

表 4-1 複合材料積層板與邊界支撐之材料常數

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_c (MPa)	b_e (m)	h_e (m)
146.503 (0.723%)*	9.223 (1.189%)	6.8356 (3.158%)	0.306 (0.189%)	2.028 (2.303%)	0.5e-2	2.1e-3

* 變異係數

表 4-2 彈性支撐複合材料積層板之自然頻率

Layup	Edge support			Center support	Method	Natural frequency λ						
	k_L	k_R	k_C	k_C		1st	2nd	3rd	4th	5th	6th	7th
$[0^\circ/90^\circ/0^\circ]$	2 (MN/m ²)	800 (N)	5 (kN/m)	5	Present*	28.95	29.68	48.89	57.18	61.02	63.11	69.93
					CLPT	28.90	29.46	48.53	57.00	60.84	63.05	69.76
$[45^\circ/-45^\circ/45^\circ]$	100 (kN/m ²)	100 (N)	1 (kN/m)	1	Present	17.60	22.93	28.95	31.33	37.59	39.79	44.58
					CLPT	17.60	22.93	28.95	31.33	37.59	39.79	44.58
$[0^\circ/90^\circ/0^\circ]$	∞	20	0	0	ANSYS	17.49	22.83	28.89	31.24	37.57	39.72	44.55
					Present	13.95	21.76	38.64	51.21	55.79	63.99	66.98
					CLPT	13.95	21.76	38.64	51.21	55.79	63.99	66.98
					Masoud and Pierre [60]	13.95	—	—	—	—	—	—
					Reddy[61]	13.948	—	—	—	—	—	—
					Present	25.90	33.33	49.59	69.39	74.04	74.79	84.85
$[0^\circ/90^\circ/0^\circ]$	∞	20	0	0	CLPT	25.90	33.33	49.59	69.39	74.04	74.79	84.85
					Masoud and Pierre [60]	25.91	—	—	—	—	—	—
					Present	31.24	38.64	55.73	82.94	84.51	88.81	99.34
					CLPT	31.24	38.64	55.73	82.94	84.51	88.81	99.34
$[0^\circ/90^\circ/0^\circ]$	∞	20	0	0	Masoud and Pierre [60]	31.24	—	—	—	—	—	—
					Present	31.24	—	—	—	—	—	—

* Material property and definition of normalized natural frequency for the analysis :

$$E_1 = 200\text{GPa}, E_2 = 10\text{GPa}, G_{12} = 6\text{GPa}, \nu_{12} = 0.25, \lambda = \sqrt{\rho h \omega^2 a^4 / D_0}, D_0 = E_1 h^3 / [12(1 - \nu_{12} \nu_{21})]$$

表 4-3(a) 簡單支撐正方形複合材料積層厚板 $[0^0/90^0]_s$ 之第一個自然頻率

Method \ h/b	Natural frequency λ				
	0.01	0.02	0.04	0.05	0.08
Present FSDT	6.603	6.541	6.321	6.174	5.652
Liew [26]	6.606	6.549	6.338	6.193	5.667
Reddy and Phan [62]	6.578	6.475	6.330	6.196	5.708
Present CLPT	6.625	6.625	6.625	6.625	6.625

(b) 彈性支撐正方形複合材料積層厚板 ($h/b=0.03$) 之自然頻率

Layup	Edge support	Method	Natural frequency λ				
			1st	2nd	3rd	4th	5th
$[0^0/90^0/0^0]$		Present FSDT	5.569	8.205	13.818	14.143	15.473
		ANSYS	5.593	8.251	13.870	14.156	15.509
$[30^0/-30^0/30^0]$	$K_L=10^8 N/m^2$ $R_L=100N$	Present FSDT	5.510	8.712	12.927	13.631	16.655
		ANSYS	5.530	8.742	12.968	13.646	16.688
$[45^0/-45^0/45^0]$		Present FSDT	5.525	9.576	13.280	13.710	17.911
		ANSYS	5.544	9.598	13.296	13.741	17.943

Material property and definition of normalized natural frequency for the analysis :

$$E_1/E_2=40, G_{12}=G_{13}=3/5E_2, G_{23}=1/2E_2, \nu_{12}=0.25, \nu_{21}=0.00625, \lambda=\sqrt{\rho h \omega^2 a^4 / D_0}, D_0=E_1 h^3 / [12(1-\nu_{12}\nu_{21})]$$

表 4-4 複合材料三明治板具有不同邊界條件之自然頻率

Material	Layup	h_c mm	Edge support	Method	Natural frequency (Hz)						
					1st	2nd	3rd	4th	5th	6th	
I	$[0^\circ / core / 0^\circ]$	6.35	Simply supported	Present	23.1	44.2	69.9	79.5	89.9	123.6	
				Analytical [63]	23	44	71	80	91	126	
				Experimen tal [63]	—	45	69	78	92	125	
				Zhou and Li [64]	23.3	44.5	71.2	78.8	91.6	125.1	
				Khare et al. [65]	23.5	45.0	72.5	80.6	93.4	128.2	
	ANSYS	23.4	44.9	70.5	80.4	91.4	125.9				
	II	$[30_3^0 / core]_s$	10	Clamped	Present	709	1163	1448	1661	2022	2215
					Masoud and Pierre [60]	778	1293	1599	—	—	—
					Watanabe et al. [66]	732	1197	1477	—	—	—
					ANSYS	759	1233	1510	1743	2103	2308
Present					733	1268	1405	1790	2069	2263	
$[0_3^0 / core]_s$		7	Clamped	Masoud and Pierre [60]	752	1312	1527	—	—	—	
				Watanabe et al. [66]	716	1240	1414	—	—	—	
				ANSYS	743	1292	1434	1829	2114	2308	
				Present	611	1082	1182	1585	1734	1925	
				Masoud and Pierre [60]	642	1170	1273	—	—	—	
$[30^0 / -30^0 / 30^0 / core]_s$	7	Clamped	Watanabe et al. [66]	616	1106	1204	—	—	—		
			ANSYS	625	1107	1208	1633	1766	1953		
			Present	694	1110	1384	1655	1744	2151		
			Masoud and Pierre [60]	716	1167	1497	—	—	—		
			Watanabe et al. [66]	693	1119	1412	—	—	—		
$[0^0 / 90^0 / 0^0 / core]_s$	3	Clamped	ANSYS	700	1119	1399	1672	1755	2170		
			Present	668.9	1109.8	1569.0	1834.7	1848.8	2382.1		
			$K_L = 4 \times 10^6 N/m^2$ $K_R = 10N$	ANSYS	673.1	1117.7	1596.3	1864.9	1873.2	2423.9	
			Present	665.0	1212.3	1497.3	1730.1	2128.0	2314.0		
			ANSYS	670.1	1227.1	1524.7	1754.9	2166.5	2351.8		

