國立交通大學
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碩士論文

依據製程能力指標 S_{pk} 應用複式抽樣方法於
供應商選擇

Bootstrap Approach for Supplier Selection Based on
Process Capability Index S_{pk}

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摘要

現今的製造業裡，許多公司藉由增加外購的比重以維持自己的核心競爭力，於是，供應商的選擇成為生產管理上重要的議題。在製造業裡，產品的良率一直是判斷製程好壞的重要因素，Boyles 提出了一個和良率有一對一對應關係的製程能力指標 S_{pk} ，過去有許多關於 S_{pk} 指標的近似分配、估計和檢定，這些結果已被應用在選擇單一供應商的研究，然而利用 S_{pk} 指標來同時檢定兩家不同供應商的議題至今尚無人研究。這篇研究的主要目的就是在兩家相互競爭的供應商中，挑選出一家具有較好製程能力的供應商，並建立一個依據 S_{pk} 指標選擇供應商的決策程序。在本篇論文中，我們將利用複式抽樣法來建構兩個供應商間製程能力差異的信賴下界，針對四個不同複式抽樣法來比較彼此之間犯錯機率及檢定力的表現，為了實務應用上方便，我們建立一個在犯錯機率為 0.05 下，針對給定不同選擇能力所需要的樣本數表格，最後我們也將研究的結果，應用在選擇兩家不同的彩色濾光片廠商。

關鍵字：複式抽樣法、信賴下界、製程良率、供應商選擇。

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Abstract

In today's manufacturing environment, many companies increase their out-sourcing level to keep their core competition. Supplier selection problems have become an important component of production management. Process yield is a standard criterion in the manufacturing industry as a common measure on process performance. Boyles proposed a index S_{pk} which provides an exact measure on the process yield for normal processes. Many studies considered the assessment of index S_{pk} for a single supplier. However, the testing procedure for two different suppliers selection based on S_{pk} has not been done today. The principal purpose of this research is to determine the more capable process between two competing suppliers and provide a supplier selection procedure based on S_{pk}. In this thesis, we implemented the bootstrap method to construct the lower confidence bound for the capability difference and the capability ratio between two given suppliers. The performance comparisons are made among the four bootstrap methods (the standard bootstrap (SB), the percentile bootstrap (PB), the biased corrected percentile bootstrap (BCPB), and the bootstrap-t (BT)) in terms of error probability and selection power. For convenience of applications, we tabulated the sample sizes required for various designated selection power. A real world case on the color filter manufacturing process is investigated to demonstrate the applicability of the proposed method in the end.

Key words: Bootstrap method, Lower confidence bound, Process yield, Supplier selection.

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這輩子寫過最完整且最令自己驕傲的一篇文章，就是這篇論文研究，能夠完成這篇論文首先要感謝彭文理老師和吳建瑋老師，從彭老師身上不僅學習到學術上的嚴謹態度，同時也學到了未來工作就業的態度；感謝吳老師不定時的與我們討論研究上的作法，幫助我們更了解論文內研究方法的意義。此外也非常感謝鍾淑馨老師在口試上給我的寶貴意見，由於這些老師的幫忙，才讓這篇論文得以完成。

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Notations

T	：target value
USL	：the upper specification limits presented by the process engineers
LSL	：the lower specification limits presented by the process engineers
m	：the midpoint between the upper and lower specification limits
d	：the half specification width
μ	：the population mean
σ	：the population standard deviation
σ^2	：the population variation
n	：the number of the sample size drawn from supplier
B	：the number of bootstrap resamples
N	：simulation replicated times
$\hat{S}_{pk1}^*$	：the $\hat{S}_{pk1}$ of bootstrap resamples from supplier I
$\hat{S}_{pk2}^*$	：the $\hat{S}_{pk2}$ of bootstrap resamples from supplier II
θ	：the difference or the ratio of two suppliers' S_{pk} index
$\hat{\theta}$	：the estimator of θ
$\hat{\theta}^*$	：the associated ordered bootstrap estimate of θ
$\overline{\hat{\theta}^*}$	：the sample average of the B bootstrap estimates
S_θ^*	：the standard deviation of the B bootstrap estimates

1. Introduction

Currently, many manufacturing industries have increased their out-sourcing level to keep their core competition. That is, they purchase various portions of components or subassemblies for their final products. In order to know if the supplier is qualified, some indices are needed. Process yield has long been one of the most standard criterion used in the manufacturing industry as a common measure on process performance. Process yield is defined as the percentage of processed product passing inspection. That is, the product characteristic must fall within the manufacturing tolerance. When product units rejected (non-conformities), additional costs would be incurred to the factory for scrapping or repairing the product. All passed product units are equally accepted by the producer, which incurs the factory no additional cost. On the other hand, consumer can save a lot of money from accepting producer, which has high level quality yield. In today's high-tech industry, traditional sampling method is not enough because of the high level quality yield. Process capability indices (PCIs) have been widely used in the manufacturing industry and provided numerical measures on process performance. We can determine whether a production process is capable and infer the process yield based on PCIs. This fact brings the issue of supplier selection based on PCIs into the main focus.

Many individuals have indicated various approaches for supplier selection or process comparison problems based on PCIs. Most of these researches focus on single supplier selection before. They consider the assessment of capability for a single process. However, there are fewer studies investigate the testing procedure for two different suppliers selection. Discussion relative to the subject based on the index S_{pk} has not been concentrated. The principal purpose of this research is to determine the more capable process between two competing suppliers and provide a supplier selection procedure based on S_{pk}.

In this thesis, we introduced process capability indices in common used, and reviewed some references about supplier selection problems based on PCIs first. Section 3 proposed to select a better supplier by comparing two S_{pk}. We formulated the hypothesis testing and introduced the bootstrap methodology. In section 4, we analyzed the error probability and selection power by comparing four different bootstrap methods. For convenience of applications, we tabulated the sample sizes required for various designated selection power in section 5. At last, we demonstrated a real world case on color filter manufacturing process, and made a conclusion in section 6 and 7.

2. Literature Review

2.1 Process Capability Indices

There are many capability indices proposed to use for evaluating a supplier's process capability. The first process capability index in the literature was C_p. It was introduced by Juran *et al.* (1974), but did not gain considerable acceptance until the early 1980s. It is defined as:

$$C_p = \frac{USL - LSL}{6\sigma} \quad ,$$

where USL is the upper specification limit, LSL is the lower specification limit, and σ is the process standard deviation. The index measures capability in terms of process variation only and did not take process location into consideration. Pearn *et al.* (1998) introduced an accuracy index C_a to measure the magnitude of process centering. It is defined as:

$$C_a = 1 - \frac{|\mu - m|}{d},$$

where μ is the process mean, $m = (USL + LSL)/2$, and $d = (USL - LSL)/2$. The index C_a measures the centering tendency. User can get alerts from it if the process mean is deviate form the midpoint. Kane (1986) proposed the capability index C_{pk}, considered process location of mean and process variation. The index C_{pk} determines process ability of reproducing items within the specified manufacturing tolerance. It is defined as:

$$C_{pk} = \min\{C_{pu}, C_{pl}\} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma}.$$

Based on the expression of process yield, Boyles (1994) considered the yield index S_{pk} for normal process, as defined in the following:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\{\frac{1}{2}\Phi(\frac{USL - \mu}{\sigma}) + \frac{1}{2}\Phi(\frac{\mu - LSL}{\sigma})\},$$

where Φ is the cumulative density function (c.d.f) of the standard normal distribution $N(0,1)$. Hsiang and Taguchi (1985) introduced the index C_{pm}, independently proposed by Chan *et al.* (1988). The index C_{pm} focuses on the product loss when one of its characteristics departs from the target value T. It is defined as:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}.$$

2.2 Process Yield Based on S_{pk}

2.2.1 Process Yield

In the past, we have to count the number of nonconforming items from a sample to calculate the yield. However, the fraction of non-conformities now is less than 0.01%, and we usually use parts per million (ppm) to express. Traditional methods for calculating the fraction nonconforming are no longer work since all reasonable sample sizes will probably have no defective items. These methods are substituted for capability indices.

Process yield has long been a standard criterion used in the manufacturing industry as a common measure on process performance. Process yield is defined as the percentage of processed product passing inspection. That is, the product characteristic must fall within the manufacturing tolerance. It can be calculated as:

$$\text{Yield} = F(USL) - F(LSL),$$

where USL and LSL are the upper and lower specification limits, and $F(x)$ is the cumulative distribution function of the process characteristic. If the process characteristic is normal distributed, then the process yield can be expressed as:

$$\text{Yield} = \Phi(\frac{USL-\mu}{\sigma}) - \Phi(\frac{\mu-LSL}{\sigma}),$$

where μ is the process mean, σ is the process standard deviation, and $\Phi(x)$ is the cumulative distribution function of the standard normal distribution $N(0,1)$.

2.2.2 Yield Assurance Based on S_{pk}

For normal distributed process, the relationship between the process yield and the index C_{pk} is $\text{Yield} \ge 2\Phi(3C_{pk}) - 1$. Thus, the index C_{pk} provides us with an approximate, rather than exact, measure of the actual process yield. Based on the expression of process yield, Boyles (1994) considered the yield index S_{pk} for normal process. This index S_{pk} provides an exact measure on the process yield. If $S_{pk} = c$, then the process yield can be expresses as $\text{Yield} = 2\Phi(3c) - 1$. There is a one-to-one correspondence between S_{pk} and the process yield. Table 1 summarizes the process yield, nonconformity (in ppm) as a function of the index S_{pk} =1.00, 1.33, 1.50, 1.67, and 2.00. For example, if a particular process the yield measure S_{pk} =1.67, then the corresponding value of nonconformities is 0.544 ppm.

Table 1. Some S_{pk} values and the corresponding values of fraction yield and nonconformities (ppm).

S_{pk}	Yield	Nonconformities
1.00	0.99730020	2699.80
1.33	0.99993393	66.07
1.50	0.99999320	6.80
1.67	0.99999946	0.54
2.00	0.99999999	0.01

2.3 Supplier Selection Problems based on PCIs

Because of the process mean μ and the process variance σ^2 are not known in real world. In order to calculate the estimator, however, data must be collected to calculate the index value, and a great degree of uncertainty may be introduced into capability assessments due to sampling errors.

The most common methods to assess the process capability are to utilize the interval estimation and hypotheses testing. Consequently, these estimating methods must be performed by using their sampling distributions. Kotz and Johnson (2002) presented a thorough review for the PCI developments during the years 1992 to 2000. Spiring *et al.* (2003) consolidated the research findings of process capability analysis for the period 1990–2002. Lee *et al.* (2002) considered an asymptotic distribution for an estimate $\hat{S}_{pk}$ of the process yield index S_{pk}. A useful approximate distribution of $\hat{S}_{pk}$ was furnished. Pearn and Chuang (2004) investigated the accuracy of the natural estimator of S_{pk} computationally, using a simulation technique to find the relative bias and the relative mean square error for some commonly used quality requirements. Chen (2005) considered to use the bootstrap simulation technique to find four approximate lower confidence limits for index S_{pk}. The simulation results show that the SB method significantly outperforms than other three methods. But, these studies considered the assessment of capability for a single process or supplier.

In a review of the problems for supplier selection based on PCIs, Tseng and Wu (1991) considered the problem for K available manufacturing processes based on the precision index C_p under a modified likelihood ratio (MLR) selection rule. Chou (1994) designed testing procedures for comparing two processes or suppliers in terms of C_p, C_{pl}, and C_{pu} when sample size are equal. Huang and Lee (1995) considered the supplier selection problem based on the index C_{pm} and developed a mathematically approximation method for selecting a subset containing the process associated with the smallest $\sigma^2 + (\mu - T)^2$ from K given independent processes. Pearn *et al.* (2004) provided useful information regarding the sample size required for a designated selection power. A two-phase selection procedure was developed to select a better supplier and to calculate the magnitude of the difference between two suppliers. Chen and Chen (2004) used four approximate confidence interval methods to present and compare for index C_{pm}. One based on the statistical theory given in Boyles (1991), and three based on the bootstrap (referred to as standard bootstrap, percentile bootstrap, and biased-corrected percentile bootstrap) for selecting a better supplier. However, the testing procedure for supplier selection based on S_{pk} has not been done today. In this thesis, because the exact sampling distribution of S_{pk} is analytical intractable, we will use bootstrap method to compare two processes based on S_{pk}.

3. Selection Method

3.1 Selecting a Better Supplier by Comparing Two S_{pk}

One of the purposes of the process capability indices can be put to use is to select between competing processes that which is more capable. Since we do not have direct observation of the entire processes, we have no idea that which process is more capable. When we have samples of product provided by two suppliers, we may use the sample data to select the supplier whose product is better. We may switch to a new supplier if we can be sure that the process capability index of the new supplier is higher than that of the present supplier.

In this thesis, we investigate the selection problem with two candidate processes based on the index S_{pk}. Let $x_{11}, x_{12}, \ldots, x_{1n}$ and $x_{21}, x_{22}, \ldots, x_{2n}$ be the measurements of two samples independently drawn from the normal distribution $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. In general, if a new supplier #2 (S2) wants to compete for the orders by claiming that its capability is better than the existing supplier #1 (S1), the new S2 has to convince purchaser with a prescribed confidence level information to justify the claim. Thus, the supplier selection decision would be based on the hypothesis testing comparing the two S_{pk} values. It is

$$H_0 : S_{pk1} \geq S_{pk2}$$

$$H_1 : S_{pk1} < S_{pk2} .$$

If the test rejects the null hypothesis $H_0 : S_{pk1} \geq S_{pk2}$, then one has sufficient information to conclude that the new S2 is better than the original S1, and the decision of the replacement would be suggested. This hypothesis testing problem can also be written as:

$$H_0 : S_{pk2} - S_{pk1} \leq 0 \quad \text{versus} \quad H_1 : S_{pk2} - S_{pk1} > 0 \quad \text{(difference testing)}$$

$$H_0 : S_{pk2} / S_{pk1} \leq 1 \quad \text{versus} \quad H_1 : S_{pk2} / S_{pk1} > 1 \qquad \text{(ratio testing).}$$

Therefore, if the lower confidence bound of $S_{pk2} - S_{pk1}$ is positive in difference testing, we can conclude that S2 has a better process capability than S1. Otherwise, we have no sufficient information to conclude that the S2 has a better process capability than S1. Similarly, if the lower confidence bound for the ratio between two process capability indices S_{pk2} / S_{pk1} is larger than 1, then S2 has a better process capability than S1. Otherwise, we have no sufficient information to conclude that the S2 has a better process capability than S1.

