

# $\mu$ 微中子在不同 CP-Violation 相位因子下振盪機率的討論

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## 摘要

CP-violation 相位因子在微中子振盪混和矩陣(Pontecorvo-Maki-Nakagawa-Sakata mixing matrix)中仍然是一個未知的參數。但是 CP-violation 相位因子對微中子振盪機率的影響在某一些特定的路徑長度下會趨近於零。這些特殊的路徑長度被稱為 magic baseline. 在本篇論文中，我們將用數值方式來計算 magic baseline 的長度，並與之前的解析解比較。最後，我們會證明在 magic baseline 下，CP-violation 相位因子並不影響我們用 neutrino factory 決定  $\theta_{23}$  的卦限(octant)。



Muon Neutrino Appearance and Disappearance Probabilities for Various  
CP-violation Phases at the Magic Baseline

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ABSTRACT

CP-violation phase is still an unknown parameter in neutrino oscillation mixing matrix. The effects of CP-violation phase on neutrino oscillation probabilities are minimal at the magic baseline. We calculate the magic baseline numerically and compare the result with that obtained by the analytic calculation. We then show that the CP-violation phase does not affect the determination of  $\theta_{23}$  octant by neutrino factories operated at the magic baseline.



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