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附錄 A：橢圓偏光參數的修正流程

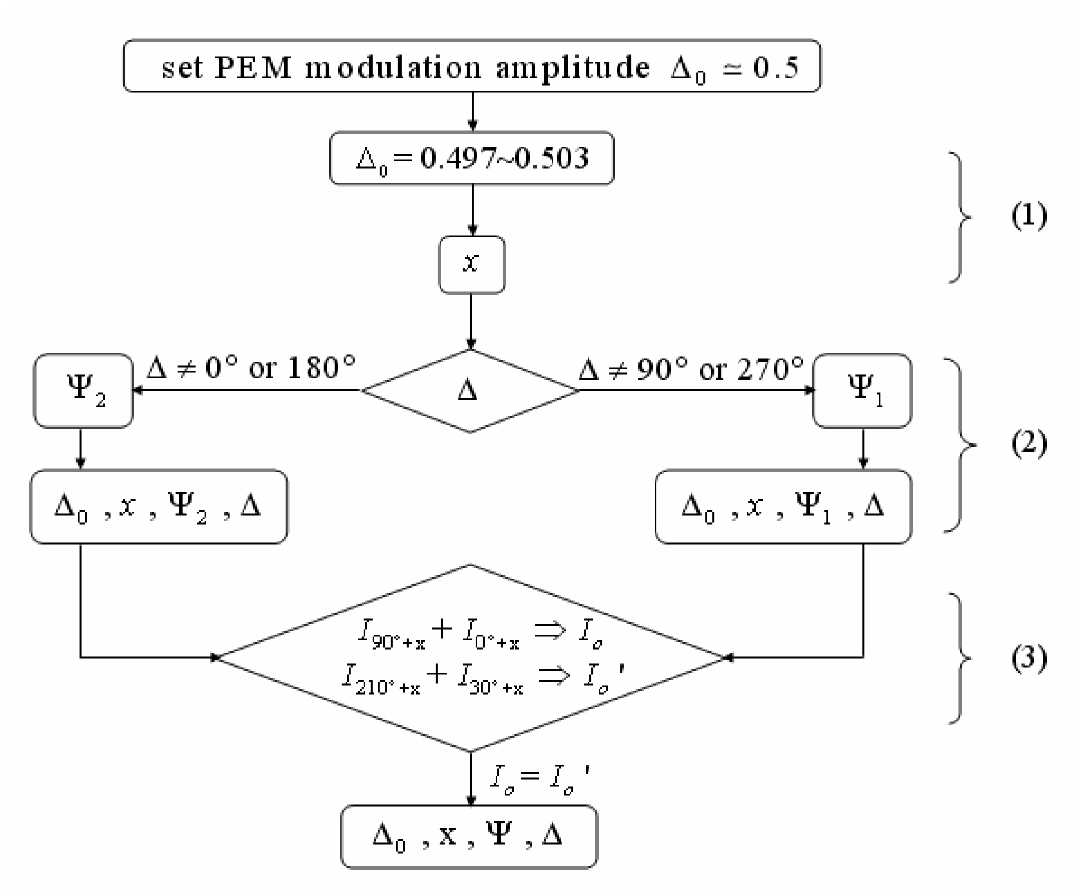

圖 A-1　橢圓偏光參數修正流程圖

將偏光片、光彈調變器與析光片之方位角分別置於 -45°、0°、45°，由光偵測器所偵測到之時變訊號為：

$$I_\theta = \frac{I_0}{2}[1 - \sin 2\Psi \cos(\Delta - \pi \sin\theta)]$$

首先我們將光彈調變器的調變振幅校正為 0.5 左右 [7]。

修正流程 (圖 A-1) 主要可分為三個部分：

(1) 我們假設 Δ_0 變動範圍為 0.497～0.503，藉由 (3.15) 式可得各組 (Δ_0，x)。

$$\frac{I_{45^o+x} - I_{225^o+x}}{I_{135^o+x} - I_{315^o+x}} = \frac{\sin[2\pi\Delta_0 \cdot \sin(x+45°)]}{\sin[2\pi\Delta_0 \cdot \cos(x+45°)]} \qquad (3.15)$$

