

國立交通大學

統計學研究所

碩士論文

藉由 K 均值分群與階層式分群程序對潛在群體
分析做參數估計

Parameter Estimation for Latent Class Models via
K-means and Hierarchical Procedures

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中華民國九十六年七月

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本研究的主要目的是藉由群聚分析的方法對潛在群體模型做參數估計。我們引用了群聚方法中的 k 均值分群和階層式分群的想法，將原本的距離測度改成相關係數或共變異數，然後對所有的主體分群，使得屬於在同一群的主體所測得的各項目能互相獨立。將估計出的潛在群體視為已知變數後，再去估計潛在群體迴歸分析模型的參數就變得容易多了。我們的模擬結果顯示出：所用的測度為相關係數或共變異數的 k 均值分群法表現得不錯，但是所用的測度為共變異數的階層式分群法表現得並不好。

關鍵字: 潛在群體迴歸、 k 均值分群、階層式分群

Parameter Estimation for Latent Class Models via K-means and Hierarchical Procedures

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Abstract

The aim of the study is to estimate the parameters of the latent class models via clustering methods. We use k-means and hierarchical ideas of clustering methods with the correlation (or covariance) among items as the distance measure to group objects such that, for all objects who belong to the same latent class, items are "independent". By viewing the estimated latent class as known variable, it becomes easy to estimate the parameters in the regression extension of latent class analysis (RLCA) model. The results of our simulation study display that the k-means method with the correlation (or covariance) measurement performed well, but the hierarchical method with the covariance measurement didn't perform well.

Key words: Regression of latent class analysis (RLCA) 、 k-means 、 hierarchical

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1. Introduction

Latent class analysis (LCA), originally described by Green (1951) and systematically developed by Lazarsfeld and Henry (1968), Goodman (1974), has been found useful for classifying objects based on their responses to a set of categorical items. The basic model postulates an underlying categorical latent variable, say, J categories, and measured items are assumed independent of one another within any category of the latent variable. Observed relationships among measured variables are thus assumed to result from the underlying classification of the data produced by the categorical latent variable. Latent class analysis may legitimately be viewed as the analog of cluster analysis. The term *cluster analysis* (first used by Tryon, 1939) encompasses a number of different algorithms and methods for grouping objects of similar kind into respective categories. In this research, instead of grouping objects of “similar kind” into respective categories, we use hierarchical and k-means ideas of clustering methods with the correlation among items as the distance measure to group objects such that, for all objects who belong to the same latent class, items are “independent”.

Recently several authors extended the LCA model to describe the effects of measured covariates on the underlying mechanism (Dayton and Macready, 1988; Van der Heijden, Dessens and Bökenholt, 1996; Bandeen-Roche, Miglioretti, Zeger and Rathouz, 1997), or on measured item distributions within latent levels (Melton, Liang and Pulver, 1994). These extended LCA models are called the regression extension of latent class analysis (RLCA) models. For the RLCA model, by using the marginalizing techniques to eliminate covariate effects from both the latent variable and measured indicators (Huang 2005), our clustering idea can be also applied to the reduced LCA model to estimate the latent class membership. By viewing the latent variable as known predictors, it becomes easy to estimate the parameters in the RLCA

model.



2. Literature review

2.1 Latent class analysis (LCA)

Latent class analysis (LCA) aims to classify objects based on their responses to a set of categorical items. To introduce the methodology, let $Y_i = (Y_{i1}, \dots, Y_{iM})^T$ denote a set of M observable polytomous indicators for the i th individual in a study sample of N persons. Y_{im} , $m=1, \dots, M$ can take values $\{1, \dots, K_m\}$, where $K_m \geq 2$. The basic model postulates an underlying categorical latent variable $S_i = 1, \dots, J$ for individual i ; within any category of the latent variable, the measured indicators are assumed to be independent of one another. Therefore, the distribution for Y_i can be expressed as follows:

$$\Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M) = \sum_{j=1}^J \{ \Pr(S_i = j) \prod_{m=1}^M \prod_{k=1}^{K_m} [\Pr(Y_{im} = k | S_i = j)]^{y_{mk}} \}, \quad (2.1)$$

where $y_{mk} = 1$ if $y_m = k$; 0 otherwise. The LCA model assumes that

$$\Pr(Y_{im} = k | S_i = j) = p_{mkj}, \quad \Pr(S_i = j) = \eta_j, \quad (2.2)$$

$i=1, \dots, N$; $m=1, \dots, M$; $k=1, \dots, K_m$; $j=1, \dots, J$. Thus, the model treats class membership probabilities, η_j , and item response probabilities conditional on class membership, p_{mkj} , as homogeneous over individuals. Heuristically, η_j is the population prevalence of class j , and p_{mkj} is the probability of an individual in class j being at level k of Y_{im} . Goodman (1974) provided an excellent overview of the LCA model, including a maximum likelihood strategy for estimating model parameters, conditions to determine local model identifiability, a strategy to test overall model fit, and the use of constraints to identify models.

2.2 Regression extension of latent class analysis (RLCA)

Huang and Bandeen-Roche (2004) extend the latent class analysis to allow both the probabilities of latent class membership and the distribution of observed responses given latent class membership to be functionally related to concomitant variables, while preserving model identifiability. By allowing covariate effects on latent class probabilities, the model can summarize the effect of risk factors on the underlying mechanism. In the case of incorporating covariates into conditional probabilities, the model can also adjust for characteristics that determine responses other than underlying classes, hence improving the accuracy of classifying individuals. For example, in evaluating functional disability, some data have suggested that women tend to rate tasks as “difficult” more readily than men independently of ability (Bandeen-Roche, Huang, Munoz, & Rubin, 1999). Without adjusting for a gender effect, the model might well classify some men and women with identical underlying functioning differently (men as “able”, women as “disabled”).

Let (x_i, z_i) be the concomitant covariates of the i th person, where $x_i = [1, x_{i1}, \dots, x_{ip}]^T$ are primary covariates hypothesized to be associated with latent class S_i , and $z_i = [z_{i1}, \dots, z_{iM}]$ with $z_{im} = [1, z_{im1}, \dots, z_{imL}]^T$, $m=1, \dots, M$ are secondary covariates used to build direct effects on measured indicators. The sets of covariates may include any combination of continuous and discrete measures and two sets of covariates may be mutually exclusive or overlap. The basis RLCA equation can be stated as follows:

$$\Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid x_i, z_i) = \sum_{j=1}^J \left\{ \eta_j(x_i^T \beta) \prod_{m=1}^M \prod_{k=1}^{K_m} p_{mkj}^{y_{mk}} (\gamma_{mj} + z_{im}^T \alpha_{mk}) \right\} \quad (2.3)$$

with $\eta_j(x_i)$ and $p_{mkj}(z_{im})$ defined as in the generalized linear framework (McCullagh and Nelder, 1989). Often, (2.3) is implemented by assuming generalized logit link functions (Agresti, 1984):

$$\log \left[\frac{\eta_j(\mathbf{x}_i^T \boldsymbol{\beta})}{\eta_J(\mathbf{x}_i^T \boldsymbol{\beta})} \right] = \beta_{0j} + \beta_{1j} x_{i1} + \cdots + \beta_{pj} x_{ip} \quad (2.4)$$

and

$$\log \left[\frac{p_{mkj}(\gamma_{mj} + z_{im}^T \boldsymbol{\alpha}_{mk})}{p_{mKj}(\gamma_{mj} + z_{im}^T \boldsymbol{\alpha}_{mk})} \right] = \gamma_{mkj} + \alpha_{1mk} z_{im1} + \cdots + \alpha_{Lmk} z_{imL}$$

$$i = 1, \dots, N; \quad m = 1, \dots, M; \quad k = 1, \dots, (K_m - 1);$$

$$j = 1, \dots, (J - 1); \quad j' = 1, \dots, J \quad (2.5)$$

Notice that in the conditional probability model (2.5), we allow unrestricted intercepts and level- and item-specific covariate coefficients, but we do not allowing the coefficients to vary across classes (i.e., α_{qmk} is dependent on m, k but independent of j). This constraint is logical if the primary purpose of modeling conditional probabilities is to prevent possible misclassification by adjusting for characteristics associated with item measurements. It is also necessary to unambiguously distinguish covariate effects on measured response probabilities from covariate effects on class probabilities. Three assumptions complete (2.3):

$$(C1) \quad \Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid S_i, \mathbf{x}_i, \mathbf{z}_i) = \Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid S_i, \mathbf{z}_i);$$

$$(C2) \quad \Pr(S_i = j \mid \mathbf{x}_i, \mathbf{z}_i) = \Pr(S_i = j \mid \mathbf{x}_i);$$

$$(C3) \quad \Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid S_i, \mathbf{z}_i) = \prod_{m=1}^M \Pr(Y_{im} = y_m \mid S_i, \mathbf{z}_{im}).$$

Huang and Bandee-Roche (2004) provided an excellent overview of the RLCA model, including model identification, Expectation-Maximization algorithm for parameter estimation, standard error calculation, convergent properties, and comparison of the RLCA model with models underlying existing latent class

modeling software.

2.3 Marginalization of the regression extension of latent class model

Here we introduce a process to “eliminate” the covariate effects, hence “marginalize” the RLCA model (2.3). The marginalization process (Huang 2005) includes two stages. Stage 1 aims to eliminate z_i . Stage 2 applies the marginalization property used Bandeen-Roche et al. (1997) to average x_i effects out of the latent prevalence.

2.3.1. Marginalizing the covariate effects on conditional probabilities

The key to marginalizing over z_i is that the process must yield random variables that follow a finite mixture distribution that is both independent of z_i and has J mixing components. One strategy for achieving such marginalization can be motivated by the properties of added variable plots for linear regression models.

Consider the linear model

$$Y = x_1^T \beta_1 + x_2^T \beta_2 + \varepsilon, \quad (2.6)$$

where ε has mean 0 and variance matrix V . Let \tilde{Y} denote the residuals of regressing Y on x_2 , and $W = V^{-1}$ be the weight matrix. Then, it is well known that if x_1 and x_2 are orthogonal (i.e., $x_1^T W x_2 = 0$), \tilde{Y} has mean $x_1^T \beta_1$ and variance V .

Hence, the simple linear regression of \tilde{Y} on x_1 yields exactly the same inferences about β_1 as if we performed the analysis on the more complicated model (2.6) (Cook and Weisberg, 1982). Viewing the just-described stability of β_1 as analogous to the desired stability of latent class dimension, J, the added variable property can be applied to model (2.5) to obtain marginalized conditional probabilities.

To present the key ideas more clearly, henceforth, the measured indicators (Y_{i1}, \dots, Y_{iM}) are assumed to be binary (i.e., $K_1 = \dots = K_M = 2$). To make the analogy to (2.6), notice that (2.5) can be viewed as fitting a logistic regression of Y_{im} on S_i

adjusting for z_{im} , separately for each m . To see this, let $S_{ij} = I(S_i = j) = 1$ if $S_i = j$; 0 otherwise, for $i = 1, \dots, N$; $j = 1, \dots, J-1$. we can reparameterize (2.5) as

$$\log it[E(Y_{im} | S_i, Z_{im}^c)] = S_i^T \gamma_m + (Z_{im}^c)^T \alpha_m \quad \text{for } i=1, \dots, N; \quad m=1, \dots, M., \quad (2.7)$$

where $S_i = [1, S_{i1}, \dots, S_{i(J-1)}]^T$;

$$Z_{im}^c = [(z_{im1} - \bar{z}_{m1}), \dots, (z_{imL} - \bar{z}_{mL})]^T, \text{ (“centered” covariate vector)}$$

$$\bar{z}_{mp} = (1/N) \sum_{i=1}^N z_{imp} \quad ;$$

$$\gamma_m = [\gamma_{m0}, \gamma_{m1}, \dots, \gamma_{m(J-1)}]^T; \text{ and } \alpha_m = [\alpha_{1m}, \dots, \alpha_{Lm}]^T.$$

Therefore, for any realization of S_i , (2.7) is a logistic regression with dependent variable: Y_{im}^p and predictors: S_i , Z_{im}^c .

Next, the problem becomes how to calculate residuals from the generalized linear model

$$\log it[E(Y_{im} | Z_{im}^c)] = (Z_{im}^c)^T \alpha_m^* \quad \text{for } i=1, \dots, N; \quad m=1, \dots, M. \quad (2.8)$$

The “pseudo-residuals” are given by

$$R_m = [R_{1m}, \dots, R_{Nm}]^T = (\hat{V}_m)^{-1} [Y_m - \hat{u}_m], \quad (2.9)$$

where “hat” represents the estimated values;

$$V_m = \text{diag}(V_{1m}, \dots, V_{Nm}), \quad V_{im} = \text{Var}(Y_{im}); \quad Y_m = [Y_{1m}, \dots, Y_{Nm}]^T; \quad u_m = E(Y_m | Z_m^c).$$

If x_i and z_{im} are independent, we can extract the Z_{im}^c from conditional probabilities by treating the residuals from the model (2.8) as new response variables and regressing them on S_i . We substitute the estimate of γ_m^* in the linear model

$$R_{im} = S_i^T \gamma_m^* + \varepsilon_{im}, \quad i=1, \dots, N; \quad m=1, \dots, M \quad (2.10)$$

for the estimate of γ_m in the model (2.7). A formal justification shows that γ_m^* and

γ_m can be very close under reasonable regularities. The above results can be extended to the cases where (Y_{i1}, \dots, Y_{iM}) are polytomous as in (2.1) and (2.3).

2.3.2. Marginalizing the covariate effects on latent prevalences

The marginalization of model (2.3) over z_i possesses the nice property that the covariates associated with latent class prevalences, x_i , can be ignored.

2.4 Hierarchical clustering methods

Hierarchical clustering techniques proceed by either a series of successive mergers or a series of successive divisions. Agglomerative hierarchical methods start with the individual objects. Thus, there are initially as many clusters as objects. The most similar objects are first grouped, and these initial groups are merged according to their similarities. Eventually, as the similarity decreases, all subgroups are fused into a single cluster.

Divisive hierarchical methods work in the opposite direction. An initial single group of objects is divided into two subgroups such that the objects in one subgroup are “far from” the objects in the other. These subgroups are then further divided into dissimilar subgroups; the process continues until there are as many subgroups as objects – that is, until each object forms a group.

The results of both agglomerative and divisive methods may be displayed in the form of a two-dimensional diagram known as a dendrogram. As we shall see, the dendrogram illustrates the mergers or divisions that have been made at successive levels.

In this research, we concentrate on agglomerative hierarchical procedures. The following are the steps in the agglomerative hierarchical clustering algorithm for grouping N objects (items or variables):

1. Start with N clusters, each containing a single object and an $N \times N$ symmetric

matrix of distances (or similarities) $D = \{d_{ik}\}$.

2. Search the distance matrix for the nearest (most similar) pair of clusters. Let the distance between “most similar” clusters U and V be d_{UV} .
3. Merge clusters U and V. Label the newly formed cluster (UV). Update the entries in the distance matrix by (a) deleting the rows and columns corresponding to cluster U and V and (b) adding a row and column giving the distances between cluster (UV) and the remaining clusters.
4. Repeat Steps 2 and 3 a total of N-1 times. All objects will be in a single cluster after the algorithm terminates. Record the identity of clusters that are merged and the levels (distances or similarities) at which the mergers take place.

2.5 Ward’s hierarchical clustering method

Ward (1963) considered hierarchical clustering procedures based on minimizing the ‘loss of information’ from joining two groups. This method is usually implemented with loss of information taken to be an increase in an error sum of squares criterion, ESS. First, for a given cluster k, let ESS_k be the sum of the squared deviation of every item in the cluster from the cluster mean (centroid). If there are currently K clusters, define ESS as the sum of the ESS_k or $ESS = ESS_1 + ESS_2 + \dots + ESS_K$. At each step in the analysis, the union of every possible pair of clusters is considered, and the two clusters whose combination results in the smallest increase in ESS (minimum loss of information) are joined. Initially, each cluster consists of single item, and, if there are N items, $ESS_k = 0$, $k=1,2,\dots,N$, so $ESS=0$. At the other extreme, when all the clusters are combined in a single group of N items, the value of ESS is given by

$$ESS = \sum_{j=1}^N (x_j - \bar{x})'(x_j - \bar{x}),$$

where x_j is the multivariate measurement associated with the j th item and \bar{x} is the mean of all the items.

The results of Ward's method can be displayed as a dendrogram. The vertical axis gives the values of ESS at which the mergers occur.

Ward's method is based on the notion that the clusters of multivariate observations are expected to be roughly elliptically shaped. It is a hierarchical precursor to nonhierarchical clustering methods that optimized some criterion for dividing data into a given number of elliptical groups.

2.6 K-means method

The K-means method is one of the more popular nonhierarchical procedures. MacQueen (1967) suggests the term K-means for describing an algorithm of his that assigns each object to the cluster having the nearest centroid (mean). In its simplest version, the process is composed of these three steps:

1. Partition the objects into K initial clusters.
2. Proceed through the list of objects, assigning an object to the cluster whose centroid (mean) is nearest. (Distance is usually computed using Euclidean distance with either standardized or unstandardized observations.) Recalculate the centroid for the cluster receiving the new object and for the cluster losing the object.
3. Repeat Step2 until no more reassignments take place.

Rather than start with a partition of all objects into K preliminary groups in Step 1, we could specify K initial centroids (seed points) and then proceed to step 2.

The final assignment of objects to clusters will be, to some extent, dependent upon the initial partition or the initial selection of seed points. Experience suggests that most major changes in assignment occur with the first reallocation step.



