Chapter 3

Temporary Lag and Anticipated Synchronization and Anti-Synchronization of Uncoupled Double Mackey-Glass Systems

Without any control scheme and coupling terms, temporary lag or anticipated synchronization and the lag or anticipated anti-synchronization of double Mackey-Glass systems with small and similar initial conditions are discovered. When all initial conditions are positive, the lag synchronization is obtained. The negative initial values make the time history inverse and temporary lag anti-synchronization occurs. The phenomena both appear intermittently.

3.1 Temporary lag and anticipated synchronization and temporary lag and anticipated anti-synchronization of uncoupled time-delayed chaotic systems

Consider the first time-delay chaotic system

$$\dot{x} = f(x, x_\tau, t) \tag{3.1}$$

and second time-delay chaotic system

$$\dot{y} = f(y, y_\tau, t) \tag{3.2}$$

where $x, y \in R^n$ are n-dimensional state vectors, $x_\tau = x(t-\tau)$, $y_\tau = y(t-\tau)$ are corresponding time-delay state vectors, and $f : R^n \to R^n$ defines a vector function in n-dimensional space. The error are defined as $e = x(t-T) - y(t)$. If the following conditions hold, the systems are in temporary lag synchronization.

$$e_i = x_{iT_j} - y_i = 0, \quad i = 1,2,\ldots., p \le n, \quad j = 1,2,\ldots., m \qquad \text{for} \quad t_{iT_{j1}} \le t \le t_{iT_{j2}} \tag{3.3}$$

where x_i, y_i are the state vectors of the system, T_j is the time which y_i lag behind x_i in the j-th intervals. When T_j is negative, we have temporary anticipated synchronization.

In the case of anti-synchronization, the states of the systems which have opposite signs, the error $e = x(t-T) + y(t)$ will converge to zero. Therefore, we can say the temporary lag anti-synchronization is achieved when the following conditions are satisfied:

$$e_i = x_{iT_j} + y_i = 0\,, \quad i = 1,2,....,p \le n\,, \quad j = 1,2,....,m \qquad \text{for } t_{iT_{j1}} \le t \le t_{iT_{j2}} \qquad (3.4)$$

where x_i, y_i are the state vectors of the system, T_j is the time which y_i lag behind x_i in the j-th intervals. When T_j is negative, we have temporary anticipated anti-synchronization.

3.2 The lag and anticipated synchronization of two identical double Mackey-Glass systems

We consider two double Mackey-Glass systems :

$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1+x_{1\tau}^n} - rx_1 \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1+x_{2\tau}^n} - rx_2 - x_1 \end{cases} \qquad (3.5)$$

and

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1+y_{1\tau}^n} - ry_1 \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1+y_{2\tau}^n} - ry_2 - y_1 \end{cases} \qquad (3.6)$$

The variables x_1, x_2, y_1, y_2 are the concentration of the mature blood cells in the blood, and $x_{1\tau}$, $x_{2\tau}$, $y_{1\tau}$, $y_{2\tau}$ are presented the request of the cells which is made after τ seconds, i.e. $x_{i\tau} = x_i(t-\tau)$, $y_{i\tau} = y_i(t-\tau)$, $(i = 1,2)$. The time delay τ indicates the difference between the time of cellular production in the bone marrow and of the release of mature cells into the blood.

In our study, we keep the delay time fixed in 20 second ($\tau = 20$) and the parameters are shown as follow: $b = 0.2$, $r = 0.1$, and $n = 10$. All the numerical simulations are implemented by Matlab. The initial conditions we choose are constant, i.e. the variable $x(t+\theta)$ maintains a constant for all $\theta \in (-\tau, 0)$.

