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O.R. Applications

Capability adjustment for gamma processes with mean shift consideration in implementing Six Sigma program

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Abstract

In the 1980s, Motorola, Inc. introduced its Six Sigma quality program to the world. Some quality practitioners questioned why the Six Sigma advocates claim it is necessary to add a 1.5σ shift to the process mean when estimating process capability. Bothe [Bothe, D.R., 2002. Statistical reason for the 1.5σ shift. Quality Engineering 14 (3), 479–487] provided a statistical reason for considering such a shift in the process mean for normal processes. In this paper, we consider gamma processes which cover a wide class of applications. For fixed sample size n, the detection power of the control chart can be computed. For small process mean shifts, it is beyond the control chart detection power, which results in overestimating process capability. To resolve the problem, we first examine Bothe's approach and find the detection power is less than 0.5 when data comes from gamma distribution, showing that Bothe's adjustments are inadequate when we have gamma processes. We then calculate adjustments under various sample sizes n and gamma parameter N, with power fixed to 0.5. At the end, we adjust the formula of process capability to accommodate those shifts which can not be detected. Consequently, our adjustments provide much more accurate capability calculation for gamma processes. For illustration purpose, an application example is presented.

Keywords: Quality management; Dynamic C_{pk}; Gamma distribution; Mean shift; Process capability index

1. Introduction

Process capability indices (PCIs), C_p, C_{pk}, C_{pm} and C_{pmk} have been proposed in the manufacturing industry providing numerical measures on whether a process is capable of reproducing items within specification limits preset in the factory (see Kane, 1986; Chan et al., 1988; Pearn et al., 1992; Kotz and Lovelace, 1998). These indices have been defined as:

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$$C_p = \frac{USL - LSL}{6\sigma}, \quad C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \quad C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}.$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation (overall process variation), and T is the target value. The index C_p considers the overall process variability relative to the manufacturing tolerance, reflecting product quality consistency. The index C_{pk} takes the magnitude of process variance as well as process departure from target value, and has been regarded as a yield-based index since it providing lower bounds on process yield. The index C_{pm} emphasizes on measuring the ability of the process to cluster around the target, which therefore reflects the degrees of process targeting (centering). Since the design of C_{pm} is based on the average process loss relative to the manufacturing tolerance, the index C_{pm} provides an upper bound on the average process loss, which has been alternatively called the Taguchi index. The index C_{pmk} is constructed from combining the modifications to C_p that produced C_{pk} and C_{pm}, which inherits the merits of both indices.

Since Motorola, Inc. introduced its Six Sigma quality initiative in the 1980s, quality practitioners have questioned why the followers of this initiative have added a 1.5σ shift to the process mean when estimating process capability. The advocates of Six Sigma have claimed that such an adjustment is necessary, but they have offered only personal experiences and three dated empirical studies as justification for this claim (see Bender, 1975; Evans, 1975; Gilson, 1951). By examining the sensitivity of control charts to detect changes of various magnitudes, Bothe (2002) provided a statistically based reason for this claim. In his study, Bothe assumed that the process data is approximately normally distributed. However, non-normal processes occur frequently, in particular, in the semiconductor industry. Pyzdek (1992) mentioned that the distributions of certain chemical processes, such as zinc plating in a hot-dip galvanizing process, are very often skewed. Choi et al. (1996) presented an example of a skewed distribution in the ‘‘active area’’ shaping stage of the wafer’s production processes. Gamma distribution (skewed), denoted as Gamma(N, θ), with various values of N and θ, covers a wide class of non-normal applications, including the manufacturing of semiconductor products, head/gimbal assembly for memory storage systems, jet-turbine engine components, flip-chips and chip-on-board, audio-speaker drivers, wood products, and many others. Therefore, it seems reasonable that we use gamma process for data analysis.

The control charts are commonly used in many industries for providing early warning for the shift in the process mean. For example, the cumulative sum chart is known to be effective on detecting sustained shifts in the process mean (see e.g. Lucas and Crosier, 2000; Luceno and Puig-Pey, 2002; Lucas, 1976). If the control chart detects a process mean shift, then the process is not under control. However, for momentary process mean shifts, it may be beyond the control chart detection power. Consequently, the undetected shifts may result in overestimating process capability. If the process mean shifts are not detected, then unadjusted C_{pk} would overestimate the actual process yield. Bothe (2002) provided a statistical reason for considering such a shift in the process mean for normal processes. However, if the capability indices are based on the assumption of a normal distribution of data but are used to deal with non-normal observations, the values of the capability indices may, in the majority of situations, misrepresent actual product quality. This paper first examines Bothe’s approach and finds that the detection power of the control chart is less than 0.5 when data comes from gamma distribution. This shows that Bothe’s adjustments are inadequate when we have gamma processes. Then, the adjustments under various sample sizes (n) and gamma parameters (N) with a fixed detection power of 0.5 are calculated. Finally, the process capability formula is adjusted to accommodate the undetected shifts. As a result, our adjustments provide significantly more accurate calculations of the capability of gamma processes. A real-world example taken from the manufacturing process of semiconductors is investigated to illustrate the applicability of the process capability index.

2. Gamma process

All of us know that the case of non-normal processes occurs frequently in practice, for example, in the semiconductor industry. Pyzdek (1992) pointed out the skewed distributions that are bounded on one side occur frequently in industry and gave several examples, such as a shearing process and a chemical dip process. The abundance of outputs from skewed distributions makes the normality assumption often unreasonable. A gamma distribution, with varied N and θ values, covers a wide class of non-normal applications. Therefore, a gamma process for data analysis has been chosen for this study. The difference between normal and gamma distributions is compared in Section 2.1. And the statistical property of gamma distribution is discussed in Section 2.2.

