LIST OF FIGURES

| Figure 2.1 | The phase portrait, Poincaré map for the fractional order | 11 |
|------------|--|----|
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.2 | The phase portrait, Poincaré map for the fractional order | 11 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 1$ | |
| Figure 2.3 | The phase portrait, Poincaré map for the fractional order | 12 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 0.1$ | |
| Figure 2.4 | The phase portrait, Poincaré map for the fractional order | 12 |
| | double van der Pol system, u versus v, $\alpha_1 = 1, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.5 | The phase portrait, Poincaré map for the fractional order | 13 |
| Figure 2.6 | double van der Pol system, u versus v, $\alpha_1 = 0.9, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ The phase portrait. Poincaré map for the fractional order | 13 |
| 8 | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.9, \beta_1 = 0.9, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.7 | The phase portrait, Poincaré map for the fractional order | 14 |
| | double van der Pol system, x versus y, $\alpha_1 = 0.1, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.8 | The phase portrait, Poincar∈ map for the fractional order | 14 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.9 | The phase portrait, Poincaré map for the fractional order | 15 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 1$ | |
| | | |

| Figure 2.10 | The phase portrait, Poincaré map for the fractional order | 15 |
|------------------------|--|----|
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 0.1$ | |
| Figure 2.11 | The phase portrait, Poincaré map for the fractional order | 16 |
| | double van der Pol system, x versus y, $\alpha_1 = 1, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.12 | The phase portrait, Poincaré map for the fractional order | 16 |
| | double van der Pol system, x versus y, $\alpha_1 = 0.9, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.13 | The phase portrait, Poincaré map for the fractional order | 17 |
| | double van der Pol system, x versus y, $\alpha_1 = 0.1, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.14 | The phase portrait, Poincaré map for the fractional order | 17 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta = 0.1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.15 | The phase portrait, Poincaré map for the fractional order | 18 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 1$ | |
| E ' 2 1(| 1896 | 10 |
| Figure 2.10 | double you der Del gystem y yourges y | 10 |
| | double van der Foi system, x versus y, $\alpha = 0.1 \ \beta = 0.1 \ \alpha = 0.1 \ \beta = 0.1$ | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 0.1$ | |
| Figure 2.17 | The phase portrait, Poincaré map for the fractional order | 19 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 1, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.18 | The phase portrait, Poincaré map for the fractional order | 19 |
| | double van der Pol system, x versus y, $\alpha_1 = 0.1, \beta_1 = 1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.19 | The phase portrait, Poincaré map for the fractional order | 20 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 1, \beta_2 = 1$ | |
| Figure 2.20 | The phase portrait, Poincaré map for the fractional order | 20 |
| | double van der Pol system,x versus y, | |

| | $\alpha = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 1$ | |
|-------------|--|----|
| Figure 2.21 | The phase portrait, Poincaré map for the fractional order | 21 |
| | double van der Pol system,x versus y, | |
| | $\alpha_1 = 0.1, \beta_1 = 0.1, \alpha_2 = 0.1, \beta_2 = 0.1$ | |
| Figure 3.1 | Lyapunov exponent diagram of the double van der Pol system | 26 |
| | for c between 1.0 and 3.0 in (a) and enlarge ib (b). | |
| Figure 3.2 | Phase portraits of the double van der Pol system. | 26 |
| Figure 3.3 | CS and AS for initial condition (x ₂ , y ₂ , u ₂ , v ₂)= (-3, 4, -3, 4) and | 26 |
| | k=1, | |
| | (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4 . | |
| Figure 3.4 | AS for initial condition (x ₂ , y ₂ , u ₂ , v ₂) = (-3, 4, -3, 4) and k=0.9, | 27 |
| | (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4 . | |
| Figure 3.5 | CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$ and | 27 |
| | k=1, | |
| | (a) e ₁ , e ₂ , e ₃ , e ₄ (b) E ₁ , E ₂ , E ₃ , E ₄ . | |
| Figure 3.6 | CS for initial condition(x ₂ , y ₂ , u ₂ , v ₂) = (3, -4, 3, -4) and k=0.9, | 27 |
| | (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4 . | |
| Figure 3.7 | CS or AS vs. the k for different initial conditions: | 28 |
| | case a: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (-3, 4, -3, -3)$ | |
| | 4) | |
| | case b: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$ | |
| - | -4) | • |
| Figure 3.8 | CS and AS for initial condition(x ₂ , y ₂ , u ₂ , v ₂) = (-3, 4, 3, 4) and k=0.97, | 28 |
| | (a) e ₁ , e ₂ , e ₃ , e ₄ (b) E ₁ , E ₂ , E ₃ , E ₄ . | |
| Figure 3.9 | CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (-3, 4, 3, 4)$ and | 29 |
| | k=1.02, | |
| | (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4 . | |
| Figure 3.10 | CS and AS for initial condition (x ₂ , y ₂ , u ₂ , v ₂) = (3, -4, -3, -4) | 29 |
| | and k=0.97,(a) e ₁ , e ₂ , e ₃ , e ₄ (b) E ₁ , E ₂ , E ₃ , E ₄ . | |