In order to estimate the yield measure S_{pk}, we consider the following natural estimator $\hat{S}_{pk}$, involving the statistics $\bar{x}=\sum_{i=1}^{n}x_i/n$, and $s=[\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)]^{1/2}$ are the sample mean and the sample standard deviation being the conventional estimators of μ and σ, respectively, obtained from a well-controlled process. The estimator is evidently

$$\hat{S}_{pk}=\frac{1}{3}\Phi^{-1}\{\frac{1}{2}\Phi(\frac{USL-\bar{x}}{s})+\frac{1}{2}\Phi(\frac{\bar{x}-LSL}{s})\}.$$

Even under the normal distribution, the exact distribution of $\hat{S}_{pk}$ is mathematically intractable. Consequently, testing the process performance can not be accomplished. Lee *et al.* (2002) obtained an approximate distribution of $\hat{S}_{pk}$ using the Taylor expansion technique. The estimator $\hat{S}_{pk}$ can be expressed approximately as:

$$\hat{S}_{pk}=S_{pk}+\frac{1}{6\sqrt{n}}[\phi(3S_{pk})]^{-1}W+O_p(n^{-1}),$$

where

$$W=-\frac{d}{2\sigma^3}Y\left[(1+\delta)\phi\left(\frac{1+\delta}{\gamma}\right)+(1-\delta)\phi\left(\frac{1-\delta}{\gamma}\right)\right]-\left(\frac{Z}{\sigma}\right)\left[\phi\left(\frac{1-\delta}{\gamma}\right)-\phi\left(\frac{1+\delta}{\gamma}\right)\right],$$

and above $\delta=(\mu-m)/d$, $\gamma=\sigma/d$, $\phi(.)$ is the probability density function (p.d.f) of the standard normal variable $N(0,1)$. $O_p(n^{-1})$ represents the error of the expansion having a leading term of order n^{-1} in probability. It is noted that the asymptotic expansion of $\hat{S}_{pk}$ is normally distributed with mean S_{pk} and variance $(a^2+b^2)/36n(\phi(3S_{pk}))^2$,

where $$a=\frac{d}{\sqrt{2}\sigma}\left[(1+\delta)\phi\left(\frac{1+\delta}{\gamma}\right)+(1-\delta)\phi\left(\frac{1-\delta}{\gamma}\right)\right],$$

$$b=\left[\phi\left(\frac{1-\delta}{\gamma}\right)-\phi\left(\frac{1+\delta}{\gamma}\right)\right].$$

Moreover, using rather complicated algebraic manipulations, Pearn *et al.* (2004) showed that, the estimator $\hat{S}_{pk}$ can be expressed in the form of:

$$\hat{S}_{pk}=S_{pk}+D_1Z+D_2Y+D_3Z^2+D_4ZY+D_5Y^2+O_p(\frac{1}{n\sqrt{n}}),$$

here Z and Y are distributed according to the joint bivariate normal distribution, and D_i, $i=1,2,...,5$, are functions of $\delta=(\mu-m)/d$ and $\gamma=\sigma/d$. Therefore, the distribution of $\hat{S}_{pk}$ may alternatively be approximated by the following polynomial combination of the distributions of Z and Y:

$$(Z,Y)\xrightarrow{d} N\left((0,0),\sum\nolimits_2\right), where \sum\nolimits_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix}$$

with the bias approximated as: $D_1Z + D_2Y + D_3Z^2 + D_4ZY + D_5Y^2$.

Both of these approximations to the distribution of $\hat{S}_{pk}$ are rather complicated and tedious. Undoubtedly, the distributions of $\hat{S}_{pk2} - \hat{S}_{pk1}$ or $\hat{S}_{pk2} / \hat{S}_{pk1}$ and the constructions of exact confidence intervals for $\hat{S}_{pk2} - \hat{S}_{pk1}$ and $\hat{S}_{pk2} / \hat{S}_{pk1}$ are much more difficult.

3.2 Bootstrap Methodology

Traditionally, statistical research work has relied on the central limit theorem and normal approximations to obtain standard errors and confidence intervals. These techniques are valid only when the statistic, or some known transformation of statistic, is asymptotically normal distribution. Unfortunately, most process data in real world are not normal distributed. More than that, the distribution of data is usually unknown. A major motivation for the traditional reliance on normal-theory methods has been computational tractability. Access to powerful computation enables the use of statistics in new and varied way. Idealized models and assumptions can now be replaced with more realistic modeling or by virtually model-free analyses. Efron (1979, 1982) introduced a nonparametric, computational intensive but effective estimation method, called the "Bootstrap", which is a data based simulation technique for statistical inference. One can use the nonparametric bootstrap method to estimate the sampling distribution of a statistic, while assuming only that the observations are independent and identically distributed. The merit of the nonparametric bootstrap approach is that it does not rely on any assumptions regarding the underlying distribution. Rather than using distribution frequency tables to compute approximate p probability vales, the bootstrap method generates a unique sampling distribution based on the actual sample rather than the analytic method.

Most of PCIs literature concluded that the performance of bootstrap limits for PCIs are quite satisfactory in the majority of the cases. After Efron (1979, 1982) introduced the bootstrap, Efron and Tibshirani (1986) further developed three bootstrap confidence intervals: the standard bootstrap (SB) confidence interval, the percentile bootstrap (PB) confidence interval, and the biased-corrected percentile bootstrap (BCPB) confidence interval. Franklin and Wasserman (1991) proposed an initial study of these three methods for obtaining confidence intervals for C_{pk} when the process was normal distributed. Franklin and Wasserman (1992) also offered three bootstrap lower confidence limits for index C_p, C_{pk}, and C_{pm}. They compared the confidence interval from bootstrap and from parametric estimates. The simulation results show that, the bootstrap confidence limits perform as good as the lower confidence limits derived by the parametric method in the normal process environment. (see Chou *et al* (1990) for C_p, Bissell (1990) for C_{pk}, and Boyles (1991) for C_{pm}). These studies indicate that the bootstrap limits for PCIs are satisfactory in the cases.

In this thesis, the following four bootstrap confidence limits are employed to determine the lower confidence bounds of difference and ratio statistics and the results are used to select the better supplier of the two candidates. For $n_1 = n_2 = n$, let two bootstrap samples of size n drawn with replacement from the two original sample be denoted by $\left\{x_{11}^*, x_{21}^*, \ldots, x_{1n}^*\right\}$ and $\left\{x_{21}^*, x_{22}^*, \ldots, x_{2n}^*\right\}$. The bootstrap sample statistics $\bar{x}_1^*$, s_1^*, $\bar{x}_2^*$, and s_2^* are computed, as well as $\hat{S}_{pk1}^*$, and $\hat{S}_{pk2}^*$. A random sample of n^n possible resamples are drawn, the statistic is calculated by each of these, and the resulting empirical distribution is referred to as the bootstrap distribution of statistic. Due to the overwhelming computation time, it is not of practical interest to chose n^n such samples. Empirical work (Eforn and Tibshirani (1986)) indicated that a roughly minimum of 1,000 bootstrap resamples is usually sufficient to compute reasonable accurate confidence interval estimates for population parameters. For accuracy purpose, we consider B=3,000 bootstrap resamples (rather than 1,000). Thus, we take B=3,000 bootstrap estimates $\hat{\theta}^*$ = $(\hat{S}_{pk2}^* - \hat{S}_{pk1}^*)$ or $(\hat{S}_{pk2}^* / \hat{S}_{pk1}^*)$ of θ = $S_{pk2} - S_{pk1}$ or S_{pk2} / S_{pk1}, respectively, then order them from the smallest to the largest $\hat{\theta}_{(l)}^* = (\hat{S}_{pk2}^* - \hat{S}_{pk1}^*)_{(l)}$ or $\hat{\theta}_{(l)}^* = (\hat{S}_{pk2}^* / \hat{S}_{pk1}^*)_{(l)}$ where $l = 1, 2, \ldots, B$.

Four types of bootstrap confidence intervals, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), the biased corrected percentile bootstrap confidence interval (BCPB), and the bootstrap-t (BT) methods introduced by Efron (1981), and Efron and Tibshirani (1986) are conducted in this paper. The generic notation $\hat{\theta}$ and $\hat{\theta}^*$ are the estimator of θ and the associated ordered bootstrap estimate. Construction of a two-sided $100(1-2\alpha)\%$ confidence limit will be described. We note that a lower $100(1-\alpha)\%$ confidence limit can be obtained by using only a lower limit. The formulation details for the four types of confidence intervals are displayed as follows.

[A] Standard Bootstrap (SB) Method

Form the B bootstrap estimates $\hat{\theta}_{(l)}^*$, $l = 1, 2, \ldots, B$, the sample average and the sample standard deviation can be obtained as

$$\bar{\hat{\theta}}^* = \frac{1}{B}\sum_{l=1}^{B}\hat{\theta}_{(l)}^*, \quad S_\theta^* = \left(\frac{1}{B-1}\sum_{l=1}^{B}[\hat{\theta}_{(l)}^* - \bar{\hat{\theta}}^*]^2\right)^{1/2}.$$

The quantity S_θ^* is an estimator of the standard deviation of $\hat{\theta}$ if the distribution of $\hat{\theta}$ is approximately normal. Thus, the $100(1-2\alpha)\%$ SB confidence interval for θ can be constructed as

$$[\bar{\hat{\theta}}^* - z_\alpha S_\theta^*,\ \bar{\hat{\theta}}^* + z_\alpha S_\theta^*],$$

where $\hat{\theta}$ is the estimator of θ for the original sample, and z_α is the upper α quantile of the standard normal distribution.

[B] Percentile Bootstrap (PB) Method

From the ordered collection of $\hat{\theta}^*_{(l)}$, $l = 1,2,\ldots,B$, the α percentage and 1-α percentage points are used to obtain the $100(1-2\alpha)\%$ PB confidence interval for θ,

$$[\hat{\theta}^*_{(\alpha B)},\ \hat{\theta}^*_{((1-\alpha)B)}].$$

[C] Biased-Corrected Percentile Bootstrap (BCPB) Method

While the percentile confidence interval is intuitively appealing, it is possible that cause sampling errors, the bootstrap distribution may be biased. In other words, it is possible that bootstrap distributions using only a sample of the complete bootstrap distribution may be shifted higher or lower than would be expected. A three steps procedure is suggested to correct for the possible bias (Efron, 1982). First, using the ordered distribution of $\hat{\theta}^*$, calculate the probability $p_0 = P[\hat{\theta}^* \le \hat{\theta}_0]$. Second, we compute the inverse of the cumulative distribution function of a standard normal based upon p_0 as $z_0 = \Phi^{-1}(p_0)$, $p_L = \Phi(2z_0 - z_\alpha)$ $p_U = \Phi(2z_0 + z_\alpha)$. Finally, executing these steps to obtain the $100(1-2\alpha)\%$ BCPB confidence interval

$$[\hat{\theta}^*_{(p_L B)},\ \hat{\theta}^*_{(p_U B)}].$$

[D] Bootstrap-t (BT) method

By using bootstrap method to approximate the distribution of a statistic of the form $(\hat{\theta}-\theta)/S_{\hat{\theta}}$, the bootstrap approximation in this case is obtained by taking bootstrap samples from the original data values, calculating the corresponding estimates $\hat{\theta}^*$ and their estimated standard error, hence finding the bootstrapped T-values $T = (\hat{\theta}^* - \hat{\theta})/S_\theta^*$. The hope is that the generated distribution will mimic the distribution of T. The $100(1-2\alpha)\%$ BT confidence interval for θ may constitute as

$$[\hat{\theta}^* - t_\alpha^* S_{\hat{\theta}}^*,\ \hat{\theta}^* - t_{1-\alpha}^* S_{\hat{\theta}}^*],$$

where t_{α}^{*} and $t_{1-\alpha}^{*}$ are the upper α and $1-\alpha$ quantiles of the bootstrap t-distribution respectively, i.e. by finding the values that satisfy the two equations $P[(\hat{\theta}^{*} - \hat{\theta}) / S_{\theta}^{*} > t_{\alpha}^{*}] = \alpha$ and $P[(\hat{\theta}^{*} - \hat{\theta}) / S_{\theta}^{*} > t_{1-\alpha}^{*}] = 1 - \alpha$, for the generated bootstrap estimates.

4. Performance Comparisons of Four Bootstrap Methods

4.1 Simulation Layout Setting

There are mainly two important characteristics, the process location relative to its specification limits, and the process spread in process capability. The closer the process location is to the mid-point of the specification limits and the smaller the process spread, the more capable the process is. A mathematical relationship among the indices C_p, C_a, and S_{pk} can be established as:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\{\frac{1}{2}\Phi(\frac{USL-\mu}{\sigma}) + \frac{1}{2}\Phi(\frac{\mu-LSL}{\sigma})\}$$

$$= \frac{1}{3}\Phi^{-1}\{\frac{1}{2}\Phi(\frac{d-|\mu-m|}{\sigma}) + \frac{1}{2}\Phi(\frac{d+|\mu-m|}{\sigma})\}$$

$$= \frac{1}{3}\Phi^{-1}\{\frac{1}{2}\Phi(\frac{1-|\mu-m|/d}{\sigma/d}) + \frac{1}{2}\Phi(\frac{1+|\mu-m|/d}{\sigma/d})\}$$

$$= \frac{1}{3}\Phi^{-1}\{\frac{1}{2}\Phi[3C_pC_a] + \frac{1}{2}\Phi[3C_p(2-C_a)]\}.$$

Based on this relationship, it is note that we can combine several different combinations of C_p and C_a for the same S_{pk} value by setting between the process centering and the magnitude of process variation. Table 2 displays many different C_a values and the corresponding process spread of the magnitude of μ.

Table 2. C_a values and ranges of μ.

C_a value	Range of μ
$C_a = 1.00$	$\mu = m$
$0.75 < C_a < 1.00$	$0 < \|\mu - m\| < d/4$
$0.50 < C_a < 0.75$	$d/4 < \|\mu - m\| < d/2$
$0.25 < C_a < 0.50$	$d/2 < \|\mu - m\| < 3d/4$
$0.00 < C_a < 0.25$	$3d/4 < \|\mu - m\| < d$
$C_a = 0.00$	$\mu = LSL$ or $\mu = USL$
$C_a < 0.00$	$\mu < LSL$ or $\mu > USL$

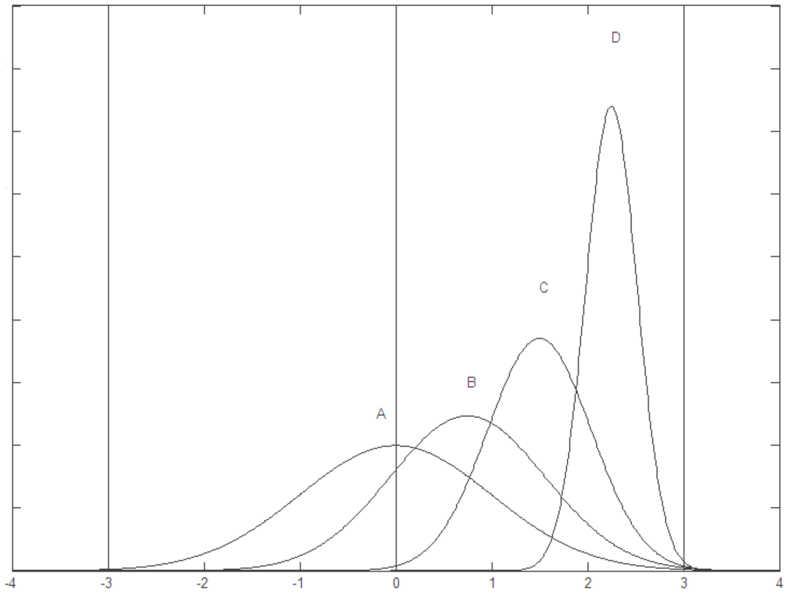


Figure 1. Four processes with S_{pk} =1.00.