2.1. The gamma distribution

In this section, we investigate the gamma distribution to study the effect on the detection power of the control chart. Observations from the gamma distribution are non-negative. The gamma distribution can be denoted as $\mathrm{Gamma}(N,\theta)$ with the probability density function given by Ross (2005) to be as follows:

$$f(x;N,\theta)=\frac{1}{\theta^N\Gamma(N)}x^{N-1}\exp\{-x/\theta\},\quad x>0,\ N>0,\ \theta>0$$

and the mean and variance are given, respectively, by

$$\mu=N\theta\quad\text{and}\quad\sigma^2=N\theta^2.$$

Denote the family of gamma distributions with mean $N\theta$ by $\mathrm{Gamma}(N,\theta)$. The gamma distributions are skewed. To see how this distribution are different from the standard normal distribution in terms of skewness and kurtosis, Table 1 presents the values of skewness and kurtosis (which are defined as the third and fourth moments of the standardized distribution, respectively) of the gamma distributions under study. Note that the case $N=1$ corresponds to the exponential distribution and the skewness and kurtosis of $\mathrm{Gamma}(N,1)$ are $2/\sqrt{N}$ and $6/N+3$ respectively. We can find in Table 1 when the N decreases, the corresponding values of skewness and kurtosis will become large and far away from the values of the standard normal distribution. The result through these distributions, we can get some insights of the effects of non-normality in terms of skewness and kurtosis.

Fig. 1 presents several gamma distributions along with a normal distribution for the same mean and variance. In this study, we let $N=0.5$, 1, 2, 3, 4, and 5, while (without loss of generality) fixing $\theta=1$. These values of N and θ correspond to the values used by Schilling and Nelson (1976). As can be seen from Fig. 1a–f, as N increases, the gamma distribution appears more nearly normal distribution. In fact, we demonstrate this convergence property in Table 1, by calculating the skewness and kurtosis. It can be seen that as N increases, the skewness and kurtosis of gamma distribution are very close to those of normal distribution. Through these distributions, we wish to get some insights of the effects of non-normality on the detection power in terms of skewness and kurtosis in Section 3.

2.2. Statistical properties of gamma distribution

The gamma distribution has a reproductive property: If X_1 and X_2 are independent random variables and each has a gamma distribution with possible different values of N_1, N_2 of N, but with common values of θ, then X_1+X_2 also has a gamma distribution (see Ross, 2005), with $N=N_1+N_2$, and with the same value

Table 1
Values of skewness and kurtosis of various gamma distributions

Distribution	Skewness	Kurtosis
$N(0,1)$	0	3
$\mathrm{Gamma}(5,1)$	0.8944	4.2
$\mathrm{Gamma}(4,1)$	1	4.5
$\mathrm{Gamma}(3,1)$	1.1547	5
$\mathrm{Gamma}(2,1)$	1.4142	6
$\mathrm{Gamma}(1,1)$	2	9
$\mathrm{Gamma}(0.5,1)$	2.8284	15