Material I : a=1.83 m, b=1.22 m, $h_f=0.41$ mm, $E_1 = E_2=68.95$ GPa, $\nu_{12}=0.33$, $\rho_f = 2768$ kg/m³,
 $G_{xz}=134.5$ MPa, $G_{yz}=51.7$ MPa, and $\rho_c = 121.8$ kg/m³.

Material II : a=0.45 m, b=0.3 m, $h_f=0.375$ mm, $E_1=105$ GPa, $E_2=8.74$ GPa, $G_{12}=4.56$ GPa, $\nu_{12}=0.327$,
 $\rho_f = 1600$ kg/m³, $G_{xz}=103$ MPa, $G_{yz}=62.1$ MPa, and $\rho_c = 16$ kg/m³.

Material III : a=0.3 m, b=0.3 m, $h_f=0.5$ mm, $E_1=150$ GPa, $E_2=10$ GPa, $G_{12}=6$ GPa, $G_{23}=2$ GPa $\nu_{12}=0.3$,
 $\rho_f = 1550$ kg/m³, $E_c=100$ MPa, $G_c=38.46$ Mpa, and $\rho_c = 50$ kg/m³.

表 4-5 彈性支撐 $[(0^\circ/90^\circ)_s/\overline{core}]_s$ 板於不同長寬比和心層材料之正規化自然頻率

E_c	K	R	a/b	Normalized natural frequency λ^*				
				1st	2nd	3rd	4th	5th
10 MPa	1	1	0.5	14.19	17.07	25.82	29.77	31.83
			1	5.28	10.82	10.86	15.3	22.19
			2	3.85	4.93	10.2	10.28	11.26
		10^6	0.5	14.22	17.87	26.22	30.51	32.49
			1	5.3	12.13	12.19	16.56	22.66
			2	3.86	5.88	11.01	11.59	12.51
	10	1	0.5	24.48	28.59	35.36	43.4	48.16
			1	12.02	18.95	19.14	24.35	28.12
			2	8.81	10.02	13.57	17.06	17.94
		10^6	0.5	24.69	28.81	35.59	43.81	48.31
			1	12.24	19.21	19.55	24.59	28.31
			2	9	10.56	14.07	17.35	18.25
	10^{10}	1	0.5	27.5	33.84	42.68	52.68	54.59
			1	15.66	25.14	25.92	32.58	36.12
			2	11.92	15.32	20.06	22.99	24.94
		10^6	0.5	27.81	34.08	42.78	52.8	54.74
			1	16.16	25.28	26.12	32.6	36.23
			2	12.47	15.64	20.31	23.1	25.01
	1	1	0.5	16.05	22.33	58.54	68.48	72.47
			1	5.52	16.47	16.52	28.18	70.5
			2	4.02	5.89	16.02	18.62	19.62
		10^6	0.5	16.12	31.26	98.7	99.93	107.76
			1	5.53	27.3	27.36	41.77	97.53
			2	4.03	8.02	26.83	27.08	29.22
10	1	0.5	47.69	52.57	87.55	95.58	102.77	
		1	16.63	27.47	27.5	38.68	74.56	
		2	12.12	13.44	23.27	24.94	26.89	
	10^6	0.5	49.45	57.45	109.85	119.94	127.05	
		1	17.2	33.96	34	47.85	99.61	
		2	12.54	14.63	29.5	31.84	33.81	
10^{10}	1	0.5	122.43	162.3	240.53	323.65	341.12	
		1	49.4	112.78	112.9	155.87	207.5	
		2	35.28	46.83	72.31	105.47	109.37	
	10^6	0.5	174.14	213.37	284.02	358.4	375.3	
		1	77.85	144.05	144.16	190.98	234.97	
		2	56.64	68.79	93.97	130.28	132.55	

* $\lambda = \omega b^2 \sqrt{\rho_f H / D_{11}}$; $H = 2h_f + h_c$; D_{11} is bending stiffness in x direction.

$K = K_{L1} \times a^3 / D_{11} = K_{L2} \times a^3 / D_{11} = K_{L3} \times b^3 / D_{22} = K_{L4} \times b^3 / D_{22}$; and $R = R_{L1} \times a / D_{11} = R_{L2} \times a / D_{11} = R_{L3} \times b / D_{22} = R_{L4} \times b / D_{22}$.

