# 國立交通大學交通運輸研究所

## 碩士論文

不可儲存商品之生產績效衡量--整合式資料包絡分析模式

**An Integrated Data Envelopment Analysis Model to Evaluate the Performance of Non-storable Commodities**

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## 摘要

在評估不可儲存商品（如運輸服務）之生產績效時，由於生產效率與生產效果之績效值並不等同，故應同時衡量兩部分，方不至於偏頗。當不可儲存商品一旦生產且有一部分產出未能同時被消費時，則其生產效果(指技術效率與服務效果的綜合效果)的績效值將小於生產效率的績效值。有鑑於此，本研究試圖建構一整合式資料包絡分析（Integrated Data Envelopment Analysis; IDEA）模式，以同時求解技術效率及服務效果的績效值。本 IDEA 模式可同時決定投入變數、生產變數及消費變數的參數值，亦可證明具有「合理性」及「唯一性」。經個案分析結果顯示，本 IDEA 模式的鑑別力較傳統 DEA 模式高。另，本文進一步建構一般化 IDEA 模式，可看出權重變動對各個 DMU 之影響，俾提出更多改善效率之方法。

**關鍵字**：整合式資料包絡分析模式，不可儲存商品，技術效率，技術效果。

# An Integrated Data Envelopment Analysis Model to Evaluate the Performance of Non-storable Commodities

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## Abstract

Efficiency and effectiveness for non-storable commodities such as transport services represent two distinct measurements. When such commodities are produced and a portion of which are not consumed instantaneously, the technical effectiveness (a combined effect of technical efficiency and service effectiveness) would be likely less than the technical efficiency. Based on this, this thesis attempts to develop an integrated data envelopment analysis (IDEA) model that can jointly determine the overall efficiency from the aspects of technical-efficiency, service-effectiveness, and technical-effectiveness. The core logic for the proposed IDEA model is to simultaneously determine the virtual multipliers associated with the variables of factor production and consumption. The underlying properties of reasonability and uniqueness of the proposed IDEA model are proven. The applicability of the proposed model is also demonstrated with a case study. It shows that our proposed IDEA model has higher discrimination power than the conventional separated DEA models.

**Key Words**: integrated DEA model, non-storable commodities, technical efficiency, technical effectiveness.

# 致謝

兩年的光陰悄悄飛逝，終於到了道離別的一刻，心中有滿足也有失落，滿足於學業的完成，失落在分離即將到來。在交通大學的兩年中，結識了許多影響深遠的好朋友以及尊長，在此謹以文字獻給陪伴我度過這兩年時光的大家。

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回想這兩年中，從大家一起進入研究所的那天，就開啟我們彼此的友誼之窗，大家在這兩年中總是不吝惜對彼此伸出援手，互相幫忙互相扶持，謝謝大家在這兩年中的陪伴，讓我知道同學之間的情誼可以這樣的深刻，如此的銘心，很不想要互道珍重，但是大家有各自美好的前程，也做好展翅高飛的準備，所以我預祝大家前程似錦、飛黃騰達。最後感謝你們陪我度過這兩年快樂又艱辛的日子。

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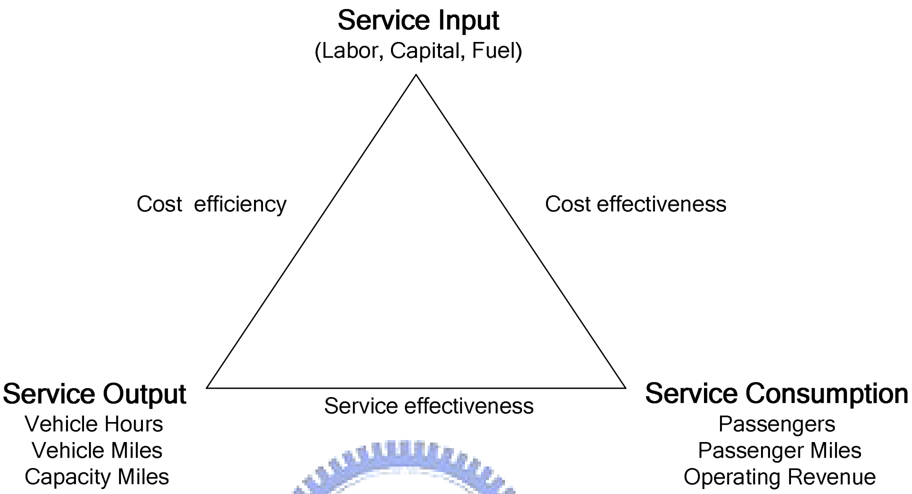
# 1. Introduction

## 1.1. Background

Data Envelopment Analysis (DEA) is a technique that provides a comprehensive insight into how comparatively well an organization performs. It can be used to rank quality level and analyze the performance with multiple inputs and outputs simultaneously. DEA imposes neither a specific functional relationship between production output and input, nor any assumptions on the specific statistical distribution of the error terms. DEA can be defined as a nonparametric method of measuring the efficiency of a Decision Making Unit (DMU).

DEA can be directly applied to evaluate the relative performance of the companies producing storable products, since these products can be stored for re-sell in the future even they cannot be sold instantly. The operating performance of such organization can be represented by its technical efficiency which is equivalent to technical effectiveness. However, in evaluating the industry producing non-storable products, such as transportation industry, technical efficiency only represent one aspect of the performance. The manager of a transport company might even more concern about technical effectiveness, which measures how many revenue passenger-miles or ton-miles are generated. Accordingly, Fielding (1985) proposed an analytical framework to evaluate the performance of a transportation industry by three aspects: cost-efficiency, service-effectiveness and cost-effectiveness, as depicted in Fig.1. In order to completely and fairly evaluate the relative performance of a transport organization, many studies employed DEA to evaluate the efficiency and effectiveness under respective aspect independently. For instance, Chiou and Chen (2006) employed DEA to evaluate the relative performance of domestic air routes operated by one airline under these three aspects respectively. However, some contradictory improvement suggestions were proposed based on the evaluating results of three independent DEA model. Lan and Lin (2003) employed a two-stage DEA model to evaluate the relative efficiency of various rail companies. They first use input-oriented DEA model to evaluate the cost-efficiency of these companies, then employ output-oriented DEA model to evaluate the service-effectiveness of these companies. The efficiency scores of cost-effectiveness aspect can be obtained as the product of the scores of cost-efficiency and service-effectiveness. Although this approach (two-stage DEA model) will not generate conflicting improvement suggestions, an unrealistic assumption have been made that the organization can be clearly divided into two departments: production and sales and be evaluated separately without any integration or coordination.

These unrealistic evaluation results of abovementioned studies are mainly rooted from their separate evaluation procedure. Therefore, a one-stage evaluation procedure is extremely essential to evaluate the performance of transportation industry for avoiding these problems. This study aims to develop an integrated DEA model to simultaneously evaluate the performance of transportation industry under three various aspects within one stage.




Source: Fielding, G.J., Babitsky, T.T. and Brenner, M.E. (1985) Performance evaluation for bus transit. *Transportation Research*, **19A**, 73-82.

Fig 1 The relationship between cost efficiency, cost effectiveness and service effectiveness

## 1.2. Purpose

Based on the abovementioned background and motivation, the main purposes of this study are listed as follows:

1. Review and summarize the related studies in evaluating the performance of transportation industry by applying DEA model.

2. Develop and validate a one-stage DEA model for simultaneously evaluating the relative performances of transportation organizations under three aspects of cost-efficiency, service-effectiveness and cost-effectiveness.

3. Propose an effective and efficient solution algorithm for the one-stage DEA model.

4. Apply the proposed one-stage DEA model to evaluate the relative performances of domestic air routes and compare the results with those of Chiou and Chen(2006).

## 1.3. Framework and organization

The flowchart of this study is shown in Figure 2. Following this chapter, the thesis is organized as follows. Chapter 2 reviews some relevant literature on DEA. Chapter 3 introduces our proposed integrated DEA models. The essential properties of the proposed models are proven in Chapter 4. A case study with the proposed IDEA models is conducted in Chapter 5. Final conclusions and future study are addressed in Chapter 6.

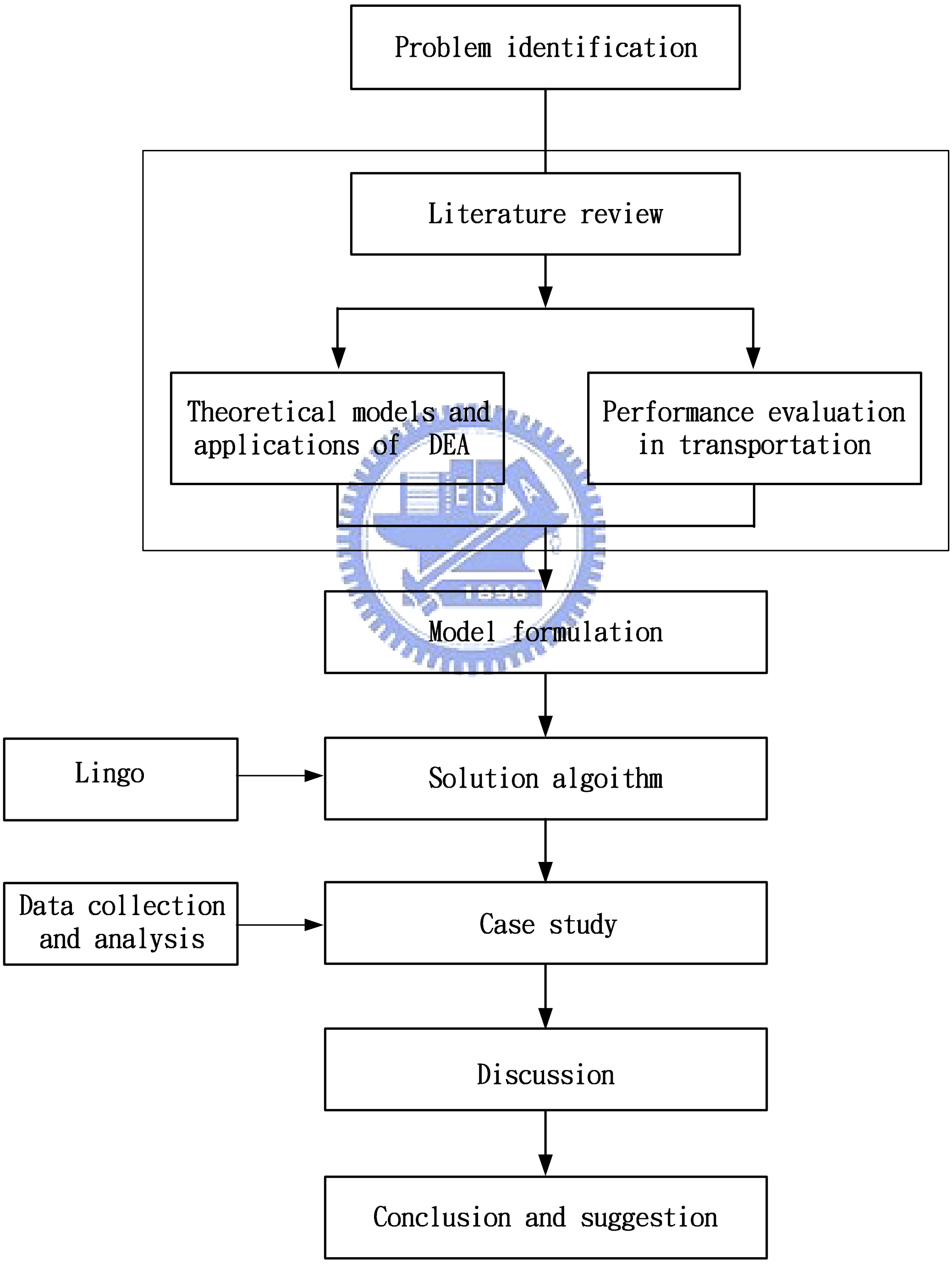



Fig 2 Research flowchart

# 2. Literature review

## 2.1. Applications of DEA in transportation

DEA model has been widely applied to evaluate the relative performance of transportation industries, such as air transportation, maritime transportation, transit, railway, etc. The related studies are reviewed and summarized as follows.

### 2.1.1. Air transportation

Adler and Berechman (2001) use DEA to determine the relative efficiency or quality ranking of various West-European and other airports. The main source of data for this study was a questionnaire whose objective was to evaluate the quality level of 26 airports.

Chiou and Chen (2006) employ DEA approach to evaluate the performance of domestic air routes from the perspectives of cost efficiency, cost effectiveness and service effectiveness. The cost efficiency indicates the relative efficiency in the production; while the service effectiveness stands for the relative efficiency in the sale. The cost effectiveness therefore represents a combined effect of the relative efficiency in both production and sale. This paper adopts this framework to evaluate air route performance.

There are three input variables: fuel cost (FC), personnel cost (PC), including the salaries of cabin and ground-handling crews, and aircraft cost (AC), including maintenance costs, depreciation costs and interest payments. The production variables include number of flights (FL) and seat-mile (SM). The service variables include passenger-mile (PM) and embarkation passengers (EP), as shown in Fig. 3. This study also uses Tobit regression to identify variables are significant or not.

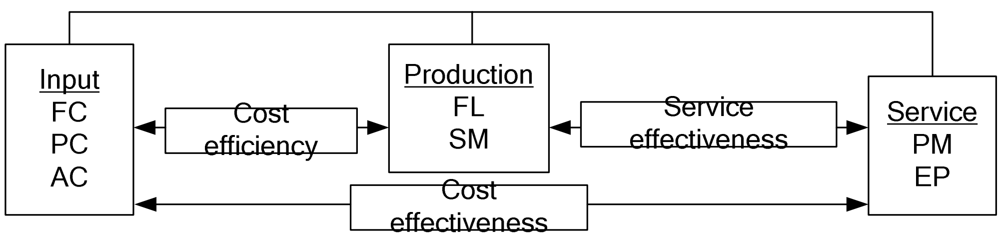



Fig 3 The analysis framework

Peck *et al.* (1998) focus on discretionary maintenance strategies and their relationship to aircraft reliability, as measured by the percentage of scheduled flights delayed because of mechanical problems. The methodology of data envelopment analysis employed to identify the various strategies employed by the major airlines over the time period 1990-1994. The output variable was defined to be the percentage of all scheduled flights arrivals delayed for mechanical reasons not including weather or scheduling problems. The input variables represent all of the reported non-overlapping categories of maintenance expenses.

Tzeng and Chiang (2000) propose a new efficiency measure in data envelopment analysis: the efficiency achievement measure. Comparing with the traditional radial measure and distance measure proposed by Chang and Guh (1995) using different sets of multipliers to compute the efficiency ratio, the efficiency achievement measure does so by using the common multipliers that obtained easily by solving fuzzy multiple objectives programming.

### 2.1.2. Maritime transportation

Tongzon (2001) applies DEA to provide an efficiency measurement for four Australian and twelve other international container ports. This study uses two output and six input measures of port performance. The output measures are cargo throughput and ship working rate. Based on the production framework, port inputs can be generalized as land, labor and capital. The major capital inputs in port operations are the number of berths, cranes and tugs. This study has shown the suitability of DEA for port efficiency evaluation.

### 2.1.3. Transit

Karlaftis (2003) uncovers production characteristics of transit firms by relating efficiency with production in a less constraining environment. In this study uses data envelopment analysis to rank efficient subsets of transit systems and then based on the results of the DEA analysis, build globally efficient frontier production functions. The results indicate that when jointly considered, there is an improvement on both the theoretical and empirical aspects of examining efficiency and production in transit systems.

Fielding et al. (1984) use three categories of statistics-service inputs, service outputs and service consumption-provided the framework to organize the much larger set of data. Cost-efficiency indicators measure service inputs (labor, capital, fuel) to the amount of service produced (service outputs: vehicle hours, vehicle miles, capacity miles, service reliability). Cost-effectiveness indicators measure the level of service consumption (passengers, passenger miles, operating revenue) against service inputs. Finally, service-effectiveness indicators measure the extent to which service outputs are consumed. Fig. 1 portrays the organizing framework.

Odeck and Alkadi (2001) focus on the performance of Norwegian bus companies subsidized by the government. The performance is evaluated from a productive efficiency point of view. The framework is DEA approach to efficiency measurement. In this study, the output variables are seat kilometers, vehicle kilometers, passenger kilometers, and passengers and the input variables are the total number of seats (TS) offered by the company, fuel consumption in liters (FC) and equipment (EQ) such as oil and tires. The average bus company is found to be exhibiting increasing return to scale. This means that the average company is smaller than the optimal size.

Viton (1998) examines the claim that US bus transit productivity has declined in recent years. These systems operated either conventional motor-bus (MB) or demand-responsive (DR) services (or both), but no other form of public transit. This paper uses a piecewise-linear best-practice DEA production frontier, computed for multi-modal bus transit between 1988 and 1992. The outputs are vehicle-miles, vehicle hours and passenger trips.

The inputs come from three sources. First is a set of variables describing the situation in which the system finds itself. These include the average fleet age and the number of directional miles provided by the MB. Second, we use a number of conventional inputs: the fleet sizes, and the number of gallons of fuel. It distinguishes four kinds of labor inputs: the number of person-hours of transportation, maintenance, administrative, capital and labor used by each mode in providing service. The final inputs are those for which there is no obvious summary physical measure. For these we use a cost measure. In this category we have the cost of tires and other materials and supplies, of services, of utilities, and of insurance.

The results do not support the pessimistic view of changes in the industry because both the efficiency and productivity approaches suggest an improvement.