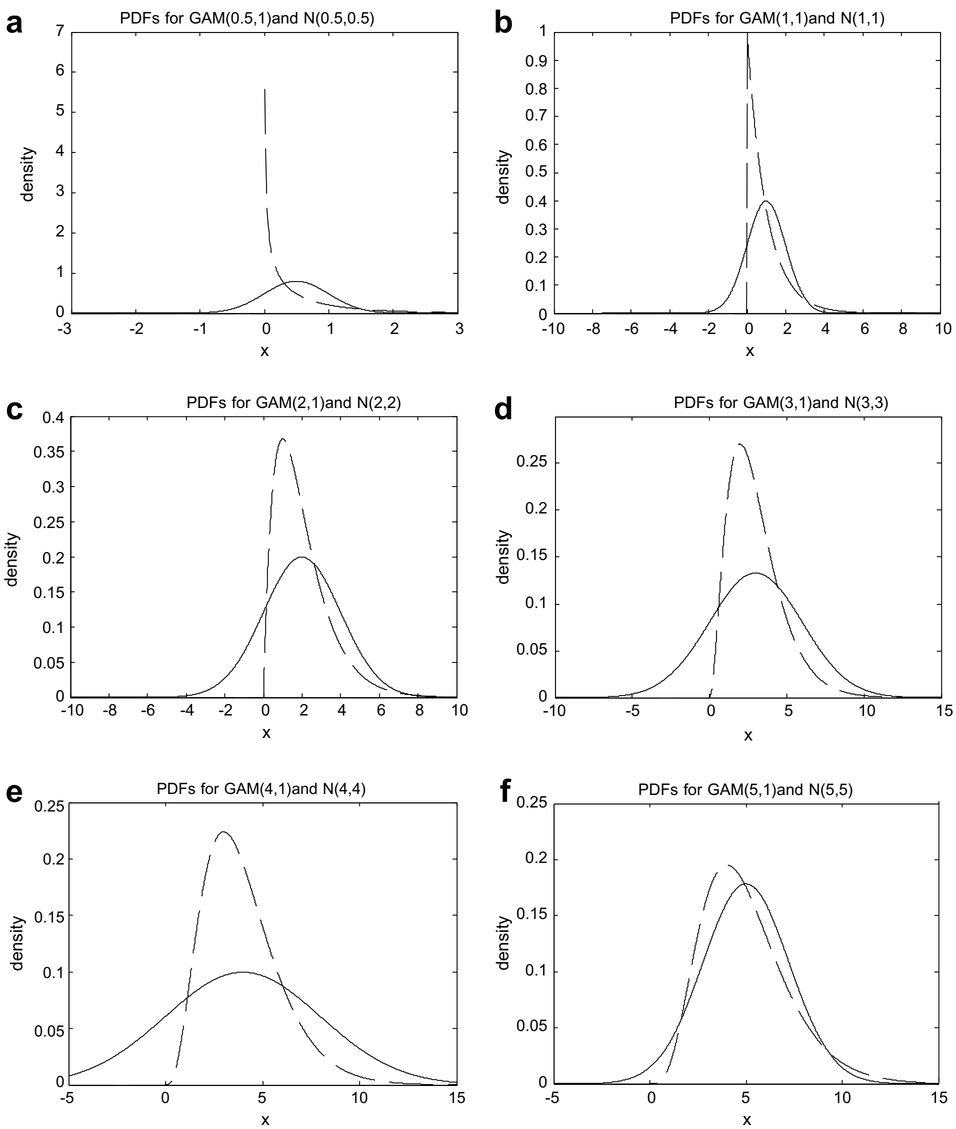

Fig. 1. (a) Probability density functions for Gamma$(0.5, 1)$ and $N(0.5, 0.5)$. (b) Probability density functions for Gamma$(1, 1)$ and $N(1, 1)$. (c) Probability density functions for Gamma$(2, 1)$ and N(2, 2). (d) Probability density functions for Gamma$(3, 1)$ and $N(3, 3)$. (e) Probability density functions for Gamma$(4, 1)$ and $N(4, 4)$. (f) Probability density functions for Gamma$(5, 1)$ and $N(5, 5)$.

of θ. Applying this property, let X_1,$X_2, \ldots, X_n$ be a sequence of independent distribution of Gamma(N, θ) and then the distribution of $X_1 + X_2 + \cdots + X_n$ is Gamma(nN, θ). Using simply statistical technique, we can conclude that $\overline{X}_n = (X_1 + X_2 + \cdots + X_n)/n \sim$ Gamma$(nN, \theta/n)$.

The standard deviation of the $\overline{X}_n$ distribution, $\sigma_{\bar{x}_n}$, is calculated from its relationship to the distribution parameters and the subgroup size n as follows:

$$\sigma_{\bar{x}_n} = \sqrt{nN \times \left(\frac{\theta}{n}\right)^2} = \sqrt{\frac{N}{n}} \cdot \theta.$$

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent distribution of Gamma$(3, 1)$ and we plot the probability density function of the average $\overline{X}_n$ for subgroup size $n = 2(1)5$ in Figs. 2a–d. We can found that the variance of

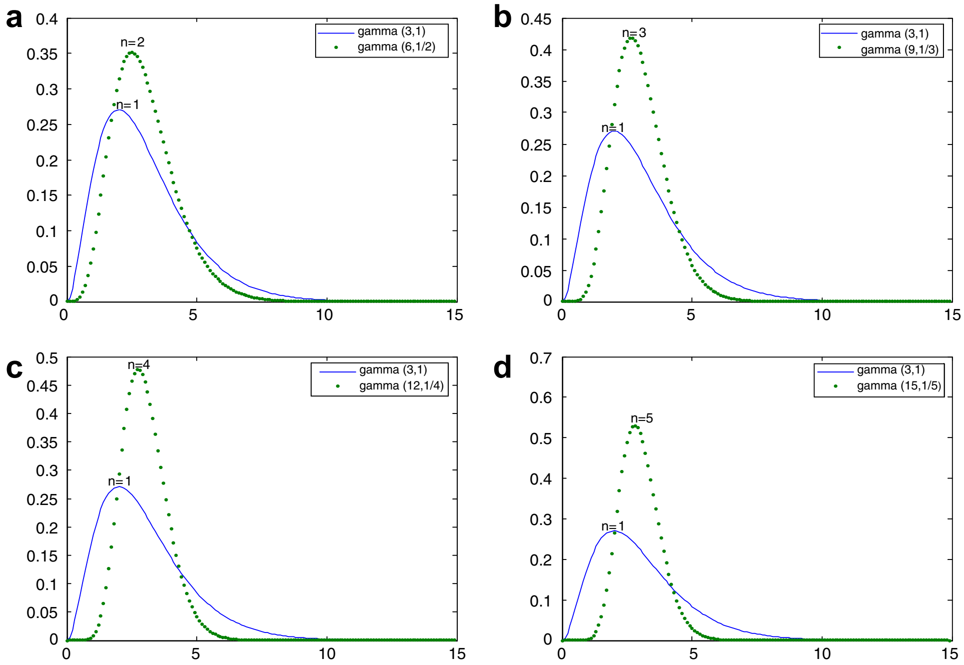

Fig. 2. (a) Probability density functions for Gamma(3, 1) and the average $\overline{X}_n$ for $n = 2$. (b). Probability density functions for Gamma(3, 1) and the average $\overline{X}_n$ for $n = 3$. (c) Probability density function for Gamma(3, 1) and the average $\overline{X}_n$ for $n = 4$. (d) Probability density function for Gamma(3, 1) and the average $\overline{X}_n$ for $n = 5$.

average $\overline{X}_n$ will get smaller as subgroup size n increases. This situation means that the distribution of $\overline{X}_n$ is more centralized when $n > 1$.