(2) 已知（Δ_0，x）由 (3.12)～(3.14) 式可得各組（Δ_0，x，Ψ，Δ)。

$$\Delta = \cot^{-1}\left\{\frac{\cos[\pi\Delta_0(\sin x+\cos x)]-C_1}{\sin[\pi\Delta_0(\sin x+\cos x)]}\right\} \tag{3.12}$$

$$\sin 2\Psi_1 = \frac{C_2}{C_2\cos[\Delta-\pi\Delta_0(\sin x+\cos x)]\cdot D_1-\sin[\Delta-\pi\Delta_0(\sin x+\cos x)]\cdot D_2} \tag{3.13}$$

$$\sin 2\Psi_2 = \frac{C_3}{C_3\cos\Delta\cdot\cos[2\pi\Delta_0\sin(30^\circ+x)]+\sin\Delta\cdot\sin[2\pi\Delta_0\sin(30^\circ+x)]} \tag{3.14}$$

其中令 $D_1=\cos[\pi\Delta_0(\sin x-\cos x)]$；$D_2=\sin[\pi\Delta_0(\sin x-\cos x)]$

$$C_1=-\frac{I_{90^\circ+x}-I_{0^\circ+x}}{I_{210^\circ+x}-I_{30^\circ+x}}\cdot\frac{\sin[2\pi\Delta_0\sin(30^\circ+x)]}{\sin[\pi\Delta_0(\sin x-\cos x)]}，C_2=\frac{I_{90^\circ+x}-I_{0^\circ+x}}{I_{90^\circ+x}+I_{0^\circ+x}}，C_3=\frac{I_{210^\circ+x}-I_{30^\circ+x}}{I_{210^\circ+x}+I_{30^\circ+x}}$$

(3) 將各組（Δ_0，x，Ψ，Δ) 與實驗所得的強度代入方程式 (3.16a)、(3.16b) 利用兩式 I_0 相等的條件作擬合可得系統誤差與修正後精確的橢圓偏光參數。

$$I_{90^\circ+x}+I_{0^\circ+x}=I_0\{1-\sin 2\Psi\cos[\Delta-\pi\Delta_0(\cos x+\sin x)]\cdot\cos[\pi\Delta_0(\cos x+\sin x)]\} \tag{3.16a}$$

$$I_{210^\circ+x}+I_{30^\circ+x}=I_0\{1-\sin 2\Psi\cos\Delta\cdot\cos[2\pi\Delta_0\sin(30^\circ+x)]\} \tag{3.16b}$$

附錄 B：利用 Mathematica 分析系統誤差

考慮系統誤差光偵測器所偵測到之時變訊號為：

$$I_\theta = \frac{I_0}{2}[1-\sin 2\Psi \cdot \cos(\Delta - 2\pi\Delta_0 \sin\theta)]$$

$I_{0°+x}$, $I_{30°+x}$, $I_{90°+x}$, $I_{210°+x}$ 之光強度如下 $data_i$ (i=0, 30, 90 and 210^o)

所示：

$$\text{data0} = \frac{\text{I0}}{2}\,(1 - \text{Sin}[2\,\Psi]\,\text{Cos}[\Delta - 2\,\pi\,\Delta_0\,\text{Sin}[x]]);$$

$$\text{data30} = \frac{\text{I0}}{2}\,(1 - \text{Sin}[2\,\Psi]\,\text{Cos}[\Delta - 2\,\pi\,\Delta_0\,\text{Sin}[\pi/6 + x]]);$$

$$\text{data90} = \frac{\text{I0}}{2}\,(1 - \text{Sin}[2\,\Psi]\,\text{Cos}[\Delta - 2\,\pi\,\Delta_0\,\text{Cos}[x]]);$$

$$\text{data210} = \frac{\text{I0}}{2}\,(1 - \text{Sin}[2\,\Psi]\,\text{Cos}[\Delta + 2\,\pi\,\Delta_0\,\text{Sin}[\pi/6 + x]]);$$

$$\text{f} = \frac{\text{data90} - \text{data0}}{\text{data90} + \text{data0}} \;\text{// FullSimplify};$$

$$\text{g} = \frac{\text{data210} - \text{data30}}{\text{data210} + \text{data30}} \;\text{// FullSimplify};$$

$$\text{m} = \begin{pmatrix} \partial_x \text{f} & \partial_{\Delta_0} \text{f} \\ \partial_x \text{g} & \partial_{\Delta_0} \text{g} \end{pmatrix} \;\text{// MatrixForm};$$

$$\text{n} = \begin{pmatrix} \partial_\Psi \text{f} & \partial_\Delta \text{f} \\ \partial_\Psi \text{g} & \partial_\Delta \text{g} \end{pmatrix} \;\text{// MatrixForm};$$

$$(*\;\; \text{m}=\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \;;\; \text{n}=\begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \;\; *)$$

