive the ML decoding together with some suboptimal decoding schemes with respect to the STC/UFHSS system.

4.4.1 Decoding with CSI Available

Assume perfect estimation of $\alpha_{i,q}$'s is available at the receiver. Let the selected received signals $\tilde{\boldsymbol{r}} = (\tilde{r}^k_{q,t,b}\ \forall\ q,t,b,k)$, the jamming indicator $\boldsymbol{x} = (x_{t,b}\ \forall\ t,b)$, path gains $\boldsymbol{\alpha} = (\alpha_{i,q}\ \forall\ i,q)$, the collided transmitted symbols indicator $\tilde{\boldsymbol{\beta}} = (\tilde{\beta}_{i,t,b}\ \forall\ b,i,t)$,and the estimated symbols $\hat{\boldsymbol{s}} = \left(\hat{s}^k_{i,t}\ \forall\ i,k,t\right)$. The optimal decoding metrics with respect to CSI available are presented in the following section.

4.3.1.1 Maximum Likelihood Decoding with JSI Available

To evaluate the system performance, we derive the ML decoding of space-time codes as follows. By (4.11), assume perfect estimation of JSI $x_{t,b}$'s are available at the receiver. The likelihood function is given by

$$\begin{aligned} & f\left\{\tilde{\boldsymbol{r}}\,\middle|\,\hat{\boldsymbol{s}},\ \boldsymbol{\alpha},\boldsymbol{x},\ \tilde{\boldsymbol{\beta}}\right\} \\ &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{b=1}^{B(t)} f\left\{\tilde{r}^k_{q,t,b} = \sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}^k_{i,t} + \tilde{\eta}^k_{q,t,b}\,\middle|\,\hat{s}^k_{i,t},\alpha_{i,q},x_{t,b},\ \tilde{\beta}_{i,t,b}\forall\ i,q,t,b,\ k\right\} \qquad (4.60) \\ &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{b=1}^{B(t)} \frac{1}{\sqrt{\pi\left(N_0 + x^2_{t,b}N_J/\rho\right)}}\exp\left\{-\frac{\left|\tilde{r}^k_{q,t,b} - \sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}^k_{i,t}\right|}{N_0 + x^2_{t,b}N_J/\rho}\right\}. \end{aligned}$$

By taking logarithm on the likehood function, $\hat{\boldsymbol{s}}^k_{i,t}$'s can be decoded in the ML sense by maximizing the following metric:

$$\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{b=1}^{B(t)}\ln\left\{\frac{1}{\sqrt{\pi\left(N_0 + x^2_{t,b}N_J/\rho\right)}}\exp\left\{-\frac{\left|\tilde{r}^k_{q,t,b} - \sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}^k_{i,t}\right|}{N_0 + x^2_{t,b}N_J/\rho}\right\}\right\}. \qquad (4.61)$$

Suppose $x_{t,b}$'s are not available at the receiver. The likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$, $\tilde{\boldsymbol{\beta}}$, and $\hat{\boldsymbol{\alpha}}$, i.e., $f\left\{\tilde{\boldsymbol{r}}\,\middle|\,\hat{\boldsymbol{s}},\ \boldsymbol{\alpha},\ \tilde{\boldsymbol{\beta}}\right\}$, can be obtained by averaging (4.61) with respect to $x_{t,b}$'s. A closed-form expression of $\ln f\left\{\tilde{\boldsymbol{r}}\left|\hat{\boldsymbol{s}},\ \boldsymbol{\alpha},\ \tilde{\boldsymbol{\beta}}\right.\right\}$ is then derived as

$$\sum_{t=1}^{L}\sum_{b=1}^{B(t)}\ln\left\{\rho\prod_{k=1}^{M}\prod_{q=1}^{m}\frac{1}{\sqrt{\pi\left(N_0+N_J/\rho\right)}}\exp\left(-\frac{\left|\tilde{r}_{q,t,b}^{k}-\sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{N_0+N_J/\rho}\right)\right. \tag{4.62}$$

$$\left.+(1-\rho)\prod_{k=1}^{M}\prod_{q=1}^{m}\frac{1}{\sqrt{\pi N_0}}\exp\left(-\frac{\left|\tilde{r}_{q,t,b}^{k}-\sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{N_0}\right)\right\}.$$

4.3.1.2 Suboptimal Decoding Schemes

Althought the ML decoding can provide the optimum error correcting performance, the computation required to execute (4.63) might limit its feasibility in practical applications. Besides, the side informations of channel and jamming, i.e., $E_b/N_0, E_b/N_J,$ and ρ are also required to be estimated. Hence, by ignoring all coefficients in (4.63) and use the approximation of $\exp(x)\approx 1+x$, we proposed a suboptimal scheme SUB3.1 with the following decoding metric:

$$\sum_{t=1}^{L}\sum_{b=1}^{B(t)}\sum_{k=1}^{M}\sum_{q=1}^{m}\ln\left(1+\left|\tilde{r}_{q,t,b}^{k}-\sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2\right). \tag{4.63}$$

4.4.2 Decoding without CSI

Suppose $\alpha_{i,q}$'s are not available at the receiver, the optimal decoding metrics of the STC/UFHSS system with respect to CSI unknown are presented in the following section.

4.3.2.1 Maximum Likelihood Decoding with JSI Available

Let $a_{1,x_{t,b}}=\frac{1}{\sqrt{\pi\left(N_0+x_{t,b}^2N_J/\rho\right)}}, a_{2,x_{t,b}}=\frac{1}{\left(N_0+x_{t,b}^2N_J/\rho\right)}$, then (4.61) can be rewritten as

$$f\left\{\tilde{\boldsymbol{r}}\left|\hat{\boldsymbol{s}},\boldsymbol{\alpha},\boldsymbol{x},\tilde{\boldsymbol{\beta}}\right.\right\} = \left\{\prod_{t=1}^{L}\prod_{b=1}^{B(t)}\prod_{k=1}^{M}a_{1,x_{t,b}}^{m}\right\}\cdot\exp\left(-\sum_{t=1}^{L}\sum^{B(t)_{b=1}}\sum_{k=1}^{M}\sum_{q=1}^{m}a_{2,x_{t,b}}\left[\left|\tilde{r}_{q,t,b}^{k}\right|^2\right.\right.$$

$$\left.\left.-2\mathrm{Re}\left(\tilde{r}_{q,t,b}^{k}\sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}_{i,t}^{k}\right)\right]+\left|\sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2.\right) \tag{4.64}$$

The $\mathrm{Re}\left(\tilde{r}^k_{q,t,b}\sum_{i=1}^n \tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}^k_{i,t}\right)$ and $\left|\sum_{i=1}^n \tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}^k_{i,t}\right|^2$ of the exponent can be rewritten as

$$\begin{aligned}
&\mathrm{Re}\left(\tilde{r}^k_{q,t,b}\sum_{i=1}^n \tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}^k_{i,t}\right) \\
&= \mathrm{Re}\left(\tilde{r}^k_{q,t,b}\sum_{i=1}^n \tilde{\beta}_{i,t,b}\left(\alpha_{R,i,q} - j\alpha_{I,i,q}\right)\hat{s}^k_{i,t}\right) \\
&= \mathrm{Re}\left(\tilde{r}^k_{q,t,b}\sum_{i=1}^n \tilde{\beta}_{i,t,b}\alpha_{R,i,q}\hat{s}^k_{i,t}\right) + \mathrm{Im}\left(\tilde{r}^k_{q,t,b}\sum_{i=1}^n \tilde{\beta}_{i,t,b}\alpha_{I,i,q}\hat{s}^k_{i,t}\right)
\end{aligned} \tag{4.65}$$

and

$$\begin{aligned}
\left|\sum_{i=1}^n \tilde{\beta}_{i,t,b}\alpha_{i,q}\hat{s}^k_{i,t}\right|^2 &= \left|\sum_{i=1}^n \tilde{\beta}_{i,t,b}\left(\alpha_{R,i,q} + j\alpha_{I,i,q}\right)\hat{s}^k_{i,t}\right|^2 \\
&= \sum_{i=1}^n \alpha_{R,i,q}\left|\tilde{\beta}_{i,t,b}\hat{s}^k_{i,t}\right|^2 + \sum_{\substack{i=1\\ i\neq l}}^n\sum_{\substack{l=1\\ l\neq i}}^n \alpha_{R,i,q}\alpha_{R,l,q}\tilde{\beta}_{i,t,b}\hat{s}^k_{i,t}\tilde{\beta}_{l,t,b}\hat{s}^k_{l,t} \\
&\quad + \sum_{i=1}^n \alpha_{I,i,q}\left|\tilde{\beta}_{i,t,b}\hat{s}^k_{i,t}\right|^2 + \sum_{\substack{i=1\\ i\neq l}}^n\sum_{\substack{l=1\\ l\neq i}}^n \alpha_{I,i,q}\alpha_{I,l,q}\tilde{\beta}_{i,t,b}\hat{s}^k_{i,t}\tilde{\beta}_{l,t,b}\hat{s}^k_{l,t}.
\end{aligned} \tag{4.66}$$

By averaging $\alpha_{i,q}$'s with respect to the conditional probability density function in (4.64), we can get

$$\begin{aligned}
&\int_{-\infty}^{\infty} f(\tilde{\boldsymbol{r}}|\hat{\boldsymbol{s}}, \boldsymbol{x}, \boldsymbol{\alpha}, \tilde{\boldsymbol{\beta}})f(\boldsymbol{\alpha})d\boldsymbol{\alpha} \\
&= \left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{b=1}^{B(t)}\left(a_{1,x_{t,b}}\exp\left(-a_{2,x_{t,b}}\left|\tilde{r}^k_{q,t,b}\right|^2\right)\right)\right\}\cdot\left\{\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}\lambda^k_{i,t,b}+1\right)^{-1}\right.\right. \\
&\exp\left(\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}u^{q,k}_{i,t,b}\right|^2}{\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}\lambda^k_{i,t,b}+1}\right)
\end{aligned} \tag{4.67}$$

where

$$\begin{aligned}
a_{1,x_{t,b}} &= \frac{1}{\sqrt{\pi(N_0 + x^2_{t,b}N_J/\rho)}} \quad a_{2,x_{t,b}} = \frac{1}{N_0 + x^2_{t,b}N_J/\rho} \\
u^{q,k}_{i,t,b} &= \left((r^k_{q,t}\tilde{\beta}_{1,t,b}\hat{s}^k_{1,t}),\ (r^k_{q,t}\tilde{\beta}_{2,t,b}\hat{s}^k_{2,t}),\ \ldots,\ (r^k_{q,t}\tilde{\beta}_{n,t,b}\hat{s}^k_{n,t})\right)\boldsymbol{v}^k_{i,t} \\
&= (u^{b,k}_{1,q,t},\ u^{b,k}_{2,q,t},\ \ldots,\ u^{b,k}_{n,q,t}).
\end{aligned}$$

$\boldsymbol{v}_{i,t}^{k}$'s and $\lambda_{i,t,b}^{k}$'s stand for the eigenvectors and eigenvalues of the following matrix, respectively:

$$\begin{bmatrix} \left|\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right|^2 & \left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right)\left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right) & \cdots & \left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right)\left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right) \\ \left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right)\left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right) & \left|\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right|^2 & \cdots & \left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right)\left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right)\left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right) & \left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right)\left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right) & \cdots & \left|\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right|^2 \end{bmatrix}.$$

The ML decoding then chooses $\hat{s}_{i,t}^{k}$'s by maximizing (4.67).

4.3.2.2 Suboptimal Decoding Schemes

To optimize the trade-off between decoding complexity and performance, we present the suboptimal scheme SUB3.2 and SUB3.3 with JSI available. The suboptimal scheme SUB3.2 which chooses $\tilde{s}_{i,t}^{k}$'s by maximizing

$$\sum_{t=1}^{L}\sum_{b=1}^{B(t)} \ln\left\{\prod_{k=1}^{M}\prod_{q=1}^{m}\left(a_{1,x_{t,b}}\exp\left(-a_{2,x_{t,b}}\left|\tilde{r}_{q,t,b}^{k}\right|^2\right)\right)\left[\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\left(\sum_{k=1}^{M}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1\right)^{-1} \cdot \exp\left(\frac{\left|\sum_{k=1}^{M}a_{2,x_{t,b}}u_{i,t,b}^{q,k}\right|^2}{\sum_{k=1}^{M}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1}\right)\right]\right]\right\}. \tag{4.68}$$

where the values of $a_{1,x_{t,b}}$, $a_{2,x_{t,b}}$, $\lambda_{i,t}^{k,q}$, and $u_{i,q,t}^{b,k}$ are the same as that in (4.67). The other suboptimal scheme SUB3.3 which uses the approximation of $\exp(x) \approx 1 + x$ is denoted by

$$\text{SUB3.3} : \max_{\hat{\boldsymbol{s}}}\sum_{t=1}^{L}\sum_{b=1}^{B(t)}\sum_{q=1}^{m}\sum_{i=1}^{n}\ln\left(1+\frac{\left|\sum_{k=1}^{M}a_{2,x_{t,b}}u_{i,t,b}^{q,k}\right|^2}{\sum_{k=1}^{M}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1}\right). \tag{4.69}$$

Suppose $\alpha_{i,q}$'s and x_t's are both not available at the receiver, and the closed-form of $f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\tilde{\boldsymbol{\beta}}\right)$ is too complexity to be written. Therefore, we propose three suboptimal decoding schemes to reduce the decoding complexity. The first one suboptimal scheme is

SUB3.4 which is represented as

$$\left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{b=1}^{B(t)}\left(a_{1,x_{t,b}=1}\exp\left(-a_{2,x_{t,b}=1}\left|\tilde{r}_{q,t,b}^{k}\right|^{2}\right)\right)\right\}\cdot\left\{\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}=1}\lambda_{i,t,b}^{k}+1\right)^{-1}\right.\right.$$

$$\left.\left.\exp\left(\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}=1}u_{i,t,b}^{q,k}\right|^{2}}{\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}=1}\lambda_{i,t,b}^{k}+1}\right)\right]\right\}+\sum_{t'=1}^{L}\sum_{b'=1}^{B(t)}\left[\frac{\rho^{nL-1}}{(1-\rho)^{-1}}\cdot\frac{a_{1,x_{i,t}=1}^{Mm(Ln-1)}}{a_{1,x_{i,t}=0}^{Mm}}\cdot\right.$$

$$\exp\left(-a_{2,x_{i,t}=1}\sum_{q=1}^{m}\sum_{k=1}^{M}\left(\sum_{t=1}^{L}\sum_{b=1}^{n}\left|r_{q,t,b}^{k}\right|^{2}+a_{3}\left|r_{q,t',b'}^{k}\right|^{2}\right)\right)\cdot\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\prod_{\substack{i=1\\i\neq i'}}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}=1}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)^{-1}\right.$$

$$\left.\cdot\exp\left(\frac{\left|a_{2,x_{i,t}=1}\left(\sum_{t=1}^{L}\sum_{b=1}^{B(t)}\sum_{k=1}^{M}u_{i,t,b}^{q,k}+a_{3}\sum_{k=1}^{M}u_{i,t',b'}^{q,k}\right)\right|^{2}}{a_{2,x_{i,t}=1}\left(\sum_{t=1}^{L}\sum_{b=1}^{n}\sum_{k=1}^{M}\lambda_{i,t,b}^{k}+a_{3}\sum_{k=1}^{M}\lambda_{i,t',b'}^{k}\right)+1}\right)\right]. \tag{4.70}$$

where

$$a_3=\frac{N_J}{N_0\rho}\qquad a_{2,x_{t,b}=1}=\frac{1}{N_0+N_J/\rho}\qquad a_{1,x_{t,b}=0}=\frac{1}{\sqrt{\pi N_0}}\qquad a_{1,x_{t,b}=1}=\frac{1}{\sqrt{\pi\left(N_0+N_J/\rho\right)}}$$

The derivation of the metric in (4.70) is similar to that in Appendix F. Another suboptimal scheme is SUB3.5 with the following decoding metric

$$\sum_{t=1}^{L}\sum_{b=1}^{B(t)}\ln\left\{(1-\rho)\left(K_{t,x_{t,b}=0}^{q,k}\right)^{Mm}\left[\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\left(\sum_{k=1}^{M}a_{2,x_{t,b}=0}\lambda_{i,t,b}^{k}+1\right)^{-1}\exp\left(\frac{\left|\sum_{k=1}^{M}a_{2,x_{t,b}=0}u_{i,t,b}^{q,k}\right|^{2}}{\sum_{k=1}^{M}a_{2,x_{t,b}=0}\lambda_{i,t,b}^{k}+1}\right)\right.\right.\right.$$

$$\left.+\rho\left(K_{t,x_{t,b}=1}^{q,k}\right)^{Mm}\left[\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\left(\sum_{k=1}^{M}a_{2,x_{t,b}=1}\lambda_{i,t,b}^{k}+1\right)^{-1}\exp\left(\frac{\left|\sum_{k=1}^{M}a_{2,x_{t,b}=1}u_{i,t,b}^{q,k}\right|^{2}}{\sum_{k=1}^{M}a_{2,x_{t,b}=1}\lambda_{i,t,b}^{k}+1}\right)\right\} \tag{4.71}$$

where

$$a_{2,x_{t,b}=0}=\frac{1}{N_0}\qquad a_{2,x_{t,b}=1}=\frac{1}{N_0+N_J/\rho}\qquad a_{1,x_{t,b}=0}=\frac{1}{\sqrt{\pi N_0}}\qquad a_{1,x_{t,b}=1}=\frac{1}{\sqrt{\pi\left(N_0+N_J/\rho\right)}}$$

$$K_{i,t,x_{t,b}=0}^{k,q}=a_{1,x_{t,b}=0}\exp\left(-a_{2,x_{t,b}=0}\left|r_{q,t}^{k}\right|^{2}\right)\qquad K_{i,t,x_{t,b}=1}^{k,q}=a_{1,x_{t,b}=1}\exp\left(-a_{2,x_{t,b}=1}\left|r_{q,t}^{k}\right|^{2}\right).$$

The other suboptimal scheme SUB3.6 is

$$\text{SUB3.6} : \max_{\hat{\boldsymbol{s}}} \sum_{q=1}^{m} \sum_{i=1}^{n} \frac{\left| \sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{b=1}^{B(t)} a_{2,x_{t,b}} u_{i,t,b}^{q,k} \right|^2}{\sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{b=1}^{B(t)} a_{2,x_{t,b}} \lambda_{i,t,b}^{k} + 1}. \tag{4.72}$$

4.5 Simulation Results and Discussions

In this section, we simulate the 4-state space-time code with two transmitter/receiver antennas, 4FSK modulation, and 1000 frequency hopping bands for used over Rayleigh fading channels with the AWGN and PBNJ to explore the performance of the STC/FHSS system. In Figure 4.6-4.34, the space-time code of STC/WFHSS system we used for simulation is

$$\left(x_1^t, x_2^t\right) = b_{t-1}\left(0,2\right) \oplus_4 a_{t-1}\left(0,3\right) \oplus_4 b_t\left(3,2\right) \oplus_4 a_t\left(2,1\right). \tag{4.73}$$

and the space-time code of STC/UFHSS and STC/UFHSS system for simulation is

$$\left(x_1^t, x_2^t\right) = b_{t-1}\left(2,2\right) \oplus_4 a_{t-1}\left(1,3\right) \oplus_4 b_t\left(1,1\right) \oplus_4 a_t\left(2,2\right). \tag{4.74}$$

For the decoding with CSI available, the impact of E_b/N_J on the bit-error-rate (BER) performance is first investigated. Observed from the performance curves in Figure 4.6- 4.11 with $E_b/N_0 = 15$ dB, $\rho = 0.2$ and $\rho = 0.05$, the ML decoding with JSI provides the best performance followed by the ML decoding without JSI and suboptimal decoding scheme. The performance of the two optimal decodings of the STC/WFHSS system are very closed no matter whether the JSI is available or not, and the SUB1.1 is observed to provide similar performance as the optimum decoding. The two optimal decodings of the STC/UFHSS and STC/OFHSS systems are similar at low and high SNRs, and the suboptimal scheme SUB2.1 and SUB3.1 approximates the ML decoding well at high E_b/N_J. The performance plots of the ML decoding with CSI available corresponding to different values of ρ and E_b/N_J are shown in Figure 4.12-4.19. Given $E_b/N_0 = 25$ dB and $E_b/N_J = 5$ dB in Figure 4.12- 4.13, we can find that the worst performance is located at $\rho = 1$. That is, the bandwidth of the PBNJ is spread as same as the system bandwidth can make the performance worst.

