Chapter 5

Robust chaos lag synchronization of double Mackey-Glass system by noise excitation of parameters

In this Chapter, the lag synchronization of two uncoupled double Mackey-Glass systems is achieved via the parameter excited method. This method is accomplished by replacing the corresponding parameters of the systems with two lag noise signals. By means of the difference of the timing between two replacements for the first system and the second system, the lag synchronization can be obtained.

5.1 Lag synchronization of double Mackey-Glass system by parameter excited method

We consider a double Mackey-Glass system described by

$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1+x_{1\tau}^n} - r_1 x_1 \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1+x_{2\tau}^n} - r_2 x_2 - kx_1 \end{cases} \tag{5.1}$$

and a second identical double Mackey-Glass system described by

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1+y_{1\tau}^n} - r_1 y_1 \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1+y_{2\tau}^n} - r_2 y_2 - ky_1 \end{cases} \tag{5.2}$$

where x_1, x_2, y_1, y_2 are state variables and $x_{i\tau} = x_i(t-\tau)$, $y_{i\tau} = y_i(t-\tau)$ $(i=1,2)$, τ is a time delay, and b, r_1, r_2, n, k are constant parameters. The parameters and the time delay are chosen as follows: $b=0.2$, $r_1=r_2=0.1$, $n=10$, $k=1$ and $\tau=20$. The initial values are given as $(x_{10}, x_{20}) = (0.1, 0.1)$ and $(y_{10}, y_{20}) = (0.2, 0.2)$.

The lag synchronization is obtained by using the control scheme called parameter excited method. The designated parameter is replaced by a Rayleigh noise signal, but there exist a time difference between two replacements for the first system and for the second system. The parameter of the first system is substituted by a noise at $t = 0\sec$, and the parameter of the second system is substituted by the noise at $t = d\sec$. In other words, the control schemes do not work sy nchronously for these two systems. The illustrations will show that the system (1) and system (2) are in lag synchronization.

5.2 Numerical simulation results of lag synchronizations

All simulations are carried out by Simulink environment of MATLAB. By replacing the corresponding parameter b, r_1, r_2 or k by a Rayleigh noise signal respectively, lag synchronizations of two uncoupled double Mackey -Glass systems can be achieved with appropriate noise strengths. Errors are defined as $e_1(t) = x_1(t) - y_1(t+d)$, $e_2(t) = x_2(t) - y_2(t+d)$, where *d* is the lag of the states of the second system lag behind the states of the first system and also the time difference of the control schemes acting on these two systems. In our study, *d* is kept a constant, *d*=30. e_1 and e_2 will converge to zero as $t \to \infty$ and the lag synchronization is obtained.

Firstly, two corresponding parameters b of systems (1) and (2) are replaced by pN where N is a Rayleigh noise and p is the noise strength. In this case, we take the noise strength

$$p = 0.25i, \quad i = 1,2,...,50 \tag{5.3}$$

The simulation results are shown in Fig. 5.1. It is found that the lag synchronization is successfully achieved with most noise strengths. Fig. 5.2 shows the error states e_1, e_2 and the time histories of $x_i, y_i \, (i = 1,2)$ with noise strength $p = 11$. Lag synchronization is accomplished when $t > 4000\sec$. It is noted that some lag synchronizations need more time (>30000sec). For instance, in Fig. 5.3, when the noise strength is taken as $p = 8.5$, the error states converge to zero at $t > 37500\sec$.

Then the corresponding parameters r_1 and r_2 are replaced by a Rayleigh noise signal pN where p is the noise strength. When the noise strength is in the range of $0.105 \le p \le 0.16$, the lag synchronization is obtained. Error e_1, e_2 and the time histories of the state variables with noise strength $p = 0.15$ are given in Fig. 5.4. As $p \ge 0.165$, the state variables of the system (5.1) and (5.2) approach zero and Fig. 5.5 shows the time histories of $x_i, y_i\,(i = 1,2)$ with noise strength $p = 0.165$. No lag synchronization is found in the rest range of the noise strength.

