國 立 交 通 大 學

財 務 金 融 研 究 所

碩 士 論 文

以GARCH-Jump模型評價障礙選擇權

Pricing Barrier Options under GARCH-Jump Model

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摘要

本篇論文是根據段教授2005年的論文,在評價核心加上跳躍，以及同一時間在資產的報酬以及波動度的相關跳躍來評價障礙型的選擇權。 既然障礙型選擇權是和資產的走勢有關，在資產上加上跳躍的影響必然會對於障礙型選擇權有所影響。因此，本篇論文探討此一現象並且去比較段教授所推導出來的論文在評價障礙型選擇權上的表現。

關鍵字：GARCH, 障礙型選擇權,波動度

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ABSTRACT

This paper follows Duan et al. (2005 Jumping Starting GARCH) that incorporating jumps in pricing kernel and correlated jumps in asset returns and volatilities. Since barrier options is a path dependent derivatives, incorporating jumps in the underlying assets should have some effects in it. Therefore, we investigate this issue in this paper, and we'll compare those models pricing performance.

Key words: Barrier options, GARCH, Volatilities

致　　　　　　　　謝

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最後要感謝我的家人，沒有你們就沒有今天的我。

以尊　丁亥年夏天

僅誌於　新竹交大

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1. Introduction

In recent years, the GARCH model has been increasingly used to investigate return time series and pricing options, such as Duan (1997), Hardle and Hafner (2000) etc. However, it seems that incorporating jumps into volatility is getting more and more important, since many empirical researches have showed that models which incorporate jumps into not only returns but also volatilities gain some improvement in explaining the return data on the S&P 500 index and Nasdaq 100 index (Eraker, Johannes and Polson, 2003).

Barrier options are widely traded these years by investor and hedger. However, barrier options are path dependent options, incorporating jumps in volatility may have some effect in pricing barrier options. Since Duan et al. (2005)* point out that the GARCH-Jump models show a better fit of the European options than the traditional GARCH models with normal innovations do, we use the same models Duan et al. derived to price barrier options and hope to have good pricing performance on the barrier options.

In this paper, we employ the GARCH-Jump models and the corresponding option pricing theory derived by Duan et al. (2005) to investigate the performance of these models in pricing barrier options. We also compare the results to the closed form solutions. This GARCH-Jump option pricing model is a generalization of the typical GARCH option pricing model with normal innovations constructed by Duan (1995). We test these models using Monte Carlo simulation and find that although these models are good in pricing European options, they seem to have some bias in

* This paper can be found on http://www.rotman.utoronto.ca/~jcduan/

pricing barrier options.

These new GARCH models are also interesting in their discrete time approximations. Duan et al. (2005) have derived a variety of continuous time limiting models base on the GARCH-Jump processes. When the GARCH process is curtailed, but jumps allowed, the limiting model nests the jump-diffusion model of Merton (1976). When the jumps are banned, both in return and volatilities, the limiting model can be thought to converge to continuous time stochastic volatility model. Finally, when jumps are permitted, the limiting models contain jumps and diffusive elements in both return and volatilities along the lines of Eraker, Johannes and Polson (2003) and Duffie, Singleton and Pan (1999).

The paper proceeds as follows. First, we show some setup of the pricing kernel and the dynamics of the underlying asset that derived by Duan (2005). We also show the updating schemes. Second, we introduce the Barrier options, and collect some closed form solutions of the Barrier Options shown by Paul Wilmott (1998) on Quantitative Finance. Third, we show some designs for pricing the barrier options. Fourth, we examine the pricing performance and following a conclusion.

2. The GARCH-Jump Option Pricing Model

2.1. Some Setups

Considering a discrete-time economy in a period of [0, T], and assuming that the dynamics of the asset price and the pricing kernel are:

$$S_{t-1} = E^P[S_t \frac{m_t}{m_{t-1}} | F_{t-1}] \tag{1}$$

Where, m_t is the marginal utility of consumption at date t, S_t is the total payout consisting of price and dividends, and F_t is the filtration.

We followed Duan et. al.(2005) to assume that the pricing kernel, m_t / m_{t-1} is given by:

$$\frac{m_t}{m_{t-1}} = e^{a+bJ_t} \qquad (2)$$

Where J_t is compounded Poisson random variable that is one standard normal random variables plus a Poisson random sum of normally distribution variables. That is,

$$J_t = X_t^{(0)} + \sum_{j=1}^{N_t} X_t^{(j)} \qquad (3)$$

Where

$$X_t^{(0)} \sim N(0,1)$$

$$X_t^{(j)} \sim N(\mu, \gamma^2) \text{ for } j = 1,2 \ldots.$$

N_t is a Poisson random variable with intensity λ. The random variables are independent for j=1,2,… and t=1,2,…T.

We also follows Duan(2005) to assume the asset price S_t following the process:

$$\frac{S_t}{S_{t-1}} = e^{\alpha_t + \sqrt{h_t}\bar{J}_t} \qquad (4)$$

Where, $\bar{J}_t$ is a standard normal random variable plus a Poisson random sum of normal random variables. That is,

$$\bar{J}_t = \bar{X}_t^{(0)} + \sum_{j=1}^{N_t} \bar{X}_t^{(j)} \qquad (5)$$

Where

$$\bar{X}_t^{(0)} \sim N(0,1)$$

$$\bar{X}_t^{(j)} \sim N(\bar{\mu}, \bar{\gamma}^2) \text{ for } j = 1,2 \ldots.$$

Furthermore, for t=1,2….T:

$$\mathrm{Corr}\left(X_t^{(i)}, \overline{X_\tau^{(j)}}\right) = \begin{cases} \rho & \text{if } i = j \text{ and } t = \tau \\ 0 & \text{otherwise} \end{cases}$$

and N_t is the same Poisson random variable as in the pricing kernel.

This is to mean that there are N_t shocks in the day t, and each shocks scale is determined by the Normal distribution. Therefore, given the k shocks in day t, the pricing kernel consists of a draw from the sum of $k + 1$ normal distributions, and the return of the asset are also consists $k + 1$ correlated normal distributions.

We assume that the single period continuously compounded interest rate is constant. Therefore, we can find some relationship between r and the pricing kernel:

$$E^P\left[\frac{m_t}{m_{t-1}}\middle|\mathcal{F}_{t-1}\right] = e^{-r} \tag{6}$$

$$E^P\left[\frac{m_t}{m_{t-1}}\frac{S_t}{S_{t-1}}\middle|\mathcal{F}_{t-1}\right] = 1 \tag{7}$$

These assumptions lead the dynamics of the asset price changed as the following proposition.

Proposition 1:

Under measure P, the dynamics of the asset price can be expressed as:

$$\frac{S_t}{S_{t-1}} = e^{\alpha_t + \sqrt{h_t}\bar{J}_t} \tag{8}$$

Where

$$\alpha_t = r - \frac{h_t}{2} - \sqrt{h_t}b\rho + \lambda\kappa(1 - K_t) \tag{9}$$

$$h_t = F(h_{t-i}, \bar{J}_{t-i} + b\rho; i = 1,2 \ldots) \tag{10}$$

$$\bar{J}_t = \bar{X}_t^{(0)} + \sum_{j=1}^{N_t} \bar{X}_t^{(j)}$$

$$\bar{X}_t^{(0)} \sim N(0,1) \ \text{ for } t = 1,2, \dots T$$

$$\bar{X}_t^{(j)} \sim N(\bar{\mu}, \bar{\gamma}^2) \ \text{ for } t = 1,2, \dots T, \text{and } j = 1,2, \dots$$

$$\kappa = \exp\left(b\mu + \frac{1}{2} b^2 \gamma^2\right)$$

$$K_t = \exp\left(\sqrt{h_t}(\bar{\mu} + b\rho\gamma\bar{\gamma}\right) + \frac{1}{2} h_t \bar{\gamma}^2) \tag{11}$$

In order to price the derivatives in the risk neutral measure, we have to derive the probability measure Q

$$dQ = e^{rT} \frac{m_T}{m_0} dP \tag{12}$$

Lemma 1:

(i) Q is a probability measure.