| Figure 3.11 | CS and AS for initial condition (x ₂ , y ₂ , u ₂ , v ₂) = (3, -4, -3, -4) and k=1.02, (a) e ₁ , e ₂ , e ₃ , e ₄ (b) E ₁ , E ₂ , E ₃ , E ₄ . | 29 |
|-------------|--|----|
| Figure 3.12 | CS or AS vs. the k for different initial conditions: | 30 |
| | case c: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (-3, 4, 3, 4)$ | |
| | case d: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (3, -4, -3, -4)$ | |
| Figure 4.1 | CS and AS for initial condition(x ₂ , y ₂ , u ₂ , v ₂)= (3, -4, 3, -4) and <i>k</i> =1, | 36 |
| | (a) e_1 , e_2 , e_3 , e_4 (b) E_1 , E_2 , E_3 , E_4 | |
| Figure 4.2 | AS for initial condition (x ₂ , y ₂ , u ₂ , v ₂) = (3, -4, 3, -4) and <i>k</i> =529, | 36 |
| | (a)e ₁ , e ₂ , e ₃ , e ₄ (b) E ₁ , E ₂ , E ₃ , E ₄ . | |
| Figure 4.3 | CS and AS for initial condition (x ₂ , y ₂ , u ₂ , v ₂) = (-3, 4, -3, 4) and <i>k</i> =1, | 36 |
| Figure 4.4 | (a) e₁, e₂, e₃, e₄ (b) E₁, E₂, E₃, E₄. CS for initial condition(x₂, y₂, u₂, v₂) = (-3, 4, -3, 4) and k=529, (a) e₁, e₂, e₃, e₄ (b) E₁, E₂, E₃, E₄. | 37 |
| Figure 4.5 | CS or AS vs. the <i>k</i> for different initial conditions: | 37 |
| | case a: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$ | |
| | case b: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (-3, 4, -3, 4)$ | |
| Figure 4.6 | CS and AS for initial condition(x ₂ , y ₂ , u ₂ , v ₂) = (3, -4, 3, -4) and <i>k</i> =1, | 38 |
| | (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4 . | |
| Figure 4.7 | CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$ and $k=2379$,(a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4 . | 38 |
| Figure 4.8 | CS and AS for initial condition (x ₂ , y ₂ , u ₂ , v ₂) = (-3, 4, -3, 4) and <i>k</i> =1 | 38 |
| | (a) e ₁ , e ₂ , e ₃ , e ₄ (b) E ₁ , E ₂ , E ₃ , E ₄ . | |

| Figure 4.9 | CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (-3, 4, -3, 4)$ | 39 |
|-------------|--|----|
| | and $k=2379$, (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4 . | |
| Figure 4.10 | CS or AS vs. the k for different initial conditions: | 39 |
| | case c: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$ | |
| | case d: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (-3, 4, -3, 4)$ | |
| Figure 5.1 | Phase portraits of the double Duffing system | 52 |
| Figure 5.2 | Time histories of state errors for E_1 , E_2 , E_3 , E_4 , | 52 |
| Figure 5.3 | Time histories of coefficients $b_1, c_1, c_2, d_1, j_1, g_1, h_1, h_2$ | 53 |
| Figure 5.4 | Time histories of state errors for $E_{1,} E_{2,} E_{3,} E_{4}$ for Case (b) | 54 |
| Figure 5.5 | Time histories of coefficients $k_1, a_1, b_1, c_1, d_1, j_1, f_1, g_1, h_1, \lambda_1$ for | 55 |
| | Case (b) | |
| | ESAN | |