Cowie and Asenova (1999) claim that the ideal output measure is passenger kilometers, unfortunately due to commercial sensitivity such figures are unavailable. Nevertheless, clearly related to passenger kilometers is operating revenue. The inputs for each company reflect capital and labor elements. Labor is simply the total staff employed, both management and operational. This study shows strong evidence of increasing returns for smaller companies. This study uses technical, managerial and organizational efficiency. The technical efficiency of each company is assessed by a comparison of all companies in the data set. The level of managerial efficiency however, can be further isolated from overall technical efficiency by separating DMUs into the different sets of interest. The difference between technical and managerial efficiency represents the level of inefficiency attributed to the organizational structure.

Karlaftis (2004) uses data envelopment analysis and globally efficient frontier production functions to investigate two important issues in transit operations: first, the relationship between the two basic dimensions of performance, namely efficiency and effectiveness; second, the relationship between performance and scale economies.

This study found that systems performing well in one dimension (e.g. efficiency) generally perform well in the other dimensions (e.g. effectiveness). This is important since the performance scores can be useful in describing transit system performance both for internal and external purposes.

This study uses two outputs: vehicle-miles (often referred to as ‘‘produced output type’’) and passenger-miles (often referred to as ‘‘consumed output type’’). Transit systems most frequently use three input quantities, namely labor, fuel, and capital to produce output.

As many authors have suggested (for example Fielding, 1987), vehicle-miles are related to service efficiency while ridership (and passenger-miles) are related to effectiveness; a combined vehicle-miles and ridership output is related to a ‘‘combined’’ or ‘‘overall’’ performance measure. As such, in this paper we estimate three separate sets of models, each of them utilizing the same inputs but different outputs: the first is an efficiency model, using total annual vehicle-miles as output; the second is an effectiveness model, using total annual ridership as the measure of output; the third is a multi-output model using both annual vehicle-miles and annual ridership as outputs to capture the combined performance.

### 2.1.4. Railway

Lan and Lin (2003) adopt various DEA approaches to investigate the technical efficiency and service effectiveness for some selected 76 railways. This paper attempts to estimate both of the technical efficiency and service effectiveness for worldwide rail systems by employing two-stage DEA. At the technical efficiency analysis stage, we use input orientation DEA by selecting length of lines, number of locomotives and cars, and number of employees as inputs and train-kilometer as output. At the service effectiveness analysis stage, we use output orientation DEA by selecting train-kilometer as input and passenger-kilometer and ton-kilometer as outputs. In addition, we perform a technical effectiveness analysis with one-stage DEA by choosing the same input factors and outputs.

Conventional DEA approaches neither consider the environmental differences across the DMUs nor account for the statistical error (data noise) and slack effects. Thus, the comparison can be seriously biased because all DMUs are not brought into a common platform. Fried et al. (2002) proposed a three-stage DEA approach with consideration of the environmental effects and statistical noise, but they still did not adjust the slack effects. Lan and Lin (2005) propose a four-stage DEA approach with further adjustment of slack effects. The empirical results show that proposed four-stage DEA approach has slightly more reasonable efficiency and effectiveness scores than those measured by Fried's three-stage DEA approach.

This paper measures the technical efficiency by selecting number of passenger cars per kilometer of lines, number of freight cars per kilometer of lines, and number of employees per kilometer of lines as input factors and passenger train-kilometer per kilometer of lines and freight-train-kilometer per kilometer of lines as output variables. In measuring the service effectiveness, on the other hand, we choose passenger-kilometers and ton-kilometers as two consumptions and passenger train kilometers and freight train-kilometers as two outputs.

## 2.2. DEA modeling

Yun *et al.* (2004) suggest a model called generalized DEA (GDEA) model, which can treat the basic DEA models (CCR model, BCC model and FDH model) in a unified way. GDEA model can make a quantitative analysis for inefficiency on the basis of surplus of inputs and slack of outputs.

DEA was suggested by Charnes, Cooper and Rhodes (CCR) which is concerned with the estimation of technical efficiency and efficient frontiers. The CCR model generalized the single output/single input ratio efficiency measure for each decision making unit to multiple outputs/multiple inputs situations by forming the ratio of a weighted sum of outputs to a weighted sum of inputs. Tulkens introduced a relative efficiency to non-convex free disposable hull (FDH) of the observed data, and formulated a mixed integer programming to calculate the relative efficiency for each DMU.

Gautam and Paul (2006) provide an alternative framework for solving DEA models which, in comparison with the standard linear programming (LP) based approach that solves one LP for each DMU. The method of projection, which we use, is Fourier–Motzkin (F–M) elimination. It is shown that the output from the F–M method improves on existing methods of (i) establishing the returns to scale status of each DMU, (ii) calculating cross-efficiencies and (iii) dealing with weight flexibility.

El-Mahgary and Lahdlma (1995) examine various two-dimensional charts for illustrating the DEA efficiency results. The identification of reference units provides a general framework that can be used to define guideline for the inefficient units. Visualizing such results should help decision-maker to better understand the result of a DEA assessment.

Cooper *et al.* (2001) examine two approaches that are presently available in the DEA literature for use in identifying and analyzing congestion. These two approaches are due to Färe et al. (Färe, R., Grosskopf, S., Lovell, C.A.K., 1985, The measurement of efficiency of production, Kluwer-Nijhoff Publishing, Boston, MA) and Cooper et al. (Cooper, W.W., Thompson, R.G., Thrall, R.M., 1996, Introduction: extensions and new developments in DEA, Annals of Operations research 66, 3-45). This study shows that FGL model might fail to give correct result.

Cherchye *et al.* (2001) respond the problem that FGL model fails to identify congestion in Cooper et al. examples. Because FGL model was originally proposed for measuring structural efficiency rather than detecting congestion.

## 2.3. Comparisons of DEA with other methods

Cullinane *et al.* (2006) apply the two leading approaches to efficiency measurement, DEA and SFA, to the same data set for the container port industry. This study suggests that a dynamic application of these frontier techniques, utilizing panel data approaches, may be more germane to ascertaining the relative efficiency levels of the international ports industry. In a dynamic context, technical efficiency can be separated not only from scale efficiency, but also from technological take-up. This paper rank order of the technical efficiency derived from applying the alternative DEA and SFA approaches ranges from 0.63 to 1.00, indicating that these approaches yield similar efficiency rankings. The hypothesis of constant returns to scale in the production frontier for the industry could not be rejected when applying the stochastic frontier model. The application of the same sort of hypothesis test to the results yielded by the application of the DEA model is not appropriate, however, as the mathematical programming nature of DEA means that the underlying model does not possess any statistical assumptions or properties per se. Applying the DEA approach does, however, yield the results that the terminals in the sample were found to exhibit a mix of decreasing, increasing and constant returns to scale at current levels of output. Compared with the stochastic parametric frontier approach, DEA imposes neither a specific functional relationship between production output and input, nor any assumptions on the specific statistical distribution of the error terms. In so doing, the data are believed to be able to "speak for themselves" and the DEA approach has the advantage of minimal specification error. However, the DEA model does not allow for measurement error or random shocks. Instead, all these factors are attributed to efficiency, a characteristic that inevitably leads to potential estimation errors. In this paper, the main objective of a port is assumed to be the minimization of the use of input(s) and maximization of the output(s). The inputs of this paper are terminal length (m), terminal area (ha), quayside gantry (number), yard gantry (number) and straddle carrier (number) and the output of this paper is container throughput (TEU).

Pels *et al.* (2001) use data envelopment analysis and stochastic production frontier analysis to determine efficiency rations for European airports. The SFA might be more flexible then DEA as SFA includes a noise term. However, this study suggests that more attention has to be paid to the "explaining" inefficiency, either using a stochastic frontier model or DEA output because the inputs used are not "standard" variable inputs. That means in the short run, they cannot be fully flexible. The estimation result of SFA is similar to DEA result. It appears that most airports are operating under increase returns to scale.

Coelli and Perelman (1999) discuss and compare a number of the different methods that have been used to estimate multi-output distance functions. This study focus upon the three most commonly used estimation methods:

(1) A parametric frontier using linear programming methods;
(2) A non-parametric piece-wise linear frontier using the linear programming method known as data envelopment analysis (DEA); and
(3) A parametric frontier using corrected ordinary least squares (COLS).

The three different estimation methods provide similar information on the relative productive performance. The correlations between the various sets of technical efficiency predictions are all positive and significant. Furthermore, the parameter estimates obtained using the two parametric estimates are also quite similar in many respects. Given these observations, it appears that a researcher can safely select one of these methods without too much concern for their choice having a large influence upon results.

## 2.4. Summary

Table 1 summarizes of the literature review, from which, one can notice several points. First, some papers only use technical efficiency to evaluate the performance of transportation. That means these papers do not consider non-storable characteristic of transportation industries. Second, some papers use two stages (technical efficiency and service effectiveness) to evaluate the performance for transportation industries, however, these papers calculate the efficiency and effectiveness scores independently. One shall calculate the efficiency scores and effectiveness at the same time because one is evaluating two different departments in one company. One should treat these two departments dependently. Third, from these papers, one could discover that most of them use labor, capital and fuel as input variable and use vehicle miles and passenger miles as output and service variables.

In this study, we will use cost efficiency, service effectiveness and cost effectiveness to evaluate the performance for transportation industry. In order to treat these three parts as an interactively dependent group, we try to formulate an integrated model to measure these three performance scores at the same time.

Table 1 **Summary of literature review**

| No | Author | Year | Industry | Approach | Evaluating aspect | Input variables | Output variables | Service variables | Model | DMU |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Cullinane et al. | 2006 | Port | DEA<br>SFA | Technical efficiency | terminal length | container throughput | - | CCR<br>BCC | Country |
| | | | | | | terminal area | | | | |
| | | | | | | quayside gantry | | | | |
| | | | | | | yard gantry | | | | |
| | | | | | | straddle carrier | | | | |
| 2 | Pels et al. | 2001 | Airport | DEA<br>SFA | Technical efficiency | Terminal size | Air transport movement | - | BCC | City |
| | | | | | | aircraft parking positions at the terminal | Passenger movement | | | |
| | | | | | | remote aircraft parking positions | - | | | |
| | | | | | | number of check-in desks | | | | |
| | | | | | | number of baggage claim | | | | |
| 3 | Karlaftis | 2003 | Transit | DEA | Technical efficiency | Operating cost | Vehicle-miles travelled | - | CCR | US City |
| | | | | | | Number of vehicles | Passengers | | | |
| | | | | | | Gallons of fuel | - | | | |
| | | | | | | Total employees | | | | |
| 4 | Adler and Berechman | 2001 | Airport | DEA | Technical efficiency | Questionnaire | Service satisfacting | - | BCC | City |
| | | | | | | Haul charge | - | | | |
| | | | | | | Connection times | | | | |
| | | | | | | Average delay time | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Number of terminals | | | | |
| | | | | | | Number of runways | | | | |
| | | | | | | Distance to the nearest major city-center | | | | |
| 5 | Tongzon | 2001 | Port | DEA | Technical efficiency | number of berths, cranes and tugs | cargo throughput | - | CCR | City |
| | | | | | | number of port authority employees | ship working rate | | | |
| | | | | | | terminal area of the ports | - | | | |
| 6 | Chiou and Chen | 2006 | Airport | DEA | Cost efficiency | fuel cost | number of flights | passenger-mile | CCR BCC | Airline |
| | | | | | Service effectiveness | personnel cost | seat-mile | embarkation passengers | | |
| | | | | | | aircraft cost | - | - | | |
| | | | | | Cost effectiveness | - | | | | |
| 7 | Fielding et al. | 1984 | Transit | DEA | Technical efficiency | labor | vehicle hours | passengers | CCR | US City |
| | | | | | | capital | vehicle miles | passenger miles | | |
| | | | | | Service effectiveness | fuel | capacity miles | operating revenue | | |
| | | | | | | - | service reliability | - | | |
| | | | | | Technical effectiveness | | - | | | |
| 8 | Yun et al | 2004 | Bank | DEA GDEA | Technical efficiency | Non-interes expense | Deposits | - | CCR BCC FDH | Bank |
| | | | | | | Interest income plus non-interest income | - | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | - | | | | |
| 9 | Peck et al. | 1998 | Airport | DEA | Technical efficiency | labor expenses on airframes | flights arrivals delayed | - | BCC | Airlines |
| | | | | | | labor expenses on aircraft engines | for mechanical reasons | | | |
| | | | | | | expenditures on airframe repairs | - | | | |
| | | | | | | expenditures on engine repairs | | | | |
| | | | | | | material expenditures on airframes | | | | |
| | | | | | | material expenditures on engines | | | | |
| 10 | Odeck and Alkadi | 2001 | Transit | DEA | Technical efficiency | total number of seats | seat kilometers | - | BCC | Bus company |
| | | | | | | fuel consumption | vehicle kilometers | | | |
| | | | | | | consumption equipment | passenger kilometers | | | |
| | | | | | | - | passengers | | | |
| 11 | Philip A. Viton | 1998 | Transit | DEA | Technical efficiency | fleet sizes | vehicle-miles | - | BCC | Transit industry |
| | | | | | | number of gallons of fuel | passenger trips | | | |
| | | | | | | number of person-hours of transportation | vehicle hours | | | |
| | | | | | | number of person-hours | - | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | of maintenance<br>number of person-hours of administrative<br>capital<br>the cost of tires and other materials<br>the cost of services<br>the cost of utilities<br>the cost of insurance | | | | |
| 12 | Cowie and Asenova | 1999 | Transit | DEA | Technical efficiency<br>Managerial efficiency<br>Organisational efficiency | total staff employed<br>fleet size<br>- | operating revenue<br>- | - | BCC | Bus company |
| 13 | Lan and Lin | 2003 | Railway | DEA | Technical efficiency<br>Service effectiveness | length of lines<br>number of locomotives and cars<br>number of employees<br>- | train-kilometer<br>- | passenger-kilometer<br>ton-kilometer<br>- | CCR<br>BCC<br>EXO<br>CAT | Railway |
| 14 | Lan and Lin | 2005 | Railway | DEA | Technical efficiency | Lines<br>Passenger cars | passenger train-kilometer<br>freight-train-kilometer | passenger-kilometers<br>ton-kilometers | BCC<br>(Four | Railway |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Service effectiveness | Freight cars<br>Employees | - | - | Stage) | |
| 15 | Tzeng and Chiang | 2000 | Airport | DEA | Technical efficiency | total capital<br>number of employees<br>total number of seats | net operation revenue<br>passenger-kilometers | - | CCR<br>BCC | Airline company |
| 16 | Karlaftis | 2004 | Transit | DEA | Technical efficiency | Number of vehicles<br>gallons of fuel | vehicle-miles | passenger-miles | BCC | City |
| | | | | | Service effectiveness | Total employees | - | - | | |
| | | | | | Technical effectiveness | - | | | | |
| 17 | Coelli and Perelman | 1999 | Railway | DEA<br>SFA<br>COLS | Technical efficiency | annual mean of monthly data on staff€ levels | passenger services | - | BCC | Company |
| | | | | | | available freight wagons | freight services | | | |
| | | | | | | coach transport capacities in tones<br>coach transport capacities in seats<br>total length of lines | - | | | |

EXO DEA: exogenously fixed inputs model

CAT DEA: To compare the performance measurements in a homogeneous environment can be formulated according to appropriate categorical variables.

COLS: A parametric frontier using corrected ordinary least squares

# 3. Methodology

## 3.1. Conventional DEA models

DEA was initially developed as a method for assessing the comparative efficiencies of organizational units. The key feature which makes the units comparable is that they perform the same function in terms of the kinds of inputs they use and the types of outputs they produce.

DEA was first developed by Charnes et al. (1978), who generalized the single-output/single-input ratio efficiency measure for each DMU. The CCR model generalized the single output/single input ratio efficiency measure for DMU to multiple outputs/multiple inputs situations by forming the ratio of a weighted sum of outputs to a weighted sum of inputs. Based on the CCR model, Banker et al. (1984) suggested a model for estimating technical efficiency and scale inefficiency in DEA by adding convexity constrain. The BCC model relaxed the constant returns to scale assumption of the CCR model and made it possible to investigate whether the performance of each DMU was conducted in region of increasing, constant or decreasing returns to scale in multiple outputs and multiple inputs situations.

The main characteristics of DEA are that (i) it can be applied to analyze multiple outputs and multiple inputs without pre-assigned weights, (ii) it can be used for measuring a relative efficiency based on the observed data without knowing information on the production function.

Two basic DEA models are CCR model and BCC model. These two basic forms are illustrated as following.

