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Decentralized decision-making and protocol design for recycled material flows

I-Hsuan Hong [a,*], Jane C. Ammons [b], Matthew J. Realff [c]

[a] Department of Industrial Engineering and Management, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan
[b] School of Industrial and Systems Engineering, Georgia Institute of Technology, 765 Ferst Drive, NW Atlanta, GA 30332-0205, USA
[c] School of Chemical and Biomolecular Engineering, Georgia Institute of Technology, 311 Ferst Drive, Atlanta, GA 30332-0100, USA

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ABSTRACT

Reverse logistics networks often consist of several tiers with independent members competing at each tier. This paper develops a methodology to examine the individual entity behavior in reverse production systems. We consider two tiers in the network, collectors and processors. The collectors determine individual flow functions that relate the flow they provide each processor to the overall vector of prices that the processors determine. Because the exact final prices are unknown, each collector solves a robust optimization formulation where the prices paid by the processors are assumed to be within given ranges. The processors compete for the flow from the collectors until the Nash equilibrium is reached in this competitive tier, which sets the vector of prices to be offered to the collectors. To demonstrate the approach, a numerical example is given for a prototypical recycling network.

1. Introduction

Maximizing the efficiency of recycled material flows is growing in urgency due to high demands in many raw material markets and the increasing concern for environmental impact of disposal. Supply chains are evolving from “open loop” unidirectional flows of materials, parts, and products from suppliers to end customers into more complex “closed loop”-linked forward and reverse arcs (Fleischmann et al., 2000; Guide and Harrison, 2003; Realff et al., 2004; de la Fuente et al., 2008). Forward production systems are being expanded to incorporate reverse production systems (RPS) that include sorting, demanufacturing and/or refurbished processes in reverse logistics systems.

Most of the research on RPS design views the system in a *centralized* way; the key assumption is that one planner has the requisite information about all the participating entities and seeks the optimal solution for the entire system (see Ammons et al., 2001; Shih, 2001; Barros et al., 1998; Assavapokee et al., 2008). Wang et al. (2004) remark upon the three major drawbacks of centralized supply chain optimization models: (1) By ignoring the independence of the supply chain members, the competitive behavior between entities may lower the system efficiency and hence a centralized model may not capture the appropriate bargaining mechanisms that can mitigate the competitive behavior; (2) The cost of information processing may be expensive and the central decision maker must gather all the information from every entity; and (3) The computation of solutions to centralized optimization models can be very challenging.

Many emerging RPS structures consist of several independent entities where individual entrepreneurs have their own profit functions and often are unwilling to reveal their own information to each other or the public. This type of system behavior is *decentralized.* Often the decision variables for each entity in a decentralized

* Corresponding author. Tel.: +886 3 513 1340; fax: +886 3 572 2392.
E-mail address: ihong@mail.nctu.edu.tw (I-H. Hong).

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system are also influenced by other entities' decisions, coupling prices between members of the same tier, and flows between supply chain tiers. In this paper, we focus on decentralized decision-making and protocol design for the RPS with two tiers. The two tiers represent the collectors, who interact directly with the source of recycled items, and the processors who purchase the items from the collectors and convert them into more fungible commodities that are sold to customers.

The concept of equilibrium has been widely applied in many fields: traffic network equilibrium (Sheffi, 1985) and economic models (Cournot, 1838; Bertrand, 1883; Stackelberg, 1934). The Cournot- (see Hobbs, 2001) and the Stackelberg-type (Savaskan et al., 2004) models are two commonly used equilibrium models in decentralized systems. However, in practice the Cournot-type model may incur information divulgence problems because it requires the collection of optimality conditions from different entities in order to establish the equilibrium solution. Conceptually, the solution procedure implies that entities need to pass the information of their optimality conditions to some invisible hand in the system, which requires the willingness to share information among participants with a centralized body in order to obtain the equilibrium solution. Furthermore, the Stackelberg-type model (a leader–follower problem) may have implicit solution problems in a multiple-entity case since the leader considers the follower's optimal response to its decision under the Stackelberg model framework. Technically, this means the leader substitutes the follower's optimal response function into its problem, and hence must have knowledge of it. This type of models is solved by the backward induction (Fudenberg and Tirole, 1991). Nevertheless, an implicit solution may be reached in a multiple-entity case due to the property of substitution for optimal responses. In addition, we doubt whether the leader will have knowledge of the follower's optimal response in real-world decentralized systems. Instead, to avoid the problems of information divulgence and implicit solutions, we develop an explicit decision-making mechanism for calculating the optimal (self-interest) acquisition prices and the independent optimal flow determination for recycled materials in a decentralized RPS.

While forward and reverse supply chains share many similarities, there are significant differences. For forward supply chain systems, the material flow volumes are usually assumed to be functions of all prices in the final market (Nicholson, 2002; Corbett and Karmarkar, 2001). Once the historical data of demand and prices are available, the quantity and price relationship can be predicted since retailers face a considerable number of customers and perfect market assumptions are not unreasonable. However, for the RPS, the number of entities in the network is relatively small compared with a forward supply chain network. The relationship of the quantity and price in certain parts of the supply chain cannot been derived due to the lack of data. Instead, we present a robust approach to determine the relationship between the material flow volume and price between the collection and processing tier of the supply chain.

The remainder of the paper is organized as follows. In Section 2, we give a brief literature review. In Section 3, we provide the formal definition of our two-tier problem: the upstream and downstream entities and their connection. In Sections 4 and 5, we develop mathematical models for upstream and downstream entities to determine the price and flow decisions in a decentralized RPS. In Section 6, we apply the algorithm to a numerical example to determine the equilibrium product prices and resulting flows, and also provide a discussion of the model and results. Section 7 presents conclusions and also suggests directions for future research.

2. Literature review

The past decade has seen an enormous increase in research on reverse logistics management issues. Flapper (1995, 1996), Fleischmann et al. (2000), and Guide and Harrison (2003) give systematic overviews and challenges of the logistic aspects of reuse and recycling in closed loop supply chains. Much of the research in RPS tends to be product, or system, specific due to the various features and complexities needed to handle the different recycling and reuse scenarios. Research on recycling and resource recovery for specific materials such as paper, plastics and sand include Pohlen and Farris (1992), Wang et al. (1995), Huttunen (1996) and Barros et al. (1998). Examples of product recovery and reuse include copy machines (Thierry et al., 1995; Thierry, 1997; Krikke, 1998), computers and electronics equipment (Jayaraman et al., 1997; Hong et al., 2006), and reusable transportation containers (Kroon and Vrijens, 1995). Chung and Wee (2008) investigate the impact of the green product design. The basic underlying assumption in these papers is that the planning of reverse logistics operations is done by a single decision maker to optimize the total system performance.

There are a growing number of research papers on forward or reverse supply chains that model the independent decision-making process of each supply chain entity, specifically the interaction between pricing decisions and material flow volume transacted in the network. Majumder and Groenevelt (2001) examine the competition behavior between an original equipment manufacturer (OEM) and the third-party local remanufacturer when the recycled products affect the demand of the original products. Guide et al. (2003) present an economic analysis for calculating the optimal acquisition prices and the optimal selling price for remanufactured products with different quality classes in one single remanufacturing firm. Savaskan et al. (2004) model three options for collecting used products, subcontracting with retailers, outsourcing to a third-party firm, and collecting by themselves, as decentralized decision-making systems with the manufacturer being the Stackelberg leader. Savaskan and Van Wassenhove (2006) analyze different reverse channel designs of direct and indirect product collection systems where the manufacturer collects used products directly from the consumers or collects via retailers. The models presented in the above papers are limited in the number of supply chain entities and their coordination. Several researchers have presented competition models with the scope of multiple entities (Corbett and Karmarkar, 2001; Nagurney and Toyasaki, 2005). Corbett and Karmarkar (2001) develop a model that considers entry decisions and post-entry competition in multi-tier serial supply chains. Nagurney and Toyasaki (2005) use a variational inequality solution approach to solve for the equilibrium network flow and *endogenous* prices of recycled materials. In this paper, instead, we consider a general RPS network structure with two tiers and multiple entities and propose an algorithm to solve independently for the *explicit* equilibrium acquisition prices and resulting network flows within the network.