Fig. 3.1 shows the time histories of double Mackey-Glass system with initial conditions (x_{10}, x_{20}) = (0.001, 0.001), (y_{10}, y_{20}) = (0.0015, 0.0015) respectively. Because the similar characteristics exist for x_1, y_1 and for x_2, y_2, we only draw the time histories of x_1, y_1 (Fig. 3.1 (a)~(f)) and the time histories of error, $e_1 = x_{1T_j} - y_1$ (Fig. 3.1 (g)~(l)). From Fig. 3.1, the temporary lag and anticipated synchronizations appear intermittently. Anticipated synchronizations are more than lag synchronization. In Table II, we marshal the length of the temporary lag (anticipated) synchronization and the lag (anticipated) of x_1 to y_1 and x_2 to y_2, which are varied in each intervals. There are two lag synchronous intervals and four anticipated synchronous intervals between 30000 seconds. Notice that the longest interval occur at the first interval, about 1200 seconds. Others are hundreds seconds long.

We also find the trend of decreasing the length of the temporary synchronization with increasing initial conditions. As the initial values increase, the time intervals for temporary lag or anticipated synchronization decrease. Table III show the lengths of the first time interval where the initial values are varied from 0.00001 to 0.1, L_1 and L_2 indicate the length of first temporary synchronization of x_1, y_1 and of x_2, y_2, respectively. From the curve fitting presented in Fig. 3.2 and Fig. 3.3, the relations between L_1, L_2 and x_{10}, x_{20} are obtained as follow:

$$L_1 = -229.93\ln(x_{10}) - 262.06 \tag{3.7}$$

and

$$L_2 = -229.88\ln(x_{20}) - 261.58 \quad (3.8)$$

They are essentially identical.

Table II. The length of temporary lag (anticipated) synchronization and the lag (lead) of x_1, x_2 to y_1, y_2.

	x_1, y_1			x_2, y_2		
	time intervals (sec)	length of temporary synchronization (sec)	lag of y_1 to x_1 (sec)	time intervals (sec)	length of temporary synchronization (sec)	lag of y_2 to x_2 (sec)
1	0—1187	1187	−17	0—1194	1194	−17
2	8730—9215	485	−37	8740—9360	620	−38
3	14630—15000	370	8	14640—15010	370	8
4	18103—18611	508	−77	18111—18658	547	−77
5	19387—19983	596	−55	19390—19990	600	−55
6	28580—29010	430	7	28530—28980	450	6

Table III. The lengths of the first time intervals of TLS and TAS where the initial values are varied from 0.00001 to 0.1.

Initial conditions $(x_{10} = x_{20}, y_{10} = y_{20})$	L_1	L_2
$(10^{-5}, 1.5\times10^{-5})$	2593	2593
$(5\times10^{-5}, 7.5\times10^{-5})$	1759	1759
$(10^{-4}, 1.5\times10^{-4})$	1683	1683
$(5\times10^{-4}, 7.5\times10^{-4})$	1806	1806
$(10^{-3}, 1.5\times10^{-3})$	1187	1186
$(5\times10^{-3}, 7.5\times10^{-3})$	843	843
(0.01,0.015)	1031	1033
(0.05,0.075)	382	382
(0.1,0.15)	231	231