(ii) For any $\mathcal{F}_t$ measurable random vairblae, X_t :

$$X_{t-1} = E^P\left[X_t \frac{m_t}{m_{t-1}} \middle| \mathcal{F}_{t-1}\right] = e^{-r} E^Q[X_t | \mathcal{F}_{t-1}]$$

Proposition 2:†

Under measure Q, the dynamics of the asset price can be expressed as:

$$\frac{S_t}{S_{t-1}} = e^{\tilde{\alpha}_t + \sqrt{h_t}\tilde{J}_t}$$

Where Q is the local risk-neutral measure, and

$$\tilde{\alpha}_t = r - \frac{h_t}{2} + \tilde{\lambda}(1 - K_t) \tag{13}$$

† Proposition 1, Lemma 1 and Proposition 2 are derived by Duan (2005), and please find their proof in Duan's paper.

$$h_t = F(h_{t-i}, \tilde{J}_{t-i} + b\rho; i = 1,2 \dots) \quad (14)$$

$$\tilde{J}_t = \tilde{X}_t^{(0)} + \sum_{j=1}^{\tilde{N}_t} \tilde{X}_t^{(j)}$$

$$\tilde{X}_t^{(0)} \sim N(0,1) \;\; \text{for } t = 1,2,\dots T$$

$$\tilde{X}_t^{(j)} \sim N(\bar{\mu} + b\rho\gamma\bar{\gamma}, \bar{\gamma}^2) \;\; \text{for } t = 1,2,\dots T, \text{and } j = 1,2,\dots$$

$$\tilde{X}_t^{(j)} \text{ are independent} \quad \text{for } t = 1,2,\dots T, \text{and } j = 0,1,2 \dots$$

$$\widetilde{N_t} \text{ has a Poisson distribution with parameter } \tilde{\lambda} = \lambda\kappa$$

Where, K_t is the same as the equation (12).

However, under measure Q, the dynamics of the asset price have the similar form that under the data generating measure P. Under measure Q, the mean of normal distribution is shifted, but the variance is the same under both measures. The Poisson random variables are also shifted under measure Q.

2.2 The Local Variance Of the Compounded Poisson

By knowing the factor that if $W = \sum_{i=1}^{N} X_i$ where X_i is a sequence of iid random variables and W is a compounded Poisson random variable, then $E[e^{tW}] = \exp[\lambda t(\phi_x(t) - 1]$ where $\phi_x(t)$ is the moment generating function. Therefore, we can derived that

$$E[W] = \lambda\, E[X] \qquad \text{and} \qquad Var[W] = \lambda\, E[X^2]$$

Since $\overline{X_t^{(j)}}$ is normal distribution, $E[X^2] = \overline{\mu^2} + \overline{\gamma^2}$, for the sake of convenience we let

$$\hat{\gamma}^2 = \bar{\mu}^2 + \bar{\gamma}^2 \quad (15)$$

Therefore, the local variance of the logarithm returns under measure P for date t conditioning in t-1 is $h_t Var^P(\bar{J}_t) = h_t(1 + \lambda\hat{\gamma}^2)$.

Note that the expected mean of $\bar{J}_t$ under measure P is $E^P(\bar{J}_t) = \lambda\bar{\mu}$

where h_t is the local scaling factor which can be any predictable processes.

We use NGARCH and TGARCH here, and will be introduced later. However, under measure Q, the local variance is become

$$h_t Var^Q(\tilde{J}_t) = h_t(1 + \tilde{\lambda}\tilde{\gamma}^2) \qquad (16)$$

which is not equal to the local variance under measure P unless $\kappa = 1$ and $b\rho\gamma = 0$.

The expected value and the variance of $\tilde{J}_t$ under measure Q are:

$$E^Q(\tilde{J}_t) = \tilde{\lambda}\bar{\mu} + b\rho\gamma\bar{\gamma} \qquad (17)$$

$$Var^Q(\tilde{J}_t) = 1 + \tilde{\lambda}\tilde{\gamma}^2 \qquad (18)$$

2.3. Updating Schemes for the Scaling Factor

We follow the Duan (2005) to set the updating schemes to be NGARCH and TGARCH.

2.3.1 The NGARCH Model

In some papers like Christoffersen and Jacobs (2004) found that NGARCH models performed the best among many GARCH option models with normal distribution. Therefore, we choose it to be one of our updating scaling factor schemes here. However, in the empirical tests GARCH (1, 1) is good enough to describe the empirical stock price, so we use GARCH (1, 1) hereafter.

The NGARCH form:

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 h_{t-1}\left[\frac{\bar{J}_{t-1} - \lambda\bar{\mu}}{\sqrt{1+\lambda\hat{\gamma}^2}} - c\right]^2 \qquad (19)$$

Where β_0 is positive, β_1 and β_2 are nonnegative to insure that the unconditional mean is positive. The unconditional mean of h_t is equals to $\beta_0/[1-\beta_1-\beta_2(1+c^2)]$. Therefore, the process is stationary if $\beta_1-\beta_2(1+c^2)<1$. When $\lambda=0$ this model reduces to the NGARCH-Normal process used by Duan (1995). Under measure Q the updating schemes can be written to this way below.

$$h_t=\beta_0+\beta_1 h_{t-1}+\beta_2^* h_{t-1}\left[\frac{\tilde{J}_{t-1}-(\tilde{\lambda}\bar{\mu}+b\rho\gamma\bar{\gamma})}{\sqrt{1+\tilde{\lambda}\tilde{\gamma}^2}}-c^*\right]^2 \tag{20}$$

$$\beta_2^*=\beta_2\left(\frac{1+\tilde{\lambda}\tilde{\gamma}^2}{1+\lambda\hat{\gamma}^2}\right) \tag{21}$$

$$c^*=\frac{c\sqrt{1+\lambda\hat{\gamma}^2}+\lambda\bar{\mu}-\tilde{\lambda}(\bar{\mu}+b\rho\gamma\bar{\gamma})-b\rho}{\sqrt{1+\tilde{\lambda}\tilde{\gamma}^2}} \tag{22}$$

$$\tilde{\gamma}^2=(\bar{\mu}+b\rho\gamma\bar{\gamma})^2+\bar{\gamma}^2 \tag{23}$$

$$\frac{\tilde{J}_{t-1}-(\tilde{\lambda}\bar{\mu}+b\rho\gamma\bar{\gamma})}{\sqrt{1+\tilde{\lambda}\tilde{\gamma}^2}}\sim N(0,1)\ \text{under measure Q} \tag{24}$$

2.3.2 The TGARCH Model

In Hardle and Hanfer's paper (2000) they found that simulated threshold GARCH option prices are substantially closer to observed market price than simulated GARCH prices and Black-Scholes prices, when a stock index series with a pronounced leverage effect. Therefore, the second model we considered here is TGARCH(1, 1) model:

$$\phi_t=\beta_0+\beta_1\phi_{t-1}+\beta_2\left|\frac{\tilde{J}_{t-1}-\lambda\bar{\mu}}{\sqrt{1+\lambda\hat{\gamma}^2}}\right|+\beta_3\max\left(-\frac{\tilde{J}_{t-1}-\lambda\bar{\mu}}{\sqrt{1+\lambda\hat{\gamma}^2}},0\right) \tag{25}$$

$$h_t = \phi_t^2$$

When λ = 0 this updating scheme reduces to the standard TGARCH model.

Under measure Q, this model becomes

$$\phi_t = \beta_0 + \beta_1\phi_{t-1} + \beta_2^* \left| \frac{\tilde{J}_{t-1} - (\tilde{\lambda}\bar{\mu} + b\rho\gamma\bar{\gamma})}{\sqrt{1+\tilde{\lambda}\tilde{\gamma}^2}} + q \right|$$

$$+\beta_3^* \max\left(-\frac{\tilde{J}_{t-1} - (\tilde{\lambda}\bar{\mu} + b\rho\gamma\bar{\gamma})}{\sqrt{1+\tilde{\lambda}\tilde{\gamma}^2}} - q\,, 0 \right) \qquad (26)$$

Where

$$h_t = \phi_t^2$$

$$\beta_j^* = \beta_j\sqrt{\frac{1+\tilde{\lambda}\tilde{\gamma}^2}{1+\lambda\hat{\gamma}^2}}\,,\ j=2,\,3$$

$$q = \frac{b\rho(1+\gamma\bar{\gamma}) + \bar{\mu}\lambda(\kappa-1)}{\sqrt{1+\tilde{\lambda}\tilde{\gamma}^2}}$$

Therefore, when the local scaling factor h_t follows either NGARCH or TGARCH, under measure Q, the updating schemes translates into a similar NGARH or TGARCH process.