3. A two-tier RPS problem: upstream and downstream

A RPS to reuse or recycle end-of-life products is a network of transportation logistics and processing functions that collect, refurbish, and demanufacture. In general, several entities in different tiers compose a network of collection and processing steps, connected by a transportation logistics system. In this paper, for simplicity, we assume a basic RPS consisting of two tiers of multiple facilities, one collection and one processing, facing sources and demand markets. Material flow allocation and product acquisition are common challenges for the reverse logistics network, where the network may be geographically dispersed. Our experience with firms or non-profit recycling organizations in scrap electronics (e-scrap) reveals several specific questions that go beyond the current reverse logistics models either in the strategic or operational level.

- What is the end-of-life product transaction mechanism between collectors and processors when they negotiate the price-flow contract?
- How do the collectors allocate their collected items to the processors if both of them are run by independent individuals?
- How do the processors determine their price offers if they bid for the collected items from collectors?

We first illustrate a two-tier network problem consisting of upstream and downstream tiers for a RPS depicted in Fig. 1. In general, the RPS is a network of several entities with functions that include collection and processing

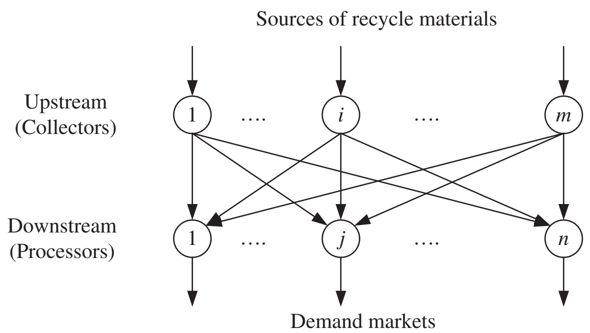

Fig. 1. A two-tier RPS network with collection and processing sites.

phases. The upstream tier represents the collection phase relating to sorting/consolidation processes and the downstream tier denotes the processing phase including refurbish/demanufacturing processes. Upstream entities collect end-of-life items from the residential or business sectors, and then independent downstream entities bid exchange prices for collected items from upstream entities. Upstream entities usually collect in distinct market segments (business or residential) or distinct geographic locations. Thus, there is no competition among the upstream entities in our model. A successful upstream entity must carefully manage its material flow allocation of collected items, i.e., design an effective, fair and transparent price-flow contracts between itself and downstream entities, to pursue its self-interest and ensure it meets the demand for material. Independent downstream entities compete for collected items from upstream entities with other members in the downstream tier. There are several value-added refurbishing/demanufacturing processes involved in the downstream entities and items are transformed to refurbished items, sub-components, or materials (e.g., used products, or raw materials) which are sold in several specific demand markets. An important issue for independent downstream entities is how to determine the optimal acquisition price, which is used to acquire the items from upstream entities.

We focus on the transaction between upstream and downstream tiers on material flow allocation and associated price decisions. Upstream entities collect end-of-life products from residential or business sectors, which may hold positive- or negative-value recycled items. In the e-scrap industry, residential or business sources may need to pay a collection fee to collectors for discarding the obsolete e-scrap items (Hong et al., 2006). We assume that the collection amount in upstream entities is a function of the collection fee that the upstream entity charges from end-of-life product sources: the higher the fee, the lower the potential amount collected from sources. We let the source supply function denote this function. Downstream entities convert end-of-life products into several valuable raw materials and used products as well as trash after refurbish/demanufacturing processes. Since the transportation cost for the recycled item is paid by the downstream entity, downstream entities have less incentive to acquire homogenous collected items from further away upstream entities than near sites. It is reasonable to assume that the valuation of homogenous items collected in different upstream entities is identical and downstream entities will compensate for transportation costs. Consequently, the net unit reward received by an upstream entity is assumed to be the acquisition price minus the unit transportation cost. In this paper, we specifically focus on the transaction of valuable items between upstream and downstream tiers and, as a result, we assume that the acquisition prices to be offered by downstream entities are positive. We also argue that the amount of raw materials resulting from the decomposition of end-of-life products and used products is relatively small compared with the quantity in the virgin raw material and brand-new product markets. This observation leads to the assumption that the selling prices

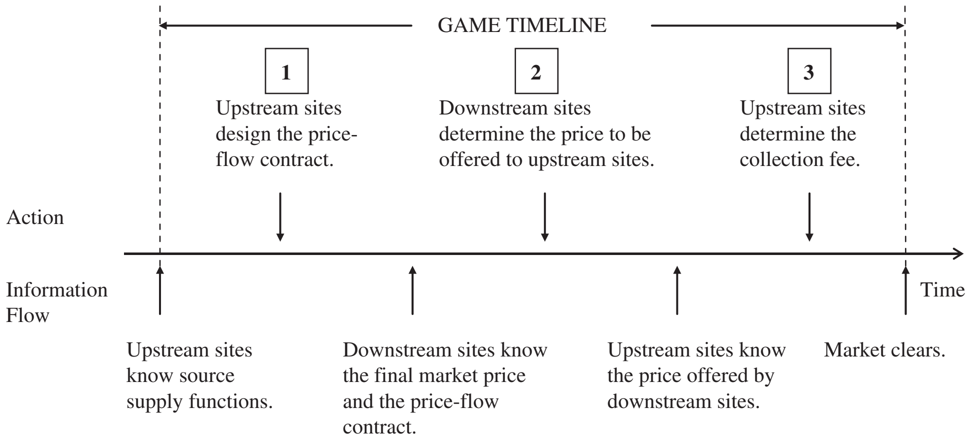

Fig. 2. The decision timeline for a two-tier problem.

of raw materials or used products in demand markets are fixed amounts, not affected by the sales quantities.

We focus on two issues: the equilibrium acquisition prices to be offered by downstream entities and the optimal price-flow contract between upstream and downstream tiers. The price-flow contract is a mechanism describing the correspondence between the acquisition prices offered by downstream entities and the flow amount supplied by the upstream entity to its subsequent downstream entities. The decision timeline for a two-tier problem is shown in Fig. 2 where the upper arrows indicate the tasks of upstream or downstream entities, and the lower arrows state information disclosure timeline. The steps of the solution algorithm are illustrated by the number in the rectangle box in Fig. 2. Upstream entities first determine the price-flow contracts and communicate them to the associated downstream entities. We assume that upstream entities are unable to change the price-flow contracts after communication. Then, downstream entities compete with other entities within the same tier for the flow from upstream entities on the basis of the price they offer for the recycled items. The downstream tier gives a single vector of prices that apply to all the upstream tier members. The decisions of downstream entities are the acquisition prices to be offered to the upstream tier. Downstream entities simultaneously choose their respective price decisions based on the price-flow contracts given by the associated upstream entities. Finally, upstream entities determine the collection fee to acquire the corresponding amount of recycled items from the source. We present our modeling for independent upstream and downstream entities in the following subsequent sections.

4. The upstream model: price-flow contract

In this section, we present a robust optimization model for the independent upstream entity to determine the robust price-flow contract between upstream and downstream tiers. For simplicity, we refer to the price-flow contract as the *flow function*. We depict the upstream and downstream sites as nodes and the material flows as links in Fig. 1. Specifically, we consider m upstream sites who are involved in the collection of end-of-life products, which can then be acquired by n downstream sites. A typical upstream site is denoted by i, and a typical downstream site by j. We first discuss the robust approach and scenario setting in the upstream model followed by the description of flow functions determined by the independent upstream site and the upstream model itself.

4.1. *The robust approach and scenario setting*

The goal of the upstream model for any particular site i, $i = 1,2,\dots,m$, is to design a "good" price-flow contract, or flow function. Due to the assumption of no information sharing in our decentralized model, upstream entities do not know the exact final acquisition prices to be offered by downstream entities. Each upstream entity predicts the possible range of acquisition prices as input information for determining flow functions. One way to forecast lower and upper bounds of acquisition prices is based upon the information of transportation costs and market prices. A possible lower bound on the price is the transportation cost between upstream and downstream tiers; otherwise, the upstream sites obtain a negative price offer since the net unit reward received by an upstream entity is the acquisition price minus the unit transportation cost. A negative unit reward is not in their interest. Therefore, we assume that the forecast acquisition prices are at least as much as the associated transportation costs in the model. Another possible lower bound is the market price if upstream sites are unwilling to sell for less than the market price. However, if downstream sites own the bargaining power, the highest market price may be a potential upper bound of the acquisition price since downstream sites are unwilling to pay more than the market price for acquiring colleted items from upstream sites.