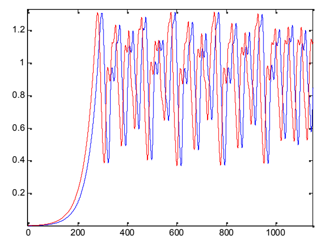
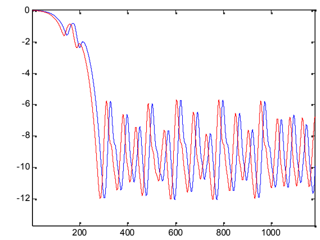
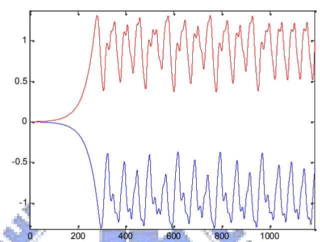
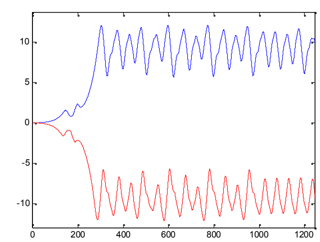
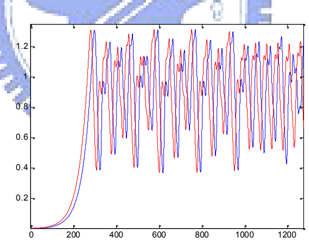
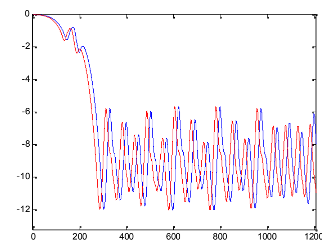
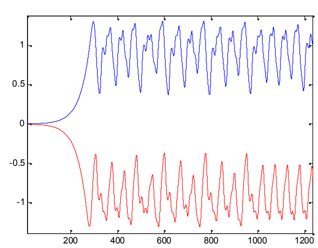
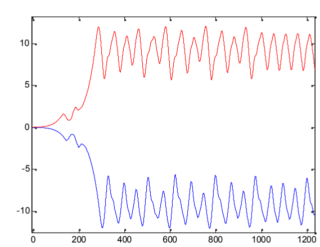
3.3 The lag and anticipated anti-synchronization of two identical double Mackey-Glass systems

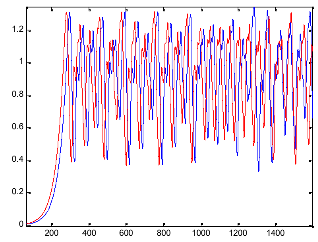
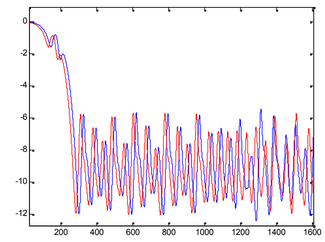
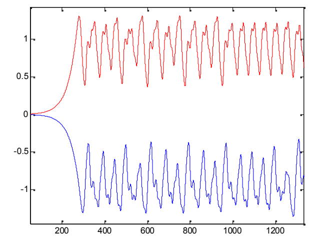
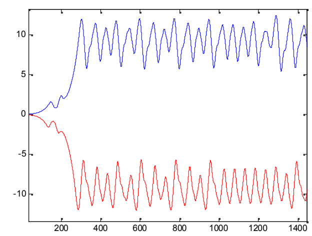
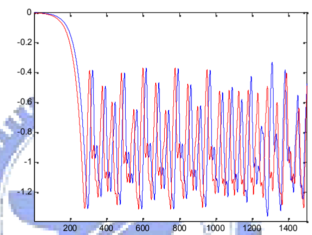
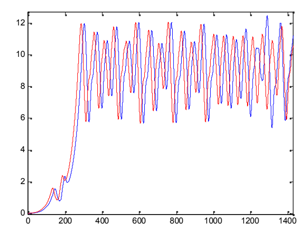
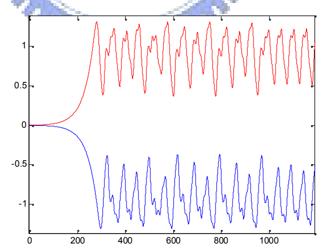
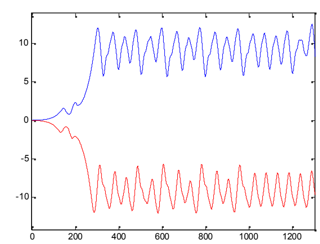
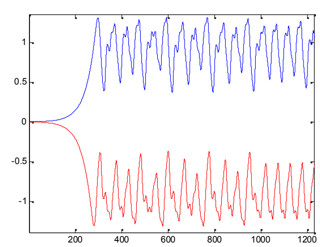
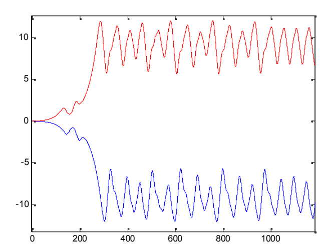
In this section, we add one, two, three or four minus sign to the initial conditions, TLS and TLAS occur alternatively.