2.4. Further Investigation of the Risk Premium

Under the physical data generating measure P, the expected total return on the underlying assets can be regarded as:

$$E^P\left[\frac{S_t}{S_{t-1}}\right] = e^{r+\eta_t}$$

Where the η_t is the risk premium and by Proposition 1 and the moment generating function of the Compounded Poisson, we can show that:

$$\eta_t = \lambda\kappa(1 - K_t) - \lambda\left(1 - e^{\bar{\mu}\sqrt{h_t} + \frac{\bar{\gamma}^2 h_t}{2}}\right) - \sqrt{h_t}b\rho \tag{27}$$

The h_t here is less than 10^{-6}, so by the Taylor's Expansion the η_t can be expressed as:

$$\eta_t = [\lambda\bar{\mu}(1 - \kappa) - b\rho(1 + \lambda\kappa\gamma\bar{\gamma})]\sqrt{h_t} + \lambda\bar{\gamma}^2(1 - \kappa)\frac{h_t}{2}$$

First, in order to have some intuition to these pricing model, we let $\kappa = 1$ and $\gamma = 0$ to see what will happened in the risk premium. In this case, the risk premium reduces to $-b\rho\sqrt{h_t}$, and jumps can't affect it. This is to mean that the jump risk is fully diversifiable, which is correspond to the assumption made by Merton (1976).

Second, when the $\kappa \neq 1$ and $\gamma = 0$ in the pricing kernel, the risk premium η_t is:

$$\eta_t = [\lambda\bar{\mu}(1 - \kappa) - b\rho]\sqrt{h_t} + \lambda\bar{\gamma}^2(1 - \kappa)\frac{h_t}{2}$$

The jump size $\bar{\gamma}$ affects minimally in the risk premium, and Naik and Lee (1990) who extends Merton's model to this situation that the jump risk is not diversifiable fully.

With $\kappa=1$ and $\gamma>0$, the risk premium is likely to be:

$$\eta_t = -b\rho\sqrt{h_t} - b\rho\lambda\gamma\bar{\gamma}\sqrt{h_t}$$

The risk premium here is uncertainty because of the jump size $\bar{\gamma}$ and the intensity λ.

Finally, when κ is release from 1, the impact of the intensity of the process on the risk premium becomes more complex.

The expected value of the pricing kernel which determined the interest rate fully can be derived by equation (2) and the compounded Poisson's moment generating function:

$$e^r = E^P\left[\frac{m_t}{m_{t-1}}\middle|\mathcal{F}_t\right] = e^{a+\frac{b^2}{2}+\lambda(\kappa-1)}$$

When κ=1 (i.e., $\mu = -b\gamma^2/2$), the effects of the jump in the pricing kernel will not affect interest rate. For all other values of κ, the jump process explicitly affects both the interest rate and asset price.

2.5. Nested Models

The first model considering that κ = 1 and γ = 0. In this case that the pricing kernel is $\eta_t = -b\rho\sqrt{h_t}$ and the jumps can't affect the risk premium. According to the risk premium, you can find that the jump risks are fully diversifiable. With $\beta_1 = \beta_2 = 0$ in the NGARCH updating scheme and $\beta_1 = \beta_2 = \beta_3 = 0$ in the TGARCH updating scheme the scaling factor remains constant. When the jump risk is fully diversifiable, the local scaling factor is constant, and innovations, conditional on the number of jumps are normal, the model here can be regarded as the discrete-time Merton model, or MERTON, for short.

The second model considering the same model we just mentioned, but release κ from 1 and γ from 0. In these kinds of setup here, it's the same as the jump risk is not been diversifiable and implies that the jump risk is been priced. We followed Duan et al. calls the generalized Merton model, or G-MERTON, for short.

The third set of models considering here are models with no jumps (λ=0), but with scaling factor being GARCH processes[‡]. In this setup, innovations are normal random variables, and the risk premium is $\eta_t = -b\rho\sqrt{h_t}$. If the volatility forecast scheme is NGARCH, the system is become NGARCH-Normal model. If the forecast process is TGARCH, the system is called TGARCH-Normal model. According to the Duan (1997), these two models, in the limit, give rise to an extended version of the Hull and White (1997) and Heston (1993) stochastic volatility models, respectively.

The fourth set of models considering here are models where $\kappa = 1$ and $\gamma = 0$, but the scaling factor are permitted to be GARCH processes and jumps are permitted. In these models, jumps risk is diversifiable ($\eta_t = -b\rho\sqrt{h_t}$), volatility is stochastic and innovations are not normal. These two models are referred to as the NGARCH-Restricted model and the TGARCH-Restricted model.

The final sets of models here are the most general models where jump risk is priced, scaling factor are stochastic GARCH processes, jumps are allowed and innovations are not normal. These two models are referred to as the NGARCH-Full and TGARCH-Full models.[§]

Therefore, we follow Duan et al. (2005) considering 8 models here, and summarized in Table 1.

‡ The GARCH processes are NGARCH or TGARCH.

§ Duan, Ritchken and Sun (2005) have investigated the limiting behavior of these models.

Table 1:
Taxonomy of models

Model	Restrictions	Condition
Jump models:		
(1) Merton	$\beta_1 = \beta_2 = \beta_3 = 0, \kappa = 1, \gamma = 0$	J,JD,SC,IN
(2) G-Merton	$\beta_1 = \beta_2 = \beta_3 = 0$	J,JP,SC,IN
Normal Models:		
(3) NGARCH-Normal	$\lambda = 0$	NJ,JD,SS,IN
(4) TGARCH-Normal	$\lambda = 0$	NJ,JD,SS,IN
Restricted Models:		
(5) NGARCH-Restricted	$\kappa = 1, \gamma = 0$	J,JD,SS,I
(6) TGARCH-Restricted	$\kappa = 1, \gamma = 0$	J,JD,SS,I
Full Models:		
(7) NGARCH-Full		J,JP,SS,I
(8) TGARCH-Full		J,JP,SS,I
J : Jumps are allowed	**NJ: Not allowed Jumps**	**JD: Jump risks are diversifiable**
JP: Jump risks are priced	**SC: Scaling factor are constant**	**SS: Scaling factors are stochastic**
I: Innovations are not Normal	**IN: Innovations are normal**	

3. Barrier Options

Barrier Options are path dependent options. Their payoffs are decided by the barrier and the strike price. Take an up-and-out call option for an example, it pays the payoff of max (S_T, K) if the stock price never higher than the barrier. If the stock price is higher than the barrier, the barrier option worth nothing and you get a rebate. Barrier Options are widely used in hedging and investing when the investor or hedger thought that they sure the direction of the market. There are 8 kinds of barrier options, including up-and-out call(put), up-and-in call(put), down-and-out call (put) and down-and-in call (put). Figure 1 is the comparison between vanilla call option and up-and-out call option. The closed form solutions of the 8 kinds of barrier options are listed in the table 2. Most of them are derived by Reiner & Rubinstein (1991), others are listed in Paul Wilmott (1998).