Next, we replace two corresponding parameters r_1 and k of the systems (5.1) and (5.2) by pN where N is a Rayleigh noise and p is the noise strength. As the noise strength in the range of $0.02 \le p \le 0.05$ and $0.105 \le p \le 0.16$, the lag synchronization can be accomplished. Fig. 5.6 and Fig. 5.7 show the error states and the time histories of the states variables of two systems with noise strengths $p = 0.03$ and $p = 0.12$ respectively. When the noise strength p is taken between two foregoing ranges, $0.05 < p < 0.105$, a phenomenon called temporary lag synchronization (TLS) is found. Fig. 5.8 shows the error states and the time histories of the state variables with $p = 0.103$. When the noise strength decreases as $p \le 0.01$, the error state e_1 converge to zero and the lag synchronization for x_1 and y_1 is achieved. However, the error state e_2 is chaotic and the lag synchronization for x_2 and y_2 can not be obtained. This phenomenon is called *partial lag synchronization*. The error states e_1, e_2 and the time histories of x_1, y_1 are shown in Fig. 5.9. When the noise strength increases to $p > 0.16$, the trajectories of x_1, y_1 approach to zero and the difference between x_2 and y_2 is chaotic. The error states e_1, e_2 and the time histories of x_1, y_1 are shown in Fig. 5.10.

In order to verify the robustness of lag synchronization, a small disturbance

$\varepsilon(x_1 - y_1)\cos t$ is added in two double Mackey-Glass systems:

$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1+x_{1\tau}^n} - r_1 x_1 + \varepsilon(x_1 - y_1)\cos t \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1+x_{2\tau}^n} - r_2 x_2 - kx_1 \end{cases} \tag{5.4}$$

and

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1+y_{1\tau}^n} - r_1 y_1 + \varepsilon(x_1 - y_1)\cos t \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1+y_{2\tau}^n} - r_2 y_2 - ky_1 \end{cases} \tag{5.5}$$

where ε is a small number which is taken as 10^{-5}. The lag synchronization is accomplished as well via the parameter excited method. In the case of replacing b, Fig. 5.11 and Fig. 5.12 show the error states and the time histories of the state variables of systems (5.4) and (5.5) with different noise strengths. One can find that the error states approach to zero in the case with $p = 8.5$ and the lag synchronization is obtain temporarily in the case with $p = 5.25$ which is defined as temporary lag synchronization (TLS). In the cases of replacing r_1, r_2 and r_1, k, Figs. 5.13 and 5.14 indicate that the error states practically approach zero which imply that lag synchronization by parameter excited method is robust in the presence of small disturbances.

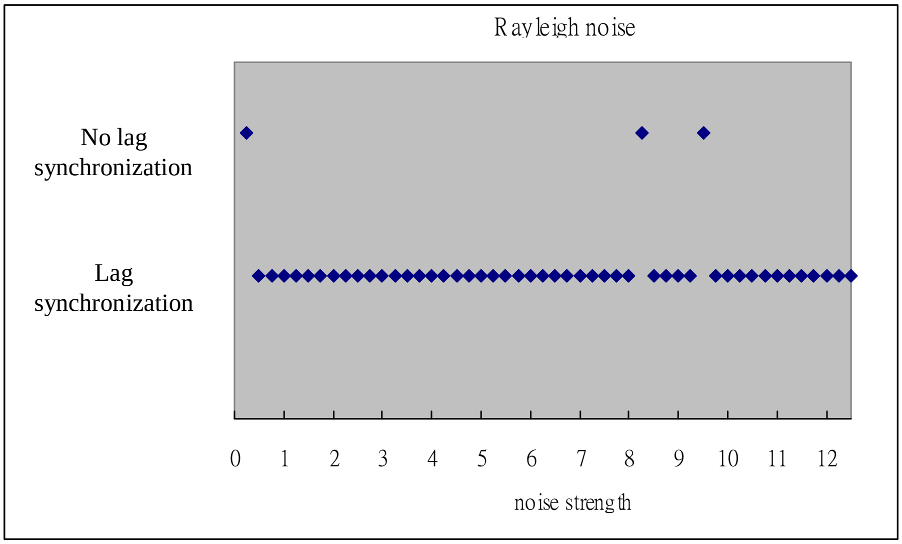

Fig. 5.1 Two corresponding parameters b are substituted by a Rayleigh noise with different noise strengths.

