國立交通大學
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碩士論文

依據製程能力指標 C_{pmk} 應用複式抽樣方法於
供應商選擇

Bootstrap Approach for Supplier Selection Based on
Process Capability Index C_{pmk}

研 究 生：邱勝亮

指導教授：彭文理 博士
吳建瑋 博士

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研 究 生：邱勝亮　　Student : Sheng-Liang Chiu
指導教授：彭文理 博士　　Advisor : Dr. W. L. Pearn
吳建瑋 博士　　Dr. Chien-Wei Wu

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國立交通大學工業工程與管理學系碩士班

摘要

製程能力指標是藉由一個指標值來衡量製程的能力與產品的品質，過去學者對於以製程能力指標來衡量兩家供應商製程的問題已經提出了一些方法。然而，依據製程能力指標 C_{pmk} 於供應商選取的問題目前尚未被研究。這個指標的建構結合了 C_{pk} 與 C_{pm} 兩個指標的優點，同時考量到製程良率以及製程損失的特性。本篇論文的研究目的就是在兩家相互競爭的供應商之間選出一家具有較好製程能力的供應商，並建立了一個依據 C_{pmk} 指標的決策程序供使用者於決策時使用。本研究是應用複式抽樣的方法針對兩供應商之製程間的檢定統計量來估計信賴下界，藉由比較四種複式抽樣信賴區間的錯誤機率和篩選檢定力後，結果發現以偏誤校正的比例複式抽樣法 (BCPB) 在相同的樣本數下有較穩定的錯誤機率以及比較顯著的篩選檢定力。所以在這四種複式抽樣方法中，BCPB 之複式抽樣法為表現比較好的方法。 最後，為了實務應用上的便利，我們提供一個供應商選取程序作為選取決策之參考。

關鍵字：製程能力分析、供應商選擇、複式抽樣法、錯誤機率、篩選檢定力。

Bootstrap Approach for Supplier Selection Based on Process Capability Index C_{pmk}

Student: Sheng-Liang Chiu Advisor： Dr. W. L. Pearn
Dr. Chien-Wei Wu

Department of Industrial Engineering and Management
National Chiao Tung University

Abstract

Process capability indices (PCIs) intended to provide single-number assessments of ability to meet specification limits on quality characteristics. Many individuals have indicated various approaches for supplier selection or process comparison problem based on PCIs. However, the method of supplier selection based on process capability index C_{pmk} is not yet investigated. The index is constructed by combining the yield-based index C_{pk} and the loss-based index C_{pm}, taking into account the process yield as well as the process loss. The principal purpose of this thesis is to determine the more capable process between two competing suppliers and provide the supplier selection procedure based on C_{pmk} index for practical applications. In this study, we apply the bootstrap method, a data-based simulation technique, to construct lower confidence bound for the statistics between two suppliers. A comparison among four bootstrap methods is also analyzed by evaluating the error probability and the selection power. The result indicates that the BCPB method is the better approach among four bootstrap methods for process comparison due to its stable error probability and larger selection power with a fixed sample size. Finally, for convenience of applications, a practical step-by-step testing procedure for engineers is implemented to refer to supplier selection decisions.

Key words: bootstrap method, error probability, process capability indices, selection power, supplier selection.

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Notations

T : target

LSL : the lower specification limits preset by the process engineers

USL : the upper specification limits preset by the process engineers

d : the half specification width

m : the midpoint between the upper and the lower specifications limits

μ : the population mean

σ^2 : the population variation

σ : the population standard deviation

$\%NC$: the fraction of Non-Conformities

n : the number of the sample size drawn from suppliers

B : the number of bootstrap resamples

N : simulation replicated times

$\hat{C}_{pmk1}$: the $\hat{C}_{pmk1}$ of bootstrap resamples from supplier I

$\hat{C}_{pmk2}$: the $\hat{C}_{pmk2}$ of bootstrap resamples from supplier II

θ : the difference or the ratio of two suppliers' C_{pmk} index

$\hat{\theta}$: the estimator of θ

$\hat{\theta}^*$: the associated ordered bootstrap estimate of θ

$\bar{\hat{\theta}}^*$: the sample average of the B bootstrap estimates

S_θ^* : the standard deviation of the B bootstrap estimates

1. Introduction

1.1 Motivation

Over past decades, there are remarkable developments in the field of process capability indices (PCIs). PCIs are intended to provide single-number assessments of ability to meet specification limits on quality characteristics (Kotz and Johnson (2002)). On the other hand, the trend of vertical integration between suppliers and manufacturers has been developed. Supplier selection problem plays a critical role in modern manufacturing environment. It has been proposed that process capability index is the most precise and effective assessment for the determination of the better supplier.

Many individuals have indicated various approaches for supplier selection or process comparison problem based on PCIs. For the index C_p, Tseng and Wu (1991) and Chou (1994) used modified likelihood ratio and likelihood ratio test respectively to compare processes. For the index C_{pk}, Chen ant Tong (2003) constructed the biased corrected percentile bootstrap (BCPB) confidence interval of $(C_{pk1} - C_{pk2})$ to select the better of two suppliers and Daniels *et al.* (2005) accessed the Bonferroni method to select suppliers. For the index C_{pm}, Huang and Lee (1995), Pearn *et al.* (2004) and Chen and Chen (2004a) suggested looking for the smallest $\gamma^2 = E(X-T)^2 = \sigma^2 + (\mu - T)^2$, two-phase selection procedure and ratio test method respectively for supplier selection. Although a lot of investigation on supplier selection based on PCIs has been done so far, discussion relative to the subject based on the index C_{pmk} has not been concentrated on. With the merit of combining process yield and process loss, more studies need to be conducted for the method of supplier selection based on C_{pmk} index.

1.2 Research Objectives

The purpose of this thesis is to determine the more capable process between two competing suppliers based on C_{pmk} index. Owing to the complexity of sampling distribution, we apply the bootstrap method, a data-based simulation technique, to construct lower confidence bound for the statistics between two suppliers. A comparison among four bootstrap methods is also analyzed by evaluating the error probability and the selection power. After the analysis of the simulation outcome, this study provides sample size tables for conducting hypothesis test and the supplier selection procedure based on C_{pmk} index for current manufacturing industries.

1.3 Research Structure

In this section, it has been shown a brief review of our research about bootstrap approach for supplier selection based on process capability index C_{pmk}. A summary of the substance for each chapter is presented below; and further, the research structure is illustrated in Figure 1.

Ch1. Introduction: Serve as an orientation for readers to understand summary knowledge about PCIs and bootstrap.

Ch2. Literature review: A review of the characteristics and formulations of PCIs in the first part. For the second part, review papers about supplier selection based on PCIs has been summarized.

Ch3. Selection Method: Introduce the approach of formulating hypothesis tests and bootstrap sampling methodology.

Ch4. Performance Comparison of Four Bootstrap Methods: Apply simulation technique to compare four bootstrap methods based on error probability and selection power analysis.

Ch5. Supplier Selection Based on BCPB Method: According to the results of performance comparison in Chapter 4, sample size tables and selection procedure are provided.

Ch6. Application Example: Take an example from FPC industry to illustrate supplier selection method in this thesis.

Ch7. Conclusion: Take a broad look at our findings for the specific supplier selection problem.

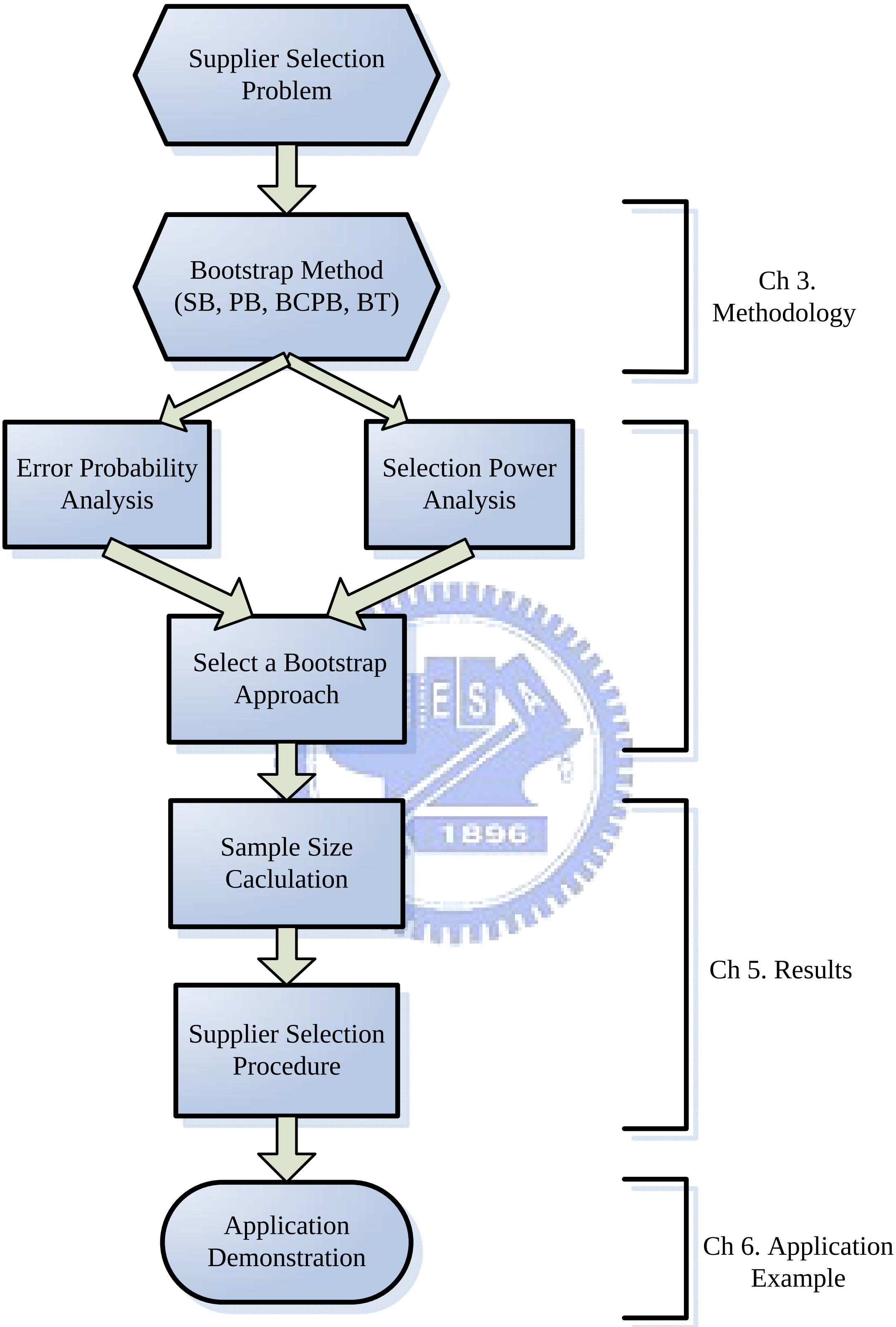

Figure 1. Illustration of research structure.

