Chapter 4

Chaos control and synchronization of double Mackey-Glass system by noise excitation of parameters

In this Chapter, the parameter excited method is applied to control chaos of a double Mackey-Glass system and to synchronize two uncoupled identical double Mackey-Glass systems. By replacing a parameter of the chaotic system by a noise signal, its chaotic motion can be eliminated. By replacing the corresponding parameters of two identical chaotic systems by a noise signal, these two chaotic systems with different initial conditions can be synchronized.

4.1 Chaos control and synchronization for uncoupled double Mackey-Glass system by parameter excited method

We consider a double Mackey-Glass system as follow:

$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1+x_{1\tau}^n} - rx_1 \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1+x_{2\tau}^n} - rx_2 - x_1 \end{cases} \tag{4.1}$$

where x_1, x_2 are state variables and $x_{i\tau} = x_i(t-\tau)$, $(i=1,2)$, τ is a time delay, and b, r, n are constant parameter. We keep the delay time fixed at 20 second ($\tau = 20$) and the parameters are taken as $b = 0.2$, $r = 0.1$, $n = 10$. The initial values are given as $(x_{10}, x_{20}) = (0.1, 0.1)$. With these data, the equilibrium point (0,0,0) of Eq. (4.1) is unstable and leads to chaotic motion. The bifurcation diagram is shown in Fig. 2.2 of Chapter 2. By replacing a parameter by a noise signal, the chaotic motion can be eliminated and the equilibrium point becomes asymptotically stable.

Next, a second identical double Mackey-Glass system is given by

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1+y_{1\tau}^n} - ry_1 \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1+y_{2\tau}^n} - ry_2 - y_1 \end{cases} \tag{4.2}$$

where the parameters and time delay τ are the same as Eq. (4.1) but with different initial values $(y_{10}, y_{20}) = (0.2, 0.2)$. We use the parameter excited method to synchronize these two identical double Mackey-Glass chaotic systems with different initial conditions. By replacing the corresponding parameters of these two chaotic systems by a noise signal, the synchronizations are achieved successfully in major cases.

4.2 Numerical simulations of chaos control

In Sections 4.2 and 4.3, the numerical simulations which carried out by Simulink environment of MATLAB are presented. The corresponding parameter is replaced by Gaussian noise, Rayleigh noise, Rician noise and uniform noise respectively and the noise strength is adjustable. With suitable noise strengths, the chaotic motions of double Mackey-Glass system can be eliminated, and the motions converge to zero.

4.2.1 Gaussian noise

The noise that has a probability density function (PDF) of the normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \tag{4.3}$$

is called Gaussian distributed noise, where μ is the mean and σ^2 is the variance of the random variable. The Simulink Communication toolbox provides the Gaussian noise generator block. In our case, we take the mean as 0 and the variance as 1. Therefore, μ is a constant vector and K is a constant matrix.

Parameter b and parameter r of Eq. (4.1) are substituted respectively by p_1F_1 where F_1 is Gaussian noise and p_1 is the noise strength. When b is replaced, the chaotic behavior is suppressed and the system is asymptotically stable at the origin as $p_1 < 0.6$. Fig 4.1 shows the time histories of the variables x_1 and x_2 with noise strength $p_1 = 0.5$. When r is replaced, the trajectories gradually increase unbounded when $p_1 > 0.5$ and chaotic behavior cannot eliminated with any noise strength.

4.2.2 Rayleigh noise

The probability density function of Rayleigh distributed noise is

$$f(x) = \begin{cases} \dfrac{x}{\sigma^2} \exp\left(\dfrac{-x^2}{2\sigma^2} \right) & x \ge 0 \\ 0 & x < 0 \end{cases} \tag{4.4}$$

where σ^2 is known as the fading envelope of the Rayleigh distribution. The Simulink Communication toolbox provides the Rayleigh noise generator block. We specify $\sigma = 1$ in the case.

Parameter r of Eq. (4.1) is substituted by $p_2 F_2$ where F_2 is Rayleigh noise and p_2 is the noise strength. The chaotic motion of the system can be eliminated when $p_2 \ge 0.165$. In other words, noise excitation of parameters makes the double Mackey-Glass system asymptotically stable at the origin. The time histories of the variables x_1 and x_2 with noise strength 0.2 are shown in Fig. 4.2.

4.2.3 **Rician noise**

The probability density function of Rician distributed noise is

$$f(x) = \begin{cases} \dfrac{x}{\sigma^2} I_0\left(\dfrac{mx}{\sigma^2} \right) \exp\left(-\dfrac{x^2 + m^2}{2\sigma^2} \right) & x \ge 0 \\ 0 & x < 0 \end{cases} \tag{4.5}$$

where σ is the standard deviation of the Gaussian distribution that underlies the Rician distribution noise, I_0 is the modified 0th-order Bessel function of the first kind given by

$$I_0(y) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{y\cos t} dt \tag{4.6}$$

And m is defined as $m^2 = m_I^2 + m_Q^2$ where m_I and m_Q are the mean values of two independent Gaussian components. The Simulink Communication toolbox provides the Rician noise generator block. We assign that $\sigma = 1$ and K-factor 2 in the case, which the K-factor has a definition as a form of $K = m^2 / 2\sigma^2$.

Parameter r of Eq. (4.1) is substituted by $p_3 F_3$ where F_3 is Rician noise and p_3 is the noise strength. The chaotic motion of the system can be eliminated as $p_3 > 0.08$. Numerical simulation, illustrated in Fig. 4.3, shows that the motion is asymptotically stabilized to the equilibrium point (0,0,0) by the noise excitation method with noise strength $p_3 = 0.1$.