Figure 1 plots four process with difference combination of (C_p, C_a) with $S_{pk} = 1.00$, *LSL*=10, *USL*=20, and *m*=15. i.e. $(C_p, C_a) = (1.00, 1.00)$ for process A, $(C_p, C_a) = (1.23661662, 0.75)$ for process B, $(C_p, C_a) = (1.85478349, 0.5)$ for process C, $(C_p, C_a) = (3.70956682, 0.25)$ for process D. These four processes have equivalent $S_{pk} = 1.00$, and all have yields equal to 99.73%, but constructed with different μ and σ. Hence, in order to make a comparative study among four bootstrap confidence limits, we take series of simulations to investigate the error probability and the selection power of difference and ratio testing statistics for the performance comparisons of four bootstrap methods. The setting values of parameters for two manufacturing suppliers used in the simulation study are given in Table 3. We investigate the performance of the methods with selected parameters for a wide range of index values and in on-target and off-target processes. For each combination, we generate 3,000 random samples, and the corresponding bootstrap confidence intervals for each of these samples are assessed in section 4.2.

Table 3. The parameter setting values for two manufacturing suppliers used in the simulation study under $S_{pk1} = S_{pk2} = 1.00$.

Case	S_{pk1}	C_{p1}	C_{a1}	S_{pk2}	C_{p2}	C_{a2}
1	1	1	1	1	1	1
2	1	1	1	1	1.23661662	3/4
3	1	1	1	1	1.85478349	1/2
4	1	1	1	1	3.70956682	1/4
5	1	1.23661662	3/4	1	1	1
6	1	1.23661662	3/4	1	1.23661662	3/4
7	1	1.23661662	3/4	1	1.85478349	1/2
8	1	1.23661662	3/4	1	3.70956682	1/4
9	1	1.85478349	1/2	1	1	1
10	1	1.85478349	1/2	1	1.23661662	3/4
11	1	1.85478349	1/2	1	1.85478349	1/2
12	1	1.85478349	1/2	1	3.70956682	1/4
13	1	3.70956682	1/4	1	1	1
14	1	3.70956682	1/4	1	1.23661662	3/4
15	1	3.70956682	1/4	1	1.85478349	1/2
16	1	3.70956682	1/4	1	3.70956682	1/4

4.2 Error Probability Analysis

The error probability is the first step which we want to investigate. It is the proportion of times that reject the null hypothesis $H_0 : S_{pk1} \geq S_{pk2}$, while $H_0 : S_{pk1} \geq S_{pk2}$ is true. Thus, we will calculate the proportion of times that the lower confidence bound of $S_{pk2} - S_{pk1}$ is positive and the lower confidence bound of S_{pk2} / S_{pk1} is larger than 1 for each case given in Table 2. We set sample size n=100 drawn with replacement , the bootstrap resamples B=3,000, and the single simulation is replicated N=3,000 times. We usually set that the probability of the error selection less than a maximum value α, referred to the α condition. The frequency of the error is a binomial random variable with N=3,000 and α=0.05. Thus, the 99% confidence interval for the error probability is $\alpha^* \pm Z_{0.005} \times \sqrt{\alpha^*(1-\alpha^*)/N} = 0.05 \pm 2.576 \times \sqrt{(0.05 \times 0.95)/3000} = 0.05 \pm 0.0103$. That is, one can have a 99% confidence that a “true 0.05 error probability” would have a range from 0.0397 to 0.061. Figure 2 and 3 show that the error probability of the four bootstrap methods for the difference and the ratio statistics with 16 different combination cases tabulated in Table 3.

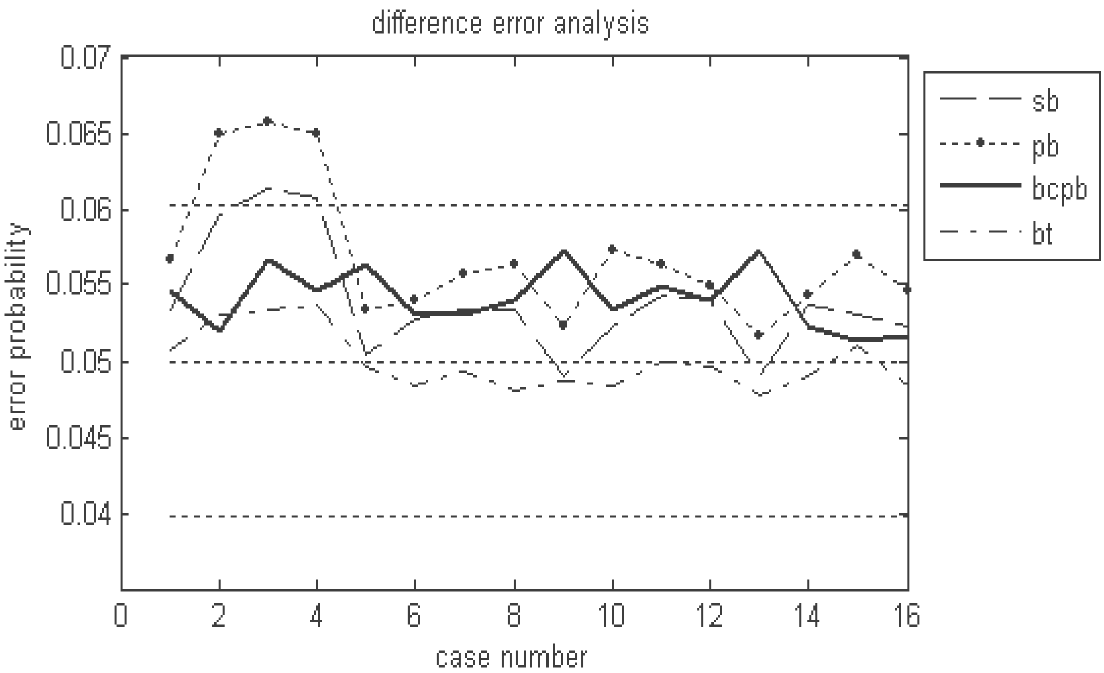


Figure 2. Error probability of four bootstraps under $S_{pk1} = S_{pk2} = 1.00$.

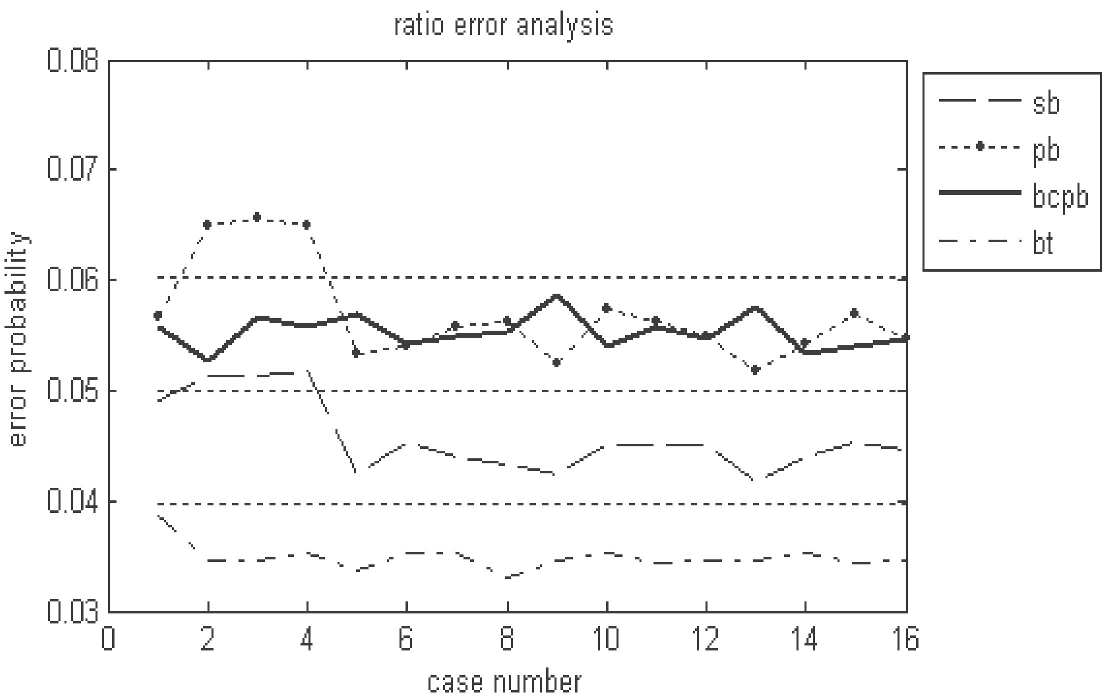


Figure 3. Error probability of four bootstraps under $S_{pk2} / S_{pk1} = 1.00$.

We have some results from Figure 2 and 3. In Figure 2, it shows that there are three cases out of the control limit (0.0397, 0.061) for the PB method for the difference statistic. There is only one occurrence out of the interval for SB method. We can note that with BCPB and BT methods, there is no case out of the control limit. That is, for different combinations of C_p and C_a for equal S_{pk} value have no significant effect in error probability with BCPB and BT methods.

As for the ratio statistic in Figure 3, there are three cases out of the control limit (0.0397, 0.061) for the PB method. For the BT method, all of these cases are behind the lower control limit. That is, BT is a conservative bootstrap method for ratio statistic. With the SB and BCPB methods, there is no occurrence out of the interval. Table 4 and 5 show the results of error probability analysis for difference and ratio test. It means that, for different combinations of C_p and C_a for equal S_{pk} value have no significant effect in error probability with SB and BCPB methods.

Table 4. The results of error probability analysis for difference test.

Bootstrap method of difference test	Mean of these 16 cases error	Standard deviation of these 16 cases error	Number of out of limits	Out of limits case
SB	0.053895	0.003684	1	3
PB	0.056896	0.004436	3	2,3,4
BCPB	0.054166	0.001966	0	None
BT	0.049917	0.001934	0	None

Table 5. The results of error probability analysis for ratio test.

Bootstrap method of ratio test	Mean of these 16 cases error	Standard deviation of these 16 cases error	Number of out of limits	Out of limits case
SB	0.045708	0.003297	0	None
PB	0.056896	0.004436	3	2,3,4
BCPB	0.055314	0.001595	0	None
BT	0.034917	0.001189	16	All

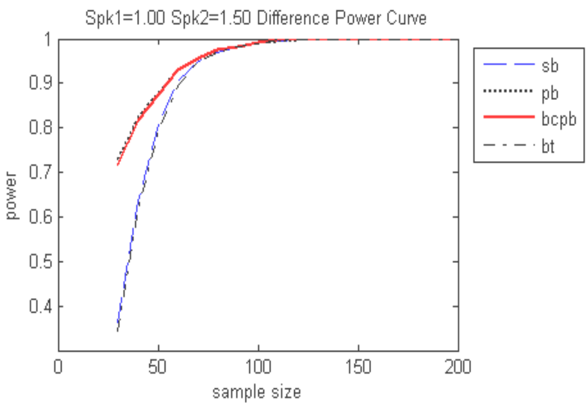
Besides that, an average lower confidence bound and the standard deviation of the lower confidence bound were calculated based on the N=3,000 different trials. Table 6 takes four of sixteen cases to show that the average lower confidence bound and the standard deviation of the lower confidence bound for each of the four different bootstrap methods. The results of all cases are summarized in Table 12.

Table 6. Simulation results of the four bootstrap methods for the difference and ratio statistics.

S_{pk1}	C_{p1}	C_{a1}	S_{pk2}	C_{p2}	C_{a2}	Bootstrap Method	Difference			Ratio		
							Error probability	Average LCB	Std. of LCB	Error probability	Average LCB	Std. of LCB
1	1	1	1	1	1	SB	0.05333	-0.16505	0.10305	0.04900	0.84596	0.08617
						PB	0.05667	-0.16490	0.10415	0.05667	0.85486	0.08679
						BCPB	0.05467	-0.16495	0.10370	0.05567	0.85475	0.08658
						BT	0.05067	-0.16466	0.10118	0.03867	0.83317	0.08529
1	1.236617	3/4	1	1	1	SB	0.05033	-0.17027	0.10184	0.04233	0.84187	0.08462
						PB	0.05333	-0.17004	0.10302	0.05333	0.85058	0.08527
						BCPB	0.05633	-0.16281	0.10183	0.05700	0.85639	0.08556
						BT	0.04967	-0.16656	0.09955	0.03367	0.83273	0.08385
1	1.854783	1/2	1	3.709567	1/4	SB	0.05400	-0.16182	0.10107	0.04500	0.84966	0.08423
						PB	0.05500	-0.16168	0.10243	0.05500	0.85829	0.08501
						BCPB	0.05400	-0.16179	0.10104	0.05467	0.85812	0.08467
						BT	0.04967	-0.16148	0.09846	0.03467	0.83720	0.08297
1	3.709567	1/4	1	3.709567	1/4	SB	0.05233	-0.16188	0.10116	0.04467	0.84960	0.08431
						PB	0.05467	-0.16167	0.10244	0.05467	0.85823	0.08507
						BCPB	0.05167	-0.16167	0.10100	0.05467	0.85817	0.08466
						BT	0.04833	-0.16150	0.09863	0.03467	0.83721	0.08316

4.3 Selection Power Analysis

After the error probability analysis, we can roughly ensure that, there is less effect for different combinations of C_p and C_a for equal S_{pk} value with difference and ratio statistic. In order to compare the performance of these four bootstrap methods, we conduct further simulations of selection power with different sample sizes *n*=30(10)200 for $S_{pk1} = 1.00$, and $S_{pk2} = 1.05(0.05)1.50$. The selection power calculates the probability of rejecting the null hypothesis $H_0 : S_{pk1} \geq S_{pk2}$ while actually $H_1 : S_{pk1} < S_{pk2}$ is true. For the difference statistic, the selection power computes the proportion of times that the lower confidence bound of $S_{pk2} - S_{pk1}$ is positive in the simulation. Similarly, for the ratio statistic, the selection power computes the proportion of times that the lower confidence bound of S_{pk2} / S_{pk1} is larger than 1. Figures 4-5 show the power of the four bootstrap methods for the difference and ratio statistic with sample size *n*=30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.50$, respectively. The power curves for $S_{pk1} = 1.00$, $S_{pk2} = 1.05(0.05)1.50$, and *n*=30(10)200 are showed in Figures 13-32.