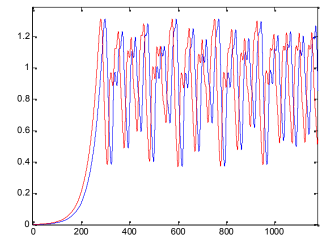
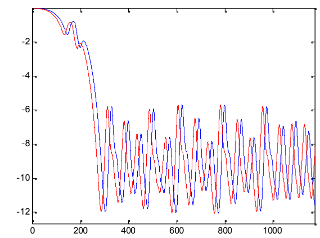
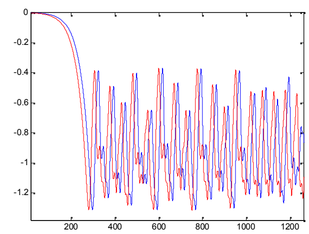
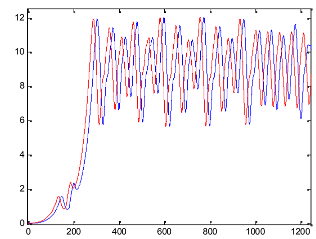
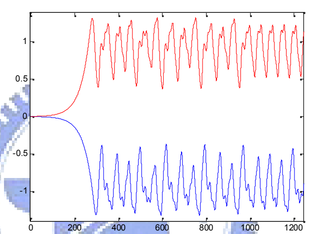
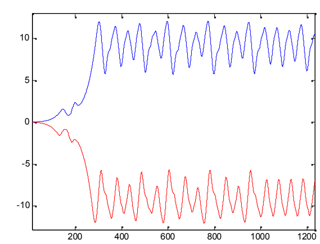
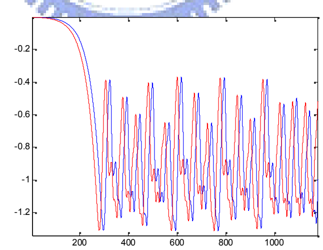
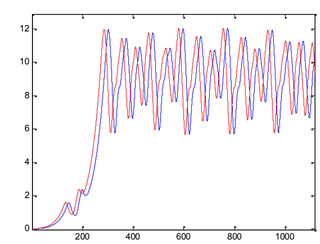
Table IV shows the results of the simulations. There are interesting phenomena. The minus sign makes the original time history inverse but with same magnitude, i.e. two time histories are symmetric to the abscissa. From Case 1~4, it is found that the inverse effect only appears when the initial condition x_{10} or y_{10} is negative. On the contrary, it does not work for x_{20} and y_{20}. The trajectories of x_1 and x_2 are upside down as x_{10} is negative, and the trajectories of y_1 and y_2 show the similar characteristics with negative y_{10}. In these two cases, the anticipated anti-synchronizations exist. Because the negative initial conditions x_{20}, y_{20} have no influence on the systems, there are still anticipated synchronizations in Case 2 and 4. Case 5~9 show the results where there are two negative initial conditions at the same time. In Case 5 and 7, only the inverses of x_1 and x_2 occur, so two systems are in anticipated anti-synchronization. Case 6 and 9 maintain anticipated synchronization because both trajectories are opposite in the former case and no inversion exists in the latter case. Case 8 shows the anticipated anti-synchronization where the trajectory of y_1 and y_2 is reversed. Finally, the simulations where there are three and four negative initial values, are presented respectively. It is easy to know that Case 10 is the same as Case 6 and Case 11 and Case 1 are quite alike.

According to the symmetric relations between cases with negative initial conditions and the original cases, the lengths of the lag anti-synchronizations and the lags of y_1 to x_1 are all invariant, just as that in Table II which is listed in Section 3.2.