Figure 1:
The European Call Options vs Up-and-Out Call Options

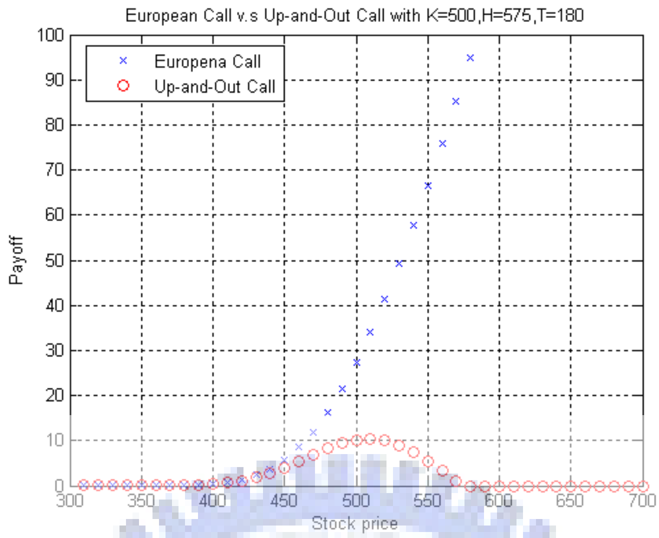


Table 2:
The Closed-form Solution of the Barrier Options

Barrier Option	Closed Form Solution
Up-and-Out Call	$Se^{-q(T-t)}\left(N(d1)-N(d3)-b\left(N(d6)-N(d8)\right)\right)-Ke^{-r(T-t)}\left(N(d2)-N(d4)-a\left(N(d5)-N(d7)\right)\right)$
Up-and-Out Put	● K>B : $Ke^{-r(T-t)}\left(1-N(d2)-a\left(N(d7)-N(d5)\right)\right)-Se^{-q(T-t)}\left(1-N(d1)-b\left(N(d8)\right)\right)$ ● K<B: $Ke^{-r(T-t)}\left(1-N(d4)-a\left(N(d7)\right)\right)-Se^{-q(T-t)}\left(1-N(d3)-b\left(N(d6)\right)\right)$
Up-and-In Call	$Se^{-q(T-t)}\left(N(d3)+b\left(N(d6)-N(d8)\right)\right)-Ke^{-r(T-t)}\left(N(d4)+a\left(N(d5)-N(d7)\right)\right)$
Up-and-In Put	● K>B : $Ke^{-r(T-t)}\left(N(d4)-N(d2)+a\left(N(d5)\right)\right)-Se^{-q(T-t)}\left(N(d3)-N(d1)+b\left(N(d6)\right)\right)$ ● K<B: $Ke^{-r(T-t)}\left(1-N(d4)-a\left(N(d5)\right)\right)-Se^{-q(T-t)}\left(1-N(d3)-b\left(N(d6)\right)\right)$
Down-and-Out Call	● K>B : $Se^{-q(T-t)}\left(N(d1)-b\left(1-N(d8)\right)\right)-Ke^{-r(T-t)}\left(N(d2)-a\left(1-N(d7)\right)\right)$ ● K<B: $Se^{-q(T-t)}\left(N(d3)-b\left(1-N(d6)\right)\right)-Ke^{-r(T-t)}\left(N(d4)-a\left(1-N(d5)\right)\right)$
Down-and-Out Put	$Ke^{-r(T-t)}\left(N(d4)-N(d2)-a\left(N(d7)-N(d5)\right)\right)-Se^{-q(T-t)}\left(N(d3)-N(d1)-b\left(N(d8)-N(d6)\right)\right)$
Down-and-In Call	● K>B : $Se^{-q(T-t)}\left(b\left(1-N(d8)\right)\right)-Ke^{-r(T-t)}\left(a\left(1-N(d7)\right)\right)$ ● K<B: $Se^{-q(T-t)}\left(N(d1)-N(d3)+b\left(1-N(d6)\right)\right)-Ke^{-r(T-t)}\left(N(d2)-N(d4)+a\left(1-N(d5)\right)\right)$
Down-and-In Put	$Ke^{-r(T-t)}\left(1-N(d4)+a\left(N(d7)-N(d5)\right)\right)-Se^{-q(T-t)}\left(1-N(d3)+b\left(N(d8)-N(d6)\right)\right)$

Where S is the initial stock price, r is the risk free rate, q is the dividend yield, T is time to maturity, K is the strike price, B is the Barrier, σ is the volatility $a = (\frac{B}{S})^{-1+(\frac{2(r-q)}{\sigma^2})}$ $b = (\frac{B}{S})^{1+(\frac{2(r-q)}{\sigma^2})}$

$$d1 = \frac{\ln\left(\frac{S}{K}\right)+\left(r-q+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad d2 = \frac{\ln\left(\frac{S}{K}\right)+\left(r-q-\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad d3 = \frac{\ln\left(\frac{S}{B}\right)+\left(r-q+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad d4 = \frac{\ln\left(\frac{S}{B}\right)+\left(r-q-\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d5 = \frac{\ln\left(\frac{S}{B}\right)-\left(r-q-\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad d6 = \frac{\ln\left(\frac{S}{B}\right)-\left(r-q-\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad d7 = \frac{\ln\left(\frac{S\cdot E}{B^2}\right)-\left(r-q-\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad d8 = \frac{\ln\left(\frac{S\cdot E}{B^2}\right)-\left(r-q+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

4. Numerical Results

4.1. Parameters Estimation

We use the parameters that estimated by Duan et al. (2005), and these parameters are shown in Appendix. The data used by Duan et al. (2005) is estimated by MLE method covering 1991 January to 1994 December S&P 500 Index.

4.2. Pricing the Barrier Option

In this paper, we only focs on the up-and-out call option, because it's the easiest path dependent derivatives. We use Monte Carlo simulation to price the options here[**]. According to the Proposition 2 and Lemma 1, we can simulate the prices of underlying assets day by day, and derive the options price easily. Actually, as long as the Proposition 2 and Lemma 1 are correct, we can price any kinds of options or exotic options.

4.3. Delta of the Barrier Option

In addition to gain some intuition of the up-and-out call options payoff here, we still want to know the results of delta in these 8 models. Since it's too hard to have an easy form of delta, we use the finite difference method to derive delta in this paper.

4.4. Results

As can be seen from Table 4 to Table 6, we show the up-and-out call options payoff under different strike prices. According to these 3 tables, we find that prices of the up-and-out call options seem to be different between these 8 models with closed-form solution. As the time to maturity is longer and the up-and-out call option price is close to the initial stock price, it seems to overprice the barrier options

[**] The random number generator used here is Fog Agner (2004), www.agner.org/random.

comparing to the closed-form solution. Since it's difficult to see the relationship between the 8 models with closed form solution, we draw Figure 2 to find out their relationship. According to Figure 2, we can find that when the time to maturity is 30 days and the barrier is higher than 525, the pricing performance of these 8 models are closer to the closed-form solution. When the time to maturity is more than 90 days and the barrier is closer to the initial stock price, these 8 models have higher pricing errors in the barrier options price. An explanation here is that our volatility used to simulate is 0.14 when the time to maturity is 30 days in this case, the underlying assets have no chance to across the barrier and the up-and-out call reduce to the vanilla call. Since these 8 models are good in pricing European options, it should have similar price in pricing vanilla calls.

Figure 3 shows the average bias comparing to the closed-form solution and we can find that Merton model is the closest to closed-form solution, and other models are almost overpricing the up-and-out call options price comparing to the closed-form solution.

However, we can find that almost all models under pricing the up-and-out call options when the strike price is closer to the initial stock price under the European options like condition.

Because the NGARCH models, NGARCH-Normal, NGARCH-Restricted and NGARCH-full model, have similar results in pricing up-and-out call options and so does TGARCH models, TGARCH-Normal, TGARCH-Restricted and TGARCH-full model, we choose the full model, Merton and Gmerton models here to have some further researches.

First, we test their pricing performance on pricing vanilla call options. Figure 4 shows the vanilla call options payoff under different stock prices. We can find that they are similar in pricing vanilla call options excepting TGARCH-full model in pricing vanilla call options under some stock price. We test their performance on pricing up-and-out call options next. Figure 5 shows the up-and-out call options under different stock prices. Although the models we chose have similar results in pricing vanilla call options, it shows a lot of differences in pricing up-and-out call options. All these 4 models overprice the up-and-out call options, and TGARCH-full model is higher than NGARCH-full model than Gmerton model followed by Merton model.

Next, we test the sensitivity of the jump in the TGARCH-full model. Although it seems that TGARCH-full model over price the up-and-out call options, TGARCH-full model's assumption is closer to the real data in the market. Therefore, we test TGARCH-full model's sensitivity by changing its jump parameters. As can be seen in Figure 6, while the jump parameter is getting larger, the up-and-out call options prices are getting smaller. It's a reasonable result, because when the jump parameters become larger, the underlying assets should have more chance to across the barrier.