3. The detection power of gamma process

The major purpose of individuals control chart is assisting on identifying shifts and drifts in processes and it is easily to be implemented. But, some assumptions should be satisfied before control charts are used. The assumptions include that the process characteristics must follow normal distributions. Actually, non-normal processes occur frequently in practice. Due to above-mentioned statements, we replace the traditional, $\mu \pm 3\sigma$, to be the upper or lower control limits by the quantile of cumulative distribution function for different parameters of Gamma(N, θ) ($F_{0.00135}$ and $F_{0.99865}$) and detect the power of gamma process under Bothe's capability adjustments.

Let $X_1, X_2, \ldots, X_n$ be a sequence observations of independent and identically distributed in Gamma(N, θ). Using the reproductive property of gamma distribution, the mean of the observations is $\overline{X}_n$ ($\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$) which is distributed in Gamma$(nN, \theta/n)$, then we can obtain that $\mu_{X_i} = \mu_{\overline{X}_n} = N \times \theta$, $\sigma_{X_i} = \sqrt{N} \times \theta$, and

Table 2
Detection power of various gamma processes

Subgroup size n	Shift δ	Distribution Gamma$(N, 1)$										
		$N = 0.5$	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 9$	$N = 10$	$N(0, 1)$
2	2.12	0.027	0.054	0.100	0.136	0.164	0.187	0.206	0.222	0.235	0.247	0.257	0.5
3	1.73	0.040	0.078	0.136	0.176	0.205	0.228	0.246	0.262	0.274	0.285	0.294	0.5
4	1.50	0.054	0.100	0.164	0.206	0.236	0.258	0.275	0.289	0.301	0.311	0.320	0.5
5	1.34	0.066	0.119	0.187	0.228	0.257	0.278	0.294	0.308	0.319	0.328	0.336	0.5
6	1.22	0.077	0.134	0.203	0.244	0.272	0.292	0.307	0.320	0.330	0.339	0.346	0.5

$\sigma_{\overline{X}_n} = (\sqrt{N} \times \theta)/\sqrt{n}$. Consequently, we derived the power of gamma process as follows. Since the type II error β is

$$\beta = P(LCL \leqslant \overline{X}_n \leqslant UCL|\mu_1 = \mu_0 + k\sigma_{X_i}) = P(F_{0.00135} \leqslant \overline{X}_n \leqslant F_{0.99865}|\mu_1 = \mu_0 + k\sigma_{X_i})$$
$$= G_{\overline{X}_n}(F_{0.99865}) - G_{\overline{X}_n}(F_{0.00135}),$$

where $1 - \beta$ is the detection power of the process, $G_{\overline{X}_n}(\cdot)$ is the cumulative distribution function of gamma distribution with that mean has shifted and μ_1 is the mean after process shift (μ_0 is the mean of the original process). The control limits LCL and UCL are calculated as $F_{0.00135}$ and $F_{0.99865}$ respectively.

Table 2 presents the detection power when data comes from gamma distribution with $N = 0.5$, 1(1)10 and $\theta = 1$. The magnitude of shift in the second column on the left is Bothe's capability adjustments determined when data comes from normal distribution and the detection power is 0.5.

From Table 2, we can find that the detection power is less than 0.5 when data comes from gamma distribution under Bothe's capability adjustments. Our study shows that the detection power gets closer to 0.5 as N increases, which is reasonable since the corresponding distributions get closer to the standard normal distribution. This is due to Bothe's (2002) approach is based on the normality assumption of the data and the detection power is 0.5. The skewness of Gamma$(N, 1)$ is $2/\sqrt{N}$. Therefore, as N decreases the gamma distribution is more skewed and the detection power is poorer. For example, when $N = 0.5$ and the subgroup size $n = 2$, the detection power is 0.027. It implies Bothe's adjustments are inadequate when we have skewed processes. Consequently, in our study, we determined the capability adjustment and calculation when process data comes from gamma distribution.