3. Models

3.1 LCA

Let (Y_{i1}, \dots, Y_{iM}) denote a set of M observable polytomous outcome indicators and S_i denote the unobservable class membership, for the i th individual in a study sample of N persons. Y_{im} can take values $\{1, \dots, K_m\}$, where $K_m \geq 2$, $m=1, \dots, M$, and S_i can take values $\{1, \dots, J\}$. The latent class analysis model is based on the concept of conditional independence in the sense that the observed variables are assumed to be statistically independent within latent classes. Therefore, the distribution for (Y_{i1}, \dots, Y_{iM}) can be expressed as the finite mixture density:

$$\Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M) = \sum_{j=1}^J \{ \eta_j \prod_{m=1}^M \prod_{k=1}^{K_m} p_{mkj}^{y_{mk}} \} , \quad (3.1)$$

where $\eta_j = \Pr(S_i = j)$ are the “latent class probabilities” of each underlying variable category, $p_{mkj} = \Pr(Y_{im} = k | S_i = j)$ are the “conditional probabilities” of the measured responses given the underlying variable category and $y_{mk} = I(y_m = k)$.

For more detail on identifiability, parameter estimations and the test overall model fit, readers may reference Goodman (1974)

3.2 RLCA

To incorporate covariate effects into LCA, let (x_i, z_i) be the associated covariate vector for the i th person, where $x_i = [1, x_{i1}, \dots, x_{ip}]^T$ are predictors associated with latent class S_i , and $z_i = [z_{i1}, \dots, z_{iM}]$; $z_{im} = [1, z_{im1}, \dots, z_{imL}]^T$ with $m=1, \dots, M$ are covariates used to build direct effects on measured indicators. The sets of covariates may include any combination of continuous and discrete measures. To marginalize

the RLCA model (3.2) , we begin by assuming that the two sets of covariates are mutually independent. The basis RLCA equation can be stated as

$$\Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid x_i, z_i) = \sum_{j=1}^J \left\{ \eta_j(x_i^T \beta) \prod_{m=1}^M \prod_{k=1}^{K_m} p_{mkj}^{y_{mk}}(\gamma_{mj} + z_{im}^T \alpha_{mk}) \right\} \quad (3.2)$$

with $\eta_j(x_i^T \beta)$ and $p_{mkj}(\gamma_{mj} + z_{im}^T \alpha_{mk})$ defined as in the generalized linear framework (McCullagh and Nelder, 1989). Often, (3.2) is implemented by assuming generalized logit (Agresti, 1984) link functions:

$$\log \left[\frac{\eta_j(x_i^T \beta)}{\eta_{j'}(x_i^T \beta)} \right] = \beta_{0j} + \beta_{1j} x_{i1} + \dots + \beta_{pj} x_{ip} \quad (3.3)$$

and

$$\log \left[\frac{p_{mkj}(\gamma_{mj} + z_{im}^T \alpha_{mk})}{p_{m'kj}(\gamma_{m'j} + z_{im}^T \alpha_{m'k})} \right] = \gamma_{mkj} + \alpha_{1mk} z_{im1} + \dots + \alpha_{Lmk} z_{imL} \quad (3.4)$$

$i = 1, \dots, N; \quad m = 1, \dots, M; \quad k = 1, \dots, (K_m - 1);$
 $j = 1, \dots, (J - 1); \quad j' = 1, \dots, J$

If the regression coefficients in (3.3) or (3.4) are set as 0, model (3.2) reduces to models studied by Melton, Liang and Pulver (1994), Dayton and Macready (1998) or an ordinary latent class analysis (3.1).

Notice that in the conditional probability model (3.4), we allow unrestricted intercepts and level- and item-specific covariate coefficients, but we do not allowing the coefficients to vary across classes (i.e., α_{qmk} is dependent on m, k but independent of j). This constraint is logical if the primary purpose of modeling conditional probabilities is to prevent possible misclassification by adjusting for characteristics associated with item measurements. Three assumptions complete (3.2):

$$(C1) \Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid S_i, x_i, z_i) = \Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid S_i, z_i);$$

$$(C2) \Pr(S_i = j \mid x_i, z_i) = \Pr(S_i = j \mid x_i);$$

$$(C3) \Pr(Y_{i1} = y_1, \dots, Y_{iM} = y_M \mid S_i, z_i) = \prod_{m=1}^M \Pr(Y_{im} = y_m \mid S_i, z_{im}).$$

For more detail on model assumptions, identifiability and parameter estimations, readers may reference Huang and Bandeen-Roche (2004).



4. Parameter estimations by clustering analysis

The parameters in (3.2) are typically estimated by maximum likelihood (ML) for a fixed number of classes, J . Viewing the class membership S_i as unobservable, the LCA model (3.1) and RLCA model (3.2) becomes a typical incomplete-data problem. Goodman (1974) provided an excellent maximum likelihood strategy for estimating model parameters in (3.1), and Hung and Bandeen-Roche (2004) had successfully used the Expectation-Maximization (EM) algorithm (Dempster, Laird, & Rubin, 1997) to computing ML estimates of the parameters in (3.2) and created a powerful computer module to implement the proposed latent class model (3.2). However implementing the EM algorithm to estimate parameters in finite-mixture models is typically time-consuming. Therefore we propose an alternative clustering analysis strategy to predict parameters in (3.1) and (3.2).

4.1 Latent class membership estimations for LCA

Latent class analysis aims to classify objects based on their responses to a set of categorical items. The basic model postulates an underlying categorical latent variable $S_i = 1, \dots, J$ for individual i ; within any category of the latent variable, the measured indicators are assumed to be independent of one another. Therefore if we can estimate the unobservable class membership S_i based on model assumption, then it is easy to predict the parameters in (3.1) by viewing the estimated class membership as known variable. We propose the following strategy to estimate S_i .

Here we apply the concept of k-means (MacQueen, 1967) and agglomerative hierarchical methods to group the objects. Notice that rather than grouping the objects into J subgroups such that the objects in one subgroup are “far from” the objects in the other, we try to group objects such that observed variables are statistically

independent within latent classes. Therefore we use sample correlation or sample covariance as the distance in k-means and agglomerative hierarchical methods, and the concept of the 'loss information' and 'minimum loss of information' in ward's hierarchical clustering method is included.

First, we introduce how to calculate the sample correlation and sample covariance matrix used in the k-means and agglomerative hierarchical clustering algorithms. For individual i , we transform the M observable polytomous outcome indicators (Y_{i1}, \dots, Y_{iM}) to the following dummy variables:

$$\tilde{Y}_i = (Y_{i11}, \dots, Y_{i1(K_1-1)}, Y_{i21}, \dots, Y_{i2(K_2-1)}, \dots, Y_{iM1}, \dots, Y_{iM(K_M-1)})$$

$$\text{with } Y_{imk} = I(Y_{im} = k), m = 1, \dots, M; k = 1, \dots, K_{m-1}.$$

Then,

$$\begin{aligned} \text{Cov}(\tilde{Y}_i) &= [\text{Cov}(Y_{imk}, Y_{iqs})] \\ &= \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1M} \\ B_{21} & B_{22} & \dots & B_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ B_{M1} & B_{M2} & \dots & B_{MM} \end{bmatrix}, \end{aligned} \quad (4.1)$$

where for the m th item and q th item, B_{mq} is the block of $(K_m - 1) \times (K_q - 1)$ covariance matrix. Various elements of the variance-covariance matrix of measured indicators are:

$$\text{Cov}(Y_{imk}, Y_{iqs}) = \begin{cases} \Pr(Y_{imk} = 1) - \Pr(Y_{imk} = 1)\Pr(Y_{iqs} = 1) & \text{if } m = q \text{ and } k = s \\ -\Pr(Y_{imk} = 1)\Pr(Y_{iqs} = 1) & \text{if } m = q \text{ and } k \neq s \\ \Pr(Y_{imk} = 1, Y_{iqs} = 1) - \Pr(Y_{imk} = 1)\Pr(Y_{iqs} = 1) & \text{if } m \neq q \end{cases} \quad (4.2)$$

These variances were estimated by replacing the probabilities with the sample averages. From sample covariance matrix, we can also calculate the sample correlation matrix as $\tilde{D}^{-\frac{1}{2}} \widehat{\text{Cov}}(\tilde{Y}_i) \tilde{D}^{-\frac{1}{2}}$, where $\tilde{D} = \text{diag}(\hat{B}_{11}, \hat{B}_{22}, \dots, \hat{B}_{MM})$.

The following are the steps in the k-means and agglomerative hierarchical clustering

algorithm separately.

The K-means algorithm:

1. Partition the objects into K initial clusters.
2. Proceed through the list of objects, assigning an object to the cluster which reaching 'minimum loss of independence'.
3. Repeat Step2 until no more reassignments take place.

In Step 1, we first specify K initial centroids (seed points) and then proceed through the list of objects, assigning an object to the cluster whose centroid (mean) is nearest. (Distance is computed using Euclidean distance.) To reach enough sample size such that the sample covariance/correlation is meaningful, it is necessary to adjust the number of objects in each initial cluster. Therefore, once existing an one initial cluster including members less than we expect, we repartition the objects “randomly” and “evenly” into K initial clusters.

Next, we shall introduce what the minimum loss of independence is in Step2. For a given cluster k, let $MCov_k$ be the mean of the absolute values of entries in non diagonal blocks of sample correlation matrix (or sample covariance matrix) for the M observable polytomous outcome indicators. For a given object, if it is assigned to some cluster j, we define the loss of independence LoI_j as the sum of the $MCov_k$ or $LoI_j = MCov_1^{(j)} + MCov_2^{(j)} + \dots + MCov_K^{(j)}$, where $MCov_k^{(j)}$ is the mean of the absolute values of non-diagonal-block entries of $Cov(\tilde{Y}_i)$ after the object being assigned to cluster j. For a given object, after assigning through K clusters, we can get $LoI_j, j= 1, \dots, K$. The smaller the value of LoI_j is, the more independent the

observed variables for objects within cluster j are. So, we take the minimum LoI_j as the ‘minimum loss of independence’ and assign a given object to the cluster corresponding to the minimum loss of independence. An example of k-means algorithm procedure can be found in Figure1.

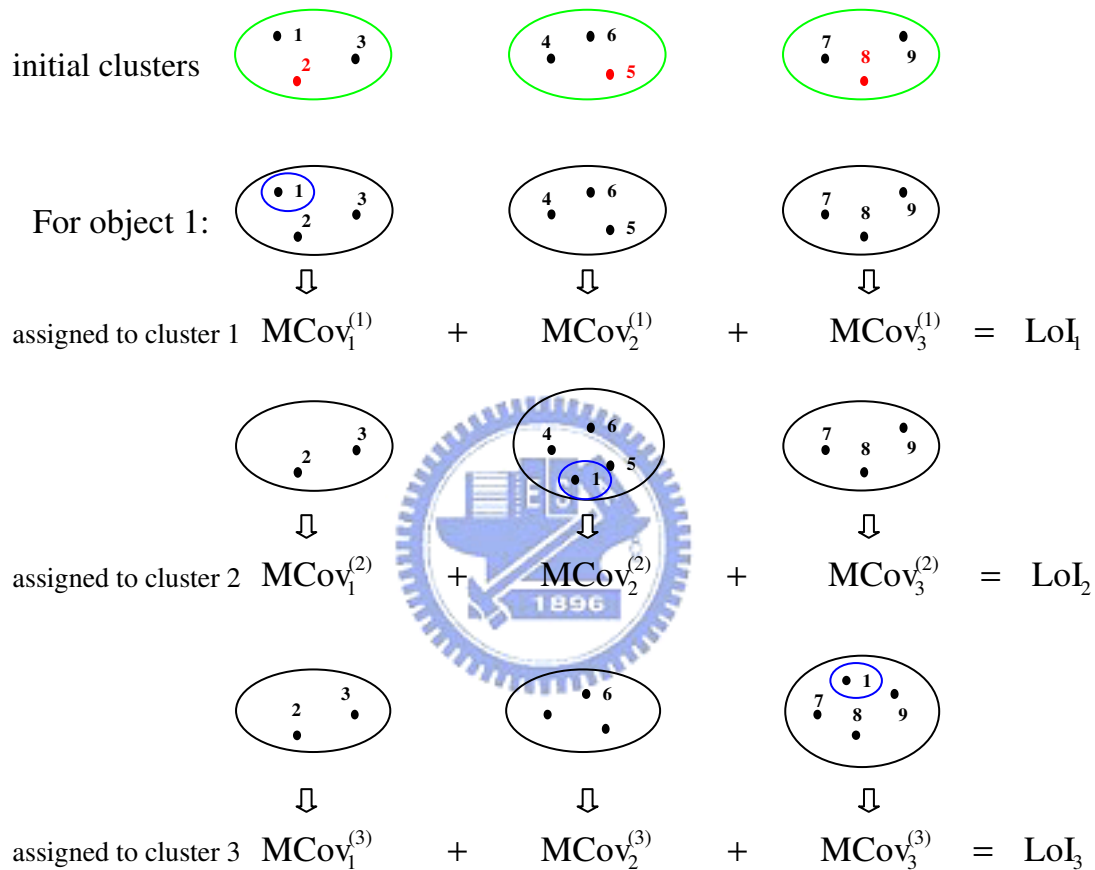


Figure 1: An example of k-means algorithm procedure.

Step1: Partition the 9 objects into 3 initial clusters.

Step2: What cluster will the object 1 be assigned to?

Assigning the object 1 to the cluster 1,2 and 3 separately, we can get LoI_1 , LoI_2 and LoI_3 . Assign the object 1 to the cluster which reaching “minimum loss of independence”.

Proceed through the objects 2-9, repeat above procedure.

Step3: Repeat Step2 until no more reassignments take place.

The agglomerative hierarchical clustering algorithm:

1. Start with N clusters, each cluster consists of a single object.
2. If there are some objects whose M items are all the same, then we merge them together. Otherwise, skip this step.
3. The union of every possible pair of clusters is considered, and the two clusters U and V whose combination results in the minimum loss of independence are merged. Label the newly formed cluster (UV).
4. Repeat Step 3 until all objects will be in a single cluster after the algorithm terminates. Record the identity of clusters that are merged and the levels (minimum loss of independence) at which the mergers take place.

Now we will introduce the minimum loss of independence in Step3 in the agglomerative hierarchical clustering algorithm. For currently K clusters, if a cluster U is merged to the other cluster V, then the K clusters decrease to K-1 clusters. We defined the loss of independence for cluster U being merged to cluster V, $LoI_{(V,U)}$, as the sum of $MCov_k$ (defined in the K-means algorithm) of each cluster:

$$LoI_{(V,U)} = MCov_1^{(V,U)} + \dots + MCov_{U-1}^{(V,U)} + MCov_{U+1}^{(V,U)} + \dots + MCov_K^{(V,U)}.$$

For currently K clusters, we can get $C_2^K LoI_{(V,U)}$, where (V, U) belong to the union of every possible pair of clusters. The smaller the value of $LoI_{(V,U)}$ is, the more independent the observed variables for objects within the newly formed cluster (UV) are. So, we take the minimum $LoI_{(V,U)}$ as the ‘minimum loss of independence’ and merge the two clusters U and V whose combination results in the minimum loss of independence. An example of hierarchical algorithm procedure can be found in Figure2.

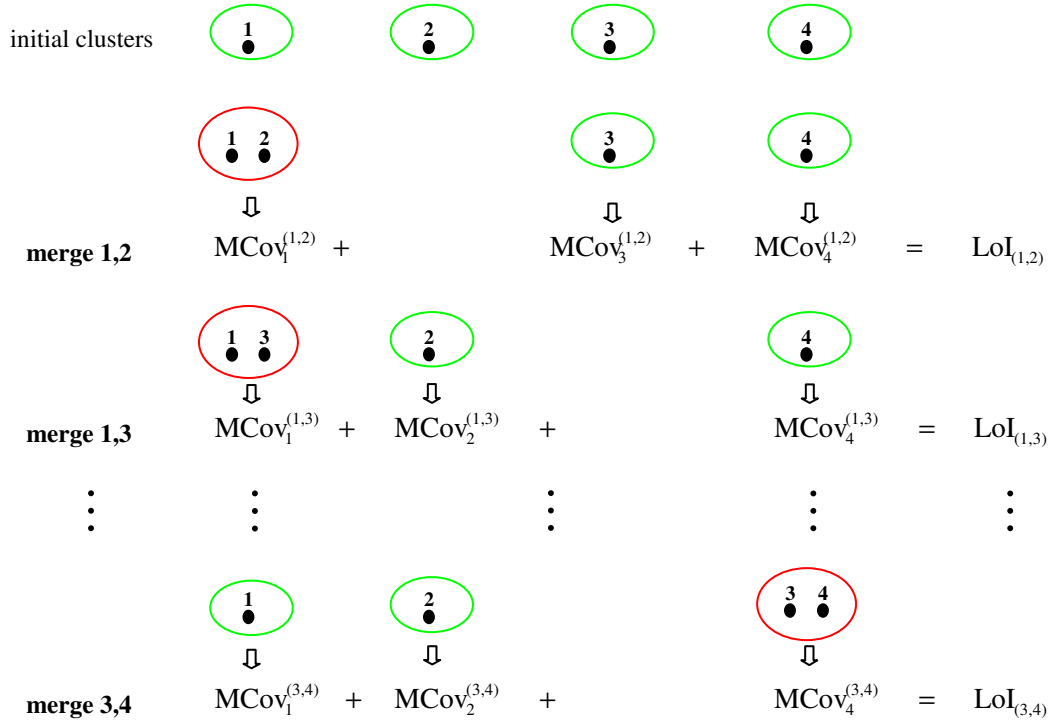


Figure 2: An example of hierarchical algorithm procedure.

Step1: Start with 4 initial clusters, each cluster consists of a single object.

Step2: Which pair of clusters will be merged?

Consider the union of 6 ($=C_2^4$) possible pair of clusters, we can get

$LoI_{(1,2)}$, $LoI_{(1,3)}$, $LoI_{(1,4)}$, $LoI_{(2,3)}$, $LoI_{(2,4)}$ and $LoI_{(3,4)}$. Merge the pair of

clusters whose combination results in the “minimum loss of independence”.