表 4-6 彈性支撐 $[(45^\circ/-45^\circ)_s/\overline{core}]_s$ 板於不同長寬比和心層材料之正規化自然頻率

E_c	K	R	a/b	Normalized natural frequency λ^*					
				1st	2nd	3rd	4th	5th	
10 MPa	1	0.5	0.5	4.25	7.35	9.91	13.66	17.79	
			1	1	2.28	3.73	6.38	8.61	9.09
			2	13.52	15.77	22.99	28.62	30.16	
		10 ⁶	0.5	4.26	9.01	10.95	14.4	18.94	
			1	2.29	4.67	8.2	9.61	9.92	
			2	21.84	24	29.23	36.51	44.17	
	10	0.5	1	9.79	13.7	17.18	20.33	21.64	
			1	1	6.04	7.59	11.14	11.28	12.86
			2	22.03	24.29	29.86	37.26	44.29	
		10 ⁶	0.5	9.99	14.29	17.4	20.65	22.22	
			1	6.23	8	11.68	11.79	13.33	
			2	24.57	30.02	37.86	46.68	49.38	
	10 ¹⁰	0.5	1	13.61	21.77	23.23	28.91	31.6	
			1	1	9.95	13.35	17.8	19.63	21.71
			2	24.97	30.11	38.06	46.83	49.5	
		10 ⁶	0.5	14.24	22.04	23.45	29.13	31.82	
			1	10.75	13.82	18.12	19.89	21.98	
			2	4.25	7.35	9.91	13.66	17.79	
	1	0.5	1	15.72	17.9	40.14	57.35	75.19	
			1	1	4.47	9.66	15.86	27.63	41.47
			2	2.38	4.83	9.05	14.93	18.95	
		10 ⁶	0.5	15.78	20.85	54.79	95.78	107.67	
			1	4.48	14.51	26.46	38.84	54.59	
			2	2.39	7.09	14.36	20.28	26.07	
1 GPa	10	0.5	46.42	48.29	61.66	94.01	96.99		
		1	1	13.46	18.16	25.47	34.17	45.25	
		2	7.19	9.23	14.13	18.47	21.02		
	10 ⁶	0.5	48.15	50.56	71.61	116.11	120.2		
		1	13.91	20.71	31.87	42.98	56.75		
		2	7.44	10.44	17.36	22.58	27.23		
10 ¹⁰	0.5	1	119.18	145.44	190.75	252.28	306.61		
		1	1	44.34	79.74	111.89	127.85	150.35	
		2	24.12	40.63	66.43	66.69	81.7		
	10 ⁶	0.5	159.11	181.32	222.14	279.95	331.72		
		1	64.37	101.08	135.07	150.4	173.39		
		2	36.13	53.83	81.22	84.21	99.27		

* $\lambda = \omega b^2 \sqrt{\rho_f H / D_{11}}$; $H = 2h_f + h_c$; D_{11} is bending stiffness in x direction.

$K = K_{L1} \times a^3 / D_{11} = K_{L2} \times a^3 / D_{11} = K_{L3} \times b^3 / D_{22} = K_{L4} \times b^3 / D_{22}$; and $R = R_{L1} \times a / D_{11} = R_{L2} \times a / D_{11} = R_{L3} \times b / D_{22} = R_{L4} \times b / D_{22}$.

表 6-1 複合材料積層板結構試片編號、疊層角度、幾何尺寸與密度

試片編號	疊層角度	長度 (m)	寬度 (m)	厚度 (m)	密度 (kg/m ³)
A1	[0°] ₆	0.195	0.195	7.5e-4	1523
A2	[0°/90°/0°] _S	0.195	0.195	7.5e-4	1541
A3	[45°/-45°/45°] _S	0.195	0.195	7.5e-4	1551
B1	[0°] ₈	0.205	0.205	1.e-3	1538
B2	[0°/90°] _{2S}	0.205	0.205	1.e-3	1540
B3	[45°/-45°] _{2S}	0.205	0.205	1.e-3	1540
C1	[0°] ₁₂	0.215	0.215	1.5e-3	1556
C2	[0°/90°/0°] _{2S}	0.215	0.215	1.5e-3	1539
C3	[45°/-45°/45°] _{2S}	0.215	0.215	1.5e-3	1551

表 6-2 複合材料積層板結構試片之編號及自然頻率實驗值

試片編號	自然頻率值 (Hz)					
A1	108	164	270	360	400	418
A2	107	232	323.5	408	468.5	595
A3	126	261	301	444	523	554
B1	120	187	311	417	467	490
B2	122	281	364	469	574	710
B3	149	321	357	540	624	646
C1	163.5	245.5	410	554.5	610	650
C2	163	378	482	618	764	920
C3	193	415	456	680	795	820

表 6-3 彈性支撐積層板層數為六層，使用不同數目頻率之反算

疊層方式	頻率 數目	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)
$[0^\circ]_6$	4	141.180 ^a	9.610	6.190	0.30000	2.300
		(-3.63%) ^b	(4.20%)	(-9.45%)	(-1.96%)	(13.41%)
	5	141.914	9.520	6.860	0.28900	2.090
		(-3.79%)	(3.22%)	(0.35%)	(-5.56%)	(3.06%)
	6	140.807	9.157	7.419	0.30374	2.076
		(-4.54%)	(-0.72%)	(8.53%)	(-0.74%)	(2.37%)
$[0^\circ/90^\circ/0^\circ]_S$	4	146.715	8.590	6.600	0.30000	1.980
		(-0.53%)	(-6.86%)	(-3.45%)	(-1.96%)	(-2.37%)
	5	147.350	9.560	5.960	0.30000	1.980
		(-0.10%)	(3.65%)	(-12.81%)	(-1.96%)	(-2.37%)
	6	145.402	9.933	6.701	0.30003	1.976
		(-1.42%)	(7.70%)	(-1.97%)	(-1.95%)	(-2.56%)
$[45^\circ/-45^\circ/45^\circ]_S$	4	139.128	9.940	4.750	0.30000	2.000
		(-5.68%)	(7.77%)	(-30.51%)	(-1.96%)	(-1.38%)
	5	138.929	9.160	7.160	0.30000	2.000
		(-5.17%)	(-0.68%)	(4.74%)	(-1.96%)	(-1.38%)
	6	139.521	9.053	7.054	0.30004	1.998
		(-5.41%)	(-1.84%)	(3.19%)	(-1.95%)	(-1.48%)