4.2.4 **Uniform noise**

The probability density function of uniform distributed noise is

$$f(x) = \begin{cases} \frac{1}{d-c} & if\ c \le x \le d \\ 0 & otherwise \end{cases} \tag{4.7}$$

The mean value of this density function μ and its variance σ are given as follow:

$$\mu = \frac{c+d}{2} \tag{4.8}$$

$$\sigma^2 = \frac{(d-c)^2}{12} \tag{4.9}$$

The Simulink Communication toolbox provides the Rician noise generator block. We specify lower bound $c = 0$ and upper bound $d = 1$ in the case.

Parameter r of Eq. (4.1) is substituted by $p_4 F_4$ where F_4 is uniform noise and p_4 is the noise strength. When $p_4 > 0.4$, the chaotic motion can be eliminated and the system is asymptotically stable at the origin. Fig. 4.4 illustrates the time histories of the states of the system with noise strength $p_4 = 0.5$.

4.3 Numerical simulations of chaos synchronizations

In this section, we use the parameter excited method by replacing the corresponding parameters by Gaussian noise, Rayleigh noise, Rician noise and uniform noise respectively, to synchronize two uncoupled double Mackey-Glass systems. The system parameter b and r are substituted by noise respectively and the noise strength is variable. The error states which are defined as $e_i = x_i - y_i$, $(i = 1,2)$ will converge to zero as $t \to \infty$ when the strength is chosen properly. The results of simulations show that the synchronizations are successfully achieved via parameter excited method in major cases.

4.3.1 Gaussian noise

We replace two corresponding parameters b and two corresponding parameters r of the systems (4.1) and (4.2) by $p_1 F_1$ respectively where F_1 is Gaussian noise and p_1 is the noise strength. When b is replaced, the trajectory of the states converge to zero with $p_1 < 0.6$. When the strength is increased, the error states oscillate. However, in a small range of 0.61~0.625, the systems show temporary chaos synchronization. Fig. 4.5 shows error e_1, e_2 and the time histories of the state variables with noise strength $p_1 = 0.625$. As r is replaced, the trajectory of the states gradually increase unbounded when the strength is larger than 0.5. In other words, Gaussian noise excitation can be used only when the noise strength $p_1 < 0.5$ to synchronize two identical double Mackey-Glass systems with different initial conditions. Fig. 4.6 shows error e_1, e_2 and the time histories of the state variables with noise strength $p_1 = 0.05$.

4.3.2 Rayleigh noise

We replace two corresponding parameters b and two corresponding parameters

r of the systems (4.1) and (4.2) by p_2F_2 respectively where F_2 is Rayleigh noise and p_2 is the noise strength. The synchronizations are successfully achieved in both cases. When b is replaced, we assume that the noise strength

$$p_2 = 0.25i\,, \quad i = 1,2,...,50 \tag{4.10}$$

Fig. 4.7 shows the result of the simulation. We find that the synchronizations of two double Mackey-Glass systems are achieved with major noise strength, but failed with minor cases. Fig. 4.8 and Fig. 4.9 show the error states and the phase portraits of the systems with $p_2 = 0.5$ and $p_2 = 9.25$. The error states approach to zero in the former case, but not in the latter case. However, if we choose the strength appropriately, the chaos synchronizations are accomplished.

When two corresponding r are substituted, only a small interval of the noise strength $0.105 \le p_2 \le 0.16$ leads to synchronization. Error e_1, e_2 and the phase portraits of the systems with $p_2 = 0.16$ are shown in Fig. 4.10. Besides, in a small range of 0.08~0.1, the systems show temporary chaos synchronization. Fig. 4.11 shows error e_1, e_2 and the time histories of the state variables with noise strength $p_2 = 0.08$. For the values of p_2 other than these ranges, chaos synchronization cannot be obtained.

4.3.3 Rician noise

We replace two corresponding parameter b and two corresponding parameters r of the system (4.1) and (4.2) with p_3F_3 respectively where F_3 is Rician noise and p_3 is the noise strength. In the case of b, we assume that the noise strength

$$p_3 = 0.25i\,, \quad i = 1,2,...,50 \tag{4.11}$$

As shown in Fig. 4.12, the Rician noise is more effective than Rayleigh noise. In the range of $0.25 \le p_3 \le 12.5$, the synchronization is achieved except p_3 takes 1.25, 4.75, 11.25, 12 and 12.25. Fig. 4.13 shows the error states and the phase portraits of the systems with noise strength $p_3 = 5$.

In the case of r, synchronization is obtained only when $0.06 \le p_3 \le 0.08$. Error e_1, e_2 and the phase portraits of the systems with noise strength $p_3 = 0.07$ are given in Fig. 4.14. The error states oscillate when $p_3 < 0.06$ and the state variables of the system converge to zero as $p_3 > 0.08$. Chaos synchronization cannot obtain for the values of p_2 other than 0.105~0.16.

4.3.4 Uniform noise

We replace two corresponding parameters b and two corresponding parameters r of the system (4.1) and (4.2) by $p_4 F_4$ respectively where F_4 is uniform noise and p_4 is the noise strength. In the case of b, we assume that the noise strength

$$p_3 = 0.25i, \quad i = 1,2,\ldots,50 \tag{4.13}$$

As shown in Fig. 4.15, the synchronization is achieved except for a few p_4 in the range of $0.25 \le p_4 \le 12.5$. Fig. 4.16 shows the error states and the phase portraits of the systems with noise strength $p_4 = 10.25$.

In the case of r, synchronization is obtained only when $0.26 \le p_4 \le 0.4$. Error e_1, e_2 and the phase portraits of the systems with noise strength $p_4 = 0.27$ are given in Fig. 4.17. The error states oscillate while $p_4 < 0.26$ and the states of the system converge to zero as $p_4 > 0.4$. Two systems cannot be synchronized with the values of the noise strength other than 0.26~0.4.