Table IV. The time histories of double Mackey-Glass system with negative initial values.

Case	Initial conditions $(x_{10}, x_{20}),(y_{10}, y_{20})$	$x_1 : blue, y_1 : red$	$x_2 : blue, y_2 : red$
0	$(0.001, 0.001)$, $(0.0015, 0.0015)$		
		Anticipated synchronization	Anticipated synchronization
1	$(-0.001, 0.001)$, $(0.0015, 0.0015)$		
		Anticipated anti-synchronization	Anticipated anti-synchronization
2	$(0.001, -0.001)$, $(0.0015, 0.0015)$		
		Anticipated synchronization	Anticipated synchronization
3	$(0.001, 0.001)$, $(-0.0015, 0.0015)$		
		Anticipated anti-synchronization	Anticipated anti-synchronization

4	$(0.001, 0.001)$, $(0.0015, -0.0015)$	 Anticipated synchronization	 Anticipated synchronization
5	$(-0.001, -0.001)$, $(0.0015, 0.0015)$	 Anticipated anti-synchronization	 Anticipated anti-synchronization
6	$(-0.001, 0.001)$, $(-0.0015, 0.0015)$	 Anticipated synchronization	 Anticipated synchronization
7	$(-0.001, 0.001)$, $(0.0015, -0.0015)$	 Anticipated anti-synchronization	 Anticipated anti-synchronization
8	$(0.001, -0.001)$, $(-0.0015, 0.0015)$	 Anticipated anti-synchronization	 Anticipated anti-synchronization

9	$(0.001, -0.001)$, $(0.0015, -0.0015)$		
		Anticipated synchronization	Anticipated synchronization
10	$(-0.001, -0.001)$, $(-0.0015, 0.0015)$		
		Anticipated synchronization	Anticipated synchronization
11	$(-0.001, -0.001)$, $(0.0015, -0.0015)$		
		Anticipated anti-synchronization	Anticipated anti-synchronization
12	$(-0.001, -0.001)$, $(-0.0015, -0.0015)$		
		Anticipated synchronization	Anticipated synchronization

The time histories and the time histories of error $e_1 = x_{1T_j} + y_1$ with initial conditions $(x_{10}, y_{10}) = (-0.001, 0.001)$, $(x_{20}, y_{20}) = (0.0015, 0.0015)$ are shown in Fig. 3.4. Comparing with Fig. 3.1, nothing is changed except the i nverse of x_1 and y_1.