2. Literature Review

2.1 Process Capability Indices

Process Capability Indices are intended to provide single-number assessments of ability to meet specification limits on quality characteristics (Kotz and Johnson (2002)). It has been proposed in the manufacturing industry to measure on whether a process is capable of reproducing items or not. A review in this section is going to describe some and current development in PCIs. The use of process capability indices began in United States during early 1980s. Many authors have promoted the use of various PCIs for evaluating a supplier's process capability. Examples include Boyles (1991), Pearn *et al.* (1992), Kushler and Hurley (1992), Kotz and Johnson (1993), Vännman and Kotz (1995), Vännman (1997), Kotz and Lovelace (1998), Pearn *et al.* (1998), Kotz and Johnson (2002), Pearn and Shu (2003) and references therein. A general acceptance of the idea that PCIs can be used only after it have been established that a process is in statistical control and an assumption that the measured characteristics should have a normal distribution (at least, approximately). Four well-known capability indices have been defined respectively as (Juran (1974), Pearn *et al.* (1998), Kane (1986), and Hsiang and Taguchi (1985)):

$$C_p = \frac{USL - LSL}{6\sigma},$$

$$C_a = 1 - \frac{|\mu - m|}{d},$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma},$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

where μ is the process mean, USL is the upper specification limit, LSL is the lower specification limit, σ is the process standard deviation, T is target value, $d = (USL - LSL)/2$, and $m = (USL + LSL)/2$. The C_p index reflects product consistency by evaluating the overall process variability relative to the manufacturing tolerance. The C_a index measures the degree of process centering, which can be regarded as a process accuracy index. The C_{pk} index evaluates process variation and the location of the process mean to offset some of the weakness in C_p and C_a, which is a yield-based index (see Boyles (1991)) providing lower bounds on process yield. The C_{pm} index incorporate with the variation of production items with respect to the target value and specification limits preset in the factory. Since the design is based on the average process loss, which has been called the Taguchi index.

Many process capability indices, such as C_p, C_a, C_{pk} and C_{pm}, have been proposed to provide numerical measures. Combining the advantages of these indices, Pearn *et al.* (1992) introduced a new capability index called C_{pmk}. It is

$$C_{pmk} = \min\left\{\frac{USL-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-LSL}{3\sqrt{\sigma^2+(\mu-T)^2}}\right\}.$$

It is constructed by combining the yield-based index C_{pk} and the loss-based index C_{pm}, taking into account the process yield as well as the process loss. When the process mean μ depart from the target value *T*, the reduced value of C_{pmk} is more significant than those of C_p, C_{pk} and C_{pm}. And it remains sensitive to the shift of process variation. Clearly $C_p \ge C_{pk} \ge C_{pmk}$ and $C_p \ge C_{pm} \ge C_{pmk}$. The relation between C_{pk} and C_{pm} is less clearcut. If the process meets the capability requirement '$C_{pmk} \ge C$', then the process must meet both capability requirements '$C_{pk} \ge C$' and '$C_{pm} \ge C$' since $C_{pm} \ge C_{pmk}$ and $C_{pk} \ge C_{pmk}$ (Pearn and Lin (2002)). While C_{pk} remains the more widely used index, C_{pmk} is considered to be an advanced and useful index for processes with two-sided specification limits.

2.2 The Method of Selecting the Better Supplier Based on PCIs

With the improvement of technology, it is more important to enhance quality and satisfy the customer's requirements. Judging the better of suppliers is the critical issue. A review of the literature indicates that many approaches have been applied for supplier selection. Tseng and Wu (1991) considered the problem for k available manufacturing processes based on the precision index C_p under a modified likelihood ratio (MLR) selection rule. Chou (1994) used the likelihood ratio test (LRT) to compare two processes for the unilateral cases that two sample sizes are equal and developed F test to compare two suppliers based on C_p. Huang and Lee (1995) selected the supplier by searching the largest C_{pm} which are used to looking for the smallest $\gamma^2 = E(X-T)^2 = \sigma^2 + (\mu-T)^2$. The purpose was to select a subset containing the processes from given independent process. Chen and Tong (2003) proposed a bootstrap re-sampling simulation method to construct the biased corrected percentile bootstrap (BCPB) confidence interval of $(C_{pk1} - C_{pk2})$ to select the better of two suppliers. Furthermore, Pearn *et al.* (2004) implemented this method which developed a two-phase selection procedure to select a better supplier and examine the magnitude of the difference between the two suppliers. Chen and Chen (2004a) judged the better of two processes based on a confidence interval for the ratio C_{pm1}/C_{pm2}. Four methods are presented and compared. One based on the statistical theory given in Boyles (1991) and three based on the bootstrap, (referred to as SB, PB and BCPB). Chen and Chen (2004b) developed approximately F test to determine whether or not two processes are equally capable based on C_{pm}. Daniels *et al.* (2005) considered the Bonferroni, Modified Bonferroni, Difference, Ratio and General Confidence Interval methods to construct confidence intervals for performing these comparisons on C_{pk} and C_{pm}. Chen and Chen (2006) applied the process incapability index C_{pp} to develop an evaluation model that assesses the quality performance of suppliers. However, difference and ratio test for supplier selection based on C_{pmk} have not been developed due to the complexity of its sampling distribution. This study applies the bootstrap re-sampling simulation to compare two processes based on C_{pmk}.

2.3 Process Yield Based on C_{pmk} Index

For most supplier selection problem in manufacturing factories, increasing the product yield or reducing the percentages of non-conforming items is the primary concern for quality improvement. Motorola's "Six Sigma" program essentially requires the process capability at least 2.0 to accommodate the possible 1.5σ process shift (see Harry (1988)), and no more than 3.4 *ppm* are defectives. The most natural measure is the proportion itself called the yield, which we refer to $Yield$ defined as:

$$Yield = \int_{LSL}^{USL} dF(x) = F(USL) - F(LSL),$$

where $F(x)$ is the cumulative distribution function of the measured characteristic X. If the process characteristic X follows $N(\mu, \sigma^2)$, then the fraction of nonconformities NC is:

$$\%NC = 1 - \Phi\left(\frac{USL-\mu}{\sigma}\right) + \Phi\left(\frac{\mu - LSL}{\sigma}\right).$$

The index C_{pk} provides bounds on process yield for a normally distributed process. Given fixed value of C_{pk}, the bounds are $2\Phi(3C_{pk}) - 1 \le yield \le \Phi(3C_{pk})$ (Boyles(1991)) or $\Phi(-3C_{pk}) \le \%NC \le 2\Phi(-3C_{pk})$ for $0 \le C_a \le 1$, where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution $N(0,1)$. For $C_{pk} = 1.00$, one would expect that the fractions of defectives is no more than 2700 *ppm*. It is presently not clear whether or not the index C_{pmk} is related to the process yield, since the relationship between C_{pmk} and process yield has not been developed. Pearn and Lin (2005) have provided a mathematically derivation of an upper bound formula on process yield in terms of the percentage of nonconformities. The bounds are:

$$0 \le \%NC \le 2\Phi(-3C_{pmk}) \text{ for } C_{pmk} \ge \sqrt{2}/3 .$$

These results conform that the two indices, C_{pk} and C_{pmk} provide the same upper bounds on the percentage of nonconformities. For instance, given $C_{pk} = C_{pmk} = 1$, the information of the process yield is $\%NC \le 2699.796\ ppm$ for various process centering measure $0 \le C_a \le 1$ and $0.750 \le C_a \le 1$. The calculation illustrates the advantage of using the index C_{pmk} compared to the index C_{pk} when measuring the process yield. The first one provides a better customer protection in terms of process yield and process centering. Table 1 displays the bounds on $\%NC$ and bounds on C_a for $C_{pk} = C_{pmk} = C$, respectively. It could be shown that the lower bound on C_a for C_{pmk} is higher than it on C_{pk} with the increasing value of C_{pmk}. The table conforms that the index C_{pmk} provides a better customer protection again.

Table 1. Bounds on $\%NC$ and C_a for $C_{pk} = C_{pmk} = C$.