Table 3
AS_{50} values for several subgroup sizes n and various N values

n	N											
	0.5	1	2	3	4	5	6	7	8	9	10	$N(0,1)$
2	4.182	3.611	3.185	2.992	2.876	2.797	2.738	2.692	2.655	2.625	2.599	2.12
3	3.127	2.732	2.443	2.313	2.236	2.182	2.143	2.113	2.088	2.067	2.050	1.73
4	2.553	2.252	2.034	1.936	1.878	1.838	1.808	1.785	1.767	1.752	1.738	1.50
5	2.188	1.944	1.769	1.690	1.644	1.612	1.588	1.570	1.555	1.543	1.532	1.34
6	1.932	1.727	1.581	1.515	1.476	1.450	1.430	1.415	1.403	1.392	1.384	1.22
7	1.741	1.565	1.439	1.383	1.350	1.327	1.310	1.297	1.286	1.278	1.270	1.13
8	1.592	1.438	1.328	1.279	1.249	1.229	1.215	1.203	1.194	1.186	1.180	1.06
9	1.473	1.336	1.237	1.194	1.168	1.150	1.137	1.127	1.118	1.112	1.106	1.00
10	1.375	1.251	1.162	1.123	1.100	1.084	1.072	1.063	1.055	1.049	1.044	0.95
11	1.292	1.179	1.099	1.063	1.042	1.027	1.016	1.008	1.001	0.996	0.991	0.90
12	1.222	1.118	1.044	1.011	0.992	0.978	0.969	0.961	0.955	0.950	0.945	0.87
13	1.160	1.064	0.996	0.966	0.948	0.936	0.927	0.920	0.914	0.909	0.905	0.83
14	1.107	1.018	0.954	0.926	0.910	0.898	0.890	0.883	0.878	0.874	0.870	0.80
15	1.059	0.976	0.917	0.891	0.875	0.864	0.857	0.850	0.846	0.842	0.838	0.77
16	1.017	0.939	0.883	0.859	0.844	0.834	0.827	0.821	0.817	0.813	0.810	0.75
17	0.979	0.905	0.853	0.830	0.816	0.807	0.800	0.795	0.790	0.787	0.784	0.73
18	0.944	0.875	0.826	0.804	0.791	0.782	0.775	0.770	0.766	0.763	0.760	0.71
19	0.913	0.847	0.801	0.780	0.768	0.759	0.753	0.748	0.744	0.741	0.738	0.69
20	0.884	0.822	0.778	0.758	0.746	0.738	0.732	0.728	0.724	0.721	0.718	0.67
21	0.858	0.798	0.756	0.738	0.726	0.719	0.713	0.709	0.705	0.702	0.700	0.65
22	0.834	0.777	0.737	0.719	0.708	0.701	0.695	0.691	0.688	0.685	0.683	0.64
23	0.811	0.757	0.718	0.701	0.691	0.684	0.679	0.675	0.672	0.669	0.667	0.63
24	0.790	0.738	0.701	0.685	0.675	0.669	0.664	0.660	0.657	0.654	0.652	0.61
25	0.771	0.721	0.685	0.670	0.660	0.654	0.649	0.646	0.643	0.640	0.638	0.60
26	0.753	0.704	0.670	0.655	0.646	0.640	0.636	0.632	0.629	0.627	0.625	0.59
27	0.736	0.689	0.656	0.642	0.633	0.627	0.623	0.619	0.617	0.615	0.613	0.58
28	0.720	0.675	0.643	0.629	0.621	0.615	0.611	0.608	0.605	0.603	0.601	0.57
29	0.704	0.661	0.631	0.617	0.609	0.604	0.599	0.596	0.594	0.592	0.590	0.56
30	0.690	0.648	0.619	0.606	0.598	0.593	0.589	0.586	0.583	0.581	0.579	0.55

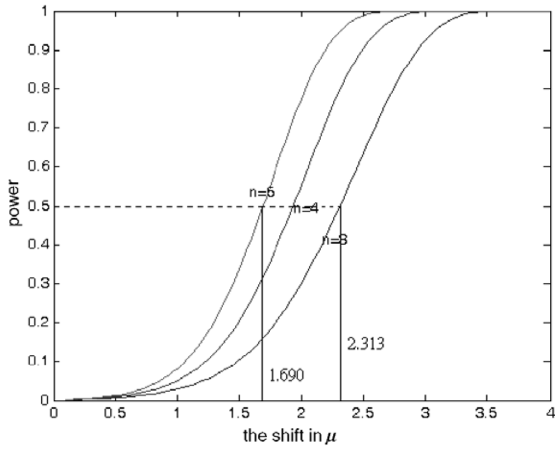

Fig. 3. Power curves for the commonly used subgroup sizes 3, 4 and 5 when $N = 3$.

4. Undetected mean shift under designated power

The undetected mean shift adjustment in Table 3 is called AS_{50} which is the magnitude of shift we need to adjust based on designated detection power is 0.5 and process data comes from gamma distribution. We develop a Matlab program (see Appendix) to determine the adjustment AS_{50}. The program reads the desired detection power (set to be 0.5), the gamma parameter N and the subgroup size n. Table 3 displays the magnitude of adjustments AS_{50} based on the detection power is 0.5 and data comes from Gamma$(N, 1)$ with various values of N (=0.5 and 1(1)10) and $n = 2(1)30$. For example, if we set $N = 3$ and $n = 5$, then the adjustment is $AS_{50} = 1.69$. We conclude that the adjustment $AS_{50} \cdot \sigma(= 1.69\sigma)$ is required based on the detection power is 0.5 and data comes from Gamma$(3, 1)$. It also shows from Table 3 that the adjustments AS_{50} get closer to Bothe's adjustments as N increases (when $n = 2(1)6$), which is reasonable since the corresponding distributions get closer to the standard normal distribution. However, we should notice that when N is small (distribution is strongly skewed), the required adjustment in the capability index formula is much greater than those for normal processes. Using the adjusted process capability formula, the engineers can determine the actual process capability more accurately.

Fig. 3 presents the power curves, these lines on the graph depict the probabilities of detecting a shift in μ for the commonly used subgroup size $n = 3$, 4, 5 (expressed in σ units on the horizontal axis) when $N = 3$. All these lines are close to zero for small shifts in μ. It can be found that the power of the chart with all three curves eventually leveling off close to 100% as the size of the shifts in excess of 3.5σ. The dashed horizontal line drawn in Fig. 3 shows that there is a 50% probability of missing a 1.69σ shift in μ when n is 5, while μ must move by 2.313σ to have this same probability when n is only 3. The shift sizes that have a 50% probability of remaining undetected, called AS_{50} values are listed in Table 3 for subgroup sizes $n = 2(1)30$. Momentary movements in μ smaller than $AS_{50}\sigma$ are more than likely to be missed by a control chart. Therefore our adjustment AS_{50} takes into account those shifts that are not detected by the control chart.