### 3.1.1. CCR

DMU k is assumed to be evaluated. And there are *i* DMUs, each utilizes *j* kinds of inputs, $(x_{1i}, x_{2i}, \ldots, x_{ji})$, and purchases *r* kinds of outputs, $(y_{1i}, y_{2i}, \ldots, y_{ri})$. The efficiency of DMU k can be estimated by following programming.

$$\underset{u,v}{Max} \quad h_k = \frac{\sum_{r=1}^{s} u_r y_{kr}}{\sum_{j=1}^{m} v_j x_{kj}}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r y_{ir}}{\sum_{j=1}^{m} v_j x_{ij}} \le 1\,, \quad i = 1,2,\cdots,n \tag{1-1}$$

$$v_j \ge 0\,, \quad j = 1,2,\cdots,m$$

$$u_r \ge 0\,, \quad r = 1,2,\cdots,s$$

The model (1-1) is an input oriented programming problem, which can be formulated as output oriented problem by following programming.

$$\underset{\omega,\mu}{Min} \quad g_k = \frac{\sum_{j=1}^{m} \omega_j x_{kj}}{\sum_{r=1}^{s} \mu_r y_{kr}}$$

$$\text{s.t.} \quad \frac{\sum_{j=1}^{m} \omega_j x_{ij}}{\sum_{r=1}^{s} \mu_r y_{ir}} \ge 1\,, \quad i = 1,2,\cdots,n \tag{1-2}$$

$$\omega_j \ge 0\,, \quad j = 1,2,\cdots,m$$

$$\mu_r \ge 0\,, \quad r = 1,2,\cdots,s$$

Then, one can transform above model (1-1) into an ordinary linear problem, show as following.

$$\underset{u,v}{Max} \quad h_k = \sum_{r=1}^{s} u_r y_{kr}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{ir} - \sum_{j=1}^{m} v_j x_{ij} \le 0\,, \quad i = 1,2,\cdots,n \tag{1-3}$$

$$\sum_{j=1}^{m} v_j x_{kj} = 1\,,$$

$$v_j \ge 0\,, \quad j = 1,2,\cdots,m$$

$$u_r \geq 0\,,\quad r = 1,2,\cdots,s$$

Because model (1-3) is a linear problem, one can transform it into dual problem as follows.

$$\underset{z,\lambda_i}{Min} \quad z$$

$$\text{s.t.} \quad zx_{kj} - \sum_{i=1}^{n} x_{ij}\lambda_i \geq 0\,,\quad j = 1,2,\cdots,m \qquad (1\text{-}4)$$

$$-y_{kr} + \sum_{i=1}^{n} y_{ir}\lambda_i \geq 0\,,\quad r = 1,2,\cdots,s$$

$$\lambda_i \geq 0\,,\quad i = 1,2,\cdots,n$$

z is a scalar, which is the efficiency of *k*th firm, and it ranges from zero to unity. If z equals to one, the firm is efficient. And if z is less then one, the firm is inefficient.

One also can transform output oriented model (1-2) into linear problem (1-5) and then one can find its dual problem (1-6), show as follows.

$$\underset{\omega,\mu}{Min} \quad g_k = \sum_{j=1}^{m} \omega_j x_{kj}$$

$$\text{s.t.} \quad -\sum_{r=1}^{s} \mu_r y_{ir} + \sum_{j=1}^{m} \omega_j x_{ij} \geq 0\,,\quad i = 1,2,\cdots,n \qquad (1\text{-}5)$$

$$\sum_{r=1}^{s} \mu_r y_{kr} = 1\,,$$

$$v_j \geq 0\,,\quad j = 1,2,\cdots,m$$

$$u_r \geq 0\,,\quad r = 1,2,\cdots,s$$

Dual problem,

$$\underset{z,\lambda_i}{Max} \quad \phi$$

$$\text{s.t.} \quad -\phi \cdot y_{kr} + \sum_{i=1}^{n} y_{ir}\lambda_i \geq 0\,,\quad r = 1,2,\cdots,s$$

$$x_{kj} - \sum_{i=1}^{n} x_{ij}\lambda_i \geq 0\,,\quad j = 1,2,\cdots,m \qquad (1\text{-}6)$$

$$\lambda_i \geq 0\,,\quad i = 1,2,\cdots,n$$

### 3.1.2. BCC

Model (1-4) and model (1-6) are input and output oriented DEA models under the assumption of constant returns to scale (CRS) production technology. Then Banker, Charnes and Cooper (1984) relaxed this CRS constrain to variable returns to scale (VRS) technology by adding convexity constraint, as following models. Then one can get BCC input (1-7) and output oriented model (1-8) as following.

$$\begin{aligned} &\underset{z,\lambda_i}{Min} \quad z \\ &\text{s.t.} \quad z x_{kj} - \sum_{i=1}^{n} x_{ij}\lambda_i \geq 0\,,\quad j = 1,2,\cdots,m \\ &\qquad -y_{kr} + \sum_{i=1}^{n} y_{ir}\lambda_i \geq 0\,,\quad r = 1,2,\cdots,s \\ &\qquad \lambda_i \geq 0\,,\quad i = 1,2,\cdots,n \\ &\qquad \sum_{i=1}^{n} \lambda_i = 1 \end{aligned} \tag{1-7}$$

$$\begin{aligned} &\underset{z,\lambda_i}{Max} \quad \phi \\ &\text{s.t.} \quad -\phi \cdot y_{kr} + \sum_{i=1}^{n} y_{ir}\lambda_i \geq 0\,,\quad r = 1,2,\cdots,s \\ &\qquad x_{kj} - \sum_{i=1}^{n} x_{ij}\lambda_i \geq 0\,,\quad j = 1,2,\cdots,m \\ &\qquad \lambda_i \geq 0\,,\quad i = 1,2,\cdots,n \\ &\qquad \sum_{i=1}^{n} \lambda_i = 1 \end{aligned} \tag{1-8}$$

Once one knows the basic models for DEA, one can use these models to evaluate relative efficiency for each DMU. One usually uses two stages DEA to evaluate non-storable commodities. That means one uses input oriented DEA model to evaluate technical efficiency and use output oriented DEA model to evaluate service effectiveness.

From these two stages DEA, one would know how to improve the efficiency in each department. If one uses two stages DEA to calculate the value of technical efficiency and service effectiveness respectively, it means one treats these two departments as independent. However, these two departments are dependent; namely, one cannot calculate the efficiency value independently. One should calculate the technical efficiency and service effectiveness at the same time for non-storable commodities. The main purpose of this research is to formulate an integrated model which can determine the efficiency value for non-storable commodities at the same time.

## 3.2. Proposed IDEA models

DEA is a useful method to evaluate the performance for firms. If we want to evaluate the performance of a transportation industry, we need to pay attention to the main characteristics of transportation, which provides non-storable commodities. That means we should not use cost efficiency only. We can use the framework proposed by Fielding et al. (1978) to evaluate the performance of transportation. This framework indicates that one needs to evaluate cost efficiency, service effectiveness and cost effectiveness jointly. Figure 1 portrays this framework.

### 3.2.1. Proposed models

#### 3.2.1.1. Cost efficiency

We use the following input-oriented DEA model to evaluate the performance of $DMU_k$ between inputs and outputs. From $h_k^{IO}$, we would know the proportion of inputs we should decrease.

$$\max_{u,v} \quad h_k^{IO} = \frac{\sum_{r=1}^{s} u_r y_{kr}}{\sum_{j=1}^{m} v_j x_{kj}} \tag{2-1}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r y_{ir}}{\sum_{j=1}^{m} v_j x_{ij}} \le 1, \quad i = 1,2,\cdots,n$$

$$v_j \ge 0, \quad j = 1,2,\cdots,m$$

$$u_r \ge 0, \quad r = 1,2,\cdots,s$$

The symbols are assigned the following means:

$DMU_i$, $i = 1,2,\cdots,n$

$x_{ij}$, observed amount of input $j = 1,2,\cdots,m$ used by $DMU_i$.

$y_{ir}$, observed amount of input $r = 1,2,\cdots,s$ used by $DMU_i$.

$x_{kj}$, observed amount of input $j = 1,2,\cdots,m$ used by $DMU_k$.

$y_{kr}$, observed amount of input $r = 1,2,\cdots,s$ used by $DMU_k$.

$v_j$, $u_r$, DEA weight on the *j* th input and *r* th output.

Then, we transform above model to a dual problem and we can get the following input-oriented DEA model. We can use this model to get the value of cost efficiency.

$$\underset{z,\lambda_i}{Min} \quad z^{IO} \tag{2-2}$$

$$\text{s.t.} \quad z^{IO} x_{kj} \geq \sum_{i=1}^{n} x_{ij}\lambda_i, \quad j = 1,2,\cdots,m$$

$$y_{kr} \leq \sum_{i=1}^{n} y_{ir}\lambda_i, \quad r = 1,2,\cdots,s$$

$$\lambda_i \geq 0, \quad i = 1,2,\cdots,n$$

### 3.2.1.2. Service effectiveness

We use an input-oriented DEA model to evaluate the performance of $DMU_k$ between outputs and services. From this result, we would know the proportion of outputs we should reduce.

$$\underset{u,w}{Max} \quad h_k^{OS} = \frac{\sum_{q=1}^{p} w_q l_{kq}}{\sum_{r=1}^{s} u_r y_{kr}} \tag{2-3}$$

$$\text{s.t.} \quad \frac{\sum_{q=1} w_q l_{iq}}{\sum_{r=1}^{s} u_r y_{ir}} \leq 1, \quad i = 1,2,\cdots,n$$

$$u_r \geq 0, \quad r = 1,2,\cdots,s$$

$$w_q \geq 0, \quad q = 1,2,\cdots,p$$

The symbols are assigned the following means:

$l_{iq}$, observed amount of input $q = 1,2,\cdots,p$ used by $DMU_i$.

$l_{kq}$, observed amount of input $q = 1,2,\cdots,p$ used by $DMU_k$.

$w_q$, DEA weight on the $q$ th service.

We also can transform this model into dual problem. Then we can get the following input-oriented DEA model.

$$\min_{z,\lambda_i} \quad z^{OS} \tag{2-4}$$

$$\begin{aligned} \text{s.t.} \quad & z^{OS} y_{kr} \geq \sum_{i=1}^{n} y_{ir}\lambda_i, \quad r = 1,2,\cdots,s \\ & l_{kq} \leq \sum_{i=1}^{n} l_{iq}\lambda_i, \quad q = 1,2,\cdots,p \\ & \lambda_i \geq 0, \quad i = 1,2,\cdots,n \end{aligned}$$

## 3.2.1.3. Integrated model

### 3.2.1.3.1. Constant returns to scale

In this part, we use individual model to develop an integrated CCR model (ICCR). We let each model decide its multiplier in the integrated model at the same time. Technical efficiency stands for production sector and service effectiveness represents sale sector; however, technical effectiveness doesn't stand for any sector as non-storable commodities are produced. That's why this study doesn't employ technical effectiveness to evaluate the performance.

$$\max_{u,v,w} \quad \left( \frac{\sum_{r=1}^{s} u_r y_{kr}}{\sum_{j=1}^{m} v_j x_{kj}} \right) + \left( \frac{\sum_{q=1}^{p} w_q l_{kq}}{\sum_{r=1}^{s} u_r y_{kr}} \right) \tag{2-7}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{ir} \leq \sum_{j=1}^{m} v_j x_{ij}, \quad i = 1,2,\cdots,n \\ & \sum_{q=1}^{p} w_q l_{iq} \leq \sum_{r=1}^{s} u_r y_{ir}, \quad i = 1,2,\cdots,n \end{aligned}$$

$$v_j \geq 0\,, \qquad j = 1,2,\cdots,m$$

$$w_q \geq 0\,, \qquad q = 1,2,\cdots,p$$

$$u_r \geq 0\,, \qquad r = 1,2,\cdots,s$$

Once we proposed the original model, we can add slack analysis in this model. In order to do slack analysis, we add slack variables in each variable. The ICCR model shows as following and ICCR model assumes production and sale sector is equal weight.

$$\underset{u,v,w}{Max} \quad \left( \frac{\sum_{r=1}^{s} u_r y_{kr}}{\sum_{j=1}^{m} v_j x_{kj}} \right) + \left( \frac{\sum_{q=1}^{p} w_q l_{kq}}{\sum_{r=1}^{s} u_r y_{kr}} \right) \tag{2-8}$$

s.t. $\displaystyle \sum_{r=1}^{s} u_r y_{ir} \leq \sum_{j=1}^{m} v_j \left( x_{kj} - s_{kj} \right), \quad i = 1,2,\cdots,n$

$$\sum_{q=1}^{p} w_q \left( l_{kq} + s_{kq} \right) \leq \sum_{r=1}^{s} u_r y_{ir}\,, \quad i = 1,2,\cdots,n$$

$$v_j \geq 0\,, \qquad j = 1,2,\cdots,m$$

$$w_q \geq 0\,, \qquad q = 1,2,\cdots,p$$

$$u_r \geq 0\,, \qquad r = 1,2,\cdots,s$$

In the revised model, we hold production variable ( $y_{ir}$ ) unchanged. That means we only have to minimize the input and maximize the service value. In other words, there wouldn't be slack value of production variable. Through this model, we can get the performance value and slack variable.

Then we can rewrite this model as following:

$$\underset{u,v,w}{Max} \quad h = \left( \sum_{r=1}^{s} u_r y_{kr} \right) \left( \sum_{r=1}^{s} u_r y_{kr} \right) + \left( \sum_{q=1}^{p} w_q l_{kq} \right) \left( \sum_{j=1}^{m} v_j x_{kj} \right)$$

$$\text{s.t.} \quad \left(\sum_{j=1}^{m} v_j x_{ij}\right)\left(\sum_{r=1}^{s} u_r y_{ir}\right) = 1 \tag{2-9}$$

$$\sum_{r=1}^{s} u_r y_{ir} \le \sum_{j=1}^{m} v_j \left(x_{kj} - s_{kj}\right), \quad i = 1,2,\cdots,n$$

$$\sum_{q=1}^{p} w_q \left(l_{kq} + s_{kq}\right) \le \sum_{r=1}^{s} u_r y_{ir}, \quad i = 1,2,\cdots,n$$

$$v_j \ge 0, \qquad j = 1,2,\cdots,m$$

$$w_q \ge 0, \qquad q = 1,2,\cdots,p$$

$$u_r \ge 0, \qquad r = 1,2,\cdots,s$$

We can use this integrated model to calculate the value of cost efficiency and service effectiveness for each DMU. Then we would know which DMU has the best performance and how does it improve its performance. This IDEA model cannot transfer to dual form, because IDEA model isn't the linear programming problem.

In eg. 2-8, *h* stands for the overall efficiency score, which is the efficiency of *k*th firm, and it ranges from zero to two. If *h* equals to two, the firm is efficient. And if *h* is less two, the firm is inefficient. If firm is inefficiency, one can check the efficiency scores, which respectively calculated from the integrated model (cost efficiency, service effectiveness and cost effectiveness) to see which part need to improve.

#### 3.2.1.3.2. **Variable returns to scale**

In order to fit the true production behavior, we are going to change CRS production technique into VRS production technique. Because our model form is a nonlinear problem, we can not use conventional way to add VRS variable ($\sum \lambda = 1$) in dual problem. We add our VRS variable in following BCC model (IBCC).

$$\underset{u,v,w}{Max} \quad \left( \frac{\sum_{r=1}^{s} u_r y_{kr} - u_0}{\sum_{j=1}^{m} v_j x_{kj}} \right) + \left( \frac{\sum_{q=1}^{p} w_q l_{kq} - u_1}{\sum_{r=1}^{s} u_r y_{kr} - u_0} \right) \tag{2-10}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{ir} - u_0 \leq \sum_{j=1}^{m} v_j \left( x_{kj} - s_{kj} \right), \quad i = 1,2,\cdots,n \\
& \sum_{q=1}^{p} w_q \left( l_{kq} + s_{kq} \right) - u_1 \leq \sum_{r=1}^{s} u_r y_{ir} - u_0, \quad i = 1,2,\cdots,n \\
& v_j \geq 0, \qquad j = 1,2,\cdots,m \\
& w_q \geq 0, \qquad q = 1,2,\cdots,p \\
& u_r \geq 0, \qquad r = 1,2,\cdots,s
\end{aligned}$$

We can use this model to get the performance value of each DMU under VRS technique. From this proposed model we can get efficiency value, slack value and we also can know each DMU is in increase, decrease or constant returns to scale.

# 4. Properties of the proposed IDEA Models

In this chapter, we prove that the proposed IDEA models exhibits two essential properties: rationality and uniqueness.

## 4.1. Rationality

### 4.1.1. Rationality for ICCR model

According to Charnes, et al. (1978), their proposed measure of the efficiency of any DMU is obtained as the maximum of a ratio of weighted outputs, subject to the condition that the similar ratio for every DMU be less than or equal to unity. Since the proposed integrated DEA model is to maximize two aspects of efficiency values, the overall efficiency value should be less than or equal to two. Our proposed measure of the efficiency of any DMU can also be obtained in a similar way. Mathematically,

$$[\text{ICCR'}] \qquad \underset{u,v,w}{Max} \left( \frac{\sum_{r=1}^{R} u_r y_{kr}}{\sum_{j=1}^{J} v_j x_{kj}} \right) + \left( \frac{\sum_{s=1}^{S} w_s l_{ks}}{\sum_{r=1}^{R} u_r y_{kr}} \right)$$

$$\text{st.} \quad \frac{\sum_{r=1}^{R} u_r y_{ir}}{\sum_{j=1}^{J} v_j x_{ij}} \le 1, \quad i = 1,2,\cdots,I$$

$$\frac{\sum_{s=1}^{S} w_s l_{is}}{\sum_{r=1}^{R} u_r y_{ir}} \le 1, \quad i = 1,2,\cdots,I$$

$$v_j \ge 0, \qquad j = 1,2,\cdots,J$$

$$w_s \ge 0, \qquad s = 1,2,\cdots,S$$

$$u_r \ge 0, \qquad r = 1,2,\cdots,R$$

Let $E_r' = \frac{x_R}{x_r}$ and $E_r'' = \frac{l_r}{l_R}$ respectively represent the technical efficiency (ratios of inputs at a given output) and service effectiveness (ratios of consumptions at a given output), where $x_R$ is the minimum input that can produce the given output and $x_r$ is the actual input being rated from the same output. Likewise, $l_R$ is the maximum yield that can be generated from the given output and $l_r$ is the actual yield being rated from the same output. Then, the overall efficiency can be calculated as $E_r = E_r' + E_r'' = \frac{x_R}{x_r} + \frac{l_r}{l_R}$.

Essentially, $0 \le E_r \le 2$.