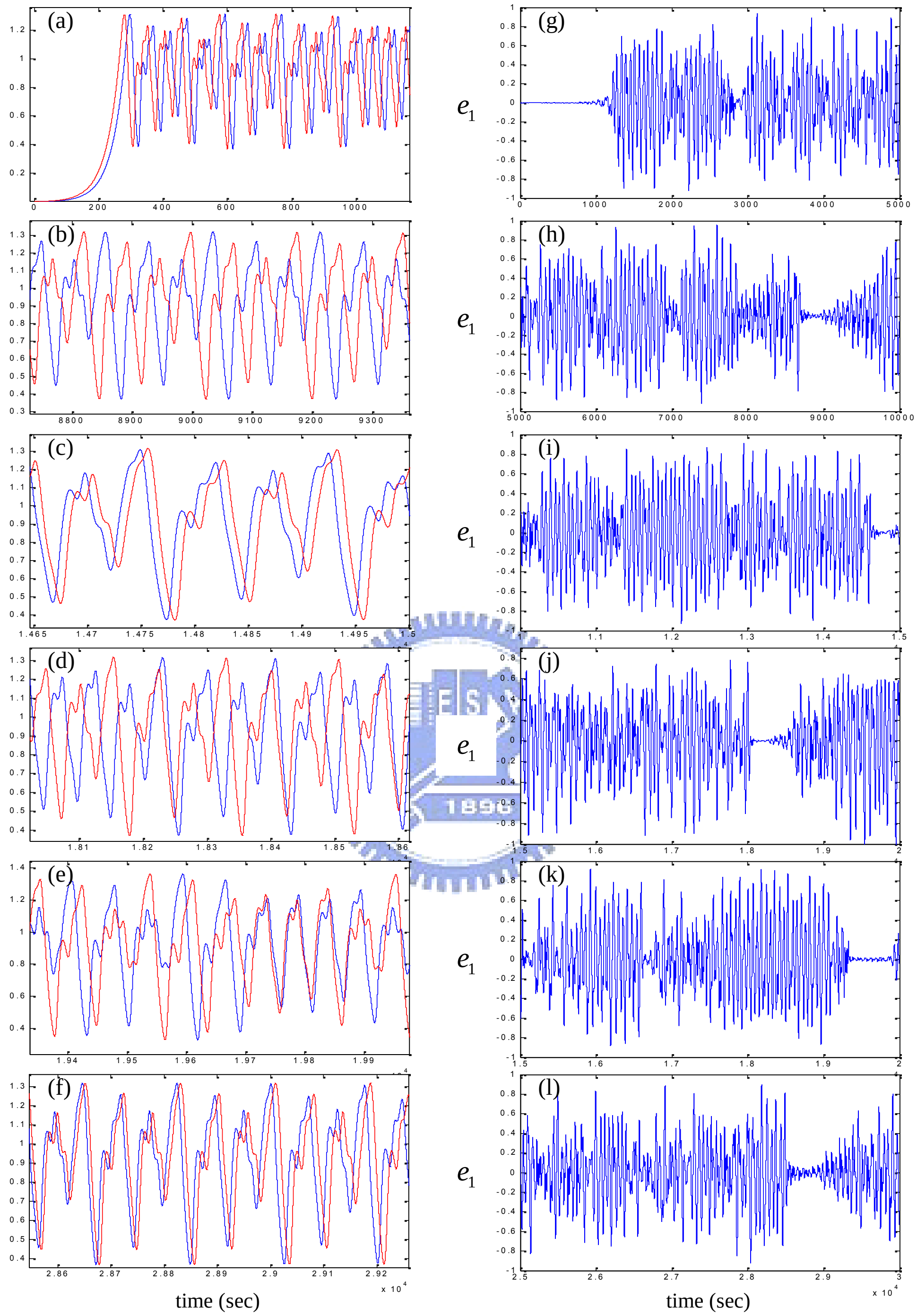


Fig. 3.1 (a)~(f) The time histories of x_1 (blue) and y_1 (red) and (g)~(l) error $e_1 = x_{1T_j} - y_1$ of double Mackey-Glass systems with initial conditions $(x_{10}, x_{20}) =$ (0.001, 0.001), $(y_{10}, y_{20}) =$ (0.0015, 0.0015).