5. Capability adjustment

5.1. Estimator of C_{pk} in the non-normal case

The index C_{pk} has been viewed as an yield-based index since it provides bounds on the process yield for a normally distributed process with a fixed value of C_{pk}. This index C_{pk} is defined as:

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

where as above USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean and σ is the process standard deviation. The proper use of process capability indices, which are statistical measures of process capability, is based on several assumptions. One of the most essential is that the process monitored is supposed to be stable and the output is approximately normally distributed. When the distribution of a process characteristic is non-normal, PCIs calculated using conventional methods could often lead to erroneous and misleading interpretation of the process's capability.

In the recent years, several approaches to the problems of PCIs for the non-normal populations have been suggested (see e.g. Pal, 2005; Ding, 2004; Pearn and Chen, 1997; Kotz and Lovelace, 1998; Somerville and Montgomery, 1996; Kocherlakota et al., 1992). Several authors used data transformation techniques such as the Box–Cox power transformation, Johnson's transformations and quantile transform techniques to solve this problem. And some authors replaced the unknown distribution by a known three or four-parameter distribution. Examples include Clements (1989), Franklin and Wasserman (1992), Shore (1998) and Polansky (1998). We did not consider the Box–Cox transformation because: (1) process characteristics might be lost after the transformation, and the transformed data is difficult to interpret. (2) In general, however, practitioners may feel uncomfortable working with transformed data and may have some difficulty in reversing the results of the calculations back to the original scale. Due to above-mentioned statements, we use the most common method for modifying PCIs in the non-normal case is the technique of quantile estimation. Analogous to the normal case, where the "natural" process width is between the 0.135 percentile and the 99.865 percentile, PCIs can be redefined in terms of their quantiles for possible modification in the non-normal case. The quantile definition for C_{pu} and C_{pl} are defined as:

$$C_{pu} = \frac{USL - \text{median}}{(\text{upper } 0.135\% \text{ point}) - \text{median}} = \frac{USL - F_{0.5}}{F_{0.99865} - F_{0.5}}$$

and

$$C_{pl} = \frac{\text{median} - LSL}{\text{median} - (\text{lower } 0.135\% \text{ point})} = \frac{F_{0.5} - LSL}{F_{0.5} - F_{0.00135}}.$$

Then the index C_{pk} would be calculated as the minimum of C_{pu} and C_{pl}, namely:

$$C_{pk} = \min\{C_{pu}, C_{pl}\} = \min\left\{\frac{USL - F_{0.5}}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - LSL}{F_{0.5} - F_{0.00135}}\right\}, \tag{1}$$

so that the normality assumption can be verified simultaneously.

We can obtain more accurate measures of these percentile points ($F_{0.00135}$, $F_{0.5}$ and $F_{0.99865}$) under consideration in the non-normal case, if we are able to find a better distributional form for the data, which provides a very satisfactory fit. This involves modeling the process data with alternative probability plot models, such as the Weibull or gamma ones (see e.g. Dudewicz and Mishra, 1998; Kotz and Lovelace, 1998). Nevertheless, an obvious disadvantage of probability plotting is that it is not a truly objective procedure. It is quite possible for two analysts to arrive at different conclusions using the same data. Accordingly, it is often desirable to supplement probability plots with goodness-of-fit tests, which possess more formal statistical foundations (see, e.g., Shapiro, 1995). Choosing proper distribution to fit the data is an important step in probability plotting. Sometimes one can use the available knowledge of the physical phenomenon or the past experience to suggest a choice of the distribution.

5.2. Modifying the assessment of C_{pk}

Since a process will experience shifts in $F_{0.5}$ (=median) of various magnitudes and not all of these will be discovered, we must take them into account when estimating outgoing quality so customers are not disappointed. Whereas the shifts of process mean ranging in size from 0 up to $AS_{50}\sigma$ are the ones likely to remain undetected (larger shifts should be detected by the control chart), a cautious method is to assume that every missed shift is as large as $AS_{50}\sigma$.

Considering the undetected process mean shift as large as $AS_{50}\sigma$, we use $F_{0.5}$ minus $AS_{50}\sigma$ to evaluate how well the process output meets the LSL and $F_{0.5}$ plus $AS_{50}\sigma$ for determining conformance to the USL when estimating the index C_{pk}. Incorporating both of these adjustments into the C_{pk} formula (see Eq. (1)) we obtained the "dynamic" C_{pk} index by making the following modifications:

$$C_{pk} = \min\left\{\frac{USL - (F_{0.5} + AS_{50}\sigma)}{F_{0.99865} - F_{0.5}}, \frac{(F_{0.5} - AS_{50}\sigma) - LSL}{F_{0.5} - F_{0.00135}}\right\}$$
$$= \min\left\{\frac{USL - F_{0.5}}{F_{0.99865} - F_{0.5}} - \frac{AS_{50}\sigma}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - LSL}{F_{0.5} - F_{0.00135}} - \frac{AS_{50}\sigma}{F_{0.5} - F_{0.00135}}\right\}. \quad (2)$$

By considering an adjustment $AS_{50}\sigma$ in this assessment for undetected shifts in process median, the estimate of dynamic index C_{pk} will decrease and the expected total number of nonconforming parts will increase. It must be noticed that this nonconforming level assumes that undetected shifts are happening almost constantly and that every one is equal to $AS_{50}\sigma$. From Table 3, the practitioners can find the AS_{50} to calculate the dynamic index C_{pk} for determining whether their process meets the preset capability requirement, and make reliable decisions to the process.