Alternately, we can also derive the overall efficiency, $E_r$, from our proposal integrated DEA model as follows. For any given output $y$,

$$\underset{u,v}{Max} \quad h_r = \frac{uy_r}{vx_r} + \frac{wl_r}{uy_r}$$

$$\begin{aligned} s.t. \quad & \frac{uy_R}{vx_R} \le 1, \\ & \frac{uy_r}{vx_r} \le 1, \\ & \frac{wl_R}{uy_R} \le 1, \\ & \frac{wl_r}{uy_r} \le 1, \\ & u, v, w \ge 0 \end{aligned}$$

Let $u^*, v^*$, $w^*$ represent the optimal pair of corresponding values. Since $x_R \le x_r$, $l_R \ge l_r$ and $y_R = y_r = y$, it implies $u^* y_r = u^* y_R = v^* x_R$ and $u^* y_r = u^* y_R = w^* l_R$. We then have the following results and relationship:

Technical efficiency= $\frac{u^* y_r}{v^* x_r} = \frac{u^* y_R}{v^* x_r} = \frac{v^* x_R}{v^* x_r} = E_r'$

Service effectiveness= $\frac{w^* l_r}{u^* y_r} = \frac{w^* l_r}{u^* y_R} = \frac{w^* l_r}{w^* l_R} = E_r''$

Thus, $h_o = \frac{u^* y_r}{v^* x_r} + \frac{w^* l_r}{u^* y_r} = E_r' + E_r'' = E_r$.

In conclusion, the efficiency scores determined by the proposed integrated DEA model are proven with an essential property of reasonability because the optimal values of the proposed model have satisfied the definition of efficiency.

### 4.1.2. Rationality for IBCC model

The proposed IBCC model is presented as follows:

$$[\text{IBCC'}] \quad \underset{u,v,w}{Max} \quad \left( \frac{\sum_{r=1}^{R} u_r y_{kr} - u_0}{\sum_{j=1}^{J} v_j x_{kj}} \right) + \left( \frac{\sum_{s=1}^{S} w_s l_{ks} - u_1}{\sum_{r=1}^{R} u_r y_{kr} - u_0} \right)$$

$$s.t. \quad \frac{\sum_{r=1}^{R} u_r y_{ir} - u_0}{\sum_{j=1}^{J} v_j x_{ij}} \le 1, \quad i = 1,2,\cdots,I$$

$$\frac{\sum_{s=1}^{S} w_s l_{is} - u_1}{\sum_{r=1}^{R} u_r y_{ir} - u_0} \le 1, \quad i = 1,2,\cdots,I$$

$$v_j \ge 0, \quad j = 1,2,\cdots,J$$

$$w_s \ge 0, \quad s = 1,2,\cdots,S$$

$$u_r \ge 0, \quad r = 1,2,\cdots,R$$

The definition of efficiency is the same as in ICCR model. We can derive the overall efficiency from our proposal integrated DEA model as follows.

$$\underset{u,v}{Max} \quad h_r = \frac{uy_r - u_0}{vx_r} + \frac{wl_r - u_1}{uy_r - u_0}$$

$$s.t. \quad \frac{uy_R - u_0}{vx_R} \le 1,$$

$$\frac{uy_r - u_0}{vx_r} \le 1,$$

$$\frac{wl_R - u_1}{uy_R - u_0} \le 1,$$

$$\frac{wl_r - u_1}{uy_r - u_0} \le 1,$$

$$u, v, w \ge 0$$

Let $u^*, v^*$, $w^*$, $u_0^*$, $u_1^*$ represent the optimal pair of corresponding values. Since $x_R \le x_r$, $l_R \ge l_r$ and $y_R = y_r = y$, it implies $u^* y_r - u_0 = u^* y_R - u_0 = v^* x_R$ and $u^* y_r - u_0 = u^* y_R - u_0 = w^* l_R - u_1$. We then have the following results and relationship:

$$\text{Technical efficiency} = \frac{u^* y_r - u_0}{v^* x_r} = \frac{u^* y_R - u_0}{v^* x_r} = \frac{v^* x_R}{v^* x_r} = E'_r$$

$$\text{Service effectiveness} = \frac{w^* l_r - u_1}{u^* y_r - u_0} = \frac{w^* l_r - u_1}{u^* y_R - u_0} = \frac{w^* l_r - u_1}{w^* l_R - u_1}$$

$$= \frac{w^*(l_r - \frac{u_1}{w^*})}{w^*(l_R - \frac{u_1}{w^*})} = \frac{l_r - \frac{u_1}{w^*}}{l_R - \frac{u_1}{w^*}}$$

$$0 < \text{Service effectiveness} = \frac{l_r - \frac{u_1}{w^*}}{l_R - \frac{u_1}{w^*}} = \tilde{E} < 1,$$

Where $u_1$ is a scale variable.

When $u_1 > 0$, we can get the result: $l_r > l_r - \frac{u_1}{w^*}$. That means DMU r needs to downsize. Then it can reach optimal scale.

When $u_1 = 0$, we can get the result: $l_r = l_r - \frac{u_1}{w^*}$. That means DMU r reaches optimal scale.

When $u_1 < 0$, we can get the result: $l_r < l_r - \frac{u_1}{w^*}$. That means DMU r needs to upsize. Then it can reach optimal scale.

In conclusion, the efficiency scores determined by the proposed integrated DEA model are proven with an essential property of rationality. Furthermore, IBBC model can determine the optimal scale of each DMU.

## 4.2. Uniqueness

To show the uniqueness of joint efficiency measurement of the proposed model, we have to prove that the virtual multipliers of ***u***, ***v***, and ***w*** determined by the proposed model are a global optimal solution, not a local one. For a nonlinear programming problem, only for the model with a convex or concave objective function under a convex feasible region (*i.e.* sufficient conditions) would the solutions, obtained via the Karush-Kuhn-Tucker (KKT) conditions (*i.e.* necessary conditions), guarantee a global optimum. In other words, the convexity or concavity of objective function together with the convexity of feasible region must be examined. For simplicity, without loss of generality, the mathematical model of [ICCR-S] and [IBCC-S] are examined and only the case of single input, output and service variable is presented.

Since all the constraints in [ICCR-S] and [IBCC-S] are linear, the feasible set defined by these constraints is convex. Then the bordered Hessian matrix of objective function of [ICCR-S] can be computed as:

$$H = \begin{vmatrix} 0 & 0 & -y^{-1}lu^{-2} \\ 0 & 2x^{-1}yv^{-3}u & -x^{-1}yv^{-2} \\ -y^{-1}lu^{-2} & -x^{-1}yv^{-2} & 2y^{-1}lu^{-3}w \end{vmatrix}$$

The signs of the first, second and third leading principal minors of $\boldsymbol{H}$ are $|H_1| \leq 0$, $|H_2| \geq 0$ and $|H_3| \leq 0$ indicating that the bordered Hessian is negative semi-definite and the objective function is a concave function. In other words, the sufficient conditions for a global maximum are proven.

The bordered Hessian matrix of objective function of [IBCC-S] can be computed as:

$$H =$$

$$\begin{vmatrix} 0 & 0 & 0 & l(uy-u_0)^{-2} & -yl(uy-u_0)^{-2} \\ 0 & 2x^{-1}v^{-3}(uy-u_0) & 0 & x(vx)^{-2} & -xy(vx)^{-2} \\ 0 & 0 & 0 & -(uy-u_0)^{-2} & y(uy-u_0)^{-2} \\ l(uy-u_0)^{-2} & x(vx)^{-2} & -(uy-u_0)^{-2} & 2(wl-u_1)(uy-u_0)^{-3} & -2y(wl-u_1)(uy-u_0)^{-3} \\ -yl(uy-u_0)^{-2} & -xy(vx)^{-2} & y(uy-u_0)^{-2} & -2y(wl-u_1)(uy-u_0)^{-3} & 2y^2(wl-u_1)(uy-u_0)^{-3} \end{vmatrix}$$

The signs of principal minors of $\boldsymbol{H}$ are $|H_1| \leq 0$, $|H_2| \geq 0$, $|H_3| \leq 0$, $|H_4| \geq 0$ and $|H_5| \leq 0$. This indicating that the bordered Hessian is negative semi-definite and the objective function is a concave function. In other words, the sufficient conditions for a global maximum are proven.

# 5. Case study

## 5.1. Data

A total of 15 domestic air routes operated by a Taiwanese airline are evaluated in this study. The basic characteristics of these routes are summarized in Table 2. Because each route has its own properties, it's worthy to evaluate the performance of each route.

Table 2 Basic characteristics of the 15 domestic routes operated by Airline U

| No. | Route | Terminal | | Major market | | Inland/offshore | |
|---|---|---|---|---|---|---|---|
| | | Origin | Destination | Business | Recreation | Inland | Offshore |
| 1 | TSA-KHH | Taipei | Kaohsiung | ✓ | | ✓ | |
| 2 | TSA-TNN | Taipei | Tainan | ✓ | | ✓ | |
| 3 | TSA-TXG | Taipei | Taichung | ✓ | | ✓ | |
| 4 | TSA-CYI | Taipei | Chiayi | | ✓ | ✓ | |
| 5 | TSA-TTT | Taipei | Taitung | | ✓ | ✓ | |
| 6 | TSA-MZG | Taipei | Makung | | ✓ | | ✓ |
| 7 | TXG-MZG | Taichung | Makung | | ✓ | | ✓ |
| 8 | CYI-MZG | Chiayi | Makung | | ✓ | | ✓ |
| 9 | TNN-MZG | Tainan | Makung | | ✓ | | ✓ |
| 10 | KHH-MZG | Kaohsiung | Makung | | ✓ | | ✓ |
| 11 | TSA-KNH | Taipei | Kinmen | | ✓ | | ✓ |
| 12 | TXG-KNH | Taichung | Kinmen | | ✓ | | ✓ |
| 13 | CYI-KNH | Chiayi | Kinmen | | ✓ | | ✓ |
| 14 | TNN-KNH | Tainan | Kinmen | | ✓ | | ✓ |
| 15 | KHH-KNH | Kaohsiung | Kinmen | | ✓ | | ✓ |

Source: Chiou and Chen (2006)

To preserve confidentiality, the airline is referred percentages, not in real values. In complying with the rule of thumb that the number of DMUs must exceed twice of the total number of input and output variables, the number of variables in each perspective is limited. Thus, in the input perspective, the original twelve items of attributed costs are aggregated into three variables: fuel cost (FC), personnel cost (PC), including the salaries of cabin and ground-handling crews, and aircraft cost (AC), including maintenance costs, depreciation costs and interest payments. The production variables include number of flights (FL) and seat-mile (SM). The service variables include passenger-mile (PM) and embarkation passengers (EP), as demonstration in Table 3.Tthe relationship between these three variables are shown in Fig. 3.

Table 3 Data of 15 domestic routes operated by Airline U (in %)

| Route | Factor variable | | | Production variable | | Consumption variable | |
|---|---|---|---|---|---|---|---|
| | FC | PC | AC | FL | SM | PM | EP |
| 1 | 32.53 | 26.14 | 26.58 | 16.02 | 33.41 | 32.06 | 25.18 |
| 2 | 14.31 | 9.72 | 11.42 | 7.49 | 14.00 | 11.70 | 10.25 |
| 3 | 5.61 | 10.91 | 8.75 | 15.50 | 4.94 | 5.17 | 9.66 |
| 4 | 7.32 | 11.07 | 10.45 | 15.16 | 8.05 | 8.14 | 9.14 |
| 5 | 10.49 | 7.04 | 9.18 | 5.52 | 10.13 | 8.98 | 8.01 |
| 6 | 6.77 | 5.00 | 5.96 | 4.06 | 6.66 | 7.03 | 6.47 |
| 7 | 2.06 | 4.61 | 3.47 | 6.65 | 2.26 | 2.27 | 3.99 |
| 8 | 0.64 | 1.74 | 1.27 | 2.45 | 0.53 | 0.57 | 1.57 |
| 9 | 1.35 | 3.66 | 2.37 | 5.13 | 1.21 | 1.42 | 3.65 |
| 10 | 3.79 | 7.87 | 5.63 | 8.92 | 3.72 | 4.25 | 7.19 |
| 11 | 9.61 | 5.09 | 7.05 | 3.99 | 8.06 | 9.78 | 7.17 |
| 12 | 2.34 | 3.11 | 3.55 | 4.46 | 2.70 | 3.50 | 3.44 |
| 13 | 0.5 | 0.75 | 0.83 | 1.05 | 0.63 | 0.69 | 0.69 |
| 14 | 0.51 | 0.76 | 0.83 | 1.05 | 0.67 | 0.81 | 0.75 |
| 15 | 2.15 | 2.53 | 2.67 | 2.54 | 3.03 | 3.62 | 2.84 |
| Total | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Source: Chiou and Chen (2006)

## 5.2. Efficiency scores

The optimal virtual multipliers corresponding to all variables are first determined by the integrated DEA models. The joint and separate efficiency scores of each route under CRS and VRS are then computed, respectively, as shown in Table 4 and Table 5. Note from Table 4 that only two routes (TNN-MZG and TSA-KNH) are evaluated as overall efficiency by the integrated CCR model and that three routes (TNN-MZG, TSA-KNH, and TXG-KNH) were evaluated as overall efficiency by separated CCR model (Chiou and Chen, 2006). Namely, the proposed integrated CCR model has higher discriminating power over the separated CCR models. By definition, the efficiency score of technical effectiveness of our integrated CCR model is equal to the product of scores of technical efficiency and service effectiveness. In contrast, the separated CCR models proposed by Chiou and Chen (2006) fail to possess this property.

Table 4 Scores of overall and separate efficiencies of each route under CRS

| Route | Integrated CCR model | | | | Separated CCR model | | |
|---|---|---|---|---|---|---|---|
| | Overall efficiency | Technical efficiency | Service effectiveness | Technical effectiveness | Technical efficiency | Service effectiveness | Technical effectiveness |
| 1 | 1.8214 | 1.0000 | 0.8214 | 0.8214 | 1.000 | 0.875 | 0.889 |
| 2 | 1.7810 | 0.9927 | 0.7884 | 0.7826 | 1.000 | 0.797 | 0.817 |
| 3 | 1.7573 | 0.9411 | 0.8162 | 0.7681 | 0.996 | 0.855 | 0.860 |
| 4 | 1.6678 | 0.8970 | 0.7708 | 0.6914 | 0.984 | 0.854 | 0.851 |
| 5 | 1.8127 | 0.9675 | 0.8452 | 0.8177 | 0.991 | 0.787 | 0.766 |
| 6 | 1.9500 | 0.9500 | 1.0000 | 0.9500 | 0.958 | 1.000 | 1.000 |
| 7 | 1.8102 | 1.0000 | 0.8102 | 0.8102 | 1.000 | 0.815 | 0.866 |
| 8 | 1.9065 | 0.9900 | 0.9165 | 0.9073 | 1.000 | 0.977 | 0.905 |
| 9 | 2.0000* | 1.0000* | 1.0000* | 1.0000* | 1.000* | 1.000* | 1.000* |
| 10 | 1.8788 | 0.9026 | 0.9761 | 0.8810 | 0.901 | 0.978 | 0.913 |
| 11 | 2.0000* | 1.0000* | 1.0000* | 1.0000* | 1.000* | 1.000* | 1.000* |
| 12 | 1.8920 | 0.8920 | 1.0000 | 0.8920 | 1.000* | 1.000* | 1.000* |
| 13 | 1.8168 | 0.9695 | 0.8473 | 0.8215 | 1.000 | 0.844 | 0.874 |
| 14 | 1.9302 | 1.0000 | 0.9302 | 0.9302 | 1.000 | 0.929 | 0.978 |
| 15 | 1.9650 | 1.0000 | 0.9650 | 0.9567 | 1.000 | 0.963 | 1.000 |

Note: "*" is indicated the efficient DMUs.

From Table 5, we notice that there are four routes evaluated as overall efficiency by the proposed integrated BCC model, whereas eight routes have been evaluated as overall efficiency by the separated BCC model (Chiou and Chen, 2006). Once again, it shows a superior discrimination power of the proposed integrated DEA model over the separated DEA models.

Using the proposed integrated BCC model, we further examine the signs of $u_0$ ($u_1$) to identify the scale property for technical efficiency (service effectiveness). The DMU is increasing returns to scale (IRS), if $u_0^* < 0$ ($u_1^* < 0$). If $u_0^*>0$ ($u_1^* > 0$), the DMU is decreasing returns to scale (DRS). If $u_0^*=0$ ($u_1^* = 0$), the DMU is constant returns to scale (CRS). The results are summarized in Table 6. Note that most DMUs are characterized with DRS in their production sector or sale sector, implying that downsizing the scale may be needed.

Table 5 Scores of overall and separate efficiencies of each route under VRS

| Route | Integrated BCC model | | | | Separated BCC model | | |
|---|---|---|---|---|---|---|---|
| | Overall efficiency | Technical efficiency | Service effectiveness | Technical effectiveness | Technical efficiency | Service effectiveness | Technical effectiveness |
| 1 | 2.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* |
| 2 | 1.8590 | 1.0000 | 0.8590 | 0.8590 | 1.0000 | 0.8600 | 0.9120 |
| 3 | 2.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* |
| 4 | 1.8919 | 1.0000 | 0.8919 | 0.8919 | 1.0000 | 0.9090 | 1.0000 |
| 5 | 1.8667 | 0.9891 | 0.8776 | 0.8680 | 0.9960 | 0.8840 | 0.9180 |
| 6 | 1.9568 | 0.9568 | 1.0000 | 0.9568 | 0.9590 | 1.0000 | 1.0000 |
| 7 | 1.8128 | 1.0000 | 0.8128 | 0.8128 | 1.0000 | 0.8290 | 0.9030 |
| 8 | 1.9073 | 0.9073 | 1.0000 | 0.9073 | 1.0000* | 1.0000* | 1.0000* |
| 9 | 2.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* |
| 10 | 1.9382 | 0.9382 | 1.0000 | 0.9382 | 0.9380 | 1.0000 | 1.0000 |
| 11 | 2.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* | 1.0000* |
| 12 | 1.9074 | 0.9090 | 0.9984 | 0.9075 | 1.0000* | 1.0000* | 1.0000* |
| 13 | 1.8321 | 0.9692 | 0.8629 | 0.8363 | 1.0000* | 1.0000* | 1.0000* |
| 14 | 1.9488 | 0.9488 | 1.0000 | 0.9488 | 1.0000* | 1.0000* | 1.0000* |
| 15 | 1.9725 | 1.0000 | 0.9725 | 0.9725 | 1.0000 | 0.9730 | 1.0000 |

Note: “*” is indicated the efficient DMUs.