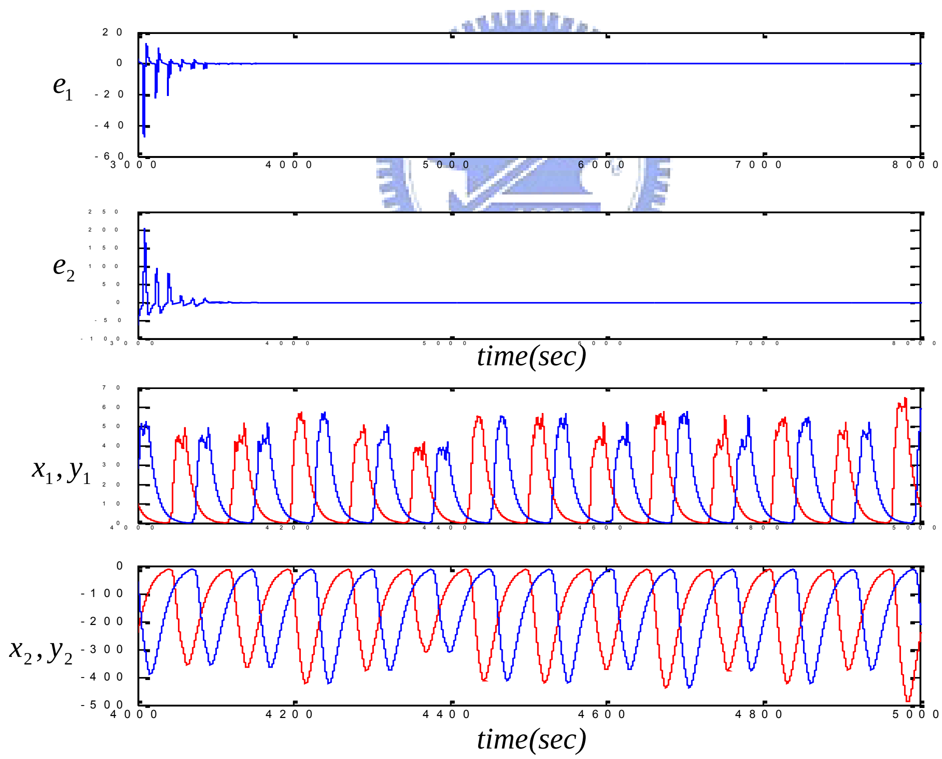

Fig. 5.2 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 11$.

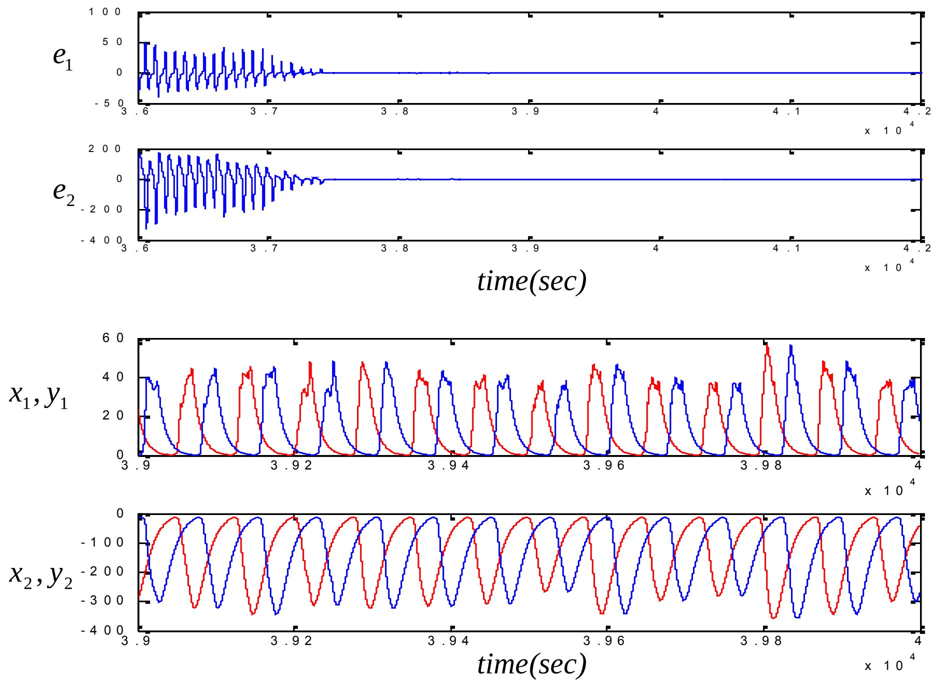

Fig. 5.3 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 8.5$.