Step3: Repeat Step2 until all objects will be in a single cluster.

The results of agglomerative hierarchical clustering method can be displayed as a dendrogram. The vertical axis gives the values of minimum loss of independence at which the mergers occur.

4.2 Latent class membership estimations for RLCA

The k-means and agglomerative hierarchical clustering algorithms are based on the model (3.1) where no covariates are incorporated. The two algorithms also work for

the model (3.2) under eliminating the covariate effects (Huang2005), hence “marginalize” the model (3.2).

The key to marginalizing over z_i is that the process must yield random variables that follow a finite mixture distribution that is both independent of z_i and has J mixing components. One strategy for achieving such marginalization can be motivated by the properties of added variable plots for linear regression models. The conditional probabilities (3.4) can be viewed as fitting a logistic regression of Y_{im} on S_i adjusting for z_{im} , separately for each m. Next the problem becomes how to calculate residuals from the generalized linear model:

$$\log it[E(Y_{im}^p | Z_{im}^c)] = (Z_{im}^c)^T \alpha_m^* \quad \text{for } i=1, \dots, N; \quad m=1, \dots, M. \quad (4.3)$$

where

“p” denotes polytomous responses; $Y_{im}^p = [Y_{im1}, \dots, Y_{im(K_m-1)}]^T$ and $Y_{imk} = I(Y_{im} = k)$; $Z_{im}^c = [(z_{im1} - \bar{z}_{m1}), \dots, (z_{imL} - \bar{z}_{mL})]^T$ (“centered” covariate vector), $\bar{z}_{mp} = (1/N) \sum_{i=1}^N z_{imp}$;

Under polytomous item responses, the pseudo-residual of ith participant’s mth response item is

$$R_{im}^p = (\hat{V}_{im}^p)^{-1} (Y_{im}^p - \hat{\mu}_{im}^p), \quad (4.4)$$

where “hat” denotes the estimated values; $R_{im}^p = [R_{im1}, \dots, R_{im(K_m)}]^T$; $V_{im}^p = \text{Var}(Y_{im}^p)$;

$\mu_m^p = E(Y_m^p | Z_m^c)$; and $i=1, \dots, N$; $m=1, \dots, M$, $k=1, \dots, K_m$. With the nice

property that the covariates associated with class prevalences x_i can be ignored and

under the assumption of x_i and z_{im} are independent, we can treating the residuals

from the model (4.1) as new response variables. Details of the above the

marginalization process can be found in Huang (2005) and in section 2.3 of this thesis.

We can classify objects based on the new response variables R_{im}^p to a set of

categorical items. The methods to classify objects are the same as the k-means and

agglomerative hierarchical clustering algorithms in section 4.1, besides the estimation of the covariance matrix $\text{Cov}(\tilde{Y}_i)$ in (4.1), evaluated as $\frac{1}{n-1} \left[\tilde{\mathbf{R}}^T \left(\mathbf{I}_n - \frac{1}{n} \mathbf{1}\mathbf{1}' \right) \tilde{\mathbf{R}} \right]$, where $\tilde{\mathbf{R}}$ is the residual matrix of n objects.

4.3 Parameter estimation by viewing estimated latent class as known variable

Denote the estimated latent class as \hat{S}_i for individual i . We transform (3.3) and (3.4) as the following form:

$$\log \left[\frac{\Pr(\hat{S}_i = j | \mathbf{x}_i)}{\Pr(\hat{S}_i = J | \mathbf{x}_i)} \right] = \beta_0 + \beta_{1j} x_{i1} + \cdots + \beta_{pj} x_{ip}$$

(4.5) and

$$\log \left[\frac{\Pr(Y_{im} = k | \hat{S}_i, \mathbf{z}_i)}{\Pr(Y_{im} = K | \hat{S}_i, \mathbf{z}_i)} \right] = \gamma_{mk0}^* + \gamma_{mk1}^* \hat{S}_{i1} + \cdots + \gamma_{mk(J-1)}^* \hat{S}_{i(J-1)} + \alpha_{1mk} z_{im1} + \cdots + \alpha_{Lmk} z_{imL}$$

$i = 1, \dots, N; \quad m = 1, \dots, M; \quad k = 1, \dots, (K_m - 1);$
 $j = 1, \dots, (J - 1)$

where $\hat{S}_{ij} = \mathbf{I}(\hat{S}_i = j)$. Notice that $\gamma_{mk0}^* = \gamma_{mkJ}$ and $\gamma_{mkj}^* = \gamma_{mkj} - \gamma_{mkJ}$ in (3.4).

Then, we can easily predict the parameters in (3.2) by multinomial logistic regression (4.5) and (4.6).

5. Simulation study

The simulation study aims to examine the performance of the proposed approach.

5.1 Generated data from the RLCA model

Here, three different RLCA (3.2) models were simulated. One was a three-class RLCA with five two-level measured indicators, two covariates associated with conditional probabilities, and two covariates associated with latent prevalences (i.e., $J=3$, $M=5$, $K_1 = \dots = K_5 = 2$, $P=L=2$). Another was a six-class RLCA with five three-level measured three-level measured indicators, two covariates associated with conditional probabilities, and two covariates associated with latent prevalences (i.e., $J=6$, $M=5$, $K_1 = \dots = K_5 = 3$, $P=L=2$). The other was a two-class RLCA with the same indicators and covariate setting as the six-class model (i.e., $J=2$, $M=5$, $K_1 = \dots = K_5 = 3$, $P=L=2$). For each model, the model parameters $\{\beta_{pj}, j=1, \dots, J-1\}$ for each $p \in \{0, 1, \dots, P\}$, $\{\gamma_{jmk}, j=1, \dots, J\}$ for all m, k , and $\{\alpha_{qmk}, m=1, \dots, M; k=1, \dots, (K_m - 1)\}$ for all q , were given. Table 1~6 shows the values of the model parameters for the three model separately. We got the covariates of 3-class model from the subjects who joined the Multidimensional Psychopathological Study on Schizophrenia (MPSS) or the Study on Etiological Factors of Schizophrenia (SEFOS). We got the covariates of 6-class model and 2-class model from the subjects who joined the Multidimensional Psychopathology Group Research Projects (MPGRP), MPSS or SEFOS. In each model, the covariates associated with conditional probabilities include variables of sex and age (year), and the covariates associated with latent prevalences include variables of occupation (with versus without occupation) and dprime, which is the sensitivity index of the Continuous Performance Task (CPT; Rosvold et al., 1956) performance.

We fit each model under several different sample sizes. For the three-class RLCA, the selected sample sizes were 100 and 500, which gave roughly 3 and 16 individuals per parameter of RLCA (3.2), respectively. For the six-class RLCA, the selected sample sizes were 300 and 1000, which gave roughly 3 and 10 individuals per parameter, respectively. For the two-class RLCA, the selected sample sizes were 150 and 700, which gave roughly 3 and 16 individuals per parameter, respectively. The observable measurements Y_i were then generated from each different model structure with 100 replications.

5.2 Simulation results

In each case, the results of simulation study are represented in five tables which include the average parameters estimates (listed in two tables), average conditional probabilities, average latent prevalences and average correlation coefficients for 100 replications, separately. We shall explain these results later. The simulation results for 3-class model with 100 sample sizes are presented from Table 7 to Table 11. The simulation results for 3-class model with 500 sample sizes are presented from Table 12 to Table 16. The simulation results for 6-class model with 300 sample sizes are presented from Table 17 to Table 21. The simulation results for 6-class model with 1000 sample sizes are presented from Table 22 to Table 26. The simulation results for 2-class model with 150 sample sizes are presented from Table 27 to Table 31. The simulation results for 3-class model with 750 sample sizes are presented from Table 32 to Table 36. According to Table 7 ~ Table 36, we can see that these results of the k-means method using correlation coefficients measurement are similar to those of k-means method using covariance measurement. So, we shall discard the results of k-means method using covariance measurement in the following discussion.

First, we discuss the simulation results which are presented from Table 12 to

Table 16 of 3-class model with 500 sample sizes.

Average parameters estimations:

Table 12 and Table 13 under the column “TRUE” include all $\{\beta_{pj}, \gamma_{jmk}, \alpha_{qmk}\}$ in simulated data. All average of $\{\beta_{pj}, \gamma_{jmk}, \alpha_{qmk}\}$ estimates got from the k-means method using correlation coefficients measurement (K_Corr) and covariance measurement (K_Cova) separately and the hierarchical method using covariance measurement (H_Cova) are also shown in Table 12 and Table 13. Why not correlation coefficients measurement for Hierarchical method? At the initial stage, correlation coefficients in two objects are always one. Table 12 and Table 13 can demonstrate that the parameters estimates got from the k-means method are not bad compared to the true parameters. But the parameters estimates got from the hierarchical procedure are poor. The bad performance of the hierarchical procedure is result from there is no provision for a reallocation of objects that may have been “incorrectly” grouped at an early stage. Furthermore, the hierarchical procedure is sensitive to cluster structure. This means that hierarchical procedure have the chance to perform more well only when there is clear cluster structure than when there is no clear cluster structure.

Table 12 and Table 13 also include the standard errors of parameters estimates in doing multinomial regressions, (4.1) and (4.2), and the average sample standard errors of the parameters estimates for 100 replications. The sample standard errors of the estimates for 100 replications include the variation of doing multinomial regression and creating cluster membership. Because we use the multinomial regression to estimate parameters under the assumption of known cluster membership, the standard errors of parameters estimates in doing multinomial regression did not include the variations of creating cluster membership. Therefore, the standard errors of parameters estimates in doing multinomial regressions should be smaller than the

sample standard errors of the estimates for 100 replications. This is demonstrated in Table 12 and Table 13. However this is not demonstrated in Table 7 and Table 8 for the 3-class model with 100 sample sizes which gave very few individuals per parameter. For the sparse data, the estimated standard errors of parameters estimates in doing multinomial regressions are not accurate. Therefore, the standard errors of parameters estimates in doing multinomial regressions are not always smaller than the sample standard errors of the estimates over 100 replications for the 3-class model with 100 sample sizes.

Average conditional probabilities:

Table 14 under the column “TRUE” displays the RLCA conditional probabilities evaluated at the sample means of the incorporated covariates:

$$p_{mkj} = \frac{\exp(\gamma_{mkj} + \bar{z}_m^T \alpha_{mk})}{1 + \sum_{i=1}^{K-1} \exp(\gamma_{mki} + \bar{z}_m^T \alpha_{mk})}, \quad k = 1, \dots, K - 1$$

$$p_{mkj} = \frac{1}{1 + \sum_{i=1}^{K-1} \exp(\gamma_{mki} + \bar{z}_m^T \alpha_{mk})}$$

where $\bar{z}_m = \frac{1}{N} \sum_{i=1}^N z_{im}$.

The average of estimated conditional probabilities over 100 replications with k-means (K_Corr and K_Cova) and hierarchical (H_Cova) methods are also shown in Table 14. The estimated conditional probabilities for k-means and hierarchical methods are:

$$\hat{p}_{mkj} = \frac{\text{the number of individuals in class } j \text{ being at level } k \text{ of } Y_{im}}{\text{the number of individuals in class } j}$$

Overall, the average conditional probabilities for the k-means method are more closed to the true conditional probabilities than the average conditional probabilities for the hierarchical method.

Average latent prevalence:

Table 15 under the column “TRUE” displays the sample average of the RLCA prevalences:

$$\eta_j^* = \frac{1}{N} \sum_{i=1}^N \eta_j(x_i^T \beta)$$

The average of estimated prevalences over 100 replications with k-means (K_Corr and K_Cova) and hierarchical (H_Cova) methods are also shown in Table 15.

The estimated prevalences are:

$$\hat{\eta}_j = \frac{\text{the number of individuals in class } j}{\text{the total number of individuals in study}}$$

Overall, the average latent prevalences for the k-means method are more closed to the true latent prevalences than the average latent prevalences for the hierarchical method.

Average correlation coefficients:

We evaluated the $MCov_k$ of the objects in the same cluster k. Table 16 displays the average of $MCov_k$ over 100 replications in each cluster k. As expected, the k-means method resulted smaller average correlation coefficients than the hierarchical method.

Next, for the 6-class model with 1000 sample sizes, we shall discuss the simulation results which are presented from Table 22 to Table 26. These tables show that the results of whether the k-means procedure or the hierarchical procedure are poor obviously comparing to the 3-class model with 500 sample sizes. Figure 3 shows the dendrogram of the hierarchical procedure for 6-class model with 1000 sample sizes. The dendrogram indicates the cluster structure in 6-class model with 1000 sample sizes. Therefore we guess the objects in 6-class model with 1000 sample sizes should be divided to two clusters, which is demonstrate in the 2-class model. For the 2-class model with 750 sample sizes, the simulation results, which are presented from

Table 32 to Table 36, are much better than the 6-class model with 1000 sample sizes. To go back to the Figure 3, the hierarchical procedure produces inversions (Morgan, Byron and Andrew P.G., 1995). An inversion occurs when an object joins an cluster at smaller covariance than that of a previous consolidation.

When we use maximum likelihood to estimate the parameters in (3.2), the maximum likelihood estimation (MLE) is relative to the number of individuals given in per parameter. For the spare data, which gave less individuals per parameter, the MLE can not be obtained or the MLE is not a good estimation .For the three models, 3-class RLCA with 100 sample sizes, 6-class RLCA with 300 sample sizes and 2-class RLCA with 150 sample sizes, which gave less individuals per parameter, the simulation results are not worse than those that gave more individuals per parameter. It demonstrates that our clustering procedure is irrelative to the number of individuals given per parameter.



6. Discussion

The k-means and hierarchical approaches are alternatives in estimating parameters in RLCA. Overall, in our simulation study, the k-means approach performed well, but the hierarchical approach didn't perform well. At the early stage of hierarchical approach, each cluster contains only very few objects (i.e. at the initial stage, only one object for each cluster). It is not appropriate to use the sample covariance to represent the association between two random variables when the sample size is small. The wrong reallocation of objects at an early stage will result in wrong reallocation of objects at following stage. For the improvement of the proposed hierarchical approach, it may be a good idea to choice alternative measurements of the association between two random variables.



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Table 1: Values of α_0 and α_{Lm} in 3-class model

	item 1	item 2	item 3	item 4	item 5
α_0					
class 1	6.7479	3.4660	0.2348	-7.6250	11.8628
class 2	-1.3035	1.6160	-2.1880	-7.6992	-0.4100
class 3	-4.3977	-1.4305	-4.0995	-20.6690	-1.1043
α_{Lm}					
z_{1m}	0.3944	-0.6396	-0.0581	1.0860	-0.8558
z_{2m}	0.0749	-0.0528	0.0289	0.3551	0.0189

Table 2: Values of β_0 and β_{pj} in 3-class model

	class 1 vs. class 3	class 2 vs. class 3
β_0		
	-1.8685	-0.2319
β_{pj}		
x_{i1}	1.1274	0.3254
x_{i2}	0.5260	0.0134

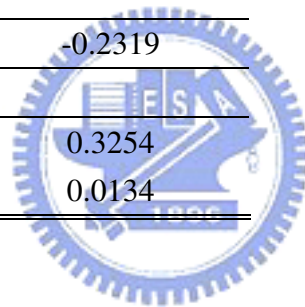


Table 3: Values of α_0 and α_{Lm} in 6-class model

	item 1	item 2	item 3	item 4	item 5
level 1 v.s. level 3					
α_0					
class 1	5.8950	4.6522	1.0752	4.0627	3.4993
class 2	-0.1017	-0.3133	-4.4863	0.8992	0.3045
class 3	1.9543	2.8491	-2.6076	2.2020	1.9947
class 4	0.8544	0.4180	0.4860	4.7660	0.5688
class 5	-5.6842	-9.5250	-3.8248	-3.9258	-1.8959
class 6	-6.0287	-2.2211	-7.4452	-6.8831	0.1988
α_{Lm}					
z_{1m}	-0.8438	-0.5129	-0.4503	-1.8826	-0.7673
z_{2m}	0.0438	-0.0442	0.0069	0.0300	0.0003
level 2 v.s. level 3					
α_0					
class 1	3.3360	2.2059	0.3966	1.4379	1.3682
class 2	0.2934	0.0936	-1.8840	1.4048	0.5097
class 3	1.9570	2.7255	0.7330	3.4921	2.9509
class 4	2.9822	1.0677	0.6211	4.6124	0.8161
class 5	-4.4138	-4.3121	-1.8665	-2.3500	-0.3425
class 6	0.4196	0.5214	-2.6499	-0.2605	0.8891
α_{Lm}					
z_{1m}	-0.2110	-0.2738	-0.7938	-0.7830	-0.5860
z_{2m}	0.0179	-0.0076	0.0185	0.0075	-0.0115

Table 4: Values of β_0 and β_{pj} in 6-class model

	class 1 vs. class 6	class 2 vs. class 6	class 3 vs. class 6	class 4 vs. class 6	class 5 vs. class 6
β_0					
	-1.0866	0.4266	-0.6706	-0.4556	0.1665
β_{pj}					
x_{i1}	0.2739	-1.7432	-0.8519	-0.2596	-0.6237
x_{i2}	0.6879	0.0102	0.5634	0.2530	0.1428

Table 5: Values of α_0 and α_{Lm} in 2-class model

	item 1	item 2	item 3	item 4	item 5
level 1 v.s. level 3					
α_0					
class 1	3.0464	2.0611	-0.4656	2.8398	1.6822
class 2	-3.7435	-3.9859	-4.8386	-2.8929	-1.2178
α_{Lm}					
z_{1m}	0.3906	0.3114	-0.0532	-0.4618	-0.2682
z_{2m}	0.0511	-0.0138	0.0243	0.0416	0.0111
level 2 v.s. level 3					
α_0					
class 1	3.0959	1.6635	0.2100	2.9031	1.5960
class 2	-0.8657	-0.9226	-1.7795	-0.7267	0.3726
α_{Lm}					
z_{1m}	0.5012	0.3124	-0.4417	0.1048	-0.2919
z_{2m}	0.0122	-0.0016	0.0192	0.0072	-0.0132