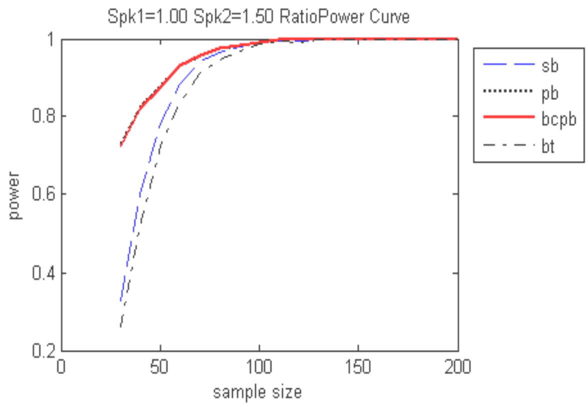


Figure 4. The selection power of the four bootstrap methods for the difference statistic with sample size n=30(10)200.

Figure 5. The selection power of the four bootstrap methods for the ratio statistic with sample size n= 30(10)200.

In Figure 4 and Figure 5, we find that PB and BCPB methods are much powerful under the same sample size. On the contrary, SB and BT methods have larger required sample size with fixed selection power. Under the two considerations of error probability above and selection power analysis, the BCPB method has more correct error probability and better selection power with fixed sample size. Consequently, we recommend the best of these four bootstrap methods is the BCPB method.

5. Supplier Selection Based on BCPB Method

5.1 Sample Size Determination with Designated Selection Power

In general, if a new supplier #2 (S2) wants to compete for the orders by claiming that its capability is better than the existing supplier #1 (S1), the new S2 has to convince purchaser with a prescribed confidence level information to justify the claim. Therefore, the sample size required for designated selection power must be determined to collect actual data from the factories. We investigate the BCPB method with *B*=3,000 bootstrap resamples, and the each simulation was then replicated with *N*=3,000 times. For convenience of applications, we tabulate the sample sizes required for various designated selection power = 0.90, 0.95, 0.975, and 0.99 under error probability $\alpha = 0.05$. The selection power calculates the probability of rejecting the null hypothesis $H_0 : S_{pk1} \geq S_{pk2}$ while actually $H_1 : S_{pk1} < S_{pk2}$ is true. Tables 7-8 show the sample size required of the BCPB method for the difference with $S_{pk1} = 1.00$ and $S_{pk2} = 1.10(0.05)1.50$ and ratio statistics with $S_{pk2} = 1.10(0.05)1.50$. We also calculate the sample size required for $S_{pk1} = 1.30$ and $S_{pk2} = 1.40(0.05)1.80$ for difference and ratio statistics in Table 9 and 10.

Table 7. Sample size required of BCPB method for the difference statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.00$, $S_{pk2} = 1.10(0.05)1.50$.

S_{pk1} S_{pk2}	1 1.1	1 1.15	1 1.2	1 1.25	1 1.3	1 1.35	1 1.4	1 1.45	1 1.5
90%	941	444	257	171	125	95	77	63	53
95%	1188	541	332	217	155	121	97	80	67
97.5%	1400	666	397	265	184	143	113	100	81
99%	1777	807	463	333	228	184	133	115	93

Table 8. Sample size required of BCPB method for the ratio statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.00$, $S_{pk2} = 1.10(0.05)1.50$.

S_{pk1}	1	1	1	1	1	1	1	1	1
S_{pk2}	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
90%	938	443	263	169	126	95	76	63	51
95%	1191	563	326	219	151	118	96	81	66
97.5%	1416	666	401	257	189	139	116	94	81
99%	1716	822	469	304	233	184	150	116	96

Table 9. Sample size required of BCPB method for the difference statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.30$, $S_{pk2} = 1.40(0.05)1.80$.

S_{pk1}	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
S_{pk2}	1.4	1.45	1.5	1.55	1.6	1.65	1.7	1.75	1.8
90%	1596	713	415	271	195	146	117	92	78
95%	1916	891	521	354	251	190	155	125	102
97.5%	2350	1088	643	413	338	230	176	147	119
99%	2925	1350	763	525	382	279	227	178	150

Table 10. Sample size required of BCPB method for the ratio statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.30$, $S_{pk2} = 1.40(0.05)1.80$.

S_{pk1}	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
S_{pk2}	1.4	1.45	1.5	1.55	1.6	1.65	1.7	1.75	1.8
90%	1596	721	421	275	198	148	121	96	77
95%	1965	917	514	350	250	189	152	126	102
97.5%	2397	1124	652	424	313	239	185	144	124
99%	2829	1297	749	512	376	278	226	189	152

For the convenience of observation, Figures 6-9 depict sample size curves based on the four sample size tables, respectively.

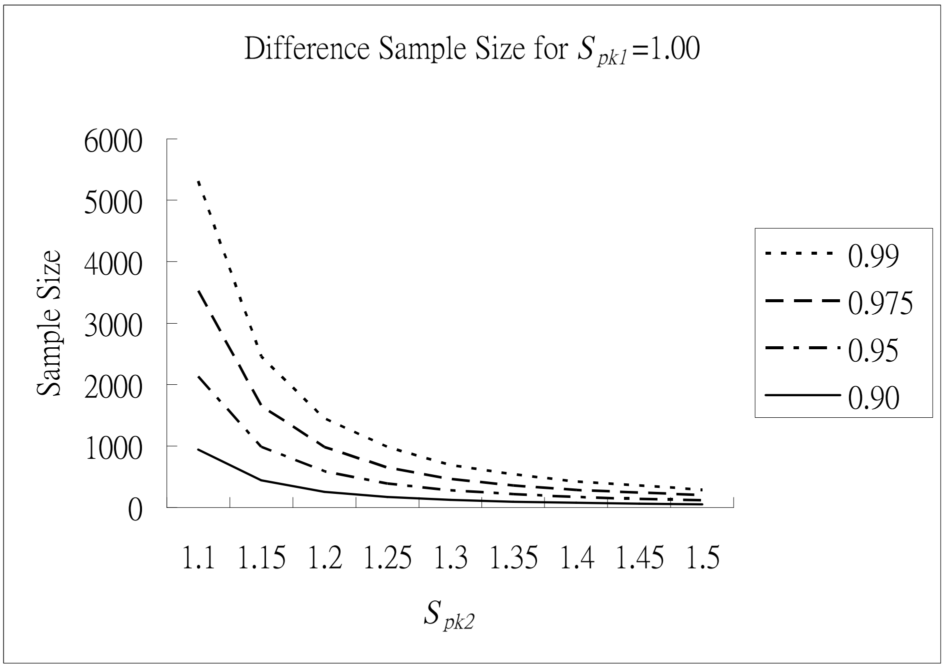


Figure 6. The sample size curve for the difference statistic under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.00$, $S_{pk2} = 1.10(0.05)1.50$.

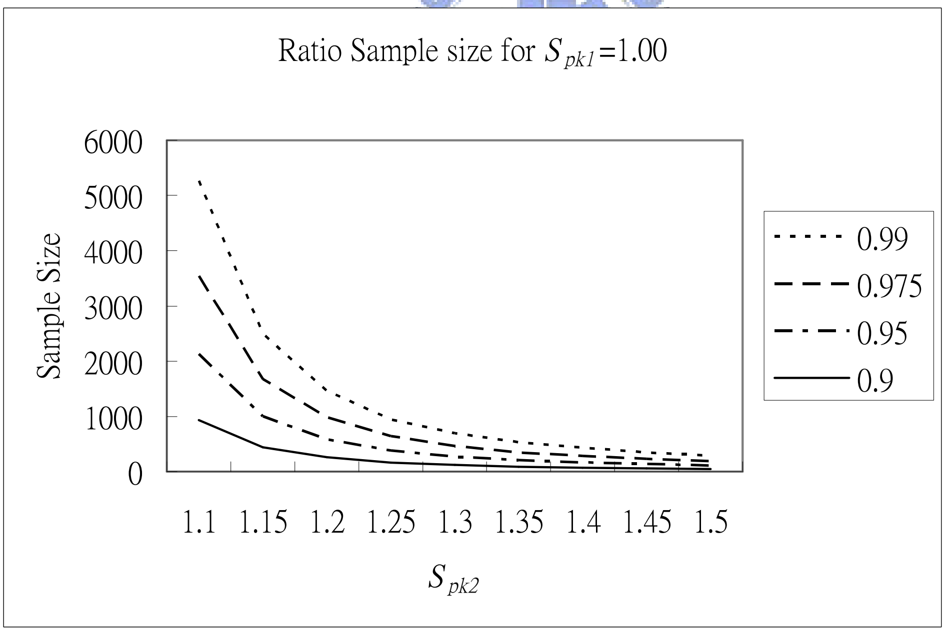


Figure 7. The sample size curve for the ratio statistic under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.00$, $S_{pk2} = 1.10(0.05)1.50$.

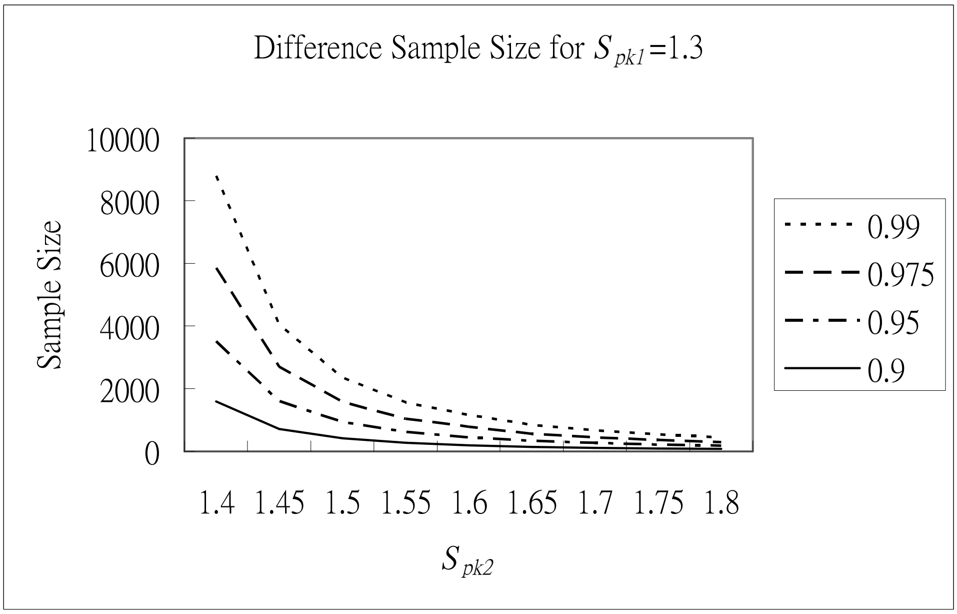


Figure 8. The sample size curve for the difference statistic under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.3$, $S_{pk2} = 1.40(0.05)1.80$.

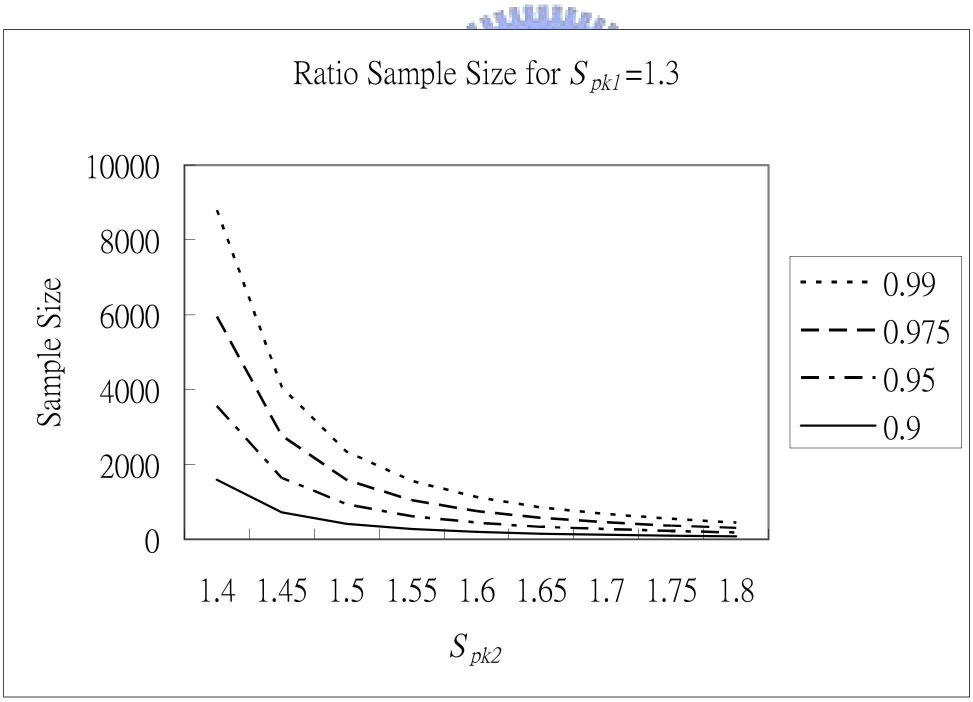


Figure 9. The sample size curve for the ratio statistic under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $S_{pk1} = 1.3$, $S_{pk2} = 1.40(0.05)1.80$.

From these figures, we can note that the larger the value of the difference $\delta = S_{pk2} - S_{pk1}$ or the ratio $\gamma = S_{pk2} / S_{pk1}$ between two suppliers, the smaller the sample size required for fixed selection power. For fixed δ or γ and S_{pk1}, the sample size required increases as designated selection power increases. Besides, the sample size required is very similar either for the difference or the ratio statistics. This phenomenon can be explained easily, since the smaller of the difference and the larger designated selection power, the more collected sample is required to account for the smaller uncertainty in the estimation.

5.2 Selecting the Better Supplier

In this supplier selection problem, the practitioner should set the present minimum requirement of S_{pk} values, and the minimal difference δ or the minimal ratio γ must be differentiated between suppliers with designated selection power. The practitioner alternatively might check Tables 7-10 for the sample size required under error probability $\alpha = 0.05$, with designated selection power = 0.90, 0.95, 0.975, 0.99. After that, based on the BCPB method, if the LCB of $\hat{S}_{pk2} - \hat{S}_{pk1}$ is positive or the LCB of $\hat{S}_{pk2} / \hat{S}_{pk1}$ is greater than 1, then we can conclude that the supplier #2 is better than the supplier #1. Otherwise, we do not have sufficient information to reject the null hypothesis $H_0 : S_{pk1} \geq S_{pk2}$. That is, we would believe that the existing supplier #1 is better than the new supplier #2.