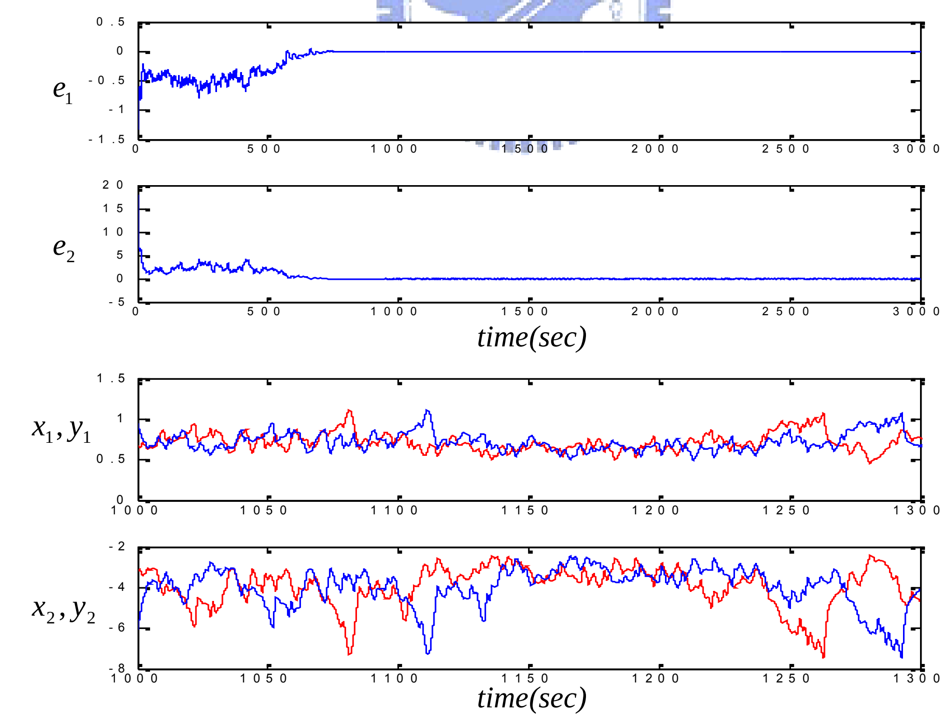

Fig. 5.4 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r are substituted by a Rayleigh noise with noise strength $p = 0.15$.

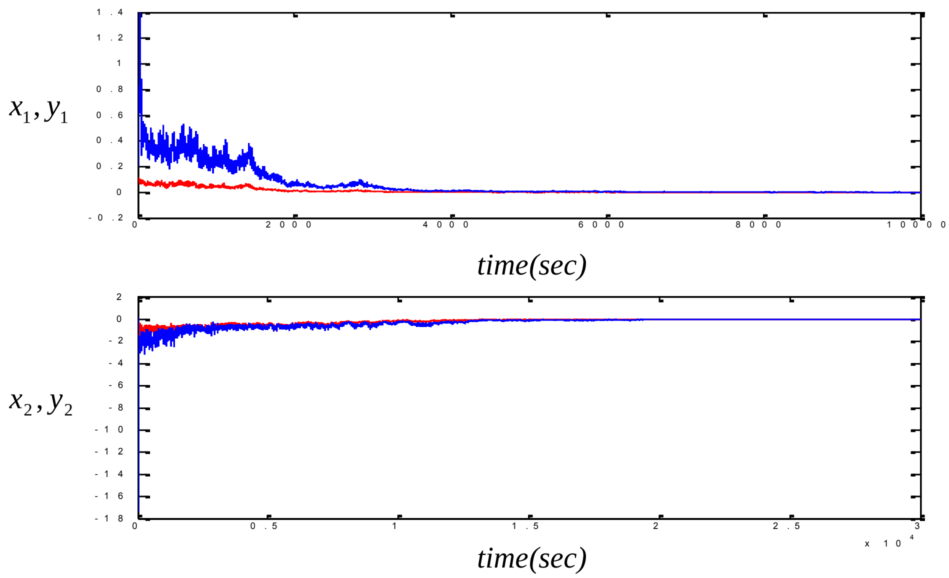

Fig. 5.5 Chaos control by parameter excited method. The time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass system when parameter r is substituted by a uniform noise with noise strength $p = 0.165$.