^a 反求所得材料常數

^b 括號內數值為百分比誤差值

表 6-4 彈性支撐積層板層數為八層，使用不同數目頻率之反算

疊層方式	頻率 數目	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)
$[0^\circ]_8$	4	137.060 ^a	9.940	6.140	0.29700	2.000
		(-7.08%) ^b	(7.77%)	(-10.18%)	(-2.94%)	(-1.38%)
	5	137.724	9.640	6.730	0.30000	2.000
		(-6.63%)	(4.52%)	(-1.55%)	(-1.96%)	(-1.38%)
	6	137.108	9.436	7.064	0.30029	1.997
		(-7.05%)	(2.31%)	(3.34%)	(-1.87%)	(-1.53%)
$[0^\circ/90^\circ]_{2S}$	4	139.541	8.840	6.840	0.30000	2.000
		(-5.40%)	(-4.15%)	(0.06%)	(-1.96%)	(-1.38%)
	5	139.725	8.820	6.820	0.30000	2.000
		(-5.27%)	(-4.37%)	(-0.23%)	(-1.96%)	(-1.38%)
	6	138.505	9.207	7.207	0.30002	1.998
		(-6.10%)	(-0.17%)	(5.43%)	(-1.94%)	(-1.48%)
$[45^\circ/-45^\circ]_{2S}$	4	142.441	9.093	7.094	0.30003	2.000
		(-3.43%)	(-1.41%)	(3.77%)	(-1.95%)	(-1.38%)
	5	143.218	8.720	6.720	0.30000	2.000
		(-2.91%)	(-5.45%)	(-1.70%)	(-1.96%)	(-1.38%)
	6	143.212	8.704	6.704	0.30002	1.999
		(-2.91%)	(-5.63%)	(-1.93%)	(-1.95%)	(-1.43%)

^a 反求所得材料常數

^b 括號內數值為百分比誤差值

表 6-5 彈性支撐積層板層數為十二層，使用不同數目頻率之反算

疊層方式	頻率 數目	E ₁ (GPa)	E ₂ (GPa)	G ₁₂ (GPa)	ν ₁₂	E _e (MPa)
[0°] ₁₂	4	152.091 ^a	9.938	6.788	0.29600	1.874
		(3.11%) ^b	(7.75%)	(-0.71%)	(-3.27%)	(-7.57%)
	5	148.824	9.690	7.100	0.30400	2.120
		(0.90%)	(5.06%)	(3.86%)	(-0.65%)	(4.54%)
	6	148.901	9.704	7.048	0.30679	2.117
		(0.95%)	(5.22%)	(3.10%)	(0.26%)	(4.39%)
[0°/90°/0°] _{2S}	4	139.281	9.710	7.710	0.30000	2.400
		(-5.57%)	(5.28%)	(12.79%)	(-1.96%)	(18.34%)
	5	138.377	9.798	7.798	0.27165	2.598
		(-6.19%)	(6.23%)	(14.07%)	(-11.23%)	(28.11%)
	6	143.73	9.573	7.188	0.296	2.044
		(-2.56%)	(3.79%)	(5.15%)	(-3.27%)	(0.79%)
[45°/-45°/45°] _{2S}	4	143.866	9.310	6.840	0.30000	2.000
		(-2.47%)	(0.94%)	(0.06%)	(-1.96%)	(-1.38%)
	5	143.833	9.590	7.430	0.29600	1.950
		(-2.49%)	(3.98%)	(8.69%)	(-3.27%)	(-3.85%)
	6	144.11	9.297	7.419	0.30024	1.96
		(-2.30%)	(0.80%)	(8.53%)	(-1.88%)	(-3.35%)

^a 反求所得材料常數

^b 括號內數值為百分比誤差值

表 6-6 複合材料積層板試片 [0°]₆ 其反求之局部與總域極小值

材料常數	Solution							
	1		2		3		4	
	Start	Final	Start	Final	Start	Final	Start	Final*
E ₁ (GPa)	32.606	140.807	27.762	140.807	146.355	140.807	36.945	140.807
E ₂ (GPa)	1.273	9.157	19.728	9.157	19.249	9.157	14.256	9.157
G ₁₂ (GPa)	0.145	7.419	4.309	7.419	8.391	7.419	3.609	7.419
ν ₁₂	0.070	0.304	0.291	0.304	0.261	0.304	0.352	0.304
E _b (MPa)	0.743	2.076	5.877	2.076	6.593	2.076	3.906	2.076
迭代次數	9		9		10		9	

* 總域極小值

表6-7 彈性支撐積層板層數為六層之反算

試片 編號	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	迭代 次數
A1	140.807 (-4.54%)*	9.157 (-0.72%)	7.419 (8.53%)	0.30374 (-0.74%)	2.076 (2.37%)	9
A2	145.402 (-1.42%)	9.933 (7.70%)	6.701 (-1.97%)	0.30003 (-1.95%)	1.976 (-2.56%)	6
A3	139.521 (-5.41%)	9.053 (-1.84%)	7.054 (3.19%)	0.30004 (-1.95%)	1.998 (-1.48%)	13

* 括號內數值為反算預測與拉伸試驗量測之百分比誤差值

表6-8 彈性支撐積層板層數為八層之反算

試片 編號	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	迭代 次數
B1	137.108 (-7.05%)*	9.436 (2.31%)	7.064 (3.34%)	0.30029 (-1.87%)	1.997 (-1.53%)	6
B2	138.505 (-6.10%)	9.207 (-0.17%)	7.207 (5.43%)	0.30002 (-1.94%)	1.998 (-1.48%)	12
B3	143.212 (-2.91%)	8.704 (-5.63%)	6.704 (-1.93%)	0.30002 (-1.95%)	1.999 (-1.43%)	7

* 括號內數值為反算預測與拉伸試驗量測之百分比誤差值

表6-9 彈性支撐積層板層數為十二層之反算

試片 編號	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	迭代 次數
C1	148.901 (0.95%)*	9.704 (5.22%)	7.048 (3.10%)	0.30679 (0.26%)	2.117 (4.39%)	6
C2	143.73 (-2.56%)	9.573 (3.79%)	7.188 (5.15%)	0.296 (-3.27%)	2.044 (0.79%)	10
C3	144.11 (-2.30%)	9.297 (0.80%)	7.419 (8.53%)	0.30024 (-1.88%)	1.96 (-3.35%)	9

* 括號內數值為反算預測與拉伸試驗量測之百分比誤差值

表6-10 複合材料積層板具有中間彈簧支撐之自然頻率實驗值

Layup	Natural frequency (Hz)						
	1st	2nd	3rd	4th	5th	6th	7th
$[0^\circ]_8$	143	183	310	418	459	474	549
$[0^\circ/90^\circ]_{2S}$	145	284	367	467	582	707	781
$[45^\circ/-45^\circ/45^\circ]_S$	158	270	309	458	534	575	677