Table 6 Returns to scale of each route

| Route | Technical efficiency | | Service effectiveness | |
|---|---|---|---|---|
| | $u_0^*$ | RTS | $u_1^*$ | RTS |
| 1 | 0.0045 | DRS | 0.1427 | DRS |
| 2 | 0.0088 | DRS | 0.1427 | DRS |
| 3 | 0.1053 | DRS | 0.5397 | DRS |
| 4 | 0.1226 | DRS | 0.5041 | DRS |
| 5 | 0.0522 | DRS | 0.2432 | DRS |
| 6 | 0.0263 | DRS | 0.0552 | DRS |
| 7 | 0.0079 | DRS | 0.0000 | CRS |
| 8 | 0.1656 | DRS | 0.0000 | CRS |
| 9 | 0.0000 | CRS | 0.0000 | CRS |
| 10 | 0.1180 | DRS | 0.1741 | DRS |
| 11 | 0.0000 | CRS | 0.0000 | CRS |
| 12 | 0.0229 | DRS | 0.0356 | DRS |
| 13 | 0.0313 | DRS | 0.0000 | CRS |
| 14 | 0.1027 | DRS | 0.0000 | CRS |
| 15 | 0.0191 | DRS | 0.0000 | CRS |

## 5.3. Slack analysis

To develop improvement strategies for the inefficient routes, slack values for each of the factor and consumption variables are computed according to [ICCR-S] models. The results are reported in Table 7. Except for two efficient routes (No. 9 and No. 11), the remaining 13 inefficient routes require either reducing factor amount or increasing consumption amount. Taking Route No. 8 as an example, decreasing FC by 0.0593%, PC by 0.0749%, and AC by 0.0806%, or increasing PM by 0.1501% alone would achieve efficiency. Consequently, the contradictory improvement suggestions based on separated DEA models by Chiou and Chen (2006) would be avoided.

Table 7 Slack values of factor and consumption variables under CRS

| Route | Factor variable | | | Consumption variable | |
|---|---|---|---|---|---|
| | FC | PC | AC | PM | EP |
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | -0.2448 | 0.0000 | 0.0000 | 0.2500 | 0.2500 |
| 3 | 0.0000 | 0.0000 | 0.0000 | 0.1709 | 0.0000 |
| 4 | -0.2500 | -0.2500 | -0.2500 | 0.2500 | 0.2500 |
| 5 | -0.1852 | -0.0687 | -0.1552 | 0.2500 | 0.2500 |
| 6 | -0.0025 | -0.0025 | -0.0025 | 0.0000 | 0.0000 |
| 7 | 0.0000 | -0.2500 | 0.0000 | 0.2500 | 0.2500 |
| 8 | -0.0593 | -0.0749 | -0.0806 | 0.1501 | 0.0000 |
| 9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | -0.2500 | -0.2500 | -0.2500 | 0.2464 | 0.0000 |
| 11 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 12 | -0.2129 | -0.2500 | -0.2500 | 0.0024 | 0.0030 |
| 13 | 0.0000 | 0.0000 | -0.2500 | 0.1615 | 0.0676 |
| 14 | -0.0261 | -0.2500 | -0.0144 | 0.0000 | 0.0000 |
| 15 | 0.0000 | 0.0000 | 0.0000 | 0.1737 | 0.2500 |

## 5.4. Weight analysis for generalized IDEA model

The above IDEA model has adopted an additive form of technical efficiency and service effectiveness, which implicitly assumes equal weights for both terms. A more generalized IDEA model with various weights can be expressed as follows:

$$\underset{u,v,w}{Max} \quad H_k = \alpha\left(\frac{\sum_{r=1}^{s} u_r y_{kr}}{\sum_{j=1}^{m} v_j x_{kj}}\right) + (1-\alpha)\left(\frac{\sum_{q=1}^{p} w_q l_{kq}}{\sum_{r=1}^{s} u_r y_{kr}}\right)$$

$$\begin{aligned}
s.t. \quad & \sum_{r=1}^{s} u_r y_{ir} \le \sum_{j=1}^{m} v_j \left(x_{kj} - s_{kj}\right), \quad i = 1,2,\cdots,n \\
& \sum_{q=1}^{p} w_q \left(l_{kq} + s_{kq}\right) \le \sum_{r=1}^{s} u_r y_{ir}, \quad i = 1,2,\cdots,n \\
& v_j \ge 0, \qquad j = 1,2,\cdots,m \\
& w_q \ge 0, \qquad q = 1,2,\cdots,p \\
& u_r \ge 0, \qquad r = 1,2,\cdots,s
\end{aligned}$$

If the weights of production and marketing department are allowed to be endogenous, some DMUs might reach efficiency by totally ignoring the performance of production department or marketing department, which might not be very reasonable in practice. Thus, this study set the weights as exogenous parameter ($\alpha$).

Where, $\alpha$ is the weight of technical efficiency, which is subjectively given by the decision maker. (1-$\alpha$) is the weight of service effectiveness. If the decision maker concerns more about the technical efficiency than the service effectiveness, then $\alpha$ can be set lager than 0.5, vice versa. Taking DMU 5 as an example, the technical efficiency and service effectiveness with various weight combinations are computed and the technical efficiency will increase and the service effectiveness will decrease as $\alpha$ gets larger. Obviously, the generalized IDEA model can provide the decision-maker with wider spectrum of information than only the equal-weight information.

In this section, we will discuss influence of weight change for each DMU. First, we will demonstrate the result of DMU 1. From Table 8 and Figure 4, we could realize that only when $\alpha$ equal to 0.1 and 0.2 weight change will change efficiency score. When $\alpha$ larger than 0.3 weight change would no effect on efficiency score no matter for technical efficiency or service effectiveness. That's because technical efficiency is reach optimal value when $\alpha$ equal to 0.3. No matter how the weight changes technical efficiency will adjust by itself in order to reach optimal value.

For relative efficient DMU such DMU 1, weight changes wouldn’t increase efficiency score by large scale. For this kind DMUs, they need to improve their efficiency by adjust their input, output and/or consumption variables rather than change their weight only. We could know that weight changing isn’t so important for efficient DMUs than for inefficient DMUs.

Table 8 The technical efficiency and service effectiveness of DMU 1 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.88245 | 0.85085 |
| 0.2 | 0.91033 | 0.84733 |
| 0.3 | 1.00000 | 0.82141 |
| 0.4 | 1.00000 | 0.82141 |
| 0.5 | 1.00000 | 0.82141 |
| 0.6 | 1.00000 | 0.82141 |
| 0.7 | 1.00000 | 0.82141 |
| 0.8 | 1.00000 | 0.82141 |
| 0.9 | 1.00000 | 0.82141 |

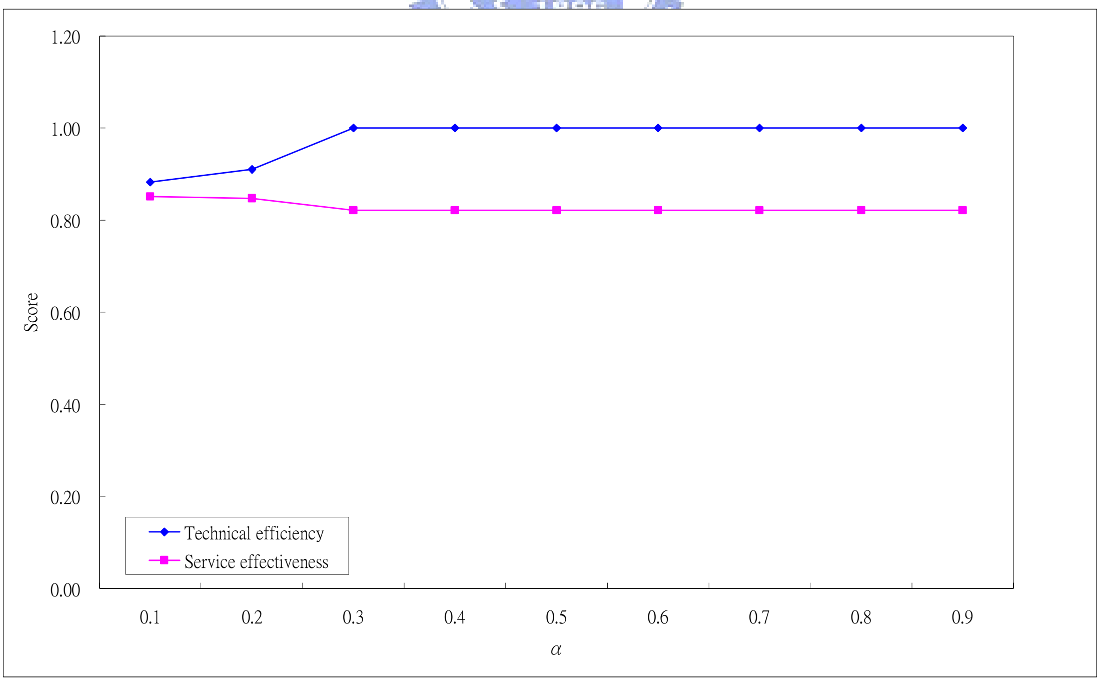



Fig 4 The shapes of technical efficiency and service effectiveness of DMU 1 with various weight combinations

The result of DMU 2 is showing in Table 9 and Figure 5. DMU 2 is relative efficiency in technical efficiency. When $\alpha$ get larger, technical efficiency will increase slowly. However the score change cause by weight change isn't significant. The reason cause this result is the same as DMU 1.

If DMU 2 wishes to increase its efficiency score, it needs to adjust is input, output and/or consumption variables rather than change its weight only.

Table 9 The technical efficiency and service effectiveness of DMU 2 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.98480 | 0.79051 |
| 0.2 | 0.98480 | 0.79051 |
| 0.3 | 0.99268 | 0.78836 |
| 0.4 | 0.99268 | 0.78836 |
| 0.5 | 0.99268 | 0.78836 |
| 0.6 | 1.00000 | 0.77920 |
| 0.7 | 1.00000 | 0.77920 |
| 0.8 | 1.00000 | 0.77920 |
| 0.9 | 1.00000 | 0.77920 |

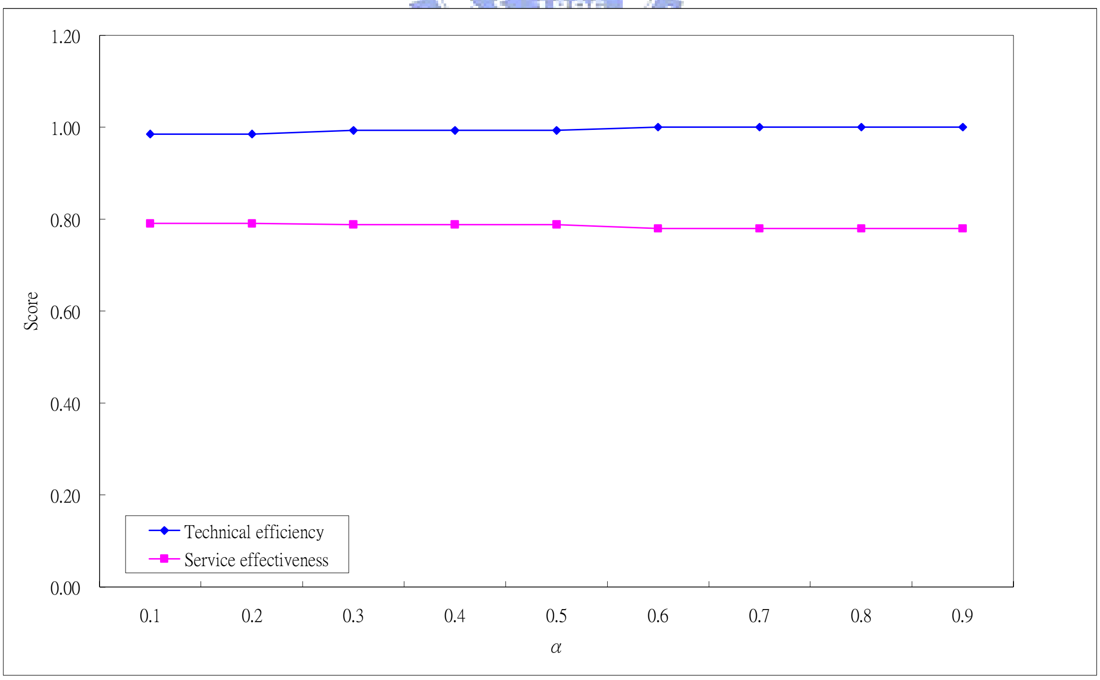



Fig 5 The shapes of technical efficiency and service effectiveness of DMU 2 with various weight combinations

The result for DMU 3 is displaying in Table 10 and Figure 6. They show that weight changing causes significant improvement in performance. That's because DMU 3 is a relative inefficiency DMU. When the weight changes, the efficiency scores will have apparent change. If DMU 3 wants to improve its efficiency score, it can either modify weight combinations or adjust input, output and/or consumption variables.

From this example, we could know that weight change is more useful for relative inefficiency DMU. However once efficiency score of technical efficiency or service effectiveness is close to unity under certain weight combination, the weight change is no more useful in improving efficiency score. In this case, when $\alpha$=0.4, no matter how the weight change the efficiency score isn't change at all. Unless the weight combination become extreme such as $\alpha$=0.9. Then the sector which gets most source will have higher efficiency score such as technical efficiency.

Table 10 The technical efficiency and service effectiveness of DMU 3 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.78654 | 0.85611 |
| 0.2 | 0.83382 | 0.84874 |
| 0.3 | 0.93082 | 0.82127 |
| 0.4 | 0.94107 | 0.81624 |
| 0.5 | 0.94107 | 0.81624 |
| 0.6 | 0.94107 | 0.81624 |
| 0.7 | 0.94107 | 0.81624 |
| 0.8 | 0.94107 | 0.81624 |
| 0.9 | 0.94355 | 0.80406 |

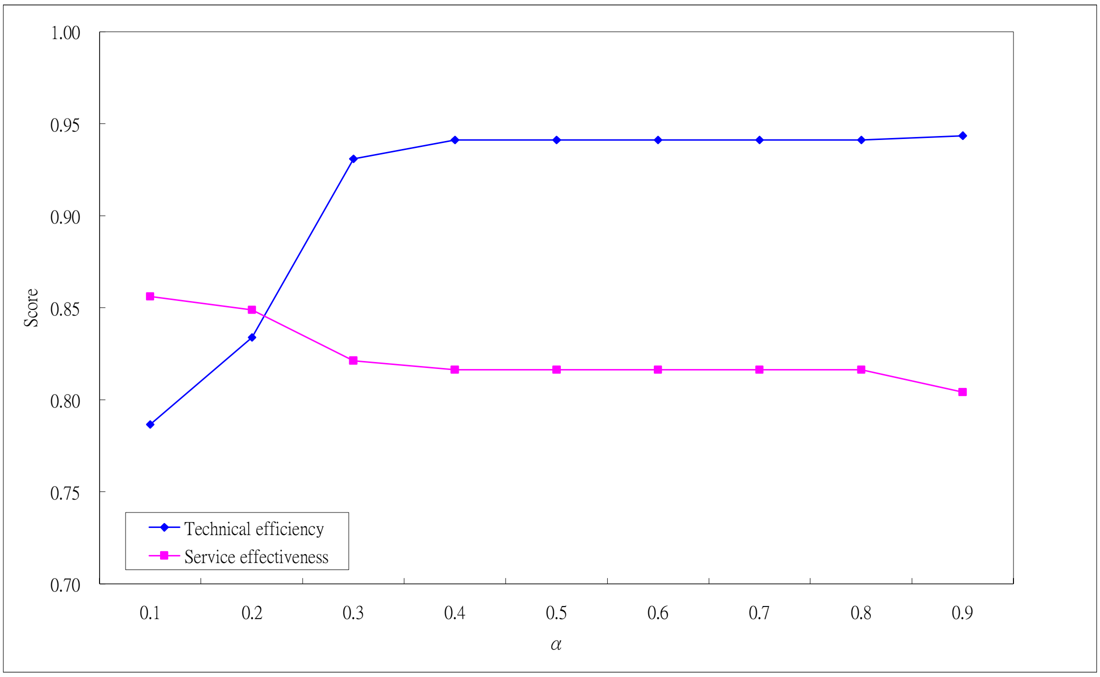



Fig 6 The shapes of technical efficiency and service effectiveness of DMU 3 with various weight combinations

Table 11 and Figure 7 are demonstrated the result of DMU 4. DMU 4 is a relative inefficient DMU as well. Its efficiency score in sensitive about weight change. It can improve it efficiency performance through weight change and/or adjusting input, output and/or consumption variables.

The result of DMU 5 is demonstrating in Table 12 and Figure 10. DMU 5 will have the same pattern as DMU 4 because DMU 5 is an inefficient DMU as well.