6. Application Example：Color Filter Supplier Selection

Thin-film transistor liquid-crystal display (TFT-LCD) is one of the potential module of the high-tech products in the communication, information and consumer electronics industries. The TFT-LCD consumes less energy and weighs less compared to a cathode-ray tube (CRT). Besides that, it has emerged as the most widely used display solution, due to its high reliability, viewing quality and performance, compact size and environment-friendly features.

The basic structure of a TFT-LCD panel may be thought of as two glass substrates sandwiching a layer of liquid crystal. The front glass substrate is fitted with a color filter, while the back glass substrate has transistors fabricated on it. When voltage is applied to a transistor, the liquid crystal is bent, allowing light to pass through to form a pixel. A light source is located at the back of the panel and is called a backlight unit. The front glass substrate is fitted with a color filter, which gives each pixel its own color. Figure 10 shows the combination of the structure.

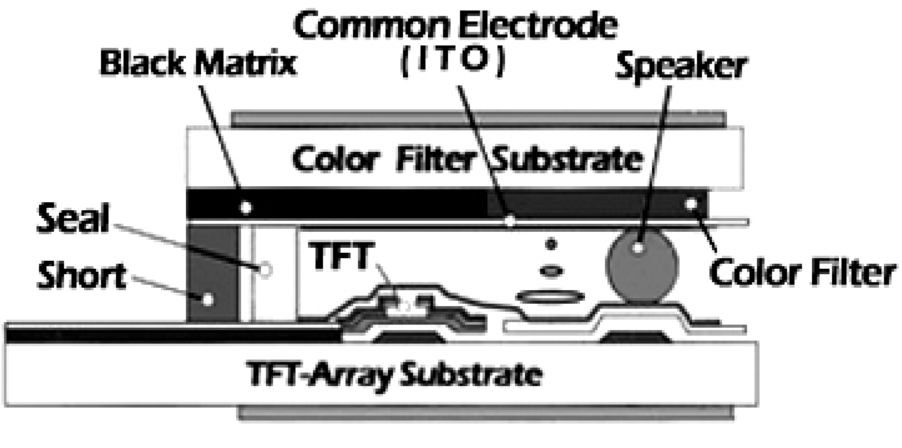


Figure 10. The combination of TFT-LCD structure.

The color filter is the most key component for a TFT-LCD. Many companies invest in producing a larger color filter to reduce the production cost. Competition in this market is very fierce. The thickness of the color filter is one of the most important quality characteristics. If the thickness of color filter is not in control, the TFT-LCD product may result in a certain degree of aberration.

The example is taken from a TFT-LCD manufacturing company, located in a science-based industrial park in Taiwan. The company would like to determine which of the two color filter suppliers has better process capability. For a particular model of the color filter investigated, the *USL* of a color filter thickness is set to 0.7mm, the *LSL* of a color filter thickness is set to 0.56mm, and the target value of a color filter thickness is set to 0.63mm.

6.1 Data Analysis and Supplier Selection

For the supplier selection problem, the practitioner should input the minimal requirement of S_{pk} value first. Second, the minimal difference of S_{pk} between these two suppliers with a designated selection power has to be set. Then we could decide the sample size based on Tables 7-10. In this case, the upper specification limit is 0.7mm, the lower specification limit is 0.56mm, and the target value is 0.63mm. The minimal requirement for the color filter product is 1.00, and the minimal difference between these two suppliers is 0.3, with selection power 0.95. By checking Tables 7-10, the sample size required for the difference statistics is 155, and for the ratio statistics is 151. We take 155 samples for S1 and S2, respectively. All sample data for two suppliers are showed in tables 15-16.

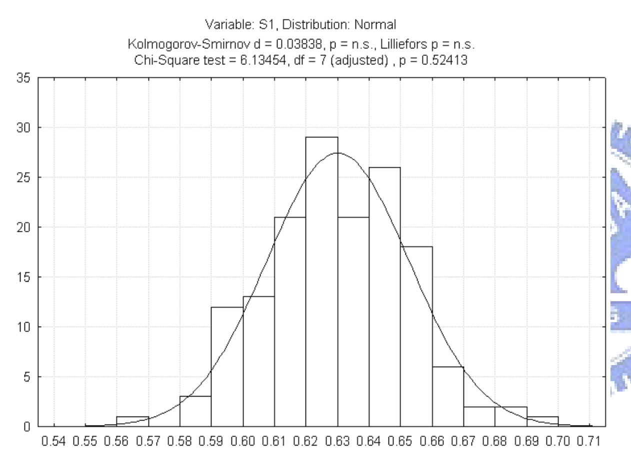


Figure 11. Histogram of data S1.

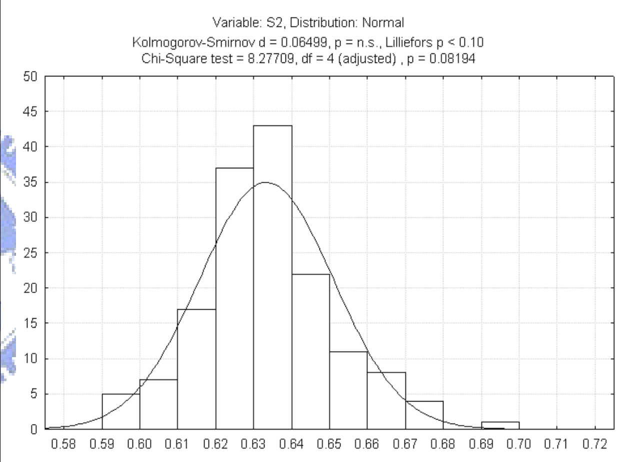


Figure 12. Histogram of data S2.

Figures 11-12 show the histogram of the 155 samples for S1 and S2. We use Kolmogorov–Smirnov test to check if these two suppliers' data are normal distributed. The statistic d for S1 is 0.038, and the statistic d for S2 is 0.065. Because both of these two p-values are greater than 0.05, we can not reject the null hypothesis. Thus, we conclude that the sample data for the two suppliers can be regarded as normal processes. We calculate the sample means, sample standard deviations, and the sample estimators $\hat{S}_{pk}$ for S1 and S2, summarized in Table 11.

Table 11. The calculated sample statistics for two suppliers.

	$\bar{x}$	s	$\hat{S}_{pk}$
S1	0.630129	0.022558	1.0344
S2	0.633369	0.017689	1.2973

We execute the Matlab program to obtain the LCB for the difference between these two processes $\hat{S}_{pk2} - \hat{S}_{pk1}$ is 0.09357, and the LCB for the ratio $\hat{S}_{pk2} / \hat{S}_{pk1}$ is 1.0865. Therefore, we can conclude that S2 is better than the present supplier S1.

7. Conclusions

Supplier selection problem is an important issue in the manufacturing industry. The decision maker usually faces the problem of selecting the better supplier between two candidates. For most manufacturing factories, process yield is the fundamental criterion for supplier selection. The index S_{pk} provides an exact measure on the process yield. However, the supplier selection problem based on index S_{pk} has not been done.

In this thesis, we compared the performance of S_{pk2} - S_{pk1} and S_{pk2}/S_{pk1} with four different bootstrap methods including the standard bootstrap (SB), the percentile bootstrap (PB), the biased-corrected percentile bootstrap (BCPB), and the bootstrap-t (BT) methods. In error probability analysis, we found that SB and BCPB methods have stable error probabilities for both difference and ratio test. PB and BCPB methods are much powerful under the same sample size in selection power analysis. Thus, the performance of BCPB method is better than the other three methods. For practitioner's convenience, the useful information about the sample size required with designated selection power based on the BCPB method was tabulated. After that, we investigated a real world case on the color filter manufacturing process, and demonstrated the applicability of the proposed method step by step in the end.

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Table 12. The error probability of four bootstrap methods for the difference and ratio statistic with 16 combinations of (C_{p1}, C_{a1}) and (C_{p2}, C_{a2}) under S_{pk1}=S_{pk2}=1.00.

USL = 20, LSL = 10, d = 5, m = 15							n = 100 (Difference Test)			n = 100 (Ratio Test)		
S_{pk1}	C_{p1}	C_{a1}	S_{pk2}	C_{p2}	C_{a2}	Bootstrap	P	LBound	Std	P	LBound	Std
1	1	1	1	1	1	SB	0.05333	-0.16505	0.10305	0.04900	0.84596	0.08617
						PB	0.05667	-0.16490	0.10415	0.05667	0.85486	0.08679
						BCPB	0.05467	-0.16495	0.10370	0.05567	0.85475	0.08658
						BT	0.05067	-0.16466	0.10118	0.03867	0.83317	0.08592
1	1	1	1	1.23661662	3/4	SB	0.05967	-0.15686	0.10305	0.05133	0.85359	0.08650
						PB	0.06500	-0.15681	0.10429	0.06500	0.86235	0.08725
						BCPB	0.05200	-0.16387	0.10345	0.05267	0.85649	0.08630
						BT	0.05300	-0.15967	0.10076	0.03467	0.83762	0.08509
1	1	1	1	1.85478349	1/2	SB	0.06133	-0.15665	0.10299	0.05133	0.85378	0.08644
						PB	0.06567	-0.15656	0.10426	0.06567	0.86256	0.08719
						BCPB	0.05667	-0.16385	0.10348	0.05667	0.85653	0.08634
						BT	0.05333	-0.15956	0.10065	0.03467	0.83773	0.08498
1	1	1	1	3.70956682	1/4	SB	0.06067	-0.15659	0.10298	0.05167	0.85384	0.08647
						PB	0.06500	-0.15649	0.10416	0.06500	0.86265	0.08715
						BCPB	0.05467	-0.16381	0.10343	0.05567	0.85663	0.08628
						BT	0.05367	-0.15951	0.10068	0.03533	0.83780	0.08505

$USL = 20$, $LSL = 10$, $d = 5$, $m = 15$							$n = 100$ (Difference Test)			$n = 100$ (Ratio Test)		
S_{pk1}	C_{p1}	C_{a1}	S_{pk2}	C_{p2}	C_{a2}	Bootstrap	P	LBound	Std	P	LBound	Std
1	1.23661662	3/4	1	1	1	SB	0.05033	-0.17027	0.10184	0.04233	0.84187	0.08462
						PB	0.05333	-0.17004	0.10302	0.05333	0.85058	0.08527
						BCPB	0.05633	-0.16281	0.10183	0.05700	0.85639	0.08556
						BT	0.04967	-0.16656	0.09955	0.03367	0.83273	0.08385
1	1.23661662	3/4	1	1.23661662	3/4	SB	0.05267	-0.16194	0.10119	0.04533	0.84953	0.08434
						PB	0.05400	-0.16179	0.10240	0.05400	0.85813	0.08505
						BCPB	0.05300	-0.16191	0.10098	0.05433	0.85794	0.08470
						BT	0.04833	-0.16155	0.09869	0.03533	0.83710	0.08319
1	1.23661662	3/4	1	1.85478349	1/2	SB	0.05333	-0.16183	0.10112	0.04400	0.84964	0.08422
						PB	0.05567	-0.16169	0.10240	0.05567	0.85823	0.08499
						BCPB	0.05300	-0.16172	0.10114	0.05500	0.85813	0.08476
						BT	0.04933	-0.16146	0.09867	0.03533	0.83718	0.08309
1	1.23661662	3/4	1	3.70956682	1/4	SB	0.05333	-0.16173	0.10112	0.04333	0.84971	0.08426
						PB	0.05633	-0.16157	0.10227	0.05633	0.85835	0.08493
						BCPB	0.05400	-0.16182	0.10091	0.05533	0.85812	0.08451
						BT	0.04800	-0.16145	0.09865	0.03300	0.83722	0.08315

USL = 20, LSL = 10, d = 5, m = 15							n = 100 (Difference Test)			n = 100 (Ratio Test)		
S_{pk1}	C_{p1}	C_{a1}	S_{pk2}	C_{p2}	C_{a2}	Bootstrap	P	LBound	Std	P	LBound	Std
1	1.85478349	1/2	1	1	1	SB	0.04900	-0.17033	0.10172	0.04233	0.84180	0.08448
						PB	0.05233	-0.17010	0.10305	0.05233	0.85053	0.08526
						BCPB	0.05733	-0.16284	0.10197	0.05867	0.85637	0.08567
						BT	0.04867	-0.16653	0.09944	0.03467	0.83273	0.08380
1	1.85478349	1/2	1	1.23661662	3/4	SB	0.05233	-0.16204	0.10118	0.04500	0.84945	0.08431
						PB	0.05733	-0.16190	0.10247	0.05733	0.85809	0.08507
						BCPB	0.05333	-0.16172	0.10104	0.05400	0.85813	0.08462
						BT	0.04833	-0.16159	0.09867	0.03533	0.83709	0.08318
1	1.85478349	1/2	1	1.85478349	1/2	SB	0.05433	-0.16185	0.10114	0.04500	0.84963	0.08428
						PB	0.05633	-0.16166	0.10238	0.05633	0.85828	0.08502
						BCPB	0.05500	-0.16161	0.10090	0.05567	0.85826	0.08466
						BT	0.05000	-0.16139	0.09861	0.03433	0.83734	0.08311
1	1.85478349	1/2	1	3.70956682	1/4	SB	0.05400	-0.16182	0.10107	0.04500	0.84966	0.08423
						PB	0.05500	-0.16168	0.10243	0.05500	0.85829	0.08501
						BCPB	0.05400	-0.16179	0.10104	0.05467	0.85812	0.08467
						BT	0.04967	-0.16148	0.09846	0.03467	0.83720	0.08297

USL = 20, LSL = 10, d = 5, m = 15							n = 100 (Difference Test)			n = 100 (Ratio Test)		
S_{pk1}	C_{p1}	C_{a1}	S_{pk2}	C_{p2}	C_{a2}	Bootstrap	P	LBound	Std	P	LBound	Std
1	3.70956682	1/4	1	1	1	SB	0.04900	-0.17023	0.10168	0.04167	0.84190	0.08449
						PB	0.05167	-0.17002	0.10293	0.05167	0.85062	0.08525
						BCPB	0.05733	-0.16279	0.10195	0.05767	0.85643	0.08570
						BT	0.04767	-0.16656	0.09947	0.03467	0.83276	0.08386
1	3.70956682	1/4	1	1.23661662	3/4	SB	0.05367	-0.16196	0.10118	0.04400	0.84952	0.08432
						PB	0.05433	-0.16170	0.10241	0.05433	0.85824	0.08504
						BCPB	0.05233	-0.16158	0.10089	0.05333	0.85827	0.08458
						BT	0.04900	-0.16162	0.09862	0.03533	0.83714	0.08316
1	3.70956682	1/4	1	1.85478349	1/2	SB	0.05300	-0.16188	0.10113	0.04533	0.84959	0.08431
						PB	0.05700	-0.16167	0.10242	0.05700	0.85826	0.08505
						BCPB	0.05133	-0.16174	0.10085	0.05400	0.85814	0.08453
						BT	0.05100	-0.16147	0.09849	0.03433	0.83723	0.08304
1	3.70956682	1/4	1	3.70956682	1/4	SB	0.05233	-0.16188	0.10116	0.04467	0.84960	0.08431
						PB	0.05467	-0.16167	0.10244	0.05467	0.85823	0.08507
						BCPB	0.05167	-0.16167	0.10100	0.05467	0.85817	0.08466.
						BT	0.04833	-0.16150	0.09863	0.03467	0.83721	0.08316