A particular price combination, $(P_{1\omega},\ldots,P_{j\omega},\ldots,P_{n\omega})$, of downstream entities refers to one scenario $\omega\in\Omega$, where $P_{j\omega}$ is the unit material price downstream site j willing to offer in price scenario ω. There are an infinite number of scenarios because the range of acquisition prices forecasted by the upstream entities fall in continuous compact intervals. In this paper, for computational convenience, we assume that a finite number of discrete points are chosen by each upstream entity to represent the price range for the flow function design. A practical approach for computation is to select k points evenly in every dimension of the price range. Thus, the scenario space Ω considered is with k^n scenarios if there are n downstream tier entities. We note that upstream entities make this relaxation for computational convenience when they design flow functions, but downstream entities still determine acquisition prices in a continuous space.

The objective of upstream entities is to construct a set of robust flow functions against the price ambiguity. In this paper, we use the measure of robust deviation defined by Kouvelis and Yu (1997), such that each upstream site is to minimize the maximum difference between the best it can obtain when price offers from downstream sites are realized and the robust objective value under the designed flow function. This differs from a stochastic approach because each upstream entity is not required to assign a probability distribution over the acquisition prices, and it has practical benefits as the knowledge of acquisition prices is limited in a decentralized RPS. This minimax optimization approach (Winston, 1994) captures a notion of "risk"—the upstream site wants to protect itself from doing very poorly in a given price realization, which is unknown before contracting with the downstream tier.

4.2. The price-flow contract: flow functions

A growing literature in operations management presents studies of supply chain contracting (see Tsay, 1999; Donohue, 2000; Cachon and Lariviere, 2001; Corbett et al., 2004; Giannoccaro and Pontrandolfo, 2004). These papers focus on the coordination between two parties of an upstream and a downstream entity (say a supplier and a buyer). In this paper, we present a general price-flow contract describing not only the coordination between tiers but also the competition within the tier. Intuitively the upstream entity tends to ship higher flow to the downstream entity who offers the higher acquisition price. Obviously the price-flow contract from upstream sites to downstream sites is dependent of acquisition prices offered by downstream entities.

We let $V_{ij}^{(\mathrm{Tr})}$ denote the unit transportation cost from site i to j. The unit reward that upstream site i receives from downstream site j is represented as the material price that the downstream entity is willing to offer while covering the associated unit transportation cost. Therefore, the unit reward of site i in price scenario ω is $P_{j\omega}-V_{ij}^{(\mathrm{Tr})}$. For the material flow from upstream site i to downstream site j for scenario ω, denoted by $x_{ij\omega}^{(\mathrm{Tr})}$, upstream site i tends to increase the flow amount on $x_{ij\omega}^{(\mathrm{Tr})}$ if downstream site j offers a higher price. Meanwhile, upstream site i may decrease the amount of $x_{ij\omega}^{(\mathrm{Tr})}$ to feed more flow to other arcs if other downstream sites provide more incentives in price offers. Our modeling implies any particular arc of material flow is not only a function of the price offered by its destination downstream site, but also the relative price offers of other downstream sites. The decision variables for upstream site i are the coefficients of material flow determination, denoted by $\alpha_{ijj'}$, from upstream site i to downstream site j affected by downstream site j' for all of downstream site pairs j and j'. Note the decision variables of $\alpha_{ijj'}$ are not dependent of price scenario ω: $\alpha_{ijj'}$ is a common set of coefficients for all of price scenarios. Using the common linear function assumption (Corbett and Karmarkar, 2001; Guide et al., 2003), the material flow from upstream site i to downstream site j in price scenario ω is represented as

$$x_{ij\omega}^{(\mathrm{Tr})}=\sum_{j'=1}^{n}\alpha_{ijj'}(P_{j'\omega}-V_{ij'}^{(\mathrm{Tr})})\quad\forall i,j. \tag{1}$$

4.3. Potential maximum flow determination

Upstream entities select several discrete price points to represent the possible acquisition price range, and they may determine different collection fees and collect different amounts from sources given different acquisition prices to be offered by downstream entities. Before constructing price-flow contracts for upstream entities, we first examine transactions between upstream sites and sources to obtain the potential maximum flow corresponding to each different price scenario pre-selected by upstream entities. Assume the collection amount in upstream site i, $i=1,2,\ldots,m$, is given by a source supply function $S_{i\omega}=a_i-b_iP_{i\omega}^{(\mathrm{Co})}$, where a_i and b_i are parameters and $a_i,\ b_i>0$. We let $P_{i\omega}^{(\mathrm{Co})}$ denote the collection fee charged by site i, and $S_{i\omega}$ be the potential maximum flow amount collected in upstream site i corresponding to price scenario ω. To ensure that the upstream site i obtains a non-negative amount of flow, we require $P_{i\omega}^{(\mathrm{Co})}\leqslant a_i/b_i$ for all price scenarios. The potential profit of upstream site i in price scenario ω, $\Pi_{i\omega}$, is

$$\begin{aligned}\prod\nolimits_{i\omega}&=S_{i\omega}\left(P_{i\omega}^{(\mathrm{Co})}+\max_{j=1,\ldots,n}\{P_{j\omega}-V_{ij}^{(\mathrm{Tr})}\}\right)\\&=(a_i-b_iP_{i\omega}^{(\mathrm{Co})})\left(P_{i\omega}^{(\mathrm{Co})}+\max_{j=1,\ldots,n}\{P_{j\omega}-V_{ij}^{(\mathrm{Tr})}\}\right),\end{aligned} \tag{2}$$

where upstream site i picks the highest price offer from downstream sites as the selling price and the only unknown variable in (2) is the collection fee of $P_{i\omega}^{(\mathrm{Co})}$ that site i charged for the material from sources corresponding to price scenario ω. For notation simplicity, we let $P_{i\omega}^{(\max)}=\max_{j=1,\ldots,n}\{P_{j\omega}-V_{ij}^{(\mathrm{Tr})}\}$. The potential profit function $\Pi_{i\omega}$ is concave in $P_{i\omega}^{(\mathrm{Co})}$ whenever $b_i>0$, so (2) is maximized when the first-order condition holds, i.e., when

$$P_{i\omega}^{(\mathrm{Co})*}=\min\{(a_i-b_i\,P_{i\omega}^{(\max)})/2b_i,a_i/b_i\}\quad\forall i,\omega. \tag{3}$$

Thus, (3) is the optimal collection fee for upstream site i in price scenario ω. The potential maximum flow amount collected in upstream site i corresponding to price scenario ω, $S^{*}_{i\omega}$, can be obtained by substituting $P^{(\mathrm{Co})*}_{i\omega}$ into $S_{i\omega}=a_i-b_iP^{(\mathrm{Co})}_{i\omega}$. Note the pre-determined collection fee $P^{(\mathrm{Co})*}_{i\omega}$ and the available flow $S^{*}_{i\omega}$ are realized by solving (3) in price scenario ω being input information for the robust optimization model presented in Section 4.4 to determine price-flow contracts.

4.4. The robust model for upstream sites

To execute the robust approach, first the optimal solution of each upstream site for each specified price scenario is found. This solution calculates the highest profit that the individual upstream site can obtain if it were to know the acquisition prices exactly. Then, we minimize the maximum deviation of the objective function value between the "ideal" and the "robust" sales profit for all price scenarios. Finally, we adjust the decision variables, α's, to ensure those returning the best sales profit for all tight and non-tight price scenarios. We let $O^{*}_{i\omega}$ denote the optimal objective value of upstream site i for price scenario ω, and $C^{(\mathrm{Tr})}_{ij}$ denote the shipment capacity between upstream site i and downstream site j. We assume that each upstream site i seeks to maximize the total profit associated with its collection and material allocation operations with the optimization problem given as follows for upstream site i for price scenario ω.