Table 11 The technical efficiency and service effectiveness of DMU 4 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.87147 | 0.78877 |
| 0.2 | 0.87147 | 0.78877 |
| 0.3 | 0.87147 | 0.78877 |
| 0.4 | 0.87147 | 0.78877 |
| 0.5 | 0.89702 | 0.77083 |
| 0.6 | 0.97748 | 0.68851 |
| 0.7 | 0.98773 | 0.67224 |
| 0.8 | 0.99026 | 0.66504 |
| 0.9 | 0.99026 | 0.66504 |

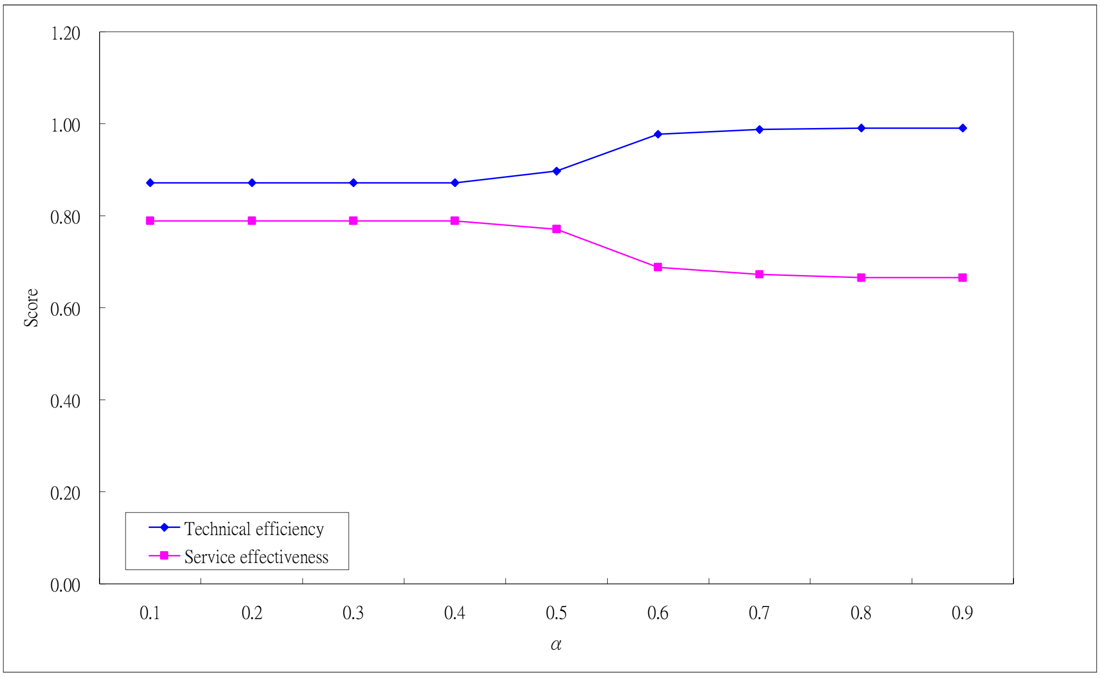



Fig 7 The shapes of technical efficiency and service effectiveness of DMU 4 with various weight combinations

Table 12 The technical efficiency and service effectiveness of DMU 5 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.940981 | 0.85118 |
| 0.2 | 0.967532 | 0.84522 |
| 0.3 | 0.967532 | 0.84522 |
| 0.4 | 0.967532 | 0.84522 |
| 0.5 | 0.967532 | 0.84522 |
| 0.6 | 0.972326 | 0.83987 |
| 0.7 | 0.972327 | 0.83987 |
| 0.8 | 0.995025 | 0.75819 |
| 0.9 | 0.996021 | 0.75031 |

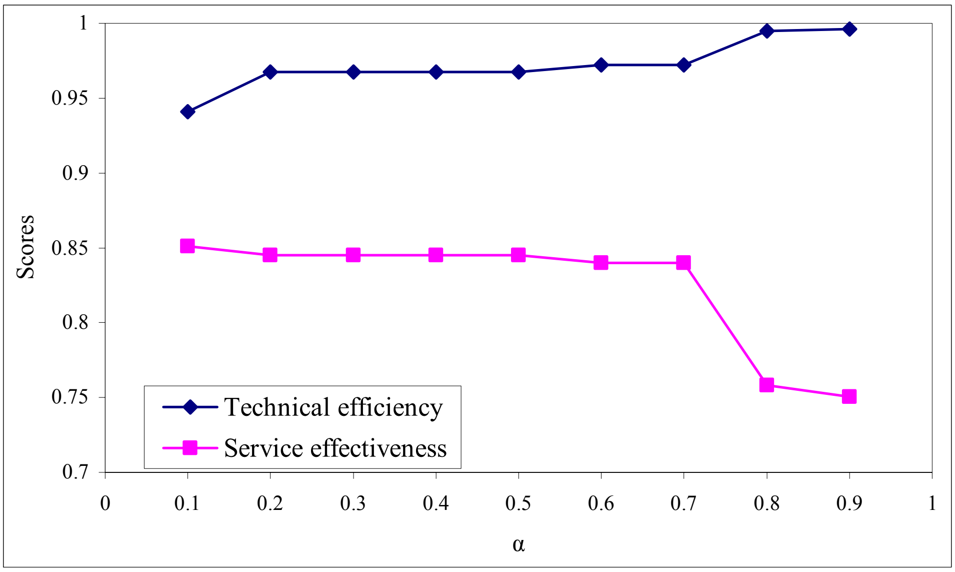



Fig 8 The shapes of technical efficiency and service effectiveness of DMU 5 with various weight combinations

The result of weight change of DMU 6 is showing in Table 13 and Figure 9. DMU 6 is a relative efficient DMU. Its service effectiveness reaches unity which means its sale department is benchmark. No matter how the weight changes it will adjust by itself unless the weight is in extreme position such as $\alpha$ =0.9. In this situation, production department get most source so technical efficiency will perform better and service effectiveness will become worse. If DMU 6 wants to improve its performance, it better focus on adjusting its input, output and/or consumption variables.

Table 13 The technical efficiency and service effectiveness of DMU 6 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.95003 | 1.00000 |
| 0.2 | 0.95003 | 1.00000 |
| 0.3 | 0.95003 | 1.00000 |
| 0.4 | 0.95003 | 1.00000 |
| 0.5 | 0.95003 | 1.00000 |
| 0.6 | 0.95003 | 1.00000 |
| 0.7 | 0.95003 | 1.00000 |
| 0.8 | 0.95003 | 1.00000 |
| 0.9 | 0.95276 | 0.98852 |

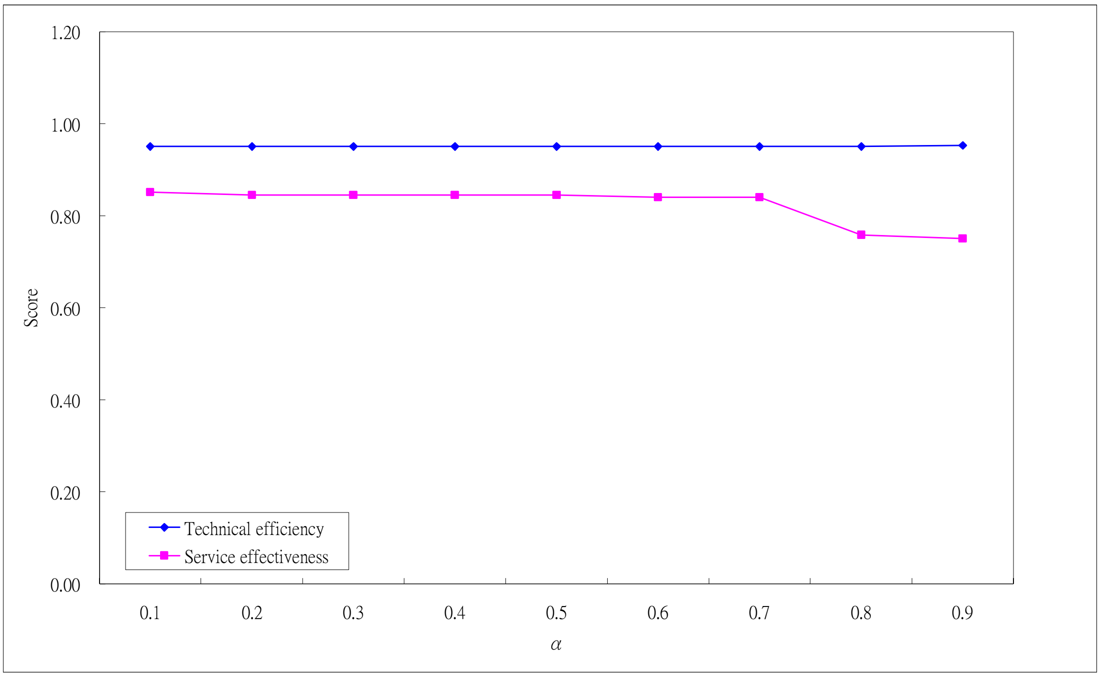



Fig 9 The shapes of technical efficiency and service effectiveness of DMU 6 with various weight combinations

Table 14 and Figure 10 demonstrate the outcomes of DMU 7. Like DMU 6, DMU 7 is a relative efficient DMU. However it performs better in technical efficiency. Only when weight combination become extreme such as $\alpha$=0.1, service effectiveness will perform better. Otherwise technical efficiency will always be benchmark. Talking about performance improvement about DMU 7, it could focus on modify its input, output and/or consumption variables rather than find optimal weight combination.

Table 14 The technical efficiency and service effectiveness of DMU 7 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.96007 | 0.81723 |
| 0.2 | 1.00000 | 0.81024 |
| 0.3 | 1.00000 | 0.81024 |
| 0.4 | 1.00000 | 0.81024 |
| 0.5 | 1.00000 | 0.81024 |
| 0.6 | 1.00000 | 0.81024 |
| 0.7 | 1.00000 | 0.81024 |
| 0.8 | 1.00000 | 0.81024 |
| 0.9 | 1.00000 | 0.81024 |

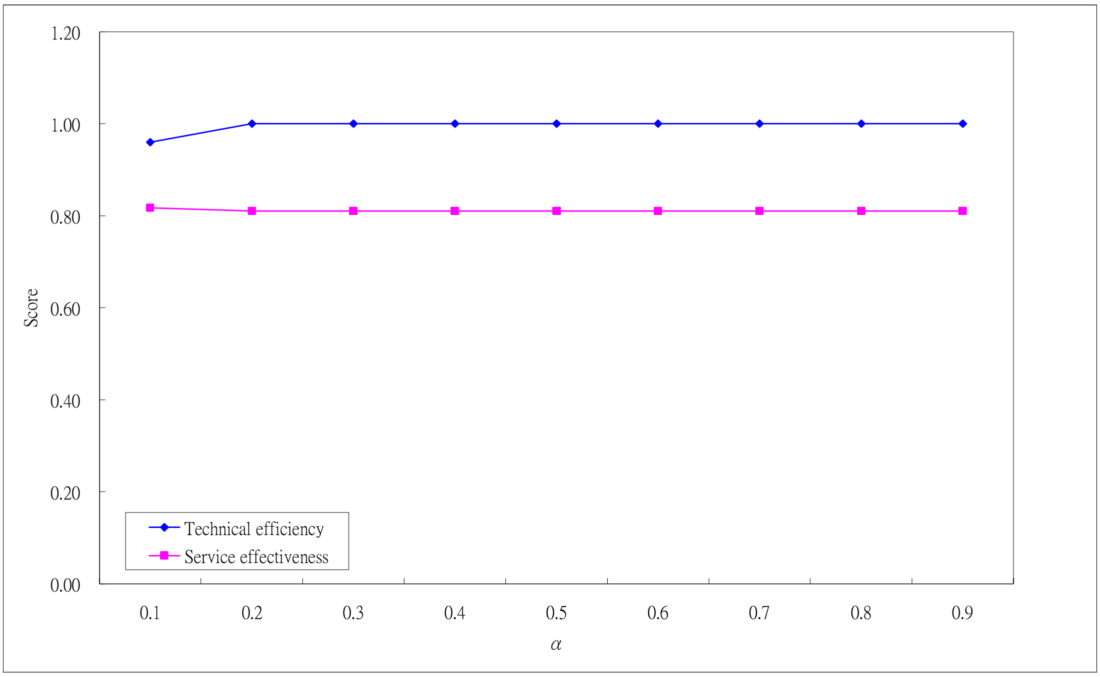



Fig 10 The shapes of technical efficiency and service effectiveness of DMU 7 with various weight combinations

Both the scores of technical efficiency and service effectiveness of different weight combination for DMU 8 and DMU 10 are demonstrating in Table 15, Table 16, Figure 11 and Figure 12. Although DMU 8 and DMU 9 looks like relative efficient DMUs in the efficient score, they are not benchmarks in both production and sale department.

They are still sensitive about weight change especially at some extreme weight combination such as $\alpha$=0.1 or 0.2. When weight combination is in extreme level, weight change will have certain influence on scores. If the weight combination is in average level such as $\alpha$=0.5 or 0.6, efficiency score will has little response about weight change.

For this kind DMUs, they can improve their performance scores both through weight change and adjusting their input, output and/or consumption variables. However the main improvement approach of this kind should focus on adjusting their variables.

Table 15 The technical efficiency and service effectiveness of DMU 8 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.90691 | 0.94448 |
| 0.2 | 0.97387 | 0.93167 |
| 0.3 | 0.97387 | 0.93167 |
| 0.4 | 0.97387 | 0.93167 |
| 0.5 | 0.99002 | 0.91646 |
| 0.6 | 0.99002 | 0.91646 |
| 0.7 | 0.99002 | 0.91646 |
| 0.8 | 0.99002 | 0.91646 |
| 0.9 | 0.99002 | 0.91646 |

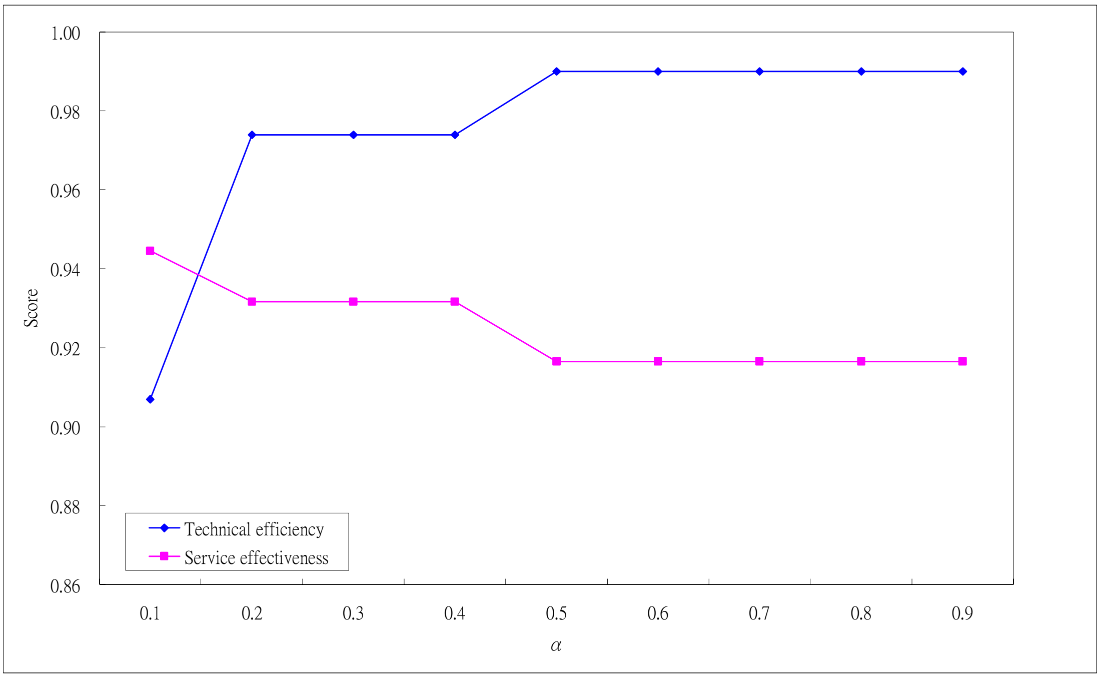



Fig 11 The shapes of technical efficiency and service effectiveness of DMU 8 with various weight combinations

Table 16 The technical efficiency and service effectiveness of DMU 10 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.87781 | 0.97950 |
| 0.2 | 0.90262 | 0.97615 |
| 0.3 | 0.90262 | 0.97615 |
| 0.4 | 0.90262 | 0.97615 |
| 0.5 | 0.90262 | 0.97615 |
| 0.6 | 0.90262 | 0.97615 |
| 0.7 | 0.90262 | 0.97615 |
| 0.8 | 0.90262 | 0.97615 |
| 0.9 | 0.90262 | 0.97615 |

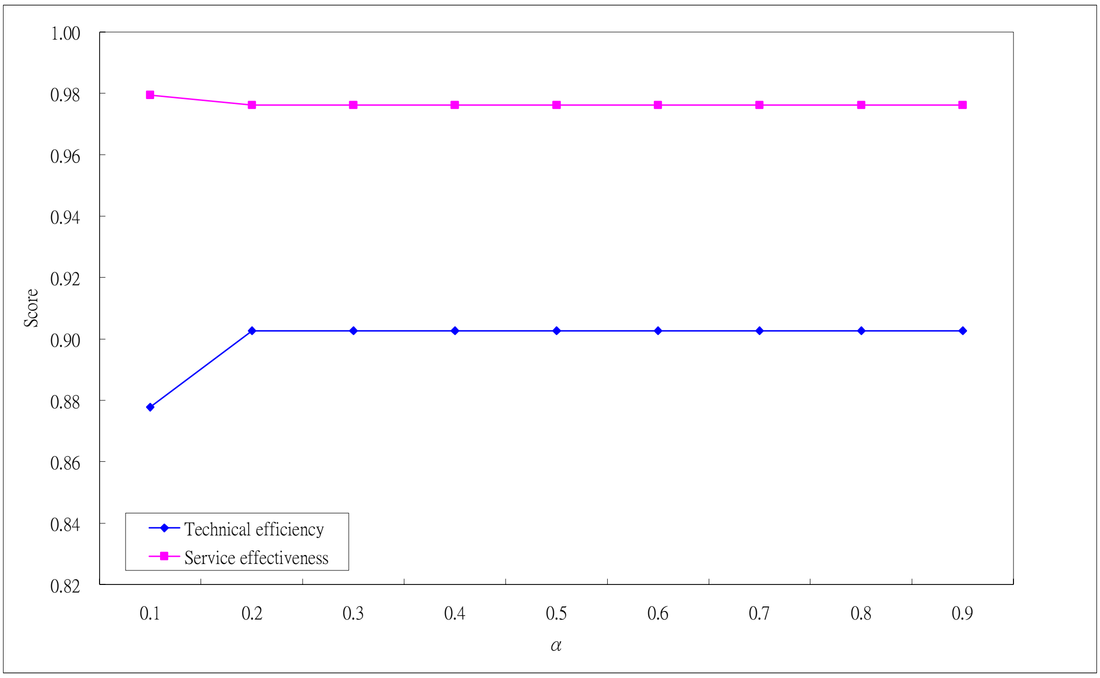



Fig 12 The shapes of technical efficiency and service effectiveness of DMU 10 with various weight combinations

The result of DMU 9 and DMU 11 is shown in Table 17, Table 18, Figure 13, and Figure 14. Both DMU 9 and DMU 11 reach overall efficiency. Their efficiency score no matter technical efficiency or service effectiveness are all equal to unit. For these kinds DMU, different weight combinations have insignificant effect.