Table 6: Values of β_0 and β_{pj} in 2-class model

class 1 vs. class 2	
β_0	
-0.9355	
β_{pj}	
x_{i1}	0.4413
x_{i2}	0.4144

Table 7: Average parameters estimations for 100 replication in 3-class model, N=100
(standard error in multinomial regression / sample standard error for 100 replication)

item 1				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-4.398	-3.528 (2.075 / 2.184)	-4.444 (10.114 / 3.814)	-4.358 (4.315 / 3.276)
class 1	11.146	8.019 (17.291 / 4.219)	9.778 (22.483 / 5.572)	7.329 (8.99 / 4.011)
class 2	3.094	3.910 (4.998 / 3.001)	7.503 (24.921 / 6.628)	8.339 (35.991 / 6.713)
sex	0.394	0.594 (1.054 / 1.213)	0.864 (4.422 / 1.785)	0.839 (2.494 / 1.922)
age	0.075	0.049 (0.046 / 0.047)	0.051 (0.233 / 0.061)	0.041 (0.057 / 0.07)

item 2				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.431	-1.291 (4.304 / 2.99)	-2.119 (7.229 / 4.003)	-2.190 (5.213 / 3.778)
class 1	4.897	5.623 (3.913 / 2.962)	7.651 (8.37 / 5.032)	6.154 (5.031 / 3.565)
class 2	3.046	3.259 (4.092 / 3.278)	2.839 (15.678 / 7.492)	0.250 (23.224 / 7.469)
sex	-0.640	-0.564 (0.608 / 0.643)	-0.715 (1.8 / 1.525)	-0.544 (0.796 / 1.019)
age	-0.053	-0.079 (0.04 / 0.036)	-0.088 (0.046 / 0.049)	-0.080 (0.04 / 0.047)

item 3				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-4.100	-5.595 (7.938 / 3.768)	-6.424 (7.862 / 4.322)	-6.369 (6.834 / 4.121)
class 1	4.334	6.440 (7.523 / 4.213)	7.838 (8.554 / 4.67)	6.709 (6.721 / 3.944)
class 2	1.912	3.426 (9.758 / 5.219)	1.985 (11.701 / 6.661)	-2.054 (19.358 / 5.666)
sex	-0.058	0.162 (0.998 / 1.374)	-0.009 (0.752 / 1.066)	0.135 (0.561 / 0.648)
age	0.029	0.001 (0.037 / 0.039)	0.002 (0.039 / 0.043)	0.008 (0.036 / 0.036)

item 4				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-20.669	-6.283 (4.109 / 5.524)	-7.280 (7.075 / 5.89)	-10.886 (8.832 / 13.008)
class 1	13.044	6.194 (5.067 / 3.709)	7.740 (10.621 / 4.95)	8.858 (8.238 / 7.563)
class 2	12.970	4.506 (4.411 / 3.837)	7.454 (22.45 / 6.907)	10.801 (NaN / 11.956)
sex	1.086	0.819 (1.34 / 1.47)	0.766 (1.723 / 1.646)	0.944 (2.687 / 2.484)
age	0.355	0.097 (0.051 / 0.109)	0.105 (0.057 / 0.077)	0.166 (0.092 / 0.26)

Table 7: (Continued)

item 5				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.104	-0.893 (1.277 / 1.401)	-0.825 (1.937 / 1.996)	-0.895 (1.542 / 1.635)
class 1	12.967	3.864 (0.817 / 1.419)	5.404 (9.679 / 4.146)	3.578 (2.704 / 2.075)
class 2	0.694	1.804 (1.819 / 2.111)	1.702 (8.345 / 5.394)	-3.136 (18.43 / 6.21)
sex	-0.856	-0.710 (0.566 / 0.622)	-0.710 (0.576 / 0.609)	-0.683 (0.588 / 0.699)
age	0.019	0.003 (0.035 / 0.037)	0.002 (0.036 / 0.041)	0.003 (0.036 / 0.04)

Table 8: Average parameters estimations for 100 replication in 3-class model, N=100
(standard error in multinomial regression / sample standard error for 100 replication)

Class 1 vs. Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.868	-1.116 (0.617 / 0.825)	-1.192 (0.634 / 1.053)	-0.650 (0.545 / 0.832)
occup	1.127	0.863 (0.549 / 0.583)	0.882 (0.555 / 0.538)	0.871 (0.528 / 0.554)
dprime	0.526	0.324 (0.17 / 0.191)	0.350 (0.174 / 0.208)	0.292 (0.154 / 0.174)

Class 2 vs. Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.232	-0.825 (0.606 / 0.682)	-1.220 (0.71 / 1.091)	-3.108 (2.574 / 4.536)
occup	0.325	0.243 (1.448 / 1.333)	-0.075 (6.056 / 2.408)	-1.609 (19.879 / 4.498)
dprime	0.013	0.097 (0.177 / 0.19)	0.118 (0.205 / 0.235)	-0.191 (1.139 / 2.937)

Table 9: Average conditional Probability for 100 replication in 3- class model, N=100
(sample standard deviance in parentheses)

Class 1				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	1.000	0.971 (0.044)	0.972 (0.031)	0.955 (0.036)
Level 2	0.000	0.029 (0.044)	0.028 (0.031)	0.045 (0.036)
item2				
Level 1	0.802	0.767 (0.100)	0.775 (0.115)	0.702 (0.089)
Level 2	0.198	0.233 (0.100)	0.225 (0.115)	0.298 (0.089)
item3				
Level 1	0.760	0.687 (0.111)	0.702 (0.143)	0.629 (0.098)
Level 2	0.240	0.313 (0.111)	0.298 (0.143)	0.371 (0.098)
item4				
Level 1	0.990	0.928 (0.049)	0.920 (0.058)	0.913 (0.046)
Level 2	0.010	0.072 (0.049)	0.080 (0.058)	0.087 (0.046)
item5				
Level 1	1.000	0.907 (0.064)	0.905 (0.076)	0.868 (0.075)
Level 2	0.000	0.093 (0.064)	0.095 (0.076)	0.132 (0.075)
Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.797	0.782 (0.180)	0.810 (0.285)	0.818 (0.340)
Level 2	0.203	0.218 (0.180)	0.190 (0.285)	0.182 (0.340)
item2				
Level 1	0.389	0.383 (0.193)	0.366 (0.308)	0.243 (0.377)
Level 2	0.611	0.617 (0.193)	0.634 (0.308)	0.757 (0.377)
item3				
Level 1	0.220	0.292 (0.209)	0.268 (0.286)	0.098 (0.266)
Level 2	0.780	0.708 (0.209)	0.732 (0.286)	0.902 (0.266)
item4				
Level 1	0.989	0.746 (0.195)	0.775 (0.314)	0.805 (0.349)
Level 2	0.011	0.254 (0.195)	0.225 (0.314)	0.195 (0.349)
item5				
Level 1	0.442	0.581 (0.196)	0.547 (0.311)	0.246 (0.326)
Level 2	0.558	0.419 (0.196)	0.453 (0.311)	0.754 (0.326)

Table 9: (Continued)

Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.151	0.217 (0.119)	0.191 (0.114)	0.144 (0.079)
Level 2	0.849	0.783 (0.119)	0.809 (0.114)	0.856 (0.079)
item2				
Level 1	0.029	0.061 (0.063)	0.059 (0.056)	0.051 (0.046)
Level 2	0.971	0.940 (0.063)	0.941 (0.056)	0.949 (0.046)
item3				
Level 1	0.040	0.044 (0.057)	0.039 (0.044)	0.033 (0.033)
Level 2	0.960	0.956 (0.057)	0.961 (0.044)	0.967 (0.033)
item4				
Level 1	0.000	0.162 (0.113)	0.141 (0.105)	0.086 (0.071)
Level 2	1.000	0.838 (0.113)	0.859 (0.105)	0.914 (0.071)
item5				
Level 1	0.284	0.273 (0.118)	0.287 (0.130)	0.279 (0.092)
Level 2	0.716	0.727 (0.118)	0.713 (0.130)	0.721 (0.092)

Table 10: Average Latent Prevalences for 100 replication in 3 class model, N=100
(sample standard deviance in parentheses)

	TRUE	K_Corr	K_Cova	H_Cova
class1	0.392	0.416 (0.087)	0.439 (0.131)	0.570 (0.103)
class2	0.286	0.229 (0.055)	0.199 (0.113)	0.078 (0.060)
class3	0.322	0.355 (0.082)	0.362 (0.092)	0.351 (0.082)

Table 11: Average Correlation Coefficients for 100 replication in 3-class model,
N=100 (total number of not NA values in parentheses)

	K_Corr	K_Cova	H_Cova
class1	0.120 (100)	0.199 (100)	0.164 (100)
class2	0.119 (100)	0.303 (100)	0.636 (81)
class3	0.149 (100)	0.183 (100)	0.189 (100)

Table 12: Average parameters estimations for 100 replication in 3-class model, N=500
(standard error in multinomial regression / sample standard error for 100 replication)

item 1				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-4.398	-3.013 (0.713 / 1.222)	-3.138 (0.749 / 1.216)	-3.192 (0.775 / 0.888)
class 1	11.146	5.468 (0.818 / 1.242)	6.166 (3.297 / 2.611)	4.878 (0.393 / 0.555)
class 2	3.094	2.751 (0.435 / 1.926)	3.387 (4.474 / 4.401)	9.473 (14.246 / 3.82)
sex	0.394	0.468 (0.304 / 0.319)	0.421 (0.321 / 0.357)	0.448 (0.336 / 0.348)
age	0.075	0.044 (0.019 / 0.022)	0.045 (0.02 / 0.02)	0.040 (0.021 / 0.023)

item 2				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.431	-0.365 (0.624 / 0.963)	-0.455 (0.633 / 1.425)	-0.621 (0.586 / 0.787)
class 1	4.897	4.336 (0.419 / 0.937)	4.275 (0.439 / 1.996)	3.930 (0.378 / 0.456)
class 2	3.046	1.938 (0.763 / 1.912)	0.851 (2.193 / 4.882)	-2.901 (3.141 / 4.75)
sex	-0.640	-0.504 (0.262 / 0.285)	-0.535 (0.262 / 0.291)	-0.439 (0.247 / 0.292)
age	-0.053	-0.069 (0.017 / 0.018)	-0.068 (0.017 / 0.022)	-0.065 (0.015 / 0.018)

item 3				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-4.100	-3.712 (0.673 / 1.149)	-3.699 (0.668 / 1.373)	-3.924 (0.641 / 1.212)
class 1	4.334	4.120 (0.433 / 1.203)	4.252 (0.429 / 1.527)	3.785 (0.404 / 1.084)
class 2	1.912	1.761 (0.846 / 2.152)	0.829 (2.586 / 4.277)	-3.955 (5.093 / 4.278)
sex	-0.058	0.031 (0.249 / 0.356)	0.021 (0.255 / 0.339)	0.138 (0.236 / 0.315)
age	0.029	0.014 (0.015 / 0.017)	0.013 (0.016 / 0.018)	0.018 (0.014 / 0.015)

item 4				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-20.669	-4.588 (0.76 / 1.494)	-4.937 (0.796 / 2.004)	-5.956 (0.946 / 1.481)
class 1	13.044	4.760 (0.434 / 1.081)	4.980 (0.494 / 1.492)	5.199 (0.42 / 0.709)
class 2	12.970	2.746 (0.438 / 1.933)	2.953 (2.499 / 4.765)	9.129 (10.859 / 4.31)
sex	1.086	0.435 (0.292 / 0.36)	0.407 (0.305 / 0.38)	0.518 (0.344 / 0.398)
age	0.355	0.078 (0.019 / 0.025)	0.082 (0.02 / 0.027)	0.097 (0.023 / 0.03)

Table 12: (Continued)

item 5				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.104	-0.870 (0.546 / 0.839)	-0.911 (0.611 / 1.017)	-0.896 (0.525 / 0.616)
class 1	12.967	3.694 (0.579 / 1.355)	4.260 (1.11 / 2.73)	2.873 (0.253 / 0.413)
class 2	0.694	1.158 (0.367 / 2.226)	1.898 (0.82 / 3.56)	-2.579 (0.774 / 4.566)
sex	-0.856	-0.721 (0.247 / 0.294)	-0.727 (0.246 / 0.274)	-0.599 (0.24 / 0.292)
age	0.019	0.006 (0.015 / 0.016)	0.006 (0.015 / 0.016)	0.009 (0.015 / 0.017)

Table 13: Average parameters estimations for 100 replication in 3-class model, N=500
(standard error in multinomial regression / sample standard error for 100 replication)

Class 1 vs. Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.868	-0.990 (0.256 / 0.567)	-1.001 (0.26 / 0.639)	-0.578 (0.225 / 0.289)
occup	1.127	0.835 (0.242 / 0.285)	0.848 (0.245 / 0.252)	0.804 (0.23 / 0.227)
dprime	0.526	0.296 (0.072 / 0.085)	0.298 (0.074 / 0.09)	0.259 (0.065 / 0.067)

Class 2 vs. Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.232	-1.181 (0.296 / 0.71)	-1.264 (0.314 / 0.902)	-2.121 (0.462 / 0.887)
occup	0.325	0.376 (0.328 / 0.446)	0.346 (0.347 / 0.459)	0.162 (0.814 / 0.867)
dprime	0.013	0.092 (0.088 / 0.123)	0.102 (0.093 / 0.152)	0.030 (0.139 / 0.172)

Table 14 : Average conditional Probability for 100 replication in 3- class model, N=500
(sample standard deviance in parentheses)

Class 1				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	1.000	0.977 (0.015)	0.977 (0.016)	0.957 (0.016)
Level 2	0.000	0.023 (0.015)	0.024 (0.016)	0.043 (0.016)
item2				
Level 1	0.797	0.762 (0.093)	0.745 (0.117)	0.698 (0.038)
Level 2	0.203	0.238 (0.093)	0.255 (0.117)	0.302 (0.038)
item3				
Level 1	0.763	0.694 (0.090)	0.703 (0.113)	0.628 (0.049)
Level 2	0.237	0.306 (0.090)	0.297 (0.113)	0.373 (0.049)
item4				
Level 1	0.992	0.937 (0.025)	0.935 (0.025)	0.921 (0.018)
Level 2	0.008	0.063 (0.025)	0.065 (0.025)	0.079 (0.018)
item5				
Level 1	1.000	0.910 (0.053)	0.903 (0.066)	0.866 (0.039)
Level 2	0.000	0.090 (0.053)	0.097 (0.066)	0.135 (0.039)
Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.803	0.737 (0.253)	0.724 (0.310)	0.962 (0.093)
Level 2	0.197	0.263 (0.253)	0.276 (0.310)	0.038 (0.093)
item2				
Level 1	0.381	0.325 (0.218)	0.330 (0.272)	0.100 (0.183)
Level 2	0.619	0.675 (0.218)	0.670 (0.272)	0.900 (0.183)
item3				
Level 1	0.222	0.264 (0.223)	0.248 (0.248)	0.061 (0.151)
Level 2	0.778	0.736 (0.223)	0.752 (0.248)	0.940 (0.151)
item4				
Level 1	0.991	0.667 (0.263)	0.647 (0.335)	0.929 (0.158)
Level 2	0.009	0.333 (0.263)	0.353 (0.335)	0.071 (0.158)
item5				
Level 1	0.444	0.554 (0.256)	0.567 (0.271)	0.207 (0.213)
Level 2	0.556	0.446 (0.256)	0.433 (0.271)	0.793 (0.213)

Table 14 : (Continued)

Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.156	0.245 (0.111)	0.230 (0.111)	0.178 (0.046)
Level 2	0.844	0.755 (0.111)	0.770 (0.111)	0.822 (0.046)
item2				
Level 1	0.029	0.071 (0.049)	0.073 (0.050)	0.061 (0.023)
Level 2	0.972	0.929 (0.049)	0.927 (0.050)	0.939 (0.023)
item3				
Level 1	0.041	0.049 (0.041)	0.050 (0.041)	0.045 (0.020)
Level 2	0.959	0.951 (0.041)	0.950 (0.041)	0.955 (0.020)
item4				
Level 1	0.000	0.192 (0.107)	0.177 (0.104)	0.106 (0.037)
Level 2	1.000	0.808 (0.107)	0.823 (0.104)	0.894 (0.037)
item5				
Level 1	0.285	0.289 (0.089)	0.282 (0.083)	0.293 (0.036)
Level 2	0.715	0.711 (0.089)	0.718 (0.083)	0.707 (0.036)

Table 15: Average Latent Prevalences for 100 replication in 3 class model , N=500
(sample standard deviance in parentheses)

	TRUE	K_Corr	K_Cova	H_Cova
class1	0.393	0.449 (0.094)	0.453 (0.114)	0.570 (0.047)
class2	0.284	0.176 (0.069)	0.177 (0.093)	0.062 (0.035)
class3	0.323	0.374 (0.081)	0.370 (0.084)	0.368 (0.036)

Table 16: Average Correlation Coefficients for 100 replication in 3-class model,
N=500 (total number of not NA values in parentheses)

	K_Corr	K_Cova	H_Cova
class1	0.061 (100)	0.086 (100)	0.109 (100)
class2	0.041 (100)	0.091 (100)	0.386 (100)
class3	0.095 (100)	0.088 (100)	0.079 (100)