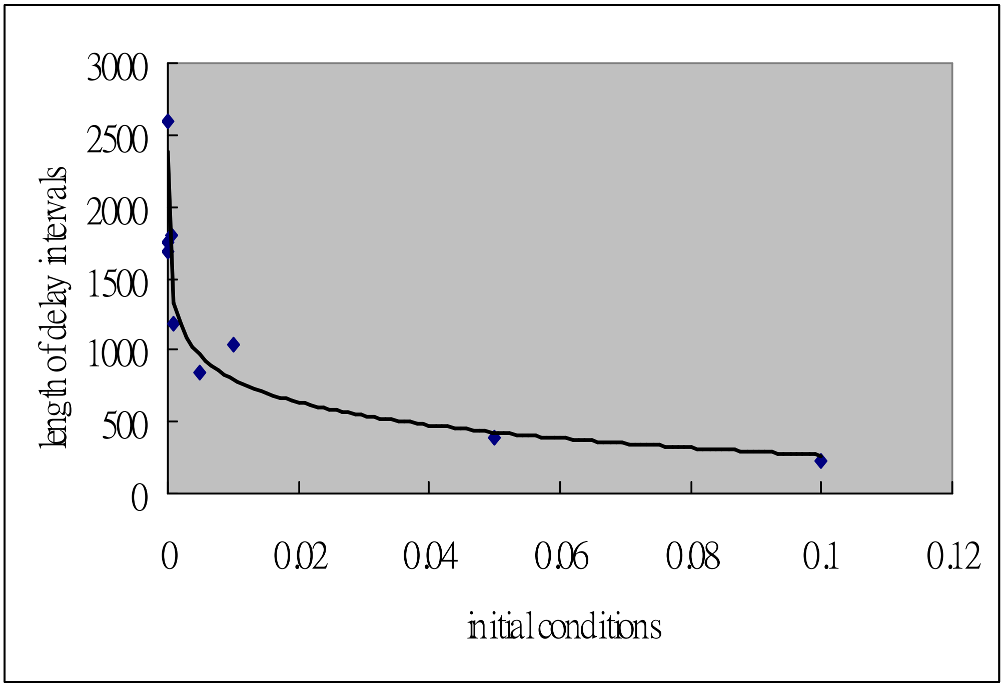


Fig. 3.2 The curve fitting of initial condition x_0 to the length of temporary lag or anticipated synchronization L_1.

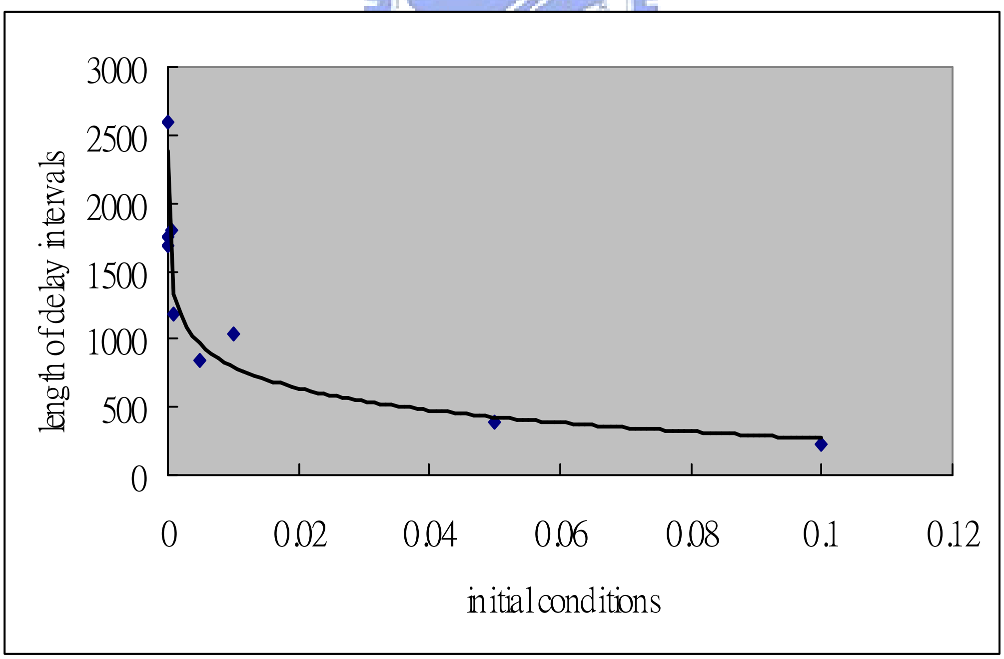


Fig. 3.3 The curve fitting of initial condition x_0 to the length of temporary lag or anticipated synchronization L_2.