Because both produce and sale sector of these kinds DMUs will adjust by themselves, weight change will have no power in improving efficiency scores. Once DMU reaches overall efficiency, weight change is not an important issue it should concern.

Table 17 The technical efficiency and service effectiveness of DMU 9 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 1.00000 | 1.00000 |
| 0.2 | 1.00000 | 1.00000 |
| 0.3 | 1.00000 | 1.00000 |
| 0.4 | 1.00000 | 1.00000 |
| 0.5 | 1.00000 | 1.00000 |
| 0.6 | 1.00000 | 1.00000 |
| 0.7 | 1.00000 | 1.00000 |
| 0.8 | 1.00000 | 1.00000 |
| 0.9 | 1.00000 | 1.00000 |

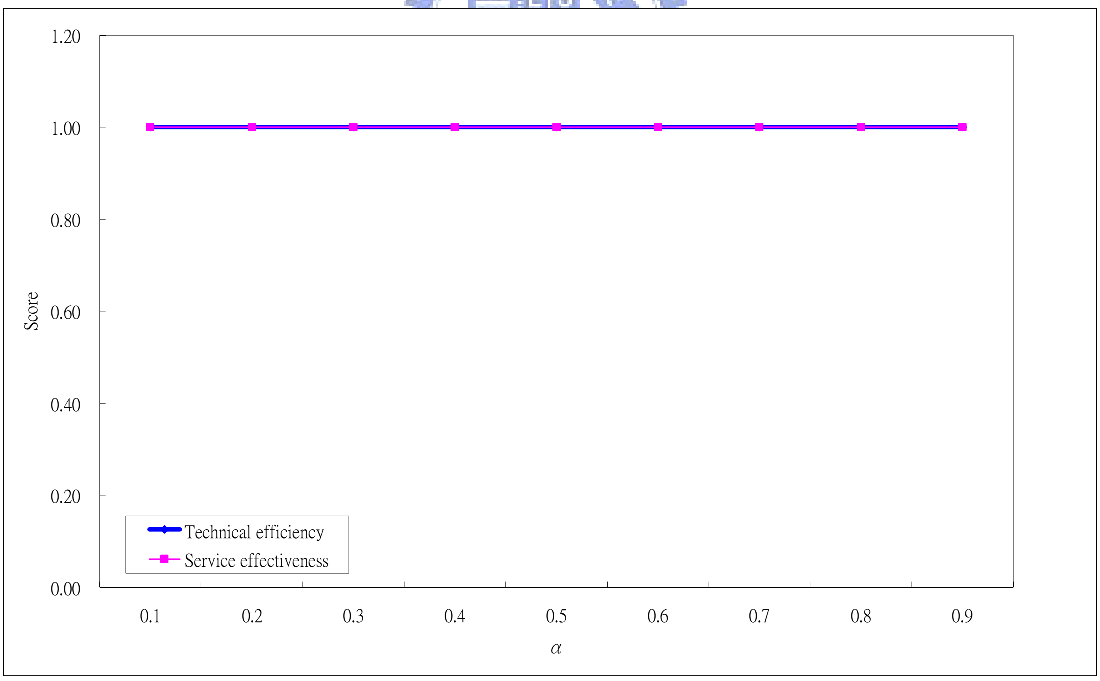



Fig 13 The shapes of technical efficiency and service effectiveness of DMU 9 with various weight combinations

Table 18 The technical efficiency and service effectiveness of DMU 11 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 1.00000 | 1.00000 |
| 0.2 | 1.00000 | 1.00000 |
| 0.3 | 1.00000 | 1.00000 |
| 0.4 | 1.00000 | 1.00000 |
| 0.5 | 1.00000 | 1.00000 |
| 0.6 | 1.00000 | 1.00000 |
| 0.7 | 1.00000 | 1.00000 |
| 0.8 | 1.00000 | 1.00000 |
| 0.9 | 1.00000 | 1.00000 |

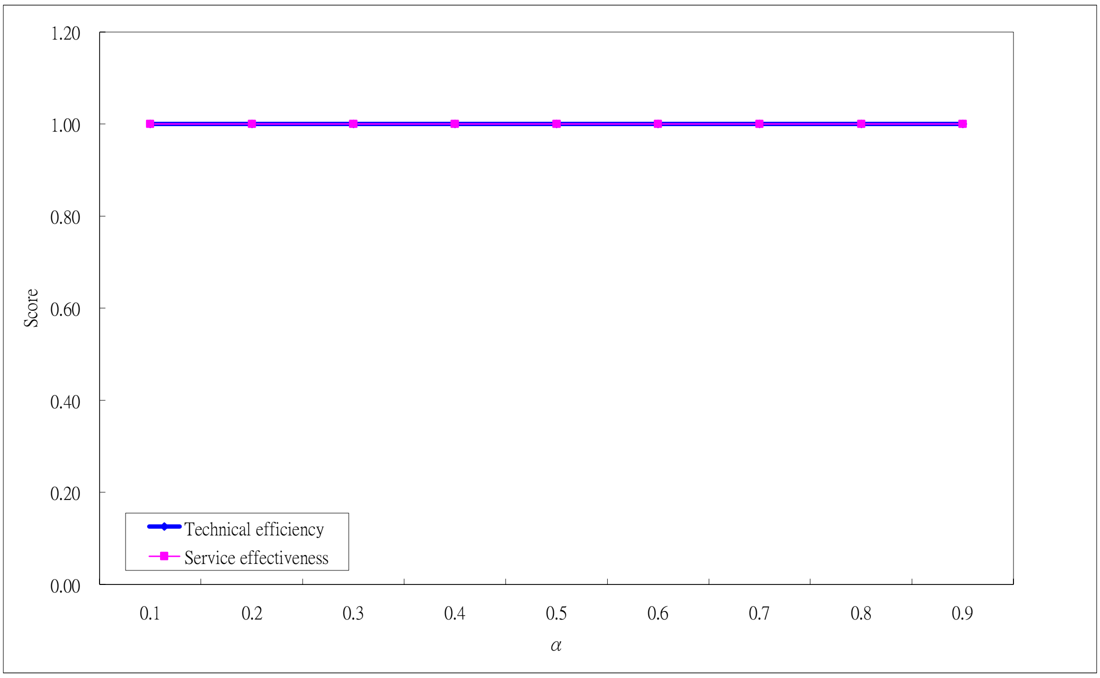



Fig 14 The shapes of technical efficiency and service effectiveness of DMU 11 with various weight combinations

Table 19 and Figure 15 are displaying the result of different weight combinations for DMU 12. There is an interesting shape on Figure 15. The efficiency scores is fixed in two value. When $\alpha$ is lying between 0.1 and 0.5 and between 0.6 and 0.9.

First, when $\alpha$ is between 0.1 and 0.5, service effectiveness is unity. Sale department will adjust by itself until the source become less and less. Which means production department get most source such as $\alpha$ is lying 0.5 and between 0.6 and 0.9. When $\alpha$ is large enough, the technical efficiency will be performance better.

The DMU like DMU 12, which is relative inefficiency in one department and efficiency in another department, can improve its performance by finding optimal weight combinations and modifying its variable values. Both these method can help to better its performance.

Table 19 The technical efficiency and service effectiveness of DMU 12 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.89205 | 1.00000 |
| 0.2 | 0.89205 | 1.00000 |
| 0.3 | 0.89205 | 1.00000 |
| 0.4 | 0.89205 | 1.00000 |
| 0.5 | 0.89205 | 1.00000 |
| 0.6 | 1.00000 | 0.85259 |
| 0.7 | 1.00000 | 0.85259 |
| 0.8 | 1.00000 | 0.85259 |
| 0.9 | 1.00000 | 0.85259 |

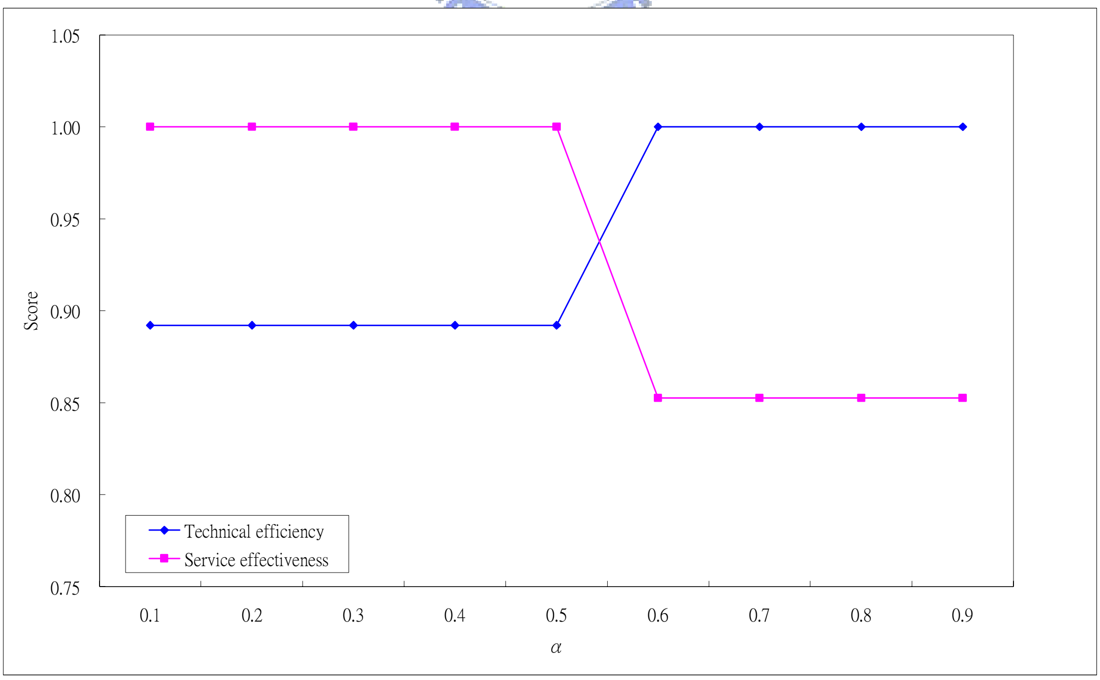



Fig 15 The shapes of technical efficiency and service effectiveness of DMU 12 with various weight combinations

Result of DMU 13, DMU 14 and DMU 15 are demonstrating in Table 20, Table 21 and Table 22. The shapes of these results are displaying in Figure 16, Figure 17 and Figure 18. From these results, we could know that all these DMU are relative efficiency. Only weight combinations are in extreme level, the efficiency scores will be different and the difference is only in a small scale.

If these kinds DMUs hope to raise its efficiency scores, they better to concentrate on adjusting their variable values.

Table 20 The technical efficiency and service effectiveness of DMU 13 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.96949 | 0.84731 |
| 0.2 | 0.96949 | 0.84731 |
| 0.3 | 0.96949 | 0.84731 |
| 0.4 | 0.96949 | 0.84731 |
| 0.5 | 0.96949 | 0.84731 |
| 0.6 | 0.96949 | 0.84731 |
| 0.7 | 0.96949 | 0.84731 |
| 0.8 | 0.96949 | 0.84731 |
| 0.9 | 0.98929 | 0.72021 |

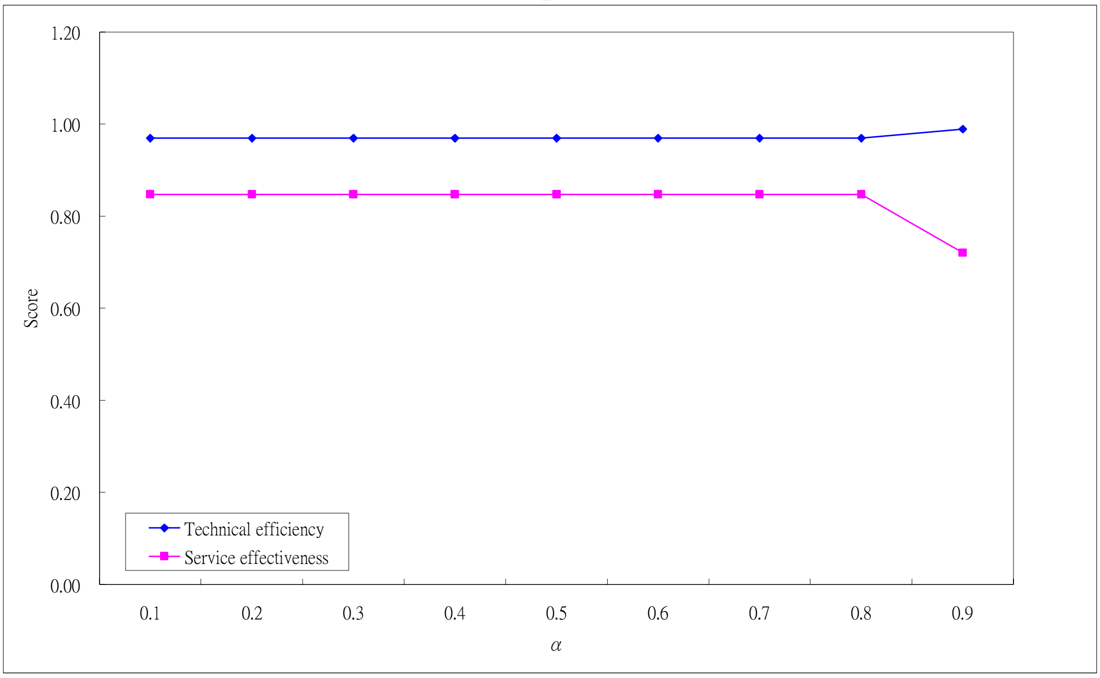



Fig 16 The shapes of technical efficiency and service effectiveness of DMU 13 with various weight combinations

Table 21 The technical efficiency and service effectiveness of DMU 14 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 0.97151 | 0.93701 |
| 0.2 | 1.00000 | 0.93016 |
| 0.3 | 1.00000 | 0.93016 |
| 0.4 | 1.00000 | 0.93016 |
| 0.5 | 1.00000 | 0.93016 |
| 0.6 | 1.00000 | 0.93016 |
| 0.7 | 1.00000 | 0.93016 |
| 0.8 | 1.00000 | 0.93016 |
| 0.9 | 1.00000 | 0.93016 |

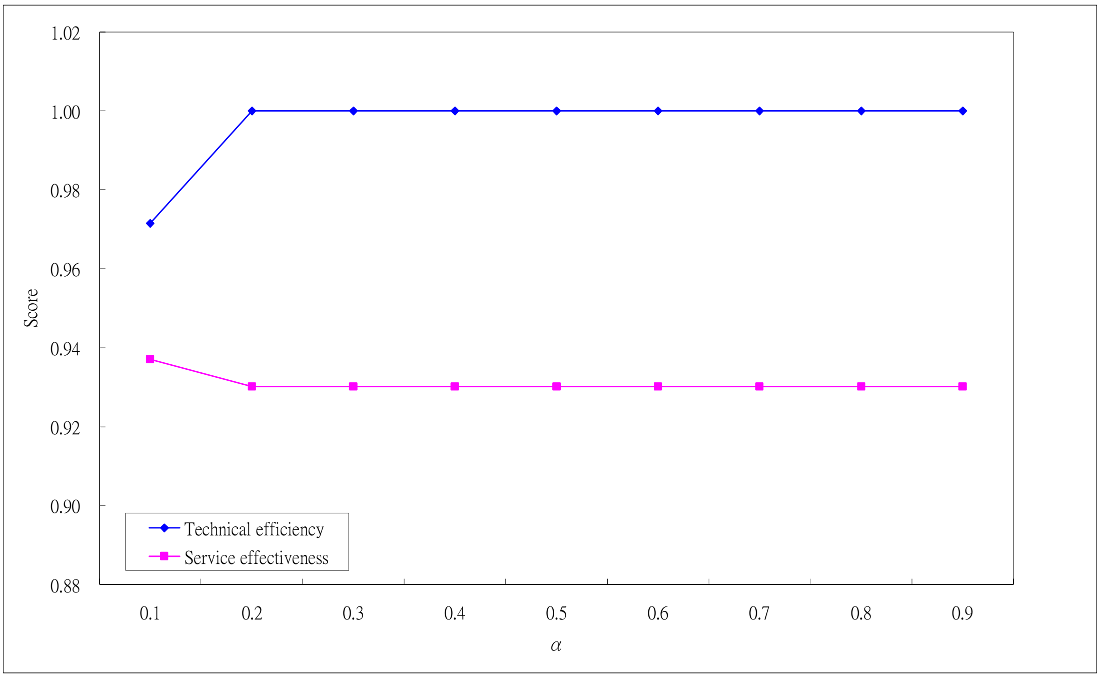



Fig 17 The shapes of technical efficiency and service effectiveness of DMU 14 with various weight combinations