On the contrary, for $E_b/N_J = 25$ dB, the small fraction bandwidth of the PBNJ can make the performance worst than the full band noise jammer at $\rho = 1$. In Figure 4.14- 4.19, the AWGN is not too small for $E_b/N_0 = 15$ dB. Hence, for $E_b/N_J = 25$dB, we can find that the performance is dominated by AWGN mostly, and the worst performance is not obvious. Figure 4.20- 4.25 show the ML decoding with CSI available corresponding to different values of ρ and E_b/N_0, an irregular relation between the BER and ρ of the ML decoding with respect to three different STC/FHSS systems is observed. E_b should thus be determined corresponding to the worst case to guarantee the designed performance for the PBNJ with uncertain ρ. The ML and suboptimal decoding schemes for E_b/N_0=15 dB and E_b/N_J=10 dB with CSI available are shown in Figure 4.26-4.28. The performance gap between the simulated decoding schemes of the STC/WFHSS system is almost the same as ρ changes, and the performance of the optimal decodings of the STC/OFHSS and STC/UFHSS system are very close with respect to different values of ρ.

For the decoding without CSI, the simulation results of the ML decoding and some suboptimal decoding schemes are provided in Figure 4.29-4.31. The ML decoding is observed to provide the optimum BER performance, and SUB1.2 provide similar performance as the optimum decoding. The SUB1.3 of the STC/WFHSS, SUB2.2 and SUB2.3 of the STC/OFHSS, SUB3.2 and SUB3.3 of the STC/UFHSS, are proposed to reduce the decoding complexity by using Viterbi algorithm. In Figure 4.32-4.34, it shows the ML decoding with JSI available and some suboptimal decoding schemes without JSI. The SUB1.4 and SUB1.5 of the STC/WFHSS, SUB2.4 and SUB2.5 of the STC/OFHSS, and SUB3.4 and SUB3.5 of the STC/UFHSS are observed to provide similar performance of the ML decoding at high E_b/N_J, respectively. Moreover, the SUB1.6, SUB2.6, and SUB3.6 are proposed by using Viterbi algorithm. Figure 4.35-4.36 show the ML decoding with CSI and JSI available with respect to different values of memory of space-time codes in Table 4.1 and Table 4.3, respectively. In Figure 4.37-4.38, it shows the performance of space-time codes in Table 4.2 and Table 4.4 with two transmitter antennas and three receiver antennas. The performance comparison of the three different STC/FHSS systems is shown in Figure 4.39.

Finally, The performance comparisons of the system we proposed with the conventional coding scheme, convolutional codes (CC) with 4FSK modulation and two transmitter/receiver antennas over Rayleigh fading channels with PBNJ are shown in Figure 4.40. The space-time code of the STC/WFHSS system we used for comparison

$$(x_1^t, x_2^t) = b_{t-2}(0,2) \oplus_4 a_{t-2}(0,3) \oplus_4 b_{t-1}(1,2) \oplus_4 a_{t-1}(2,2) \oplus_4 b_t(3,3) \oplus_4 a_t(2,2). \quad (4.75)$$

and the space-time code of the STC/UFHSS and STC/OFHSS system is

$$(x_1^t, x_2^t) = b_{t-2}(1,1) \oplus_4 at-2(2,2) \oplus_4 b_{t-1}(3,0) \oplus_4 a_{t-1}(2,0) \oplus_4 b_t(2,2) \oplus_4 a_t(3,3). \quad (4.76)$$

Consider a (2, 1) code with the following generator matrix

$$G(D) = \left[\ 1 + D + D^2 + D^4 \quad 1 + D^3 + D^4\ \right]. \quad (4.77)$$

where D denotes the operator for time delay. The memory of the CC equals to 4 for the same decoding complexity with respect to the system we proposed, and we combined CC with space-time block codes (STBC) [16][17] and FHSS with Alamouti schemes for the same special diversity and bandwidth efficiency as our proposed system. In Figure 4.40, we can find that the STC/FHSS system provides much better performance than the CC/STBC/FHSS system.

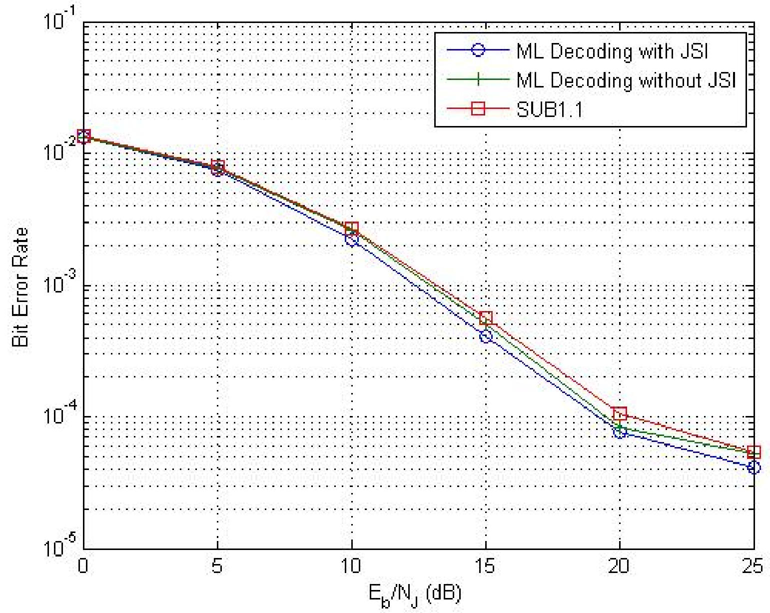

Figure 4.6: Performance plots of STC/WFHSS with CSI available for $\rho = 0.2$ and $E_b/N_0 =$ 15 dB.

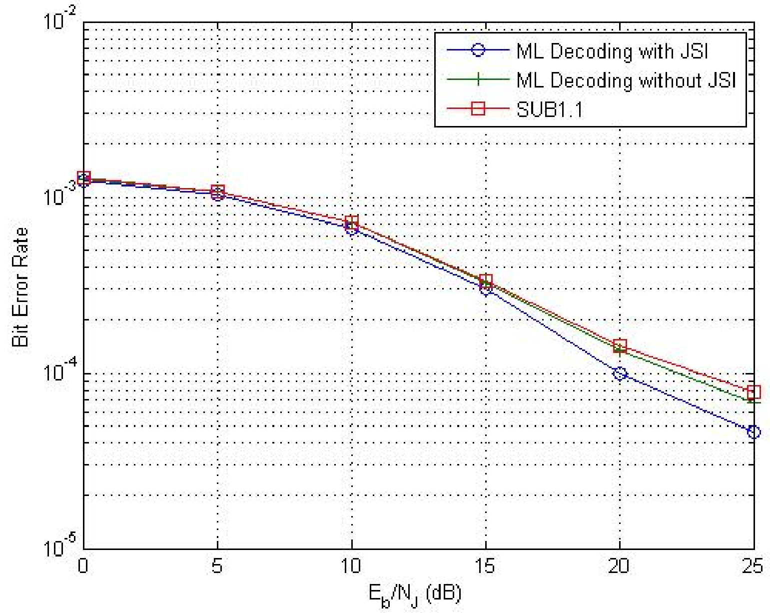

Figure 4.7: Performance plots of STC/WFHSS with CSI available for $\rho = 0.05$ and $E_b/N_0 =$ 15 dB.

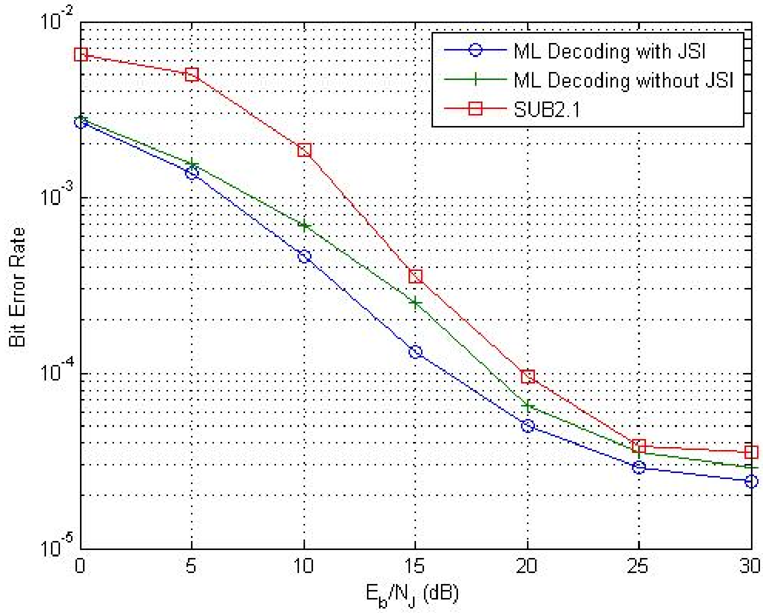

Figure 4.8: Performance plots of STC/OFHSS with CSI available for $\rho = 0.2$ and $E_b/N_0 =$ 15 dB.

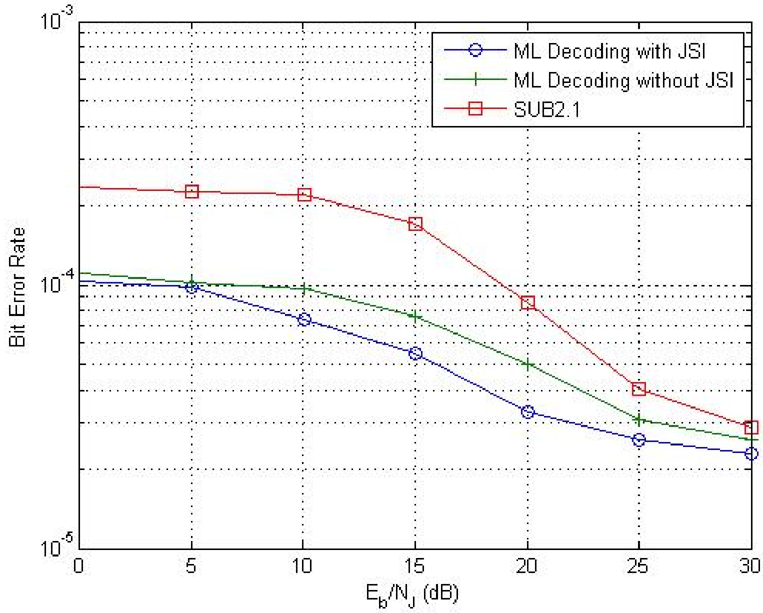

Figure 4.9: Performance plots of STC/OFHSS with CSI available for $\rho = 0.05$ and $E_b/N_0 =$ 15 dB.

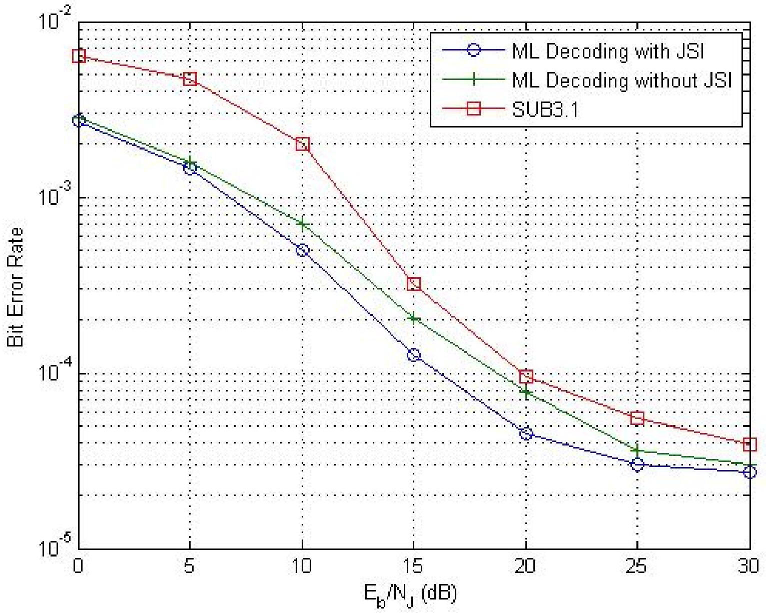

Figure 4.10: Performance plots of STC/UFHSS with CSI available for $\rho = 0.2$ and $E_b/N_0 =$ 15 dB.

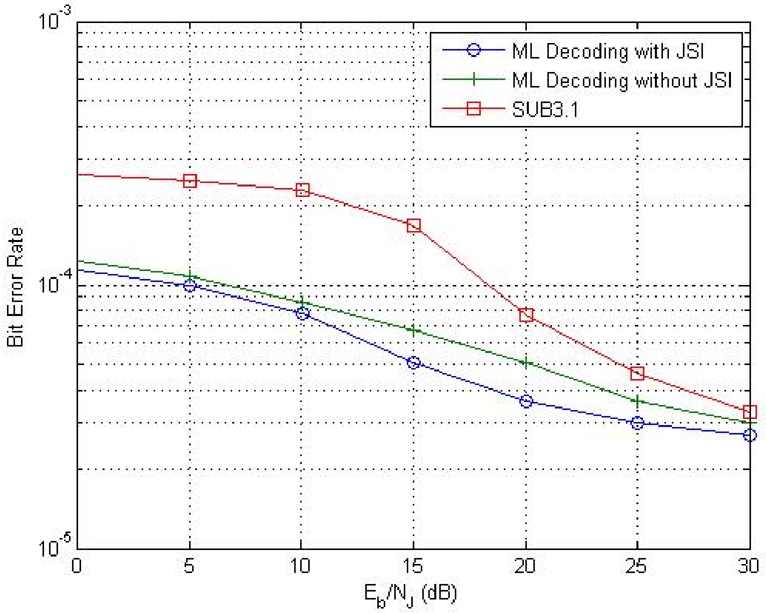

Figure 4.11: Performance plots of STC/UFHSS with CSI available for $\rho = 0.05$ and $E_b/N_0 =$ 15 dB.

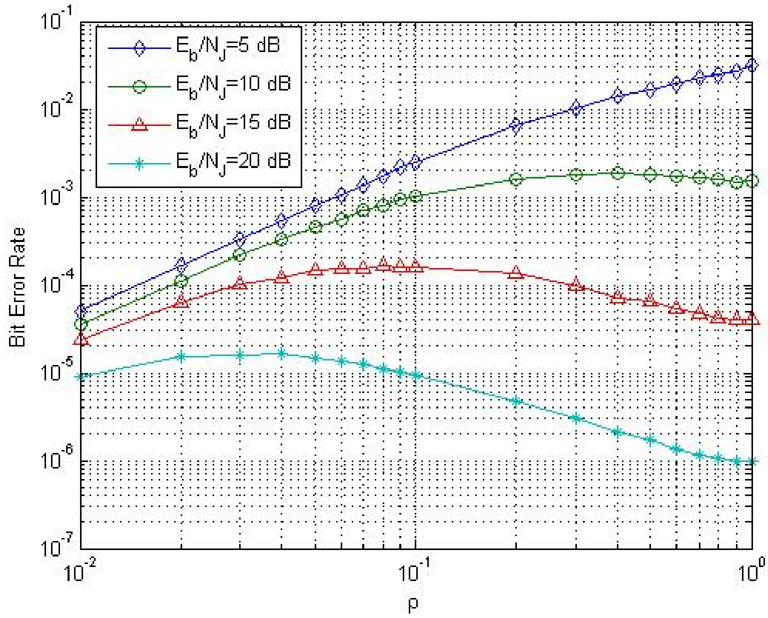

Figure 4.12: BER v.s. ρ of the ML decoding of STC/WFHSS for $E_b/N_0 = 25$ dB with CSI available.

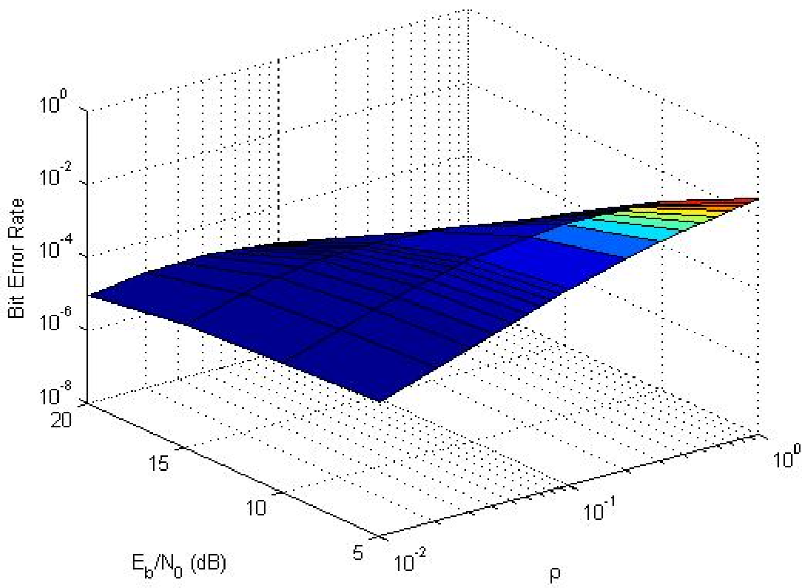

Figure 4.13: 3-dimension plots of the BER v.s. ρ for the ML decoding of STC/WFHSS for $E_b/N_0 = 25$ dB with CSI available.

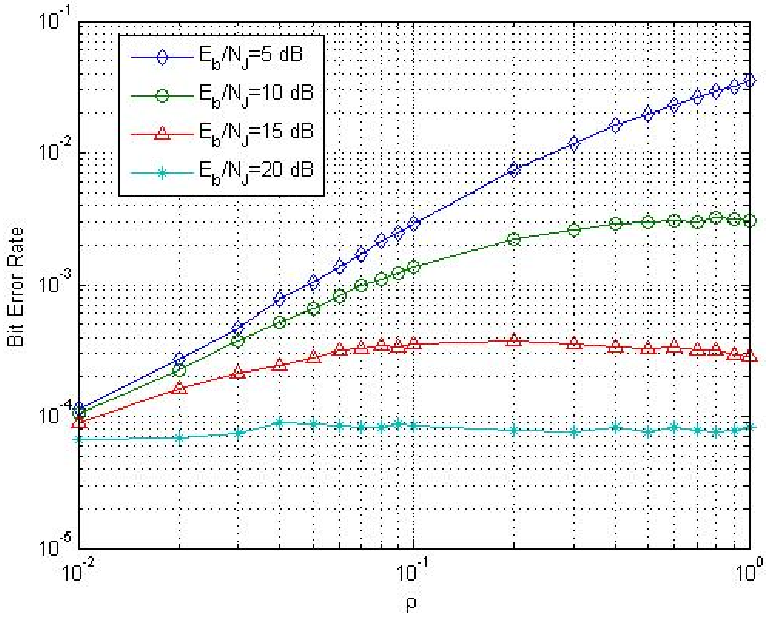

Figure 4.14: BER v.s. ρ of the ML decoding of STC/WFHSS for $E_b/N_0 = 15$ dB with CSI available.

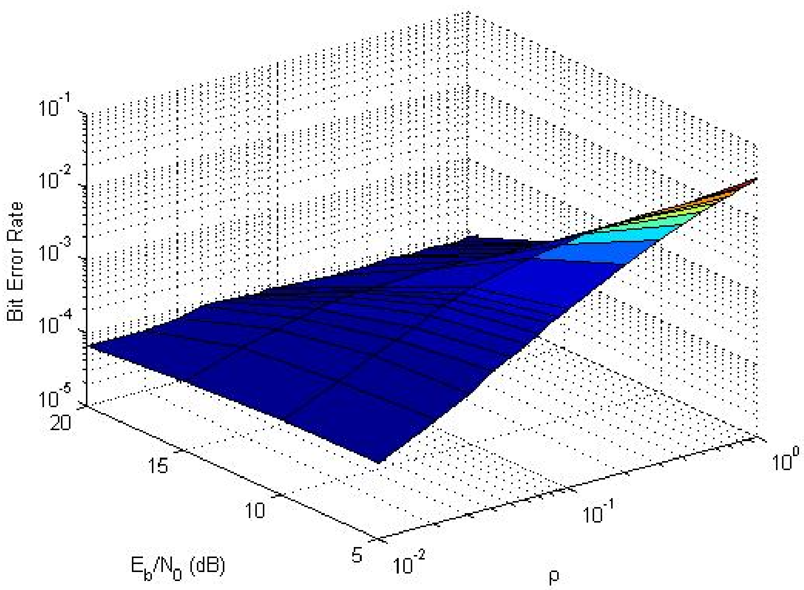

Figure 4.15: 3-dimension plots of the BER v.s. ρ for the ML decoding of STC/WFHSS for $E_b/N_0 = 15$ dB with CSI available.