Table 17: Average parameters estimations for 100 replication in 6-class model, N=300
(standard error in multinomial regression / sample standard error for 100 replication)

item 1				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-6.029	1.737 (1.172 / 7.757)	1.844 (1.25 / 9.46)	-18.193 (0.587 / 8.816)
class 1	11.924	7.056 (1.015 / 10.156)	11.540 (0.512 / 13.163)	20.592 (0.473 / 9.543)
class 2	5.927	-0.524 (0.734 / 10.784)	6.671 (0.988 / 48.88)	-4.585 (NaN / 9.622)
class 3	7.983	2.137 (0.904 / 8.185)	3.102 (0.771 / 11.635)	-5.584 (NaN / 10.72)
class 4	6.883	4.156 (0.868 / 15.471)	3.248 (NaN / 12.919)	-6.038 (NaN / 8.985)
class 5	0.345	-6.390 (0.844 / 8.454)	-11.281 (0.813 / 11.652)	5.012 (0.215 / 7.417)
sex	-0.844	-0.062 (0.493 / 0.679)	0.021 (0.556 / 1.54)	0.007 (0.45 / 0.605)
age	0.044	0.025 (0.031 / 0.033)	0.031 (0.035 / 0.035)	0.012 (0.027 / 0.027)
level 2 v.s. level 3				
intercept	0.420	1.944 (1.041 / 6.915)	3.062 (1.169 / 7.594)	-6.591 (0.833 / 19.015)
class 1	2.916	3.160 (0.846 / 11.05)	4.487 (NaN / 12.456)	7.261 (0.583 / 18.744)
class 2	-0.126	-0.824 (0.645 / 9.806)	4.154 (0.678 / 48.578)	-8.932 (0.19 / 22.377)
class 3	1.537	1.734 (0.837 / 8.172)	1.403 (0.666 / 11.047)	-14.967 (0.076 / 21.241)
class 4	2.563	5.146 (0.844 / 14.696)	4.037 (0.768 / 12.696)	-13.289 (0.097 / 21.671)
class 5	-4.833	-4.491 (0.626 / 7.344)	-8.430 (0.794 / 10.288)	-3.427 (0.484 / 20.184)
sex	-0.211	0.195 (0.454 / 0.658)	0.321 (0.523 / 1.471)	0.272 (0.46 / 0.6)
age	0.018	0.013 (0.029 / 0.033)	0.018 (0.033 / 0.037)	0.007 (0.028 / 0.032)
item 2				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-2.221	-2.164 (1.175 / 12.823)	-6.534 (1.081 / 14.855)	-17.409 (0.411 / 5.298)
class 1	6.873	12.051 (NaN / 39.33)	14.547 (0.786 / 15.798)	19.805 (0.382 / 6.667)
class 2	1.908	1.197 (NaN / 16.979)	4.786 (0.634 / 19.049)	-5.189 (NaN / 8.769)
class 3	5.070	3.983 (0.885 / 14.828)	9.973 (0.639 / 15.834)	-4.981 (NaN / 6.646)
class 4	2.639	2.165 (0.762 / 18.335)	6.946 (0.598 / 18.129)	-4.645 (NaN / 6.238)
class 5	-7.304	-5.225 (0.619 / 14.321)	-6.648 (0.353 / 14.421)	-0.197 (NaN / 3.487)
sex	-0.513	-0.056 (0.476 / 0.551)	-0.125 (0.481 / 0.558)	-0.077 (0.358 / 0.363)
age	-0.044	-0.042 (0.03 / 0.034)	-0.043 (0.03 / 0.031)	-0.032 (0.022 / 0.023)

Table 17: (Continued)

level 2 v.s. level 3				
intercept	0.521	-0.177 (0.9 / 4.198)	0.688 (0.859 / 9.632)	-14.797 (0.567 / 12.287)
class 1	1.685	-0.313 (0.649 / 20.431)	2.958 (NaN / 10.485)	15.760 (0.457 / 12.282)
class 2	-0.428	-2.130 (0.481 / 8.227)	-1.686 (0.457 / 13.856)	-6.299 (0.118 / 16.771)
class 3	2.204	2.565 (0.641 / 5.587)	3.966 (0.5 / 12.154)	-7.688 (0.052 / 15.491)
class 4	0.546	3.276 (0.605 / 16.832)	0.820 (0.511 / 13.506)	-7.565 (0.065 / 14.053)
class 5	-4.833	-3.179 (0.586 / 6.489)	-5.827 (0.507 / 11.646)	2.191 (0.214 / 12.536)
sex	-0.274	-0.016 (0.365 / 0.444)	-0.056 (0.372 / 0.448)	-0.001 (0.323 / 0.351)
age	-0.008	-0.007 (0.023 / 0.025)	-0.008 (0.023 / 0.026)	-0.007 (0.02 / 0.021)

item 3				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-7.445	-7.876 (1.11 / 14.908)	-13.160 (0.847 / 14.946)	-17.015 (0.425 / 2.719)
class 1	8.520	9.461 (0.974 / 14.59)	14.795 (0.548 / 14.646)	16.709 (0.387 / 3.143)
class 2	2.959	3.506 (0.646 / 16.817)	5.581 (0.344 / 18.171)	-4.871 (NaN / 7.409)
class 3	4.838	4.650 (NaN / 15.864)	8.864 (0.461 / 15.489)	-3.483 (NaN / 7.756)
class 4	7.931	9.387 (0.905 / 17.781)	9.811 (0.429 / 16.866)	-3.928 (NaN / 6.145)
class 5	3.620	-1.814 (0.583 / 16.499)	1.398 (0.336 / 13.851)	-0.858 (0.098 / 6.024)
sex	-0.450	0.024 (0.49 / 0.669)	-0.047 (0.442 / 0.553)	-0.026 (0.348 / 0.465)
age	0.007	0.001 (0.03 / 0.032)	0.003 (0.028 / 0.034)	-0.001 (0.022 / 0.02)
level 2 v.s. level 3				
intercept	-2.650	-3.047 (0.752 / 6.555)	-5.241 (0.793 / 12.547)	-14.044 (0.507 / 7.571)
class 1	3.046	3.135 (0.511 / 6.924)	5.938 (0.638 / 12.714)	14.007 (0.39 / 7.439)
class 2	0.766	0.205 (0.448 / 9.96)	2.768 (0.533 / 10.725)	-4.661 (0.116 / 13.138)
class 3	3.383	4.565 (NaN / 8.395)	6.147 (0.542 / 13.399)	2.191 (0.153 / 18.248)
class 4	3.271	1.230 (0.415 / 7.296)	4.325 (0.554 / 13.633)	-0.970 (0.131 / 15.97)
class 5	0.783	0.804 (0.575 / 6.619)	2.992 (0.616 / 12.729)	4.074 (0.295 / 11.242)
sex	-0.794	-0.495 (0.313 / 0.332)	-0.547 (0.319 / 0.378)	-0.443 (0.286 / 0.304)
age	0.019	0.012 (0.02 / 0.018)	0.014 (0.02 / 0.02)	0.008 (0.018 / 0.017)

Table 17: (Continued)

item 4				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-6.883	-0.819 (1.134 / 6.3)	-0.232 (1.11 / 8.601)	-17.852 (0.6 / 7.573)
class 1	10.946	7.891 (1.221 / 8.307)	14.228 (NaN / 23.778)	20.849 (0.478 / 9.591)
class 2	7.782	2.412 (0.745 / 7.801)	5.210 (0.597 / 12.414)	-3.024 (NaN / 12.86)
class 3	9.085	3.977 (0.741 / 7.998)	5.279 (0.78 / 11.649)	-3.640 (NaN / 15.571)
class 4	11.649	5.875 (0.721 / 8.92)	6.106 (0.577 / 11.194)	-6.823 (NaN / 8.339)
class 5	2.957	-3.419 (0.756 / 7.367)	-6.178 (0.687 / 9.665)	3.662 (0.163 / 7.743)
sex	-1.883	-0.876 (0.476 / 0.645)	-1.053 (0.516 / 0.749)	-0.647 (0.412 / 0.506)
age	0.030	0.017 (0.029 / 0.031)	0.020 (0.031 / 0.031)	0.003 (0.025 / 0.026)
level 2 v.s. level 3				
intercept	-0.261	-0.793 (1.045 / 5.424)	0.123 (1.025 / 6.465)	-14.601 (0.734 / 14.925)
class 1	1.698	5.215 (1.108 / 7.847)	4.434 (NaN / 21.747)	16.064 (0.52 / 13.532)
class 2	1.665	2.730 (0.683 / 7.125)	3.804 (0.547 / 10.582)	-2.888 (0.165 / 25.005)
class 3	3.753	5.776 (0.715 / 7.502)	5.606 (0.747 / 10.134)	-6.064 (0.126 / 15.897)
class 4	4.873	5.226 (0.708 / 8.282)	5.971 (0.546 / 10.546)	-7.845 (0.081 / 14.333)
class 5	-2.089	-1.519 (0.627 / 6.265)	-4.647 (0.606 / 10.304)	4.238 (0.361 / 17.071)
sex	-0.783	-0.132 (0.434 / 0.557)	-0.289 (0.479 / 0.632)	-0.107 (0.418 / 0.503)
age	0.008	-0.002 (0.027 / 0.029)	0.001 (0.029 / 0.027)	-0.009 (0.026 / 0.026)
item 5				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	0.199	0.580 (0.973 / 5.077)	2.523 (0.939 / 10.278)	-8.121 (0.699 / 11.996)
class 1	3.300	2.947 (0.815 / 4.854)	4.531 (0.908 / 12.099)	10.991 (0.519 / 12.657)
class 2	0.106	0.541 (0.662 / 8.4)	0.139 (0.594 / 14.111)	-7.807 (NaN / 14.952)
class 3	1.796	1.273 (0.712 / 6.059)	1.275 (0.616 / 11.492)	-5.210 (NaN / 17.885)
class 4	0.370	0.195 (0.707 / 6.955)	-0.008 (0.515 / 12.751)	-4.438 (NaN / 16.454)
class 5	-2.095	-2.681 (0.727 / 5.521)	-5.033 (0.647 / 11.443)	2.450 (0.514 / 14.639)
sex	-0.767	-0.533 (0.38 / 0.372)	-0.570 (0.385 / 0.427)	-0.452 (0.346 / 0.384)
age	0.000	0.002 (0.024 / 0.027)	0.003 (0.024 / 0.026)	-0.005 (0.021 / 0.022)

Table 17: (Continued)

	level 2 v.s. level 3			
intercept	0.889	0.695 (0.923 / 4.54)	3.891 (0.917 / 11.262)	-0.849 (0.734 / 17.149)
class 1	0.479	1.045 (0.878 / 5.248)	-0.487 (0.928 / 10.74)	2.432 (0.496 / 16.137)
class 2	-0.379	-0.338 (0.649 / 6.642)	-1.726 (0.58 / 13.667)	-4.965 (0.146 / 26.834)
class 3	2.062	2.744 (0.678 / 5.879)	0.590 (0.608 / 11.761)	-5.547 (0.2 / 24.622)
class 4	-0.073	0.008 (0.715 / 6.208)	-0.948 (0.582 / 13.179)	-4.960 (0.173 / 26.296)
class 5	-1.232	-1.516 (0.586 / 5.423)	-4.696 (0.593 / 11.888)	-1.360 (0.614 / 17.428)
sex	-0.586	-0.458 (0.364 / 0.437)	-0.494 (0.371 / 0.473)	-0.384 (0.361 / 0.433)
age	-0.011	-0.011 (0.023 / 0.023)	-0.012 (0.023 / 0.024)	-0.014 (0.022 / 0.022)



Table 18: Average parameters estimations for 100 replication in 6-class model, N=300
(standard error in multinomial regression / sample standard error for 100 replication)

Class 1 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.087	-0.246 (0.496 / 1.158)	-0.484 (0.523 / 1.157)	3.735 (1.868 / 2.251)
occup	0.274	0.423 (0.502 / 0.483)	0.666 (1.754 / 1.231)	2.415 (16.987 / 5.856)
dprime	0.688	0.308 (0.147 / 0.266)	0.407 (0.157 / 0.233)	0.341 (0.344 / 0.384)

Class 2 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	0.427	0.116 (0.506 / 1.404)	0.023 (0.518 / 1.505)	-0.110 (4.87 / 2.914)
occup	-1.743	-0.350 (2.121 / 1.241)	-0.081 (2.769 / 1.725)	-5.095 (53.653 / 28.913)
dprime	0.010	-0.021 (0.161 / 0.312)	0.064 (0.168 / 0.331)	0.066 (0.527 / 0.594)

Class 3 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.671	0.193 (0.475 / 1.112)	0.180 (0.501 / 1.492)	-0.511 (8.394 / 3.529)
occup	-0.852	-0.175 (0.554 / 0.528)	-0.007 (1.799 / 1.303)	-1.257 (NaN / 11.866)
dprime	0.563	0.118 (0.147 / 0.247)	0.190 (0.157 / 0.277)	0.049 (0.52 / 0.561)

Class 4 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.456	-0.182 (0.525 / 1.49)	-0.066 (0.524 / 1.271)	-4.779 (15.206 / 25.384)
occup	-0.260	0.083 (0.578 / 0.642)	0.186 (1.838 / 1.209)	-0.282 (54.282 / 12.484)
dprime	0.253	0.099 (0.161 / 0.331)	0.128 (0.168 / 0.27)	-0.801 (1.33 / 11.908)

Class 5 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	0.167	0.970 (0.421 / 1.239)	0.855 (0.43 / 1.098)	2.301 (1.902 / 2.069)
occup	-0.624	-0.126 (0.525 / 0.498)	0.062 (1.781 / 1.25)	2.165 (17.061 / 5.927)
dprime	0.143	-0.147 (0.138 / 0.259)	-0.092 (0.146 / 0.221)	0.114 (0.358 / 0.38)

Table 19: Average conditional Probability for 100 replication in 6-class model, N=300
(sample standard deviance in parentheses)

Class 1				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.955	0.874 (0.079)	0.914 (0.077)	0.563 (0.118)
Level 2	0.044	0.114 (0.072)	0.079 (0.068)	0.345 (0.094)
Level 3	0.001	0.012 (0.017)	0.007 (0.014)	0.093 (0.036)
item2				
Level 1	0.727	0.703 (0.160)	0.758 (0.136)	0.345 (0.159)
Level 2	0.235	0.241 (0.134)	0.200 (0.111)	0.438 (0.106)
Level 3	0.038	0.056 (0.048)	0.042 (0.043)	0.218 (0.065)
item3				
Level 1	0.508	0.440 (0.236)	0.531 (0.155)	0.237 (0.111)
Level 2	0.318	0.369 (0.169)	0.327 (0.132)	0.373 (0.052)
Level 3	0.174	0.191 (0.103)	0.142 (0.072)	0.390 (0.084)
item4				
Level 1	0.929	0.855 (0.083)	0.900 (0.083)	0.523 (0.127)
Level 2	0.056	0.127 (0.080)	0.087 (0.076)	0.360 (0.097)
Level 3	0.015	0.018 (0.018)	0.013 (0.019)	0.117 (0.041)
item5				
Level 1	0.883	0.820 (0.083)	0.861 (0.088)	0.525 (0.122)
Level 2	0.078	0.124 (0.069)	0.095 (0.070)	0.309 (0.083)
Level 3	0.039	0.056 (0.037)	0.044 (0.036)	0.166 (0.049)
Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.439	0.385 (0.244)	0.457 (0.273)	0.044 (0.172)
Level 2	0.384	0.424 (0.203)	0.378 (0.226)	0.214 (0.363)
Level 3	0.177	0.191 (0.217)	0.165 (0.260)	0.742 (0.387)
item2				
Level 1	0.071	0.170 (0.253)	0.211 (0.282)	0.028 (0.148)
Level 2	0.397	0.372 (0.251)	0.363 (0.245)	0.096 (0.244)
Level 3	0.532	0.457 (0.313)	0.427 (0.338)	0.876 (0.284)

Table 19 : (Continued)

item3				
Level 1	0.009	0.134 (0.237)	0.104 (0.196)	0.024 (0.136)
Level 2	0.156	0.266 (0.184)	0.277 (0.206)	0.096 (0.238)
Level 3	0.834	0.600 (0.250)	0.619 (0.266)	0.880 (0.277)
item4				
Level 1	0.361	0.360 (0.225)	0.409 (0.252)	0.070 (0.193)
Level 2	0.498	0.443 (0.206)	0.443 (0.232)	0.183 (0.331)
Level 3	0.141	0.197 (0.170)	0.148 (0.200)	0.747 (0.373)
item5				
Level 1	0.335	0.400 (0.243)	0.411 (0.280)	0.057 (0.167)
Level 2	0.306	0.311 (0.181)	0.302 (0.196)	0.354 (0.444)
Level 3	0.359	0.289 (0.240)	0.287 (0.276)	0.589 (0.460)
Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.609	0.500 (0.186)	0.505 (0.192)	0.031 (0.154)
Level 2	0.360	0.428 (0.178)	0.430 (0.181)	0.114 (0.291)
Level 3	0.031	0.072 (0.159)	0.065 (0.145)	0.855 (0.338)
item2				
Level 1	0.217	0.182 (0.172)	0.194 (0.175)	0.015 (0.104)
Level 2	0.714	0.668 (0.175)	0.653 (0.206)	0.068 (0.225)
Level 3	0.069	0.150 (0.120)	0.153 (0.176)	0.917 (0.256)
item3				
Level 1	0.020	0.086 (0.151)	0.088 (0.140)	0.017 (0.107)
Level 2	0.705	0.562 (0.210)	0.573 (0.206)	0.235 (0.376)
Level 3	0.275	0.351 (0.200)	0.340 (0.202)	0.748 (0.391)
item4				
Level 1	0.242	0.330 (0.176)	0.346 (0.210)	0.061 (0.192)
Level 2	0.732	0.602 (0.185)	0.589 (0.217)	0.101 (0.242)
Level 3	0.026	0.068 (0.122)	0.065 (0.123)	0.837 (0.326)
item5				
Level 1	0.319	0.351 (0.163)	0.368 (0.183)	0.102 (0.256)
Level 2	0.618	0.538 (0.185)	0.503 (0.194)	0.311 (0.404)
Level 3	0.063	0.111 (0.079)	0.129 (0.138)	0.587 (0.444)