參數 x 和 $\delta\Delta_0$ 的變動對強度 f 和 g 的影響如下：

$$\begin{pmatrix} df \\ dg \end{pmatrix} = \begin{pmatrix} \partial_x f & \partial_{\Delta_0} f \\ \partial_x g & \partial_{\Delta_0} g \end{pmatrix} \cdot \begin{pmatrix} dx \\ d\Delta_0 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \cdot \begin{pmatrix} dx \\ d\Delta_0 \end{pmatrix}$$

參數 Ψ 和 Δ 的變動對強度 f 和 g 的影響如下：

$$\begin{pmatrix} df \\ dg \end{pmatrix} = \begin{pmatrix} \partial_\Psi f & \partial_\Delta f \\ \partial_\Psi g & \partial_\Delta g \end{pmatrix} \cdot \begin{pmatrix} d\Psi \\ d\Delta \end{pmatrix} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \cdot \begin{pmatrix} d\Psi \\ d\Delta \end{pmatrix}$$

$n11$, $n12$, $n21$, $n22$, $m11$, $m12$, $m21$, $m22$ 如下頁所示。

由以上兩式可合併為：

$$\begin{pmatrix} d\Psi \\ d\Delta \end{pmatrix} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}^{-1} . \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} . \begin{pmatrix} dx \\ d\Delta_0 \end{pmatrix}$$

$$= \frac{1}{\det(n)} \begin{pmatrix} n_{22} & -n_{12} \\ -n_{21} & n_{11} \end{pmatrix} . \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} . \begin{pmatrix} dx \\ d\Delta_0 \end{pmatrix}$$

可得誤差 (x，$\delta\Delta_0$) 對 (Ψ，Δ) 的影響如下：

(* set $\Delta_0 = 0.5 + \delta\Delta_0$; $d\Delta_0 = d\delta\Delta_0$ *)

$$\begin{pmatrix} d\Psi \\ d\Delta \end{pmatrix} = \frac{1}{\det(n)} \begin{pmatrix} n_{22} & -n_{12} \\ -n_{21} & n_{11} \end{pmatrix} . \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} . \begin{pmatrix} dx \\ d\delta\Delta_0 \end{pmatrix}$$

(* $\det(n) = n_{11} * n_{22} - n_{12} - n_{21}$ *)

(* if $\delta\Delta_0 = x = 0$ *)

$$\begin{pmatrix} d\Psi \\ d\Delta \end{pmatrix} = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} . \begin{pmatrix} dx \\ d\delta\Delta_0 \end{pmatrix}$$

$$q_{11} = \frac{\pi}{8} \mathrm{Sin}[2\Delta]\,\mathrm{Tan}[2\Psi]\left(1 + \left(\mathrm{Cos}[\Delta] - \sqrt{3}\,\mathrm{Sin}[\Delta]\right)\mathrm{Sin}[2\Psi]\right);$$

$$q_{12} = -\frac{\pi}{4} \mathrm{Sin}[2\Delta]\,\mathrm{Tan}[2\Psi]\left(-1 + (\mathrm{Cos}[\Delta] + \mathrm{Sin}[\Delta])\,\mathrm{Sin}[2\Psi]\right);$$

$$q_{21} = -\frac{\pi}{2} \mathrm{Sin}[\Delta]\left(\mathrm{Sin}[\Delta] + \mathrm{Cos}[\Delta]\left(\sqrt{3}\,\mathrm{Cos}[\Delta] + \mathrm{Sin}[\Delta]\right)\mathrm{Sin}[2\Psi]\right);$$

$$q_{22} = \pi\,\mathrm{Sin}[\Delta]\left(-\mathrm{Sin}[\Delta] + \mathrm{Cos}[\Delta]\,(-\mathrm{Cos}[\Delta] + \mathrm{Sin}[\Delta])\,\mathrm{Sin}[2\Psi]\right);$$

可得 (3.8a)～(3.8d) 式。

$$m_{11} = -\frac{2\pi\Delta_0\,(\mathrm{Cos}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] - \mathrm{Cos}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])\,\mathrm{Sin}[2\Psi]^2\,(-\mathrm{Sin}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] + \mathrm{Cos}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])}{(-2 + (\mathrm{Cos}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] + \mathrm{Cos}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])\,\mathrm{Sin}[2\Psi])^2} - \frac{2\pi\Delta_0\,\mathrm{Sin}[2\Psi]\,(\mathrm{Sin}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] + \mathrm{Cos}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])}{-2 + (\mathrm{Cos}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] + \mathrm{Cos}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])\,\mathrm{Sin}[2\Psi]};$$