C	C_{pk}		C_{pmk}	
	Bounds on $\%NC$	Bounds on C_a	Bounds on $\%NC$	Bounds on C_a
1	2699.796	$0 \le C_a \le 1$	2699.796	$0.750 \le C_a \le 1$
1.33	66.334	$0 \le C_a \le 1$	66.334	$0.812 \le C_a \le 1$
1.5	6.795	$0 \le C_a \le 1$	6.795	$0.833 \le C_a \le 1$
1.67	0.554	$0 \le C_a \le 1$	0.554	$0.850 \le C_a \le 1$
2.00	0.002	$0 \le C_a \le 1$	0.002	$0.875 \le C_a \le 1$

3. Selection Method

3.1 Difference Test on Comparing Two C_{pmk} Indices

The process capability indices can be used to determine the more capable of competing processes. Since we have no direct observation of the entire processes, we do not know which process is more capable (Chou (1994)). In practice, real process measurements μ and σ^2 are unknown. We could gather sample data to determine index values. The indices calculated from the sample data cannot be immediately used to determine which supplier is better because sampling errors may lead to an uncorrected result. The difference hypothesis testing approach is used here to enhance reliability.

We investigate the selection problem for cases with two candidate processes based on the C_{pmk} index. Let π_i be the population assumed to be normally distributed with mean μ_i and variance σ_i^2, $i=1,2$, and $x_{i1}, x_{i2}, \ldots, x_{in_i}$ are the independent random samples from π_i, $i=1,2$. In most applications, if a new supplier II wants to compete for the orders by claiming that its capability is better than the existing supplier I, then the new S2 must furnish convincing information justifying the claim with a prescribed level of confidence. Thus, the supplier selection decisions would be based on the hypothesis testing comparing the two C_{pmk} values. It is

$$H_0 : C_{pmk1} \geq C_{pmk2}$$

$$H_1 : C_{pmk1} < C_{pmk2}\,.$$

If the test rejects the null hypothesis $H_0 : C_{pmk1} \geq C_{pmk2}$, then one has sufficient information to conclude that the new S2 is superior to the original S1, and the decision of the replacement would be suggested. In the difference hypothesis testing, this hypothesis test problem can be rewritten as

$$H_0 : C_{pmk2} - C_{pmk1} \leq 0$$

$$H_1 : C_{pmk2} - C_{pmk1} > 0\,.$$

The test statistic is given by

$$\hat{\theta} = (\hat{C}_{pmk2} - \hat{C}_{pmk1})\,.$$

Then, we apply bootstrap methodology to obtain the confidence interval for

$\theta = C_{pmk2} - C_{pmk1}$. *The decision rule:* if the lower confidence bound for the difference between two process capability indices $C_{pmk2} - C_{pmk1}$ is positive, then S2 has a better process capability than S1. Otherwise, we do not have sufficient information to conclude that the S2 has a better process capability than S1.

For a normally distributed process that is demonstrably stable (under statistical control), Pearn *et al.* (1992) considered the natural estimator of C_{pmk}

$$\hat{C}_{pmk} = \min\left\{ \frac{USL - \overline{X}}{3\sqrt{S_n^2 + (\overline{X} - T)^2}}, \frac{\overline{X} - LSL}{3\sqrt{S_n^2 + (\overline{X} - T)^2}} \right\},$$

where

$$\overline{X} = \sum_{i=1}^{n} X_i / n \text{ and } S_n^{\,2} = \sum_{i=1}^{n} (X_i - \overline{X})^2 / n$$

are the MLEs of μ and σ^2, respectively. We note that

$$S_n^{\,2} + (\overline{X} - T)^2 = \sum_{i=1}^{n} (X_i - T)^2 / n$$

which is the major part of the denominator of $\hat{C}_{pmk}$, is the uniformly minimum variance unbiased estimator (UMVUE) of $\sigma^2 + (\mu - T)^2 = E\left[(X - T)^2\right]$ is the denominator of C_{pmk}.Under the assumption of normality, Pearn *et al.* (1992) obtained the *r*-th moment and the first two moments, as well as the mean and the variance of $\hat{C}_{pmk}$ for the common cases with T=m. Evidently, $\hat{C}_{pmk}$ is a biased estimator of C_{pmk}. Chen and Hsu (1995) showed that the estimator $\hat{C}_{pmk}$ is consistent, and asymptotically unbiased. Furthermore, Vännman (1997) provided a simplified C.D.F. form of the estimator $\hat{C}_{pmk}$. It may be expressed in terms of a mixture of the chi-square and the normal distribution. The explicit form of the C.D.F. for $\hat{C}_{pmk}$ can, therefore, be expressed (using our notation) as

$$F_{\hat{C}_{pmk}}(x) = 1 - \int_0^{b\sqrt{n}/(1+3x)} G\left(\frac{(b\sqrt{n} - t)^2}{9x^2} - t^2 \right) \times \left[\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n}) \right] dt,$$

for *x>0,* where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-squared distribution χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0,1)$. Based on the estimation of C_{pmk}, Pearn and Lin (2002) implemented a testing hypothesis using the natural estimator of C_{pmk},

$H_0 : C_{pmk} \le C$ (Process is not capable.)

$H_1 : C_{pmk} > C$ (Process is capable.)

and provided an efficient Maple computer program to calculate the p-values and critical values. Besides, Pearn and Shu (2004) developed an efficient algorithm to compute the lower confidence bounds on C_{pmk} based on the estimation. However, their investigations are all developed for evaluating whether a single supplier's process conforms to a customer's requirement. For the comparison between two suppliers, it's difficult to construct the exact confidence interval for $\theta = C_{pmk2} - C_{pmk1}$ because of the complexity of the sampling distribution of $\hat{C}_{pmk2} - \hat{C}_{pmk1}$. Thus, we apply a nonparametric, data-based simulation technique for statistical inference.

3.2 Ratio Test on Comparing Two C_{pmk} Indices

Similarly, we apply a nonparametric, data-based simulation technique for hypothesis testing, due to the complexity of the sampling distribution of $\hat{C}_{pmk2} / \hat{C}_{pmk1}$. Besides the difference test, we construct the ratio test on comparing two C_{pmk} indices to make the decisions of supplier selection more reliable.

Equivalently, in the ratio testing approach, the test hypothesis problem can be rewritten as:

$$H_0 : C_{pmk2} / C_{pmk1} \le 1$$

$$H_1 : C_{pmk2} / C_{pmk1} > 1 .$$

The test statistic is given by

$$\hat{\theta} = (\hat{C}_{pmk2} / \hat{C}_{pmk1}) .$$

We also apply bootstrap methodology to obtain the confidence interval for $\theta = C_{pmk2} / C_{pmk1}$. Similarly, *the decision rule* is that if the lower confidence bound for the ratio between two process capability indices C_{pmk2} / C_{pmk1} is greater than 1, S2 has a better process capability than S1. Otherwise, if the lower confidence bound of the ratio statistic is less than 1, we would conclude that S1 has a better process capability than S2.

3.3 Bootstrap Methodology

Generally speaking, the bootstrap is a data-based simulation technique for statistical inference. The method introduced by Efron (1979, 1982) is a nonparametric, computational intensive but effective estimation method. The essence of the nonparametric bootstrap is that it does not rely on any distributional assumptions about the underlying population. For the most common application of the method, it is usually used to estimate a population standard error and confidence interval. In our study, the method is appropriate to apply to construct estimated confidence interval due to the complexity of the sampling distributions of $\hat{C}_{pmk2} - \hat{C}_{pmk1}$ and $\hat{C}_{pmk2} / \hat{C}_{pmk1}$. In order to select a better supplier accurately, our purpose for applying the bootstrap is to determine the lower confidence bounds of difference and ratio statistics precisely. In this method, B new samples, each of the same size, are drawn with replacement from the available sample. The statistic of interest in our case is to calculate $\hat{\theta}^*_{(l)}$, $l = 1, 2, \ldots, B$, $\hat{\theta}^* = (\hat{C}^*_{pmk2} - \hat{C}^*_{pmk1})$ or $(\hat{C}^*_{pmk2} / \hat{C}^*_{pmk1})$. Then, we generate a bootstrap distribution for the statistic $\hat{C}_{pmk2} - \hat{C}_{pmk1}$ or $\hat{C}_{pmk2} / \hat{C}_{pmk1}$.

The process of re-sampling bootstrap method is as follows. For $n_1 = n_2 = n$, let two bootstrap samples of size n drawn with replacement from the two original samples be denoted by $\{x^*_{11}, x^*_{21}, \ldots, x^*_{1n}\}$ $\{x^*_{21}, x^*_{22}, \ldots, x^*_{2n}\}$. The bootstrap sample statistics $\bar{x}^*_1$, s^*_1, $\bar{x}^*_2$, s^*_2, $\hat{C}^*_{pmk1}$ and $\hat{C}^*_{pmk2}$ are computed. In theory, there are n^n possible re-samples drawn. Due to the overwhelming computation time, it is not of practical interest to choose n^n such samples. Eforn and Tibshirani (1986) indicated that a roughly minimum of 1,000 bootstrap re-samples is usually sufficient to compute reasonably accurate confidence interval estimates for population parameters. In our investigation, we take B = 3,000 bootstrap re-samples for accuracy purpose. Thus, we take a sample of size $n =$ 100 and B = 3,000 to estimate $\hat{\theta}^* = (\hat{C}^*_{pmk2} - \hat{C}^*_{pmk1})$ or $(\hat{C}^*_{pmk2} / \hat{C}^*_{pmk1})$ of $\theta = C_{pmk2} - C_{pmk1}$ or C_{pmk2} / C_{pmk1}, respectively, then order them from the smallest to the largest $\hat{\theta}^*_{(l)} = (\hat{C}^*_{pmk2} - \hat{C}^*_{pmk1})_{(l)}$ or $(\hat{C}^*_{pmk2} / \hat{C}^*_{pmk1})_{(l)}$ where $l = 1, 2, \ldots, B$.