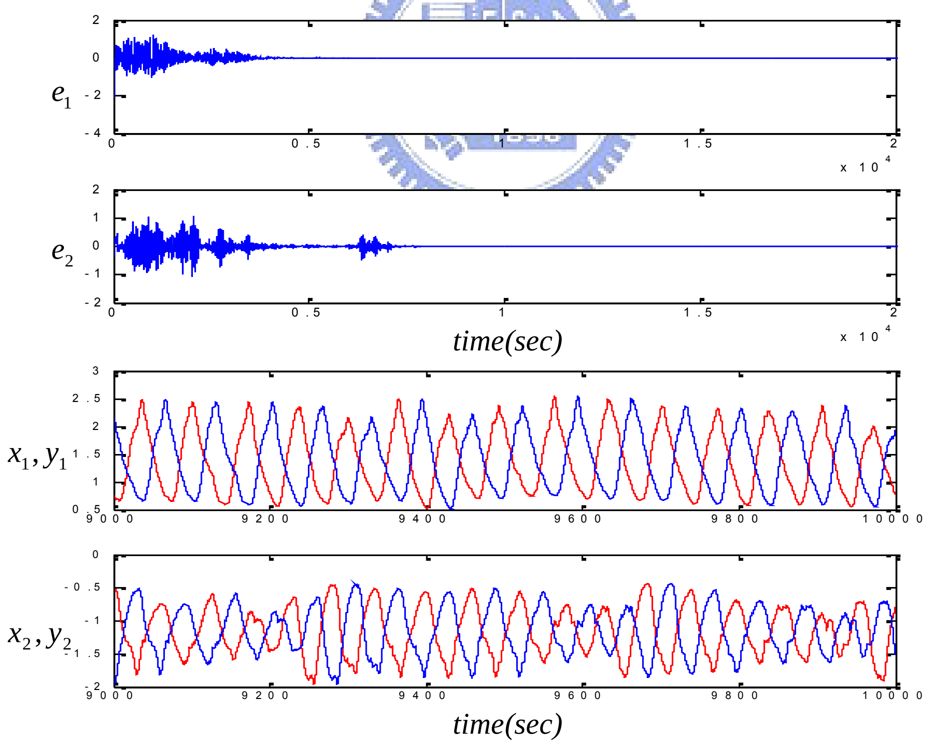

Fig. 5.6 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.03$.

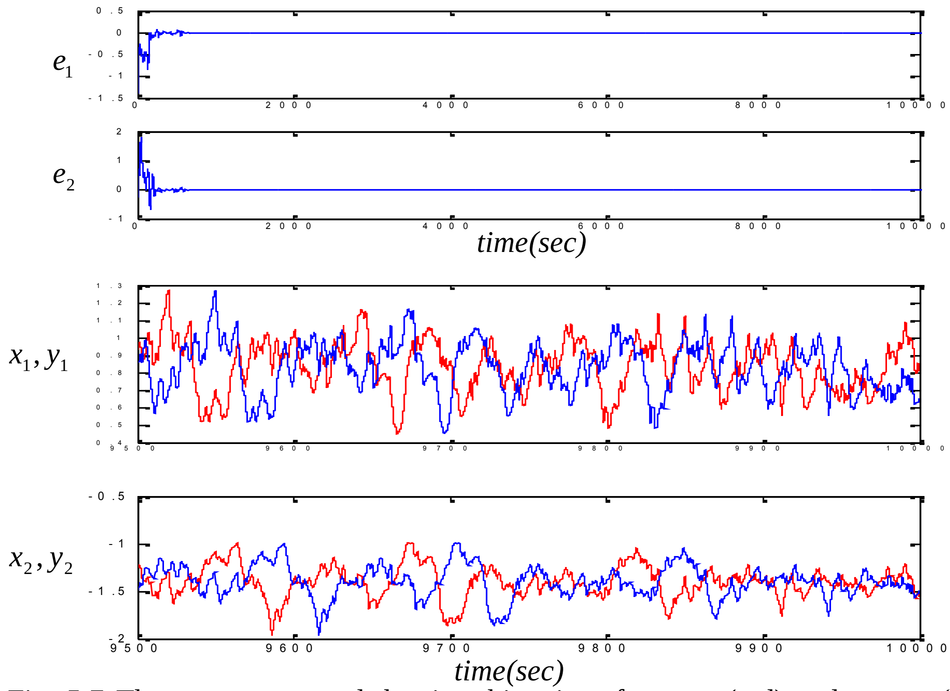

Fig. 5.7 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.12$.