表6-11 具有中心彈簧支撐之 $[0^{\circ}]_8$ 積層板，使用不同數目實驗頻率之反算

No. of measured natural frequencies	Starting point no.	Stage	Identified mechanical property						Number of iterations	
			E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_c (MPa)	k_C (N/m)		
6	1	Initial	72.738	10.611	10.343	0.23114	6.272	15.033	6	
		Final	136.34	8.563	5.59	0.3	3.061	3.643		
	2	Initial	394.832	34.915	7.625	0.37145	15.168	6.887	6	
		Final	136.343	8.563	5.593	0.3	3.058	3.644		
	3	Initial	278.223	8.977	18.075	0.31906	7.187	16.213	10	
		Final	165.22	6.164	1.925	0.29764	1.951	18.007		
	4	Initial	272.79	38.101	4.232	0.39451	8.252	10.868	9	
		Final	136.343	8.563	5.593	0.3	3.058	3.644		
	5	Initial	23.313	21.975	8.017	0.43736	4.375	12.574	5	
		Final	136.342	8.563	5.592	0.3	3.059	3.644		
	6	Initial	315.04	7.485	17.158	0.14178	9.933	2.138	8	
		Final	136.343	8.563	5.593	0.3	3.058	3.644		
Global minimum			136.343 (-6.94%)*	8.563 (-7.16%)	5.593 (-18.18%)	0.3 (-1.96%)	3.058 (50.79%)	3.644 (-5.72%)	Probability 0.9959	
7	1	Initial	175.047	2.213	13.979	0.31602	14.829	11.284	6	
		Final	138.492	8.625	6.625	0.3	2.118	3.811		
	2	Initial	294.196	15.885	7.743	0.29797	1.583	0.695	8	
		Final	138.495	8.626	6.626	0.3	2.118	3.811		
	3	Initial	146.693	26.308	18.35	0.16604	15.901	15.63	6	
		Final	138.492	8.625	6.625	0.30001	2.118	3.811		
	4	Initial	245.851	1.123	13.69	0.24554	6.003	17.796	8	
		Final	138.491	8.625	6.625	0.3	2.119	3.811		
	Global minimum			138.492 (-5.47%)	8.625 (-6.48%)	6.625 (-3.09%)	0.30001 (-1.96%)	2.118 (4.44%)	3.811 (-1.40%)	Probability 0.9921

* 括號內數值為反算預測與拉伸試驗量測之百分比誤差值

表6-12 具有中心彈簧支撐之積層板，使用實驗頻率之反算結果

Layup	Identified mechanical property					
	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_c (MPa)	k_C (N/m)
$[0^\circ/90^\circ]_{2S}$	142.181 (-2.95%)*	8.76 (-5.02%)	6.439 (-5.81%)	0.3 (-1.96%)	2.126 (4.83%)	3.834 (-0.80%)
$[45^\circ/-45^\circ/45^\circ]_S$	147.149 (0.44%)	9.927 (7.63%)	6.833 (-0.04%)	0.3 (-1.96%)	2 (-1.38%)	3.777 (-2.28%)

* 括號內數值為反算預測與拉伸試驗量測之百分比誤差值

表6-13 複合材料積層板邊界部分支撐之自然頻率實驗值

Layup	Geometry	Support condition	Natural frequency (Hz)					
			1st	2nd	3rd	4th	5th	6th
$[0^\circ/90^\circ/0^\circ]_{2S}$	square	A	154	361	436	611	660	765
		B	161	373	476	578	702	720
$[45^\circ/-45^\circ/45^\circ]_{2S}$	square	A	177	378	435	596	655	818
		B	178	404	451	668	716	787
$[0^\circ]_{12}$	square	A	152	241	386	416	601	616
		B	158	244	405	535	561	600
$[0^\circ/90^\circ/0^\circ]_{2S}$	rectangle	C	110	245	327	435	467	606
$[45^\circ/-45^\circ/45^\circ]_{2S}$			253	347	394	514	578	253

表6-14 Gr/ep $[0^\circ/90^\circ/0^\circ]_{2S}$ A型式部份彈性支撐之自然頻率理論值

Young's modulus of elastic pad (MPa)	Spring constant		Natural frequency (Hz)					
	K_L (kNm ²)	K_R (N)	1st	2nd	3rd	4th	5th	6th
1.0	2.38095	4.96032	151.3163	340.3899	391.3919	575.7812	592.2373	653.2147
15.0	35.7143	74.40476	170.972	394.3665	499.4967	663.1584	786.6243	990.3991

表6-15 Gr/ep $[0^\circ/90^\circ/0^\circ]_{2S}$ A型式部份彈性支撐 $E_e=1$ Mpa 之理論頻率反算

Starting point no.	Stage	System parameter					No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (Mpa)	
1	Initial	194.538	0.849	16.519	0.21825	4.433	12
	Final	146.5032	9.2231	6.836	0.306	1.0	
2	Initial	224.703	17.862	0.225	0.41863	13.719	10
	Final	87.7715	3.3359	3.0379	0.4682	19.7024	
3	Initial	234.44	22.888	16.971	0.19005	4.258	12
	Final	146.503	9.2232	6.836	0.306	1.0	
4	Initial	251.766	22.601	7.614	0.10768	14.206	11
	Final	146.503	9.223	6.836	0.306	1.0	
5	Initial	165.37	17.196	3.731	0.09233	10.058	7
	Final	87.7696	3.3364	3.0385	0.4682	19.7025	
6	Initial	163.239	35.29	16.643	0.01602	7.718	11
	Final	146.5037	9.2218	6.8358	0.3061	1.0	
7	Initial	62.819	20.423	7.079	0.26991	12.009	7
	Final	87.7599	3.3384	3.0404	0.4682	19.7024	
8	Initial	52.512	17.035	17.208	0.02261	17.514	9
	Final	87.7607	3.3382	3.0403	0.4682	19.7024	
9	Initial	196.172	25.595	11.252	0.01915	13.345	10
	Final	146.5032	9.2231	6.836	0.306	1.0	
Global minimum		146.503 (0%)*	9.223 (0%)	6.836 (0%)	0.306 (0%)	1.0 (0%)	Probability 0.9923

