Interaction Between Negative and Positive Index Medium Waveguides

Wei Yan, Linfang Shen, Yu Yuan, and Tzong Jer Yang

***Abstract*—The coupling between negative and positive index medium waveguides is investigated theoretically in this paper. A coupled mode theory is developed for such a waveguide system and its validity is verified. Interesting phenomena in the coupled waveguides are demonstrated, which occur in the case when the negative index medium waveguide in isolation guides its mode backward. A new type of coupled mode solution that varies exponentially with the coupling length is found in the special case when the propagation constants of two individual waveguides are nearly the same. A coupler operating in this case is insensitive to the coupling length, and its coupling efficiency can reach 100% as long as the coupling length is long enough. However, when the propagation constants of the two individual waveguides differ greatly, the coupled mode solution is still a periodic function of the coupling length, but the coupled power is output backward. In addition, the modes in the composite waveguide system are also studied using the coupled mode theory, and their fundamental properties are revealed.**

***Index Terms*—Backward waves, coupled mode theory, negative index media (NIM), waveguides.**

I. Introduction

NEGATIVE INDEX MEDIA (NIM) that have simultaneously negative permittivity and permeability, have attracted intensive interest recently. These media exhibit several extraordinary effects such as negative refraction, backward waves, and evanescent wave amplification [1]–[3], seeming to challenge several concepts well established for familiar positve index media (PIM) in electromagnetism and optics. NIM materials are not generally found in nature and, thus, need to be artificially constructed. Though the concept of NIM was already proposed by Vesalogo in 1968 [1], it was not until 2001 that the first experimental demonstration of negative index behavior was accomplished by Shelby *et al.*, with a material made by a 2-D array of repeated unit cells of copper strips and split-ring resonators [4]. So far, a variety of NIM structures have been proposed [4]–[8] in the microwave regime. More recently, NIM structures operating in the infrared and visible frequency range have also been reported [9]–[13], e.g., a NIM formed by an array of pairs of parallel gold rods was demonstrated to have a negative refraction index at a wavelength of 1.5 μm [10].

While the study of various NIM structures has been a subject of growing interest during the past few years, several research works have involved waveguides with NIM components, and interesting properties of guidance were reported [14]–[16]. For a NIM slab waveguide, the portion of a guided mode inside the slab has a Poynting vector contradirectional with the direction of the phase velocity of the mode, while the portion of the guided mode outside the slab has a Poynting vector parallel to the phase velocity. The total energy flow of the guided mode may be codirectional or contradirectional with the phase flow. No fundamental mode exists in a NIM slab, but it may still support a single-mode propagation under certain conditions. Moreover, the interaction between PIM and NIM waveguides was investigated in [17], and the phenomenon of anti-directional coupling was demonstrated, which offers a new possibility in the design of future devices and components. The bound modes of a NIM and PIM composite waveguide were also studied in [18], [19].

The coupling of waves in planar dielectric waveguides provides a means of reflectionless signal transfer from one waveguide to another thus plays an important role in integrated optics. Compared to the conventional waveguide system [20]–[22], the coupled NIM and PIM waveguide system receives less attention. In this paper, we will study carefully the coupling between planar NIM and PIM waveguides. For this purpose, a coupled mode theory is developed for such a waveguide system. Our analysis will show that there exist various possibilities of coupled mode solution for the NIM and PIM waveguide system, and the interaction of the guided modes of individual waveguides may even be equivalent to the interference of evanescent modes in the composite waveguide for particular cases. In Section II, the coupled mode theory for parallel NIM and PIM waveguides is formulated, and various coupled mode solutions are discussed in Section III. In Section IV, the guidance properties of a NIM and PIM composite waveguide are analyzed. Also, the validity of the coupled mode theory is examined in this section. Section V concludes the paper.

II. Coupled Mode Equations

Consider two parallel planar waveguides as illustrated in Fig. 1. The upper waveguide layer of width a is a PIM with relative permittivity $\varepsilon_a > 0$ and permeability $\mu_a > 0$,

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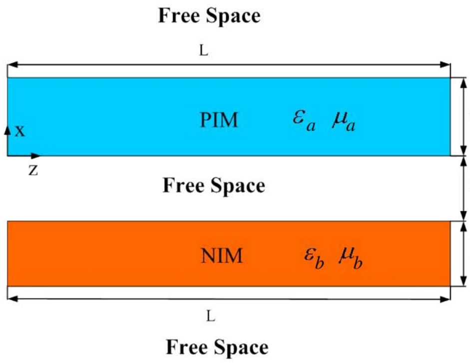

Fig. 1. Schematic of the coupled waveguide structure.