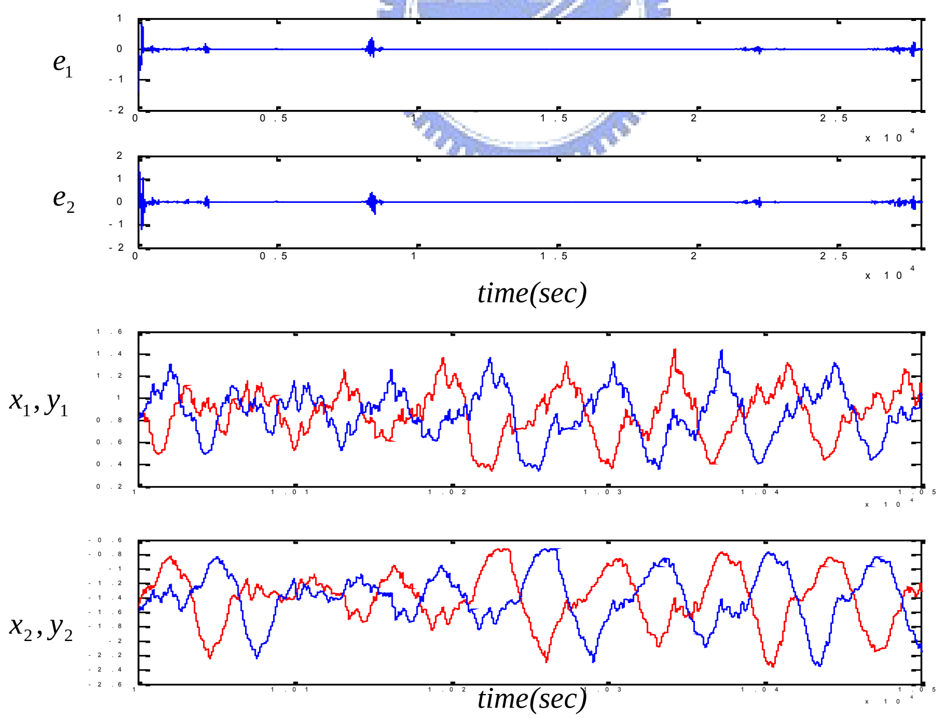

Fig. 5.8 Temporary lag synchronization. The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.103$.

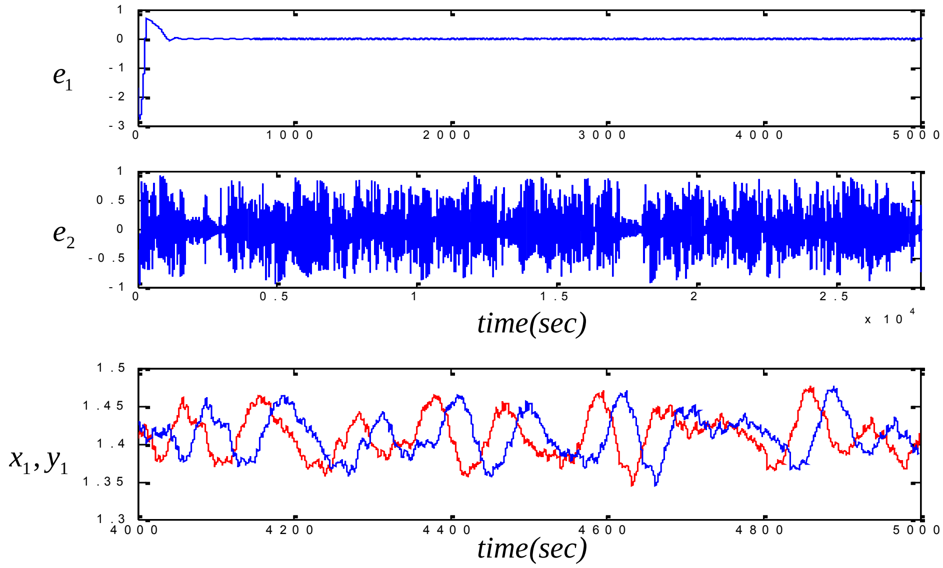

Fig. 5.9 Partial lag synchronization. The error states and the time histories of x_1 (red) and y_1 (blue) of the double Mackey-Glass systems when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.005$.

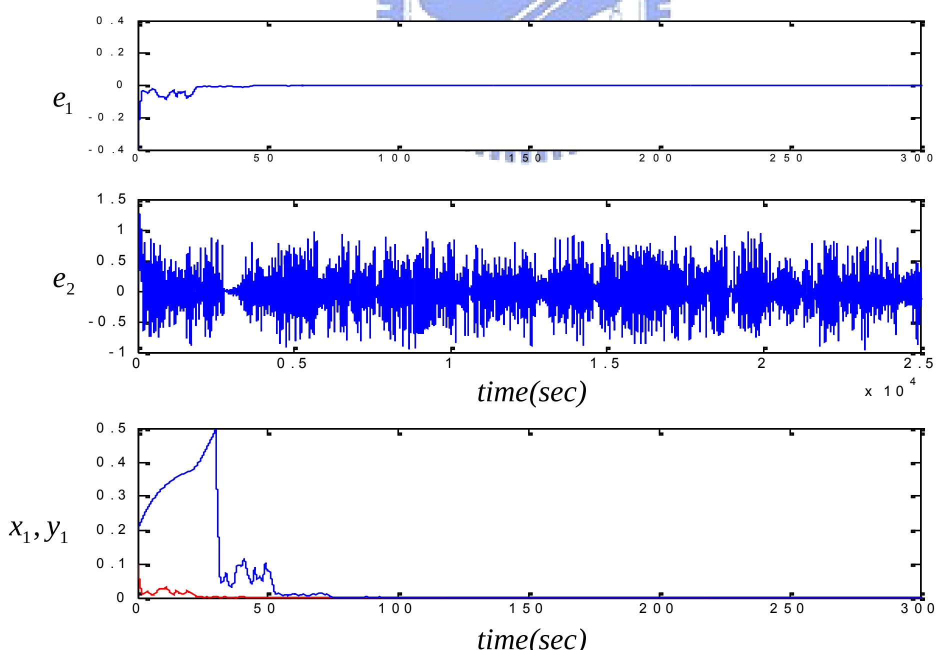

Fig. 5.10 Partial chaos control by parameter excited method. The error states and the time histories of x_1 (red) and y_1 (blue) of the double Mackey-Glass systems when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 1$.