Finally, we check their performance on the delta. Figure 7 shows the delta of the up-and-out call options under different stock prices. When the time to maturity is 30 days, we find the similar delta between the 4 models, excepting the TGARCH full model. When the time to maturity is higher than 90 days, we can find the full models have large bias in delta. We can also find that the full models under-estimating the delta at first than over-estimating the delta when the stock price is large. Although these 4 models show a lot of differences in pricing up-and-out call options, they show similar delta excepting the full models. Therefore, we may use the simple models such as MERTON or GMERTON to hedge our portfolio.

5. Conclusion

In this paper, we use Duan et al. (2005) Jumping Starting GARCH that incorporates jumps in the pricing kernel and correlate jumps in returns and volatilities to price up-and-out call options. We use the lemmas and propositions derived by Duan et al. (2005) to simulate the payoff of the up-and-out call options, and found that these models are not pricing well in up-and-out call options, a simplest path dependent derivatives. At the same time, in their performance of estimating delta, Merton and GMerton model shows similar results. Therefore, full models are not adaptive to hedge the portfolio. Although using Duan's model is good in pricing vanilla call options, it shows some biases in pricing up-and-out call options. Our results are similar to Hirsa (2002), who found that regardless of the closeness of the vanilla fist to different models, prices of up-and-out call options differ noticeably. Therefore, pricing the path-dependent options such as up-and-out call by GARCH-Jump models will cause the model risk.

Table 3:

The Up-and-Out Call Options Prices for Eight Models when T=30

The table shows the price of the up-and-out call options that is simulated by Monte Carlo simulation with 1,000,000 sample paths. We simulate the up-and-out call options by the same random variables between the 8 models for the purpose of comparison. We assume the initial stock price is 500, maturity is 30 days, barrier is 575, risk free rate is 0.057, volatility is 0.14, and the strike prices are 450 to 550. The first value is the barrier option's price, and the second row is the average biases[††] that compared with the Closed-form solution.

Maturity		T=30								
Barrier	Strike	Closed-Form	Merton	G-Merton	NGARCH-Nor	NGARCH-Res	NGARCH-Full	TGARCH-Nor	TGARCH-Res	TGARCH-Full
575	450	52.0565	52.3085	52.2106	52.4824	52.4256	52.4606	52.3544	52.2948	52.3548
			0.0048	0.003	0.0082	0.0071	0.0078	0.0057	0.0046	0.0057
	475	27.9169	28.0035	27.6817	28.4128	28.3773	28.2113	27.4388	27.3406	27.3823
			0.0031	-0.0084	0.0178	0.0165	0.0105	-0.0171	-0.0206	-0.0191
	500	9.2252	9.1313	8.3725	8.9152	8.8427	8.3652	5.2126	4.4801	4.1831
			-0.0102	-0.0924	-0.0336	-0.0415	-0.0932	-0.435	-0.5144	-0.5466
	525	1.4084	1.3487	0.9988	0.89	0.8947	0.6846	0.0231	0.0076	0.003
			-0.0424	-0.2908	-0.3681	-0.3647	-0.5139	-0.9836	-0.9946	-0.9979
	550	0.0728	0.0699	0.0381	0.0224	0.0282	0.0152	0	0	0
			-0.0398	-0.4766	-0.6923	-0.6126	-0.7912	-1	-1	-1

[††] The average bias is defined as the model price minus the closed form solution and then divided by the closed form solution.

Maturity		T=30									
Barrier	Strike	Closed-Form	Merton	G-Merton	NGARCH-Nor	NGARCH-Res	NGARCH-Full	TGARCH-Nor	TGARCH-Res	TGARCH-Full
550	450	49.8488	50.7137	51.2934	51.9101	51.7118	52.065	52.3536	52.2945	52.3548
			0.0174	0.029	0.0414	0.0374	0.0445	0.0502	0.0491	0.0503
	475	26.2627	26.7987	26.9886	27.9806	27.8612	27.9127	27.4382	27.3404	24.3823
			0.0204	0.0276	0.0654	0.0609	0.0628	0.0448	0.041	-0.0716
	500	8.1246	8.3164	7.90359	8.62316	8.49425	8.1634	5.21221	4.4799	4.18305
			0.0236	-0.0272	0.0614	0.0455	0.0048	-0.3585	-0.4486	-0.4851
	525	0.8602	0.9234	0.7539	0.7376	0.7132	0.5795	0.0228	0.0075	0.003
			0.0735	-0.1236	-0.1425	-0.1709	-0.3263	-0.9735	-0.9913	-0.9965
525	450	32.857	36.1239	39.1322	39.6618	40.0183	42.074	51.7035	52.065	52.2534
			0.0994	0.191	0.2071	0.218	0.2805	0.5736	0.5846	0.5903
	475	15.0422	16.9994	18.7902	19.7165	19.9522	21.1637	26.9997	27.1858	27.3139
			0.1301	0.2492	0.3107	0.3264	0.407	0.7949	0.8073	0.8158
	500	2.6276	3.2863	3.6587	4.3243	4.3762	4.6443	4.9853	4.4002	4.1478
			0.2507	0.3924	0.6457	0.6655	0.7675	0.8973	0.6746	0.5786

Table 4:

he Up-and-Out Call Options Prices for Eight Models when T=90

The table shows the price of the up-and-out call options that is simulated by Monte Carlo simulation with 1,000,000 sample paths. We simulate the up-and-out call options by the same random variables between the 8 models for the purpose of comparison. We assume the initial stock price is 500, maturity is 90 days, barrier is 575, risk free rate is 0.057, volatility is 0.14, and the strike prices are 450 to 550. The first value is the barrier option's price, and the second row is the average biases[‡‡] that compared with the Closed-form solution.

Maturity		T=90									
Barrier	Strike	Closed-Form	Merton	G-Merton	NGARCH-Nor	NGARCH-Res	NGARCH-Full	TGARCH-Nor	TGARCH-Res	TGARCH-Full
575	450	48.9952	50.7381	52.7505	55.3889	55.2972	56.4895	56.7679	57.0709	57.2355
			0.0356	0.0766	0.1305	0.1286	0.153	0.1586	0.1648	0.1682
	475	28.505	29.6824	30.7445	33.4526	33.3902	33.6952	33.5823	33.3629	33.1389
			0.0413	0.0786	0.1736	0.1714	0.1821	0.1781	0.1704	0.1626
	500	12.855	13.5353	13.712	15.626	15.5502	15.2093	14.5964	13.5835	12.7142
			0.0529	0.0667	0.2156	0.2097	0.1831	0.1355	0.0567	-0.011
	525	3.8457	4.1789	4.0462	4.7124	4.6385	4.0423	3.6611	2.701	1.9851
			0.0866	0.0521	0.2254	0.2062	0.0511	-0.048	-0.2977	-0.4838
	550	0.4737	0.5763	0.5302	0.605	0.5873	0.4381	0.345	0.1764	0.0842
			0.2166	-0.0026	-0.0028	-0.0027	-0.0023	-0.002	-0.0014	-0.001

‡‡ The average bias is defined as the model price minus the closed form solution and then divided by the closed form solution.