and the lower one of width b is a NIM with relative permittivity $\varepsilon_b < 0$ and permeability $\mu_b < 0$. The core layers are separated by a distance of d and they are surrounded by free space. We assume that the individual PIM waveguide supports only one guided mode with electric and magnetic fields $\{\mathbf{e}_1(x)\exp(j\beta_1 z), \mathbf{h}_1(x)\exp(j\beta_1 z)\}$ (β_1 is the propagation constant of the guided mode), whereas the individual NIM waveguide may allow multimode propagation. The modal fields of the individual NIM waveguide are denoted by $\{\mathbf{e}_m(x)\exp(j\beta_m z), \mathbf{h}_m(x)\exp(j\beta_m z)\}$ (β_m are the propagation constants of the modes), where $m = 2, 3, \cdots, N$. We consider an electromagnetic field $\{\mathbf{E}(x,z), \mathbf{H}(x,z)\}$ which satisfies Maxwell's equations plus the boundary conditions of the entire structure. Its transverse field can be written as

$$\mathbf{E}_t(x,z) = \sum_{m=1}^{N} u_m(z)\left[\mathbf{e}_{mt}(x) + \delta\mathbf{e}_m(x)\right] \tag{1}$$

$$\mathbf{H}_t(x,z) = \sum_{m=1}^{N} u_m(z)\left[\mathbf{h}_{mt}(x) + \delta\mathbf{h}_m(x)\right] \tag{2}$$

where $\delta\mathbf{e}_m$ and $\delta\mathbf{h}_m$ $(m = 1, 2, \cdots, N)$ are the corrections of the modal field in the other guide due to the induced polarization. The correction fields $\delta\mathbf{e}_m$ are introduced as in [23], and $\delta\mathbf{h}_m$ are treated in a similar manner. The longitudinal components of the vectorial field are expressed in the form

$$\mathbf{E}_z(x,z) = \sum_{m=1}^{N} u_m(z)\frac{\varepsilon_m(x)}{\varepsilon(x)}\mathbf{e}_{mz}(x) \tag{3}$$

$$\mathbf{H}_z(x,z) = \sum_{m=1}^{N} u_m(z)\frac{\mu_m(x)}{\mu(x)}\mathbf{h}_{mz}(x) \tag{4}$$

where $\varepsilon_1 = \varepsilon^{(a)}$, $\mu_1 = \mu^{(a)}$; $\varepsilon_m = \varepsilon^{(b)}$, $\mu_m = \mu^{(b)}$ for $m \geq 2$. $\{\varepsilon, \mu\}$, $\{\varepsilon^{(a)}, \mu^{(a)}\}$, and $\{\varepsilon^{(b)}, \mu^{(b)}\}$ represent the profiles of the entire structure, and of the individual PIM and NIM waveguides, respectively.

We utilize a variational principle for the coupled system under consideration following Haus *et al.* [21], and obtain the differential equations in the form

$$P\frac{d}{dz}U = -jPBU - jKU \tag{5}$$

where $U = \text{col}[u_1, u_2, \cdots, u_N]$, and $B = \text{diag}[\beta_1, \beta_2, \cdots, \beta_N]$. The matrices P and K have elements

$$P_{mn} = \bar{P}_{mn} + (\delta P_{mn} + \delta P_{nm}^*) + \Delta P_{mn} \tag{6}$$

$$K_{mn} = \bar{K}_{mn} + \delta K_{mn} + (\beta_m - \beta_n)\delta P_{mn} + \Delta K_{mn} \tag{7}$$

with

$$\bar{P}_{mn} = \int [\mathbf{e}_{mt}^* \times \mathbf{h}_{nt} + \mathbf{e}_{nt} \times \mathbf{h}_{mt}^*] \cdot \hat{z}dx \tag{8}$$

$$\begin{aligned}\bar{K}_{mn} = \omega \int [&\Delta\varepsilon_n \mathbf{e}_m^* \cdot \mathbf{e}_n - (\Delta\varepsilon_m \Delta\varepsilon_n/\varepsilon)\mathbf{e}_{zn} \cdot \mathbf{e}_{zm}^* \\ &+ \Delta\mu_n \mathbf{h}_m^* \cdot \mathbf{h}_n - (\Delta\mu_m \Delta\mu_n/\mu) \\ &\times \mathbf{h}_{zn} \cdot \mathbf{h}_{zm}^*]\, dx\end{aligned} \tag{9}$$

and

$$\delta P_{mn} = \int [\delta\mathbf{e}_n \times \mathbf{h}_{mt}^* + \mathbf{e}_{mt}^* \times \delta\mathbf{h}_n] \cdot \hat{z}dx \tag{10}$$

$$\Delta P_{mn} = \int [\delta\mathbf{e}_m^* \times \delta\mathbf{h}_n + \delta\mathbf{e}_n \times \delta\mathbf{h}_m^*] \cdot \hat{z}dx \tag{11}$$