Maximize $O_{i\omega}$

$$\textbf{Subject to}: \quad O_{i\omega}=\sum_{j=1}^{n}x^{(\mathrm{Tr})}_{ij\omega}(P_{j\omega}-V^{(\mathrm{Tr})}_{ij}+P^{(\mathrm{Co})^*}_{i\omega}), \tag{4}$$

$$x^{(\mathrm{Tr})}_{ij\omega}=\sum_{j'=1}^{n}\alpha_{ijj'}(P_{j'\omega}-V^{(\mathrm{Tr})}_{ij'}) \quad \forall j, \tag{5}$$

$$x^{(\mathrm{Tr})}_{ij\omega}\leqslant C^{(\mathrm{Tr})}_{ij} \quad \forall j, \tag{6}$$

$$\sum_{j=1}^{n}x^{(\mathrm{Tr})}_{ij\omega}\leqslant S^{*}_{i\omega}, \tag{7}$$

$$x^{(\mathrm{Tr})}_{ij\omega}\geqslant 0 \quad \forall j, \tag{8}$$

$$\alpha_{ijj'}>0 \quad \forall j,j', \quad j=j', \tag{9}$$

$$\alpha_{ijj'}\leqslant 0 \quad \forall j,j', \quad j\neq j'. \tag{10}$$

The objective function (4) is the sum of the sales profits and collection fees. Constraints (5) are the material flow function definitions for emanating arcs from upstream site i. Constraints (6) and (7) provide capacity limits for each arc and for the recycled item source. Constraints (8), (9), and (10) are sign restrictions for unknown variables. Obviously, the material flow variables, $x^{(\mathrm{Tr})}_{ij\omega}$, are non-negative, and the sign restrictions for $\alpha's$ require that the upstream site has more incentive to ship more flow on the arc where its destination price offer is increased, but less incentive when other downstream sites offer higher prices competing the material flow.

Next, we determine the robust flow function, or a common set of coefficients, $\alpha's$, to be evaluated in every price scenario $\omega\in\Omega$ for site i. Thus, for each price scenario we subtract the robust objective function value ($R_{i\omega}$) using the common set of robust coefficients from the optimal objective value ($O^{*}_{i\omega}$) of realization of acquisition price offers. The min–max robust optimization model over all price scenarios for upstream site i can be stated as

Minimize δ_i

$$\textbf{Subject to}: \delta_i\geqslant O^{*}_{i\omega}-R_{i\omega} \quad \forall\omega, \tag{11}$$

$$R_{i\omega}=\sum_{j=1}^{n}x^{(\mathrm{Tr})}_{ij\omega}(P_{j\omega}-V^{(\mathrm{Tr})}_{ij}+P^{(\mathrm{Co})^*}_{i\omega}) \quad \forall\omega, \tag{12}$$

$$x^{(\mathrm{Tr})}_{ij\omega}=\sum_{j'=1}^{n}\alpha_{ijj'}(P_{j'\omega}-V^{(\mathrm{Tr})}_{ij'}) \quad \forall j,\omega, \tag{13}$$

$$x^{(\mathrm{Tr})}_{ij\omega}\leqslant C^{(\mathrm{Tr})}_{ij} \quad \forall j,\omega, \tag{14}$$

$$\sum_{j=1}^{n}x^{(\mathrm{Tr})}_{ij\omega}\leqslant S^{*}_{i\omega} \quad \forall\omega, \tag{15}$$

$$x^{(\mathrm{Tr})}_{ij\omega}\geqslant 0 \quad \forall j,\omega, \tag{16}$$

$$\alpha_{ijj'}>0 \quad \forall j,j', \quad j=j', \tag{17}$$

$$\alpha_{ijj'}\leqslant 0 \quad \forall j, \quad j',j\neq j'. \tag{18}$$

The minimum maximum deviation δ^{*}_{i} of upstream site i is realized after solving the min–max robust optimization model. The final step of the upstream model is solving the following model to optimality to ensure that the decision variables, α's, return the best sales profit for non-effective price scenarios (non-tight price scenarios in (11)) for upstream site i. The model for each upstream site i is

Maximize $\sum_{\omega\in\Omega}R_{i\omega}$

Subject to : $\delta^{*}_{i}\geqslant O^{*}_{i\omega}-R_{i\omega} \quad \forall\omega.$

Constraints set of (12)–(18).

Given the robust solution values for α, the upstream site models determine robust flow functions for each independent upstream site. Thus, the robust flow function describing the flow shipment from upstream site i to downstream site j, denoted by $x^{(\mathrm{Tr})}_{ij}$, is represented as

$$x^{(\mathrm{Tr})}_{ij}=\sum_{j'=1}^{n}\alpha_{ijj'}(p_{j'}-V^{(\mathrm{Tr})}_{ij'}) \quad \forall i,j, \tag{19}$$

where $p_{j'}$ is the acquisition price offered by downstream site j'. Note that the price scenario ω is not an argument in the flow function at this point, and that (19) describes the material flow relationship of the amount and acquisition price between upstream and downstream tiers. Each upstream site provides each downstream site with a robust flow function to govern the material flow transactions between them. The general form of the price-flow contract for each arc shown in (19) represents not only the coordination between tiers but also the competition within the tier. For example, in the price-flow contract of arc (i,j) associated with upstream site i, site i specifies both how downstream site j's acquisition price affects the flow on arc (i,j) and how other downstream sites' acquisition prices influence the flow on arc (i,j). If an upstream entity determines its price-flow contract based on only the price of the specific downstream j it is considering, the price-flow contract is found by setting the rest of the α's equal to zero in the upstream model.

In the upstream model, each upstream entity first predicts the possible range of acquisition prices and chooses several discrete points (scenarios) to represent the price range for further flow function design. Then, each upstream entity considers the potential available input flow by pre-determining the optimal collection fee corresponding to each price scenario. The procedure of the upstream model ends when flow functions are determined by min–max robust optimization programs. In Section 5, we present downstream site models to solve for the equilibrium acquisition prices between these two tiers.

5. The downstream model: the equilibrium price

Downstream sites are involved in transactions with upstream sites and customers in final demand markets since they wish to obtain recycled items from upstream tier and sell the materials or sub-components after refurbished/demanufacturing processes. Downstream sites make decisions on their own acquisition prices subject to their constraints of processing capacities, transportation capacities, and technology restrictions. We develop an equilibrium model of competitive downstream sites to determine the Nash equilibrium price where no downstream site can improve its objective function value by a unilateral change in its price solution. In this paper, we utilize the *relaxation algorithm* (see Krawczyk and Uryasev, 2000; Contreras et al., 2004) to find the Nash equilibrium price solution.

5.1. The optimization model for the downstream site

The independent downstream site maximizes its objective function associated with the purchasing, processing cost and sales revenue and is subject to constraints imposed on the processing, transportation capacity, and demand restrictions. Required notation for the downstream model in addition to the upstream model notation is listed as follows.

Downstream model parameters:

$P_j^{(\mathrm{Sa})}$ Net selling price offered per standard unit of material to downstream site j;

$C_j^{(\mathrm{Pr})}$ Maximum amount of material that can be processed at downstream site j;

$C_{ij}^{(\mathrm{Tr})}$ Maximum amount of material that can be shipped from upstream site i to downstream site j;

$V_{ij}^{(\mathrm{Tr})}$ Transportation cost per standard unit per distance from upstream site i to downstream site j.

Downstream model variables:

p_j Price offered per standard unit by downstream site j;

$x_{ij}^{(\mathrm{Tr})}$ The material flow from upstream site i to downstream site j.

Using this notation, the optimization model for downstream site j can be stated as

$$\textbf{Maximize} \quad \sum_{i=1}^{m}(P_j^{(\mathrm{Sa})} - p_j)x_{ij}^{(\mathrm{Tr})}, \tag{20}$$

$$\begin{aligned}
\textbf{Subject to}: \quad & x_{ij}^{(\mathrm{Tr})} = \sum_{j'=1}^{n}\alpha_{ijj'}(p_{j'} - V_{ij'}^{(\mathrm{Tr})}) && \forall i \quad \text{Flow definition,}\\
& \sum_{i=1}^{m} x_{ij}^{(\mathrm{Tr})} \leqslant C_j^{(Pr)} && \quad \text{Processing capacity,}\\
& x_{ij}^{(\mathrm{Tr})} \leqslant C_{ij}^{(\mathrm{Tr})} && \forall i \quad \text{Transportation capacity,}\\
& x_{ij}^{(\mathrm{Tr})} \geqslant 0 && \forall i \quad \text{Variable restrictions,}\\
& p_j \geqslant 0.
\end{aligned}$$

We assume that recycled items coming from different upstream sites are homogeneous. Thus, the total flows shipped to downstream site j, which is denoted by $x_j^{(\mathrm{Tr})}$, is the sum of flows from different upstream sites to downstream site j and is expressed as follows:

$$x_j^{(\mathrm{Tr})} = \sum_{i=1}^{m} x_{ij}^{(\mathrm{Tr})} = \sum_{i=1}^{m}\sum_{j'=1}^{n}\alpha_{ijj'}(p_{j'} - V_{ij'}^{(\mathrm{Tr})}) \quad \forall j. \tag{21}$$

Here, the material flow variable for recycled items shipped to downstream site j, $x_j^{(\mathrm{Tr})}$, is the function of acquisition prices of all downstream sites. Hence in order for the downstream site to compute its optimal price bid, it must know the bids of the other downstream sites, and only this. The optimization model of (20) for downstream site j can be generally transformed into the model shown in (22), expressed in acquisition price variables where $\mathbf{p} = (p_1, \ldots, p_n)$ are the collective price actions and where ϕ_j is the payoff (or objective) function of downstream site j. Let g_d^j denote the row d of constraint function and b_d^j the right-hand-side parameter of row d in downstream site j's optimization model.

$$\begin{aligned}
\textbf{Maximize} \quad & \phi_j(\mathbf{p})\\
\textbf{Subject to}: \quad & g_1^j(\mathbf{p}) \leqslant b_1^j\\
& \vdots\\
& g_r^j(\mathbf{p}) \leqslant b_r^j\\
& p_j \geqslant 0.
\end{aligned} \tag{22}$$

Next, we show the convex property of downstream site models for the existence and uniqueness of the Nash equilibrium acquisition price solution.