Table 13. Selection power of the four bootstrap methods for difference statistic with sample size n = 30(10)200.

n	S_{pk1}	1	1	1	1	1	1	1	1	1	1
	S_{pk2}	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
30	SB	0.07467	0.11967	0.16467	0.21033	0.26133	0.30167	0.32867	0.35700	0.36100	0.36167
	PB	0.10067	0.15900	0.22267	0.29000	0.36367	0.43733	0.51933	0.60000	0.66667	0.72967
	BCPB	0.09433	0.14833	0.21000	0.27867	0.35200	0.42633	0.50700	0.58633	0.65900	0.71800
	BT	0.06567	0.09767	0.14000	0.18433	0.23633	0.27500	0.30100	0.33000	0.33467	0.34300
40	SB	0.09067	0.14833	0.22067	0.30367	0.38867	0.46300	0.53133	0.58800	0.62167	0.63500
	PB	0.10333	0.16367	0.24233	0.33033	0.42700	0.51800	0.61033	0.68600	0.76167	0.82267
	BCPB	0.09800	0.15900	0.23700	0.32667	0.41367	0.51033	0.60333	0.67600	0.75300	0.81733
	BT	0.08100	0.13300	0.19867	0.27700	0.35867	0.43700	0.50567	0.55833	0.59400	0.61533
50	SB	0.09200	0.16700	0.25133	0.34500	0.45600	0.55767	0.64500	0.72233	0.78400	0.80867
	PB	0.10600	0.17833	0.26500	0.36733	0.47700	0.58233	0.67400	0.76033	0.83200	0.88000
	BCPB	0.09833	0.17333	0.26033	0.35867	0.46500	0.57533	0.66767	0.75600	0.82667	0.87600
	BT	0.08267	0.15200	0.23400	0.31933	0.42867	0.52867	0.61600	0.69267	0.76700	0.79367
60	SB	0.10333	0.19033	0.29300	0.39533	0.51700	0.62033	0.73133	0.80800	0.86400	0.90367
	PB	0.11400	0.19933	0.30067	0.41367	0.52900	0.63567	0.74700	0.82500	0.88567	0.93167
	BCPB	0.11100	0.20067	0.29767	0.40500	0.52267	0.63233	0.73867	0.82000	0.88000	0.92967
	BT	0.09600	0.17667	0.27267	0.37767	0.49667	0.60200	0.70833	0.79067	0.85233	0.89400
70	SB	0.10833	0.19967	0.30167	0.43267	0.56367	0.70133	0.79933	0.87167	0.92200	0.95100
	PB	0.11400	0.20800	0.31100	0.44367	0.57600	0.71233	0.81000	0.88233	0.93067	0.95667
	BCPB	0.11133	0.20400	0.30867	0.43733	0.57100	0.70867	0.80367	0.87900	0.92933	0.95700
	BT	0.10167	0.18567	0.29133	0.40767	0.54733	0.68933	0.78433	0.86267	0.91367	0.94900
80	SB	0.11000	0.21000	0.34500	0.49133	0.63300	0.75533	0.84067	0.90200	0.94300	0.97033
	PB	0.11600	0.21900	0.35700	0.50233	0.64400	0.76167	0.84800	0.90700	0.94867	0.97567
	BCPB	0.11367	0.21967	0.35200	0.49800	0.6420	0.76200	0.84767	0.90600	0.94833	0.97533
	BT	0.10600	0.20100	0.32767	0.47833	0.61767	0.74267	0.83233	0.89500	0.93800	0.96800
90	SB	0.11700	0.23267	0.36000	0.51600	0.66567	0.78567	0.87267	0.92833	0.96367	0.98233
	PB	0.12233	0.23867	0.37000	0.52433	0.67167	0.79367	0.88000	0.93233	0.96433	0.98367
	BCPB	0.12067	0.23467	0.36967	0.52233	0.66967	0.78733	0.87633	0.92733	0.96333	0.98267
	BT	0.11333	0.21933	0.34833	0.50400	0.65800	0.77600	0.86567	0.92500	0.96100	0.98100
100	SB	0.12567	0.24400	0.39567	0.57167	0.72767	0.84167	0.91400	0.95633	0.98067	0.99067
	PB	0.13000	0.24967	0.40300	0.57967	0.73367	0.84433	0.91967	0.95800	0.98133	0.99167
	BCPB	0.12800	0.24633	0.40167	0.57100	0.73167	0.84167	0.91933	0.95667	0.98000	0.99200
	BT	0.11900	0.23500	0.38733	0.56300	0.71867	0.83300	0.91000	0.95233	0.97933	0.99000

n	S_{pk1}	1	1	1	1	1	1	1	1	1	1
	S_{pk2}	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
110	SB	0.13000	0.27033	0.44200	0.62133	0.76167	0.87000	0.93567	0.97333	0.98933	0.99667
	PB	0.13200	0.27700	0.44667	0.62833	0.76633	0.87433	0.93900	0.97333	0.99100	0.99667
	BCPB	0.13200	0.27133	0.44700	0.62100	0.76633	0.87200	0.93867	0.97333	0.98933	0.99700
	BT	0.12567	0.26000	0.42667	0.61633	0.75300	0.86500	0.93200	0.97167	0.98833	0.99700
120	SB	0.13033	0.26633	0.44900	0.63033	0.77800	0.87800	0.94700	0.97700	0.98900	0.99733
	PB	0.13367	0.26967	0.45300	0.63500	0.78233	0.88433	0.94900	0.97767	0.98867	0.99700
	BCPB	0.13200	0.26867	0.45000	0.63300	0.78033	0.88500	0.94767	0.97700	0.98867	0.99767
	BT	0.12700	0.26133	0.43667	0.62267	0.76700	0.87633	0.94500	0.97467	0.98733	0.99733
130	SB	0.14067	0.29600	0.47900	0.65967	0.81467	0.90700	0.96033	0.98600	0.99400	0.99867
	PB	0.14367	0.30300	0.48600	0.66667	0.81633	0.91133	0.96200	0.98667	0.99533	0.99867
	BCPB	0.14300	0.30000	0.47733	0.66100	0.81633	0.90800	0.96067	0.98633	0.99533	0.99867
	BT	0.13800	0.28767	0.47200	0.65300	0.80800	0.90133	0.95867	0.98533	0.99467	0.99867
140	SB	0.14233	0.31300	0.51133	0.70433	0.84600	0.92967	0.97000	0.98867	0.99700	0.99933
	PB	0.14667	0.31633	0.51800	0.71100	0.85267	0.93167	0.97267	0.98933	0.99767	0.99933
	BCPB	0.14833	0.31567	0.51600	0.70733	0.85167	0.92967	0.97233	0.98933	0.99767	0.99933
	BT	0.13667	0.30700	0.50367	0.69567	0.83867	0.92500	0.96900	0.98800	0.99733	0.99933
150	SB	0.15133	0.31600	0.53033	0.72433	0.86900	0.94267	0.97700	0.99233	0.99833	0.99933
	PB	0.15467	0.32033	0.53267	0.73067	0.87000	0.94533	0.97900	0.99300	0.99833	0.99933
	BCPB	0.15400	0.31800	0.53200	0.72933	0.86833	0.94533	0.97800	0.99267	0.99833	0.99933
	BT	0.15100	0.31000	0.52333	0.72067	0.86567	0.94167	0.97567	0.99233	0.99833	0.99933
160	SB	0.15167	0.32600	0.52800	0.73167	0.86867	0.94933	0.98100	0.99500	0.99900	1.00000
	PB	0.15700	0.33167	0.53467	0.73633	0.87100	0.95267	0.98133	0.99467	0.99900	1.00000
	BCPB	0.15600	0.33167	0.53200	0.73733	0.87100	0.95233	0.98233	0.99467	0.99900	1.00000
	BT	0.14800	0.32233	0.52067	0.72733	0.86667	0.94833	0.98033	0.99467	0.99900	1.00000
170	SB	0.16500	0.34467	0.56933	0.77000	0.89200	0.96067	0.99000	0.99800	0.99967	1.00000
	PB	0.16700	0.34733	0.57167	0.77267	0.89400	0.96233	0.99067	0.99800	0.99967	1.00000
	BCPB	0.16567	0.34567	0.56967	0.77033	0.89467	0.96200	0.98900	0.99833	0.99967	1.00000
	BT	0.16133	0.34000	0.55933	0.76200	0.89300	0.96167	0.98933	0.99767	0.99967	1.00000
180	SB	0.17500	0.37333	0.60000	0.78233	0.90867	0.96733	0.99067	0.99833	0.99933	0.99967
	PB	0.17700	0.37700	0.60567	0.78867	0.91000	0.96833	0.99067	0.99833	0.99933	1.00000
	BCPB	0.17467	0.37467	0.60467	0.78633	0.91067	0.97133	0.99167	0.99833	0.99933	1.00000
	BT	0.17067	0.36900	0.59333	0.77567	0.90700	0.96667	0.99067	0.99800	0.99933	0.99967

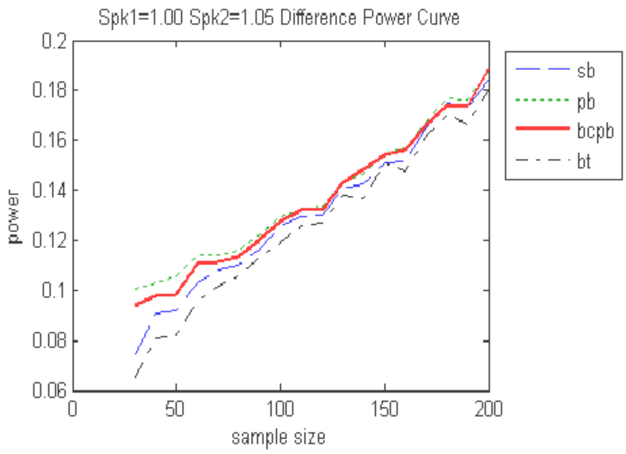
n	S_{pk1}	1	1	1	1	1	1	1	1	1	1
	S_{pk2}	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
190	SB	0.17300	0.36667	0.62100	0.80633	0.92400	0.97267	0.99367	0.99900	0.99967	0.99967
	PB	0.17633	0.37100	0.62600	0.80933	0.92633	0.97400	0.99367	0.99900	0.99967	0.99967
	BCPB	0.17367	0.36800	0.61967	0.80900	0.92600	0.97133	0.99433	0.99900	0.99967	0.99967
	BT	0.16667	0.36100	0.61033	0.80667	0.92433	0.97133	0.99233	0.99867	0.99967	0.99967
200	SB	0.18500	0.39133	0.63233	0.82867	0.92967	0.98267	0.99467	0.99833	1.00000	1.00000
	PB	0.18800	0.39667	0.63300	0.83167	0.93233	0.98233	0.99467	0.99833	1.00000	1.00000
	BCPB	0.18900	0.39567	0.63267	0.83333	0.93267	0.98233	0.99467	0.99833	1.00000	1.00000
	BT	0.18067	0.38400	0.62667	0.82400	0.93033	0.98233	0.99467	0.99833	1.00000	1.00000

Table 14. Selection power of the four bootstrap methods for ratio statistic with sample size n = 30(10)200.

n	S_{pk1}	1	1	1	1	1	1	1	1	1	1
	S_{pk2}	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
30	SB	0.06033	0.08933	0.12967	0.17767	0.22200	0.25667	0.28700	0.31000	0.32333	0.32600
	PB	0.10067	0.15900	0.22267	0.29000	0.36367	0.43733	0.51933	0.60000	0.66667	0.72967
	BCPB	0.09600	0.15067	0.21367	0.28133	0.35533	0.43233	0.51067	0.59167	0.66233	0.72333
	BT	0.03367	0.05400	0.07933	0.10900	0.14967	0.18267	0.20333	0.22767	0.24467	0.26100
40	SB	0.06967	0.12333	0.18700	0.26300	0.34300	0.41367	0.48700	0.54567	0.57967	0.60133
	PB	0.10333	0.16367	0.24233	0.33033	0.42700	0.51800	0.61033	0.68600	0.76167	0.82267
	BCPB	0.09967	0.16167	0.23933	0.33067	0.41833	0.51567	0.60700	0.68100	0.75900	0.81967
	BT	0.04500	0.07733	0.12900	0.18733	0.25400	0.32600	0.38800	0.45600	0.49400	0.52167
50	SB	0.07467	0.13900	0.21900	0.30400	0.41100	0.51300	0.60567	0.67733	0.75000	0.78300
	PB	0.10600	0.17833	0.26500	0.36733	0.47700	0.58233	0.67400	0.76033	0.83200	0.88000
	BCPB	0.10033	0.17467	0.26267	0.36333	0.47033	0.57767	0.67067	0.75967	0.82900	0.87700
	BT	0.05100	0.09400	0.16200	0.24000	0.31633	0.42300	0.51867	0.59300	0.66367	0.72367
60	SB	0.08700	0.16067	0.25500	0.36367	0.47600	0.58133	0.69167	0.77433	0.83733	0.88433
	PB	0.11400	0.19933	0.30067	0.41367	0.52900	0.63567	0.74700	0.82500	0.88567	0.93167
	BCPB	0.11200	0.20000	0.30000	0.40867	0.52500	0.63500	0.74033	0.82133	0.88333	0.93033
	BT	0.05733	0.11633	0.20300	0.29800	0.39833	0.50933	0.61400	0.70900	0.78467	0.83633
70	SB	0.09300	0.17300	0.27667	0.39667	0.52767	0.66200	0.76800	0.85067	0.90467	0.94133
	PB	0.11400	0.20800	0.31100	0.44367	0.57600	0.71233	0.81000	0.88233	0.93067	0.95667
	BCPB	0.11267	0.20567	0.30967	0.43933	0.57567	0.71033	0.80467	0.87933	0.92933	0.95700
	BT	0.06633	0.13000	0.22333	0.32900	0.44800	0.58367	0.70400	0.79933	0.86667	0.91467
80	SB	0.09800	0.18933	0.31100	0.45167	0.59733	0.72167	0.81933	0.88533	0.93367	0.96400
	PB	0.11600	0.21900	0.35700	0.50233	0.64400	0.76167	0.84800	0.90700	0.94867	0.97567
	BCPB	0.11467	0.21933	0.35267	0.50300	0.64500	0.76400	0.84967	0.90733	0.94967	0.97533
	BT	0.07200	0.14433	0.25067	0.39000	0.53233	0.66000	0.77067	0.84900	0.90600	0.94467
90	SB	0.10233	0.20233	0.33300	0.48033	0.63200	0.75733	0.85433	0.91667	0.95400	0.97833
	PB	0.12233	0.23867	0.37000	0.52433	0.67167	0.79367	0.88000	0.93233	0.96433	0.98367
	BCPB	0.12000	0.23500	0.37133	0.52400	0.67300	0.78900	0.87733	0.92733	0.96433	0.98300
	BT	0.07633	0.16033	0.28467	0.41667	0.56733	0.71067	0.81100	0.88833	0.93767	0.96767
100	SB	0.11167	0.22100	0.36333	0.53600	0.70100	0.81933	0.90000	0.94667	0.97633	0.98967
	PB	0.13000	0.24967	0.40300	0.57967	0.73367	0.84433	0.91967	0.95800	0.98133	0.99167
	BCPB	0.13067	0.24733	0.40233	0.57133	0.73433	0.84133	0.91933	0.95633	0.98067	0.99233
	BT	0.08533	0.18133	0.31133	0.47067	0.64133	0.77067	0.87100	0.93000	0.96500	0.98567