$$\begin{aligned}\delta K_{mn} = \int [&\Delta\varepsilon_m \mathbf{e}_m^* \cdot \delta\mathbf{e}_n + \Delta\varepsilon_n \delta\mathbf{e}_m^* \cdot \mathbf{e}_n \\ &+ \Delta\mu_n \delta\mathbf{h}_m^* \cdot \mathbf{h}_n + \Delta\mu_m \mathbf{h}_m^* \cdot \delta\mathbf{h}_n]\, dx\end{aligned} \tag{12}$$

$$\begin{aligned}\Delta K_{mn} = \int [&\varepsilon\delta\mathbf{e}_m^* \cdot \delta\mathbf{e}_n + \mu\delta\mathbf{h}_m^* \cdot \delta\mathbf{h}_n + (\nabla_t \times \delta\mathbf{h}_n) \\ &\cdot \delta\mathbf{e}_m^* - (\nabla_t \times \delta\mathbf{e}_n) \cdot \delta\mathbf{h}_m^*]\, dx\end{aligned} \tag{13}$$

where $\Delta\varepsilon_m = \varepsilon - \varepsilon_m$ and $\Delta\mu_m = \mu - \mu_m$. Note that $\Delta P_{mn} = \delta K_{mn} = \Delta K_{mn} = 0$ for $m \neq n$.

The above coupled-mode equations are similar to those derived by Haus *et al.* in [23], but our formulation considers a more general case, in which the permeability of the coupled waveguides is allowed to vary with the transverse coordinate (x). Furthermore, our formulation includes the factor $\varepsilon_i/\varepsilon(\mu_i/\mu)$ in the expression for $E_z(H_z)$. We let $H = PB + K = \bar{H} + \delta H$, here $\bar{H} = \bar{P}\bar{B} + \bar{K}$, then (5) can be written as $PdU/dz = -jHU$. In the case if the waveguide system is lossless, $\bar{H}$ can be proved to be Hermitian in the same manner as in [21]. As the matrix δH has off-diagonal elements $\delta H_{mn} = \beta_m \delta P_{mn} + \beta_n \delta P_{nm}^*$; thus, the matrix H is also Hermitian for the lossless case. So the power conservation holds for our formulation [24].

III. Coupled Mode Solutions

Let us consider the wave propagation and coupling along two lossless planar waveguides over a finite length L (see Fig. 1). For simplicity, we assume that the individual NIM waveguide also supports a singled mode (i.e., $N = 2$). The single-mode operation for a certain polarization in the NIM waveguide can be achieved through suppressing the appearance of surface modes. We consider an initial power to be injected into the PIM waveguide at $z = 0$. The excited mode in the NIM waveguide will have the same direction of phase flow as the guided mode in the PIM waveguide, i.e., $\beta_2 > 0$. As the energy flows of the mode in the core and cladding of the NIM waveguide are in the opposite directions, the total energy flow may be codirectional or contradirectional with the phase velocity of the mode, i.e., $\bar{P}_{22} > 0$ or $\bar{P}_{22} < 0$. The coupled power in the NIM waveguide will be output at the waveguide end $z = L$ if $\bar{P}_{22} > 0$. Otherwise, the coupled power will be output at the other end $z = 0$, if $\bar{P}_{22} < 0$. Correspondingly, the boundary condition is $u_2 = 0$ at $z = 0$ for $\bar{P}_{22} > 0$, and it becomes $u_2 = 0$ at $z = L$ for $\bar{P}_{22} < 0$.

We rewrite (5) in the form

$$\frac{d}{dz}U = -jBU - jCU \tag{14}$$

where $C = P^{-1}K = P^{-1}H - B$, with the superscript "$-1$" denoting an inverse matrix. The coefficients C_{mm} ($m = 1, 2$) in (14) are equivalent to a modification of the parameters β_m, and our analysis shows that $|C_{mm}| \ll \beta_m$ in general. The coupling coefficients C_{12} and C_{21} are especially interesting. These coupling coefficients are given by $C_{12} = (P_{22}H_{12} - P_{12}H_{22})/(P_{11}P_{22} - P_{12}P_{21})$ and $C_{21} = (P_{11}H_{21} - P_{21}H_{11})/(P_{11}P_{22} - P_{12}P_{22})$, where $H_{11} = P_{11}\beta_1 + K_{11}$ and $H_{22} = P_{22}\beta_2 + K_{22}$. Since $|P_{12}P_{21}| \ll |P_{11}P_{22}|$, $|K_{22}| \ll |P_{22}\beta_2|$, and $P_{11} \approx \bar{P}_{11}$ we find $C_{12} \approx (H_{12} - P_{12}\beta_2)/\bar{P}_{11}$. Similarly, we obtain $C_{21} \approx (H_{21} - P_{21}\beta_1)/\bar{P}_{22}$. Note that $H_{12} = H_{21}$ and $P_{12} = P_{21}$ in the lossless case. If β_1 and β_2 differ negligibly, i.e., $\beta_1 \approx \beta_2$, then C_{12} and C_{21} have the same sign for $\bar{P}_{22} > 0$, or they have opposite signs for $\bar{P}_{22} < 0$, which never happens for coupled conventional waveguides. As an example, the coupling coefficients as a function of the NIM layer thickness b are illustrated in Fig. 2(a), and the value of $(\beta_2 - \beta_1)$ is shown in Fig. 2(b). The guided wave is assumed to be TE-polarized, and the parameters of the two-waveguide system are as follows: $\varepsilon_1 = 2.1$, $\mu_1 = 1$, and $a = 0.3035\lambda$, corresponding to the waveguide parameter $V = 1$ for the PIM waveguide; $\varepsilon_2 = -1.8$, and $\mu_2 = -1$; $d = 2a$. As seen from Fig. 2(a), in the region of $0.36\lambda < b < 0.55\lambda$ where $\bar{P}_{22} > 0$, the coupling coefficients are both positive; but they have opposite signs in the region of $1.12\lambda < b < 1.63\lambda$ where $\bar{P}_{22} < 0$. The shaded areas in Fig. 2 indicate the regions where the single-mode propagation in the individual NIM waveguide is not available or only the surface mode exists. In what follows, we concentrate on the analysis of the coupling between PIM and NIM waveguides for TE polarization (results for TM polarization can be obtained from duality). We will consider two cases for waveguide system: i) $\bar{P}_{22} < 0$; ii) $\bar{P}_{22} > 0$.