Proposition 1. *The optimization model for downstream site j, $j = 1, \ldots, n$, has a strictly concave objective function with respect to p_j and a convex constraint set.*

Proof. Trivially the set of linear constraints is a convex set with respect to price variables (Nemhauser and Wolsey, 1999).

Again, the format of the material flow variable $x_j^{(\mathrm{Tr})}$ is written as $\sum_{i=1}^{m}\sum_{j'=1}^{n}\alpha_{ijj'}(p_{j'} - V_{ij'}^{(\mathrm{Tr})})$ where all coefficients α's are given by upstream sites and $V_{ij}^{(\mathrm{Tr})}$ are transportation cost parameters. The only unknown variables are all p's.

The variable $x_j^{(\mathrm{Tr})}$ can be rewritten as

$$x_j^{(\mathrm{Tr})} = \alpha_1^j p_1 + \cdots + \alpha_n^j p_n + C_j, \tag{23}$$

where C_j is a constant and $\alpha_{j'}^j$ is the coefficient term with $p_{j'}$ for downstream site j's model. The interpretation of (23) is that the material flows shipped to downstream site j are a function of price p_j and also functions of other price variables offered by other downstream sites. Clearly, the material flow is increasing as the price offer increases but decreasing when the competitors' prices increase, if there exists a price effect between downstream site j and other downstream sites. Thus, we have the following inequality relations

$$\alpha_j^j > 0 \quad \text{and} \quad \alpha_{j'}^j \leqslant 0 \quad \forall j', j' \neq j. \tag{24}$$

From (24), the sign of second derivative of the objective function ϕ_j can be determined as $\partial^2\phi_j/\partial p_j^2 < 0$. □

Proposition 2. *The impact of the change of acquisition price* $p_j, j = 1,2,\ldots,n$, *on the material flow* $x_j^{(\mathrm{Tr})}$ *is greater than the total impact on* $x_j^{(\mathrm{Tr})}$ *due to the price changes from the rest of downstream sites.*

Proof. For every upstream site i, we have $\alpha_{ijj'} > 0$ for all j, j' when j is equal to j', and $\alpha_{ijj'} \leqslant 0$ for all j, j' when j is not equal to j'. In order to ensure a positive robust objective function for each arc between the upstream and downstream tier in all of price scenarios, ω, we have $|\alpha_{ijj}| > \sum_{j'=1, j' \neq j}^{n} |\alpha_{ijj'}|$ for all i and j. Therefore, $|\alpha_j^j| > \sum_{j'=1, j' \neq j}^{n} |\alpha_{j'}^j|$ for all j and it completes the proof. □

In Section 5.2, we provide some concepts and the required notation for the illustration of the downstream site model algorithm.

5.2. Definitions and concepts

There are $j = 1, \ldots, n$ downstream sites participating in competing the material flows with the price. Each downstream site j, $j = 1, \ldots, n$, can adopt an individual price setting denoted by $p_j \in P_j$, where P_j is the set of price actions that downstream site j can choose. All downstream entities, when acting together, can take a collective action, which is a vector $\mathbf{p} = (p_1, \ldots, p_n)$. Denote the collective price action set by P, and, by definition, $P \subseteq P_1 \times P_2 \times \cdots \times P_n$. Let $\mathbf{p} = (p_1, \ldots, p_n)$ and $\mathbf{q} = (q_1, \ldots, q_n)$ be elements of the collective price action set $P_1 \times P_2 \times \cdots \times P_n$.

The following notation and terminology are based upon Krawczyk and Uryasev (2000) and Contreras et al. (2004). An element $(q_j|\mathbf{p}) \equiv (p_1, \ldots, p_{j-1}, q_j, p_{j+1}, \ldots, p_n)$ of the collective price action set can be interpreted as a set of price actions where the jth downstream entity selects price offer q_j while the remaining entities are taking price p_j, $j = 1,2,\ldots,j-1,j+1,\ldots,n$. A price action set $\mathbf{p}^* = (p_1^*, \ldots, p_n^*)$ is called the *Nash equilibrium price* if, for downstream site j,

$$\phi_j(\mathbf{p}^*) = \max_{(p_j|p^*) \in P} \phi_j(p_j|\mathbf{p}^*).$$

Note that at the Nash equilibrium solution, no entity can improve its individual objective value by a unilateral change in its price decisions. In order to compute the Nash equilibrium, we introduce the Nikaido–Isoda function (Nikaido and Isoda, 1955). This function transforms an equilibrium problem into an optimization problem (Contreras et al., 2004). The *Nikaido–Isoda function* $\Psi : (P_1 \times \cdots \times P_n) \times (P_1 \times \cdots \times P_n) \to \Re$ is defined as

$$\Psi(\mathbf{p}, \mathbf{q}) = \sum_{j=1}^{n} [\phi_j(q_j|\mathbf{p}) - \phi_j(\mathbf{p})].$$

Each summand of the Nikaido–Isoda function can be viewed as the change in the objective function value when its price action changes from p_j to q_j for all sites j in the downstream tier, while all other downstream sites continue to choose according to price vector $\mathbf{p}$. This means that one entity changes its price action while others do not. Thus, the function represents the sum of these changes in objective functions. Krawczyk and Uryasev (2000) claim that the function is non-positive for all feasible $\mathbf{q}$ when $\mathbf{p}^*$ is a Nash equilibrium solution, since no entity can improve its objective function value at equilibrium by unilaterally alternating its solution. This observation is used to construct a termination condition for the relaxation algorithm, such that when an ε is chosen, the Nash equilibrium is obtained when $\max_{\mathbf{q} \in P} \Psi(\mathbf{p}^s, \mathbf{q}) < \varepsilon$, where s is the iterative step of the relaxation algorithm.

Finally we introduce the optimum response function, which returns the set of downstream entities' price actions whereby they all try to unilaterally maximize their respective objective function values. The *optimum response function* (Krawczyk and Uryasev, 2000) at the price vector $\mathbf{p}$ is expressed in (25) and it is a collective function mapping the previous price decisions to the next price decisions of all downstream entities. Technically, each downstream entity unilaterally alters its price solution on the basis of knowledge of other downstream entities' previous price solutions and its own objective function.

$$Z(\mathbf{p}) = \arg \max_{q \in P} \Psi(\mathbf{p}, \mathbf{q}) \quad \mathbf{p}, Z(\mathbf{p}) \in P. \tag{25}$$

Next, we illustrate the relaxation algorithm to solve for the Nash equilibrium acquisition prices between upstream and downstream tiers.

5.3. The relaxation algorithm

The relaxation algorithms are used by Krawczyk and Uryasev (2000) and Contreras et al. (2004) for different applications. We apply the relaxation algorithm to iteratively search for the Nash equilibrium acquisition price solution of downstream site models. At each iteration of the algorithm, downstream sites wish to move to a price point that represents an improvement on the current price point. Having an initial estimate price vector, $\mathbf{p}^0$, the relaxation algorithm is shown as follows:

$$\mathbf{p}^{s+1} = (1 - \beta_s)\mathbf{p}^s + \beta_s Z(\mathbf{p}^s) \quad s = 0, 1, 2, \ldots, \tag{26}$$

where $0<\beta_s\leqslant 1$. The iterative step $s+1$ is constructed as a weighted average of the improvement price point $Z(\mathbf{p}^s)$ and the current price point $\mathbf{p}^s$. The optimum response function $Z(\mathbf{p}^s)$ returns the next best move of price solutions by solving the quadratic convex model shown in (20); in turn, each downstream site is trying to maximize its objective function by unilaterally moving its price solution given others' price solution. The iteration at each step is constructed as a weighted average of the improvement solution $Z(\mathbf{p})$and the current price solution $\mathbf{p}$. By taking a sufficient number of iterations, the algorithm converges to the *Nash equilibrium* price $\mathbf{p}^*$ with a specified precision.