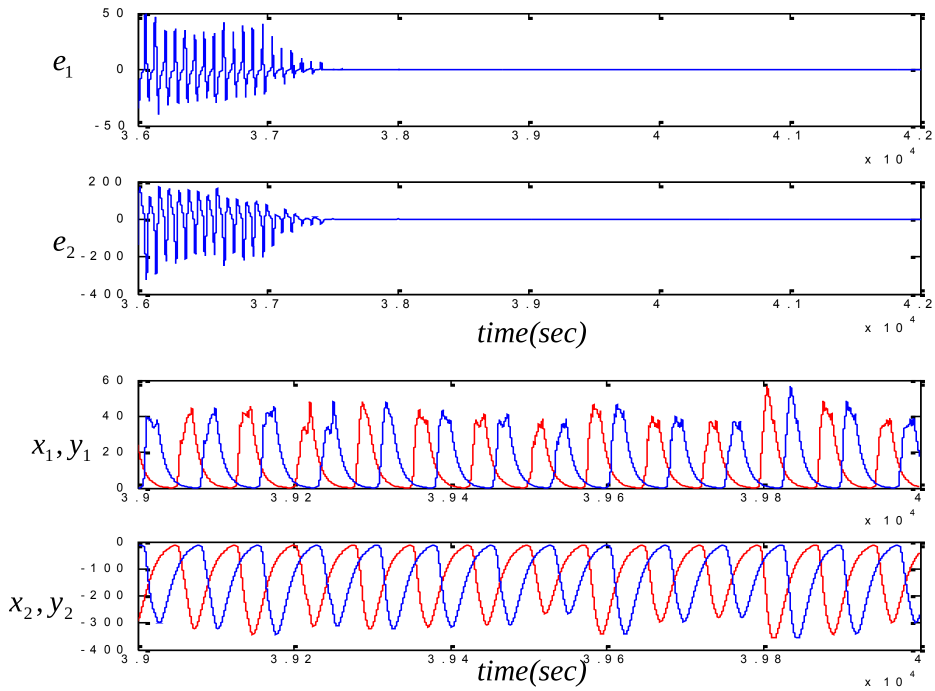

Fig. 5.11 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 8.5$ in presence of a small disturbance.

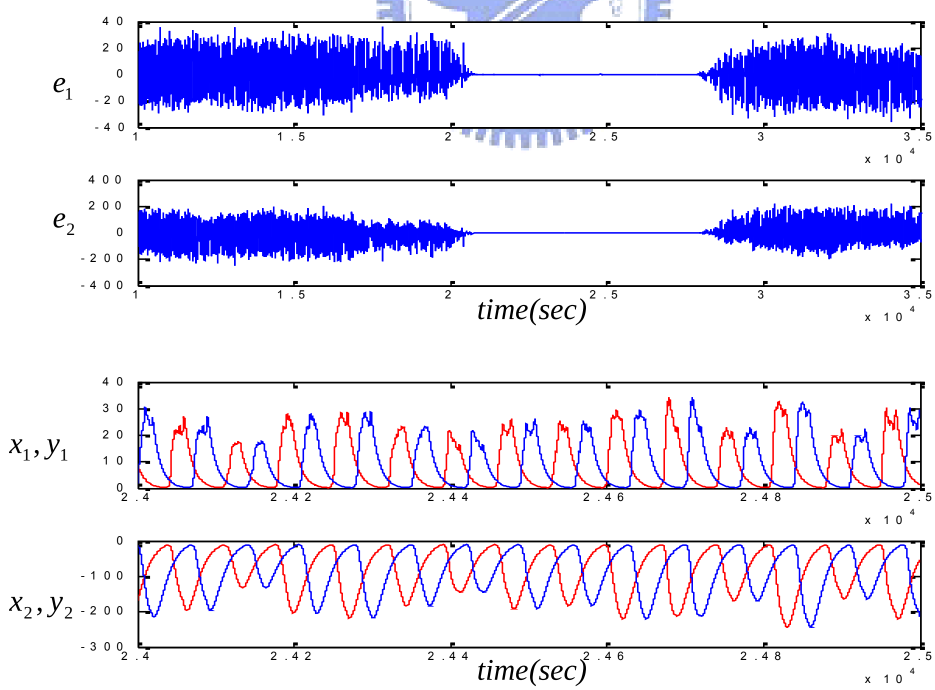

Fig. 5.12 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 5.25$ in presence of a small disturbance.