A. *Case (i):* $\bar{P}_{22} < 0$

The solution satisfying $u_1 = u_1(0)$ at $z = 0$ and $u_2 = 0$ at $z = L$ is shown to be

$$u_1(z) = A\{\psi - j\Delta \tanh[\psi(z - L)]\} e^{j\phi z} \tag{15}$$
$$u_2(z) = jAC_{21} \tanh[\psi(z - L)] e^{j\phi z} \tag{16}$$

for the case if $\Delta^2 \leq -C_{12}C_{21}$, or else it becomes

$$u_1(z) = A\{\psi \cos[\psi(z - L)] - j\Delta \sin[\psi(z - L)]\} e^{j\phi z} \tag{17}$$
$$u_2(z) = AC_{21} \sin[\psi(z - L)] e^{j\phi z} \tag{18}$$

where $\psi = \sqrt{|\Delta^2 + C_{12}C_{21}|}$, $\Delta = (\beta_1' - \beta_2')/2$, and $\phi = (\beta_1' + \beta_2')/2$ with $\beta_m' = \beta_m + C_{mm}$ ($m = 1, 2$). Since $C_{12}C_{21} < 0$ at least when $\beta_1 \approx \beta_2$ (i.e., $\Delta \approx 0$), the first type of solution, i.e., (15) and (16), is possible to occur for the case with $\bar{P}_{22} < 0$. Note that (15) and (16) are a new type of the solution for coupled mode equations, which never happens for a conventional waveguide system.

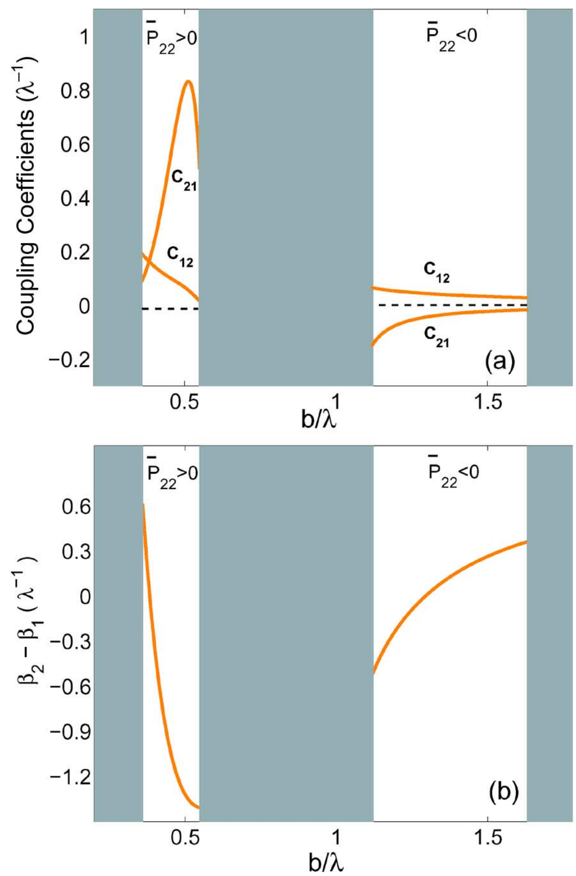

Fig. 2. (a) Coupling coefficients versus the NIM layer thickness b. (b) Propagation constant difference $(\beta_2 - \beta_1)$ versus the NIM layer thickness b. The shaded areas indicate the regions where the single-mode propagation in the individual NIM waveguide is not available.