It is also interesting to note that the concept of the algorithm itself matches the idea of a decentralized view on downstream sites. In each iteration, every entity can access all entities' previous price actions and determines its best move in price decision based on its own interests and constraints. In other words, the problem is a calculation of the succession of price decisions, where entities choose their optimum response given the price decisions of the competitors in the previous iteration. The following corollary shows that existence and uniqueness of the Nash equilibrium solution and the convergence of the relaxation algorithm to downstream sites' problems.

Corollary 1. *There exists a unique Nash equilibrium price solution for downstream sites to which the relaxation algorithm converges.*

Proof. See Appendix A.

We note that the usual solution space naturally defines the convex set P if there are no other pairing price restrictions. The necessary sign restrictions of α's ((9) and (10)) are possible and reasonable in real-world problems. The Relaxation Algorithm states that each downstream site is trying to maximize its objective function by unilaterally moving its price solution given others' previous price solutions. The algorithm itself essentially is in an iterative scheme and continues until all of downstream sites find the Nash equilibrium acquisition price solution where no downstream site is willing to alter its acquisition price based on the price-flow contracts given by associated upstream sites. In Section 6, we summarize the solution algorithm consisting of upstream and downstream models and explicitly illustrate the decision-making procedures under this framework of Sections 4 and 5.

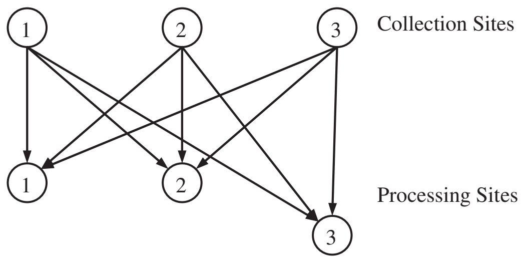

Fig. 3. The reverse production system for the example.

6. A numerical example

This example, depicted in Fig. 3, illustrates the application of the above upstream and downstream models. There are three collection sites, $i=1$, 2, and 3 in the upstream tier and three processing sites, $j=1$, 2, and 3 in the downstream tier. The collection sites collect end-of-life products from sources and ship them to processing sites. The transportation costs per unit flow between collection and processing sites are given in Table 1.

The final market prices for processing sites, $j=1$, 2, and 3 are \$105, \$110, and \$150, respectively. The collection amount functions in collection sites, $i=1$, 2, and 3 are given by $S_1=350-5P_1^{(\mathrm{Co})}$, $S_2=320-4P_2^{(\mathrm{Co})}$, and $S_3=330-5P_3^{(\mathrm{Co})}$, respectively. Clearly, the collection amount decreases as the collection site charges a higher collection fee per unit. In this example, we model the case that the transportation costs (or the distances) from Processing site 3 to collection sites are relatively larger than the costs of other arcs; however, the final market price for Processing site 3 provides the higher incentive to attract recycled items.

We assume that all collection sites predict the ranges of acquisition prices offered by processing sites, $j=1$, 2, and 3 as (\$60, \$70, and \$110)$\pm$20 in the upstream model. In other words, collection sites, $i=1$, 2, and 3 predict the price offered by processing sites to acquire obsolete products collected in collection sites are within the ranges of [\$40, \$80], [\$50, \$90], and [\$90, \$130] for processing sites, $j=1$, 2, and 3, respectively. We choose 5 evenly distributed points in each price range. The upstream model yields the following robust flow functions:

$$x_{11}^{(\mathrm{Tr})}=2.73(p_1-10)-.10(p_2-20)-1.36(p_3-75),$$
$$x_{12}^{(\mathrm{Tr})}=-.10(p_1-10)+2.73(p_2-20)-1.36(p_3-75),$$
$$x_{13}^{(\mathrm{Tr})}=-.35(p_1-10)-.35(p_2-20)+3.30(p_3-75),$$
$$x_{21}^{(\mathrm{Tr})}=2.72(p_1-12)-1.22(p_3-67.5),$$
$$x_{22}^{(\mathrm{Tr})}=-.64(p_1-12)+2.23(p_2-18)-.44(p_3-67.5),$$
$$x_{23}^{(\mathrm{Tr})}=-.88(p_1-12)+2.65(p_3-67.5),$$
$$x_{31}^{(\mathrm{Tr})}=2.78(p_1-15)-.73(p_2-22)-.31(p_3-64.5),$$
$$x_{32}^{(\mathrm{Tr})}=-.08(p_1-15)+2.67(p_2-22)-1.06(p_3-64.5),$$
$$x_{33}^{(\mathrm{Tr})}=-1.14(p_1-15)+2.92(p_3-64.5).$$

We apply the relaxation algorithm to obtain the Nash equilibrium acquisition prices determined by processing

Table 1
The unit transportation costs between entities

	$V_{11}^{(\mathrm{Tr})}$	$V_{12}^{(\mathrm{Tr})}$	$V_{13}^{(\mathrm{Tr})}$	$V_{21}^{(\mathrm{Tr})}$	$V_{22}^{(\mathrm{Tr})}$	$V_{23}^{(\mathrm{Tr})}$	$V_{31}^{(\mathrm{Tr})}$	$V_{32}^{(\mathrm{Tr})}$	$V_{33}^{(\mathrm{Tr})}$
Unit transportation cost	10	20	75	12	18	67.5	15	22	64.5

Table 2
The calculation of the relaxation algorithm for the example

Iteration(s)	p_1^s	p_2^s	p_3^s	$Z(p_1^s)$	$Z(p_2^s)$	$Z(p_3^s)$	α_s	p_1^{s+1}	p_2^{s+1}	p_3^{s+1}
1	60.00	60.00	60.00	58.73	65.79	116.72	1.00	58.73	65.79	116.72
2	58.73	65.79	116.72	68.95	76.36	116.67	0.99	68.85	76.25	116.67
3	68.85	76.25	116.67	69.47	76.90	118.23	0.98	69.46	76.88	118.20
4	69.46	76.88	118.20	69.77	77.22	118.33	0.97	69.76	77.21	118.32
5	69.76	77.21	118.32	69.81	77.26	118.37	0.96	69.81	77.25	118.37
6	69.81	77.25	118.37	69.82	77.27	118.38	0.95	69.82	77.27	118.38
7	69.82	77.27	118.38	69.82	77.27	118.38	0.94	69.82	77.27	118.38

Table 3
The resulting material flows between collection and processing sites

	$x_{11}^{(\mathrm{Tr})}$	$x_{12}^{(\mathrm{Tr})}$	$x_{13}^{(\mathrm{Tr})}$	$x_{21}^{(\mathrm{Tr})}$	$x_{22}^{(\mathrm{Tr})}$	$x_{23}^{(\mathrm{Tr})}$	$x_{31}^{(\mathrm{Tr})}$	$x_{32}^{(\mathrm{Tr})}$	$x_{33}^{(\mathrm{Tr})}$
Material flow	98.3	91.1	101.8	95.2	72.6	84.1	95.8	86.2	94.4

Table 4
The equilibrium collection fees

	$P_1^{(\mathrm{Co})*}$	$P_2^{(\mathrm{Co})*}$	$P_3^{(\mathrm{Co})*}$
Equilibrium collection fee	11.76	17.02	10.72

sites. The detailed steps of the relaxation algorithm are illustrated in Table 2. At iteration 7, the $\max_{\mathbf{q}\in P} \Psi(\mathbf{p}^s, \mathbf{q})$ approaches to zero, which indicates p_1^7, p_2^7, and p_3^7 are the Nash equilibrium acquisition prices for processing sites 1, 2, and 3.

The corresponding material flows between collection and processing sites and equilibrium collection fees of collection sites are listed in Tables 3 and 4, respectively.