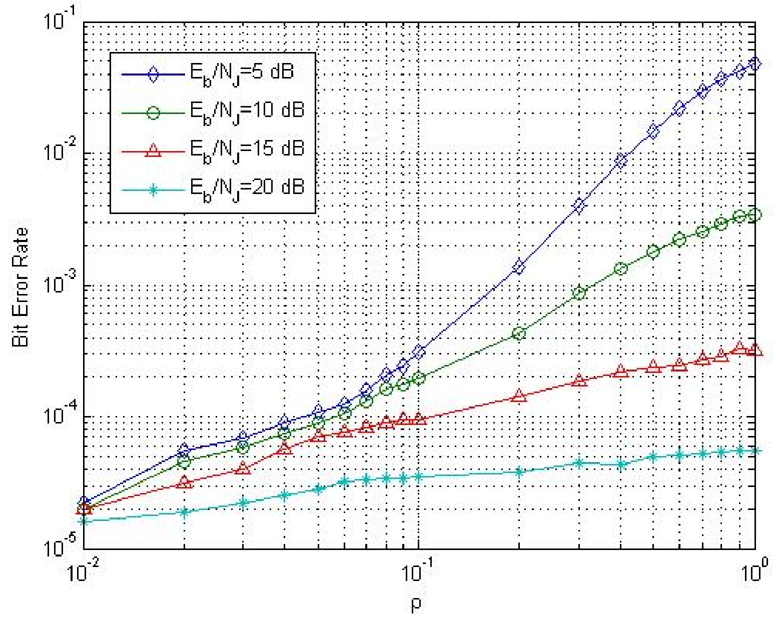

Figure 4.16: BER v.s. ρ of the ML decoding of STC/OFHSS for $E_b/N_0 = 15$ dB with CSI available.

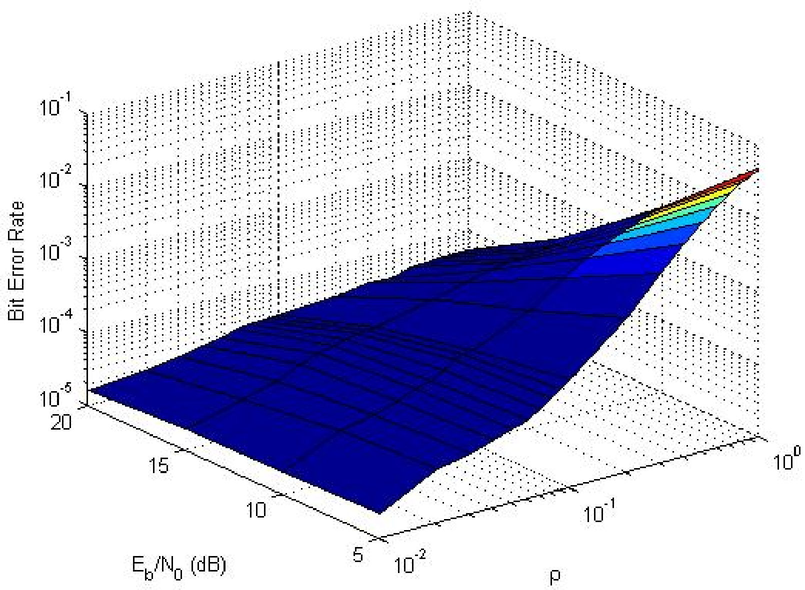

Figure 4.17: 3-dimension plots of the BER v.s. ρ for the ML decoding of STC/OFHSS for $E_b/N_0 = 15$ dB with CSI available.

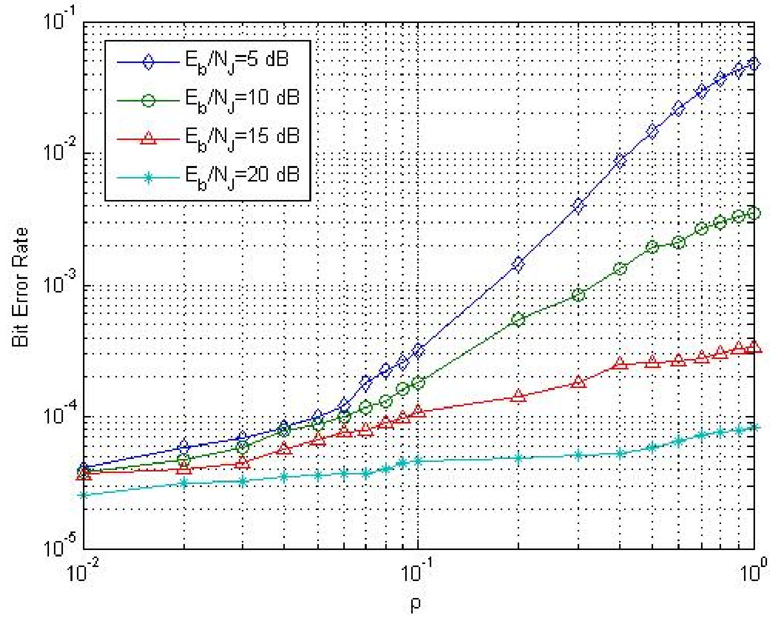

Figure 4.18: BER v.s. ρ of the ML decoding of STC/UFHSS for $E_b/N_0 = 15$ dB with CSI available.

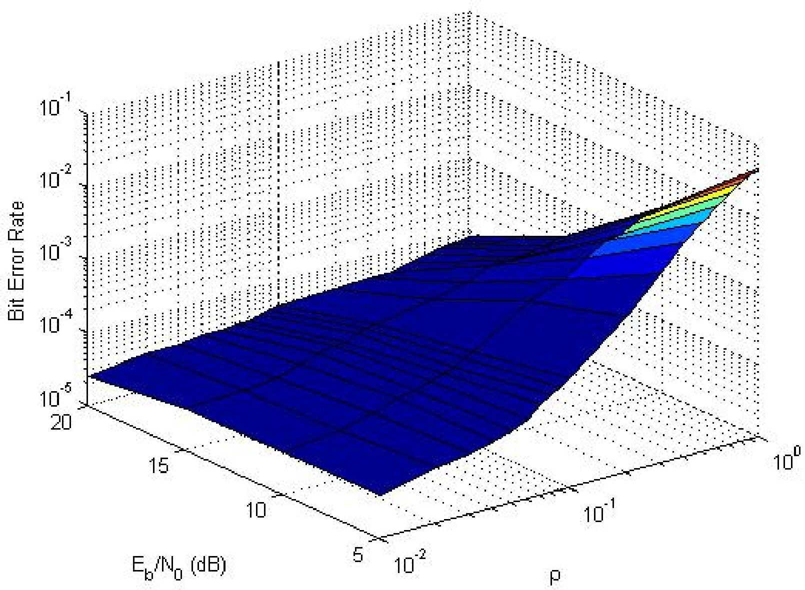

Figure 4.19: 3-dimension plots of the BER v.s. ρ for the ML decoding of STC/UFHSS for $E_b/N_0 = 15$ dB with CSI available.

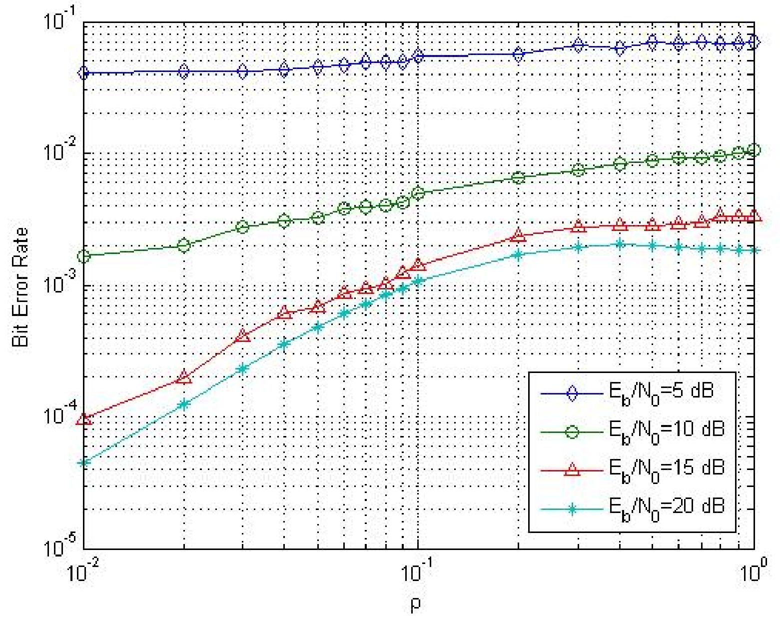

Figure 4.20: BER v.s. ρ of the ML decoding of STC/WFHSS for $E_b/N_J = 10$ dB with CSI available.

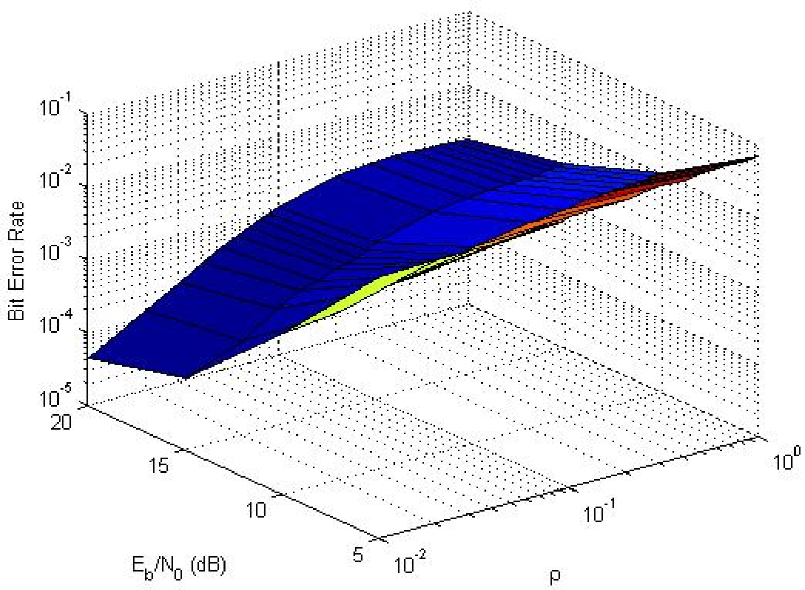

Figure 4.21: 3-dimension plots of the BER v.s. ρ for the ML decoding of STC/WFHSS for $E_b/N_J = 10$ dB with CSI available.

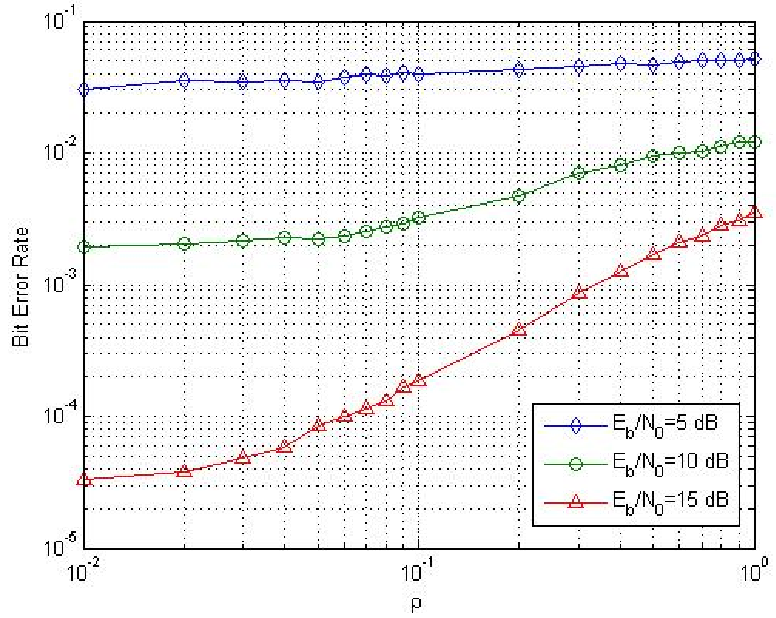

Figure 4.22: BER v.s. ρ of the ML decoding of STC/OFHSS for $E_b/N_J = 10$ dB with CSI available.

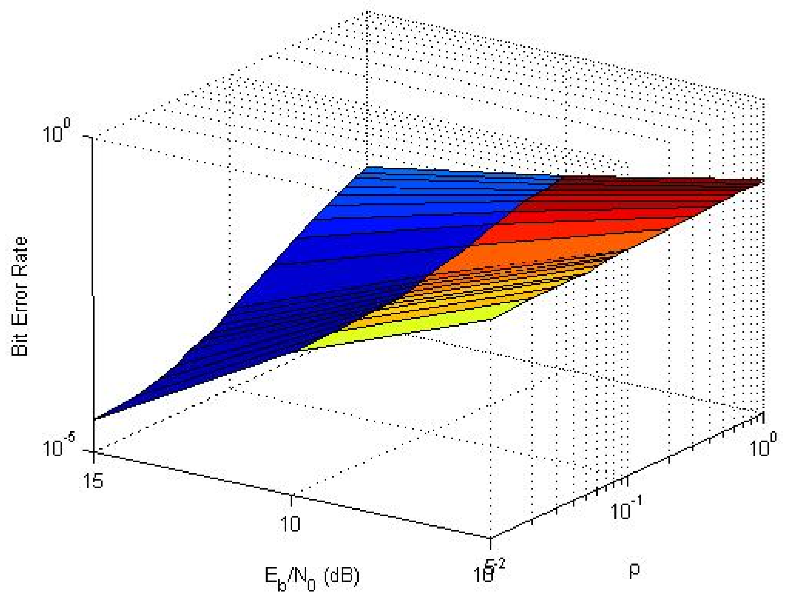

Figure 4.23: 3-dimension plots of the BER v.s. ρ for the ML decoding of STC/OFHSS for $E_b/N_J = 10$ dB with CSI available.

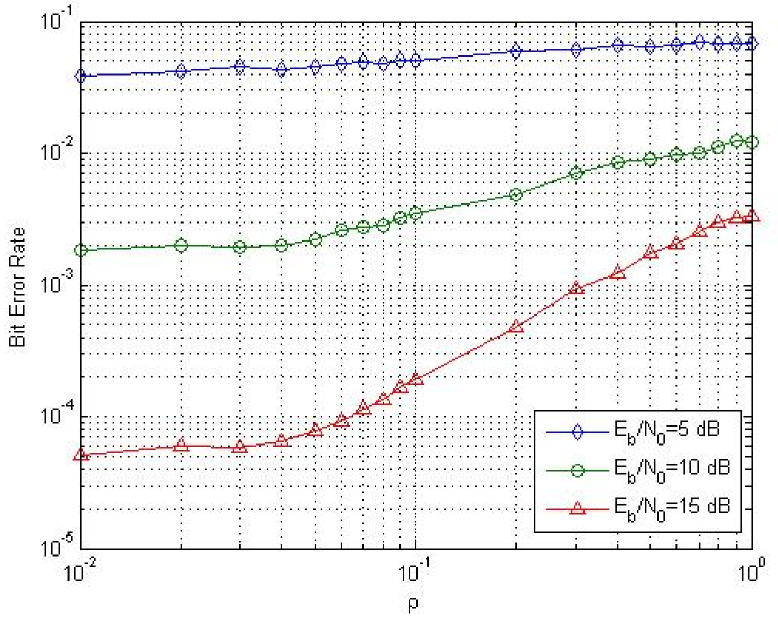

Figure 4.24: BER v.s. ρ of the ML decoding of STC/UFHSS for $E_b/N_J = 10$ dB with CSI available.

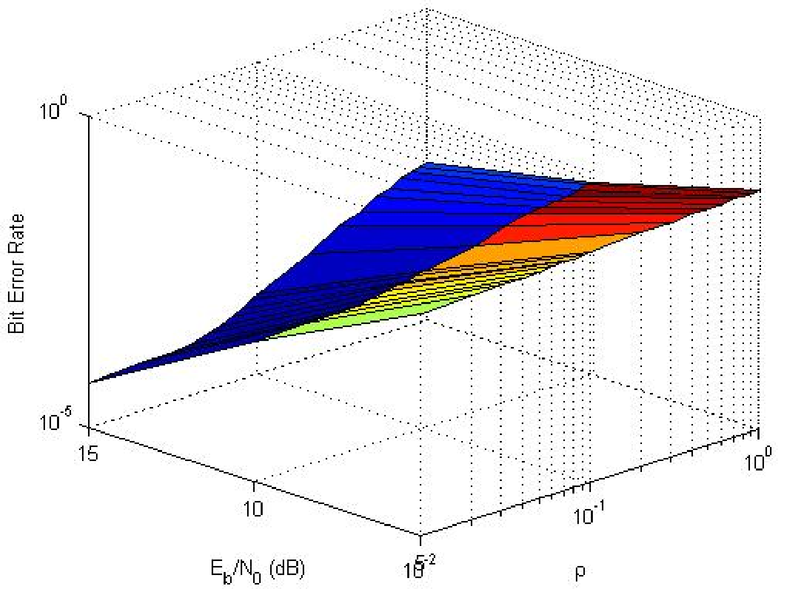

Figure 4.25: 3-dimension plots of the BER v.s. ρ for the ML decoding of STC/UFHSS for $E_b/N_J = 10$ dB with CSI available.

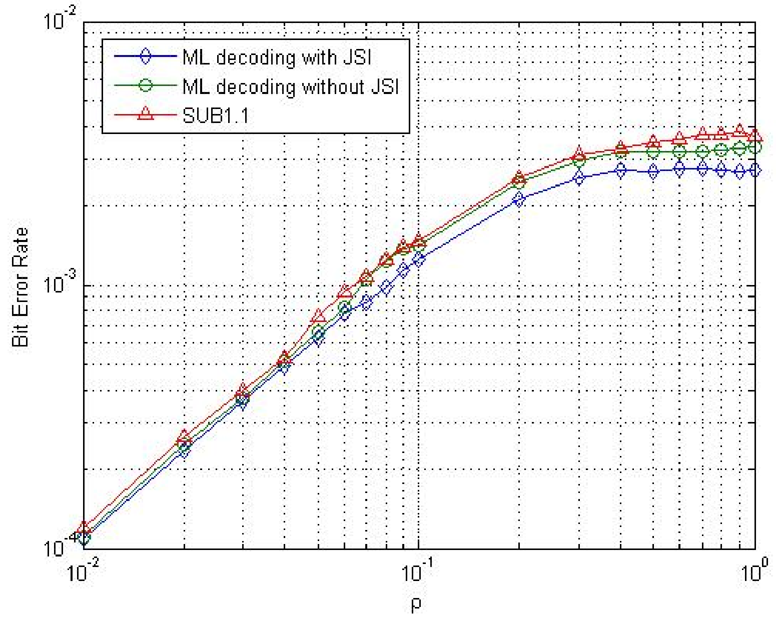

Figure 4.26: BER v.s. ρ for the ML and SUB1.1 decodings of the STC/WFHSS for $E_b/N_0 =$ 15 dB and $E_b/N_J = 10$ dB with CSI available.

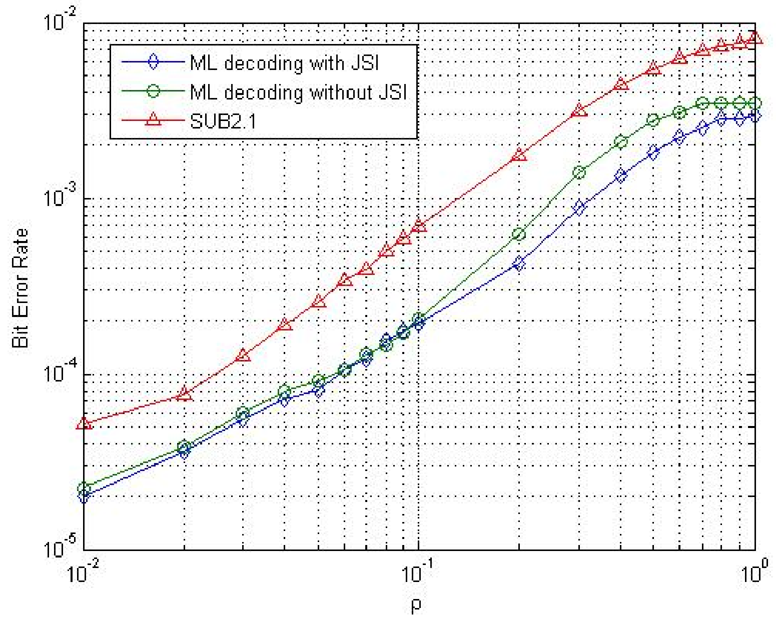

Figure 4.27: BER v.s. ρ for the ML and SUB2.1 decodings of the STC/OFHSS for $E_b/N_0 =$ 15 dB and $E_b/N_J = 10$ dB with CSI available.