The coupling efficiency (or the fraction of the coupled power), which describes the net power transfer between the two waveguides, is determined by $\eta = |\bar{P}_{22}|u_2(0)^2/\bar{P}_{11}u_1(0)^2$. In the case of $\Delta^2 \leq -C_{12}C_{21}$, we have $\eta = |\bar{P}_{22}|C_{21}^2/\bar{P}_{11}[\psi^2 \coth^2(\psi L) + \Delta^2]$; thus, the output power by the NIM waveguide increases as the length L increases, and it approaches its maximum of $\eta_{max} = |\bar{P}_{22}C_{21}/\bar{P}_{11}C_{12}|$ as $L > 1/\sqrt{|C_{12}C_{21}|}$. Evidently, η_{max} can reach nearly 100% when $\beta_1 \approx \beta_2$. In the other case if $\Delta^2 \geq -C_{12}C_{21}$, where β_1 and β_2 differ greatly, the coupling efficiency becomes $\eta = |\bar{P}_{22}|C_{21}^2/\bar{P}_{11}[\psi^2 \cot^2(\psi L) + \Delta^2]$. In this case, we find that η varies periodically with L, and it has a maximum of $\eta_{max} = |\bar{P}_{22}C_{21}^2/\bar{P}_{11}\Delta^2|$ at the lengths of $L = (n - 1/2)\pi/\psi$, here n is an integer. Since $\Delta^2 \geq -C_{12}C_{21}$, we have $\eta_{max} \leq |\bar{P}_{22}C_{21}/\bar{P}_{11}C_{12}|$. For this case our numerical analysis indicates that η_{max} is always less than 1. To illustrate the typical coupling behaviors, Fig. 3 shows the fraction of the coupled power for TE modes as a function of L for the waveguide systems with $b = 1.2947\lambda$ and 1.228λ. The other parameters of the waveguide systems are the same as in Fig. 2. The propagation constant of the individual NIM waveguide has $\beta_2 = \beta_1$ for $b = 1.2947\lambda$ and $\beta_2 = 0.9816\beta_1$ for $b = 1.228\lambda$,

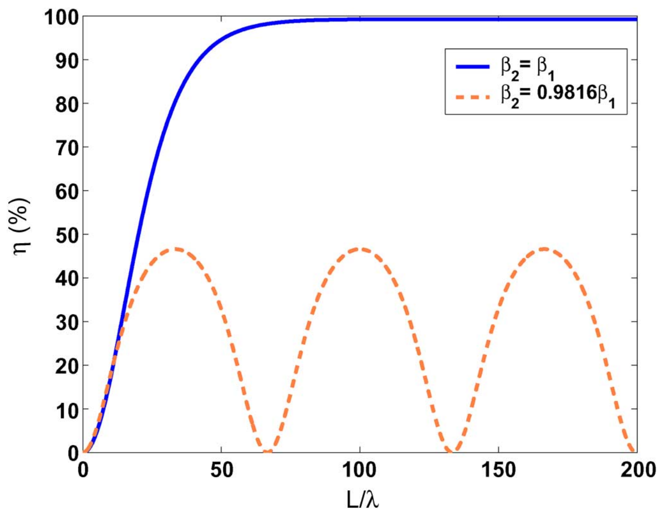

Fig. 3. Fraction of coupled power η (%) versus the coupling length L.

corresponding to two typical cases discussed above. Here, we should point out that a NIM is inherently lossy, i.e., both ϵ_b and μ_b have an imaginary part. Thus, the propagation constant (β_2) of a guided mode in the individual NIM waveguide also has an imaginary part. In the case if the NIM is severely lossy, the new type of coupled mode solution will not occur even when β_1 is equal to the real part of β_2. But such a NIM with considerable loss is not suitable for application in devices. By using low loss materials to structure NIM, it is possible to reduce the loss of a NIM to very low level at certain frequencies, which is the case of our interest.

B. *Case (ii):* $\bar{P}_{22} > 0$

The solution with $u_1 = u_1(0)$ and $u_2 = 0$ at $z = 0$ is given by

$$u_1(z) = A\left[\psi\cos(\psi z) - i\Delta\sin(\psi z)\right]e^{i\phi z} \tag{19}$$
$$u_2(z) = AC_{21}\sin(\psi z)e^{i\phi z} \tag{20}$$

where ϕ, Δ, and ψ are defined above. In this case the coupling efficiency is determined by $\eta = |\bar{P}_{22}|u_2(L)^2/\bar{P}_{11}u_1(0)^2$, and it follows that $\eta = \bar{P}_{22}C_{21}^2\sin^2(\psi L)/\bar{P}_{11}\psi^2$, which is identical to that for two parallel conventional waveguides. The maximum of the coupling efficiency is $\eta_{max} = \bar{P}_{22}C_{21}^2/\bar{P}_{11}\psi^2$, occurring at the lengths of $L = \pi(n - 1/2)\psi$, where n is an integer, and it reaches nearly 100% when β_2 is equal to β_1.