Four types of bootstrap confidence intervals, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), the biased corrected percentile bootstrap confidence interval (BCPB), and the bootstrap-*t* (BT) method introduced by Efron (1981) and Efron and Tibshiraniwill (1986) are conducted in this paper. The generic notations $\hat{\theta}$ and $\hat{\theta}^*$ will be used to denote the estimator of θ and the associated ordered bootstrap estimate. Construction of a two-sided $100(1-2\alpha)\%$ confidence limit will be described. We note that a lower $100(1-\alpha)\%$ confidence limit can be obtained by using only the lower limit. The formulation details for the four types of confidence intervals are displayed as follows.

[A] Standard Bootstrap (SB) Method

From the B bootstrap estimates $\hat{\theta}^*_{(l)}$, $l = 1, 2, \ldots, B$, the sample average and the sample standard deviation can be obtained as:

$$\bar{\hat{\theta}}^* = \frac{1}{B}\sum_{l=1}^{B}\hat{\theta}^*_{(l)}, \quad S^*_{\theta} = \left(\frac{1}{B-1}\sum_{l=1}^{B}[\hat{\theta}^*_{(l)} - \bar{\hat{\theta}}^*]^2\right)^{1/2}.$$

The quantity S^*_{θ} is an estimator of the standard deviation of $\hat{\theta}$ if the distribution of $\hat{\theta}$ is approximately normal. Thus, the $100(1-2\alpha)\%$ SB confidence interval for θ can be constructed as:

$$[\bar{\hat{\theta}}^* - z_{\alpha}S^*_{\theta}, \quad \bar{\hat{\theta}}^* + z_{\alpha}S^*_{\theta}],$$

where $\hat{\theta}$ is the estimated θ for the original sample, and z_{α} is the upper α quantile of the standard normal distribution.

[B] Percentile Bootstrap (PB) Method

From the ordered collection of $\hat{\theta}^*_{(l)}$, $l = 1, 2, \ldots, B$, the α percentage and $1-\alpha$ percentage points are used to obtained the $100(1-2\alpha)\%$ PB confidence interval for θ,

$$[\hat{\theta}^*_{(\alpha B)}, \ \hat{\theta}^*_{((1-\alpha)B)}].$$

[C] Biased-Corrected Percentile Bootstrap (BCPB) Method

While the percentile confidence interval is intuitively appealing it is possible that due to sampling errors, the bootstrap distribution may be biased. In other words, it is possible that bootstrap distributions obtained using only a sample of the complete bootstrap distribution may be shifted higher or lower than would be expected. A three steps procedure is suggested to correct for the possible bias (Efron (1982)). First, using the ordered distribution of $\hat{\theta}^*$, calculate the probability $p_0 = P[\hat{\theta}^* \le \hat{\theta}_0]$. Second, we compute the inverse of the cumulative distribution function of a standard normal based upon p_0 as $z_0 = \Phi^{-1}(p_0)$, $p_L = \Phi(2z_0 - z_{\alpha})$ $p_U = \Phi(2z_0 + z_{\alpha})$. Finally, executing these steps to obtain the $100(1-2\alpha)\%$ BCPB confidence interval,

$$[\hat{\theta}^*_{(p_L B)}, \ \hat{\theta}^*_{(p_U B)}].$$

[D] Bootstrap-t (BT) Method

By using bootstrapping to approximate the distribution of a statistic of the form $(\hat{\theta}-\theta)/S_{\hat{\theta}}$, the bootstrap approximation in this case is obtained by taking bootstrap samples from the original data values, calculating the corresponding estimates $\hat{\theta}^*$ and their estimated standard error, and hence finding the bootstrapped T-values $T=(\hat{\theta}^* - \hat{\theta})/S_{\theta}^*$. The hope is then that the generated distribution will mimic the distribution of T. The $100(1-2\alpha)\%$ BT confidence interval for θ may constitute as

$$[\hat{\theta}^* - t_{\alpha}^* S_{\hat{\theta}}^*,\ \hat{\theta}^* - t_{1-\alpha}^* S_{\hat{\theta}}^*],$$

where t_{α}^* and $t_{1-\alpha}^*$ are the upper α and $1-\alpha$ quantiles of the bootstrap *t*-distribution respectively, i.e. by finding the values that satisfy the two equations $P[(\hat{\theta}^* - \hat{\theta})/S_{\theta}^* > t_{\alpha}^*] = \alpha$ and $P[(\hat{\theta}^* - \hat{\theta})/S_{\theta}^* > t_{1-\alpha}^*] = 1-\alpha$, for the generated bootstrap estimates.

4. Performance Comparisons of Four Bootstrap Methods

4.1 Simulation Layout Setting

When establishing the formula of the process capability index C_{pmk}, there are two basic process characteristics. One is the process location in relation to its target value, the other is the process spread (overall process variation). By observing the formula, we know that the closer the process output is to the target value and the smaller the process spread the process is more capable. Comparing the pair of indices (C_{pmk}, C_{pm}), similarly to (C_{pk}, C_p), there is the relation $C_{pmk} = C_{pm} \times C_a$. Based on the relationship, there are several combinations of C_{pm} and C_a for the same C_{pmk}. On the assumption that $T = m$ (which is quite common in many practical situations) where m is the midpoint between LSL and USL, there are still several combinations of σ and C_a for the same C_{pmk} similarly. We can trade off between the magnitude of process variation and the degree of process centering. Table 2 displays various C_a values and the corresponding ranges of the departure magnitude of μ.

Table 2. C_a values and ranges of μ.

C_a value	Range of μ
$C_a = 1.00$	$\mu = m$
$0.75 < C_a < 1.00$	$0 < \lvert\mu - m\rvert < d/4$
$0.50 < C_a < 0.75$	$d/4 < \lvert\mu - m\rvert < d/2$
$0.25 < C_a < 0.50$	$d/2 < \lvert\mu - m\rvert < 3d/4$
$0.00 < C_a < 0.25$	$3d/4 < \lvert\mu - m\rvert < d$
$C_a = 0.00$	$\mu = LSL$ or $\mu = USL$
$C_a < 0.00$	$\mu < LSL$ or $\mu > USL$

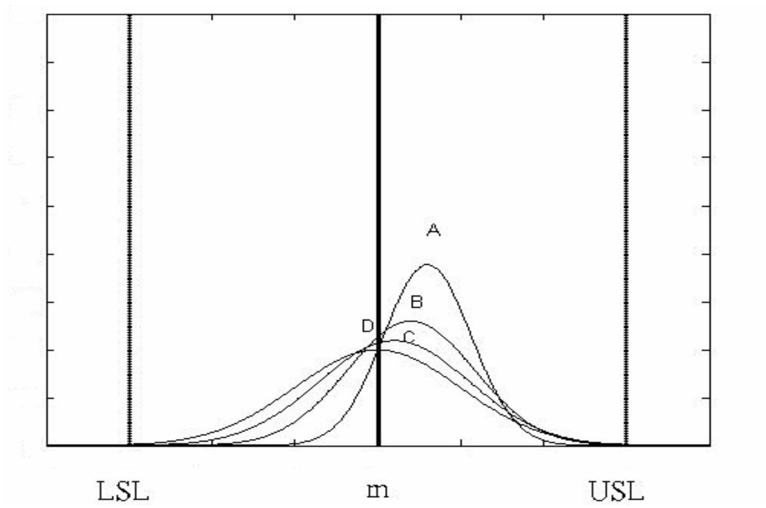

Figure 2. Four processes with $C_{pmk}=1.00$.

Table 3. Parameter values for two manufacturing suppliers used in the simulation study under $C_{pmk1}=C_{pmk2}=1.00$.

Case#	C_{pmk1}	μ_1	C_{a1}	σ_1	C_{pmk2}	μ_2	C_{a2}	σ_2
1	1	0.6	0.8000	0.5292	1	0.6	0.8000	0.5292
2	1	0.6	0.8000	0.5292	1	0.4	0.8667	0.7688
3	1	0.6	0.8000	0.5292	1	0.2	0.9333	0.9117
4	1	0.6	0.8000	0.5292	1	0	1.0000	1.0000
5	1	0.4	0.8667	0.7688	1	0.6	0.8000	0.5292
6	1	0.4	0.8667	0.7688	1	0.4	0.8667	0.7688
7	1	0.4	0.8667	0.7688	1	0.2	0.9333	0.9117
8	1	0.4	0.8667	0.7688	1	0	1.0000	1.0000
9	1	0.2	0.9333	0.9117	1	0.6	0.8000	0.5292
10	1	0.2	0.9333	0.9117	1	0.4	0.8667	0.7688
11	1	0.2	0.9333	0.9117	1	0.2	0.9333	0.9117
12	1	0.2	0.9333	0.9117	1	0	1.0000	1.0000
13	1	0	1.0000	1.0000	1	0.6	0.8000	0.5292
14	1	0	1.0000	1.0000	1	0.4	0.8667	0.7688
15	1	0	1.0000	1.0000	1	0.2	0.9333	0.9117
16	1	0	1.0000	1.0000	1	0	1.0000	1.0000

Figure 2 plots four processes with varied combinations of (C_a,σ) with $C_{pmk}=1.00$, LSL=-3, USL=3 and m=0, i.e. $(C_a,\sigma)=(0.8,0.5292)$ for process A, $(C_a,\sigma)=(0.8667,0.7688)$ for process B, $(C_a,\sigma)=(0.9333,0.9117)$ for process C and $(C_a, \sigma) = (1,1)$ for process D (from right to left in plot). These four processes are equivalent according to C_{pmk} (i.e. $C_{pmk} = 1.00$ for all four processes), and all have yields exceeding 99.73% but differ substantially with the magnitude of process variation and the degree of process centering. After setting the simulation environment, we make two investigations among the four bootstrap confidence limits to select a method which make our difference and ration testing more reliable. One is error probability analysis. We inspect the error probability from simulation results to know the magnitude of stability among four bootstrap methods. The other is selection power analysis. We choose a method with larger testing power by constructing the power curves. The sets of parameter values for two manufacturing suppliers used in the simulation study are given in Table 3. The selected parameters are chosen so as to investigate the performance of the methods for a wide range of index values and for both on-target and off-target processes. For each combination, a sample of size $n = 100$ was drawn with B=3000 bootstrap replications, and the single simulation was then replicated $N = 3,000$ times. Further analysis will be shown in Sections 4.2 and 4.3.