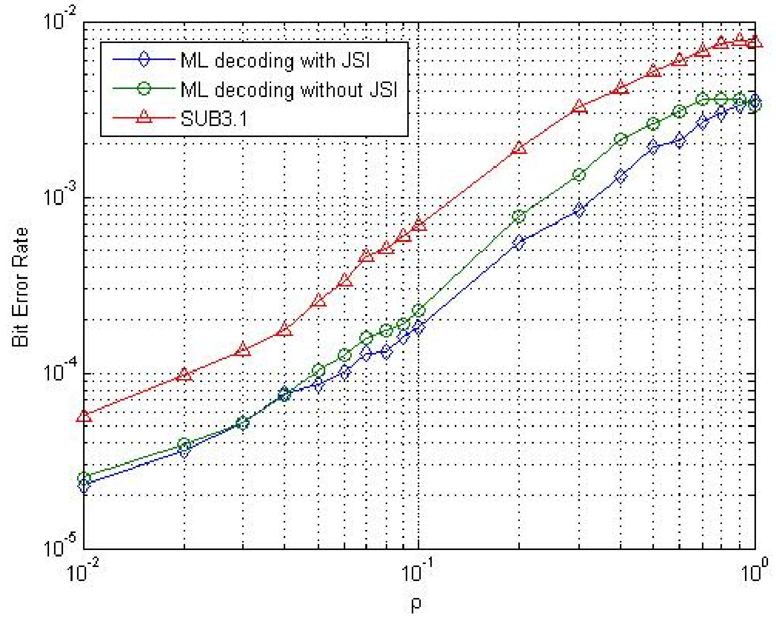

Figure 4.28: BER v.s. ρ for the ML and SUB3.1 decodings of the STC/UFHSS for $E_b/N_0 =$ 15 dB and $E_b/N_J = 10$ dB with CSI available.

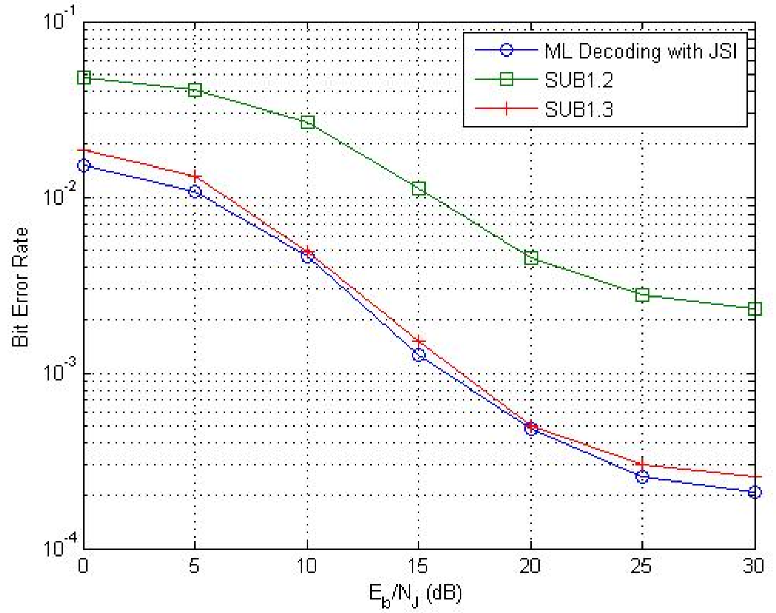

Figure 4.29: Performance plots of the STC/WFHSS without CSI for $rho = 0.2$, $E_b/N_0 = 20$ dB, and JSI available.

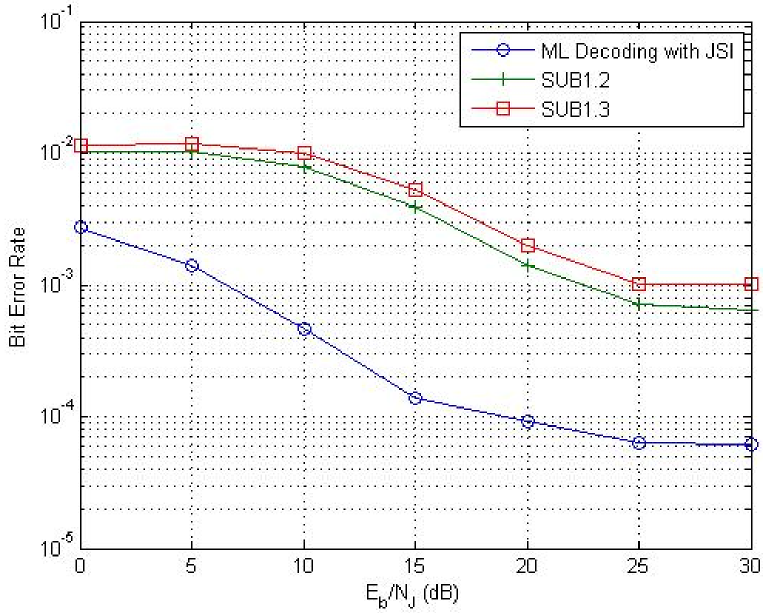

Figure 4.30: Performance plots of the STC/OFHSS without CSI for $\rho = 0.2$, $E_b/N_0 = 15$ dB, and JSI available.

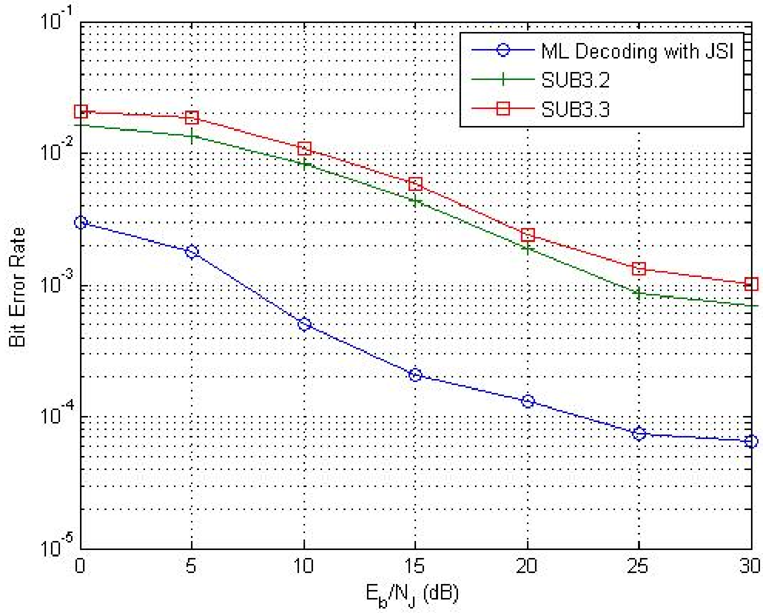

Figure 4.31: Performance plots of the STC/UFHSS without CSI for $\rho = 0.2$, $E_b/N_0 = 15$ dB, and JSI available.

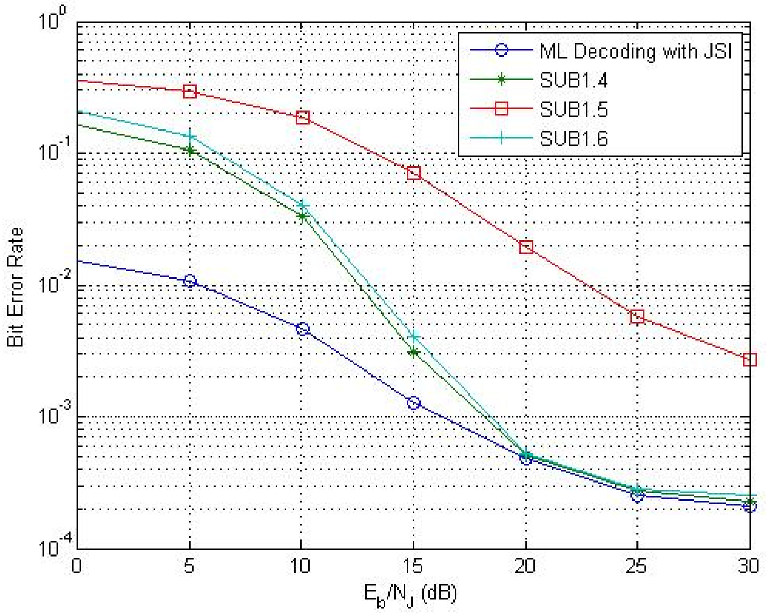

Figure 4.32: Performance plots of the STC/WFHSS without CSI for $\rho = 0.2$ and $E_b/N_0 = 20$ dB.

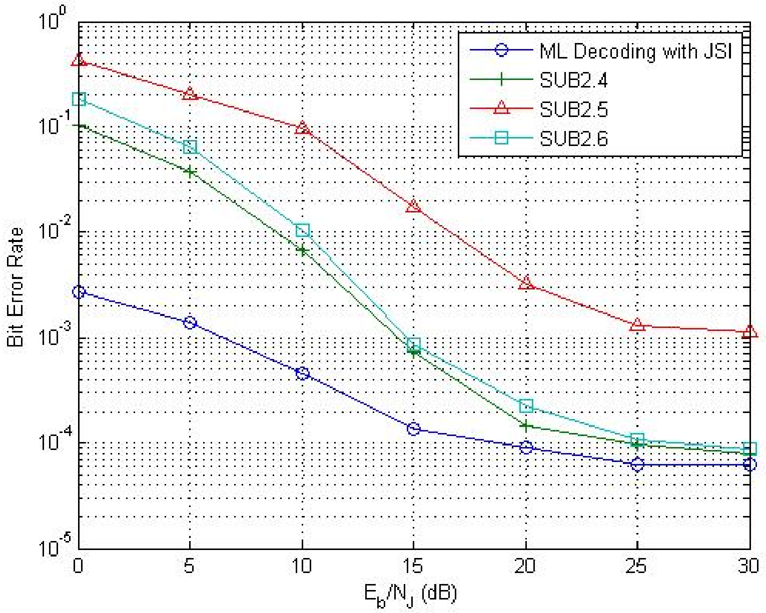

Figure 4.33: Performance plots of the STC/OFHSS without CSI for $\rho = 0.2$ and $E_b/N_0 = 15$ dB.

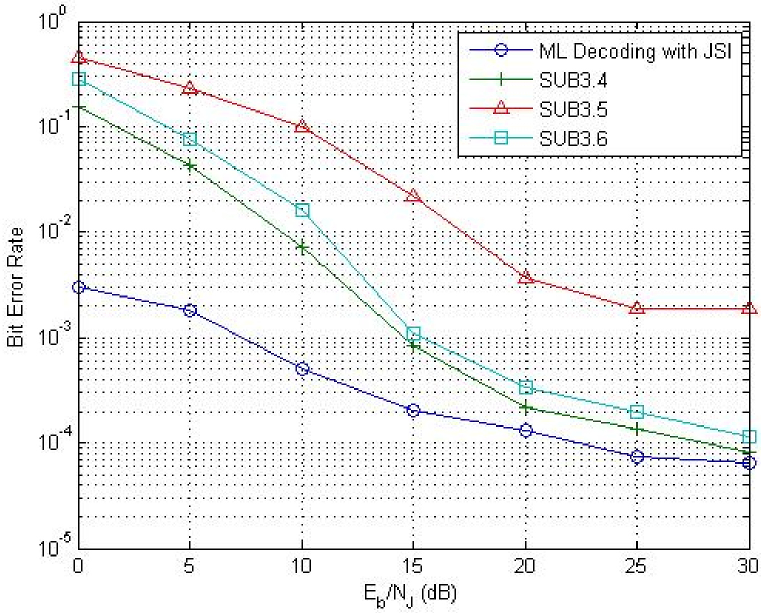

Figure 4.34: Performance plots of the STC/UFHSS without CSI for $\rho = 0.2$ and $E_b/N_0 = 15$ dB.

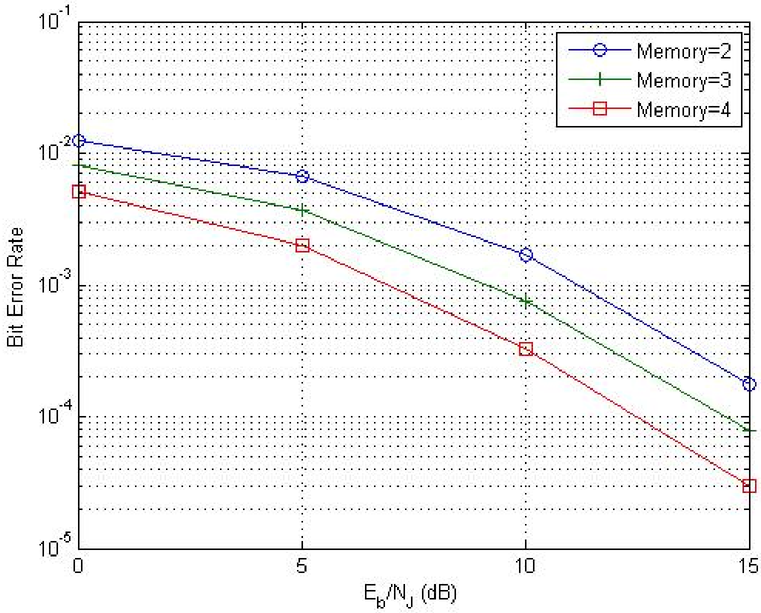

Figure 4.35: Performance of the STC/WFHSS with CSI and JSI available for $\rho = 0.2$,$E_b/N_0 = 15$ dB and 2 receiver antennas.

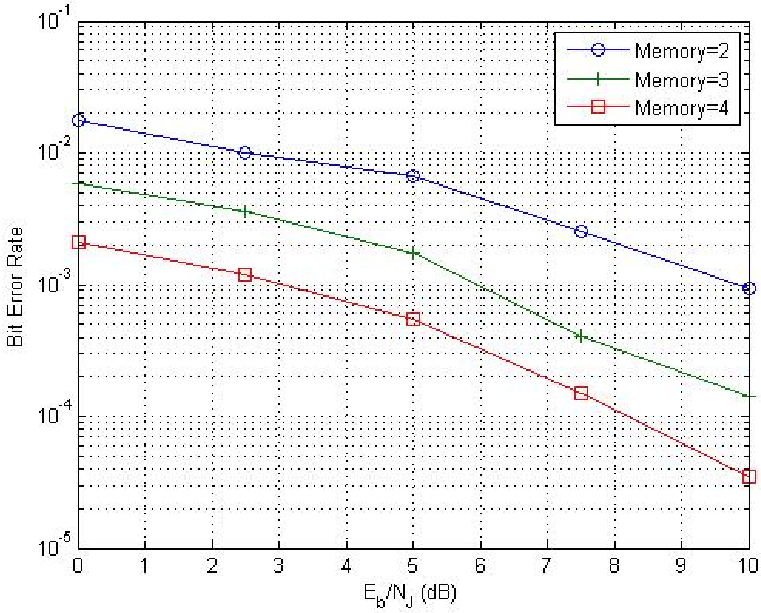

Figure 4.36: Performance of the STC/WFHSS with CSI and JSI available for $\rho = 0.2$, $E_b/N_0 = 15$ dB and 3 receiver antennas.

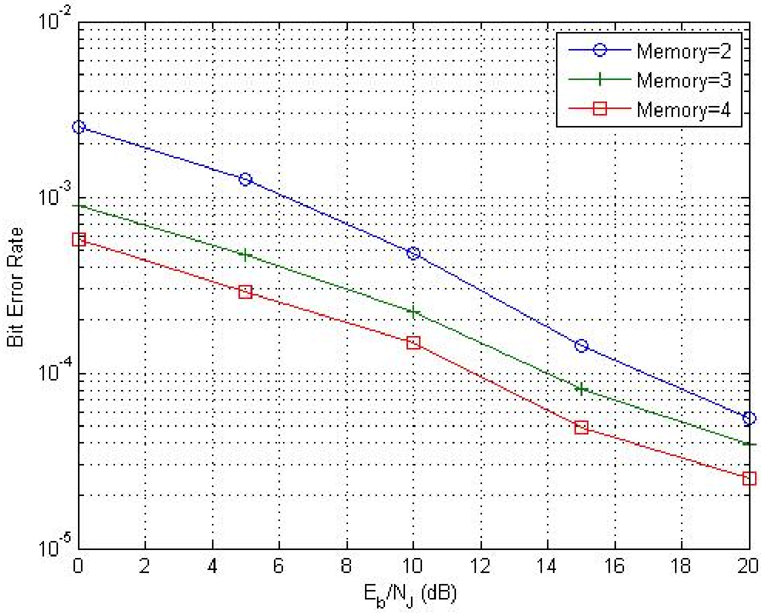

Figure 4.37: Performance of the STC/OFHSS with CSI and JSI available for $\rho = 0.2$,$E_b/N_0 = 15$ dB and 2 receiver antennas.

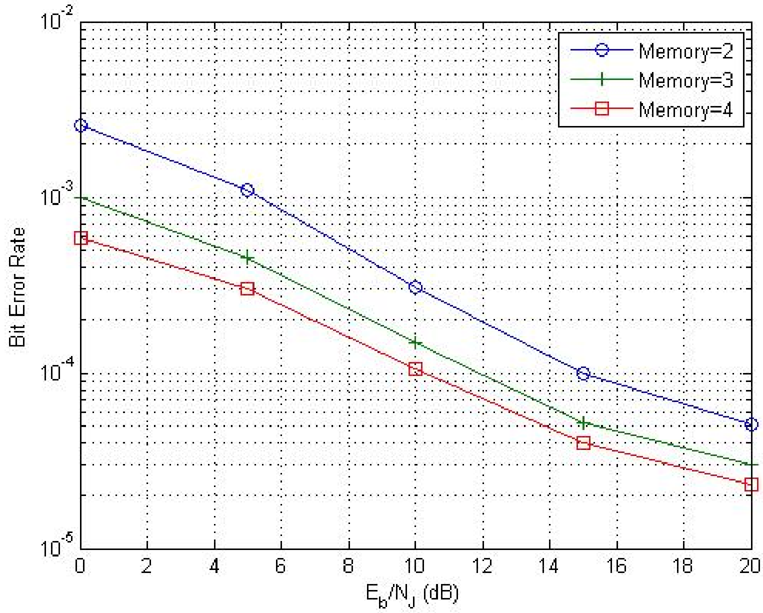

Figure 4.38: Performance of the STC/OFHSS with CSI and JSI available for $\rho = 0.2, E_b/N_0 = 15$ dB and 3 receiver antennas.

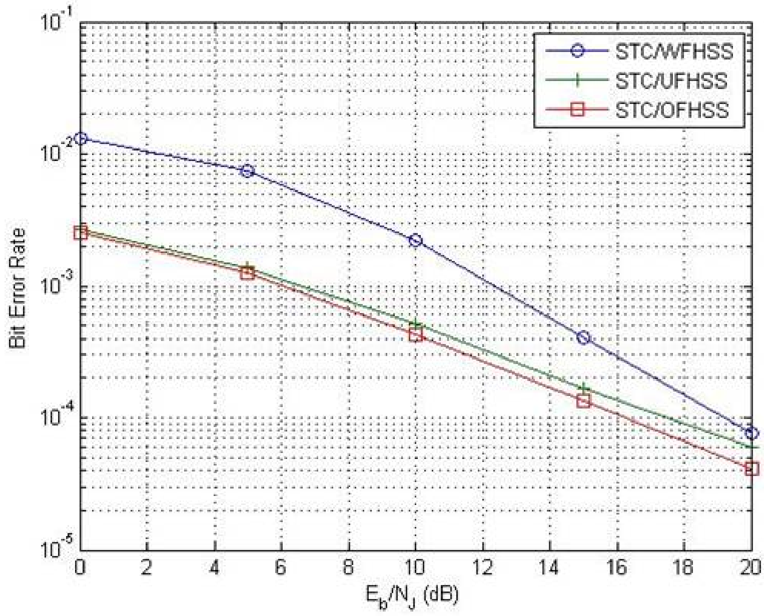

Figure 4.39: Performance of the STC/WFHSS, STC/UFHSS, and STC/OFHSS systems with memory=2 and CSI available for $\rho = 0.2$ and $E_b/N_0 = 15$ dB.