* Values in the parentheses denote percentage difference between identified and actual data

表6-16 Gr/ep $[0^\circ/90^\circ/0^\circ]_{2S}$ A型式部份彈性支撐 $E_e=15$ Mpa之理論頻率反算

Starting point no.	Stage	System parameter					No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (Mpa)	
1	Initial	194.551	21.895	16.967	0.13555	7.055	12
	Final	146.5032	9.2231	6.8361	0.306	15.0	
2	Initial	175.037	25.389	14.534	0.27722	8.494	12
	Final	146.5034	9.2231	6.8364	0.3059	15.0	
3	Initial	48.043	25.994	3.448	0.25919	8.535	13
	Final	146.503	9.223	6.836	0.306	15.0	
4	Initial	299.286	9.985	8.23	0.37452	19.554	10
	Final	146.503	9.223	6.836	0.306	15.0	
Global minimum		146.503	9.223	6.836	0.306	15.0	Probability
		(0%)*	(0%)	(0%)	(0%)	(0%)	0.9921

* Values in the parentheses denote percentage difference between identified and actual data.

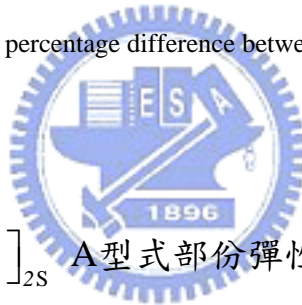


表6-17 GI/ep $[0^\circ/90^\circ/0^\circ]_{2S}$ A型式部份彈性支撐之自然頻率理論值

Young's modulus of elastic pad (MPa)	Spring constant		Natural frequency (Hz)					
	K_L (kN/m ²)	K_R (N)	1st	2nd	3rd	4th	5th	6th
1.0	2.38095	4.96032	87.91181	208.9391	235.5438	344.3321	384.733	428.8373
15.0	35.7143	74.40476	101.7888	236.4288	272.623	382.7989	466.449	557.0408

表6-18 $G_I/ep[0^\circ/90^\circ/0^\circ]_{2S}$ A型式部份彈性支撐以理論頻率之反算

Case	Actual E_e (MPa)	No. of starting points	Average no. of iteration	System parameter				
				E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (Mpa)
1	1.0	7	8	43.5(0%)*	11.5(0%)	3.45(0%)	0.27(0%)	1.0(0%)
2	15.0	8	10	43.5(0%)	11.5(0%)	3.45(0%)	0.27(0%)	15.0(0%)

*Values in the parentheses denote percentage difference between identified and actual data.

表6-19 $Gr/ep[0^\circ/90^\circ/0^\circ]_{2S}$ A型式部份彈性支撐以實驗頻率之反算

Starting point no.	Stage	System parameter					No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (Mpa)	
1	Initial	44.645	34.586	4.881	0.42941	5.985	8
	Final	141.164	8.982	6.982	0.30002	2.211	
2	Initial	160.933	0.683	17.640	0.37422	3.853	8
	Final	141.162	8.982	6.983	0.30001	2.211	
3	Initial	238.425	0.880	19.186	0.03901	4.472	11
	Final	141.158	8.983	6.984	0.30001	2.211	
4	Initial	55.035	16.731	17.782	0.03920	16.305	10
	Final	109.693	5.521	3.529	0.30015	18.005	
5	Initial	51.864	7.121	0.005	0.17907	6.870	10
	Final	141.167	8.981	6.982	0.30001	2.211	
6	Initial	213.219	36.831	9.558	0.33293	1.689	5
	Final	141.171	8.980	6.981	0.30002	2.211	
Global minimum		141.167	8.981	6.982	0.30001	2.211	Probability
		(-3.64%)*	(-2.62%)	(2.14%)	(-1.96%)	(9.02%)	0.9959

* Values in the parentheses denote percentage difference between identified and measured data.

表6-20 Gr/ep複材板不同型式部份彈性支撐以實驗頻率之反算

Layup	Support condition	Identified system parameter				
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (Mpa)
$[0^\circ/90^\circ/0^\circ]_{2S}$	B	141.896 (-3.14%)*	9.041 (-1.97%)	7.045 (3.06%)	0.30035 (-1.85%)	1.984 (-2.17%)
	A	148.425 (1.31%)	8.505 (-7.78%)	6.506 (-4.83%)	0.30002 (-1.95%)	1.992 (-1.78%)
$[45^\circ/-45^\circ/45^\circ]_{2S}$	B	144.040 (-1.68%)	9.853 (6.83%)	6.994 (2.31%)	0.29998 (-1.97%)	2.021 (-0.35%)
	A	148.983 (1.69%)	9.817 (6.44%)	7.005 (2.47%)	0.30015 (-1.91%)	1.965 (-3.11%)
$[0^\circ]_{12}$	B	143.770 (-1.87%)	9.447 (2.43%)	7.499 (9.70%)	0.30421 (-0.58%)	1.901 (-6.26%)
	C	142.187 (-2.95%)	9.750 (5.71%)	6.355 (-7.04%)	0.29751 (-2.77%)	2.144 (5.72%)
$[0^\circ/90^\circ/0^\circ]_{2S}$	C	143.608 (-1.98%)	9.326 (1.12%)	6.199 (-9.32%)	0.29996 (-1.97%)	1.996 (-1.58%)

* Values in the parentheses denote percentage difference between identified and measured data.