The preceding example demonstrates a two-tier and single-commodity decentralized RPS problem that can be solved using the models given in Sections 4 and 5. First, the upstream model for each collection site provides the flow functions used to contract with processing sites. Then, we solve for the Nash equilibrium acquisition prices between collection and processing sites by the relaxation algorithm. Finally, we obtain the corresponding material flows between these two tiers and the equilibrium collection fees of collection sites. We note the solution of equilibrium acquisition prices may not return a value located within the prediction price range in some extreme cases. If the equilibrium price is above the range, it may lead to infeasible flows due to the flow capacity of sites or arcs. To avoid this, we assume that the collection sites conservatively predict the range of acquisition prices so that the equilibrium acquisition prices are within corresponding price ranges. Otherwise, they may cause a significant loss due to a potential penalty of unsatisfactory supply to processing sites. We leave the penalty mechanism design for future work and summarize our current results in Section 7.

7. Conclusions and extensions

This paper presents a decentralized perspective for reverse production systems where each independent entity considers its own objective function and is subject to its own constraints. Meanwhile, the objective function of each entity not only depends on its own decision variables but also depends on decision variables of other entities. In this paper, we focus on a two-tier reverse production system involving the price and material flow decisions where the price-flow contract is determined by upstream entities and the acquisition prices of material flows transacted between upstream and downstream sites are determined by downstream entities. We apply the min–max robust optimization on each of the independent upstream models to generate the flow functions, which are used to contract with downstream sites.

Downstream entities compete for material flows from the upstream tier. The iterated relaxation algorithm is used to solve for the Nash equilibrium acquisition prices between upstream and downstream sites. Note that the algorithm itself matches the idea of a decentralized decision-making process where every downstream entity can access all entities' previous price actions and determines its next best move for its price decision. We also show the existence and uniqueness of the Nash equilibrium price under reasonable assumptions about the underlying functions of each entity. Then, the equilibrium acquisition prices from downstream entities are communicated to associated upstream entities, who then determine the flows to the downstream entities and the collection fees to acquire recycled items.

In this paper, upstream entities determine price-flow contracts by using a robust optimization approach, but there are other criteria, such as an expected value, a max–min objective, a Bayesian belief, which may be used by different upstream entities. Also, many reverse production systems have network structures that involve more than two types of entities we have discussed here, and with more than one type of item to be picked up and recycled. For example, computers, printers, monitors, and other auxiliary equipment are available from sources and may be converted into commodities such as steel and copper through a supply chain that involves multiple processors engaged in size reduction and smelting. The extension of our approach to these multi-tier problems with multiple item types requires further refinement of the models we have developed.

It is worth mentioning that the mechanism proposed in this paper may also exist in conventional supply chains where new products may not be necessarily traded in an open-market. Instead, several types of commodities are transacted between upstream and downstream tiers in a similar way to the model presented in this paper. For example, purchasing agents might place bids for agricultural products ranging from flowers and livestock using the type of flow function approach described in this paper.

Finally, the reverse supply chains differ general forward chains in that, in the former, the government may involve more with policy making or evaluation. The model presented in this paper is a prototype decentralized RPS model and can be used as a tool to analyze the issues of the government-subsidy, price fluctuation, comparison of centralized vs. decentralized systems, and other situations where the individual behavior of the supply chain components might be an important overall factor in the system behavior.

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Appendix A

We present required definitions as follows (see Contreras et al., 2004). Let $\Psi : P\times P \to \Re$ be a function defined on a product $P\times P$, where P is a convex closed subset of the Euclidean space $\Re^n$. Further, we consider that $\Psi(\mathbf{p},\mathbf{q})$ is *weakly convex–concave* if it satisfies the following inequalities.

$$\beta\Psi(\mathbf{x},\mathbf{z}) + (1-\beta)\Psi(\mathbf{y},\mathbf{z}) \geqslant \Psi(\beta\mathbf{x} + (1-\beta)\mathbf{y},\mathbf{z}) + \beta(1-\beta)r_z(\mathbf{x},\mathbf{y}) \quad \forall \mathbf{x},\mathbf{y},\mathbf{z}\in P, 0\leqslant\beta\leqslant 1$$

$$\text{and } \frac{r_z(\mathbf{x},\mathbf{y})}{||\mathbf{x}-\mathbf{y}||} \to 0, \quad \text{as} \quad ||\mathbf{x}-\mathbf{y}|| \to 0, \quad \forall \mathbf{z}\in P,$$

and

$$\beta\Psi(\mathbf{z},\mathbf{x}) + (1-\beta)\Psi(\mathbf{z},\mathbf{y}) \leqslant \Psi(\mathbf{z},\beta\mathbf{x} + (1-\beta)\mathbf{y}) + \beta(1-\beta)\mu_z(\mathbf{x},\mathbf{y}) \quad \forall \mathbf{x},\mathbf{y},\mathbf{z}\in P, 0\leqslant\beta\leqslant 1$$

$$\text{and } \frac{\mu_z(\mathbf{x},\mathbf{y})}{||\mathbf{x}-\mathbf{y}||} \to 0, \quad \text{as} \quad ||\mathbf{x}-\mathbf{y}|| \to 0, \quad \forall \mathbf{z}\in P,$$

where $r_{\mathbf{z}}(\mathbf{x},\mathbf{y})$ and $\mu_{\mathbf{z}}(\mathbf{x},\mathbf{y})$ are called the *residual terms*.

In other words, a function Ψ of two vector arguments is referred to as *weakly convex–concave* if it satisfies weak convexity with respect to its first argument and weak concavity with respect to its second argument. The notions of weak convexity and concavity are *relaxations* of strict convexity and concavity (Berridge and Krawczyk, 1997). The residual terms, to be chosen at will, ensure that there are many concave functions, which are weakly convex and many convex functions, which are weakly concave.

Theorem 1. (Uryasev and Rubinstein, 1994) *There exists a unique Nash equilibrium point to which the relaxation algorithm converges if*:

(1) *P is a convex compact subset of $\Re^n$*,
(2) *the Nikaido–Isoda function $\Psi : P\times P \to \Re$ is a weakly convex–concave function and $\Psi(\mathbf{p},\mathbf{p}) = 0$ for $\mathbf{p}\in P$*,
(3) *the optimum response function $Z(\mathbf{p})$ is single-valued and continuous on P*,
(4) *the residual term $r_z(\mathbf{x},\mathbf{y})$ is uniformly continuous on P with respect to z for all $\mathbf{x},\mathbf{y}\in P$*,
(5) *the residual terms satisfy $r_{\mathbf{y}}(\mathbf{x},\mathbf{y}) - \mu_{\mathbf{x}}(\mathbf{y},\mathbf{x}) \geqslant \lambda(||\mathbf{x}-\mathbf{y}||)$ $\;\mathbf{x},\mathbf{y}\in P$ where $\lambda(0)=0$ and λ is a strictly increasing function*,
(6) *the relaxation parameters β_s satisfy* (a) $\beta_s > 0$, (b) $\sum_{s=0}^{\infty}\beta_s = \infty$, *and* (c) $\beta_s \to 0$ *as* $s\to\infty$.

Proof of Corollary 1. We need to verify that our downstream models satisfy the conditions (1)–(6) in Theorem 1.

Condition (1): It is trivially satisfied.