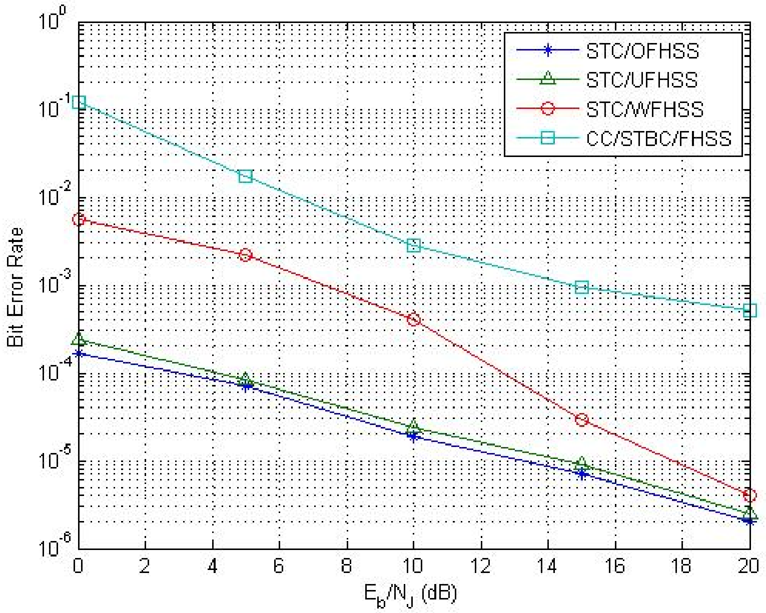

Figure 4.40: Performance of the STC/WFHSS, STC/UFHSS, STC/OFHSS, CC/STBC/FHSS systems for $\rho = 0.2$ and $E_b/N_0 = 20dB$.

Chapter 5

Conclusions

In this thesis, we investigate issues related to the performance of the STC/FHSS system in wireless PBNJ environments. There are three types of STC/FHSS systems we proposed for performance evaluation; STC/WFHSS system, STC/OFHSS system, and STC/UFHSS system. The actual performance of the proposed STC/FHSS system with arbitrary hopping patterns can then be lower and upper bounded by the evaluated performance of the STC/WFHSS and STC/OFHSS systems, respectively. In addition, the performance of the STC/UFHSS system could approach to the upper bounded performance at large frequency hopping bands and few transmitter antennas.

Based on these three proposed system model, the corresponding ML decoding is derived no matter whether CSI is available or not. Althought the ML decoding with respect to different reception conditions has been derived, however, the decoding complexity of the optimal decodings are too high and might limit its feasibility in practical applications. Besides, the complicated arithmetic of the ML decoding with CSI unknown not only repuires high computational complexity but also excludes the use of the efficient Viterbi algorithm. To optimize the trade-off between decoding complexity and performance, there are several suboptimal decoding schemes: SUB1.1-SUB1.6 for STC/WFHSS system, SUB2.1-SUB2.6 for STC/OFHSS system, and SUB3.1-SUB3.6 for STC/UFHSS system are proposed with acceptable BER performance. We also proposed two design criteria for constructing good space-time codes with respect to the wireless channels in the PBNJ environments. One of the design criteria is presented for the case of low diversity, and the other is for the case of high diversity. Good spacd-time codes are then given via a computer search. Verified by the simulation results, our system outperforms the conventional coding scheme with SISO channel coding in terms of both bandwidth efficiency and signal-to-noise ratio.

Althought we have presented the three types of STC/FHSS systems for wireless jamming channels, there are several related issues that remain to be investigated. The coding scheme we consider in the proposed system is STC techniques. We could consider other multi-input and multi-output coding schemes, e.g., differential space-time coding and layered space-time coding, for further error correction in wireless jamming channels. Besides, the frequency hopping we used is slow frequency hopping. Fast frequency hopping could also be considered for high frequency diversity gain. Finally, partial-band noise jamming and multitone noise jamming are the two most effective jamming strategies for inverestigation. Therefore, we may consider the system we proposed used in the multitone jamming environments to improve the system performance.

Appendix A

Independent of the Jamming Noise in Time, Frequency, and Space Domains

Assume $\eta_q(t)$ is the AWGN of the qth receiver antenna at time t with zero mean and variance $\frac{N_0}{2}$. The power spectral density $S_\eta(f)$ and autocorrelation $R_\eta(\tau)$ of $\eta(t)$ are

$$S_\eta(f) = \frac{N_0}{2}, \qquad R_\eta(\tau) = \frac{N_0}{2}\delta(\tau). \tag{A.1}$$

Figure A.1 shows the MFSK detector in the qth receiver antenna where T_s is the symbol duration and $f_k(t)$'s $\forall\ 1 \le k \le M$ are the orthonormal bases. Here we show the noise is independent in time, frequency, and space domains.

(1) Noise independent in time domain:

$$\begin{aligned} E\left[\eta_{q,t_1}^k \eta_{q,t_2}^k\right] &= E\left[\int_0^{T_s}\int_0^{T_s} \eta_q(t_1)\,\eta_q(t_2)\, f_k(t_1)\, f_k(t_2)\, dt_1 dt_2\right] \\ &= \int_0^{T_s}\int_0^{T_s} E\left[\eta_q(t_1)\,\eta_q(t_2)\right] f_k(t_1)\, f_k(t_2)\, dt_1 dt_2 \\ &= \frac{N_0}{2}\int_0^{T_s}\int_0^{T_s} \delta(\tau)\, f_k(t_1)\, f_k(t_2)\, dt_1 dt_2 \end{aligned} \tag{A.2}$$

where $t_2 = t_1 + \tau$, and $E[\cdot]$ denotes the expectation. If $\tau \neq 0$, we can get $E\left[\eta_q(t_1)\,\eta_q(t_1)\right] = 0$. That is, η_{q,t_1}^k and η_{q,t_2}^k are uncorrelated. Therefore, $\eta_{q,t}^k$'s are independent in time domain as they are assume to be Gaussian noise.

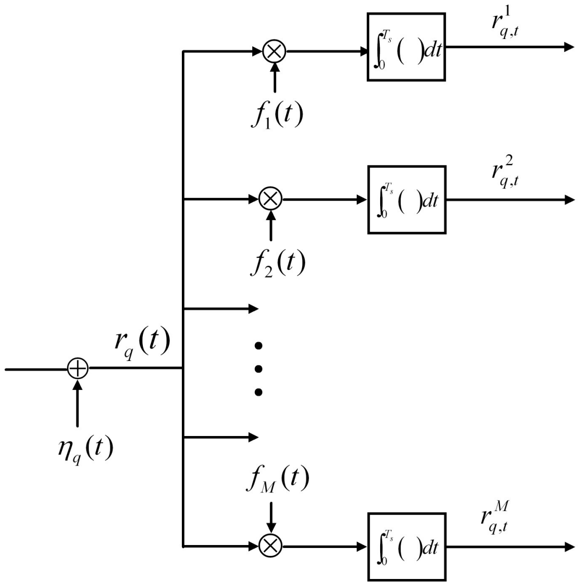

Figure A.1: MFSK detector.

(2) Noise independent in frequency domain:

$$\begin{aligned}
E\left[\eta_{q,t}^{k_1}\eta_{q,t}^{k_2}\right] &= E\left[\int_0^{T_s}\int_0^{T_s}\eta_q(t)\,\eta_q(\tau)\,f_{k_1}(t)\,f_{k_2}(\tau)\,dtd\tau\right] \\
&= \int_0^{T_s}\int_0^{T_s} E\left[\eta_q(t)\,\eta_q(\tau)\right] f_{k_1}(t)\,f_{k_2}(\tau)\,dtd\tau \\
&= \frac{N_0}{2}\int_0^{T_s}\delta(t-\tau)\,f_{k_1}(t)\,f_{k_2}(\tau)\,dtd\tau \\
&= \frac{N_0}{2}\int_0^{T_s} f_{k_1}(t)\,f_{k_2}(t)\,dt
\end{aligned} \tag{A.3}$$

where k_1 and k_2 $\forall\ 1 \leq k_1, k_2 \leq M, k_1 \neq k_2$ are denoted different frequency slots. The $f_{k_1}(t)$ and f_{k_2} are both orthonormal bases, and hence we can get

$$\int_0^{T_s} f_{k_1}(t)\,f_{k_2}(t)\,dt = 0. \tag{A.4}$$

Therefore, $E\left[\eta_{q,t}^{k_1}\eta_{q,t}^{k_2}\right] = 0$, and $\eta_{q,t}^{k}$'s are independent in frequency domain.

(3) Noise independent in space domain:

$$\begin{aligned} E\left[\eta_{q_1,t}^k \eta_{q_2,t}^k\right] &= E\left[\int_0^{T_s}\int_0^{T_s} \eta_{q_1}(t)\,\eta_{q_2}(\tau)\,f_k(t)\,f_k(\tau)\,dtd\tau\right] \\ &= \int_0^{T_s}\int_0^{T_s} E\left[\eta_{q_1}(t)\,\eta_{q_2}(\tau)\right] f_k(t)\,f_k(\tau)\,dtd\tau \\ &= \int_0^{T_s}\int_0^{T_s} E\left[\eta_{q_1}(t)\right]\cdot E\left[\eta_{q_2}(\tau)\right] f_{k_1}(t)\,f_{k_2}(\tau)\,dtd\tau \\ &= 0 \end{aligned} \tag{A.5}$$

where q_1 and q_2 $\forall\, 1 \le q_1, q_2 \le m, q_1 \ne q_2$ are denoted different receiver antennas. Therefore, $\eta_{q,t}^k$'s are independent in space domain due to $E\left[\eta_{q_1,t}^k \eta_{q_2,t}^k\right] = 0$.

Appendix B

Derivation of the ML Decoding of STC/WFHSS Systems without CSI

The derived in (4.23) is discussed in this Appendix by averaging $\alpha_{i,q}$'s with respect to the conditional probability density function defined in (4.19), and we can get

$$
\begin{aligned}
&\int_{-\infty}^{\infty} f(\boldsymbol{r}|\hat{\boldsymbol{s}}, \boldsymbol{x}, \boldsymbol{\alpha}) f(\boldsymbol{\alpha}) d\boldsymbol{\alpha} \\
&= \left\{ \prod_{t=1}^{L} \prod_{k=1}^{M} (a_{1,x_t}^{m}) \exp\left(-\sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} a_{2,x_t} |r_{q,t}^{k}|^2 \right) \right\} \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp\left\{ \sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} a_{2,x_t} \right. \\
&\quad \left[2\mathrm{Re}\left(r_{q,t}^{k} \sum_{i=1}^{n} \alpha_{R,i,q} \hat{s}_{i,t}^{k} \right) + 2\mathrm{Im}\left(r_{q,t}^{k} \sum_{i=1}^{n} \alpha_{I,i,q} \hat{s}_{i,t}^{k} \right) - \left(\sum_{i=1}^{n} \alpha_{R,i,q}^2 |\hat{s}_{i,t}^{k}|^2 \right. \right. \\
&\quad \left. \left. \left. + \sum_{i=1}^{n} \alpha_{I,i,q}^2 |\hat{s}_{i,t}^{k}|^2 + \sum_{\substack{i=1 \\ i\neq l}}^{n} \sum_{\substack{l=1 \\ l\neq i}}^{n} \alpha_{R,i,q} \alpha_{R,l,q} \hat{s}_{i,t}^{k} \hat{s}_{l,t}^{k} + \sum_{\substack{i=1 \\ i\neq l}}^{n} \sum_{\substack{l=1 \\ l\neq i}}^{n} \alpha_{I,i,q} \alpha_{I,l,q} \hat{s}_{i,t}^{k} \hat{s}_{l,t}^{k} \right) \right] \right\} \\
&\quad \cdot \frac{1}{\sqrt{2\pi}\sigma_{1,1}} \exp\left(-\frac{\alpha_{R,1,1}^2}{2\sigma_{1,1}^2} \right) \cdots \frac{1}{\sqrt{2\pi}\sigma_{n,m}} \exp\left(-\frac{\alpha_{R,n,m}^2}{2\sigma_{n,m}^2} \right) \cdot \frac{1}{\sqrt{2\pi}\sigma_{1,1}} \exp\left(-\frac{\alpha_{I,1,1}^2}{2\sigma_{1,1}^2} \right) \cdots \\
&\quad \frac{1}{\sqrt{2\pi}\sigma_{n,m}} \exp\left(-\frac{\alpha_{I,n,m}^2}{2\sigma_{n,m}^2} \right) d\alpha_{R,1,1} \cdots d\alpha_{R,n,m} \cdot d\alpha_{I,1,1} \cdots d\alpha_{I,n,m}
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\prod_{t=1}^{L}\prod_{k=1}^{M} a_{1,x_t}^{m}\right)\cdot\left(\prod_{q=1}^{m}\prod_{i=1}^{n}\frac{1}{2\pi\sigma_{i,q}^{2}}\right)\cdot\exp\left(-\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m} a_{2,x_t}|r_{q,t}^{k}|^{2}\right)\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \\
&\exp\Bigg\{\sum_{q=1}^{m}\Bigg[\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\Bigg[2\mathrm{Re}\left(r_{q,t}^{k}\sum_{i=1}^{n}\alpha_{R,i,q}\hat{s}_{i,t}^{k}\right)+2\mathrm{Im}\left(r_{q,t}^{k}\sum_{i=1}^{n}\alpha_{I,i,q}\hat{s}_{i,t}^{k}\right) \\
&-\Bigg(\sum_{i=1}^{n}\alpha_{R,i,q}^{2}|\hat{s}_{i,t}^{k}|^{2}+\sum_{i=1}^{n}\alpha_{I,i,q}^{2}|\hat{s}_{i,t}^{k}|^{2}+\sum_{\substack{i=1\\ i\neq l}}^{n}\sum_{\substack{l=1\\ l\neq i}}^{n}\alpha_{R,i,q}\alpha_{R,l,q}\hat{s}_{i,t}^{k}\hat{s}_{l,t}^{k} \\
&+\sum_{\substack{i=1\\ i\neq l}}^{n}\sum_{\substack{l=1\\ l\neq i}}^{n}\alpha_{I,i,q}\alpha_{I,l,q}\hat{s}_{i,t}^{k}\hat{s}_{l,t}^{k}\Bigg)\Bigg]\Bigg]-\sum_{i=1}^{n}\frac{\alpha_{R,i,q}^{2}}{2\sigma_{i,q}^{2}}-\sum_{i=1}^{n}\frac{\alpha_{I,i,q}^{2}}{2\sigma_{i,q}^{2}}\Bigg\}\, d\alpha_{R,1,1}\cdots dd\alpha_{I,n,m}. \qquad \text{(B.1)}
\end{aligned}
$$

For the real part of the exponent could be represented by $R(\alpha_{R,1,q},\alpha_{R,2,q},\cdots,\alpha_{R,n,q})$

$$
\begin{aligned}
&R(\alpha_{R,1,q},\alpha_{R,2,q},\cdots,\alpha_{R,n,q}) \\
&= 2\mathrm{Re}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left(a_{2,x_t}r_{q,t}^{k}\sum_{i=1}^{n}\alpha_{R,i,q}\hat{s}_{i,t}^{k}\right)\right)-\Bigg(\sum_{t=1}^{L}\sum_{k=1}^{M}\left(\sum_{i=1}^{n}a_{2,x_t}\alpha_{R,i,q}^{2}|\hat{s}_{i,t}^{k}|^{2}\right) \\
&\quad+\sum_{t=1}^{L}\sum_{k=1}^{M}\left(\sum_{\substack{i=1\\ i\neq l}}^{n}\sum_{\substack{l=1\\ l\neq i}}^{n}a_{2,x_t}\alpha_{R,i,q}\alpha_{R,l,q}\hat{s}_{i,t}^{k}\hat{s}_{l,t}^{k}\right)\Bigg)-\left(\sum_{i=1}^{n}\frac{\alpha_{R,i,q}^{2}}{2\sigma_{i,q}^{2}}\right) \\
&= 2\mathrm{Re}\left(\sum_{i=1}^{n}\alpha_{R,i,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}r_{q,t}^{k}\hat{s}_{i,t}^{k}\right)\right)-\Bigg(\sum_{i=1}^{n}\alpha_{R,i,q}^{2}\Bigg(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}|\hat{s}_{i,t}^{k}|^{2} \\
&\quad+\frac{1}{2\sigma_{i,q}^{2}}\Bigg)+\sum_{\substack{i=1\\ i\neq l}}^{n}\sum_{\substack{l=1\\ l\neq i}}^{n}\alpha_{R,i,q}\alpha_{R,l,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}\hat{s}_{i,t}^{k}\hat{s}_{l,t}^{k}\right)\Bigg) \\
&= \boldsymbol{B}_{R,q}\boldsymbol{\Lambda}_{R,q}^{T}-\boldsymbol{\Lambda}_{R,q}\left(\boldsymbol{A}+\boldsymbol{I}\right)\boldsymbol{\Lambda}_{R,q}^{T} \qquad \left(\sigma_{i,q}^{2}=\frac{1}{2}\ \forall\ i,q\right) \qquad \text{(B.2)}
\end{aligned}
$$

where

$$
\boldsymbol{\Lambda}_{R,q}=(\alpha_{R,1,q},\alpha_{R,2,q},\ldots,\alpha_{R,n,q})
$$

$$\begin{aligned}
\boldsymbol{B}_{R,q} &= 2\mathrm{Re}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} r_{q,t}^{k} \hat{s}_{1,t}^{k}\right), \left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} r_{q,t}^{k} \hat{s}_{2,t}^{k}\right), \ldots, \left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} r_{q,t}^{k} \hat{s}_{n,t}^{k}\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} \left[2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{1,t}^{k}\right), 2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{2,t}^{k}\right), \ldots, 2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{n,t}^{k}\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} \boldsymbol{B}_{R,q,t}^{k},
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{A} &= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} \begin{bmatrix} |\hat{s}_{1,t}^{k}|^2 & \hat{s}_{1,t}^{k}\hat{s}_{2,t}^{k} & \cdots & \hat{s}_{1,t}^{k}\hat{s}_{n,t}^{k} \\ \hat{s}_{2,t}^{k}\hat{s}_{1,t}^{k} & |\hat{s}_{2,t}^{k}|^2 & \cdots & \hat{s}_{2,t}^{k}\hat{s}_{n,t}^{k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{s}_{n,t}^{k}\hat{s}_{1,t}^{k} & \hat{s}_{n,t}^{k}\hat{s}_{2,t}^{k} & \cdots & |\hat{s}_{n,t}^{k}|^2 \end{bmatrix} \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} \boldsymbol{A}_{t}^{k}.
\end{aligned}$$

and I is an identity matrix. $\boldsymbol{B}_{R,q,t}^{k}$ and $\boldsymbol{A}_{t}^{k}$ are defined as

$$\boldsymbol{B}_{R,q,t}^{k} = \left[2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{1,t}^{k}\right), 2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{2,t}^{k}\right), \ldots, 2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{n,t}^{k}\right)\right] \tag{B.3}$$

$$\boldsymbol{A}_{t}^{k} = \begin{bmatrix} |\hat{s}_{1,t}^{k}|^2 & \hat{s}_{1,t}^{k}\hat{s}_{2,t}^{k} & \cdots & \hat{s}_{1,t}^{k}\hat{s}_{n,t}^{k} \\ \hat{s}_{2,t}^{k}\hat{s}_{1,t}^{k} & |\hat{s}_{2,t}^{k}|^2 & \cdots & \hat{s}_{2,t}^{k}\hat{s}_{n,t}^{k} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{s}_{n,t}^{k}\hat{s}_{1,t}^{k} & \hat{s}_{n,t}^{k}\hat{s}_{2,t}^{k} & \cdots & |\hat{s}_{n,t}^{k}|^2 \end{bmatrix}. \tag{B.4}$$