6. Application

The manufacture of integrated circuits (ICs) includes the front-end process of wafer and the back-end process of integrated circuit packaging. In an integrated circuit packaging factory, the manufacturing process generally contains the following main steps: die sawing, die mounting, wire bonding, molding, trimming and forming, marking, plating and testing (Fig. 4). Wire bonding is the most common means of providing an electrical connection from the IC device to the lead-frame and it uses ultra-thin gold or aluminum wire to form the electrical inter-connection between the chip and the package leads (Fig. 5). High-speed wire bonding equipment consists of a handling system to feed the lead-frame into the work area. Image recognition systems ensure the die is orientated to match the bonding diagram for a particular device. Wires are bonded one at a time, and two wire bonds are formed at each interconnection: one at the die (first bond) and the other at the lead-frame (second bond). The first bond involves the formation of a ball which is placed within the bond

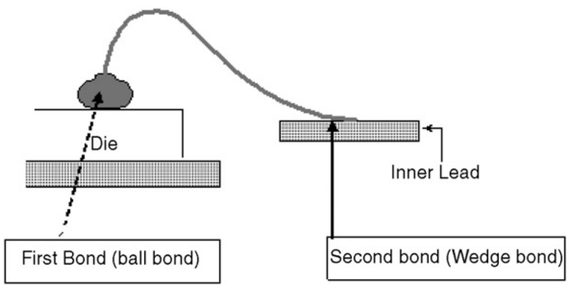

Fig. 4. Wire bonding process.

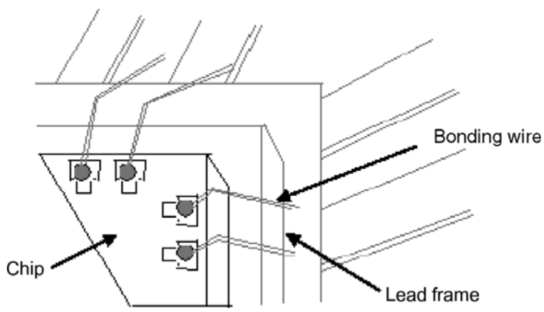

Fig. 5. The position of the chip in the wire bonding process.

Table 4
The 100 observations are collected from the historical data

2.891	4.035	4.495	2.890	2.312	3.158	5.228	3.334	5.896	5.639
3.842	1.590	1.954	1.842	0.680	2.752	1.301	2.260	0.889	2.381
0.619	2.788	1.050	3.750	3.508	6.123	6.549	5.954	2.207	4.417
4.805	1.516	2.227	2.797	1.636	1.066	0.940	4.101	4.542	1.295
1.770	3.492	5.706	3.722	6.644	2.472	1.383	4.494	1.694	2.892
2.111	3.591	2.093	3.222	2.891	2.582	0.665	3.234	1.102	1.083
1.508	1.811	2.803	6.659	0.923	6.229	3.177	2.333	1.311	4.419
2.495	0.921	4.061	9.725	1.600	4.281	3.360	1.131	1.618	4.489
3.696	1.982	2.413	5.480	1.992	2.573	1.845	4.620	6.221	1.694
4.882	1.380	3.982	2.260	2.366	2.899	3.782	2.336	1.175	3.055

pad opening on the die, under load and ultrasonic energy within a few milliseconds and forms a ball bond at the bond pad metal.

In the wire bonding process, one of the most important factors which directly relates to its level of quality is the ball size. Since the process may easily shut down when the width between the two bond balls is too small, the size of the bond ball must be taken into consideration. Therefore, the proposed USL and LSL for the ball size are 8 mil and 0.5 mil (1 mil = 1/1000 in. = 0.0254 mm), respectively. As shown in Table 4, a part of historical data is collected. Fig. 6 displays the histogram, and Fig. 7 displays the normal probability plot of these historical data. From the Figs. 6 and 7, it is evident to conclude the data collected from the factory are not normal distributed. The data analysis results justify that the process is significantly away from the normal distribution. By the goodness-of-fit tests, the historical data indicates that the process pretty approximates to be

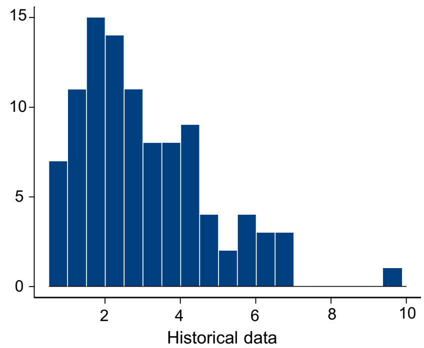

Fig. 6. Histogram plot of the historical data.

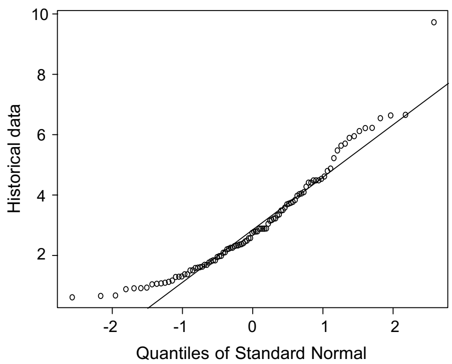

Fig. 7. Normal probability plot of the historical data.

distributed as gamma. The parameters N and θ of this gamma process could be estimated from the historical data, giving $\hat{N} = 3$ and $\hat{\theta} = 1$.