Table 19 : (Continued)

Class 4				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.164	0.476 (0.255)	0.408 (0.271)	0.024 (0.129)
Level 2	0.811	0.451 (0.235)	0.493 (0.273)	0.122 (0.291)
Level 3	0.025	0.073 (0.129)	0.099 (0.230)	0.854 (0.329)
item2				
Level 1	0.085	0.251 (0.241)	0.202 (0.227)	0.010 (0.064)
Level 2	0.607	0.525 (0.234)	0.520 (0.251)	0.058 (0.189)
Level 3	0.307	0.224 (0.220)	0.278 (0.286)	0.932 (0.220)
item3				
Level 1	0.330	0.341 (0.362)	0.204 (0.252)	0.018 (0.109)
Level 2	0.466	0.322 (0.236)	0.397 (0.235)	0.177 (0.335)
Level 3	0.203	0.337 (0.262)	0.399 (0.277)	0.804 (0.351)
item4				
Level 1	0.581	0.522 (0.252)	0.440 (0.246)	0.028 (0.150)
Level 2	0.414	0.395 (0.216)	0.419 (0.219)	0.069 (0.210)
Level 3	0.005	0.083 (0.112)	0.141 (0.256)	0.903 (0.265)
item5				
Level 1	0.360	0.460 (0.250)	0.404 (0.241)	0.123 (0.282)
Level 2	0.343	0.316 (0.198)	0.341 (0.217)	0.322 (0.422)
Level 3	0.296	0.224 (0.230)	0.255 (0.264)	0.555 (0.458)
Class 5				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.009	0.056 (0.052)	0.025 (0.029)	0.012 (0.024)
Level 2	0.019	0.176 (0.086)	0.098 (0.070)	0.046 (0.062)
Level 3	0.972	0.767 (0.108)	0.877 (0.086)	0.943 (0.078)
item2				
Level 1	0.000	0.012 (0.014)	0.003 (0.008)	0.001 (0.006)
Level 2	0.009	0.119 (0.084)	0.075 (0.057)	0.019 (0.042)
Level 3	0.991	0.869 (0.087)	0.922 (0.060)	0.980 (0.044)
item3				
Level 1	0.018	0.015 (0.019)	0.014 (0.019)	0.006 (0.019)
Level 2	0.157	0.135 (0.065)	0.141 (0.065)	0.052 (0.068)
Level 3	0.825	0.850 (0.069)	0.845 (0.070)	0.941 (0.073)

Table 19 : (Continued)

item4				
Level 1	0.019	0.061 (0.052)	0.036 (0.036)	0.010 (0.025)
Level 2	0.075	0.146 (0.077)	0.095 (0.059)	0.038 (0.055)
Level 3	0.907	0.792 (0.110)	0.869 (0.076)	0.952 (0.073)
item5				
Level 1	0.070	0.109 (0.063)	0.104 (0.053)	0.064 (0.060)
Level 2	0.248	0.255 (0.093)	0.235 (0.098)	0.171 (0.136)
Level 3	0.682	0.636 (0.115)	0.661 (0.123)	0.765 (0.163)
Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.002	0.317 (0.297)	0.282 (0.292)	0.006 (0.038)
Level 2	0.709	0.450 (0.249)	0.494 (0.288)	0.319 (0.420)
Level 3	0.289	0.234 (0.248)	0.224 (0.270)	0.675 (0.419)
item2				
Level 1	0.009	0.181 (0.304)	0.121 (0.246)	0.002 (0.020)
Level 2	0.529	0.412 (0.247)	0.438 (0.257)	0.098 (0.251)
Level 3	0.462	0.407 (0.279)	0.441 (0.286)	0.900 (0.254)
item3				
Level 1	0.001	0.138 (0.286)	0.065 (0.207)	0.000 0.000
Level 2	0.080	0.230 (0.201)	0.182 (0.188)	0.056 (0.207)
Level 3	0.919	0.632 (0.302)	0.753 (0.259)	0.944 (0.207)
item4				
Level 1	0.001	0.297 (0.284)	0.257 (0.285)	0.016 (0.075)
Level 2	0.400	0.342 (0.215)	0.341 (0.238)	0.110 (0.273)
Level 3	0.600	0.361 (0.298)	0.401 (0.331)	0.874 (0.283)
item5				
Level 1	0.272	0.340 (0.252)	0.330 (0.238)	0.110 (0.251)
Level 2	0.404	0.362 (0.199)	0.396 (0.233)	0.392 (0.438)
Level 3	0.324	0.299 (0.238)	0.274 (0.245)	0.498 (0.454)

Table 20: Average Latent Prevalences for 100 replication in 6-class model, N=300
(sample standard deviance in parentheses)

	TRUE	K_Corr	K_Cova	H_Cova
class1	0.264	0.233 (0.078)	0.217 (0.086)	0.795 (0.192)
class2	0.138	0.122 (0.047)	0.132 (0.079)	0.029 (0.111)
class3	0.195	0.183 (0.074)	0.199 (0.097)	0.026 (0.089)
class4	0.119	0.132 (0.055)	0.156 (0.101)	0.024 (0.096)
class5	0.159	0.205 (0.074)	0.183 (0.051)	0.109 (0.063)
class6	0.125	0.125 (0.055)	0.113 (0.060)	0.017 (0.012)

Table 21: Average Correlation Coefficients for 100 replication in 6-class model,
N=300. (total number of not NA values in parentheses)

	K_Corr	K_Cova	H_Cova
class1	0.159 (100)	0.168 (100)	0.211 (100)
class2	0.191 (100)	0.198 (100)	0.848 (61)
class3	0.153 (100)	0.149 (100)	0.858 (61)
class4	0.183 (100)	0.184 (100)	0.913 (43)
class5	0.256 (100)	0.226 (100)	0.406 (100)
class6	0.214 (100)	0.216 (100)	0.863 (71)

Table 22: Average parameters estimations for 100 replication in 6-class model, N=1000
(standard error in multinomial regression / sample standard error for 100 replication)

item 1				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-6.029	-1.350 (0.712 / 3.492)	-0.415 (0.753 / 4.672)	-11.734 (0.533 / 6.209)
class 1	11.924	5.659 (0.828 / 4.682)	9.291 (0.603 / 7.259)	14.051 (0.414 / 6.747)
class 2	5.927	1.459 (0.547 / 5.186)	1.597 (0.501 / 7.569)	-2.041 (NaN / 9.953)
class 3	7.983	3.223 (0.571 / 3.695)	2.845 (0.525 / 11.486)	-5.018 (NaN / 7.408)
class 4	6.883	2.915 (0.62 / 6.534)	2.536 (0.511 / 8.388)	-6.204 (NaN / 6.04)
class 5	0.345	-3.843 (0.599 / 4.203)	-6.763 (NaN / 7.542)	4.023 (0.531 / 7.237)
sex	-0.844	0.041 (0.277 / 0.401)	-0.017 (0.297 / 0.476)	0.079 (0.231 / 0.25)
age	0.044	0.035 (0.018 / 0.02)	0.041 (0.019 / 0.023)	0.012 (0.015 / 0.016)
level 2 v.s. level 3				
intercept	0.420	-0.098 (0.637 / 3.172)	0.863 (0.666 / 4.14)	-2.396 (0.715 / 10.572)
class 1	2.916	2.536 (0.833 / 4.525)	5.074 (0.476 / 6.726)	3.354 (0.534 / 10.36)
class 2	-0.126	0.594 (0.494 / 4.979)	0.332 (0.409 / 6.99)	-6.030 (0.373 / 15.412)
class 3	1.537	2.386 (0.533 / 3.561)	1.924 (0.484 / 11.036)	-11.606 (0.216 / 10.8)
class 4	2.563	3.101 (0.591 / 6.67)	2.093 (0.471 / 7.575)	-11.039 (0.255 / 14.127)
class 5	-4.833	-3.281 (0.426 / 4.406)	-4.573 (0.5 / 4.285)	-1.356 (0.658 / 10.929)
sex	-0.211	0.242 (0.254 / 0.392)	0.202 (0.276 / 0.43)	0.299 (0.237 / 0.273)
age	0.018	0.018 (0.016 / 0.018)	0.025 (0.018 / 0.021)	0.006 (0.015 / 0.015)
item 2				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-2.221	-2.008 (0.771 / 5.309)	-2.634 (0.672 / 5.215)	-16.332 (0.259 / 4.974)
class 1	6.873	7.532 (0.687 / 7.625)	8.602 (0.543 / 7.344)	18.353 (0.253 / 5.901)
class 2	1.908	0.827 (0.669 / 7.332)	0.766 (0.498 / 8.021)	-5.787 (NaN / 4.308)
class 3	5.070	3.128 (0.646 / 5.314)	3.895 (0.496 / 6.749)	-2.914 (NaN / 7.342)
class 4	2.639	2.352 (0.675 / 6.556)	2.055 (0.441 / 8.23)	-3.939 (NaN / 5.43)
class 5	-7.304	-3.709 (1.036 / 6.232)	-7.158 (0.434 / 5.833)	1.092 (0.076 / 4.923)
sex	-0.513	0.129 (0.249 / 0.243)	0.096 (0.25 / 0.271)	0.046 (0.192 / 0.194)
age	-0.044	-0.038 (0.016 / 0.017)	-0.042 (0.016 / 0.016)	-0.032 (0.012 / 0.013)

Table 22: (Continued)

level 2 v.s. level 3				
intercept	0.521	-0.665 (0.484 / 4.612)	-0.121 (0.495 / 3.301)	-14.460 (0.487 / 7.302)
class 1	1.685	2.125 (0.429 / 4.665)	3.346 (0.457 / 5.542)	15.551 (0.423 / 7.481)
class 2	-0.428	0.809 (0.345 / 6.072)	-0.214 (0.345 / 6.264)	-7.503 (0.058 / 9.435)
class 3	2.204	2.497 (0.341 / 4.588)	1.777 (0.346 / 3.815)	-4.110 (0.119 / 12.241)
class 4	0.546	1.368 (0.363 / 5.041)	0.925 (0.354 / 5.278)	-5.993 (0.093 / 10.39)
class 5	-4.833	-2.001 (0.496 / 5.127)	-3.038 (0.553 / 3.873)	7.875 (0.528 / 8.345)
sex	-0.274	0.103 (0.191 / 0.203)	0.070 (0.193 / 0.213)	0.094 (0.175 / 0.2)
age	-0.008	-0.006 (0.012 / 0.013)	-0.006 (0.012 / 0.015)	-0.006 (0.011 / 0.012)

item 3				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-7.445	-4.267 (0.683 / 7.031)	-6.275 (0.623 / 5.74)	-15.196 (0.375 / 5.66)
class 1	8.520	6.414 (0.61 / 7.781)	7.516 (0.465 / 5.632)	15.660 (NaN / 7.94)
class 2	2.959	-0.495 (0.64 / 12.376)	2.130 (0.454 / 9.158)	-5.156 (NaN / 7.839)
class 3	4.838	2.073 (0.565 / 7.372)	4.334 (0.504 / 6.554)	-1.426 (NaN / 9.861)
class 4	7.931	4.478 (0.497 / 9.631)	3.845 (0.598 / 7.312)	-0.868 (NaN / 11.42)
class 5	3.620	-0.937 (0.814 / 7.222)	0.705 (0.682 / 6.616)	3.167 (0.284 / 8.308)
sex	-0.450	-0.045 (0.245 / 0.287)	-0.048 (0.23 / 0.281)	-0.072 (0.186 / 0.185)
age	0.007	0.003 (0.015 / 0.018)	0.001 (0.015 / 0.016)	-0.003 (0.012 / 0.013)
level 2 v.s. level 3				
intercept	-2.650	-1.705 (0.434 / 4.563)	-1.573 (0.476 / 2.675)	-13.615 (0.381 / 9.351)
class 1	3.046	2.206 (0.447 / 5.462)	2.291 (0.391 / 2.653)	13.022 (NaN / 9.366)
class 2	0.766	-1.087 (NaN / 8.133)	-0.027 (0.425 / 3.721)	-3.081 (0.104 / 14.995)
class 3	3.383	2.371 (0.295 / 4.12)	2.161 (0.359 / 3.131)	2.840 (0.261 / 15.501)
class 4	3.271	1.246 (0.301 / 5.021)	1.314 (0.508 / 3.693)	3.606 (0.265 / 16.471)
class 5	0.783	-0.554 (0.33 / 4.42)	-0.435 (0.388 / 2.679)	10.427 (0.382 / 10.083)
sex	-0.794	-0.478 (0.168 / 0.167)	-0.496 (0.168 / 0.182)	-0.427 (0.153 / 0.153)
age	0.019	0.012 (0.011 / 0.01)	0.011 (0.011 / 0.011)	0.007 (0.01 / 0.009)

Table 22: (Continued)

item 4				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-6.883	-1.018 (0.664 / 2.902)	-0.351 (0.677 / 4.27)	-14.773 (0.403 / 6.513)
class 1	10.946	4.980 (0.615 / 3.237)	6.890 (0.771 / 5.578)	17.302 (0.313 / 7.112)
class 2	7.782	1.768 (0.51 / 4.709)	1.865 (0.502 / 6.252)	-2.597 (NaN / 8.553)
class 3	9.085	2.698 (0.592 / 3.736)	1.710 (0.518 / 5.186)	-5.185 (NaN / 7.701)
class 4	11.649	2.769 (0.609 / 4.364)	2.508 (0.572 / 6.271)	-4.791 (NaN / 9.252)
class 5	2.957	-2.581 (0.545 / 3.182)	-4.056 (0.57 / 4.825)	7.666 (0.431 / 5.937)
sex	-1.883	-0.846 (0.255 / 0.304)	-0.967 (0.265 / 0.328)	-0.567 (0.218 / 0.225)
age	0.030	0.021 (0.016 / 0.016)	0.022 (0.017 / 0.018)	0.004 (0.014 / 0.016)
level 2 v.s. level 3				
intercept	-0.261	0.224 (0.595 / 3.029)	0.629 (0.598 / 3.997)	-12.377 (0.428 / 11.687)
class 1	1.698	1.826 (0.607 / 3.344)	3.416 (0.732 / 5.679)	13.211 (NaN / 11.715)
class 2	1.665	1.546 (0.458 / 5.141)	1.553 (0.459 / 5.301)	4.561 (0.131 / 19.127)
class 3	3.753	2.689 (0.537 / 3.485)	2.241 (0.475 / 4.993)	-6.065 (0.081 / 14.281)
class 4	4.873	1.742 (0.576 / 4.291)	1.433 (0.5 / 5.438)	-4.001 (0.096 / 15.3)
class 5	-2.089	-2.616 (0.409 / 3.572)	-3.551 (0.441 / 4.477)	8.364 (0.458 / 11.555)
sex	-0.783	-0.144 (0.235 / 0.271)	-0.243 (0.244 / 0.305)	-0.075 (0.222 / 0.226)
age	0.008	0.000 (0.015 / 0.014)	0.002 (0.015 / 0.016)	-0.006 (0.014 / 0.015)
item 5				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	0.199	-0.146 (0.554 / 3.284)	1.120 (0.556 / 3.736)	-8.056 (0.422 / 9.196)
class 1	3.300	3.172 (0.462 / 3.377)	2.625 (0.483 / 4.781)	10.676 (0.302 / 10.066)
class 2	0.106	0.084 (0.456 / 5.887)	0.452 (0.413 / 5.994)	-4.471 (NaN / 14.093)
class 3	1.796	1.673 (0.663 / 4.025)	0.502 (0.396 / 4.967)	-3.616 (NaN / 12.054)
class 4	0.370	-0.057 (0.407 / 6.762)	0.217 (0.437 / 6.276)	-5.374 (NaN / 11.865)
class 5	-2.095	-1.963 (0.435 / 3.643)	-3.409 (0.458 / 3.913)	5.647 (0.432 / 9.2)
sex	-0.767	-0.503 (0.211 / 0.237)	-0.555 (0.21 / 0.241)	-0.441 (0.186 / 0.238)
age	0.000	0.005 (0.013 / 0.013)	0.004 (0.013 / 0.011)	-0.006 (0.012 / 0.012)

Table 22: (Continued)

	level 2 v.s. level 3			
intercept	0.889	0.673 (0.537 / 4.004)	1.909 (0.522 / 4.74)	0.468 (0.447 / 15.952)
class 1	0.479	0.784 (0.501 / 3.959)	-0.230 (0.514 / 5.11)	0.867 (0.317 / 15.836)
class 2	-0.379	-0.530 (0.458 / 6.344)	-0.877 (0.379 / 6.786)	-8.928 (0.114 / 19.48)
class 3	2.062	2.903 (0.574 / 6.413)	1.225 (0.378 / 6.41)	-3.861 (0.102 / 23.144)
class 4	-0.073	-1.121 (0.42 / 7.132)	-0.640 (0.427 / 7.106)	-10.335 (0.041 / 19.255)
class 5	-1.232	-1.346 (0.376 / 4.383)	-2.735 (0.36 / 4.973)	-1.514 (0.382 / 16.218)
sex	-0.586	-0.445 (0.202 / 0.203)	-0.486 (0.202 / 0.211)	-0.388 (0.191 / 0.211)
age	-0.011	-0.011 (0.013 / 0.013)	-0.010 (0.013 / 0.012)	-0.015 (0.012 / 0.013)



Table 23: Average parameters estimations for 100 replication in 6-class model, N=1000
(standard error in multinomial regression / sample standard error for 100 replication)

Class 1 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-1.087	-0.378 (0.243 / 0.71)	-0.509 (0.251 / 0.815)	4.088 (1.046 / 1.829)
occup	0.274	0.512 (0.27 / 0.332)	0.577 (0.283 / 0.386)	1.672 (7.947 / 4.06)
dprime	0.688	0.375 (0.074 / 0.16)	0.408 (0.077 / 0.162)	0.327 (0.247 / 0.359)