It is clear that $\boldsymbol{A}_{t}^{k}$ is nonnegative definite Hermitian, and the eigenvalues of $\boldsymbol{A}_{t}^{k}$ are nongative real numbers. Therefore, we can get

$$\boldsymbol{V}_{t}^{k}\boldsymbol{A}_{t}^{k}\boldsymbol{V}_{t}^{k^H} = \boldsymbol{D}_{t}^{k} \tag{B.5}$$

where $\boldsymbol{V}_{t}^{k}$ is a unitary matrix and $\boldsymbol{D}_{t}^{k}$ is a real diagonal matrix. The rows of $\boldsymbol{V}_{t}^{k}$, forming a complete orthonormal basis of an N-dimensional vector space, are the eigenvectors of $\boldsymbol{A}_{t}^{k}$. The diagonal elements of $\boldsymbol{D}_{t}^{k}$ are the eigenvalues $\lambda_{i,t}^{k} \geq 0,\ \forall\ 1 \leq \mathrm{i} \leq n$. Equation (B.2) can be rewritten as

$$
\begin{aligned}
& R(\alpha_{R,1,q}, \alpha_{R,2,q}, \ldots, \alpha_{R,n,q}) \\
&= \boldsymbol{B}_{R,q}\boldsymbol{\Lambda}_{R,q}^T - \boldsymbol{\Lambda}_{R,q}\left(\boldsymbol{A}+\boldsymbol{I}\right)\boldsymbol{\Lambda}_{R,q}^T \\
&= \left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\boldsymbol{B}_{R,q,t}^k\right)\boldsymbol{\Lambda}_{R,q}^T - \boldsymbol{\Lambda}_{R,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\boldsymbol{A}_t^k + \boldsymbol{I}\right)\boldsymbol{\Lambda}_{R,q}^T \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{B}_{R,q,t}^k\boldsymbol{\Lambda}_{R,q}^T - \boldsymbol{\Lambda}_{R,q}\left(\boldsymbol{A}_t^k + \frac{1}{a_{2,x_t}LM}\boldsymbol{I}\right)\boldsymbol{\Lambda}_{R,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{B}_{R,q,t}^k\left(\boldsymbol{V}_t^k\boldsymbol{V}_t^{kT}\right)\boldsymbol{\Lambda}_{R,q}^T - \boldsymbol{\Lambda}_{R,q}\left(\boldsymbol{V}_t^k\boldsymbol{D}_t^k\boldsymbol{V}_t^{kT} + \frac{1}{a_{2,x_t LM}}\boldsymbol{I}\right)\boldsymbol{\Lambda}_{R,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{B}_{R,q,t}^k\boldsymbol{V}_t^k\left(\boldsymbol{V}_t^{kT}\boldsymbol{\Lambda}_{R,q}^T\right) - \left(\boldsymbol{\Lambda}_{R,q}\boldsymbol{V}_t^k\right)\left(\boldsymbol{D}_t^k + \frac{1}{a_{2,x_t LM}}\boldsymbol{I}\right)\left(\boldsymbol{V}_t^{kT}\boldsymbol{\Lambda}_{R,q}^T\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{Z}_{q,t}^k\boldsymbol{Y}_{R,q}^T - \boldsymbol{Y}_{R,q}\left(\boldsymbol{D}_t^k + \frac{1}{a_{2,x_t LM}}\boldsymbol{I}\right)\boldsymbol{Y}_{R,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\sum_{i=1}^{n} z_{i,q,t}^k y_{R,i,q} - \sum_{i=1}^{n}\left(\lambda_{i,t}^k + \frac{1}{a_{2,x_t LM}}\right) y_{R,i,q}^2\right] \\
&= \sum_{i=1}^{n}\left[\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left(z_{i,q,t}^k y_{R,i,q} - \left(\lambda_{i,t}^k + \frac{1}{a_{2,x_t}LM}\right) y_{R,i,q}^2\right)\right]
\end{aligned} \tag{B.6}
$$

where

$$
\begin{aligned}
\boldsymbol{Z}_{q,t}^k &= \boldsymbol{B}_{R,q,t}^k\boldsymbol{V}_t^k \\
&= \left(2\mathrm{Re}\left(r_{q,t}^k\hat{s}_{1,t}^k\right),\ 2\mathrm{Re}\left(r_{q,t}^k\hat{s}_{2,t}^k\right),\ \ldots,\ 2\mathrm{Re}\left(r_{q,t}^k\hat{s}_{n,t}^k\right)\right)V_t^k \\
&= \left(z_{1,q,t}^k,\ z_{2,q,t}^k,\ \ldots,\ z_{n,q,t}^k\right)
\end{aligned}
$$

and

$$
\begin{aligned}
\boldsymbol{Y}_{R,q} &= \boldsymbol{\Lambda}_{R,q}\boldsymbol{V}_t^k \\
&= \left(\alpha_{R,1,q},\ \alpha_{R,2,q},\ \ldots,\ \alpha_{R,n,q}\right)\boldsymbol{V}_t^k \\
&= \left(y_{R,1,q},\ y_{R,2,q},\ \ldots,\ y_{R,n,q}\right).
\end{aligned}
$$

By equation $(B.1)$, we average $\alpha_{R,i,q}$ for the real part of the exponent

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}(z_{i,q,t}^{k} y_{R,i,q} - \left(\lambda_{i,t}^{k} + \frac{1}{a_{2,x_t}LM}\right) y_{R,i,q}^{2})\right)\right] dy_{R,1,1}\cdots dy_{R,n,m}$$

$$= \sqrt{\frac{\pi^{nm}}{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M} a_{2,x_t}\lambda_{i,t}^{k}+1\right)}}\exp\left(\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{a_{2,x_t}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M} a_{2,x_t} z_{i,q,t}^{k}\right)^{2}}{4\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M} a_{2,x_t}\lambda_{i,t}^{k}+1}\right). \tag{B.7}$$

$$\left(\int_{-\infty}^{\infty}\exp\{-(ax^2+bx+c)\}dx = \sqrt{\frac{\pi}{a}}\exp\left\{\frac{b^2-4ac}{4a}\right\}\right)$$

Next, the imaginary part of the exponent in $(B.1)$ are defined by $I(\alpha_{I,1,q},\alpha_{I,2,q},\cdots,\alpha_{I,n,q})$

$$\begin{aligned}
& I(\alpha_{I,1,q},\alpha_{I,2,q},\cdots,\alpha_{I,n,q}) \\
&= 2\mathrm{Im}\left(\sum_{i=1}^{n}\alpha_{R,i,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} r_{q,t}^{k}\hat{s}_{i,t}^{k}\right)\right) - \left(\sum_{i=1}^{n}\alpha_{R,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}|\hat{s}_{i,t}^{k}|^{2}\right.\right. \\
&\quad \left.\left. + \frac{1}{2\sigma_{i,q}^{2}}\right) + \sum_{\substack{i=1\\ i\neq l}}^{n}\sum_{\substack{l=1\\ l\neq i}}^{n}\alpha_{R,i,q}\alpha_{R,l,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\hat{s}_{i,t}^{k}\hat{s}_{l,t}^{k}\right)\right) \\
&= \boldsymbol{B}_{I,q}\boldsymbol{\Lambda}_{I,q}^{T} - \boldsymbol{\Lambda}_{I,q}\left(\boldsymbol{A}+\boldsymbol{I}\right)\boldsymbol{\Lambda}_{I,q}^{T}
\end{aligned} \tag{B.8}$$

where

$$\boldsymbol{\Lambda}_{I,q} = (\alpha_{I,1,q},\alpha_{I,2,q},\ldots,\alpha_{I,n,q})$$

$$\begin{aligned}
\boldsymbol{B}_{I,q} &= 2\mathrm{Im}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} r_{q,t}^{k}\hat{s}_{1,t}^{k}\right),\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} r_{q,t}^{k}\hat{s}_{2,t}^{k}\right),\ldots,\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t} r_{q,t}^{k}\hat{s}_{n,t}^{k}\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[2\mathrm{Im}\left(r_{q,t}^{k}\hat{s}_{1,t}^{k}\right), 2\mathrm{Im}\left(r_{q,t}^{k}\hat{s}_{2,t}^{k}\right),\ldots, 2\mathrm{Im}\left(r_{q,t}^{k}\hat{s}_{n,t}^{k}\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\boldsymbol{B}_{I,q,t}^{k}.
\end{aligned}$$

Equation $(B.8)$ can be written as

$$
\begin{aligned}
& I(\alpha_{I,1,q}, \alpha_{I,2,q}, \ldots, \alpha_{I,n,q}) \\
&= \boldsymbol{B}_{I,q}\boldsymbol{\Lambda}_{I,q}^T - \boldsymbol{\Lambda}_{I,q}\left(\boldsymbol{A}+\boldsymbol{I}\right)\boldsymbol{\Lambda}_{I,q}^T \\
&= \left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\boldsymbol{B}_{I,q,t}^k\right)\boldsymbol{\Lambda}_{I,q}^T - \boldsymbol{\Lambda}_{I,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\boldsymbol{A}_t^k + \boldsymbol{I}\right)\boldsymbol{\Lambda}_{I,q}^T \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{B}_{I,q,t}^k\boldsymbol{\Lambda}_{I,q}^T - \boldsymbol{\Lambda}_{I,q}\left(\boldsymbol{A}_t^k + \frac{1}{a_{2,x_t}LM}\boldsymbol{I}\right)\boldsymbol{\Lambda}_{I,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{B}_{I,q,t}^k\left(\boldsymbol{V}_t^k\boldsymbol{V}_t^{kT}\right)\boldsymbol{\Lambda}_{I,q}^T - \boldsymbol{\Lambda}_{I,q}\left(\boldsymbol{V}_t^k\boldsymbol{D}_t^k\boldsymbol{V}_t^{kT} + \frac{1}{a_{2,x_t}LM}\boldsymbol{I}\right)\boldsymbol{\Lambda}_{I,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{B}_{I,q,t}^k\boldsymbol{V}_t^k(\boldsymbol{V}_t^{kT}\boldsymbol{\Lambda}_{I,q}^T) - (\boldsymbol{\Lambda}_{I,q}\boldsymbol{V}_t^k)\left(\boldsymbol{D}_t^k + \frac{1}{a_{2,x_t}LM}\boldsymbol{I}\right)(\boldsymbol{V}_t^{kT}\boldsymbol{\Lambda}_{I,q}^T)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\boldsymbol{W}_{q,t}^k\boldsymbol{Y}_{I,q}^T - \boldsymbol{Y}_{I,q}\left(\boldsymbol{D}_t^k + \frac{1}{a_{2,x_t}LM}\boldsymbol{I}\right)\boldsymbol{Y}_{I,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left[\sum_{i=1}^{n} w_{i,q,t}^k y_{I,i,q} - \sum_{i=1}^{n}\left(\lambda_{i,t}^k + \frac{1}{a_{2,x_t}LM}\right)y_{I,i,q}^2\right] \\
&= \sum_{i=1}^{n}\left[\sum_{t=1}^{L}\sum_{k=1}^{M} a_{2,x_t}\left(w_{i,q,t}^k y_{I,i,q} - \left(\lambda_{i,t}^k + \frac{1}{a_{2,x_t}LM}\right)y_{I,i,q}^2\right)\right] \qquad \text{(B.9)}
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{W}_{q,t}^k &= \boldsymbol{B}_{I,q,t}^k\boldsymbol{V}_t^k \\
&= (2\mathrm{Im}(r_{q,t}^k\hat{s}_{1,t}^k),\ 2\mathrm{Im}(r_{q,t}^k\hat{s}_{2,t}^k),\ \ldots,\ 2\mathrm{Im}(r_{q,t}^k\hat{s}_{n,t}^k))V_t^k \\
&= (w_{1,q,t}^k,\ w_{2,q,t}^k,\ \ldots,\ w_{n,q,t}^k) \qquad \text{(B.10)}
\end{aligned}
$$

and

$$
\begin{aligned}
\boldsymbol{Y}_{I,q} &= \boldsymbol{\Lambda}_{I,q}\boldsymbol{V}_t^k \\
&= (\alpha_{I,1,q},\ \alpha_{I,2,q},\ \ldots,\ \alpha_{I,n,q})\boldsymbol{V}_t^k \\
&= (y_{I,1,q},\ y_{I,2,q},\ \ldots,\ y_{I,n,q}). \qquad \text{(B.11)}
\end{aligned}
$$

By equation $(B.1)$, we average $\alpha_{R,i,q}$ for the imaginary part of the exponent

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}(w_{i,q,t}^{k}y_{I,i,q}-\left(\lambda_{i,t}^{k}+\frac{1}{a_{2,x_t}LM}\right)y_{I,i,q}^{2})\right)\right]dy_{I,1,1}\cdots dy_{I,n,m}$$

$$=\sqrt{\frac{\pi^{nm}}{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}\lambda_{i,t}^{k}+1\right)}}\exp\left(\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{a_{2,x_t}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}w_{i,q,t}^{k}\right)^{2}}{4\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}\lambda_{i,t}^{k}+1}\right) \tag{B.12}$$

Hence, the probability density function $f\left(\boldsymbol{r}\left|\hat{\boldsymbol{s}},\boldsymbol{x}\right.\right)$ conditioned on JSI available can be expressed as

$$\left(\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}a_{1,x_t}\cdot\exp-a_{2,x_t}|r_{q,t}^{k}|^{2}\right)\cdot\left(\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}\lambda_{i,t}^{k}+1\right)\right)^{-1}$$
$$\cdot\exp\left(\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left[\left|\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}z_{i,q,t}^{k}\right|^{2}+\left|\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}w_{i,q,t}^{k}\right|^{2}\right]}{4\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}\lambda_{i,t}^{k}+1}\right). \tag{B.13}$$

A closed-form expression of the decoding metric is derived as

$$f(\boldsymbol{r}|\boldsymbol{s},\boldsymbol{x})$$
$$=\left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\left[a_{1,x_t}\cdot\exp\left(-a_{2,x_t}|r_{q,t}^{k}|^{2}\right)\right]\right\}\cdot\left\{\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}\lambda_{i,t}^{k}+1\right)^{-1}\right.\right.$$
$$\left.\left.\cdot\exp\left(\frac{\left|\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}u_{i,q,t}^{k}\right|^{2}}{\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_t}\lambda_{i,t}^{k}+1}\right)\right]\right\} \tag{B.14}$$

where

$$\left|\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}u_{i,q,t}^{k}\right|^{2}=\left|\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}z_{i,q,t}^{k}\right|^{2}+\left|\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_t}w_{i,q,t}^{k}\right|^{2}. \tag{B.15}$$

Appendix C

Derivation of the Design Criteria of STC/WFHSS Systems

The derived results of the pairwise error probability in (4.34) and (4.37) are discussed in this Appendix, respectively. First of all, by averaging x_t's with respect to the conditional pairwise error probability defined in (4.33), and then we can get

$$
\begin{aligned}
&\mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \alpha_{i,q}\ \forall\ i, q\right) \\
&= \sum_{x_1} \cdots \sum_{x_L} \mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \alpha_{i,q}, x_t\ \forall\ i, q, t\right) \mathrm{P_r}\left(x_1\right) \cdots P_r\left(x_L\right) \\
&\leq \sum_{x_1} \cdots \sum_{x_L} \frac{1}{2} \exp\left(-\sum_{t=1}^{L} a_{x_t} d_t^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})\right) \mathrm{P_r}\left(x_1\right) \cdots \mathrm{P_r}\left(x_L\right) \\
&= \frac{1}{2} \prod_{t=1}^{L} \left\{\rho \exp\left(-\frac{d_t^2(s, \tilde{s})}{4(N_0 + N_J/\rho)}\right) + (1-\rho) \exp\left(-\frac{d_t^2(s, \tilde{s})}{4N_0}\right)\right\} \\
&= \frac{1}{2} \prod_{t=1}^{L} \left\{\frac{\rho}{(1-\rho)} \exp\left(\frac{N_J/\rho}{4(N_0 + N_J/\rho)} d_t^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})\right) + 1\right\} \cdot \prod_{t=1}^{L} \left\{(1-\rho) \exp\left(-\frac{1}{4N_0} d_t^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})\right)\right\} \\
&\cong \frac{1}{2} \left\{\prod_{t=1}^{L} \left[\frac{\rho}{(1-\rho)} \exp\left(B_t\right)\right] + \sum_{t'=1}^{L} \left[\prod_{\substack{t=1 \\ t \neq t'}}^{L} \frac{\rho}{(1-\rho)} \exp\left(B_t\right)\right]\right\} \cdot C
\end{aligned}
\tag{C.1}
$$

where

$$d_t^2(s,\tilde{s}) = \sum_{q=1}^{m}\sum_{k=1}^{M}\left|\sum_{i=1}^{n}\alpha_{i,q}(s_{i,t}^k - \tilde{s}_{i,t}^k)\right|^2$$

$$B_t = \frac{N_J/\rho}{4(N_0+N_J/\rho)}\sum_{q=1}^{m}\sum_{k=1}^{M}\left|\sum_{i=1}^{n}\alpha_{i,q}(s_{i,t}^k - \tilde{s}_{i,t}^k)\right|^2$$

$$C = \prod_{t=1}^{L}\left\{(1-\rho)\exp\left(-\frac{\sum_{q=1}^{m}\sum_{k=1}^{M}\left|\sum_{i=1}^{n}\alpha_{i,q}(s_{i,t}^k - \tilde{s}_{i,t}^k)\right|^2}{4N_0}\right)\right\}.$$

The pairwise error probability can be obtained by averaging (C.1) with respect to $\alpha_{i,q}$'s, i.e.,

$$\begin{aligned}\mathrm{P_r}\left(\boldsymbol{s}\longrightarrow\tilde{\boldsymbol{s}}\right) &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}P_r\left(\boldsymbol{s}\longrightarrow\tilde{\boldsymbol{s}}|\boldsymbol{\alpha}\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}\\ &= X+Y \end{aligned}\tag{C.2}$$

where

$$\begin{aligned} X &= \frac{1}{2}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\sum_{t'=1}^{L}\left[\prod_{\substack{t=1\\t\neq t'}}^{L}\frac{\rho}{(1-\rho)}\exp\left(B_t\right)\right]\cdot C\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}\\ Y &= \frac{1}{2}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\prod_{t=1}^{L}\left[\frac{\rho}{(1-\rho)}\exp\left(B_t\right)\right]\cdot C\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}. \end{aligned}$$

The derivation of Y is similar to that of X. Therefore, we only discuss the derivation of X in this Appendix. Replace the values of B_t and C in X, and we can get

$$X = \frac{1}{2}\frac{\rho^{L-1}}{(1-\rho)^{-1}}\sum_{t'=1}^{L}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\exp\left(-a_1\sum_{q=1}^{m}\left(d_q^2\left(\boldsymbol{s},\tilde{\boldsymbol{s}}\right)\right)\right)\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}\tag{C.3}$$

where

$$\begin{aligned} a_1 &= \frac{1}{4\left(N_0+N_J/\rho\right)}\\ d_q^2\left(\boldsymbol{s},\tilde{\boldsymbol{s}}\right) &= \sum_{t=1}^{L}\sum_{k=1}^{M}\left|\sum_{i=1}^{n}\alpha_{i,q}\left(s_{i,t}^k-\tilde{s}_{i,t}^k\right)\right|^2 + \frac{N_J}{N_0\rho}\sum_{k=1}^{M}\left|\sum_{i=1}^{n}\alpha_{i,q}\left(s_{i,t'}^k-\tilde{s}_{i,t'}^k\right)\right|^2. \end{aligned}$$

The value of $d_q^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})$ can be rewritten as

$$
\begin{aligned}
& d_q^2(\boldsymbol{s}, \tilde{\boldsymbol{s}}) \\
&= \sum_{t=1}^{L} \sum_{k=1}^{M} \left| \sum_{i=1}^{n} \alpha_{i,q} (s_{i,t}^k - \tilde{s}_{i,t}^k) \right|^2 + \frac{N_J}{N_0 \rho} \sum_{k=1}^{M} \left| \sum_{i=1}^{n} \alpha_{i,q} (s_{i,t'}^k - \tilde{s}_{i,t'}^k) \right|^2 \\
&= \sum_{i=1}^{n} \sum_{i'=1}^{n} \alpha_{i,q} \alpha_{i',q}^* \left(\sum_{t=1}^{L} \sum_{k=1}^{M} \left(s_{i,t'}^k - \tilde{s}_{i,t'}^k\right) \left(s_{i',t'}^k - \tilde{s}_{i',t'}^k\right) + a_2 \sum_{k=1}^{M} \left(s_{i,t'}^k - \tilde{s}_{i,t'}^k\right) \left(s_{i',t'}^k - \tilde{s}_{i',t'}^k\right) \right) \\
&= \boldsymbol{\Lambda}_q \left(\sum_{t=1}^{L} \boldsymbol{A}_t + a_2 \boldsymbol{A}_{t'} \right) \boldsymbol{\Lambda}_q^T
\end{aligned} \tag{C.4}
$$

where $a_2 = \frac{N_J}{N_0 \rho}$, $\boldsymbol{\Lambda}_q = (\alpha_{1,q}, \alpha_{2,q}, \ldots, \alpha_{n,q})$ and $\boldsymbol{A_t}$ is denoted by the following matrix

$$
\begin{pmatrix}
\sum_{k=1}^{M} \left|s_{1,t}^k - \tilde{s}_{1,t}^k\right|^2 & \sum_{k=1}^{M} \left(s_{1,t}^k - \tilde{s}_{1,t}^k\right)\left(s_{2,t}^k - \tilde{s}_{2,t}^k\right) & \cdots & \sum_{k=1}^{M} \left(s_{1,t}^k - \tilde{s}_{1,t}^k\right)\left(s_{n,t}^k - \tilde{s}_{n,t}^k\right) \\
\sum_{k=1}^{M} \left(s_{2,t}^k - \tilde{s}_{2,t}^k\right)\left(s_{1,t}^k - \tilde{s}_{1,t}^k\right) & \sum_{k=1}^{M} \left|s_{2,t}^k - \tilde{s}_{2,t}^k\right|^2 & \cdots & \sum_{k=1}^{M} \left(s_{2,t}^k - \tilde{s}_{2,t}^k\right)\left(s_{n,t}^k - \tilde{s}_{n,t}^k\right) \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{k=1}^{M} \left(s_{n,t}^k - \tilde{s}_{n,t}^k\right)\left(s_{1,t}^k - \tilde{s}_{1,t}^k\right) & \sum_{k=1}^{M} \left(s_{n,t}^k - \tilde{s}_{n,t}^k\right)\left(s_{2,t}^k - \tilde{s}_{2,t}^k\right) & \cdots & \sum_{k=1}^{M} \left|s_{n,t}^k - \tilde{s}_{n,t}^k\right|^2
\end{pmatrix} \tag{C.5}
$$