IV. Guidance of the Composite Waveguide

It is interesting to analyze the guidance characteristic of the composite two-core waveguide, as the coupling between guided modes of two individual waveguides may always be viewed as the beating of the modes of the composite structure. The propagation constants of the modes in the composite waveguide are easily derived from (14), and are given by

$$\beta_\pm = \frac{\beta_1' + \beta_2'}{2} \pm \frac{\sqrt{(\beta_1' - \beta_2')^2 + 4C_{12}C_{21}}}{2} \tag{21}$$

where β_m' $(m = 1, 2)$ are defined in the previous section. It is known that the exact dispersion relation for the planar composite waveguide can be derived analytically for both TE and TM polarizations. The electric field $(\mathbf{E})$ for TE modes in the five regions of the waveguide structure (see Fig. 1) can be written as

$$\begin{aligned}
\mathbf{E}_1 &= \hat{y}A_1\exp(-\alpha x + i\beta z), \quad x > a(\text{free space})\\
\mathbf{E}_2 &= \hat{y}\left[A_2\exp(ik_P x) + B_2\exp(-ik_P x)\right]\exp(i\beta z)\\
&\quad 0 < x \le a(\text{PIM})\\
\mathbf{E}_3 &= \hat{y}\left[A_3\exp(-\alpha x) + B_3\exp(\alpha x)\right]\exp(i\beta z)\\
&\quad -d < x \le 0(\text{free space})\\
\mathbf{E}_4 &= \hat{y}\left[A_4\exp(ik_N x) + B_4\exp(-ik_N x)\right]\exp(i\beta z)\\
&\quad -(d+b) < x \le -d(\text{NIM})\\
\mathbf{E}_5 &= \hat{y}A_5\exp(\alpha x + i\beta z), \quad x \le -(d+b)(\text{free space})
\end{aligned}$$

where $\alpha = \sqrt{\beta^2 - k_0^2}$, $k_P = \sqrt{\varepsilon_a\mu_a k_0^2 - \beta^2}$, $k_N = \sqrt{\varepsilon_b\mu_b k_0^2 - \beta^2}$, and k_0 is the wave number in free space. The nonzero components of the magnetic field $(\mathbf{H})$ can be obtained straightforwardly from $\mathbf{E}$. The dispersion relation of TE modes is determined by imposing matching conditions on the parallel components of $\mathbf{E}$ and $\mathbf{H}$ at the interfaces $x = -(b+d)$, $-d$, 0, and a. Following this procedure yields

$$\begin{aligned}
&\left[(f_1 - 1/f_1) - 2\cot(k_P a)\right]\left[(f_2 - 1/f_2) - 2\cot(k_N b)\right]e^{\alpha d}\\
&\qquad = (f_1 + 1/f_1)(f_2 + 1/f_2)e^{-\alpha d}
\end{aligned} \tag{22}$$

with $f_1 = k_P/(\alpha\mu_a)$ and $f_2 = k_N/(\alpha\mu_b)$. This equation is the exact dispersion relation for TE modes in the composite waveguide. The exact dispersion relation for TM modes can be obtained by substitution of $\mu \leftrightarrow \varepsilon$ in (22). However, it is very difficult to solve analytically the transcendental (22). In contrast, the formula (21) enables one to gain more insight into the characteristic of the waveguide system. In the following we focus on the analysis of TE modes in the composite waveguide (results for TM modes can be obtained from duality). We will show that (21) is a good approximation to (22). Without losing generality, the interacted modes in the individual PIM and NIM waveguides are assumed to have positive propagation constants; thus, the real part of the propagation constant of each related mode in the composite waveguide is also positive. All results calculated from (21) will be compared with the exact results obtained from (22). This provides an approach to examine the validity of the coupled mode theory.

We first consider the waveguide system with $\bar{P}_{22} < 0$. In the special case with $\beta_1 \approx \beta_2$, where $C_{12}C_{21} < 0$, one sees from (21) that the propagation constants have an imaginary part and be conjugate for any separation distances d. Note that both the NIM and PIM in the waveguide system are assumed to be lossless here. So these modes in the composite waveguide are evanescent waves. The appearance of the evanescent modes is only a consequence of the special coupling between the PIM and NIM waveguides, which corresponds to the new type of coupled mode solution. In the case if β_1 and β_2 differ substantially, the propagation constants may also have an imaginary part for small d, namely, when the coupling between the individual waveguides is strong. However, as d is so increased