表6-21 複材積層板單邊彈性支撐之自然頻率實驗值

Plate	Natural frequency (Hz)						
	1st	2nd	3rd	4th	5th	6th	7th
$[0^\circ]_{60}$	32	242	468	850	1365.3	1727.3	2428.3
$[0^\circ/90^\circ]_{15S}$	38	258	1107.6	1296	1878	2403	2854

表6-22 Gr/ep或Gl/ep 複材板單邊彈性支撐之自然頻率理論值

Plate	Natural frequency (Hz)						
	1st	2nd	3rd	4th	5th	6th	7th
Gr/ep $[0^\circ]_{60}$	33.463	269.657	485.637	940.953	1481.512	1942.108	2478.084
Gl/ep $[0^\circ/90^\circ]_{15S}$	30.373	179.833	706.198	872.672	1014.772	1553.637	2102.837

表6-23 Gr/ep $[0^\circ]_{60}$ 複材板單邊彈性支撐之以理論頻率之反算

Starting point no.	Stage	System parameter							No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	K_L (MN/m ²)	K_R (N)	
1	Initial	240.951	7.131	7.785	4.867	0.148	0.923	9.618	14
	Final	146.503	9.223	6.836	1.123	0.306	0.200	1.000	
2	Initial	176.399	33.017	19.164	5.988	0.414	0.676	3.285	27
	Final	147.224	9.226	6.782	0.773	0.042	0.849	1.000	
3	Initial	315.448	13.891	15.921	1.343	0.452	0.347	4.959	13
	Final	146.503	9.223	6.836	1.123	0.306	0.200	1.000	
4	Initial	212.761	26.673	9.967	0.553	0.462	0.984	0.223	21
	Final	146.503	9.223	6.836	1.123	0.306	0.200	1.000	
5	Initial	236.916	25.043	8.280	2.866	0.280	0.684	3.213	10
	Final	146.503	9.223	6.836	1.123	0.306	0.200	1.000	
6	Initial	207.711	20.402	7.439	5.396	0.187	0.014	4.420	20
	Final	146.503	9.223	6.836	1.123	0.306	0.200	1.000	
Global minimum		146.503 (0%)*	9.223 (0%)	6.836 (0%)	1.123 (0%)	0.306 (0%)	0.200 (0%)	1.000 (0%)	Probability 0.9959

* Values in the parentheses denote percentage difference between identified and measured data.

表6-24 $G_I/ep [0^\circ / 90^\circ]_{15S}$ 複材板單邊彈性支撐以理論頻率之反算

Starting point no.	Stage	System parameter							No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	K_L (MN/m ²)	K_R (N)	
1	Initial	17.719	20.155	14.522	9.597	0.231	0.309	2.725	17
	Final	43.500	11.500	3.450	1.100	0.270	0.200	1.000	
2	Initial	149.719	32.706	7.236	6.218	0.448	0.264	6.781	14
	Final	43.500	11.500	3.450	1.100	0.270	0.200	1.000	
3	Initial	237.142	4.124	14.110	5.717	0.066	0.327	9.706	15
	Final	43.500	11.500	3.450	1.100	0.270	0.200	1.000	
4	Initial	53.421	24.312	5.411	3.046	0.208	0.521	1.813	25
	Final	43.500	11.500	3.450	1.100	0.270	0.200	1.000	
Global minimum		43.500 (0%)*	11.500 (0%)*	3.450 (0%)*	1.100 (0%)*	0.270 (0%)*	0.200 (0%)*	1.000 (0%)*	Probability 0.9921

* Values in the parentheses denote percentage difference between identified and measured data.



表6-25 複材積層板單邊彈性支撐以實驗頻率之反算

Plate	Identified system parameter						
	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	K_L (MN/m ²)	K_R (N)
$[0^\circ]_{60}$	145.620 (-0.6%)*	8.6 (-6.75%)*	6.164 (-9.38%)*	1.0 (-1.19%)*	0.291 (-4.9%)*	74	886
$[0^\circ / 90^\circ]_{15S}$	135.008 (-7.85%)*	8.603 (-6.72%)*	6.300 (-7.84%)*	1.203 (18.89%)*	0.307 (0.34%)*	94.161	1189.2
Average	140.314 (-4.22%)*	8.602 (-6.74%)*	6.232 (-8.84%)*	1.102 (-1.91%)*	0.299 (-2.29%)*	84.081	1037.6

* Values in the parentheses denote percentage difference between identified and measured data.

表6-26 彈性支撐Gr/ep三明治板之自然頻率值

Layup	Natural frequency (Hz)							
	1st	2nd	3rd	4th	5th	6th	7th	8th
	207 ^a	329	475	518	552	687	741	770
$[0_3^c/core(I)/0_3^c]$	209.27 ^b	327.57	476.96	535.07	549.82	700.38	749.36	783.96
	-1.08% ^c	0.44%	-0.41%	-3.19%	0.40%	-1.91%	-1.12%	-1.78%
	227	439	501	635	704	724	819	838
$[0^\circ/90^\circ/0^\circ/core(I)/0^\circ/90^\circ/0^\circ]$	227.47	444.54	472.59	610.07	702.25	740.2	817.86	836.14
	-0.21%	-1.25%	6.01%	4.09%	0.25%	-2.19%	0.14%	0.22%
	263.5	407.4	694	761.9	842.8	1038.8	1082.8	1292.3
$[0_3^c/core(II)/0_3^c]$	257.67	399.86	678.54	737.72	816.77	1004.14	1053.78	1246.45
	2.26%	1.89%	2.28%	3.28%	3.19%	3.45%	2.75%	3.68%
	273.9	614.8	713.8	920.5	1094.6	1223.3	1301.9	1361.5
$[0^\circ/90^\circ/0^\circ/core(II)/0^\circ/90^\circ/0^\circ]$	268.34	603.82	699.51	898.51	1076.56	1203.19	1274.79	1339.94
	2.07%	1.82%	2.04%	2.45%	1.68%	1.67%	2.13%	1.61%