$$m_{12} = \frac{2\pi\,\mathrm{Sin}[2\Psi]\,(\mathrm{Cos}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] - \mathrm{Sin}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])}{-2 + (\mathrm{Cos}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] + \mathrm{Cos}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])\,\mathrm{Sin}[2\Psi]} - \frac{2\pi\,(\mathrm{Cos}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] - \mathrm{Cos}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])\,\mathrm{Sin}[2\Psi]^2\,(\mathrm{Cos}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] + \mathrm{Sin}[x]\,\mathrm{Sin}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])}{(-2 + (\mathrm{Cos}[\Delta - 2\pi\,\mathrm{Cos}[x]\,\Delta_0] + \mathrm{Cos}[\Delta - 2\pi\,\mathrm{Sin}[x]\,\Delta_0])\,\mathrm{Sin}[2\Psi])^2};$$

$$m_{21} = \frac{\pi\Delta_0\,\mathrm{Sin}[2\Psi]\,\mathrm{Sin}[\Delta]\,\left(\sqrt{3}\,\mathrm{Cos}[x] - \mathrm{Sin}[x]\right)\left(\mathrm{Cos}\left[\pi\Delta_0\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\right] - \mathrm{Cos}[\Delta]\,\mathrm{Sin}[2\Psi]\right)}{\left(-1 + \mathrm{Cos}[\Delta]\,\mathrm{Cos}\left[\pi\Delta_0\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\right]\mathrm{Sin}[2\Psi]\right)^2};$$

$$m_{22} = \frac{\pi\,\mathrm{Sin}[\Delta]\,\mathrm{Sin}[2\Psi]\,\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\left(\mathrm{Cos}\left[\pi\Delta_0\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\right] - \mathrm{Cos}[\Delta]\,\mathrm{Sin}[2\Psi]\right)}{\left(-1 + \mathrm{Cos}[\Delta]\,\mathrm{Cos}\left[\pi\Delta_0\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\right]\mathrm{Sin}[2\Psi]\right)^2};$$

$$n_{11} = -\frac{2\,\mathrm{Cos}[2\Psi]\,\mathrm{Sin}[\pi\,(\mathrm{Cos}[x] - \mathrm{Sin}[x])\,\Delta_0]\,\mathrm{Sin}[\Delta - \pi\,(\mathrm{Cos}[x] + \mathrm{Sin}[x])\,\Delta_0]}{(-1 + \mathrm{Cos}[\pi\,(\mathrm{Cos}[x] - \mathrm{Sin}[x])\,\Delta_0]\,\mathrm{Cos}[\Delta - \pi\,(\mathrm{Cos}[x] + \mathrm{Sin}[x])\,\Delta_0]\,\mathrm{Sin}[2\Psi])^2};$$

$$n_{12} = \frac{-4\,\mathrm{Cos}[\Delta - \pi\,(\mathrm{Cos}[x] + \mathrm{Sin}[x])\,\Delta_0]\,\mathrm{Sin}[2\Psi]\,\mathrm{Sin}[\pi\,(\mathrm{Cos}[x] - \mathrm{Sin}[x])\,\Delta_0] + 2\,\mathrm{Sin}[2\Psi]^2\,\mathrm{Sin}[2\pi\,(\mathrm{Cos}[x] - \mathrm{Sin}[x])\,\Delta_0]}{(-2 + 2\,\mathrm{Cos}[\pi\,(\mathrm{Cos}[x] - \mathrm{Sin}[x])\,\Delta_0]\,\mathrm{Cos}[\Delta - \pi\,(\mathrm{Cos}[x] + \mathrm{Sin}[x])\,\Delta_0]\,\mathrm{Sin}[2\Psi])^2};$$

$$n_{21} = \frac{2\,\mathrm{Cos}[2\Psi]\,\mathrm{Sin}[\Delta]\,\mathrm{Sin}\left[\pi\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\Delta_0\right]}{\left(-1 + \mathrm{Cos}[\Delta]\,\mathrm{Cos}\left[\pi\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\Delta_0\right]\mathrm{Sin}[2\Psi]\right)^2};$$

$$n_{22} = \frac{4\,\mathrm{Cos}[\Delta]\,\mathrm{Sin}[2\Psi]\,\mathrm{Sin}\left[\pi\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\Delta_0\right] - 2\,\mathrm{Sin}[2\Psi]^2\,\mathrm{Sin}\left[2\pi\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\Delta_0\right]}{\left(-2 + 2\,\mathrm{Cos}[\Delta]\,\mathrm{Cos}\left[\pi\left(\mathrm{Cos}[x] + \sqrt{3}\,\mathrm{Sin}[x]\right)\Delta_0\right]\mathrm{Sin}[2\Psi]\right)^2};$$