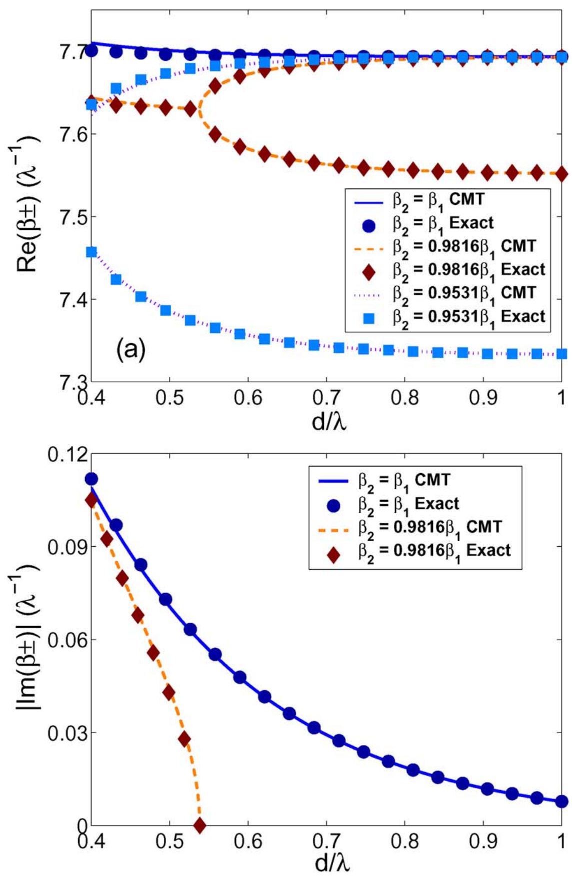

Fig. 4. Propagation constants $\beta_\pm$ versus the distance d between the core layers for the composite waveguides with various NIM layer thicknesses. The NIM waveguide in isolation guides a single mode backward ($\bar{P}_{22} < 0$) for all cases. (a) Real parts of $\beta_\pm$; (b) imaginary parts of $\beta_\pm$. The lines represent the results obtained from coupled mode theory, and the marks represent the results calculated from (22).

that $|C_{12}C_{21}| \leq (\beta_1 - \beta_2)^2$, the propagation constants will become real. In this case both modes in the composite waveguide are propagating waves. In the case if β_1 and β_2 differ greatly, the propagation constants are always real for any d. To illustrate these behaviors, Fig. 4 shows the propagation constants for the composite waveguides with different NIM layer thicknesses $b = 1.2947\lambda, 1.228\lambda$ and 1.1559λ. The other parameters are the same as in Fig. 2. The propagation constants of the individual NIM waveguide are $\beta_2 = \beta_1$, $0.9816\beta_1$, and $0.9531\beta_1$ for the three cases, respectively. In Fig. 4, the exact results (labelled by "Exact") for the three cases are also included for comparison with the results (labelled by "CMT") obtained from (21), and the excellent agreement is observed.

Next, we consider the waveguide system with $\bar{P}_{22} > 0$. In this case, our numerical analysis indicates that $C_{12}C_{21} > 0$, and the propagation constants of the modes of the composite waveguide are always real, as illustrated in Fig. 5, where the thicknesses of the NIM layers are $b = 0.3832\lambda$, 0.3895λ and 0.4007λ, corresponding to the propagation constants of the guided modes in the individual NIM waveguides $\beta_2 = \beta_1$, $0.9816\beta_1$, and $0.9531\beta_1$, respectively. The other parameters are the same as in Fig. 2. The exact results of the propagation constants for the

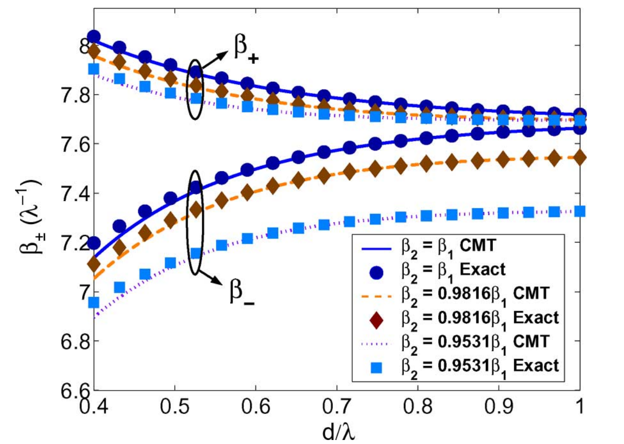

Fig. 5. Propagation constants $\beta_\pm$ versus the distance d between the core layers for the composite waveguides with various NIM layer thicknesses. The NIM waveguide in isolation guides a single mode forward ($\bar{P}_{22} > 0$) for all cases. The lines represent the results obtained from coupled mode theory, and the marks represent the results calculated from (22).

composite waveguides are also included in Fig. 5 for comparison. One sees that the results obtained from (21) are in good agreement with the exact results.