It is clear that $\boldsymbol{A}_t$ is nonnegative definite Hermitian, and the eigenvalues of $\boldsymbol{A}_t$ are nonnegative real numbers. Therefore, we can get

$$
\boldsymbol{V}_t \boldsymbol{A}_t \boldsymbol{V}_t^H = \boldsymbol{D}_t \tag{C.6}
$$

where $\boldsymbol{V}_t$ is a unitary matrix and $\boldsymbol{D}_t$ is a real diagonal matrix. The rows of $\boldsymbol{V}_t$, forming a complete orthonormal basis of an N-dimensional vector space, are the eigenvectors of $\boldsymbol{A}_t$. The diagonal elements of $\boldsymbol{D}_t$ are the eigenvalues $\lambda_{i,t} \geq 0$, $\forall\ 1 \leq i \leq n$. Equation (E.4) can be rewritten as

$$
\begin{aligned}
d_q^2(\boldsymbol{s}, \tilde{\boldsymbol{s}}) &= \Lambda_q \left(\sum_{t=1}^{L} \left(\boldsymbol{V}_t \boldsymbol{D}_t \boldsymbol{V}_t^H\right) + a_2 \left(\boldsymbol{V}_{t'} \boldsymbol{D}_{t'} \boldsymbol{V}_{t'}^H\right) \right) \boldsymbol{\Lambda}_q^T \\
&= \sum_{t=1}^{L} \left(\left(\boldsymbol{\Lambda}_q \boldsymbol{V}_t\right) \boldsymbol{D}_t \left(\boldsymbol{\Lambda}_q \boldsymbol{V}_t\right)^H \right) + a_2 \left(\left(\boldsymbol{\Lambda}_q \boldsymbol{V}_{t'}\right) \boldsymbol{D}_{t'} \left(\boldsymbol{\Lambda}_q \boldsymbol{V}_{t'}\right)^H \right) \\
&= \sum_{i=1}^{n} \left(\sum_{t=1}^{L} \lambda_{i,t} + a_2 \lambda_{i,t'} \right) |y_{i,q}|^2
\end{aligned} \tag{C.7}
$$

where $y_{i,q} = (\alpha_{1,q}, \alpha_{2,q}, \ldots, \alpha_{n,q})\, \boldsymbol{v}_{i,t}^{q}$ and $\boldsymbol{v}_{i,t}^{q}$'s stand for the eigenvectors of the matrix $\boldsymbol{A}_t$. Then, the equation (E.3) with respect to small values of rm can be derived as

$$\begin{aligned}
&\frac{1}{2}\frac{\rho^{L-1}}{(1-\rho)^{-1}}\sum_{t'=1}^{L}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\exp\left(-a_1\sum_{q=1}^{m}\left(d^2\left(\boldsymbol{s},\tilde{\boldsymbol{s}}\right)\right)\right)\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}\\
&=\frac{1}{2}\frac{\rho^{L-1}}{(1-\rho)^{-1}}\sum_{t'=1}^{L}\int_{0}^{\infty}\cdots\int_{0}^{\infty}\exp\left(-a_1\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\lambda_{i,t}+a_2\lambda_{i,t'}\right)\left|y_{i,q}\right|^2\right)\\
&\quad\cdot f\left(\left|y_{1,1}\right|\right)\cdots\cdot f\left(\left|y_{n,m}\right|\right)d\left|y_{1,1}\right|\cdots\cdot d\left|y_{n,m}\right|\\
&=\frac{1}{2}\frac{\rho^{L-1}}{(1-\rho)^{-1}}\sum_{t'=1}^{L}\prod_{q=1}^{m}\prod_{i=1}^{n}\left(1+a_1\left(\sum_{t=1}^{L}\lambda_{i,t}+a_2\lambda_{i,t'}\right)\right)^{-1}. \qquad \text{(C.8)}
\end{aligned}$$

where $|y_{i,q}|$ are independent Rayleigh-distributed random variables. The value of Y in (E.2) with similar derivation of X, and we can get

$$\begin{aligned}
&\frac{1}{2}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\prod_{t=1}^{L}\left[\frac{\rho}{(1-\rho)}\exp\left(B_t\right)\right]\cdot C\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}\\
&=\frac{\rho^L}{2}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(1+a_1\sum_{t=1}^{L}\lambda_{i,t}\right)^{-1}. \qquad \text{(C.9)}
\end{aligned}$$

where $\lambda_{i,t}$'s are the eigenvalues of A_t. From (E.5) and (E.6), the pairwise error probability can be approximated to

$$\begin{aligned}
\mathrm{P_r}\left(\boldsymbol{s}\longrightarrow\tilde{\boldsymbol{s}}\right) \cong\ &\frac{\rho^L}{2}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(1+a_1\sum_{t=1}^{L}\lambda_{i,t}\right)^{-1}\\
&+\frac{\rho^{L-1}(1-\rho)}{2}\sum_{t'=1}^{L}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(1+a_1\left(\sum_{t=1}^{L}\lambda_{i,t}+a_2\lambda_{i,t'}\right)\right)^{-1}. \qquad \text{(C.10)}
\end{aligned}$$

At high SNR's and SJR's, the above equation can be simplified as

$$\begin{aligned}
&\mathrm{P_r}\left(\boldsymbol{s}\longrightarrow\tilde{\boldsymbol{s}}\right)\\
&\cong\frac{\rho^L}{2}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(a_1\sum_{t=1}^{L}\lambda_{i,t}\right)^{-1}+\frac{\rho^{L-1}(1-\rho)}{2}\sum_{t'=1}^{L}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(a_1\left(\sum_{t=1}^{L}\lambda_{i,t}+a_2\lambda_{i,t'}\right)\right)^{-1}\\
&\cong\frac{\rho^L}{2}\left(\frac{1}{4(N_0+N_J/\rho)}\right)^{-mr}\cdot\left[\prod_{q=1}^{m}\prod_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}\right)^{-1}+\sum_{t'=1}^{L}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(\left(\sum_{t=1}^{L}\lambda_{i,t}+\lambda_{i,t'}\right)\right)^{-1}\right] \qquad \text{(C.11)}
\end{aligned}$$

The above equation is the pairwise error probability with respect to the case of low diversity. Next, we derive the pairwise error probability for the case of high diversity. For a large of

rm, according to the central limit theorem, the $d_q^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})$ in (C.7) approaches a Gaussian random variable D with the mean u_D and variance σ_D^2

$$u_D = \sum_{i=1}^{r} \sum_{q=1}^{m} \left(\sum_{t=1}^{L} \lambda_{i,t} + a\lambda_{i,t'} \right) \qquad \sigma_D^2 = \sum_{i=1}^{r} \sum_{q=1}^{m} \left(\sum_{t=1}^{L} \lambda_{i,t} + a\lambda_{i,t'} \right)^2 . \tag{C.12}$$

Then, the equation (E.3) can be rewritten as

$$\begin{aligned} X &= \frac{1}{2} \frac{\rho^{L-1}}{(1-\rho)^{-1}} \sum_{t'=1}^{L} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp\left(-a_1 \sum_{q=1}^{m} \left(d^2(s, \tilde{s}) \right) \right) f(\alpha)\, d\alpha \\ &\leq \frac{1}{2} \frac{\rho^{L-1}}{(1-\rho)^{-1}} \sum_{t'=1}^{L} \int_{D=0}^{\infty} \cdots \int_{D=0}^{\infty} \exp(-a_1 D) f(D)\, dD \\ &= \frac{1}{2} \frac{\rho^{L-1}}{(1-\rho)^{-1}} \sum_{t'=1}^{L} \left\{ \exp\left(\frac{1}{2} a_1^2 \sigma_D^2 - a_1 u_D \right) \right\} \cdot Q\left(a_1 \sigma_D - \frac{u_D}{\sigma_D} \right). \end{aligned} \tag{C.13}$$

By using the inequality

$$Q(x) \leq \frac{1}{2} \exp\left(-x^2/2 \right) \qquad \forall\, x \geq 0. \tag{C.14}$$

The equation in (C.13) can be approximated as

$$X \leq \frac{1}{2} \frac{\rho^{L-1}}{(1-\rho)^{-1}} \sum_{t'=1}^{L} \exp\left(-m \frac{\left| \sum\limits_{i=1}^{r} \left(\sum\limits_{t=1}^{L} \lambda_{i,t} + a\lambda_{i,t'} \right) \right|^2}{\sum\limits_{i=1}^{r} \left(\sum\limits_{t=1}^{L} \lambda_{i,t} + a\lambda_{i,t'} \right)^2} \right). \tag{C.15}$$

The value of Y in (E.2) with the similar derivation can be expressed as

$$Y \leq \frac{\rho^L}{2} \sum_{t'=1}^{L} \exp\left(-m \frac{\left| \sum\limits_{i=1}^{r} \sum\limits_{t=1}^{L} \lambda_{i,t} \right|^2}{\sum\limits_{i=1}^{r} \left(\sum\limits_{t=1}^{L} \lambda_{i,t} \right)^2} \right). \tag{C.16}$$

The pairwise error probability can then be bounded by

$$
\begin{aligned}
& P\left(\boldsymbol{s} \longrightarrow \tilde{\boldsymbol{s}}\right) \\
& \leq \frac{\rho^L}{2} \exp\left(-m \frac{\left|\sum_{i=1}^{r}\sum_{t=1}^{L}\lambda_{i,t}\right|^2}{\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}\right)^2}\right) + \frac{\rho^L}{2}\left(1-\rho\right)\sum_{t'=1}^{L}\exp\left(-m \frac{\left|\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}+a\lambda_{i,t'}\right)\right|^2}{\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}+a\lambda_{i,t'}\right)^2}\right) \\
& \cong \frac{\rho^L}{2}\left[\exp\left(-m \frac{\left|\sum_{i=1}^{r}\sum_{t=1}^{L}\lambda_{i,t}\right|^2}{\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}\right)^2}\right) + \exp\left(-m \frac{\left|\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}+\lambda_{i,t'}\right)\right|^2}{\sum_{i=1}^{r}\left(\sum_{t=1}^{L}\lambda_{i,t}+\lambda_{i,t'}\right)^2}\right)\right]. \qquad \text{(C.17)}
\end{aligned}
$$

Appendix D

Derivation of the ML Decoding of STC/OFHSS Systems without CSI

The derived in (4.49) is discussed in this Appendix by averaging $\alpha_{i,q}$'s with respect to the conditional probability density function defined in (4.46), and we can get

$$\int_{-\infty}^{\infty} f(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x},\boldsymbol{\alpha})f(\boldsymbol{\alpha})d\boldsymbol{\alpha}$$
$$= \left(\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{i=1}^{n}\prod_{q=1}^{m} a_{1,x_{i,t}}\text{exp}\left(-a_{2,x_{i,t}}|r_{i,q,t}^{k}|^2\right)\right)\cdot\left(\prod_{i=1}^{n}\prod_{q=1}^{m}\frac{1}{2\pi\sigma_{i,q}^2}\right)\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}$$
$$\text{exp}\left\{\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{i=1}^{n} a_{2,x_{i,t}}\left[2\text{Re}\left(r_{i,q,t}^{k}\alpha_{R,i,q}\hat{s}_{i,t}^{k}\right)+2\text{Im}\left(r_{i,q,t}^{k}\alpha_{I,i,q}\hat{s}_{i,t}^{k}\right)\right.\right.$$
$$\left.\left.-\left(\alpha_{R,i,q}^2+\alpha_{I,i,q}^2\right)\left|\hat{s}_{i,t}^{k}\right|^2\right]\right\}\cdot\text{exp}\left(-\frac{\alpha_{R,1,1}^2}{2\sigma_{1,1}^2}\right)\ldots exp\left(-\frac{\alpha_{I,n,m}^2}{2\sigma_{n,m}^2}\right)d\alpha_{R,1,1}\cdots d\alpha_{I,n,m}. \quad \text{(D.1)}$$

For the real part of the exponent could be represented by $R(\alpha_{R,1,q},\alpha_{R,2,q},\cdots,\alpha_{R,n,q})$

$$R\left(\alpha_{R,1,q},\alpha_{R,2,q},\ \ldots,\alpha_{R,n,q}\right) = 2\text{Re}\left(\sum_{i=1}^{n}\alpha_{R,i,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right) -$$
$$\left(\sum_{i=1}^{n}\alpha_{R,i,q}^2\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^2+\frac{1}{2\sigma_{i,q}^2}\right)\right). \quad \text{(D.2)}$$

By equation $(D.1)$ with $\sigma_{i,q}^2 = 1/2$, we average $\alpha_{R,i,q}$ for the real part of the exponent

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\left(2\mathrm{Re}\left(\sum_{i=1}^{n}\alpha_{R,i,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right)\right.\right.$$

$$\left.\left.-\left(\sum_{i=1}^{n}\alpha_{R,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)\right)\right)\right]d\alpha_{R,1,1}\cdots d\alpha_{R,n,m}$$

$$=\sqrt{\frac{\pi^{nm}}{\prod\limits_{q=1}^{m}\prod\limits_{i=1}^{n}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)}}\exp\left(\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\mathrm{Re}\left|\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}}{\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1}\right). \tag{D.3}$$

Next, we also average $\alpha_{I,i,q}$ for the imaginary part of the exponent in (D.1)

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\left(2\mathrm{Im}\left(\sum_{i=1}^{n}\alpha_{I,i,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right)\right.\right.$$

$$=\sqrt{\frac{\pi^{nm}}{\prod\limits_{q=1}^{m}\prod\limits_{i=1}^{n}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)}}\exp\left(\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\mathrm{Im}\left|\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}}{\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1}\right) \tag{D.4}$$

Hence, the probability density function $f(\boldsymbol{r}\,|\hat{\boldsymbol{s}},\boldsymbol{x})$ conditioned on JSI available can be expressed as

$$\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n}\left(a_{1,x_{i,t}}\cdot\exp\left(-a_{2,x_{i,t}}\left|r_{i,q,t}^{k}\right|^{2}\right)\right)\cdot\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\left(\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)^{-1}\right.$$

$$\left.\cdot\exp\left(\frac{\left|\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right|^{2}}{\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}a_{2,x_{i,t}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1}\right)\right). \tag{D.5}$$

Appendix E

Derivation of the Design Criteria of STC/OFHSS Systems

The derived results of the pairwise error probability in (4.57) and (4.59) are discussed in this Appendix, respectively. First of all, by averaging $x_{i,t}$'s with respect to the conditional pairwise error probability defined in (4.56), and then we can get

$$\begin{aligned}
&\mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \alpha_{i,q}\ \forall\ i, q\right) \\
&= \sum_{x_{1,1}} \cdots \sum_{x_{n,L}} \mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \alpha_{i,q}, x_{i,t}\ \forall\ i, q, t\right) \mathrm{P_r}\left(x_{1,1}\right) \cdots \mathrm{P_r}\left(x_{n,L}\right) \\
&\leq \sum_{x_{1,1}} \cdots \sum_{x_{n,L}} \frac{1}{2} \exp\left(-\sum_{t=1}^{L} \sum_{i=1}^{n} a_{x_{i,t}} d_{i,t}^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})\right) \mathrm{P_r}\left(x_{1,1}\right) \cdots \mathrm{P_r}\left(x_{n,L}\right) \\
&= \frac{1}{2} \prod_{t=1}^{L} \prod_{i=1}^{n} \left\{(1-\rho) \exp\left(-\frac{d_{i,t}^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})}{4N_0}\right) + \rho \exp\left(-\frac{d_{i,t}^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})}{4\left(N_0 + N_J/\rho\right)}\right)\right\} \\
&= \frac{1}{2} \prod_{t=1}^{L} \left\{\frac{\rho}{(1-\rho)} \exp\left(\frac{N_J/\rho}{4\left(N_0 + N_J/\rho\right)} d_{i,t}^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})\right) + 1\right\} \cdot \prod_{t=1}^{L} \left\{(1-\rho) \exp\left(-\frac{d_{i,t}^2(\boldsymbol{s}, \tilde{\boldsymbol{s}})}{4N_0}\right)\right\} \\
&\cong \frac{1}{2} \left\{\prod_{t=1}^{L} \prod_{i=1}^{n} \left[\exp\left(B_{i,t}\right)\right] + \sum_{t'=1}^{L} \sum_{i'=1}^{n} \left[\prod_{t=1}^{L} \prod_{i=1}^{n} \exp\left(B_{i,t}\right)\right] \cdot \left[\exp\left(B_{i',t'}\right)\right]^{-1}\right\} \cdot C \qquad \text{(E.1)}
\end{aligned}$$

where

$$B_{i,t} = \ln\left(\frac{\rho}{(1-\rho)}\right) + \frac{N_J/\rho}{4(N_0+N_J/\rho)} \sum_{q=1}^{m}\sum_{k=1}^{M} \left|\alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2$$

$$C = \prod_{t=1}^{L}\left\{(1-\rho)\exp\left(-\frac{\sum_{q=1}^{m}\sum_{k=1}^{M}\left|\alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2}{4N_0}\right)\right\}.$$

The pairwise error probability can be obtained by averaging (E.1) with respect to $\alpha_{i,q}$'s, i.e.,

$$\begin{aligned} \mathrm{P_r}(\boldsymbol{s} \to \tilde{\boldsymbol{s}}) &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \mathrm{P_r}(\boldsymbol{s}\to\tilde{\boldsymbol{s}}|\boldsymbol{\alpha}) f(\boldsymbol{\alpha})\, d\boldsymbol{\alpha} \\ &= X + Y \end{aligned} \qquad \text{(E.2)}$$

where

$$X = \frac{1}{2}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\sum_{t'=1}^{L}\sum_{i'=1}^{n}\left[\prod_{t=1}^{L}\prod_{i=1}^{n}\exp(B_{i,t})\right]\cdot\left[\exp(B_{i',t'})\right]^{-1}\cdot C\right) f(\boldsymbol{\alpha})\, d\boldsymbol{\alpha}$$

$$Y = \frac{1}{2}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\left[\prod_{t=1}^{L}\prod_{i=1}^{n}\exp(B_{i,t})\right]\cdot C\right) f(\boldsymbol{\alpha})\, d\boldsymbol{\alpha}.$$

The derivation of Y is similar to that of X. Therefore, we only discuss the derivation of X here. Replace the values of B_t and C in X, we can get

$$X = \frac{1}{2}\frac{\rho^{Ln-1}}{(1-\rho)^{-1}}\sum_{t'=1}^{L}\sum_{i'=1}^{n}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left(-a_1\sum_{q=1}^{m} d_{i',t',q}^2(s,\tilde{s})\right) f(\boldsymbol{\alpha})\, d\boldsymbol{\alpha} \qquad \text{(E.3)}$$

where

$$\begin{aligned} d_{i',t',q}^2(\boldsymbol{s},\tilde{\boldsymbol{s}}) &= \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{i=1}^{n}\left|\alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2 + \frac{N_J}{N_0\rho}\sum_{k=1}^{M}\left|\alpha_{i',q}\left(s_{i',t'}^k - \tilde{s}_{i',t'}^k\right)\right|^2 \\ a_1 &= \frac{1}{4\left(N_0 + N_J/\rho\right)}. \end{aligned}$$