Class 2 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	0.427	-0.113 (0.25 / 1.019)	-0.059 (0.249 / 1.126)	-0.194 (1.899 / 1.574)
occup	-1.743	-0.072 (0.335 / 0.435)	-0.029 (0.341 / 0.481)	-6.252 (NaN / 11.262)
dprime	0.010	0.026 (0.082 / 0.206)	0.053 (0.083 / 0.221)	0.047 (0.376 / 0.506)

Class 3 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.671	0.244 (0.223 / 0.724)	0.217 (0.227 / 0.899)	-0.584 (1.449 / 3.653)
occup	-0.852	-0.119 (0.289 / 0.38)	-0.046 (0.301 / 0.433)	-3.761 (4.941 / 10.524)
dprime	0.563	0.157 (0.072 / 0.156)	0.186 (0.074 / 0.176)	-0.054 (0.432 / 1.322)

Class 4 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.456	-0.334 (0.262 / 1.061)	-0.395 (0.269 / 1.039)	-1.203 (2.722 / 3.065)
occup	-0.260	0.079 (0.335 / 0.437)	0.109 (0.352 / 0.541)	-3.291 (8.172 / 10.7)
dprime	0.253	0.095 (0.085 / 0.237)	0.124 (0.088 / 0.212)	-0.079 (0.45 / 0.558)

Class 5 vs. Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	0.167	0.688 (0.208 / 0.828)	0.603 (0.212 / 0.836)	3.041 (1.033 / 1.261)
occup	-0.624	-0.052 (0.291 / 0.342)	-0.041 (0.306 / 0.395)	1.440 (7.964 / 4.086)
dprime	0.143	-0.075 (0.071 / 0.159)	-0.055 (0.073 / 0.152)	0.095 (0.248 / 0.293)

Table 24: Average conditional Probability for 100 replication in 6-class model, N=1000
(sample standard deviance in parentheses)

Class 1				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.955	0.879 (0.057)	0.910 (0.056)	0.566 (0.118)
Level 2	0.044	0.111 (0.053)	0.084 (0.051)	0.340 (0.092)
Level 3	0.001	0.010 (0.009)	0.005 (0.008)	0.095 (0.029)
item2				
Level 1	0.727	0.710 (0.143)	0.733 (0.122)	0.335 (0.122)
Level 2	0.235	0.232 (0.115)	0.222 (0.099)	0.446 (0.072)
Level 3	0.038	0.059 (0.038)	0.045 (0.034)	0.219 (0.060)
item3				
Level 1	0.509	0.485 (0.215)	0.504 (0.160)	0.258 (0.173)
Level 2	0.317	0.343 (0.150)	0.345 (0.118)	0.359 (0.084)
Level 3	0.175	0.172 (0.080)	0.151 (0.057)	0.384 (0.092)
item4				
Level 1	0.927	0.858 (0.064)	0.893 (0.064)	0.524 (0.130)
Level 2	0.057	0.120 (0.058)	0.094 (0.058)	0.359 (0.098)
Level 3	0.016	0.022 (0.014)	0.014 (0.013)	0.118 (0.035)
item5				
Level 1	0.883	0.829 (0.066)	0.850 (0.071)	0.532 (0.129)
Level 2	0.078	0.119 (0.052)	0.103 (0.057)	0.309 (0.086)
Level 3	0.039	0.052 (0.025)	0.047 (0.027)	0.159 (0.046)
Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.436	0.395 (0.230)	0.376 (0.239)	0.058 (0.134)
Level 2	0.385	0.430 (0.181)	0.441 (0.215)	0.246 (0.327)
Level 3	0.178	0.176 (0.204)	0.183 (0.246)	0.696 (0.373)
item2				
Level 1	0.071	0.144 (0.178)	0.140 (0.201)	0.000 (0.002)
Level 2	0.396	0.461 (0.202)	0.413 (0.193)	0.033 (0.138)
Level 3	0.533	0.395 (0.250)	0.447 (0.261)	0.967 (0.138)

Table 24 : (Continued)

item3				
Level 1	0.009	0.123 (0.233)	0.129 (0.243)	0.015 (0.076)
Level 2	0.155	0.248 (0.177)	0.230 (0.172)	0.120 (0.289)
Level 3	0.836	0.629 (0.256)	0.641 (0.264)	0.865 (0.306)
item4				
Level 1	0.356	0.346 (0.239)	0.357 (0.230)	0.036 (0.135)
Level 2	0.500	0.459 (0.217)	0.458 (0.196)	0.310 (0.415)
Level 3	0.143	0.195 (0.212)	0.185 (0.199)	0.655 (0.424)
item5				
Level 1	0.333	0.376 (0.240)	0.386 (0.239)	0.065 (0.185)
Level 2	0.306	0.325 (0.197)	0.300 (0.187)	0.265 (0.399)
Level 3	0.361	0.299 (0.267)	0.314 (0.256)	0.670 (0.439)
Class 3				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.606	0.484 (0.131)	0.496 (0.155)	0.051 (0.143)
Level 2	0.362	0.465 (0.119)	0.423 (0.134)	0.133 (0.254)
Level 3	0.032	0.051 (0.091)	0.081 (0.186)	0.816 (0.320)
item2				
Level 1	0.217	0.151 (0.093)	0.184 (0.124)	0.027 (0.089)
Level 2	0.714	0.696 (0.100)	0.655 (0.160)	0.082 (0.191)
Level 3	0.069	0.152 (0.098)	0.161 (0.169)	0.890 (0.250)
item3				
Level 1	0.020	0.071 (0.059)	0.083 (0.052)	0.051 (0.119)
Level 2	0.703	0.584 (0.145)	0.570 (0.149)	0.256 (0.334)
Level 3	0.277	0.346 (0.151)	0.347 (0.167)	0.693 (0.353)
item4				
Level 1	0.239	0.304 (0.134)	0.314 (0.137)	0.036 (0.120)
Level 2	0.735	0.629 (0.138)	0.592 (0.163)	0.093 (0.231)
Level 3	0.026	0.067 (0.107)	0.094 (0.176)	0.871 (0.297)
item5				
Level 1	0.318	0.300 (0.138)	0.336 (0.135)	0.079 (0.185)
Level 2	0.618	0.586 (0.165)	0.544 (0.165)	0.378 (0.440)
Level 3	0.064	0.114 (0.085)	0.120 (0.140)	0.543 (0.470)

Table 24 : (Continued)

Class 4				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.163	0.415 (0.277)	0.412 (0.288)	0.018 (0.113)
Level 2	0.812	0.479 (0.262)	0.445 (0.273)	0.166 (0.300)
Level 3	0.026	0.106 (0.206)	0.143 (0.286)	0.816 (0.315)
item2				
Level 1	0.085	0.217 (0.245)	0.206 (0.266)	0.010 (0.100)
Level 2	0.607	0.487 (0.195)	0.468 (0.226)	0.050 (0.168)
Level 3	0.308	0.296 (0.215)	0.325 (0.266)	0.940 (0.193)
item3				
Level 1	0.331	0.272 (0.325)	0.206 (0.244)	0.054 (0.161)
Level 2	0.465	0.310 (0.187)	0.352 (0.192)	0.280 (0.366)
Level 3	0.205	0.418 (0.249)	0.442 (0.254)	0.666 (0.405)
item4				
Level 1	0.577	0.478 (0.243)	0.463 (0.265)	0.043 (0.186)
Level 2	0.418	0.378 (0.191)	0.376 (0.215)	0.123 (0.290)
Level 3	0.005	0.144 (0.190)	0.161 (0.252)	0.833 (0.328)
item5				
Level 1	0.359	0.440 (0.268)	0.423 (0.267)	0.078 (0.251)
Level 2	0.343	0.274 (0.190)	0.285 (0.194)	0.217 (0.400)
Level 3	0.298	0.286 (0.287)	0.291 (0.301)	0.704 (0.447)
Class 5				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.009	0.035 (0.046)	0.018 (0.014)	0.011 (0.009)
Level 2	0.019	0.134 (0.075)	0.093 (0.053)	0.055 (0.029)
Level 3	0.972	0.831 (0.099)	0.889 (0.062)	0.934 (0.031)
item2				
Level 1	0.000	0.008 (0.027)	0.003 (0.004)	0.001 (0.002)
Level 2	0.009	0.087 (0.046)	0.067 (0.036)	0.015 (0.013)
Level 3	0.991	0.905 (0.063)	0.931 (0.037)	0.984 (0.013)

Table 24 : (Continued)

item3				
Level 1	0.018	0.013 (0.009)	0.014 (0.010)	0.007 (0.010)
Level 2	0.156	0.130 (0.040)	0.135 (0.029)	0.065 (0.034)
Level 3	0.826	0.858 (0.044)	0.851 (0.032)	0.929 (0.038)
item4				
Level 1	0.018	0.051 (0.040)	0.036 (0.022)	0.013 (0.011)
Level 2	0.074	0.111 (0.053)	0.087 (0.047)	0.033 (0.023)
Level 3	0.908	0.838 (0.079)	0.877 (0.062)	0.955 (0.027)
item5				
Level 1	0.070	0.092 (0.050)	0.086 (0.040)	0.056 (0.026)
Level 2	0.247	0.235 (0.084)	0.221 (0.076)	0.170 (0.072)
Level 3	0.683	0.673 (0.115)	0.694 (0.104)	0.773 (0.086)
Class 6				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.002	0.248 (0.231)	0.236 (0.220)	0.024 (0.068)
Level 2	0.709	0.465 (0.205)	0.508 (0.232)	0.406 (0.345)
Level 3	0.289	0.288 (0.253)	0.256 (0.269)	0.569 (0.351)
item2				
Level 1	0.009	0.109 (0.225)	0.098 (0.181)	0.001 (0.014)
Level 2	0.528	0.405 (0.206)	0.437 (0.189)	0.069 (0.165)
Level 3	0.463	0.487 (0.250)	0.465 (0.240)	0.929 (0.166)
item3				
Level 1	0.001	0.098 (0.232)	0.071 (0.185)	0.010 (0.061)
Level 2	0.079	0.209 (0.158)	0.206 (0.164)	0.055 (0.183)
Level 3	0.920	0.692 (0.244)	0.723 (0.239)	0.936 (0.209)
item4				
Level 1	0.001	0.245 (0.218)	0.221 (0.213)	0.013 (0.071)
Level 2	0.397	0.390 (0.191)	0.403 (0.224)	0.129 (0.318)
Level 3	0.602	0.364 (0.249)	0.376 (0.281)	0.858 (0.336)
item5				
Level 1	0.271	0.322 (0.194)	0.306 (0.185)	0.094 (0.242)
Level 2	0.403	0.386 (0.198)	0.423 (0.200)	0.484 (0.468)
Level 3	0.326	0.292 (0.244)	0.271 (0.236)	0.422 (0.462)

Table 25: Average Latent Prevalences for 100 replication in 6-class model, N=1000
(sample standard deviance in parentheses)

	TRUE	K_Corr	K_Cova	H_Cova
class1	0.269	0.253 (0.074)	0.237 (0.080)	0.791 (0.205)
class2	0.137	0.114 (0.046)	0.128 (0.064)	0.007 (0.007)
class3	0.196	0.208 (0.064)	0.220 (0.080)	0.061 (0.201)
class4	0.118	0.117 (0.060)	0.121 (0.073)	0.004 (0.003)
class5	0.158	0.189 (0.054)	0.174 (0.046)	0.129 (0.023)
class6	0.123	0.120 (0.046)	0.120 (0.056)	0.008 (0.006)

Table 26: Average Correlation Coefficients for 100 replication in 6-class model,
N=1000 (total number of not NA values in parentheses)

	K_Corr	K_Cova	H_Cova
class1	0.098 (100)	0.114 (100)	0.201 (99)
class2	0.124 (100)	0.123 (100)	0.806 (75)
class3	0.096 (100)	0.097 (100)	0.770 (59)
class4	0.119 (100)	0.121 (100)	0.909 (41)
class5	0.194 (100)	0.170 (100)	0.203 (100)
class6	0.148 (100)	0.130 (100)	0.733 (72)

Table 27: Average parameters estimations for 100 replication in 2-class model, N=150
(standard error in multinomial regression / sample standard error for 100 replication)

item 1				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-3.743	-2.311 (1.31 / 1.699)	-3.144 (1.56 / 2.493)	-8.656 (1.518 / 3.18)
class 1	6.790	5.153 (1.123 / 3.005)	6.753 (1.119 / 3.854)	9.142 (1.471 / 2.639)
sex	0.391	0.347 (0.557 / 0.607)	0.438 (0.59 / 0.587)	0.318 (0.462 / 0.501)
age	0.051	0.025 (0.034 / 0.034)	0.025 (0.036 / 0.037)	0.012 (0.027 / 0.029)
level 2 v.s. level 3				
intercept	-0.866	-0.477 (1.126 / 1.117)	-0.915 (1.18 / 1.819)	-6.543 (1.795 / 3.919)
class 1	3.962	2.879 (1.063 / 2.845)	4.161 (0.847 / 3.717)	6.875 (1.682 / 3.868)
sex	0.501	0.465 (0.499 / 0.491)	0.535 (0.525 / 0.508)	0.475 (0.495 / 0.537)
age	0.012	0.003 (0.031 / 0.028)	0.005 (0.032 / 0.031)	-0.003 (0.029 / 0.026)
item 2				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-3.986	-3.624 (1.853 / 4.279)	-5.861 (3.782 / 4.607)	-7.697 (2.202 / 2.056)
class 1	6.047	5.916 (1.601 / 4.183)	8.354 (3.688 / 4.391)	8.314 (2.184 / 1.597)
sex	0.311	0.325 (0.548 / 0.584)	0.358 (0.571 / 0.625)	0.234 (0.45 / 0.49)
age	-0.014	-0.029 (0.033 / 0.036)	-0.033 (0.034 / 0.035)	-0.024 (0.027 / 0.028)
level 2 v.s. level 3				
intercept	-0.923	-0.706 (1.001 / 1.628)	-0.758 (1.047 / 1.113)	-7.976 (0.961 / 3.775)
class 1	2.586	2.332 (0.602 / 1.479)	2.655 (0.713 / 1.141)	8.251 (0.913 / 3.689)
sex	0.312	0.333 (0.45 / 0.5)	0.375 (0.468 / 0.553)	0.287 (0.404 / 0.46)
age	-0.002	-0.009 (0.027 / 0.028)	-0.012 (0.028 / 0.027)	-0.006 (0.024 / 0.025)
item 3				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-4.839	-5.092 (3.648 / 3.836)	-7.544 (5.633 / 4.615)	-9.040 (5.332 / 1.977)
class 1	4.373	5.380 (3.15 / 4.303)	7.505 (5.33 / 4.309)	7.959 (5.706 / 1.355)
sex	-0.053	-0.145 (0.554 / 0.538)	-0.088 (0.541 / 0.543)	0.009 (0.47 / 0.45)
age	0.024	0.013 (0.033 / 0.036)	0.014 (0.032 / 0.037)	0.009 (0.028 / 0.031)

Table 27: (Continued)

level 2 v.s. level 3				
intercept	-1.780	-1.701 (0.963 / 1.892)	-1.752 (0.989 / 1.022)	-8.078 (5.289 / 2.635)
class 1	1.990	1.880 (0.44 / 1.922)	1.963 (0.462 / 0.977)	7.638 (5.262 / 2.769)
sex	-0.442	-0.432 (0.421 / 0.426)	-0.419 (0.424 / 0.447)	-0.353 (0.384 / 0.371)
age	0.019	0.013 (0.025 / 0.025)	0.013 (0.025 / 0.025)	0.009 (0.023 / 0.022)

item 4				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-2.893	-1.920 (1.265 / 2.065)	-2.417 (1.358 / 2.062)	-7.992 (1.16 / 3.369)
class 1	5.733	5.273 (1.285 / 3.45)	6.018 (1.278 / 3.56)	8.597 (1.111 / 3.245)
sex	-0.462	-0.391 (0.545 / 0.602)	-0.375 (0.569 / 0.649)	-0.180 (0.448 / 0.518)
age	0.042	0.019 (0.033 / 0.036)	0.022 (0.035 / 0.041)	0.009 (0.027 / 0.03)
level 2 v.s. level 3				
intercept	-0.727	-0.260 (1.104 / 1.783)	-0.549 (1.163 / 1.778)	-6.510 (2.002 / 4.212)
class 1	3.630	3.431 (1.257 / 3.34)	4.134 (1.174 / 3.586)	7.282 (1.924 / 4.165)
sex	0.105	0.206 (0.494 / 0.52)	0.207 (0.513 / 0.537)	0.320 (0.471 / 0.548)
age	0.007	-0.009 (0.03 / 0.033)	-0.006 (0.032 / 0.037)	-0.015 (0.028 / 0.031)

item 5				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-1.218	-1.255 (1.158 / 2.374)	-1.211 (1.125 / 1.698)	-7.060 (1.495 / 4.058)
class 1	2.900	3.006 (0.633 / 1.89)	3.051 (0.555 / 1.591)	7.925 (1.417 / 3.832)
sex	-0.268	-0.307 (0.488 / 0.5)	-0.301 (0.484 / 0.487)	-0.238 (0.437 / 0.485)
age	0.011	0.007 (0.03 / 0.033)	0.006 (0.029 / 0.03)	0.001 (0.026 / 0.024)
level 2 v.s. level 3				
intercept	0.373	0.291 (1.217 / 2.464)	0.332 (1.072 / 1.102)	-5.144 (2.757 / 4.114)
class 1	1.223	1.311 (0.73 / 2.243)	1.455 (0.518 / 1.245)	6.207 (2.639 / 4.317)
sex	-0.292	-0.308 (0.475 / 0.523)	-0.309 (0.472 / 0.517)	-0.289 (0.468 / 0.57)
age	-0.013	-0.014 (0.029 / 0.032)	-0.015 (0.029 / 0.03)	-0.019 (0.028 / 0.027)

