整數階與分數階雙 Duffing 系統的渾沌，非耦合及實用同步與反同步

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摘要

由相圖龐卡萊映射圖及分岐圖等數值方法來研究整數階與分數階雙 Duffing 系統的渾沌行為。我們可以得知在整數階時及系統每方程階數在 0.1、0.2、0.3、0.4、0.5、0.6、0.7 時，系統具有渾沌現象。藉由參數驅動方法可得對於兩個非耦合雙 Duffing 系統的渾沌同步。此方法稱為參數激發渾沌同步。首先，以第三個渾沌系統的渾沌狀態變量之函數來取代兩個非耦合雙 Duffing 系統中對應參數，可得到同步和反同步的結果。然後，再以白噪音，Rayleigh 噪音，Rician 噪音或均勻噪音取代兩個非耦合雙 Duffing 系統中對應參數。由數值模擬的結果，可得同步或反同步的結果，取決於系統的初值與驅動的強度。更可獲致完全同步與反同步輪換現象。

我們提出一種兩具不確定參數之渾沌系統廣義同步的新方法。藉基於概率觀念之實用漸近穩定性定理，我們可以嚴格的証明誤差動力學和參數動力學之共通零解實際上漸近穩定。兩個相同的渾沌系統可完成廣義同步而且參數估計值可以趨近到不確定值。我們稱之為實用廣義同步。給出了數值模擬例子。兩個雙 Duffing 系統以一個雙 van der Pol 系統為目標系統而達廣義同步。最後，証明此種廣義同步的高度強健性。

Chaos, Uncoupled and Pragmatical Synchronization and Anti-synchronization of Integral Order and Fractional Order Double Duffing Systems

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ABSTRACT

Chaos in integral order and in fractional order double Duffing systems is numerically studied by phase portraits, Poincaré maps and bifurcation diagrams. It is found that chaotic behaviors exist in integral order and in the fractional order double Duffing system with orders 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7 for each equation of the system. The chaos synchronization by driving corresponding parameters of two uncoupled identical chaotic double Duffing systems is presented. The method is named parameter excited chaos synchronization. First, replacing two corresponding parameters of the identical systems by the same function of chaotic state variables of a third chaotic system, the synchronization or anti-synchronization of two uncoupled systems can be obtained. Second, by replacing two corresponding parameters of two uncoupled identical double Duffing chaotic dynamical systems by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. Numerical simulations are illustrated for both synchronization and anti-synchronization of which the occurrence depends significantly on initial conditions and on driving strength. Alternative complete synchronization and anti-synchronization is also discovered.

A scheme is proposed to achieve generalized synchronization of two chaotic systems with uncertain parameters. By the pragmatical asymptotical stability theorem using the concept of probability, we can prove strictly that the common null solution of error dynamics and of parameter dynamics is actually asymptotically stable. Two identical chaotic systems are in generalized synchronization and the estimated parameters approach the uncertain values. It is called pragmatical generalized synchronization. As numerical example, two double Duffing chaotic systems are in generalized synchorization with a double van der Pol chaotic system as a goal system. Finally the high robustness of the generalized synchronization is obtained.

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