Maturity		T=90								
Barrier	Strike	Closed-Form	Merton	G-Merton	NGARCH-Nor	NGARCH-Res	NGARCH-Full	TGARCH-Nor	TGARCH-Res	TGARCH-Full
550	450	35.0238	37.4957	41.0641	43.4057	43.6727	47.745	49.5278	53.1675	55.2248
			0.0706	0.1725	0.2393	0.2469	0.3632	0.4141	0.518	0.5768
	475	18.2923	19.897	22.0488	24.4771	24.6869	27.1165	28.1342	30.4193	31.6212
			0.0877	0.2054	0.3381	0.3496	0.4824	0.538	0.663	0.7287
	500	6.3749	7.1898	8.0009	9.6455	9.7533	10.612	10.9385	11.5991	11.6896
			0.1278	0.2551	0.513	0.53	0.6647	0.7159	0.8195	0.8337
	525	0.8822	1.1202	1.2338	1.6554	1.6736	1.7561	1.7742	1.6699	1.4517
			0.2698	0.3985	0.8764	0.8971	0.9906	1.0111	0.8929	0.6455
525	450	14.4125	16.6894	19.432	18.9635	19.301	23.0088	24.0195	29.0013	33.5756
			0.158	0.3483	0.3158	0.3392	0.5964	0.6666	1.0122	1.3296
	475	5.6046	6.7735	8.1633	8.4988	8.7177	10.7	11.2155	14.2173	17.0216
			0.2086	0.4565	0.5164	0.5555	0.9091	1.0011	1.5367	2.0371
	500	0.8482	1.1581	1.4446	1.712	1.7809	2.2554	2.3677	3.235	4.0782
			0.3654	0.7031	1.0184	1.0996	1.659	1.7914	2.814	3.8081

Table 5:

The Up-and-Out Call Options Prices for Eight Models when T=180

The table shows the price of the up-and-out call options that is simulated by Monte Carlo simulation with 1,000,000 sample paths. We simulate the up-and-out call options by the same random variables between the 8 models for the purpose of comparison. We assume the initial stock price is 500, maturity is 180 days, barrier is 575, risk free rate is 0.057, volatility is 0.14, and the strike prices are 450 to 550. The first value is the barrier option's price, and the second row is the average biases[§§] that compared with the Closed-form solution.

Maturity		T=180								
Barrier	Strike	Closed-Form	Merton	G-Merton	NGARCH-Nor	NGARCH-Res	NGARCH-Full	TGARCH-Nor	TGARCH-Res	TGARCH-Full
575	450	36.8678	39.5887	43.8064	46.3548	46.6499	52.4465	48.1346	53.0689	57.6044
			0.0738	0.1882	0.2573	0.2653	0.4226	0.3056	0.4394	0.5625
	475	21.6156	23.4652	26.1991	28.8344	29.0888	32.7949	29.7932	33.2098	36.2077
			0.0856	0.212	0.334	0.3457	0.5172	0.3783	0.5364	0.6751
	500	10.157	11.2499	12.6254	14.7048	14.8782	16.6574	15.0556	16.8834	18.2961
			0.1076	0.243	0.4478	0.4648	0.64	0.4823	0.6622	0.8013
	525	3.2562	3.7685	4.2306	5.2936	5.3754	5.8965	5.3492	5.9746	6.2917
			0.1573	0.2992	0.6257	0.6508	0.8109	0.6428	0.8348	0.9322
	550	0.429	0.5656	0.631	0.8675	0.8828	0.9437	0.8606	0.9498	0.9518
			0.3184	0.4709	1.0221	1.0578	1.1998	1.0061	1.214	1.2186

[§§] The average bias is defined as the model price minus the closed form solution and then divided by the closed form solution.

Maturity		T=180								
Barrier	Strike	Closed-Form	Merton	G-Merton	NGARCH-Nor	NGARCH-Res	NGARCH-Full	TGARCH-Nor	TGARCH-Res	TGARCH-Full
550	450	20.9424	23.2627	26.888	27.3836	27.6595	33.5707	28.9517	33.6653	39.3246
			0.1108	0.2839	0.3076	0.3207	0.603	0.3824	0.6075	0.8778
	475	10.4585	11.8694	14.0083	14.9973	15.2137	18.8218	15.7976	18.8756	22.5656
			0.1349	0.3394	0.434	0.4547	0.7997	0.5105	0.8048	1.1576
	500	3.5371	4.1946	5.0453	5.8618	5.9801	7.5226	6.1239	7.5487	9.2621
			0.1859	0.4264	0.6572	0.6907	1.1268	0.7313	1.1341	1.6186
	525	0.4831	0.6511	0.7876	1.0296	1.0604	1.3493	1.0661	1.357	1.7049
			0.3478	0.6303	1.1312	1.195	1.793	1.2068	1.8089	2.5291
525	450	6.7981	8.1258	9.7727	9.226	9.3663	12.0998	9.6186	11.3578	13.8339
			0.1953	0.4376	0.3571	0.3778	0.7799	0.4149	0.6707	1.035
	475	2.4002	3.0056	3.7246	3.7731	3.8607	5.1674	3.9065	4.8165	6.1219
			0.2522	0.5518	0.572	0.6085	1.1529	0.6276	1.0067	1.5506
	500	0.3379	0.4794	0.6068	0.6969	0.7184	0.9966	0.711	0.921	1.2349
			0.4188	0.7958	1.0624	1.1261	1.9494	1.1042	1.7257	2.6546

Figure 2:

The Up-and-Out Call Options payoff V.S. Strike Price

This Figure presents the up-and-out call options price under different strike price. The price here is simulated by Monte Carlo Simulation with 50,000 sample paths. We use the same random variables in the simulation between these 8 models. We assume that the initial stock price is 500, the risk free rate is 0.057, and volatility is 0.14.

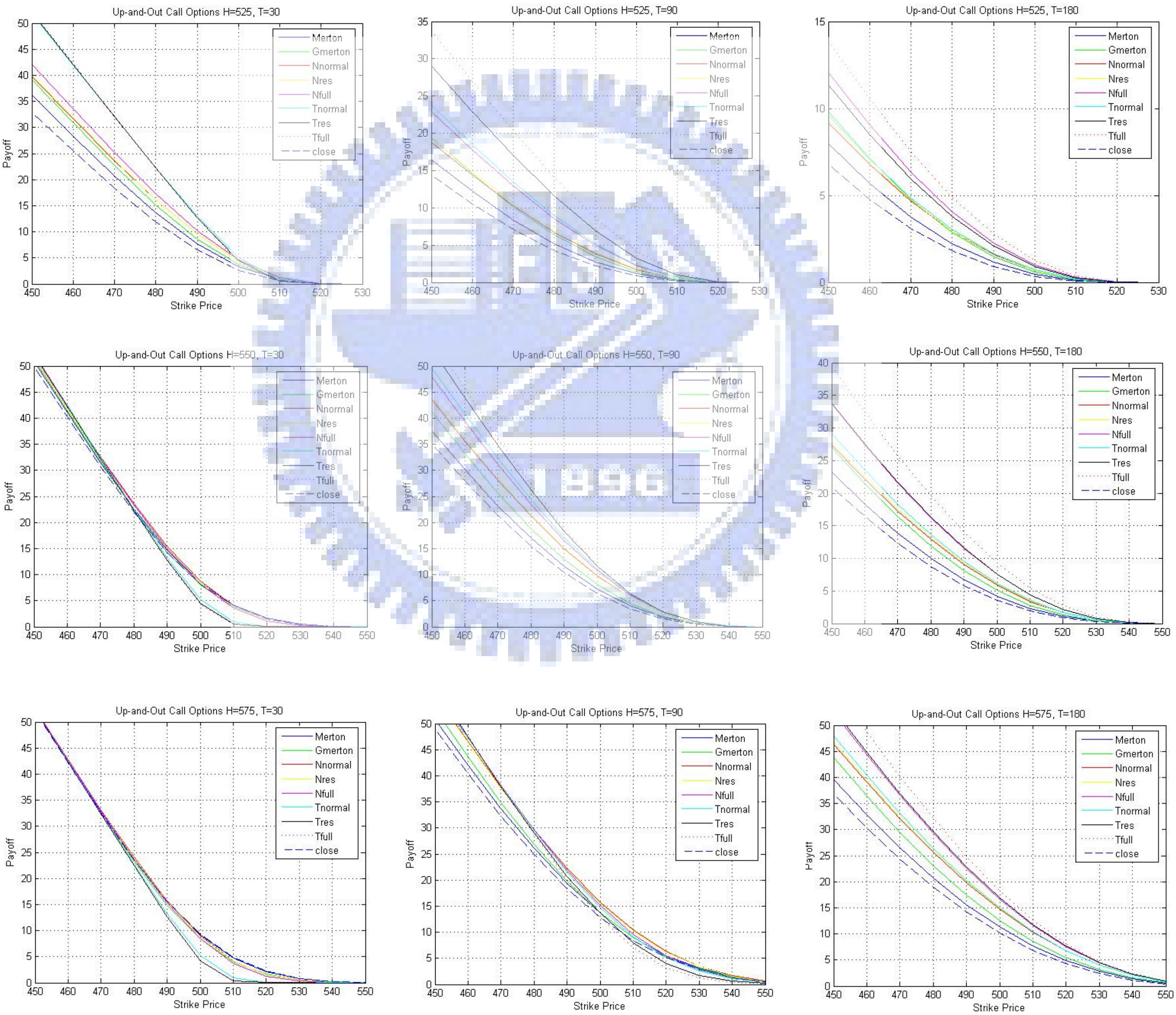


Figure 3:

The Average Bias V.S. Strike Price

This Figure presents the up-and-out call options average bias that compare to the closed-form solution. The price here is simulated by Monte Carlo Simulation with 50,000 sample paths. We use the same random variables in the simulation between these 8 models. We assume that the initial stock price is 500, the risk free rate is 0.057, and volatility is 0.14.