The value of $d^2_{i',t',q}\left(\boldsymbol{s},\tilde{\boldsymbol{s}}\right)$ can be rewritten as

$$
\begin{aligned}
d^2_{i',t',q}(\boldsymbol{s},\tilde{\boldsymbol{s}}) &= \left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s^k_{i,t}-\tilde{s}^k_{i,t}\right|^2\right)\sum_{\substack{i=1\\ \mathrm{i}\neq \mathrm{i}'}}^{n}\left|\alpha_{i,q}\right|^2 \\
&+\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s^k_{i',t}-\tilde{s}^k_{i',t}\right|^2+a_2\sum_{k=1}^{M}\left|s^k_{i',t'}-\tilde{s}^k_{i',t'}\right|^2\right)\left|\alpha_{i',q}\right|^2 \\
&= b_1\sum_{\substack{i=1\\ i\neq i'}}^{n}\left|\alpha_{i,q}\right|^2+b_2\left|\alpha_{i',q}\right|^2
\end{aligned}
\tag{E.4}
$$

where

$$
a_2=\frac{N_J}{N_0\rho}
$$

$$
b_1=\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s^k_{i,t}-\tilde{s}^k_{i,t}\right|^2 \qquad b_2=\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s^k_{i',t}-\tilde{s}^k_{i',t}\right|^2+a_2\sum_{k=1}^{M}\left|s^k_{i',t'}-\tilde{s}^k_{i',t'}\right|^2.
$$

Then, the equation (E.3) with respect to small values of rm can be derived as

$$
\begin{aligned}
&\frac{1}{2}\frac{\rho^{Ln-1}}{\left(1-\rho\right)^{-1}}\sum_{t'=1}^{L}\sum_{i'=1}^{n}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left(-a_1\sum_{q=1}^{m}\left(b_1\sum_{\substack{i=1\\ i\neq i'}}^{n}\left|\alpha_{i,q}\right|^2+b_2\left|\alpha_{i',q}\right|^2\right)\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha} \\
&=\frac{1}{2}\frac{\rho^{Ln-1}}{\left(1-\rho\right)^{-1}}\sum_{t'=1}^{L}\sum_{i'=1}^{r}\left[\prod_{q=1}^{m}\left(\prod_{\substack{i=1\\ i\neq i'}}^{r}\left(1+a_1b_1\right)\right)^{-1}\cdot\left(1+a_1b_2\right)^{-1}\right].
\end{aligned}
\tag{E.5}
$$

The value of Y in (E.2) with respect to the similar derivation, and we can get

$$
\begin{aligned}
&\frac{1}{2}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\left[\prod_{t=1}^{L}\prod_{i=1}^{n}\exp\left(B_{i,t}\right)\right]\cdot C\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha} \\
&=\frac{1}{2}\rho^{Ln}\prod_{q=1}^{L}\prod_{i=1}^{r}\left(1+a_1\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s^k_{i,t}-\tilde{s}^k_{i,t}\right|^2\right)^{-1}.
\end{aligned}
\tag{E.6}
$$

From (E.5) and (E.6), the pairwise error probability can be bounded by

$$
\begin{aligned}
\mathrm{P_r}\left(\boldsymbol{s}\longrightarrow\tilde{\boldsymbol{s}}\right) \;\cong\; &\frac{\rho^{Ln}}{2}\prod_{q=1}^{m}\prod_{i=1}^{r}\left(1+a_1\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s^k_{i,t}-\tilde{s}^k_{i,t}\right|^2\right)^{-1} \\
&+\frac{1}{2}\frac{\rho^{Ln-1}}{\left(1-\rho\right)^{-1}}\sum_{t'=1}^{L}\sum_{i'=1}^{n}\left[\prod_{q=1}^{m}\left(\prod_{\substack{i=1\\ i\neq i'}}^{r}\left(1+a_1b_1\right)\right)^{-1}\cdot\left(1+a_1b_2\right)^{-1}\right].
\end{aligned}
\tag{E.7}
$$

At high SNR's and SJR's, the above equation can be simplified as

$$\begin{aligned}
\mathrm{P_r}\left(\boldsymbol{s} \longrightarrow \tilde{\boldsymbol{s}}\right) \leq\; & \frac{1}{2}\rho^{Ln}\left(\frac{1}{4\left(N_0+N_J/\rho\right)}\right)^{mn}\left[\prod_{q=1}^{L}\prod_{i=1}^{r}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)\right. \\
& +\frac{(1-\rho)}{\rho}\sum_{t'=1}^{L}\sum_{i'=1}^{n}\left[\prod_{q=1}^{m}\left(\prod_{\substack{i=1\\ i\neq i'}}^{r}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)\right)\right. \\
& \left.\left.\cdot\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i',t}^{k}-\tilde{s}_{i',t}^{k}\right|^2+a_2\sum_{k=1}^{M}\left|s_{i',t'}^{k}-\tilde{s}_{i',t'}^{k}\right|^2\right)\right]^{-1} \\
\cong\; & \frac{1}{2}\rho^{Ln}\left(\frac{1}{4\left(N_0+N_J/\rho\right)}\right)^{mn}\left[\prod_{q=1}^{L}\prod_{i=1}^{r}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)\right. \\
& +\sum_{t'=1}^{L}\sum_{i'=1}^{n}\left[\prod_{q=1}^{m}\left(\prod_{\substack{i=1\\ i\neq i'}}^{r}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)\right)\right. \\
& \left.\left.\cdot\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i',t}^{k}-\tilde{s}_{i',t}^{k}\right|^2+\sum_{k=1}^{M}\left|s_{i',t'}^{k}-\tilde{s}_{i',t'}^{k}\right|^2\right)\right]^{-1}. \qquad \text{(E.8)}
\end{aligned}$$

The above equation is the pairwise error probability with respect to the case of low diversity. Next, we derive the pairwise error probability for the case of high diversity. For a large of rm, assume $N_J \gg N_0$, which corresponds to

$$\exp\left(-\frac{d_{i,t}^2(s,\tilde{s})}{4\left(N_0+N_J/\rho\right)}\right) \gg \exp\left(-\frac{d_{i,t}^2(s,\tilde{s})}{4N_0}\right). \qquad \text{(E.9)}$$

By (E.1), the pairwise error probability can be approximated as

$$\begin{aligned}
& \mathrm{P_r}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} \mid \alpha_{i,q}\ \forall\ i,q\right) \\
& \cong \frac{1}{2}\prod_{t=1}^{L}\prod_{i=1}^{n}\left\{\rho\exp\left(-\frac{d_{i,t}^2(\boldsymbol{s},\tilde{\boldsymbol{s}})}{4\left(N_0+N_J/\rho\right)}\right)\right\} \\
& = \frac{1}{2}\rho^{Ln}\exp\left(-\frac{1}{4\left(N_0+N_J/\rho\right)}\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)\left|\alpha_{i,q}\right|^2\right). \qquad \text{(E.10)}
\end{aligned}$$

According to the central limit theorem, the $\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2$ approaches a Gaussian random variable D with the mean u_D and variance σ_D^2

$$u_D=\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right) \qquad \sigma_D^2=\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)^2 \qquad \text{(E.11)}$$

By averaging (E.10) with respect to the Gaussian random variable D, we then have

$$\begin{aligned} \mathrm{P_r}\left(\boldsymbol{s} \longrightarrow \tilde{\boldsymbol{s}}\right) \;\cong\; & \frac{\rho^{\mathrm{L}}}{2} \exp\left(\frac{1}{2}\left(\frac{1}{4\left(N_0+N_J/\rho\right)}\right)^2 \sigma_D^2 - \left(\frac{1}{4\left(N_0+N_J/\rho\right)}\right) u_D\right) \\ & \cdot \mathrm{Q}\left(\left(\frac{1}{4\left(N_0+N_J/\rho\right)}\right)\sigma_D - \frac{u_D}{\sigma_D}\right). \end{aligned} \tag{E.12}$$

By using the inequality

$$Q\left(a\right) \leq \frac{1}{2}\exp\left(-x^2/2\right) \qquad \forall\; x \geq 0. \tag{E.13}$$

The above equation can be approximated as

$$\mathrm{P_r}\left(\boldsymbol{s} \longrightarrow \tilde{\boldsymbol{s}}\right) \cong \frac{1}{2}\rho^{L} \exp\left(-\frac{\left(\sum\limits_{q=1}^{m}\sum\limits_{i=1}^{n}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)\right)^2}{2\sum\limits_{q=1}^{m}\sum\limits_{i=1}^{n}\left(\sum\limits_{t=1}^{L}\sum\limits_{k=1}^{M}\left|s_{i,t}^{k}-\tilde{s}_{i,t}^{k}\right|^2\right)^2}\right). \tag{E.14}$$

Appendix F

Derivation of the ML Decoding of STC/UFHSS Systems without CSI

The derived in (4.67) is discussed in this Appendix by averaging $\alpha_{i,q}$'s with respect to the conditional probability density function defined in (4.64), and we can get

$$\begin{aligned}
&\int_{-\infty}^{\infty} f(\tilde{\boldsymbol{r}}|\hat{\boldsymbol{s}},\boldsymbol{x},\boldsymbol{\alpha},\tilde{\boldsymbol{\beta}}) f(\boldsymbol{\alpha}) d\boldsymbol{\alpha} \\
&= \left(\prod_{t=1}^{L}\prod_{k=1}^{M} a_{1,x_{t,b}}^{m}\right)\cdot\left(\prod_{q=1}^{m}\prod_{i=1}^{n}\frac{1}{2\pi\sigma_{i,q}^2}\right)\cdot\exp\left(-\sum_{t=1}^{L}\sum_{b=1}^{B(t)}\sum_{k=1}^{M}\sum_{q=1}^{m} a_{2,x_{t,b}}|\tilde{r}_{q,t,b}^{k}|^2\right) \\
&\quad \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left\{\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}\left[2\mathrm{Re}\left(\tilde{r}_{q,t,b}^{k}\sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{R,i,q}\hat{s}_{i,t}^{k}\right)\right.\right. \\
&\quad +2\mathrm{Im}\left(\tilde{r}_{q,t,b}^{k}\sum_{i=1}^{n}\tilde{\beta}_{i,t,b}\alpha_{I,i,q}\hat{s}_{i,t}^{k}\right)-\sum_{i=1}^{n}\alpha_{R,i,q}\left|\tilde{\beta}_{i,t,b}\hat{s}_{i,t}^{k}\right|^2+\sum_{i=1}^{n}\alpha_{I,i,q}\left|\tilde{\beta}_{i,t,b}\hat{s}_{i,t}^{k}\right|^2 \\
&\quad \left.\left.+\sum_{\substack{i=1\\i\neq l}}^{n}\sum_{\substack{l=1\\l\neq i}}^{n}\alpha_{R,i,q}\alpha_{R,l,q}\tilde{\beta}_{i,t,b}\hat{s}_{i,t}^{k}\tilde{\beta}_{l,t,b}\hat{s}_{l,t}^{k}+\sum_{\substack{i=1\\i\neq l}}^{n}\sum_{\substack{l=1\\l\neq i}}^{n}\alpha_{I,i,q}\alpha_{I,l,q}\tilde{\beta}_{i,t,b}\hat{s}_{i,t}^{k}\tilde{\beta}_{l,t,b}\hat{s}_{l,t}^{k}\right]\right\} \\
&\quad \cdot\exp\left(-\frac{\alpha_{R,1,1}^2}{2\sigma_{1,1}^2}\right)\ldots\exp\left(-\frac{\alpha_{I,n,m}^2}{2\sigma_{n,m}^2}\right)d\alpha_{R,1,1}\cdots d\alpha_{I,n,m}. \qquad \text{(F.1)}
\end{aligned}$$

For the real part of the exponent could be represented by $R(\alpha_{R,1,q}, \alpha_{R,2,q}, \cdots, \alpha_{R,n,q})$

$$
\begin{aligned}
& R\left(\alpha_{R,1,q}, \alpha_{R,2,q}, \ \ldots, \alpha_{R,n,q}\right) \\
&= 2\mathrm{Re}\left(\sum_{i=1}^{n} \alpha_{R,i,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}} \tilde{r}_{q,t,b}^{k} \tilde{\beta}_{i,t,b} \hat{s}_{i,t}^{k}\right)\right) - \\
&\quad \left(\sum_{i=1}^{n} \alpha_{R,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}} \left|\tilde{\beta}_{i,t,b} \hat{s}_{i,t}^{k}\right|^{2} + \frac{1}{2\sigma_{i,q}^{2}}\right)\right) + \\
&\quad \sum_{\substack{i=1 \\ i\neq l}}^{n}\sum_{\substack{l=1 \\ l\neq i}}^{n} \alpha_{R,i,q}\alpha_{R,l,q}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}} \tilde{\beta}_{i,t,b} \hat{s}_{i,t}^{k} \tilde{\beta}_{l,t,b} \hat{s}_{l,t}^{k}\right) \\
&= \boldsymbol{B}_{R,q}\boldsymbol{\Lambda}_{R,q}^{T} - \boldsymbol{\Lambda}_{R,q}\left(\boldsymbol{A}+\boldsymbol{I}\right)\boldsymbol{\Lambda}_{R,q}^{T} \qquad \left(\sigma_{i,q}^{2} = \frac{1}{2}\ \forall\ i,q\right)
\end{aligned}
\tag{F.2}
$$

where

$$\boldsymbol{\Lambda}_{R,q} = \left(\alpha_{R,1,q}, \alpha_{R,2,q}, \ldots, \alpha_{R,n,q}\right)$$

$$
\begin{aligned}
\boldsymbol{B}_{R,q} &= \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}} 2\mathrm{Re}\left[\left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right), \left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right), \ldots, \left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}} \boldsymbol{B}_{R,t,b}^{q,k}.
\end{aligned}
\tag{F.3}
$$

The matrix $\boldsymbol{A}$ is

$$
\begin{aligned}
&\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}
\begin{bmatrix}
\left|\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right|^{2} & \left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right)\left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right) & \cdots & \left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right)\left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right) \\
\left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right)\left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right) & \left|\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right|^{2} & \cdots & \left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right)\left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right) \\
\vdots & \vdots & \ddots & \vdots \\
\left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right)\left(\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right) & \left(\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right)\left(\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right) & \cdots & \left|\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right|^{2}
\end{bmatrix} \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}} A_{t,b}^{k}.
\end{aligned}
\tag{F.4}
$$

It is clear that $\boldsymbol{A}_{t,b}^{k}$ is nonnegative definite Hermitian, and the eigenvalues of $\boldsymbol{A}_{t,b}^{k}$ are nonnegative real numbers. Therefore, we can get

$$\boldsymbol{V}_{t,b}^{k}\boldsymbol{A}_{t,b}^{k}{\boldsymbol{V}_{\boldsymbol{t,b}}^{\boldsymbol{k}}}^{H} = \boldsymbol{D}_{t,b}^{k} \tag{F.5}$$

where $\boldsymbol{V}_{t,b}^{k}$ is a unitary matrix and $\boldsymbol{D}_{t,b}^{k}$ is a real diagonal matrix. The rows of $\boldsymbol{V}_{t,b}^{k}$, forming a complete orthonormal basis of an N-dimensional vector space, are the eigenvectors of $\boldsymbol{A}_{t,b}^{k}$. The diagonal elements of $\boldsymbol{D}_{t,b}^{k}$ are the eigenvalues $\lambda_{i,t,b}^{k} \geq 0,\ \forall\ 1 \leq \mathrm{i} \leq n$. Equation $(F.2)$ can be rewritten as

$$
\begin{aligned}
&R\left(\alpha_{R,1,q}, \alpha_{R,2,q},\ \ldots,\alpha_{R,n,q}\right). \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}\left[\boldsymbol{B}_{R,t,b}^{q,k}\left(\boldsymbol{V}_{t,b}^{k}\boldsymbol{V}_{t,b}^{kT}\right)\Lambda_{R,q}^{T} - \boldsymbol{\Lambda}_{R,q}\left(\boldsymbol{V}_{t,b}^{k}\boldsymbol{D}_{t,b}^{k}\boldsymbol{V}_{t,b}^{kT} + \frac{1}{a_{2,x_{t,b}}LMB(t)}\boldsymbol{I}\right)\boldsymbol{\Lambda}_{R,q}^{T}\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}\left[\boldsymbol{Z}_{i,b}^{q,k}\boldsymbol{Y}_{R,q}^{T} - \boldsymbol{Y}_{R,q}\left(\boldsymbol{D}_{t,b}^{k} + \frac{1}{a_{2,x_{t,b}}LMB(t)}\boldsymbol{I}\right)\boldsymbol{Y}_{R,q}^{T}\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)} a_{2,x_{t,b}}\left[\sum_{i=1}^{n}\left(z_{i,t,b}^{q,k}y_{R,i,q} - \left(\lambda_{i,t,b}^{k} + \frac{1}{a_{2,x_{t,b}}LMB(t)}\right)y_{R,i,q}^{2}\right)\right] \qquad \text{(F.6)}
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{Z}_{q,t}^{k} &= \boldsymbol{B}_{R,q,t}^{k}\boldsymbol{V}_{t,b}^{k} \\
&= \left[2\mathrm{Re}\left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right), 2\mathrm{Re}\left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right), \ldots, 2\mathrm{Re}\left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right)\right]\boldsymbol{V}_{t,b}^{k} \\
&= \left(z_{1,t,b}^{q,k}, z_{2,t,b}^{q,k}, \ldots, z_{n,t,b}^{q,k}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
\boldsymbol{Y}_{R,q} &= \boldsymbol{\Lambda}_{R,q}\boldsymbol{V}_{t,b}^{k} \\
&= \left(\alpha_{R,1,q}, \alpha_{R,2,q}, \ldots, \alpha_{R,n,q}\right)\boldsymbol{V}_{t,b}^{k} \\
&= \left(y_{R,1,q}, y_{R,2,q}, \ldots, y_{R,n,q}\right).
\end{aligned}
$$

By equation $(F.1)$, we average $\alpha_{R,i,q}$ for the real part of the exponent

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\left(z_{i,t,b}^{q,k}y_{R,i,q}-\left(\lambda_{i,t,b}^{k}+\frac{1}{a_{2,x_{t,b}}LMB(t)}\right)y_{R,i,q}^{2}\right)\right)\right]$$

$$\cdot dy_{R,1,1}\cdots dy_{R,n,m}$$

$$=\sqrt{\frac{\pi^{nm}}{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1\right)}}\exp\left(\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}z_{i,t,b}^{q,k}\right|^{2}}{4\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1}\right). \tag{F.7}$$

Next, we also average $\alpha_{I,i,q}$ for the imaginary part of the exponent in (F.1)

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\left(w_{i,t,b}^{q,k}y_{R,i,q}-\left(\lambda_{i,t,b}^{k}+\frac{1}{a_{2,x_{t,b}}LMB(t)}\right)y_{R,i,q}^{2}\right)\right)\right]$$

$$\cdot dy_{R,1,1}\cdots dy_{R,n,m}$$

$$=\sqrt{\frac{\pi^{nm}}{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1\right)}}\exp\left(\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}w_{i,t,b}^{q,k}\right|^{2}}{4\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1}\right) \tag{F.8}$$

where

$$w_{i,t,b}^{q,k} = \left[2\mathrm{Im}\left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{1,t,b}\hat{s}_{1,t}^{k}\right),2\mathrm{Im}\left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{2,t,b}\hat{s}_{2,t}^{k}\right),\ldots,2\mathrm{Im}\left(\tilde{r}_{q,t,b}^{k}\tilde{\beta}_{n,t,b}\hat{s}_{n,t}^{k}\right)\right]v_{i,t}^{k}.$$

$v_{i,t}^{k}$ and $\lambda_{i,t,b}^{k}$ are the eigenvectors and eigenvalues of the matrix $A_{t,b}^{k}$, respectively. Hence, the probability density function $f\left(\tilde{\boldsymbol{r}}\middle|\hat{\boldsymbol{s}},\boldsymbol{x},\tilde{\boldsymbol{\beta}}\right)$ conditioned on JSI available can be expressed as

$$\left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{b=1}^{B(t)}\left(a_{1,x_{t,b}}\exp\left(-a_{2,x_{t,b}}\left|\tilde{r}_{q,t,b}^{k}\right|^{2}\right)\right)\right\}\cdot\left\{\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1\right)^{-1}\right.\right.$$

$$\exp\left(\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}u_{i,t,b}^{q,k}\right|^{2}}{\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}\lambda_{i,t,b}^{k}+1}\right) \tag{F.9}$$

where

$$\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}u_{i,t,b}^{q,k}\right|^{2}=\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}z_{i,t,b}^{q,k}\right|^{2}+\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{b=1}^{B(t)}a_{2,x_{t,b}}w_{i,t,b}^{q,k}\right|^{2}.$$

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