4.2 Error Probability Analysis

When deciding whether rejects the null hypothesis H_0 or not, we might make a mistake. Generally speaking, hypothesis tests are usually evaluated and compared through their probabilities of making mistakes. In this section, we measure these error probabilities from four bootstrap methods to determine which methods of testing have smaller and stable error probabilities.

In our analysis of simulation, the error probability is the proportion of times that rejecting the null hypothesis $H_0 : C_{pmk1} \geq C_{pmk2}$, while actually $H_0 : C_{pmk1} \geq C_{pmk2}$ is true. That is, we will calculate the proportion of times that the LCB of $C_{pmk2} - C_{pmk1}$ is positive and the LCB of C_{pmk2} / C_{pmk1} is larger than 1 when $C_{pmk1} = C_{pmk2} = 1.00$. While generating the simulation, a sample size n=100 drawn with B=3000 bootstrap replications, the single simulation was replicated N=3000 times and type I error $\alpha = 0.05$ for each case given in Table 2. Usually, it is required that the probability of the error selection be less than a maximum value α^*, generally referred to as the α^*–condition. The frequency of error selection is a binomial random variable with $N = 3000$ and $\alpha^* = 0.05$. Thus, a 99% confidence interval for the error probability is

$$\alpha^* \pm Z_{0.005} \times \sqrt{\alpha^*(1-\alpha^*)/N} = 0.05 \pm 2.576 \times \sqrt{(0.05 \times 0.95)/3000} = 0.05 \pm 0.0103.$$

That is, one could be 99% confident that a "true 0.05% error probability" would have a proportion of range from 0.0397 to 0.0603. Thus, from the results of simulation for each case, we could depict confidence interval (0.0397, 0.0603). Figure 3 and Figure 4 show the error probability of four bootstrap methods for the difference and the ratio statistics with 16 combinations tabulated in Table 3, respectively.

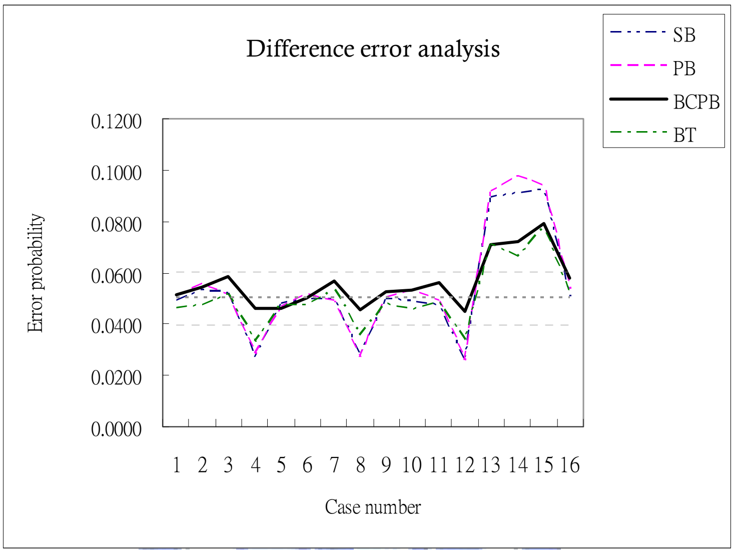

Figure 3. Error probability of four bootstraps under $C_{pmk1} = C_{pmk2} = 1.00$.

Figure 4. Error probability of four bootstraps under $C_{pmk1} / C_{pmk2} = 1.00$.

According to error probability analysis, it is shown that for the difference test, there are 7 combinations out of the 16 cases which were outside the interval (0.0397, 0.0610) for the SB, PB, BT methods. In contrast, 5 out of the 16 cases are beyond the interval for BCPB method. As for the ratio test, there are 11, 7 and 13 cases out of the 16 combinations outside the interval (0.0397, 0.0603) respectively. However, the BCPB method has only 5 out of 16 cases beyond these limits. Consequently, we know that the BCPB method has smaller and stable error probabilities for both difference and ratio test. Tables 4 and 5 show the results of error probability analysis for difference and ratio test.

Table 4. The results of error probability analysis for difference test.

Bootstrap method of difference test	Mean of these 16 cases error	Standard deviation of these 16 cases error	Number of out of limits	Out of limits case
SB	0.0532	0.020882	6	4,8,12,13,14,15
PB	0.0548	0.021841	6	4,8,12,13,14,15
BCPB	0.0560	0.010114	3	13,14,15
BT	0.0502	0.012275	6	4,8,12,13,14,15

Table 5. The results of error probability analysis for ratio test.

Bootstrap method of ratio test	Mean of these 16 cases error	Standard deviation of these 16 cases error	Number of out of limits	Out of limits case
SB	0.0423	0.017227	12	4,5,6,7,8,9,11,12,13,14,15
PB	0.0548	0.021841	6	4,8,12,13,14,15
BCPB	0.0548	0.009686	3	13,14,15
BT	0.0286	0.008759	14	1,2,3,4,5,6,7,8,9,10,11,12,13,16

In addition, an average lower bound and the standard deviation of the lower bound were calculated based on the $N = 3000$ distinct trials. The complete data for error probability with 16 combinations of (C_{a1}, σ_1) and (C_{a2}, σ_2) under $C_{pmk1} = C_{pmk2} = 1$ is tabulated in Appendix A. Error probability analysis information. Table 6 displays the particular four combinations of the average lower bound and the standard deviation of the lower bound for each of the four bootstrap confidence intervals.

Table 6. Simulation results of the four bootstrap methods for the difference and ratio statistics.

							Difference Statistic			Ratio Statistic		
C_{pmk1}	μ_1	C_{a1}	C_{pmi}	μ_2	C_{a2}	**Bootstrap Method**	**Error Prob.**	**Average LCB**	**Standard Deviation of LCB**	**Error Prob.**	**Average LCB**	**Standard Deviation of LCB**
1	**0.6**	**0.8**	**1**	**0.6**	**0.8**	SB	0.0493	-0.18447	0.11463	0.0407	0.82844	0.09346
						PB	0.0510	-0.18434	0.11593	0.0510	0.83943	0.09462
						BCPB	0.0513	-0.18454	0.11551	0.0510	0.83920	0.09432
						BT	0.0460	-0.18403	0.11246	0.0287	0.81287	0.09165
1	**0.6**	**0.8**	**1**	**0.2**	**0.93**	SB	0.0520	-0.19235	0.12165	0.0430	0.82041	0.10078
						PB	0.0517	-0.19182	0.12271	0.0517	0.83182	0.10174
						BCPB	0.0587	-0.19097	0.12447	0.0577	0.83254	0.10349
						BT	0.0517	-0.19237	0.12104	0.0350	0.80434	0.10057
1	**0.2**	**0.93**	**1**	**0.2**	**0.93**	SB	0.0470	-0.20028	0.12669	0.0370	0.81347	0.10233
						PB	0.0493	-0.20016	0.12756	0.0493	0.82669	0.10386
						BCPB	0.0560	-0.20039	0.1305	0.0573	0.82669	0.10622
						BT	0.0483	-0.20011	0.12702	0.0287	0.79479	0.10185
1	**0.2**	**0.93**	**1**	**0**	**1**	SB	0.0253	-0.22557	0.12012	0.0203	0.79196	0.09364
						PB	0.0263	-0.22695	0.12106	0.0263	0.80345	0.09512
						BCPB	0.0450	-0.20843	0.12521	0.0413	0.81858	0.09958
						BT	0.0337	-0.21407	0.12064	0.0163	0.78465	0.09414

4.3. Selection Power Analysis

Power is broadly defined as the probability that a statistical significance test will reject the null hypothesis for a specified value of an alternative hypothesis. Another way to define it is the ability of a test to detect an effect, given that the effect actually exists. If a study that is inefficiently precise or lacks power to reject a false null hypothesis, it will waste time and money in practical situation.

Therefore, in this section, we conduct selection power analysis to compare the performance of those four bootstrap methods. It is essential to apply a method which is efficiently precise and with power in hypothesis testing. Further simulations of selection power analysis are implemented with sample sizes n=10(10)200 for $C_{pmk1} = 1.00$ and $C_{pmk2} = 1.05(0.05)1.50$. The selection power computes the probability of rejecting the null hypothesis $H_0 : C_{pmk1} \geq C_{pmk2}$ while actually $H_1: C_{pmk1} < C_{pmk2}$ is true. For the difference statistic, the selection power computes the proportion of times that the LCB of $C_{pmk2} - C_{pmk1}$ is positive in the simulation. Similarly, for the ratio statistic, the selection power computes the proportion of times that the LCB of C_{pmk2} / C_{pmk1} is larger than 1. Figures 5-6 are the power curves of the four bootstrap methods for the difference and ratio statistic for on-target process with sample size $n =$ 10(10)200, $C_{pmk1} = 1.00$, $C_{pmk2} = 1.50$, $\mu_1 = 0$, $\mu_2 = 0$, respectively.