n	S_{pk1}	1	1	1	1	1	1	1	1	1	1
	S_{pk2}	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
110	SB	0.11500	0.24067	0.40933	0.58933	0.73600	0.85033	0.92233	0.96767	0.98667	0.99500
	PB	0.13200	0.27700	0.44667	0.62833	0.76633	0.87433	0.93900	0.97333	0.99100	0.99667
	BCPB	0.13300	0.27500	0.45000	0.62400	0.76733	0.87333	0.93867	0.97400	0.99000	0.99700
	BT	0.09233	0.19833	0.35533	0.52867	0.68567	0.80800	0.90300	0.95567	0.98000	0.99267
120	SB	0.11500	0.24067	0.41833	0.59867	0.75367	0.86400	0.94000	0.97167	0.98567	0.99567
	PB	0.13367	0.26967	0.45300	0.63500	0.78233	0.88433	0.94900	0.97767	0.98867	0.99700
	BCPB	0.13300	0.26800	0.45167	0.63400	0.78000	0.88600	0.94833	0.97733	0.98833	0.99767
	BT	0.09033	0.19967	0.36433	0.54367	0.71100	0.82467	0.91933	0.96133	0.98133	0.99167
130	SB	0.12733	0.27133	0.45167	0.63467	0.79167	0.89233	0.95333	0.98333	0.99367	0.99833
	PB	0.14367	0.30300	0.48600	0.66667	0.81633	0.91133	0.96200	0.98667	0.99533	0.99867
	BCPB	0.14333	0.30200	0.47767	0.66300	0.81667	0.91000	0.96100	0.98700	0.99567	0.99867
	BT	0.10167	0.22900	0.40033	0.57700	0.75467	0.86867	0.93767	0.97567	0.99067	0.99733
140	SB	0.12700	0.29100	0.47967	0.67967	0.82733	0.92100	0.96567	0.98600	0.99633	0.99900
	PB	0.14667	0.31633	0.51800	0.71100	0.85267	0.93167	0.97267	0.98933	0.99767	0.99933
	BCPB	0.14967	0.31867	0.51800	0.70933	0.85300	0.93000	0.97267	0.98900	0.99767	0.99933
	BT	0.10267	0.24567	0.42800	0.63600	0.79367	0.90433	0.95533	0.98133	0.99300	0.99867
150	SB	0.13700	0.29500	0.50533	0.70533	0.84933	0.93633	0.97267	0.99200	0.99700	0.99933
	PB	0.15467	0.32033	0.53267	0.73067	0.87000	0.94533	0.97900	0.99300	0.99833	0.99933
	BCPB	0.15367	0.31933	0.53233	0.72833	0.86967	0.94467	0.97867	0.99233	0.99833	0.99933
	BT	0.11333	0.25233	0.45467	0.66567	0.82200	0.91867	0.96467	0.98833	0.99433	0.99933
160	SB	0.14000	0.30400	0.50533	0.71100	0.85633	0.94333	0.97800	0.99367	0.99833	0.99967
	PB	0.15700	0.33167	0.53467	0.73633	0.87100	0.95267	0.98133	0.99467	0.99900	1.00000
	BCPB	0.15700	0.33200	0.53267	0.73633	0.87100	0.95200	0.98300	0.99500	0.99933	1.00000
	BT	0.11600	0.26100	0.46367	0.66500	0.83000	0.92600	0.97400	0.99167	0.99800	0.99967
170	SB	0.15033	0.32300	0.54433	0.74500	0.88067	0.95333	0.98667	0.99733	0.99967	1.00000
	PB	0.16700	0.34733	0.57167	0.77267	0.89400	0.96233	0.99067	0.99800	0.99967	1.00000
	BCPB	0.16667	0.34600	0.56967	0.77200	0.89567	0.96233	0.98900	0.99833	0.99967	1.00000
	BT	0.12667	0.28933	0.49500	0.71167	0.85500	0.94033	0.98067	0.99667	0.99933	1.00000
180	SB	0.15833	0.35167	0.57767	0.76367	0.89800	0.96133	0.98967	0.99767	0.99933	0.99967
	PB	0.17700	0.37700	0.60567	0.78867	0.91000	0.96833	0.99067	0.99833	0.99933	1.00000
	BCPB	0.17533	0.37533	0.60433	0.78800	0.91033	0.97133	0.99167	0.99833	0.99933	1.00000
	BT	0.13367	0.31100	0.53800	0.73300	0.87667	0.95100	0.98567	0.99700	0.99900	0.99967

n	S_{pk1}	1	1	1	1	1	1	1	1	1	1
	S_{pk2}	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
190	SB	0.15900	0.34333	0.59067	0.79300	0.91600	0.96900	0.99167	0.99833	0.99967	0.99967
	PB	0.17633	0.37100	0.62600	0.80933	0.92633	0.97400	0.99367	0.99900	0.99967	0.99967
	BCPB	0.17200	0.36833	0.62100	0.81000	0.92667	0.97233	0.99433	0.99900	0.99967	0.99967
	BT	0.13433	0.30900	0.55467	0.76400	0.89467	0.96267	0.99033	0.99767	0.99967	0.99967
200	SB	0.16733	0.37000	0.60800	0.81467	0.92267	0.97900	0.99367	0.99800	0.99967	1.00000
	PB	0.18800	0.39667	0.63300	0.83167	0.93233	0.98233	0.99467	0.99833	1.00000	1.00000
	BCPB	0.18867	0.39533	0.63367	0.83233	0.93333	0.98300	0.99467	0.99833	1.00000	1.00000
	BT	0.14333	0.33433	0.57067	0.78433	0.90933	0.96967	0.99367	0.99733	0.99900	1.00000



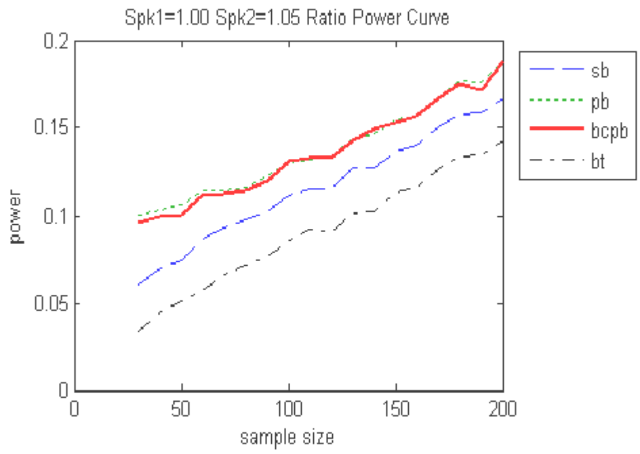
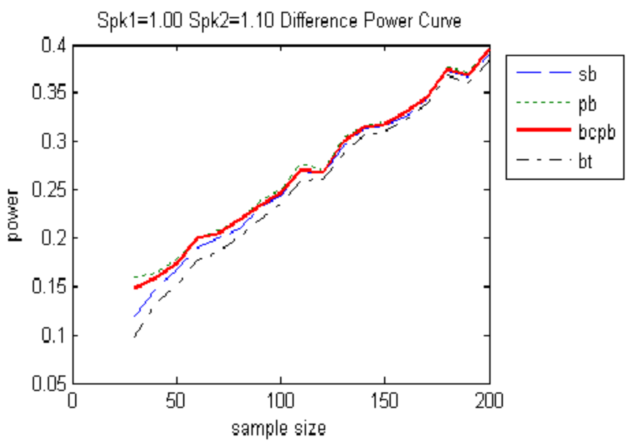


Figure 13. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.05$.

Figure 14. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.05$.



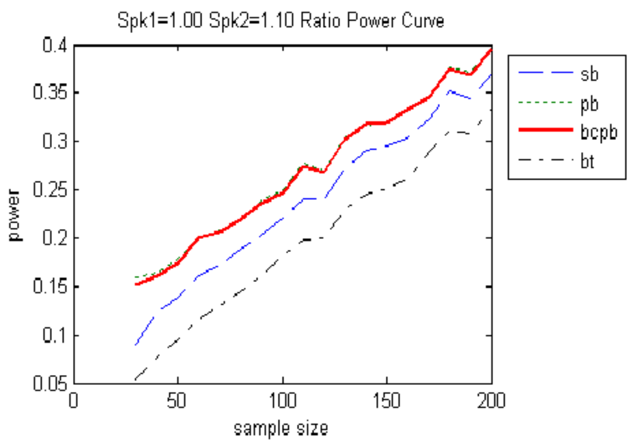


Figure 15. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.10$.

Figure 16. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.10$.

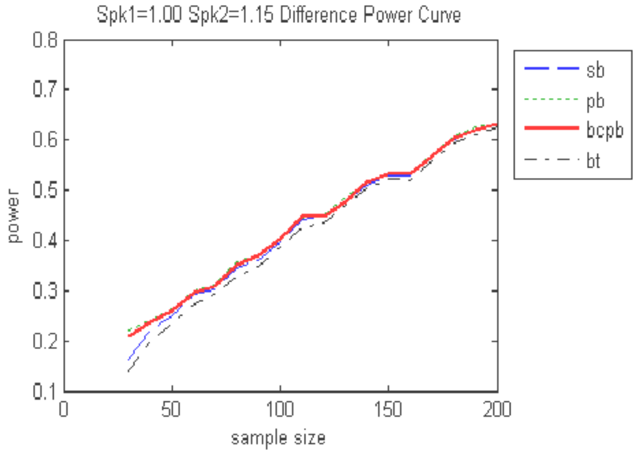


Figure 17. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.15$.

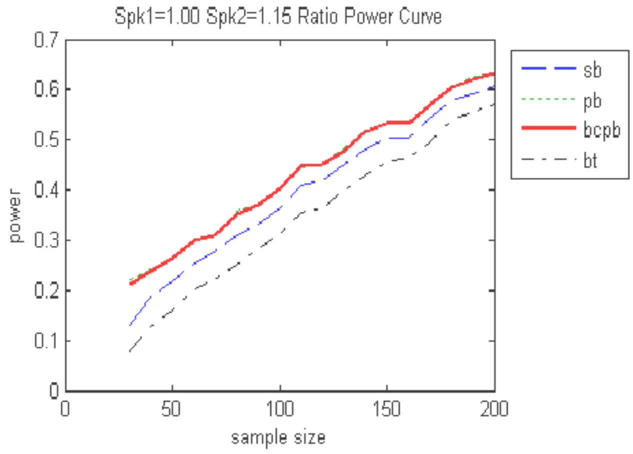


Figure 18. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.15$.

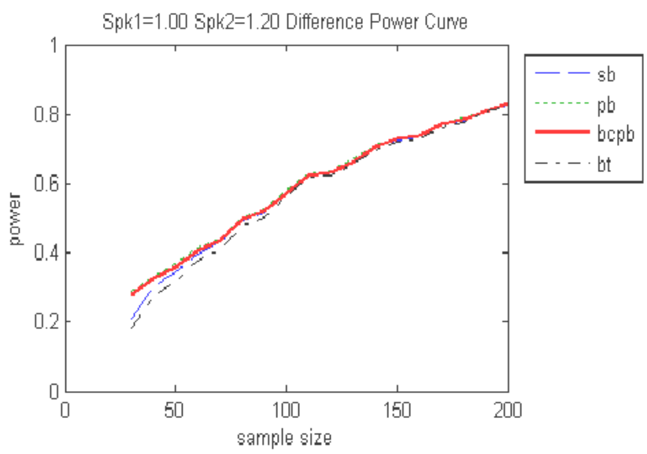


Figure 19. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.20$.

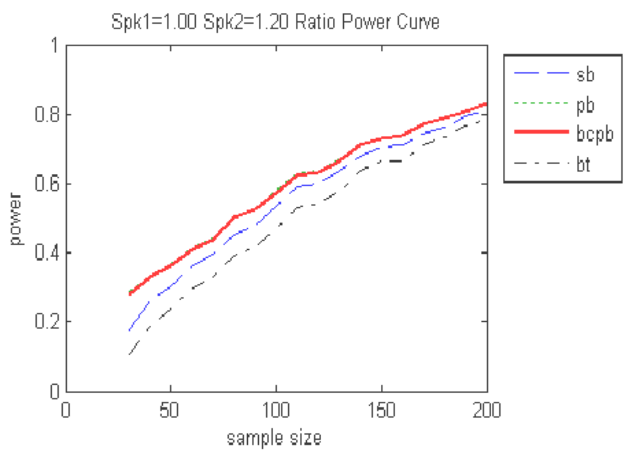


Figure 20. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.20$.

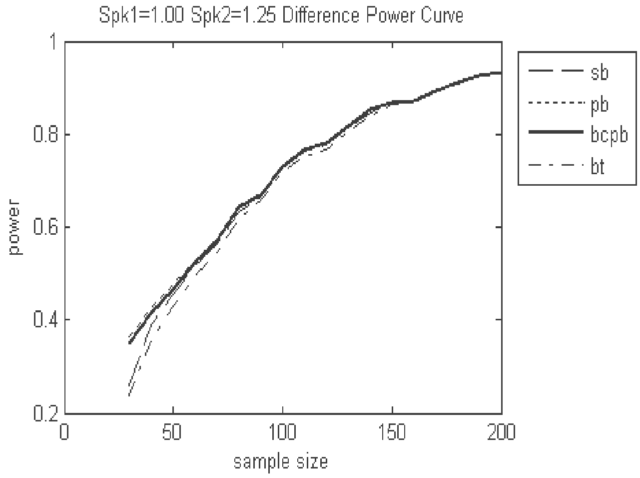


Figure 21. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.25$.