Table 28: Average parameters estimations for 100 replication in 2-class model, N=150
(standard error in multinomial regression / sample standard error for 100 replication)

Class 1 vs. Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.935	-0.596 (0.345 / 0.749)	-0.439 (0.337 / 0.635)	5.091 (4.067 / 12.64)
occup	0.441	0.294 (0.399 / 0.452)	0.306 (0.407 / 0.438)	2.366 (22.899 / 4.745)
dprime	0.414	0.282 (0.109 / 0.13)	0.309 (0.108 / 0.12)	0.648 (1.301 / 6.422)

Table 29: Average conditional Probability for 100 replication in 2-class model, N=150
(sample standard deviance in parentheses)

Class 1				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.765	0.706 (0.086)	0.696 (0.087)	0.489 (0.066)
Level 2	0.229	0.245 (0.060)	0.266 (0.070)	0.314 (0.051)
Level 3	0.005	0.049 (0.052)	0.038 (0.038)	0.197 (0.043)
item2				
Level 1	0.457	0.421 (0.094)	0.418 (0.075)	0.278 (0.082)
Level 2	0.464	0.457 (0.072)	0.465 (0.053)	0.405 (0.057)
Level 3	0.079	0.122 (0.073)	0.117 (0.065)	0.317 (0.053)
item3				
Level 1	0.325	0.324 (0.156)	0.311 (0.114)	0.203 (0.087)
Level 2	0.441	0.418 (0.112)	0.423 (0.084)	0.349 (0.054)
Level 3	0.234	0.258 (0.094)	0.266 (0.082)	0.448 (0.063)
item4				
Level 1	0.683	0.626 (0.105)	0.620 (0.077)	0.443 (0.068)
Level 2	0.304	0.321 (0.094)	0.331 (0.057)	0.339 (0.050)
Level 3	0.012	0.053 (0.055)	0.049 (0.050)	0.218 (0.041)
item5				
Level 1	0.647	0.617 (0.069)	0.601 (0.074)	0.454 (0.067)
Level 2	0.258	0.269 (0.064)	0.282 (0.057)	0.312 (0.051)
Level 3	0.095	0.114 (0.051)	0.118 (0.052)	0.234 (0.045)

Table 29: (Continued)

Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.082	0.156 (0.116)	0.114 (0.086)	0.021 (0.070)
Level 2	0.415	0.390 (0.090)	0.362 (0.093)	0.094 (0.193)
Level 3	0.504	0.455 (0.110)	0.524 (0.126)	0.885 (0.224)
item2				
Level 1	0.009	0.057 (0.070)	0.026 (0.051)	0.006 (0.038)
Level 2	0.304	0.312 (0.080)	0.282 (0.076)	0.056 (0.167)
Level 3	0.687	0.631 (0.120)	0.692 (0.107)	0.937 (0.177)
item3				
Level 1	0.014	0.041 (0.047)	0.018 (0.024)	0.001 (0.010)
Level 2	0.202	0.214 (0.086)	0.196 (0.080)	0.028 (0.099)
Level 3	0.784	0.745 (0.109)	0.786 (0.091)	0.971 (0.102)
item4				
Level 1	0.098	0.156 (0.102)	0.117 (0.076)	0.030 (0.116)
Level 2	0.356	0.350 (0.073)	0.326 (0.087)	0.082 (0.204)
Level 3	0.547	0.494 (0.116)	0.557 (0.124)	0.888 (0.250)
item5				
Level 1	0.173	0.187 (0.097)	0.178 (0.081)	0.051 (0.118)
Level 2	0.368	0.361 (0.103)	0.345 (0.073)	0.112 (0.216)
Level 3	0.459	0.451 (0.127)	0.477 (0.101)	0.837 (0.283)

Table 30: Average Latent Prevalences for 100 replication in 2-class model, N=150
(sample standard deviance in parentheses)

	TRUE	K_Corr	K_Cova	H_Cova
class1	0.56	0.552 (0.139)	0.6 (0.114)	0.951 (0.100)
class2	0.44	0.448 (0.139)	0.4 (0.114)	0.0491 (0.100)

Table 31: Average Correlation Coefficients for 100 replication in 2-class model,
N=150 (total number of not NA values in parentheses)

	K_Corr	K_Cova	H_Cova
class1	0.143 (100)	0.148 (100)	0.289 (100)
class2	0.234 (100)	0.190 (100)	0.790 (77)



Table 32: Average parameters estimations for 100 replication in 2-class model, N=700
 (standard error in multinomial regression / sample standard error for 100 replication)

item 1				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-3.743	-2.596 (0.62 / 1.675)	-2.905 (0.653 / 0.648)	-6.079 (0.808 / 3.425)
class 1	6.790	5.113 (0.64 / 1.702)	5.576 (0.463 / 1.361)	6.356 (0.748 / 3.424)
sex	0.391	0.483 (0.261 / 0.272)	0.483 (0.272 / 0.278)	0.397 (0.203 / 0.221)
age	0.051	0.023 (0.017 / 0.017)	0.027 (0.017 / 0.017)	0.010 (0.013 / 0.014)
level 2 v.s. level 3				
intercept	-0.866	-0.518 (0.507 / 1.579)	-0.537 (0.519 / 0.56)	-4.249 (0.53 / 4.284)
class 1	3.962	2.780 (0.609 / 1.659)	3.015 (0.43 / 1.273)	4.695 (0.425 / 4.273)
sex	0.501	0.568 (0.226 / 0.217)	0.572 (0.23 / 0.229)	0.542 (0.216 / 0.22)
age	0.012	-0.002 (0.014 / 0.016)	0.000 (0.015 / 0.017)	-0.009 (0.014 / 0.016)
item 2				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-3.986	-3.193 (0.784 / 2.404)	-4.007 (0.853 / 2.45)	-9.006 (0.526 / 0.993)
class 1	6.047	5.347 (0.607 / 2.411)	6.260 (0.664 / 2.353)	9.243 (0.525 / 1.003)
sex	0.311	0.361 (0.253 / 0.262)	0.364 (0.265 / 0.275)	0.283 (0.203 / 0.209)
age	-0.014	-0.027 (0.016 / 0.017)	-0.027 (0.017 / 0.018)	-0.018 (0.013 / 0.015)
level 2 v.s. level 3				
intercept	-0.923	-0.789 (0.46 / 1.709)	-0.768 (0.477 / 0.456)	-7.517 (0.659 / 2.993)
class 1	2.586	2.503 (0.252 / 1.801)	2.578 (0.229 / 0.404)	7.731 (0.632 / 2.964)
sex	0.312	0.353 (0.205 / 0.206)	0.358 (0.212 / 0.225)	0.313 (0.182 / 0.197)
age	-0.002	-0.010 (0.013 / 0.012)	-0.010 (0.013 / 0.012)	-0.005 (0.012 / 0.011)
item 3				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-4.839	-4.617 (0.882 / 2.337)	-5.492 (0.895 / 2.712)	-9.030 (1.53 / 1.304)
class 1	4.373	5.510 (0.605 / 4.204)	5.178 (0.706 / 2.65)	7.760 (1.526 / 1.208)
sex	-0.053	0.009 (0.263 / 0.263)	-0.010 (0.246 / 0.251)	0.122 (0.213 / 0.203)
age	0.024	0.011 (0.017 / 0.021)	0.015 (0.016 / 0.015)	0.009 (0.013 / 0.013)

Table 32: (Continued)

level 2 v.s. level 3				
intercept	-1.780	-1.714 (0.441 / 1.885)	-1.707 (0.45 / 0.443)	-7.492 (1.833 / 2.753)
class 1	1.990	1.791 (0.188 / 2.159)	2.018 (0.199 / 0.267)	7.038 (1.812 / 2.699)
sex	-0.442	-0.380 (0.19 / 0.189)	-0.405 (0.193 / 0.192)	-0.312 (0.175 / 0.184)
age	0.019	0.012 (0.012 / 0.012)	0.013 (0.012 / 0.012)	0.009 (0.011 / 0.012)

item 4				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-2.893	-2.240 (0.584 / 2.094)	-2.381 (0.618 / 0.615)	-8.290 (0.847 / 2.815)
class 1	5.733	4.886 (0.497 / 2.155)	5.050 (0.406 / 0.766)	8.598 (0.829 / 2.847)
sex	-0.462	-0.331 (0.251 / 0.25)	-0.396 (0.262 / 0.26)	-0.108 (0.202 / 0.227)
age	0.042	0.020 (0.016 / 0.016)	0.024 (0.017 / 0.016)	0.012 (0.013 / 0.012)
level 2 v.s. level 3				
intercept	-0.727	-0.575 (0.5 / 1.92)	-0.481 (0.514 / 0.495)	-7.349 (0.499 / 3.442)
class 1	3.630	3.138 (0.469 / 1.94)	3.071 (0.372 / 0.575)	7.935 (0.469 / 3.451)
sex	0.105	0.158 (0.224 / 0.216)	0.118 (0.228 / 0.217)	0.304 (0.21 / 0.232)
age	0.007	-0.005 (0.014 / 0.014)	-0.002 (0.015 / 0.014)	-0.010 (0.013 / 0.013)

item 5				
	TRUE	K_Corr	K_Cova	H_Cova
level 1 v.s. level 3				
intercept	-1.218	-1.278 (0.499 / 2.513)	-1.048 (0.512 / 0.57)	-6.118 (0.734 / 3.634)
class 1	2.900	3.091 (0.303 / 2.388)	2.757 (0.237 / 0.713)	6.772 (0.69 / 3.536)
sex	-0.268	-0.223 (0.217 / 0.218)	-0.239 (0.219 / 0.225)	-0.134 (0.197 / 0.227)
age	0.011	0.004 (0.014 / 0.013)	0.005 (0.014 / 0.014)	0.001 (0.012 / 0.012)
level 2 v.s. level 3				
intercept	0.373	0.081 (0.469 / 2.414)	0.413 (0.476 / 0.46)	-5.206 (0.462 / 4.279)
class 1	1.223	1.668 (0.297 / 2.615)	1.286 (0.223 / 0.965)	6.171 (0.403 / 4.188)
sex	-0.292	-0.254 (0.211 / 0.216)	-0.257 (0.21 / 0.203)	-0.207 (0.209 / 0.232)
age	-0.013	-0.016 (0.013 / 0.013)	-0.016 (0.013 / 0.013)	-0.017 (0.013 / 0.013)

Table 33: Average parameters estimations for 100 replication in 2-class model, N=700
(standard error in multinomial regression / sample standard error for 100 replication)

Class 1 vs. Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
intercept	-0.935	-0.679 (0.158 / 0.749)	-0.640 (0.149 / 0.266)	3.201 (0.412 / 0.903)
occup	0.441	0.319 (0.193 / 0.221)	0.362 (0.184 / 0.185)	0.861 (4.131 / 2.246)
dprime	0.414	0.309 (0.05 / 0.087)	0.338 (0.048 / 0.061)	0.174 (0.144 / 0.172)

Table 34: Average conditional Probability for 100 replication in 2-class model, N=700
(sample standard deviance in parentheses)

Class 1				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.758	0.707 (0.080)	0.717 (0.036)	0.467 (0.021)
Level 2	0.237	0.256 (0.044)	0.259 (0.028)	0.322 (0.021)
Level 3	0.006	0.037 (0.047)	0.023 (0.018)	0.212 (0.020)
item2				
Level 1	0.460	0.425 (0.061)	0.434 (0.038)	0.266 (0.019)
Level 2	0.461	0.462 (0.037)	0.471 (0.027)	0.405 (0.023)
Level 3	0.078	0.113 (0.070)	0.096 (0.031)	0.330 (0.025)
item3				
Level 1	0.322	0.356 (0.195)	0.307 (0.027)	0.190 (0.014)
Level 2	0.439	0.392 (0.124)	0.436 (0.027)	0.342 (0.021)
Level 3	0.239	0.252 (0.101)	0.257 (0.030)	0.469 (0.023)
item4				
Level 1	0.675	0.625 (0.082)	0.639 (0.042)	0.427 (0.020)
Level 2	0.313	0.332 (0.050)	0.331 (0.029)	0.346 (0.018)
Level 3	0.013	0.043 (0.055)	0.030 (0.019)	0.228 (0.022)
item5				
Level 1	0.642	0.613 (0.092)	0.614 (0.070)	0.443 (0.023)
Level 2	0.263	0.279 (0.083)	0.280 (0.076)	0.319 (0.018)
Level 3	0.095	0.108 (0.044)	0.106 (0.023)	0.238 (0.020)

Table 34: (Continued)

Class 2				
	TRUE	K_Corr	K_Cova	H_Cova
item 1				
Level 1	0.078	0.130 (0.109)	0.093 (0.044)	0.049 (0.070)
Level 2	0.414	0.391 (0.058)	0.397 (0.031)	0.189 (0.201)
Level 3	0.508	0.480 (0.102)	0.510 (0.043)	0.763 (0.240)
item2				
Level 1	0.010	0.042 (0.070)	0.015 (0.024)	0.000 (0.004)
Level 2	0.304	0.301 (0.055)	0.284 (0.033)	0.032 (0.090)
Level 3	0.686	0.657 (0.110)	0.701 (0.048)	0.967 (0.090)
item3				
Level 1	0.013	0.024 (0.032)	0.014 (0.020)	0.001 (0.007)
Level 2	0.198	0.210 (0.069)	0.188 (0.027)	0.027 (0.087)
Level 3	0.788	0.766 (0.088)	0.797 (0.040)	0.972 (0.089)
item4				
Level 1	0.094	0.137 (0.096)	0.102 (0.039)	0.013 (0.038)
Level 2	0.356	0.339 (0.060)	0.341 (0.029)	0.049 (0.118)
Level 3	0.551	0.524 (0.116)	0.557 (0.044)	0.938 (0.142)
item5				
Level 1	0.170	0.188 (0.085)	0.172 (0.042)	0.057 (0.104)
Level 2	0.372	0.357 (0.077)	0.367 (0.034)	0.110 (0.171)
Level 3	0.458	0.454 (0.115)	0.462 (0.039)	0.833 (0.227)

Table 35: Average Latent Prevalences for 100 replication in 2-class model, N=700
(sample standard deviance in parentheses)

	TRUE	K_Corr	K_Cova	H_Cova
class1	0.554	0.545 (0.159)	0.573 (0.058)	0.966 (0.027)
class2	0.446	0.455 (0.159)	0.427 (0.058)	0.034 (0.027)

Table 36: Average Correlation Coefficients for 100 replication in 2-class model,
N=700 (total number of not NA values in parentheses)

	K_Corr	K_Cova	H_Cova
class1	0.098 (100)	0.089 (100)	0.288 (100)
class2	0.179 (100)	0.143 (100)	0.610 (99)

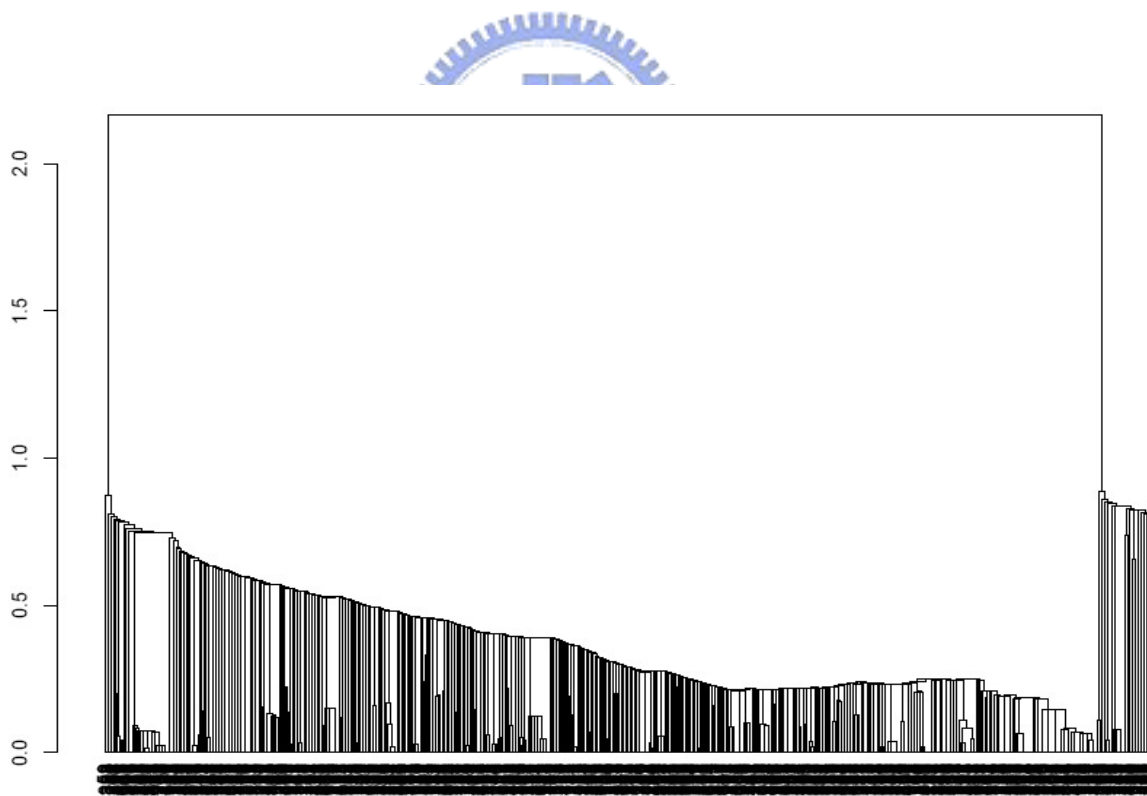


Figure 3: Dendrogram of Hierarchical procedure for 6-class model,
N=1000