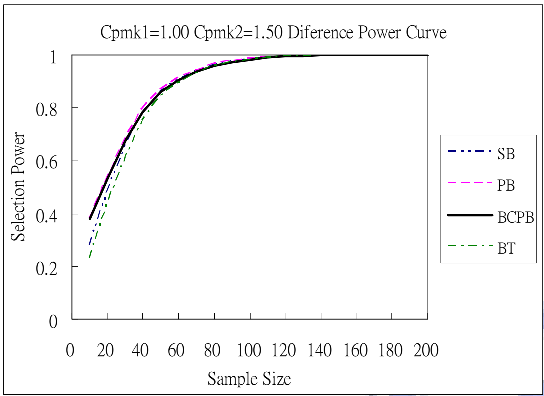

Figure 5. The selection power for the difference statistic for on-target process with sample size n=10(10)200, $C_{pmk1} = 1$, C_{pmk2}=1.50, $\mu_1 = \mu_2 = 0$.

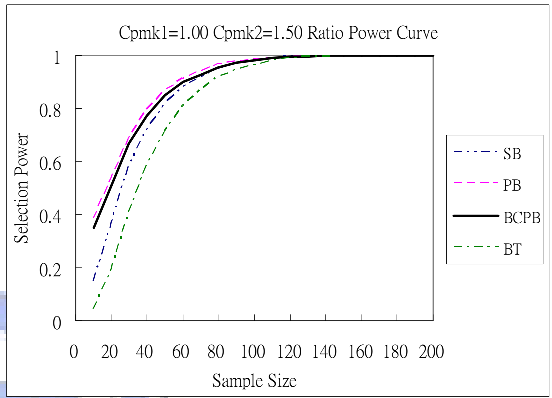

Figure 6. The selection power for the ratio statistic for on-target process with sample size n =10(10)200, $C_{pmk1} = 1$, C_{pmk2}=1.50, $\mu_1 = \mu_2 = 0$.

Similarly, Figures 7-8 are the power curves of the four bootstrap methods for the difference and ratio statistic for off-target process with sample size $n =$ 10(10)200, $C_{pmk1} = 1.00$, $C_{pmk2} = 1.50$, $\mu_1 = 0.4$, $\mu_2 = 0.4$, respectively.

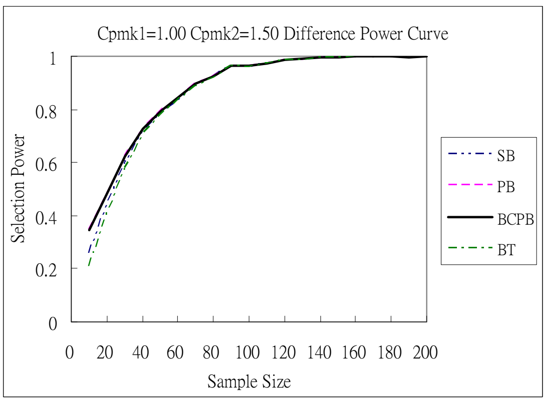

Figure 7. The selection power for the difference statistic for off-target process with sample size n=10(10)200, $C_{pmk1} = 1$, C_{pmk2}=1.50, $\mu_1 = \mu_2 = 0.4$.

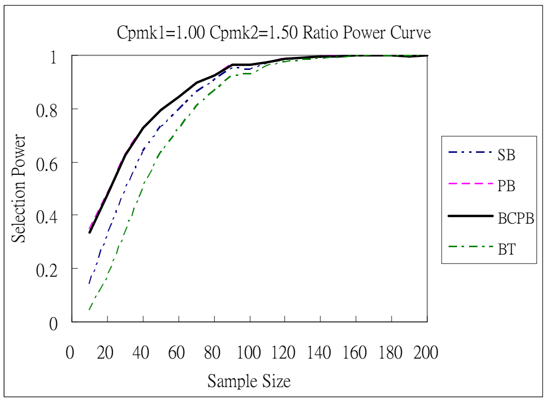

Figure 8. The selection power for the ratio statistic for off-target process with sample size n = 10(10)200, $C_{pmk1} = 1$, C_{pmk2}=1.50, $\mu_1 = \mu_2 = 0.4$.

According to Figures 5-8, it is shown that the PB and BCPB methods have larger selection power with fixed sample size in contrast with the SB and BT methods. In other words, we only need smaller required sample size for the PB and BCPB methods when conducting hypothesis test. By evaluating both error probability and selection power, the BCPB method has more stable error probability and larger selection power with fixed sample size. Consequently, we suggest that the BCPB method among four bootstrap methods is the better approach for further analysis. Besides Figures 5-8, the complete data and power curves are shown in Appendix B.

5. Supplier Selection Based on BCPB Method

5.1 Sample Size Determination with Designated Selection Power

It is a practical and important issue to determine an appropriate sample size for supplier selection problem in manufacturing industries. A study that lacks of power to reject a false null hypothesis is a waste of time and money. On the other hand, an investigation collects too many samples or has too much power to reject Ho is also wasteful for manufacturing. Thus, the requirement of appropriate sample size with designed selection power should be determined carefully before conducting the hypothesis test.

According to error probability and selection power, it has been suggested that the best of those four bootstrap methods in our study is the BCPB method. Thus, the simulation technique was applied to investigate the BCPB method with B =3,000 bootstrap replications, and the single simulation was then replicated N =3,000 times. For practical application and engineers' convenience, we calculate the requirement of sample size for difference and ratio hypothesis test by using MATLAB program. For both of the two tests, it has been investigated with $C_{pmk1}=1.00$ and 1.33 for supplier I and $C_{pmk2}=1.15(0.05)1.50$ and 1.48(0.05)1.83 for supplier II. The designated selection power = 0.90, 0.95, 0.975 and 0.99 which computes the probability of rejecting the null hypothesis $H_0: C_{pmk1} \geq C_{pmk2}$ while actually $H_1: C_{pmk1} < C_{pmk2}$ is true. Tables 7-10 display the sample size required of the BCPB method for the difference and ratio statistic with various selection powers.

Table 7. Sample size required of BCPB method for the difference statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.00$, $C_{pmk2} = 1.15(0.05)1.50$.

C_{pmk1}	1	1	1	1	1	1	1	1
C_{pmk2}	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
90%	505	288	189	140	107	83	69	58
95%	627	365	246	174	138	107	87	74
97.50%	745	430	284	207	164	125	104	87
99%	918	538	369	263	200	158	130	115

Table 8. Sample size required of BCPB method for the ratio statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.00$, $C_{pmk2} = 1.15(0.05)1.50$.

C_{pmk1}	1	1	1	1	1	1	1	1
C_{pmk2}	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
90%	504	293	199	143	109	83	72	58
95%	624	378	241	176	135	110	89	73
97.50%	765	446	291	214	156	127	105	88
99%	945	531	364	248	203	157	128	108

Table 9. Sample size required of BCPB method for the difference statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.33$, $C_{pmk2} = 1.48(0.05)1.83$.

C_{pmk1}	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
C_{pmk2}	1.48	1.53	1.58	1.63	1.68	1.73	1.78	1.83
90%	820	465	313	226	171	136	109	91
95%	1027	582	397	275	212	168	136	113
97.50%	1215	713	475	332	250	196	159	137
99%	1485	863	580	413	307	248	198	165

Table 10. Sample size required of BCPB method for the ratio statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.33$, $C_{pmk2} = 1.48(0.05)1.83$.

C_{pmk1}	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
C_{pmk2}	1.48	1.53	1.58	1.63	1.68	1.73	1.78	1.83
90%	815	469	315	228	175	136	111	91
95%	1059	582	386	275	213	169	140	116
97.50%	1262	732	467	338	270	212	167	137
99%	1912	1032	692	538	420	362	198	166

For the convenience of observation, Figures 9-12 depict sample size curves based on the four sample size tables, respectively.

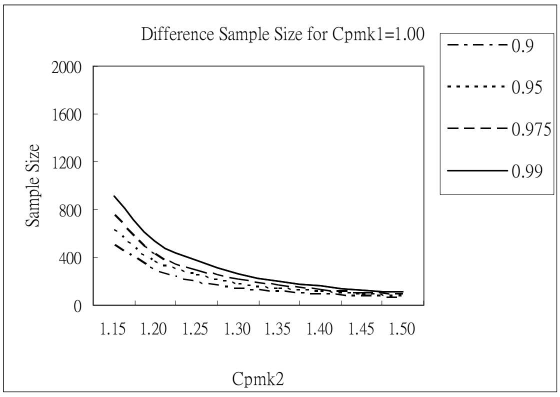

Figure 9. The sample size curve for the difference statistic under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.00$, $C_{pmk2} = 1.15(0.05)1.50$.

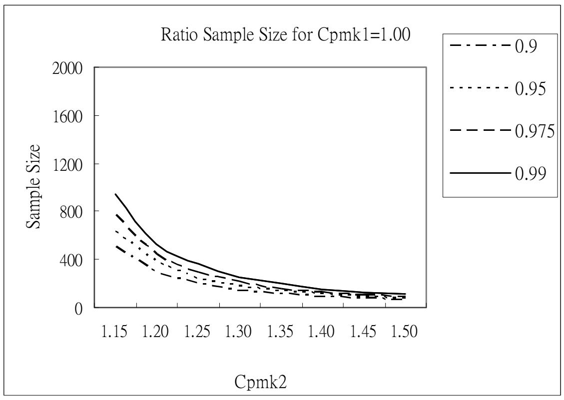

Figure 10. The sample size curve for the ratio statistic under $\alpha = 0.05$,with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.00$, $C_{pmk2} = 1.15(0.05)1.50$.