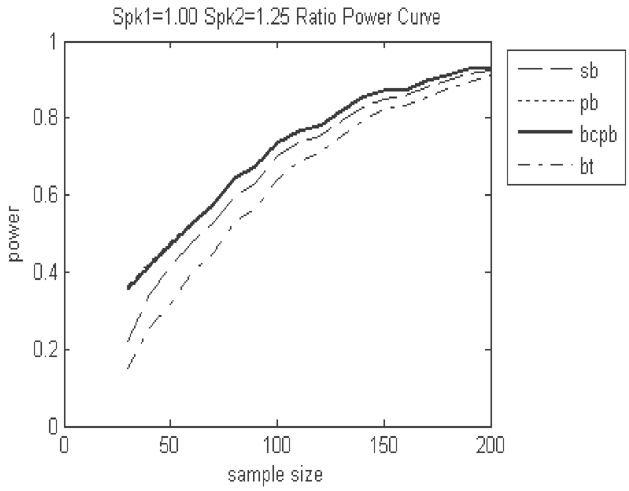


Figure 22. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.25$.

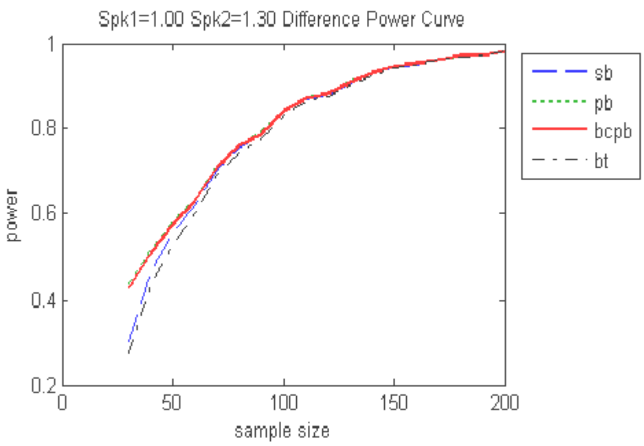


Figure 23. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.30$.

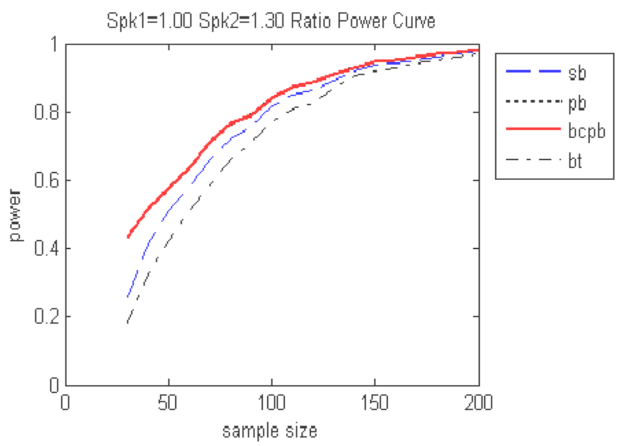


Figure 24. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.30$.

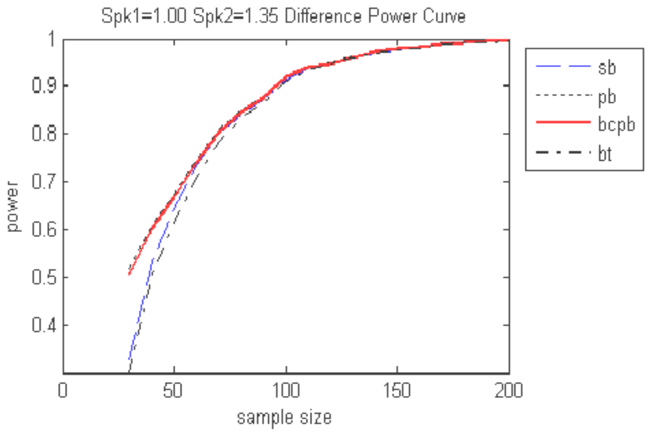


Figure 25. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.35$.

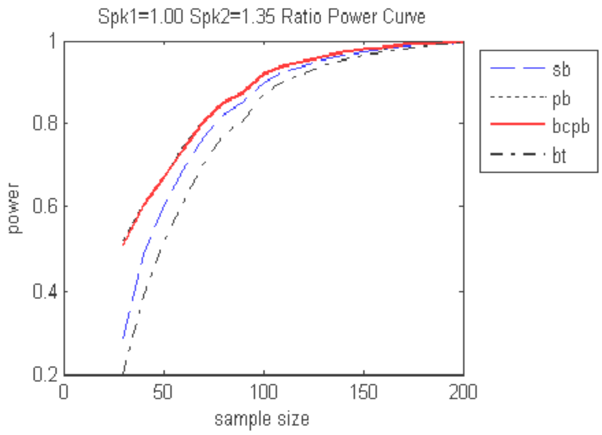


Figure 26. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.35$.

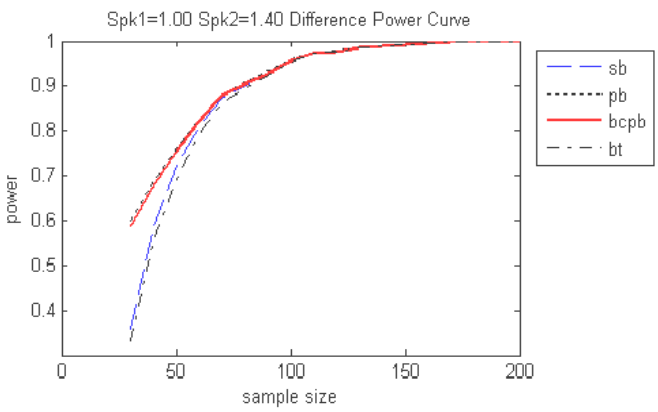


Figure 27. The difference statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.40$.

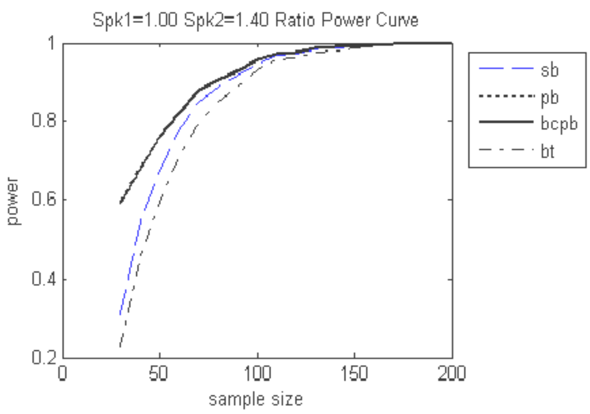


Figure 28. The ratio statistic with sample size n = 30(10)200, $S_{pk1} = 1.00$, $S_{pk2} = 1.40$.

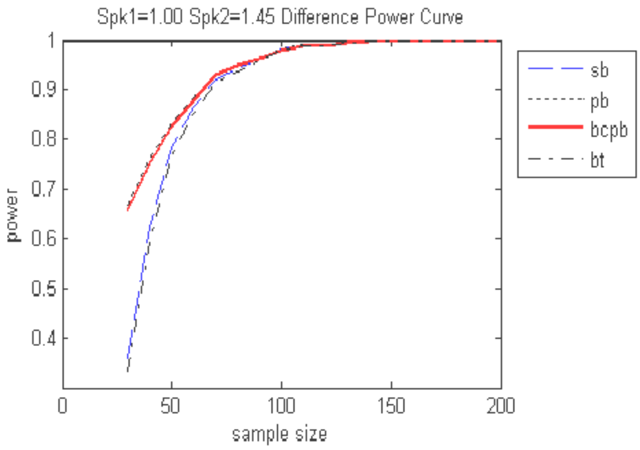


Figure 29. The difference statistic with sample size n = 30(10)200, $S_{pk1}=1.00$, $S_{pk2}=1.45$.

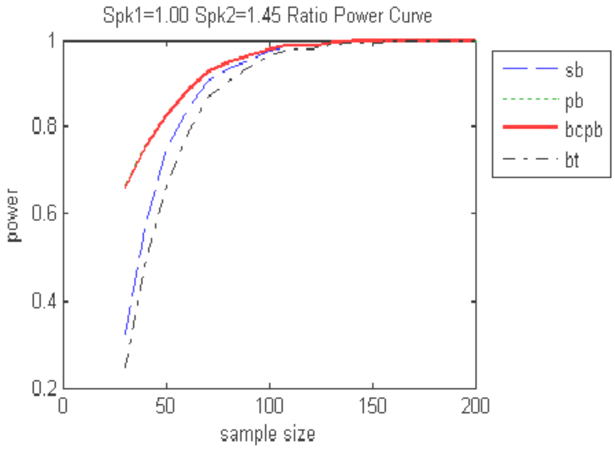


Figure 30. The ratio statistic with sample size n = 30(10)200, $S_{pk1}=1.00$, $S_{pk2}=1.45$.

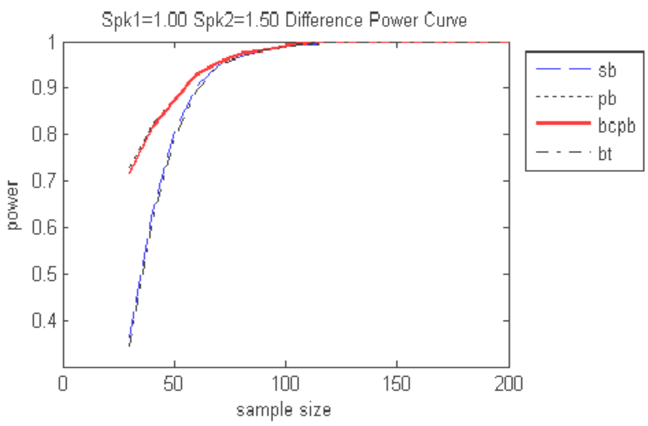


Figure 31. The difference statistic with sample size n = 30(10)200, $S_{pk1}=1.00$, $S_{pk2}=1.50$.

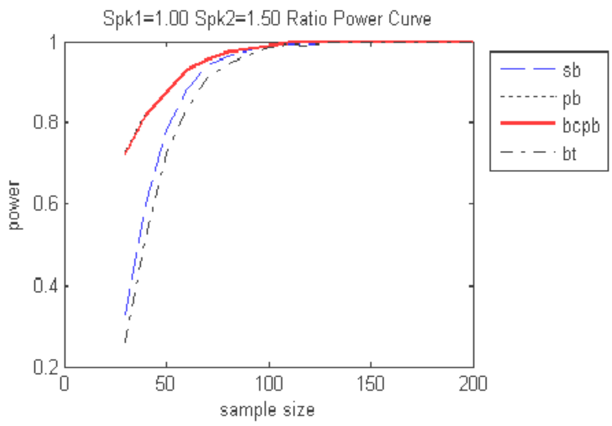


Figure 32. The ratio statistic with sample size n = 30(10)200, $S_{pk1}=1.00$, $S_{pk2}=1.50$.

Table 15. Sample data for supplier I.

0.61353	0.61552	0.61328	0.59529	0.62445	0.63116	0.65012	0.62375	0.65633	0.64408
0.59435	0.59097	0.62614	0.60378	0.67782	0.61163	0.64556	0.63773	0.62692	0.60146
0.63778	0.62606	0.60157	0.64230	0.60499	0.64875	0.62426	0.62012	0.58552	0.61908
0.65880	0.62996	0.64063	0.60310	0.65685	0.66685	0.66855	0.64960	0.59610	0.63888
0.63341	0.61983	0.66149	0.61941	0.61144	0.64516	0.61356	0.65074	0.66107	0.61672
0.62376	0.61054	0.62855	0.64133	0.64026	0.63572	0.65403	0.64628	0.64720	0.62483
0.63104	0.62912	0.60898	0.63509	0.64023	0.65852	0.65104	0.59553	0.66196	0.68931
0.60724	0.68533	0.62786	0.61883	0.63945	0.64187	0.61097	0.59440	0.62939	0.61212
0.62433	0.63652	0.62281	0.63842	0.64935	0.58631	0.63108	0.62256	0.63475	0.64225
0.63025	0.61676	0.62397	0.61954	0.64509	0.60708	0.64991	0.56073	0.62406	0.62531
0.64096	0.62553	0.65768	0.62151	0.65017	0.61804	0.62479	0.60577	0.63215	0.67966
0.63620	0.59486	0.61919	0.62155	0.69332	0.66096	0.62870	0.61128	0.64926	0.60463
0.65656	0.59263	0.58933	0.64777	0.59966	0.63912	0.61977	0.65170	0.62790	0.64034
0.62508	0.63078	0.59323	0.62059	0.60731	0.59209	0.63595	0.62983	0.65414	0.63975
0.59757	0.64739	0.63923	0.60957	0.64516	0.65291	0.64188	0.65894	0.65173	0.65041
0.65713	0.64089	0.61251	0.64204	0.60451					

Table 16. Sample data for supplier II.

0.63319	0.59254	0.62833	0.62404	0.62587	0.63986	0.62282	0.63197	0.65559	0.62487
0.63279	0.62496	0.64915	0.66193	0.67187	0.61781	0.67295	0.62161	0.63615	0.62000
0.59515	0.66258	0.61672	0.61852	0.63554	0.63414	0.63669	0.65318	0.61482	0.60397
0.63083	0.62965	0.63395	0.63709	0.65171	0.64944	0.62016	0.62190	0.60291	0.61077
0.62443	0.63228	0.63950	0.61063	0.60707	0.63941	0.63165	0.63531	0.61413	0.64547
0.60578	0.60800	0.62913	0.64539	0.62872	0.64082	0.63443	0.67411	0.64527	0.65435
0.63899	0.62116	0.59434	0.63356	0.69966	0.62779	0.62603	0.65974	0.63938	0.60937
0.63111	0.64093	0.62817	0.63136	0.61867	0.65489	0.65627	0.63971	0.67058	0.63195
0.64312	0.64829	0.66236	0.62152	0.63707	0.62309	0.62204	0.61500	0.62732	0.64333
0.63400	0.61434	0.62025	0.62176	0.63370	0.63883	0.66446	0.61982	0.65033	0.62862
0.59215	0.63196	0.62114	0.63214	0.59213	0.64364	0.63453	0.64603	0.63416	0.62022
0.66804	0.61699	0.63538	0.65878	0.63233	0.64984	0.63401	0.64590	0.65861	0.63060
0.62151	0.64389	0.62741	0.62732	0.63666	0.64178	0.66165	0.64287	0.66001	0.61175
0.63604	0.63689	0.62332	0.63232	0.62386	0.65078	0.61185	0.64486	0.60997	0.62715
0.66274	0.64410	0.61564	0.62236	0.64297	0.62569	0.64367	0.63724	0.61243	0.62582
0.62871	0.64923	0.62483	0.63170	0.63772					