Condition (2): From (23), we have that the material flows shipped to downstream site j, $j = 1,\dots,n$, is expressed as $x_j^{(\mathrm{Tr})} = \alpha_1^j p_1 + \cdots + \alpha_n^j p_n + C_j$. After algebra manipulations, the objective function of downstream site j is simply expressed in (27), where V_j is a constant parameter for downstream site j.

$$\phi_j(\mathbf{p}) = (V_j - p_j)(\alpha_1^j p_1 + \cdots + \alpha_n^j p_n + C_j). \tag{27}$$

For any solution $(z_j|\mathbf{p}) = (p_1,\dots,p_{j-1},z_j,p_{j+1},\dots,p_n)\in P$ and $(z_j|\mathbf{q}) = (q_1,\dots,q_{j-1},z_j,q_{j+1},\dots,q_n)\in P$,

$$\begin{aligned}
&\beta\phi_j(z_j|\mathbf{p}) + (1-\beta)\phi_j(z_j|\mathbf{q}) \\
&\quad = \beta(V_j - z_j)(\alpha_1^j p_1 + \cdots + \alpha_{j-1}^j p_{j-1} + \alpha_j^j z_j \\
&\qquad + \alpha_{j+1}^j p_{j+1} + \cdots + \alpha_n^j p_n + C_j) \\
&\qquad + (1-\beta)(V_j - z_j)(\alpha_1^j q_1 + \cdots + \alpha_{j-1}^j q_{j-1} \\
&\qquad + \alpha_j^j z_j + \alpha_{j+1}^j q_{j+1} + \cdots + \alpha_n^j q_n + C_j) \\
&\quad = (V_j - z_j)[\alpha_1^j[\beta p_1 + (1-\beta)q_1] + \cdots \\
&\qquad + \alpha_{j-1}^j[\beta p_{j-1} + (1-\beta)q_{j-1}] + \alpha_j^j z_j \\
&\qquad + \alpha_{j+1}^j[\beta p_{j+1} + (1-\beta)q_{j+1}] \\
&\qquad + \cdots + \alpha_n^j[\beta p_n + (1-\beta)q_n] + C_j] \\
&\quad = \phi_j(z_j|\beta\mathbf{p} + (1-\beta)\mathbf{q}).
\end{aligned}$$

Thus,

$$\beta\phi_j(z_j|\mathbf{p}) + (1-\beta)\phi_j(z_j|\mathbf{q}) = \phi_j(z_j|\beta\mathbf{p} + (1-\beta)\mathbf{q}) \quad \forall j. \tag{28}$$

Since the objective functions ϕ_j for all downstream sites are concave, we have

$$\beta\phi_j(\mathbf{p}) + (1-\beta)\phi_j(\mathbf{q}) \leqslant \phi_j(\beta\mathbf{p} + (1-\beta)\mathbf{q}) \quad \forall j. \tag{29}$$

Combining (28) and (29), the following inequality is satisfied:

$$\beta[\phi_j(z_j|\mathbf{p})-\phi_j(\mathbf{p})]+(1-\beta)[\phi_j(z_j|\mathbf{q})-\phi_j(\mathbf{q})]\geqslant\phi_j(z_j|\beta\mathbf{p}+(1-\beta)\mathbf{q})-\phi_j(\beta\mathbf{p}+(1-\beta)\mathbf{q})\quad\forall j. \tag{30}$$

Summing up all inequalities of (30) for all downstream site j, it implies that

$$\beta\sum_{j=1}^{n}[\phi_j(z_j|\mathbf{p})-\phi_j(\mathbf{p})]+(1-\beta)\sum_{j=1}^{n}[\phi_j(z_j|\mathbf{q})-\phi_j(\mathbf{q})]\geqslant\sum_{j=1}^{n}[\phi_j(z_j|\beta\mathbf{p}+(1-\beta)\mathbf{q})-\phi_j(\beta\mathbf{p}+(1-\beta)\mathbf{q})]. \tag{31}$$

The definition of the Nikaido–Isoda function is referred to (31), we have

$$\beta\Psi(\mathbf{p},\mathbf{z})+(1-\beta)\Psi(\mathbf{q},\mathbf{z})\geqslant\Psi(\beta\mathbf{p}+(1-\beta)\mathbf{q},\mathbf{z}). \tag{32}$$

From (32), the Nikaido–Isoda function Ψ is convex with respect to the first argument. Based on the same algebra manipulation, the function Ψ is also concave with respect to the second argument which is

$$\beta\Psi(\mathbf{z},\mathbf{p})+(1-\beta)\Psi(\mathbf{z},\mathbf{q})\leqslant\Psi(\mathbf{z},\beta\mathbf{p}+(1-\beta)\mathbf{q}). \tag{33}$$

From (32) and (33), the Nikaido–Isoda function Ψ is *convex–concave* which is also *weakly convex–concave*. Then, the optimization model for each downstream site j satisfies Condition (2).

Condition (3): The optimum response function $Z(\mathbf{p})$ is single-valued and continuous on P by solving the concave quadratic convex constrained model.

Condition (4): If the Nikaido–Isoda function $\Psi(\mathbf{p},\mathbf{q})$ is twice continuously differentiable with respect to both arguments on the set $P\times P$, the residual term is given by (Uryasev and Rubinstein, 1994)

$$r_z(\mathbf{p},\mathbf{q})=\tfrac{1}{2}\langle A(\mathbf{p},\mathbf{p})(\mathbf{p}-\mathbf{q}),\mathbf{p}-\mathbf{q}\rangle+o(\|\mathbf{p}-\mathbf{q}\|^2),$$

where $\langle\bullet,\bullet\rangle$ is the notation of inner product and $A(\mathbf{p},\mathbf{p})=\Psi_{pp}(\mathbf{p},\mathbf{q})|_{q=p}$ is the Hessian of the Nikaido–Isoda function with respect to the first argument evaluated at $\mathbf{q}=\mathbf{p}$. Moreover, if the function $\Psi(\mathbf{p},\mathbf{q})$ is convex with respect to $\mathbf{p}$, then $o(\|\mathbf{p}-\mathbf{q}\|^2)=0$ (Uryasev, 1988). The residual term of $r_{\mathbf{z}}(\mathbf{p},\mathbf{q})$ is a polynomial expression which is *continuous* on P. Furthermore, $r_{\mathbf{z}}(\mathbf{p},\mathbf{q})$ is *uniformly continuous* on P since P is *compact* (Bartle, 1976).

Condition (5): Assuming that $\Psi(\mathbf{p},\mathbf{q})$ is twice continuously differentiable, in order to prove this condition, it suffices to show that $\Psi_{pp}(\mathbf{p},\mathbf{q})|_{q=p}-\Psi_{qq}(\mathbf{p},\mathbf{q})|_{q=p}$ is positive definite (Krawczyk and Uryasev, 2000), where $\Psi_{pp}(\mathbf{p},\mathbf{q})|_{q=p}$ is the Hessian of the Nikaido–Isoda function with respect to the first argument and $\Psi_{qq}(\mathbf{p},\mathbf{q})|_{q=p}$ is the Hessian of the Nikaido–Isoda function with respect to the second argument, both evaluated at $\mathbf{q}=\mathbf{p}$.

The Hessian matrices of the Nikaido–Isoda function are shown in (34) and (35), respectively.

$$\Psi_{pp}(\mathbf{p},\mathbf{q})|_{q=p}=\begin{pmatrix}2\alpha_1^1 & \alpha_2^1+\alpha_1^2 & \cdots & \alpha_n^1+\alpha_1^n\\ \alpha_1^2+\alpha_2^1 & 2\alpha_2^2 & & \vdots\\ \vdots & & \ddots & \\ \alpha_1^n+\alpha_n^1 & \cdots & & 2\alpha_n^n\end{pmatrix}, \tag{34}$$

$$\Psi_{qq}(\mathbf{p},\mathbf{q})|_{q=p}=\begin{pmatrix}-2\alpha_1^1 & 0 & \cdots & 0\\ 0 & -2\alpha_2^2 & & \vdots\\ \vdots & & \ddots & \\ 0 & \cdots & & -2\alpha_n^n\end{pmatrix}, \tag{35}$$

$$\Psi_{pp}(\mathbf{p},\mathbf{q})|_{q=p}-\Psi_{qq}(\mathbf{p},\mathbf{q})|_{q=p}=\begin{pmatrix}4\alpha_1^1 & \alpha_2^1+\alpha_1^2 & \cdots & \alpha_n^1+\alpha_1^n\\ \alpha_1^2+\alpha_2^1 & 4\alpha_2^2 & & \vdots\\ \vdots & & \ddots & \\ \alpha_1^n+\alpha_n^1 & \cdots & & 4\alpha_n^n\end{pmatrix}.$$

As discussed in Proposition 1, we require that $\alpha_j^j>0$ and $\alpha_{j'}^j\leqslant 0\ \forall j',j'\neq j$. Since all pivots for the matrix of $\Psi_{pp}(\mathbf{p},\mathbf{q})|_{q=p}-\Psi_{qq}(\mathbf{p},\mathbf{q})|_{q=p}$ are positive, $\Psi_{pp}(\mathbf{p},\mathbf{q})|_{q=p}-\Psi_{qq}(\mathbf{p},\mathbf{q})|_{q=p}$ is positive definite (Strang 1986). We can conclude that the optimization models of downstream sites satisfy Condition (5).

Condition (6): In order for the algorithm to converge, we may choose any sequence (β_s) satisfying the Condition (6) of Theorem 1.

All conditions in Theorem 1 are satisfied for the optimization model of downstream sites. It completes the proof. □

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