Finally we analyze qualitatively the total energy flow of the modes in the composite waveguide, which is given by $S = U^+PU$, where the superscript "+" denotes the conjugated transpose of matrix. For the waveguide system with $\bar{P}_{22} > 0$, P is a positive Hermitian matrix, which leads to $S > 0$, indicating that the modes of the composite waveguide are always forward modes, i.e., the total energy flows of the modes are codirectional with their phase flows. Therefore, the guided modes shown in Fig. 5 are all forward modes. This is verified by the exact calculation of the total energy flows of the modes in the composite waveguide. On the other hand, for the waveguide system with $\bar{P}_{22} < 0$, P is an indefinite Hermitian matrix, and it can be written as $P = R^+JR$, where $J = \mathrm{diag}[1, -1]$, and $R^+ = [\mathbf{r}_1, \mathbf{r}_2]\mathrm{diag}[|\sqrt{|\lambda_1|}, \sqrt{|\lambda_2|}]$, with λ_n and $\mathbf{r}_n$ ($n = 1, 2$) being the eigenvalues and orthogonal eigenvectors of P, respectively. Let $V = RU = \mathrm{col}[v_1, v_2]$, then the coupled mode equations are expressed as $dV/dz = jTV$, where T is an anti-Hermitian matrix, given by $T = J[R^+]^{-1}(PB+K)R^{-1} = J[R^+]^{-1}HR^{-1}$, with $T_{12} = -T_{21}^*$. We also have $S = V^+JV = |v_1|^2 - |v_2|^2 = (|v_1/v_2|^2 - 1)|v_2|^2$. Solving the coupled mode equations, we find

$$\beta_\pm = \frac{(T_{11} + T_{22}) \pm \sqrt{(T_{11} - T_{22})^2 - 4|T_{12}|^2}}{2} \tag{23}$$

and

$$|v_1/v_2| = |\xi \pm \sqrt{\xi^2 - 1}| \tag{24}$$

where $\xi = |(T_{11} - T_{22})/2T_{21}| \geq 0$. Note that it is not necessary to take the same sign simultaneously in (23) and (24). As seen from (23), when $\xi \leq 1$, the propagation constants of two modes in the composite waveguide have an imaginary part. In this case, we have $|v_1/v_2| = 1$ and then $S = 0$, which indicates that the modes in the composite waveguide are indeed evanescent waves, though they have a nonzero real part of the propagation constants. In the case when $\xi > 1$, $\beta_\pm$ are both real, i.e., the modes in the composite waveguide are both propagating waves. In this case, one has $|v_1/v_2| > 1$ and then $S > 0$, if we take "+" in (24), while $|v_1/v_2| < 1$ and then $S < 0$, if choosing "−" in (24). Thus, one of the two modes in the composite waveguide is a forward mode, and the other must be a backward mode, whose total energy flow is contradirectional with its phase flow. Since $|P_{12}|, |P_{21}| \ll |P_{11}|, |P_{22}|$ in general cases (i.e., except in the case of super strong coupling), the matrix $P \approx \mathrm{diag}[P_{11}, P_{22}]$; thus, we have approximately $R \approx \mathrm{diag}[\sqrt{|P_{11}|}, \sqrt{|P_{22}|}]$. Substituting this equation into the expression of T, we find that $T_{11} \approx \beta_1, T_{22} \approx \beta_2$. So we infer that $S > 0$ for β_+ and $S < 0$ for β_-, if $\beta_1 > \beta_2$, otherwise $S < 0$ for β_+ and $S > 0$ for β_-, if $\beta_1 < \beta_2$. Therefore, in Fig. 4, the guided modes with propagation constant β_+ are forward modes, while the ones with propagation constant β_- are backward modes. This is also verified by the exact numerical calculation of the total energy flow.

V. Discussion and Conclusions

The wave propagation and coupling in parallel planar NIM and PIM waveguides have been studied theoretically. The coupled mode equations for such a waveguide system has been developed, which has been verified through calculation of the propagation constants of the modes in the composite waveguide. It has been shown that if the NIM waveguide in isolation guides its mode forward, the properties of the NIM and PIM waveguide system are similar to those for a conventional waveguide system. However, in the case if the NIM waveguide guides its mode backward, interesting phenomena then appear. When the propagation constants of the individual waveguides are nearly equal, there exist only evanescent modes in the composite waveguide. The solution of the coupled mode equations for this case varies exponentially with the coupling length, which never happens for a conventional waveguide system. A coupler operating in this case is very easy to fabricate, as its coupling efficiency is insensitive to the coupling length. When the propagation constants β_1 and β_2 differs significantly, the modes in the composite waveguide may change from evanescent waves to propagating ones as d increases. In the latter case the solution of the coupled mode equations is also a periodic function of the coupling length, but the coupled power is output in a direction opposite to that of input power. Correspondingly, the modes in the composite waveguide are propagating waves, and one of the modes is a forward mode while the other is a backward one. Finally we should indicate that as the NIM medium is strongly dispersive, the various phenomena mentioned above may happen in the same waveguide system at different frequencies.

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