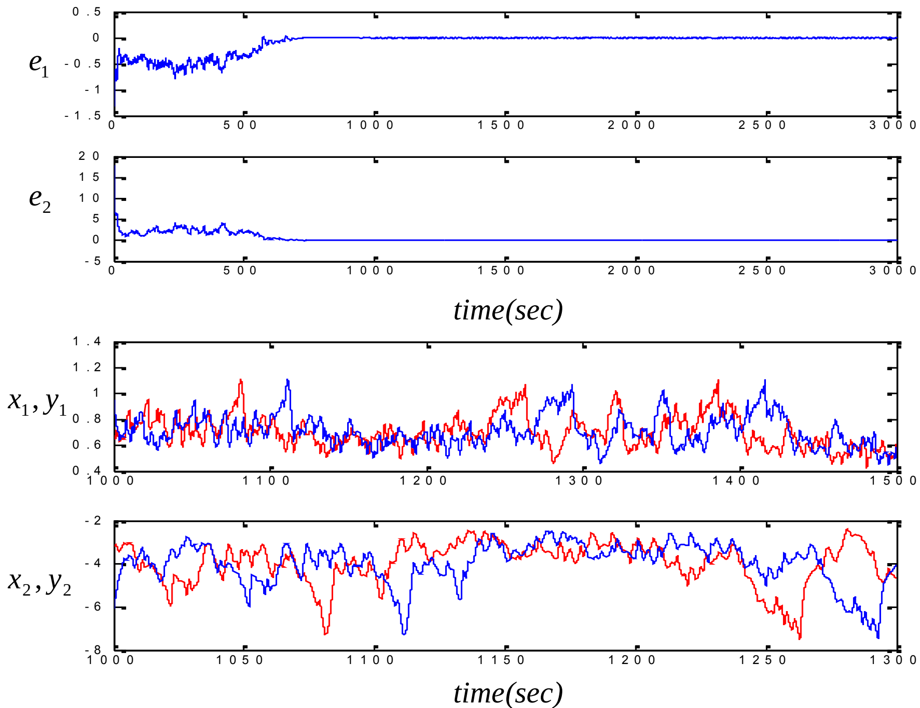

Fig. 5.13 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters r are substituted by a Rayleigh noise with noise strength $p = 0.15$ in presence of a small disturbance.

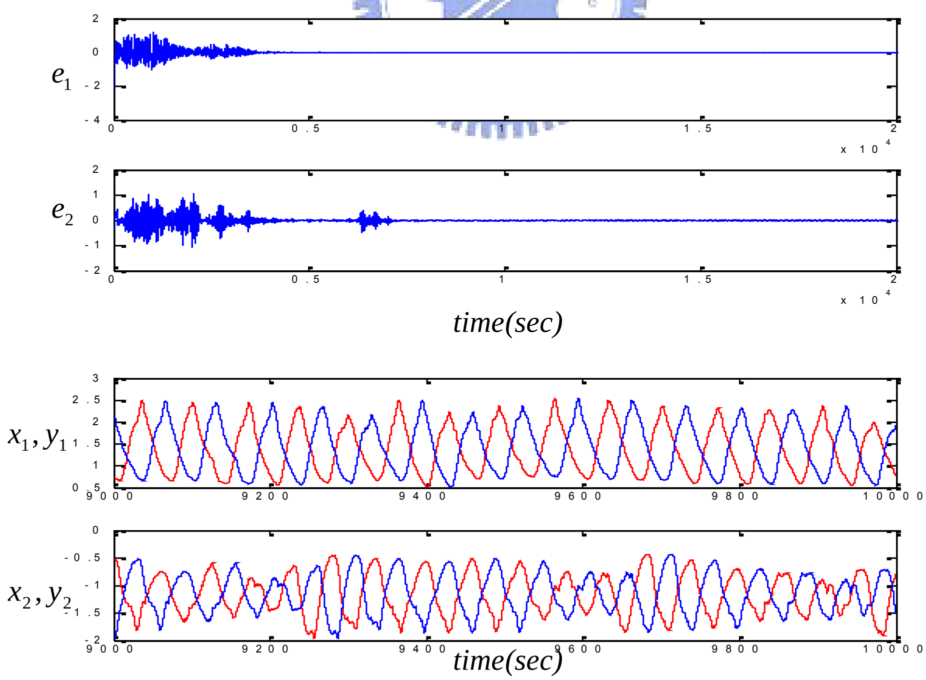

Fig. 5.14 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.03$ in presence of a small disturbance.