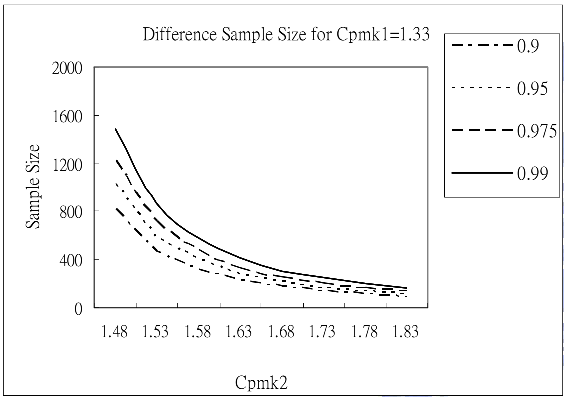

Figure 11. The sample size curve for the difference statistic under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.33$, $C_{pmk2} = 1.48(0.05)1.83$.

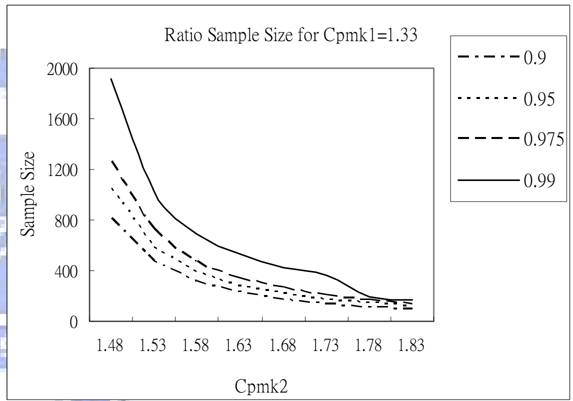

Figure 12. The sample size curve for the ratio statistic under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pmk1} = 1.33$, $C_{pmk2} = 1.48(0.05)1.83$.

Clearly, the sample size calculated from MATLAB program indicates that the larger the value of the difference $\delta = C_{pmk2} - C_{pmk1}$ between supplier I and supplier II, the smaller the sample size required for fixed selection power. And for fixed difference $\delta = C_{pmk2} - C_{pmk1}$ and C_{pmk1}, sample size increases with various selection power. Besides, if the minimum requirement of C_{pmk} value is larger, more samples are needed to distinguish between two competing processes. The results could be explained that more precise to reject the false null hypothesis more cost and time is required for practitioners.

5.2 Selection Procedure of Two Competing Suppliers

Supplier selection problem has become a more and more important issue in manufacturing industry. It is worthwhile to compare two processes with a standard and effective selection procedure. For engineers' or practitioners' convenience, the complete testing procedure for two suppliers' processes is summarized in step form as follows:

Step 1.

Determine (a) the minimum requirement of C_{pmk} value (b) the minimum acceptable difference between two C_{pmk} indices, $\delta = C_{pmk2} - C_{pmk1}$ (c) required selection power. Then, the designated sample size can be obtained by checking Tables 8-10.

Step 2.

Take a random sample with sample size n from each supplier's process and calculate the sample mean $\overline{X} = \sum_{i=1}^{n} X_i / n$ and sample variance $S_n^{\ 2}$ = $\sum_{i=1}^{n} (X_i - \overline{X})^2 / n$.

Step 3.

Apply the Shapiro-Wilk test to confirm whether the sample data for the two suppliers are taken from normal processes.

Step 4.

Calculate the sample estimators, $\hat{C}_{pmk1}$ and $\hat{C}_{pmk2}$. Based on the BCPB method, we implement the Matlab program to obtain the LCB of $\hat{C}_{pmk2} - \hat{C}_{pmk1}$ for the difference test and the LCB of $\hat{C}_{pmk2} / \hat{C}_{pmk1}$ for the ratio test.

Step 5.

The decision rule is : If the LCB of $\hat{C}_{pmk2} - \hat{C}_{pmk1}$ is positive or the LCB of $\hat{C}_{pmk2} / \hat{C}_{pmk1}$ is larger than 1, it can be concluded that the supplier II is better than the supplier I. Otherwise, the existing supplier I is not better than the new supplier II.

6. Application Example

6.1 Application Example of FPC

FPC stands for Flexible Printed Circuits, which is the electronic component being much lighter and thinner than Rigid Printed Circuit (RPC). Since Flexible Printed Circuit has excellent working efficiency and strong heat-resistance, FPC is widely used as a core component of all electronic goods. It is usually applied to the products like cameras, laptops, peripheral equipments, mobile phones, video, audio units, printers, DVD, TFT LCD, satellite equipment, military equipments and medical instruments, etc. Commercialized in the 1950s, it has been steadily developed and improved so far, global sales exceed $5.6 billion USD in 2004. With the increasing demand of electronic goods, there will be large growth in the entire FPC industry.

A flexible printed circuit consists of three layers of material: a base layer of dielectric, a central conductor layer and a top dielectric layer called a coverlayer (if a film) or covercoat (if a liquid coating) (Thomas (1996)). The coverlayer may be absent in low cost circuitry. Openings or apertures are provided in the coverlayer to allow contact with the conductor layer at desired terminal or pad locations. It's typical for each pad – termination site, an enlarged area on a conductor, usually at the end. The termination sites have a throughhole to receive a FPC connector pin or other hardware item which is soldered to the pad. Most throughholes holes are created either by NC drilling or die punching. As the improvement of technology, the digital electronic products are thinner and smaller. The electronic components of FPC are also required to correspond to its specification. Among these components, the FPC connector is one of the critical parts in assembling electronic goods. Despite the low cost of FPC connector in most FPC components, the connector plays an important role in product reliability. With the gradual increasing requirements for part and device reliability, the need to evaluate process capability and product failure rates is now greater than ever. Consequently, in order to make sure the reliable connection between FPC Connector and FPC board, the thickness of FPC board was investigated to ensure specification and quality reliability to meet customer requirements. Figures 13-14 illustrate a particular type of FPC board and a kind of FPC Connector. (Figures are taken from http://www.bestfpc.com/.)

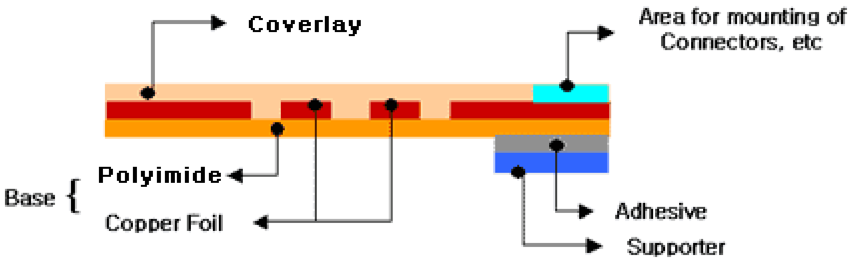

Figure 13. Coverlay Type - Single Sided FPC.

Figure 14. 0.5 mm SMT FPC Connector —— Straight.

The application example is taken from a corporation in Taipei, Taiwan. In order to enhance the product quality, the company desires to determine the more capable electronic components between two competing suppliers manufacturing FPC boards. For the SMT type of 0.5 mm FPC Connector, the USL, LSL, and the target value of FPC board thickness are 0.33 mm, 0.3 mm and 0.27 mm, respectively. The layout of the SMT type of 0.5 mm FPC Connector and 0.3 mm thickness FPC is shown in Figure 15.

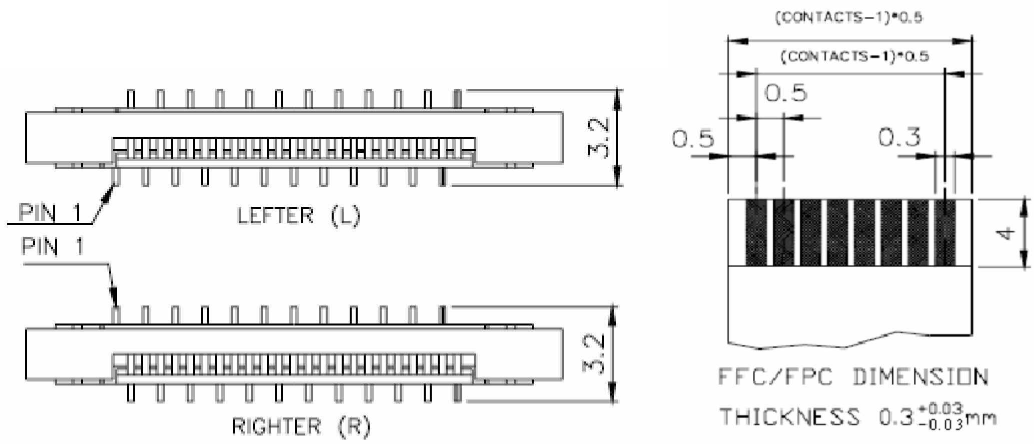

Figure 15. The layout of the SMT type of 0.5 mm FPC Connector and 0.3 mm thickness FPC.

6.2 Data Analysis and Supplier Selection

For the supplier selection problem applied to this application example, we first select two batches of goods from two competing FPC manufacturers. According to the selection procedure mentioned in Chapter 5.2, the complete procedure for two FPC board suppliers' processes is summarized in step form as follow:

Step1.

In the application case, we determine (a) the minimum requirement of C_{pmk} value is 1.00. (b) the minimum acceptable difference between two C_{pmk} indices, $\delta = C_{pmk2} - C_{pmk1}$ =0.35 (c) required selection power with 0.95. Then, by checking Tables 7-8, the sample size of difference test is 138 and of ratio test is 135. By the way, we take 138 samples for Suppler I and Supplier II, respectively.

Step2.

Take a random sample from each supplier's process (the thickness of FPC board). Tables 11-12 are the sample data of two suppliers and the sample mean $\overline{X} = \sum_{i=1}^{n} X_i / n$ and sample variance $S_n^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2 / n$ are shown in Table 14.