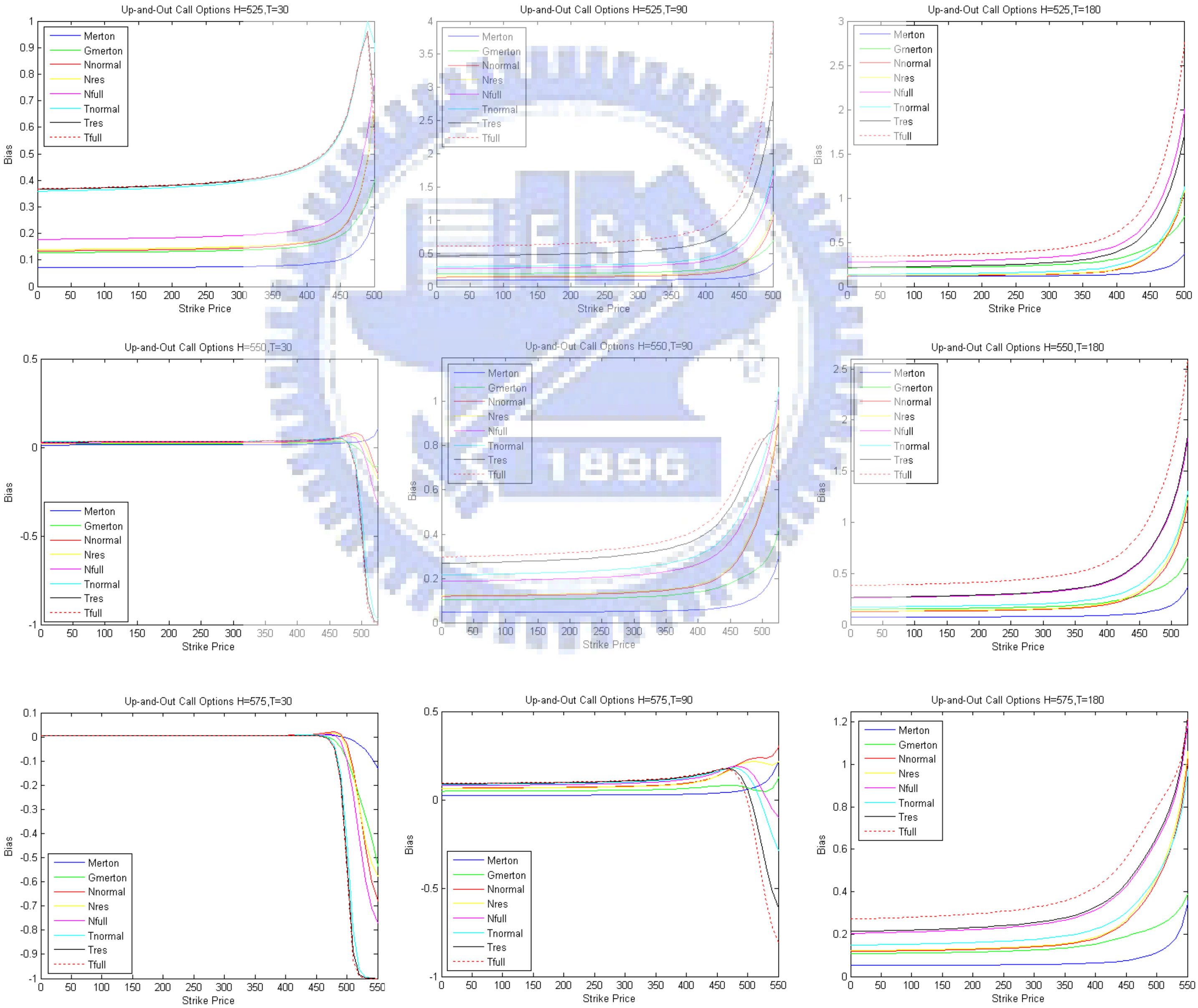


Figure 4:

European Call Options Price V.S. Stock Price

The figure shows the European call options price under different stock prices. The price here is simulated by Monte Carlo simulation with 50,000 sample paths for the Merton, Gmerton, TGARCH-full, NGARCH-full and the closed form solution. We use the same random variables in the simulation between the 4 models for the purpose of comparison. We assume the strike price is 500, risk free rate is 0.057, volatility is 0.14, and time to maturity is 30, 90 and 180 days.

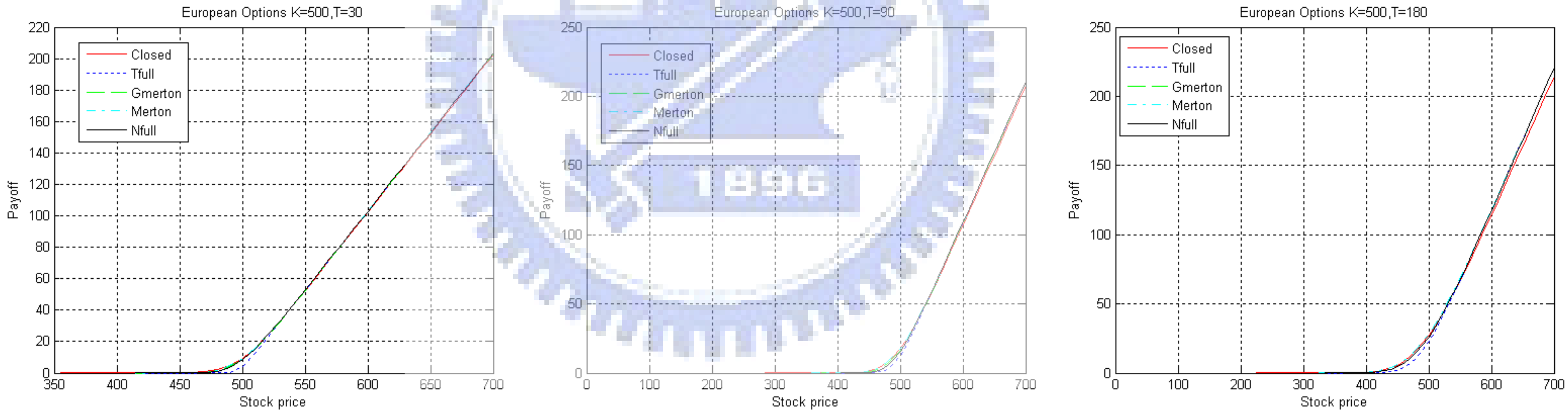


Figure 5:

Up-and-Out Call Options Price V.S. Stock Price

The figure shows the up-and-out call options price under different stock prices. The price here is simulated by Monte Carlo simulation with 50,000 sample paths for the Merton, Gmerton, TGARCH-full, NGARCH-full and the closed form solution. We use the same random variables in the simulation between the 4 models for the purpose of comparison. We assume the strike price is 500, risk free rate is 0.057, volatility is 0.14, and time to maturity is 30, 90 and 180 days.