^a Experimental natural frequency

^b Theoretical natural frequency

^c Percentage difference

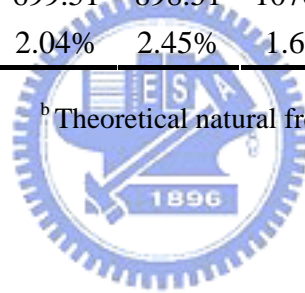


表6-27 彈性支撐Gr/ep $[0_3^{\circ}/core(I)/0_3^{\circ}]$ 三明治板以理論頻率之反算

Starting point no.	Stage	System parameter							No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_c (MPa)	ν_c	E_e (Mpa)	
1	Initial	172.2333	1.7197	1.8401	0.3956	67.3468	0.4880	0.0487	23
	Final	146.5030	9.2230	6.8360	0.3060	27.6504	0.3000	2.0280	
2	Initial	219.5101	33.7476	8.3422	0.3833	83.8477	0.2117	1.0114	20
	Final	129.5830	8.1650	7.9054	0.4811	97.0487	0.3066	0.3368	
3	Initial	369.7264	19.4123	0.9951	0.2674	38.7409	0.1706	8.7940	30
	Final	146.5030	9.2230	6.8360	0.3060	27.6504	0.3000	2.0280	
4	Initial	14.6463	10.7449	4.9484	0.4198	50.1803	0.4227	0.2273	21
	Final	146.5030	9.2230	6.8360	0.3060	27.6503	0.3000	2.0280	
5	Initial	186.6666	13.7427	16.8817	0.1155	89.6623	0.3149	5.2605	20
	Final	146.5030	9.2230	6.8360	0.3060	27.6504	0.3000	2.0280	
6	Initial	294.1296	2.3754	1.5370	0.2186	80.2237	0.1777	9.3716	28
	Final	146.5030	9.2230	6.8360	0.3060	27.6504	0.3000	2.0280	
Global minimum		146.5030 (0%)*	9.2230 (0%)	6.8360 (0%)	0.3060 (0%)	27.6503 (0.001%)	0.3000 (0%)	2.0280 (0%)	Probability 0.9959

* Values in the parentheses denote percentage difference between identified and actual data.

表6-28 彈性支撐GI/ep複合材料三明治板之自然頻率理論值

Layup	Core E_{core} (Mpa)	Edge support E_e (Mpa)	Natural frequency (Hz)							
			1st	2nd	3rd	4th	5th	6th	7th	8th
$[0_3/core/0_3]$	10	1	137.760	248.617	289.078	360.512	396.309	450.950	476.802	503.603
	2000	50	249.747	468.066	712.232	846.992	886.014	1215.907	1382.185	1472.517
$[45^-/-45^+/45^-/core/45^-/-45^+/45^-]$	10	1	145.433	276.220	282.657	377.141	430.243	434.084	500.883	512.625
	2000	50	277.753	600.628	637.175	984.942	1150.070	1164.418	1494.390	1600.772

表6-29 彈性支撐GI/ep三明治板以理論頻率之反算

Layup	Core E_{core} (MPa)	Edge support E_e (Mpa)	No. of starting points	Average no. of iteration	System parameter						
					E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_c (MPa)	ν_c	E_e (MPa)
$[0_3/core/0_3]$	10	1	6	11	38.6001 (2.6e-4%)*	8.27 (0%)	4.14 (0%)	0.26 (0%)	9.988 (-0.12%)	0.2985 (-0.5%)	1.0 (0%)
	2000	50	4	13	38.5964 (-0.01%)	8.2684 (-0.02%)	4.1391 (-0.02%)	0.26 (0%)	2002.2 (0.11%)	0.30 (0%)	50.0004 (8e-4%)
$[45^-/-45^+/45^-/core/45^-/-45^+/45^-]$	10	1	7	16	38.6 (0%)	8.27 (0%)	4.14 (0%)	0.26 (0%)	9.99 (-0.1%)	0.2986 (-0.47%)	1.0 (0%)
	2000	50	4	13	38.5951 (-0.01%)	8.2732 (0.04%)	4.1403 (0.01%)	0.2601 (0.04%)	1999.6 (-0.02%)	0.30 (0%)	50.0054 (0.01%)

* Values in the parentheses denote percentage difference between identified and actual data.

表6-30 彈性支撐Gr/ep $[0_3^0/\text{core(I)}/0_3^0]$ 三明治板以實驗頻率之反算

Starting point no.	Stage	System parameter							No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_c (MPa)	ν_c	E_e (MPa)	
1	Initial	40.801	13.805	7.861	0.2307	71.168	0.3894	2.015	14
	Final	150.208	8.669	7.430	0.3004	26.978	0.2999	2.153	
2	Initial	297.168	9.511	7.943	0.1310	96.603	0.2893	1.986	17
	Final	150.205	8.668	7.430	0.3003	26.976	0.3000	2.153	
3	Initial	21.834	15.872	0.017	0.2817	38.429	0.4310	2.009	19
	Final	150.209	8.668	7.430	0.3001	26.981	0.3000	2.153	
4	Initial	40.801	13.805	7.861	0.3073	71.168	0.2894	2.015	15
	Final	150.208	8.669	7.430	0.3004	26.978	0.2999	2.153	
Global minimum		150.205	8.668	7.430	0.3003	26.976	0.3000	2.153	Probability
		(2.53%)*	(-6.02%)*	(8.69%)*	(-1.85%)*	(-2.44%)*	(-0.01%)*	(6.18%)*	0.9921

* Values in the parentheses denote percentage difference between identified and measured data.

表6-31 彈性支撐Gr/ep三明治板以實驗頻率之反算

Layup	No. of starting points	Average no. of iteration	System parameter						
			E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_c (MPa)	ν_c	E_e (MPa)
$[0^\circ/90^\circ/0^\circ/\text{core(I)}/0^\circ/90^\circ/0^\circ]$	4	21	153.893 (5.04%)*	8.194 (-11.16%)*	6.285 (-8.06%)*	0.2984 (-2.47%)*	28.007 (1.29%)*	0.3073 (2.43%)*	2.184 (7.71%)*
$[0_3^0/\text{core(II)}/0_3^0]$	4	28	150.549 (-2.17%)*	9.100 (11.07%)*	6.738 (7.21%)*	0.3015 (-1.48%)*	4.457 (13.12%)*	0.3684 (-3.04%)*	2.239 (2.52%)*
$[0^\circ/90^\circ/0^\circ/\text{core(II)}/0^\circ/90^\circ/0^\circ]$	6	18	157.930 (4.90%)*	8.599 (-5.52%)*	6.559 (-2.67%)*	0.3010 (-0.17%)*	3.455 (-12.31%)*	0.3701 (-2.60%)*	2.084 (-6.93%)*

* Values in the parentheses denote percentage difference between identified and measured data.