Table 22 The technical efficiency and service effectiveness of DMU 15 with various weight combinations

| $\alpha$ | Technical efficiency | Service effectiveness |
|---|---|---|
| 0.1 | 1.00000 | 0.96504 |
| 0.2 | 1.00000 | 0.96504 |
| 0.3 | 1.00000 | 0.96504 |
| 0.4 | 1.00000 | 0.96504 |
| 0.5 | 1.00000 | 0.96504 |
| 0.6 | 1.00000 | 0.96504 |
| 0.7 | 1.00000 | 0.96504 |
| 0.8 | 1.00000 | 0.96504 |
| 0.9 | 1.00000 | 0.96504 |

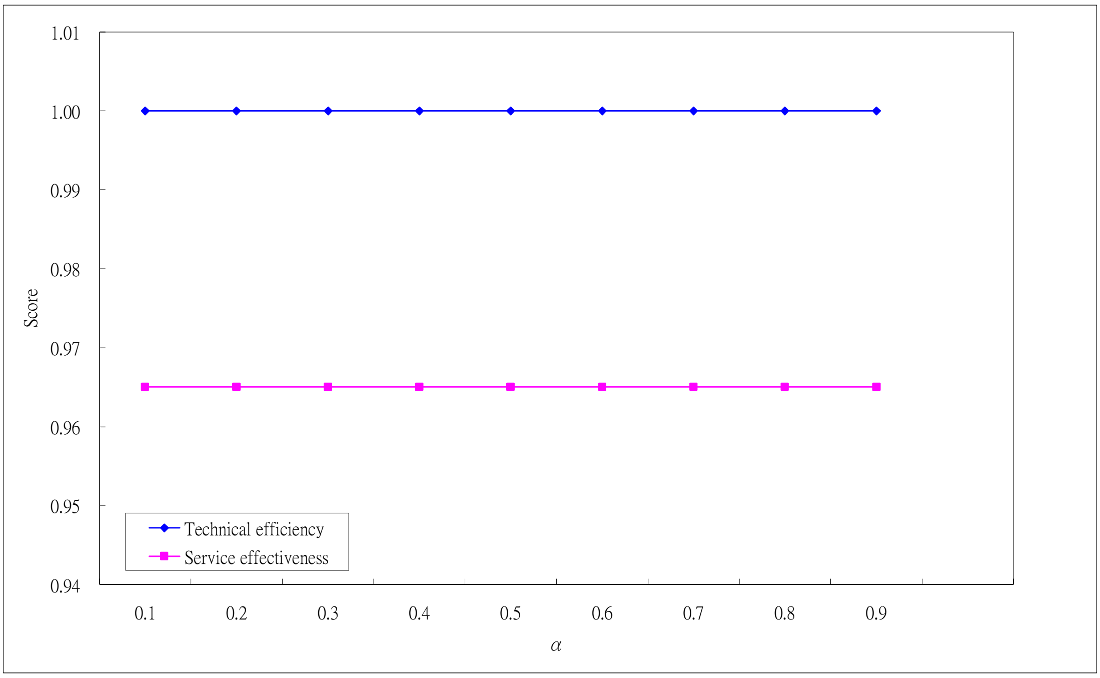



Fig 18 The shapes of technical efficiency and service effectiveness of DMU 15 with various weight combinations

## 5.5. Overall weight analysis

From above analysis, we could know that change weight will influence the performance of DMU. Then, Table 23 and Figure 19 will demonstrate the technical efficiency of each DMU with various weight combinations and Table 24 and Figure 20 is the result of service effectiveness. Obviously, the generalized integrated DEA model can provide the decision-maker with wider spectrum of information than only the equal-weight information.

Table 23 Technical efficiency of each DMU with various weight combinations

| | $\alpha$ (Weight) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Route | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 1 | 0.8824 | 0.9103 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | 0.9848 | 0.9848 | 0.9927 | 0.9927 | 0.9927 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 3 | 0.7865 | 0.8338 | 0.9308 | 0.9411 | 0.9411 | 0.9411 | 0.9411 | 0.9411 | 0.9436 |
| 4 | 0.8715 | 0.8715 | 0.8715 | 0.8715 | 0.8970 | 0.9775 | 0.9877 | 0.9903 | 0.9903 |
| 5 | 0.9410 | 0.9675 | 0.9675 | 0.9675 | 0.9675 | 0.9723 | 0.9723 | 0.9950 | 0.9960 |
| 6 | 0.9500 | 0.9500 | 0.9500 | 0.9500 | 0.9500 | 0.9500 | 0.9500 | 0.9500 | 0.9528 |
| 7 | 0.9601 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 8 | 0.9069 | 0.9739 | 0.9739 | 0.9739 | 0.9900 | 0.9900 | 0.9900 | 0.9900 | 0.9900 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 10 | 0.8778 | 0.9026 | 0.9026 | 0.9026 | 0.9026 | 0.9026 | 0.9026 | 0.9026 | 0.9026 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 12 | 0.8920 | 0.8920 | 0.8920 | 0.8920 | 0.8920 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 13 | 0.9695 | 0.9695 | 0.9695 | 0.9695 | 0.9695 | 0.9695 | 0.9695 | 0.9695 | 0.9893 |
| 14 | 0.9715 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

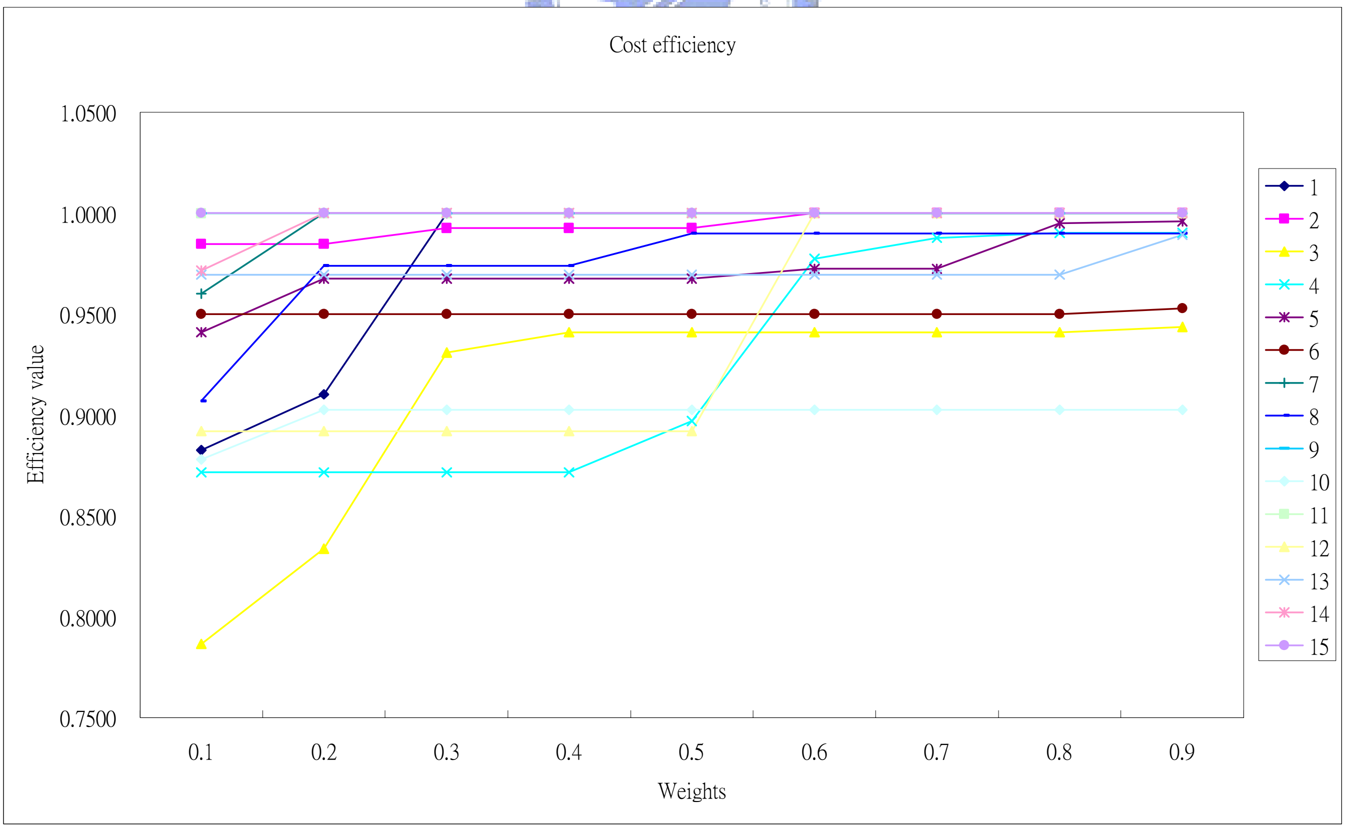



Fig 19 The shapes of technical efficiency of Each DMU with various weight combinations

Table 24 Service effectiveness of each DMU with various weight combinations

| | $\alpha$ (Weight) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Route | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 1 | 0.8508 | 0.8473 | 0.8214 | 0.8214 | 0.8214 | 0.8214 | 0.8214 | 0.8214 | 0.8214 |
| 2 | 0.7905 | 0.7905 | 0.7884 | 0.7884 | 0.7884 | 0.7792 | 0.7792 | 0.7792 | 0.7792 |
| 3 | 0.8561 | 0.8487 | 0.8213 | 0.8162 | 0.8162 | 0.8162 | 0.8162 | 0.8162 | 0.8041 |
| 4 | 0.7888 | 0.7888 | 0.7888 | 0.7888 | 0.7708 | 0.6885 | 0.6722 | 0.6650 | 0.6650 |
| 5 | 0.8512 | 0.8452 | 0.8452 | 0.8452 | 0.8452 | 0.8399 | 0.8399 | 0.7582 | 0.7503 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9885 |
| 7 | 0.8172 | 0.8102 | 0.8102 | 0.8102 | 0.8102 | 0.8102 | 0.8102 | 0.8102 | 0.8102 |
| 8 | 0.9445 | 0.9317 | 0.9317 | 0.9317 | 0.9165 | 0.9165 | 0.9165 | 0.9165 | 0.9165 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 10 | 0.9795 | 0.9761 | 0.9761 | 0.9761 | 0.9761 | 0.9761 | 0.9761 | 0.9761 | 0.9761 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.8526 | 0.8526 | 0.8526 | 0.8526 |
| 13 | 0.8473 | 0.8473 | 0.8473 | 0.8473 | 0.8473 | 0.8473 | 0.8473 | 0.8473 | 0.7202 |
| 14 | 0.9370 | 0.9302 | 0.9302 | 0.9302 | 0.9302 | 0.9302 | 0.9302 | 0.9302 | 0.9302 |
| 15 | 0.9650 | 0.9650 | 0.9650 | 0.9650 | 0.9650 | 0.9650 | 0.9650 | 0.9650 | 0.9650 |

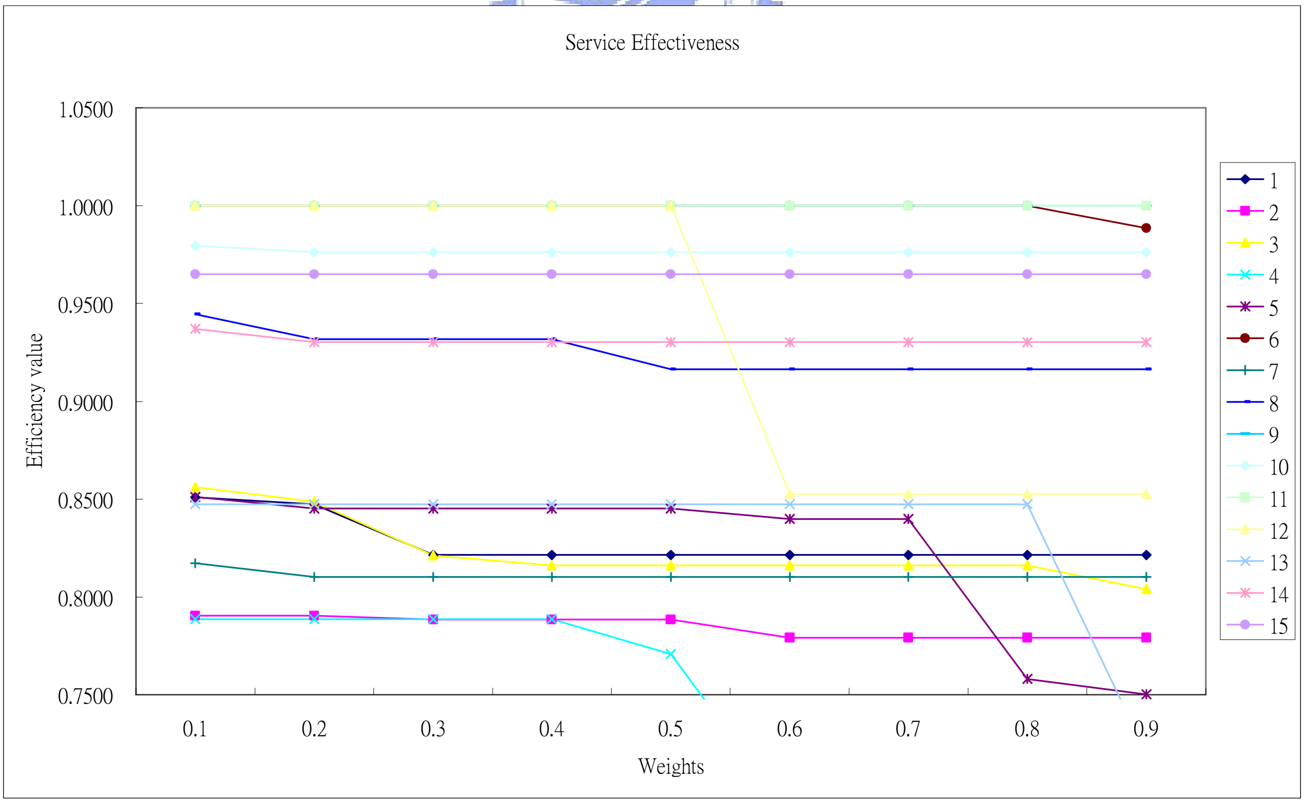



Fig 20 The shapes of Service effectiveness of Each DMU with various weight combinations

From Table 23, Table 24, Figure 19 and Figure 20, we can realize that efficient DMUs, such DMU 9 and DMU 11, has low sensitivity about weight change because no matter how the weight change the efficient DMU will adjust by itself. Only the inefficient DMUs, such as DMU 5, will have high sensitivity about weight change.

Through weight analysis, each DMU would have its own suggestion on improvement approach. Different DMU would have different features, and weight analysis provides another way in analyzing each DMU. Each DMU can find best method in improving its performance.

From above two figures, we could know that the changes of the efficiency score no matter technical efficiency or service effectiveness are not very huge. If DMUs try to improve its performance, they better focus on adjusting the variable.

# 6. Conclusions and suggestions

## 6.1. Conclusions

This paper develops integrated DEA (IDEA) models, under constant-returns-to-scale (ICCR model) and variable-returns-to-scale (IBCC model) technologies, to measure the overall efficiency and separate efficiencies for non-storable commodities, from various aspects of technical efficiency, service effectiveness, and technical effectiveness. Some major findings can be concluded as follows:

(1) The proposed IDEA model, either ICCR or IBCC, is proven with rationality and uniqueness properties. The property of rationality suggests that the scores obtained from this integrated model are efficient values rather than meaningless figures. The property of uniqueness guarantees that the efficiency scores obtained from this model are global maximum rather than local maximum.

(2) Our proposed IDEA models can be employed to measure the overall efficiency of non-storable commodities such as transportation services. The applicability of the proposed IDEA model has been demonstrated by a case study, from which the IDEA model has revealed higher discrimination power than the conventional separated DEA models.

(3) Compared with conventional separated DEA model, the proposed IDEA model can explain for non-storable commodities more explicitly. Because the IDEA model can jointly account for the production and sale departments of non-storable commodities, it is superior to conventional DEA models.

## 6.2. Suggestions

Some directions for future studies can be identified as follows.

(1) The weight analysis of the proposed IDEA model is worthy to make a further study because the weight in this study is an exogenous variable, not an endogenous variable. One could add the weight variable into the integrated DEA model and let the model decide the optimal weight for each department.

(2) An additive form of proposed IDEA model is derived in this paper, other forms of IDEA models or even multi-objective IDEA models deserve further exploration.

(3) The present paper only demonstrates the overall efficiency measure for two departments -- production (technical efficiency) and sale (service effectiveness). The proposed IDEA model can easily be extended to evaluate the overall performance of an organization with more than two departments that are vertically and/or horizontally coordinated, e.g., the supply chain managing of a firm, the mails processing of the post office, among others.

(4) More applications to other non-storable cases with the proposed IDEA model and more comparisons with other types of DEA models are also worthy of further study.

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