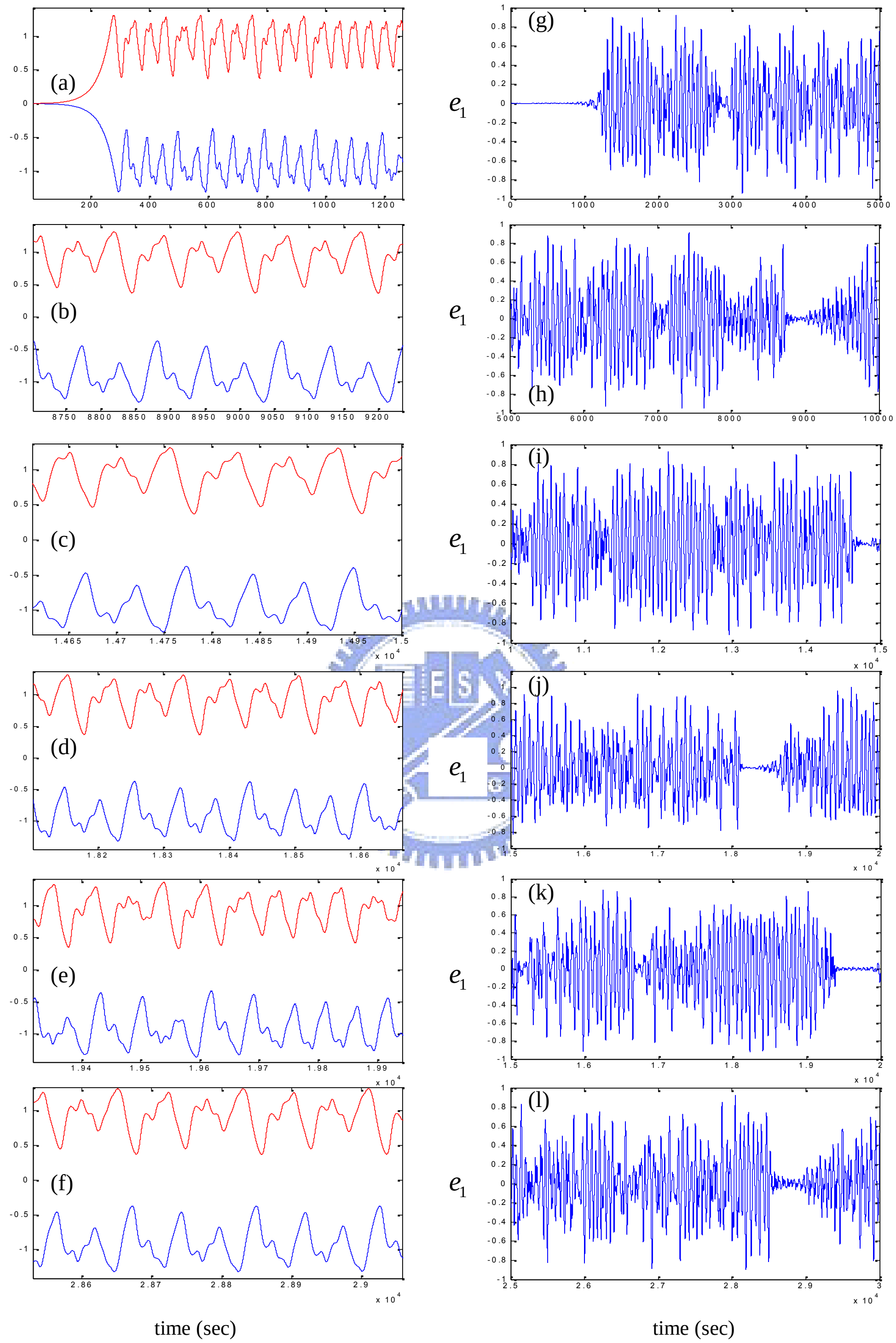


Fig. 3.4 (a)~(f) The time histories of x_1 (blue) and y_1 (red) and (g)~(l) error $e_1 = x_{1T_j} + y_1$ of double Mackey-Glass systems with initial conditions $(x_{10}, x_{20}) = (-0.001, 0.001)$, $(y_{10}, y_{20}) = (0.0015, 0.0015)$.