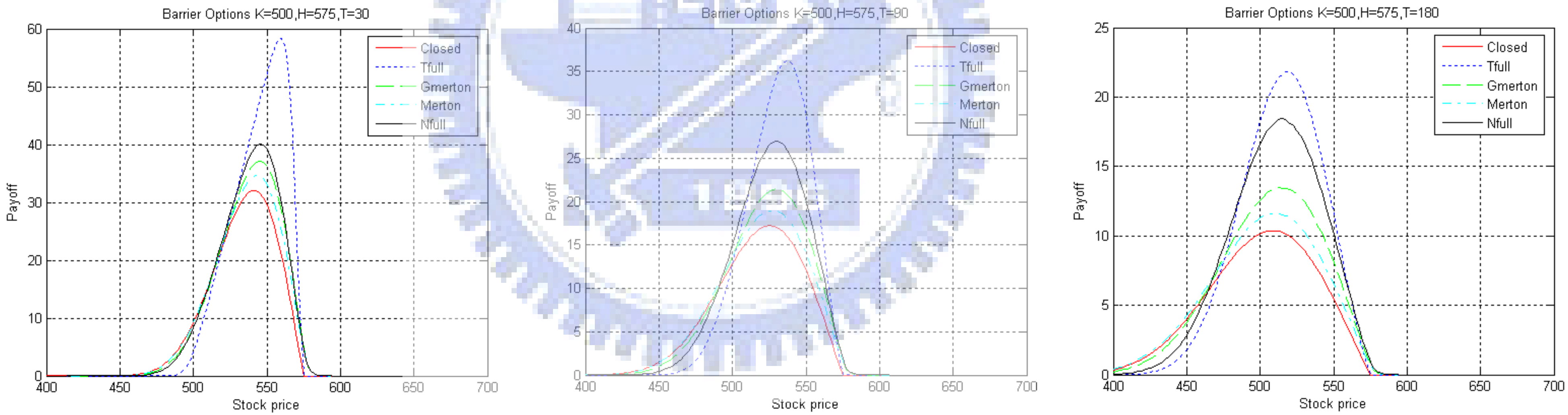


Figure 6:

Price of Barrier Options under different parameter under TGARCH-Full Model H575,K=500,T=90

The figure shows the price of the up-and-out call options price under different stock prices. We simulate the up-and-out call options by Monte Carlo simulation with 50,000 sample paths for the TGARCH-full model. We use the same random variables in the simulation for the purpose of comparison. We assume the strike price is 500, risk free rate is 0.057, barrier is 575, volatility is 0.14, and time to maturity is 90 days.

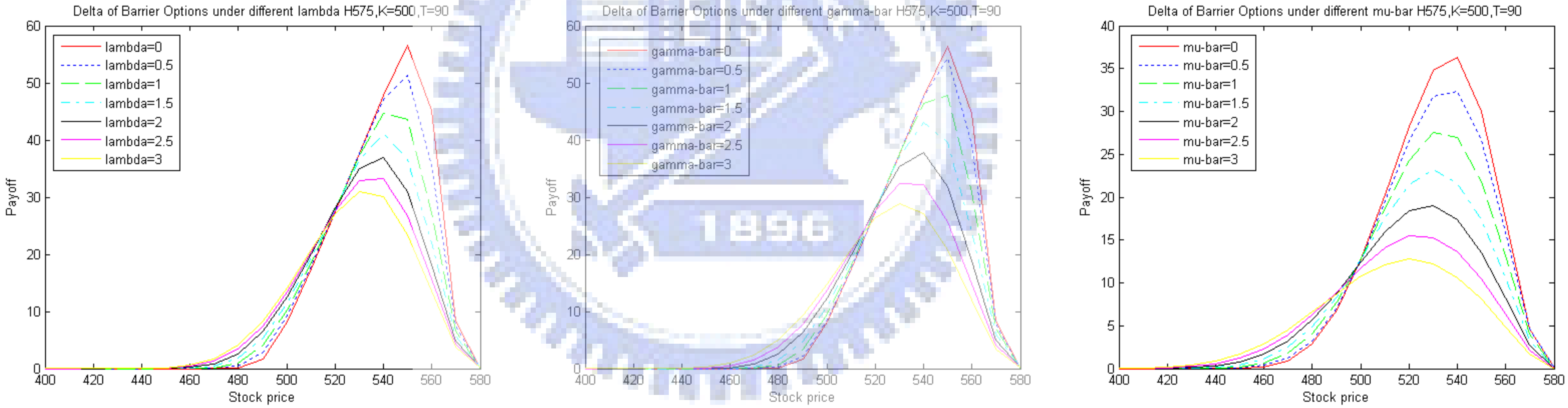
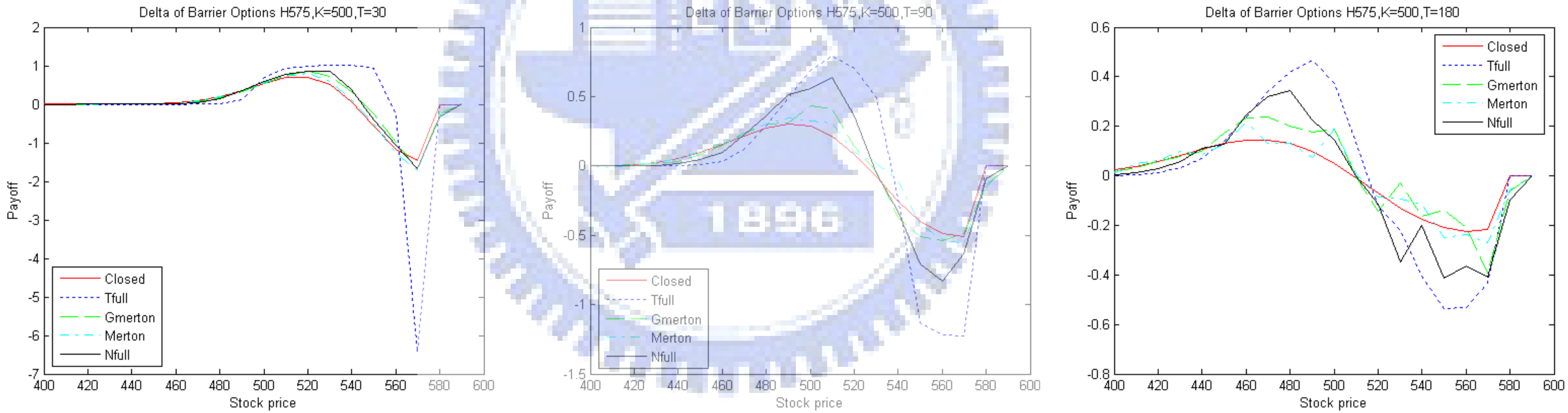


Figure 7:

Up-and-Out Call Options Delta V.S. Stock Price

The figure shows the delta of up-and-out call options under different stock prices. We estimate the delta by the finite difference method***. The price is simulated by Monte Carlo simulation with 50,000 sample paths for the Merton, Gmerton, TGARCH-full, NGARCH-full and the closed form solution. We use the same random variables in the simulation for the purpose of comparison. We assume the strike price is 500, risk free rate is 0.057, barrier is 575, volatility is 0.14, and time to maturity is 30, 90 and 180 days.



*** $\text{delta} = \frac{\partial S}{\partial C} = \frac{S(\Delta)-S(0)}{\Delta}$ and we choose Δ=0.1 here. Where S(0) is the initial stock price.

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Appendix

The Estimates for the Eight Models

These parameters are estimated by Duan et al. (2005)

	Jump		NGARCH			TGARCH		
Parameter	Merton	G-Merton	Normal	Restricted	Full	Normal	Restricted	Full
β_0	6.41E-06	6.41E-06	1.83E-06	1.65E-07	1.65E-07	-1.10E-04	-3.45E-05	-3.45E-05
β_1	-		0.84795	0.84431	0.84431	0.95765	0.96597	0.96597
β_2	-		0.07962	0.07560	0.07560	2.56E-04	5.75E-05	5.75E-05
β_3	-		-	-	-	5.09E-04	1.53E-04	1.53E-04
c	-		0.66425	0.77139	0.77139	-	-	-
λ	1.4365	1.4365	-	2.20226	2.20226	-	2.1304	2.1304
$\bar{\gamma}$	2.0705	2.0705	-	2.09608	2.0968	-	2.158	2.158
$\bar{\mu}$	0.12941	0.12941	-	0.0332	0.0332	-	0.054841	0.054841
$b\rho$	-	-0.02572	-	-0.0723	-0.01246	-0.0293	-0.0950	-0.0162
κ	-	0.9818	-	1	0.8513	-	1	0.9008
γ	-	1	-	0	1	-	0	1