Accordingly, it is appropriate to use this approach and we can obtain more accurate measures of the three quantiles ($F_{0.00135}$, $F_{0.5}$ (=median), and $F_{0.99865}$) for

$$\sigma = \sqrt{\frac{N}{n}} \times \theta = \sqrt{\frac{3}{10}} = 0.547$$

under consideration. Then "dynamic" C_{pk} index can be calculated as follows:

$$\begin{aligned} \text{dynamic } C_{pk} &= \min\left\{\frac{USL - F_{0.5} - AS_{50}\sigma}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - AS_{50}\sigma - LSL}{F_{0.5} - F_{0.00135}}\right\} \\ &= \min\left\{\frac{8 - 2.67 - 1.123(0.547)}{10.87 - 2.67}, \frac{2.67 - 1.123(0.547) - 0.5}{2.67 - 0.211}\right\} = \min\{0.58, 0.63\} = 0.58, \end{aligned}$$

with $AS_{50} = 1.123$ for $n = 10$ from Table 3. Compared it to the value of the following conventional index:

$$C_{pk} = \left\{\frac{USL - F_{0.5}}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - LSL}{F_{0.5} - F_{0.00135}}\right\} = \{0.65, 0.88\} = 0.65$$

calculated by a traditional capability study (the shift of process mean is not considered), we can find that the value of the modified C_{pk} is much smaller. This result indicates if the process mean shifts that are not detected then unadjusted C_{pk} would overestimate the actual process yield which is not derisible. Our adjustment takes into account those shifts that are not detected so that the practitioner would be able to keep its quality promise for this process. As the adjusted process capability drops below the desired quality level, the practitioner should stop the process because the process does not meet his preset capability requirement.

As the subgroup size n increases, the shift in process mean have a higher probability of detection. For example, if $n = 15$, the AS_{50} would be 0.891 for Gamma(3, 1) from Table 3, and then the "dynamic" C_{pk} index is

$$\begin{aligned} \text{dynamic } C_{pk} &= \min\left\{\frac{USL - F_{0.5} - AS_{50}\sigma}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - AS_{50}\sigma - LSL}{F_{0.5} - F_{0.00135}}\right\} \\ &= \min\left\{\frac{8 - 2.67 - 0.891(0.547)}{10.87 - 2.67}, \frac{2.67 - 0.891(0.547) - 0.5}{2.67 - 0.211}\right\} = \min\{0.6, 0.68\} = 0.6. \end{aligned}$$

Changing n from 10 to 15 increases the dynamic C_{pk} index from 0.58 to 0.6, and the total number of nonconforming parts would be reduced.

7. Conclusion

In this paper, we considered the problem of how to determine the adjustments for process capability with mean shift when data follows the gamma distribution. We first examined Bothe's approach and found the detection power is less than 0.5 when data comes from the gamma distribution, showing that Bothe's adjustments are inadequate when we have gamma processes. For gamma processes, we calculated the adjustments for various sample sizes (n) and gamma parameter (N) with detection power fixed to 0.5. For small value of N (distribution is strongly skewed), the required adjustment in the capability index formula is much greater than those for normal processes. Using the adjusted process capability formula, the engineers can determine the actual process capability more accurately. Tables are also provided for engineers/practitioners to use in their in-plant applications. A real-world semi-conductor production plant is investigated and presented to illustrate the applicability of the proposed approach.

Appendix. Matlab program for determining the adjustment AS_{50}

```
% n is the sample size
% N and theta are the parameters of gamma distribution
% power is the detection power of the control chart
```

```
clear all
power = 0.5;
for n=2:1:30;
   for N=[0.5 1 2 3 4 5 6 7 8 9 10];
      theta = 1;
      AS_50_Upper = 5;
      AS_50_Lower = 0.5;
      Sigma = sqrt(N.*(thetâ2));
%The upper and lower limits of the control chart
   F_99865 = gaminv(0.99865,n.*N,theta/n);
   F_00135 = gaminv(0.00135,n.*N,theta/n);
%Bisection Method
   B_AS_50_Upper = gamcdf(F_99865-AS_50_Upper.*Sigma,n.*N,theta/n)- gamcdf(F_00135-AS_50_Upper.
   *Sigma,n.*N,theta/n);
   P_AS_50_Upper = 1-B_AS_50_Upper;
   B_AS_50_Lower = gamcdf(F_99865-AS_50_Lower.*Sigma,n.*N,theta/n)- gamcdf(F_00135-AS_50_Lo-
   wer.*Sigma,n.*N,theta/n);
   P_AS_50_Lower = 1-B_AS_50_Lower;
   AS_50 = (AS_50_Lower+AS_50_Upper)/2;
   B_AS_50 = gamcdf(F_99865-AS_50.*Sigma,n.*N,theta/n)- gamcdf(F_00135- AS_50.*Sigma,n.*N,theta/
   n);
   P_AS_50 = 1-B_AS_50;
   while (abs(P_AS_50-power) > 0.0001)
      if P_AS_50 > power
         AS_50_Upper = AS_50;
         AS_50 = (AS_50_Lower+AS_50_Upper)/2;
         B_AS_50 = gamcdf(F_99865-AS_50.*Sigma,n.*N,theta/n)- gamcdf(F_00135- AS_50.*Sigma,n.*N,
         theta/n);
         P_AS_50 = 1-B_AS_50;
      else
         AS_50_Lower = AS_50;
         AS_50 = (AS_50_Lower+AS_50_Upper)/2;
         B_AS_50 = gamcdf(F_99865-AS_50.*Sigma,n.*N,theta/n)- gamcdf(F_00135- AS_50.*Sigma,n.*N,
         theta/n);
         P_AS_50 = 1-B_AS_50;
      end
     end
     fprintf ('%g ',AS_50);
    end
    fprintf ('%g \n', n);
end
```

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