Table 11. Sample data of supplier I.

0.28526	0.30794	0.29385	0.30208	0.29504	0.28902	0.29142	0.30913	0.32125
0.29273	0.28734	0.31174	0.28497	0.29523	0.30050	0.30055	0.31728	0.30538
0.29301	0.30420	0.28484	0.30078	0.30638	0.29491	0.30095	0.28915	0.30173
0.29795	0.29788	0.30193	0.31085	0.31528	0.29322	0.30191	0.28961	0.30702
0.30222	0.31251	0.28887	0.28862	0.31151	0.29932	0.29288	0.29574	0.29401
0.29157	0.29692	0.30447	0.29865	0.28198	0.30181	0.29390	0.30284	0.29126
0.30338	0.30874	0.28563	0.29846	0.31236	0.29021	0.30681	0.29976	0.28916
0.29828	0.29725	0.29337	0.26961	0.31548	0.30249	0.29545	0.31697	0.29654
0.28809	0.29012	0.28484	0.30190	0.30130	0.30373	0.29387	0.29891	0.29759
0.30038	0.30922	0.29843	0.30404	0.28696	0.30866	0.30816	0.30147	0.30661
0.29519	0.30214	0.29895	0.29137	0.29180	0.30232	0.30074	0.27449	0.27868
0.30786	0.31494	0.30843	0.30240	0.29895	0.29834	0.29819	0.28830	0.29386
0.30933	0.29587	0.28777	0.30473	0.30292	0.30098	0.28573	0.30603	0.29530
0.30267	0.29290	0.30210	0.29892	0.29607	0.28127	0.29729	0.29566	0.30026
0.29265	0.28585	0.30191	0.27615	0.28655	0.29777	0.29640	0.29550	0.28984
0.30095	0.29664	0.30181						

Table 12. Sample data of supplier II.

0.29604	0.30263	0.30055	0.30057	0.30148	0.30684	0.30558	0.29259	0.30955
0.30842	0.31091	0.31227	0.31455	0.29358	0.30306	0.29415	0.29071	0.30884
0.29320	0.30920	0.29910	0.29363	0.29812	0.30103	0.29592	0.30699	0.29734
0.30383	0.30101	0.31328	0.31072	0.29676	0.29550	0.28476	0.29495	0.32367
0.30149	0.29201	0.29069	0.30562	0.30186	0.29710	0.31395	0.30993	0.29973
0.30611	0.30105	0.29845	0.30132	0.29917	0.30968	0.29224	0.29531	0.30479
0.29776	0.29703	0.29779	0.30713	0.29841	0.29716	0.31045	0.29204	0.29955
0.29462	0.30019	0.30284	0.30128	0.30731	0.30679	0.28966	0.30441	0.30105
0.29399	0.30472	0.31237	0.30539	0.30691	0.31480	0.30288	0.30298	0.30667
0.30983	0.29624	0.30411	0.30047	0.31015	0.30555	0.29452	0.30478	0.29512
0.30132	0.29027	0.30454	0.28694	0.30540	0.30597	0.30010	0.30686	0.29425
0.30904	0.29812	0.29751	0.31952	0.30396	0.30605	0.30190	0.29506	0.29733
0.30849	0.29205	0.30282	0.30116	0.30037	0.30211	0.30859	0.29342	0.29764
0.29869	0.30208	0.30394	0.31120	0.29960	0.30924	0.29495	0.30477	0.29275
0.30038	0.30475	0.29904	0.29880	0.30601	0.29862	0.30565	0.30646	0.30059
0.30042	0.29493	0.30681						

Step3.

Apply the Shapiro-Wilk test to confirm whether the sample data for the two suppliers are taken from normal processes. With the Shapiro-Wilk test p-value >0.1 for two suppliers' samples, we could conclude that the sample data for the two suppliers is taken from normal processes. The outcome of the Shapiro-Wilk test is shown in Table 13. Besides, Figures 16-19 display the histogram and normal probability plot of the 138 samples for two suppliers.

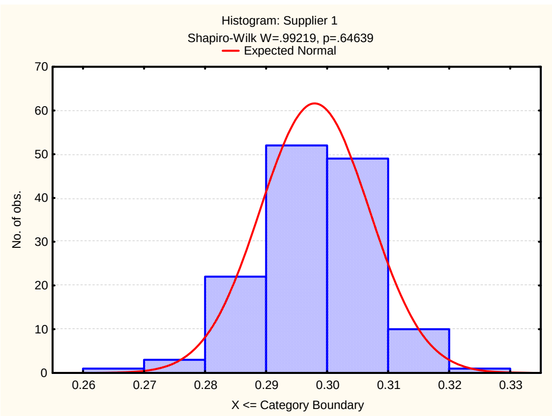

Figure 16. Histogram of supplier I data.

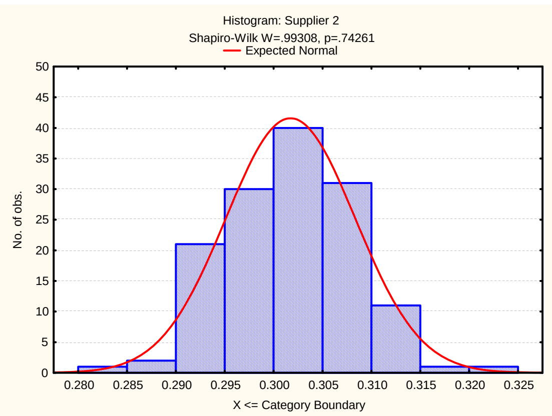

Figure 17. Histogram of supplier II data.

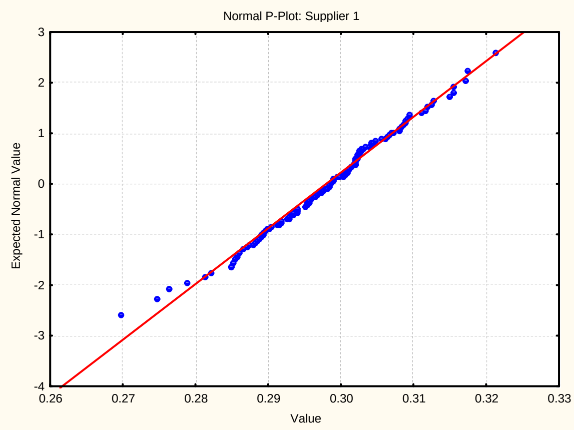

Figure 18. Normal probability plot for Supplier I.

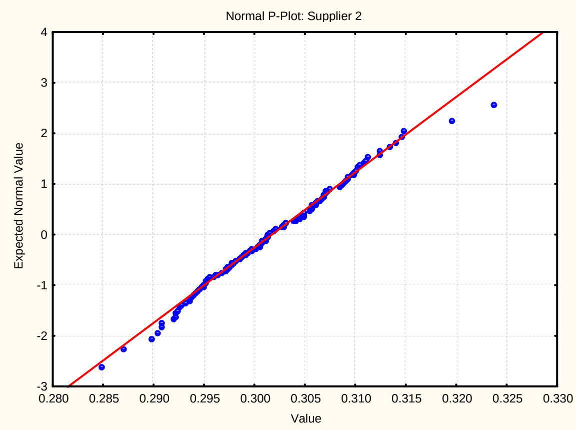

Figure 19. Normal probability plot for Supplier II.

Table 13. The outcome of Shapiro-Wilk test.

population	statistic W	p-value	conclusion
Supplier I	0.99219	0.64639	Normal
Suppler II	0.99308	0.74261	Normal

Step4.

We calculate the sample means, sample standard deviations and the sample estimators $\hat{C}_{pmk}$ for supplier I and supplier II. The sample statistics are summarized in Table 14. Based on the BCPB method, we implement the Matlab program to obtain the LCB of $\hat{C}_{pmk2} - \hat{C}_{pmk1}$ = 0.13943 for the difference test and the LCB of $\hat{C}_{pmk2} / \hat{C}_{pmk1}$ = 1.1244 for the ratio test.

Table 14. The sample statistics for two suppliers.

Population	$\bar{X}$	S	$\hat{C}_{pmk}$
Supplier I	0.29797	0.0089350	1.0175
Suppler II	0.30173	0.0066209	1.3770

Step5.

The decision rule is: (a) The LCB of $\hat{C}_{pmk2} - \hat{C}_{pmk1}$ =0.13943 is positive. (b) The LCB of $\hat{C}_{pmk2} / \hat{C}_{pmk1}$ =1.1244 is larger than 1. Consequently, it could be concluded that for process capability of the FPC board thickness, the supplier II is better than the existing supplier I.

7. Conclusions

In the field of supplier selection, it has been promoted the use of various PCIs for evaluating a supplier's process capability. In the presence of all, many researchers have indicated varied approach for supply selection based on the indices C_{pk} and C_{pm}. In order to take into account the process yield as well as the process loss, this study implements bootstrap approach for supplier selection based on C_{pmk}. The findings is that the BCPB method among four bootstrap methods is the better approach for processes comparison based on the index C_{pmk}. One possible conclusion is that the BCPB method has smaller and stable error probabilities for both difference and ratio test. And this method also has larger selection power with fixed sample size. Thus, it is recommended to apply this method to further analysis.

By the result of simulation, we implement the BCPB method to develop a practical step-by-step testing procedure for engineers to refer to supplier selection decisions. We readily acknowledge that the processes from both suppliers should be in statistical control and have a normal distribution in our research. The approach outlined in this study could be replicated in many manufacturing plants for the decision of selecting the better supplier. Finally, a practical application example in FPC industry is also investigated and applied the selection procedure to illustrate decision steps in supplier selection problem.

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