Numerical Study of Transmission Improvement in a Photonic Crystal Waveguide Bend by Mode-Matching Technique

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Abstract—**An approach to improve the transmission efficiency of photonic crystal (PC) waveguide bends through maximization of the overlap integral of fields between the straight waveguide and the waveguide bend is presented. By shifting the lattice points around the bend corner, the bound state in the waveguide bend and the guided mode in the straight waveguide are matching, and the transmission of a PC waveguide bend can be significantly improved. The transmission of a 120° PC waveguide bend with mode matching is dramatically improved from 5.7% to 87.5%. The bound state in a PC waveguide bend is similar to a cavity mode; therefore, the PC waveguide bend performs a narrowband transmission. Frequency shift of the spectra for a PC waveguide bend due to this lattice shifting can also be observed.**

Index Terms—**Bound state, cavity mode, mode matching, photonic crystal (PC), transmission, waveguide.**

I. INTRODUCTION

PHOTONIC crystal (PC) waveguides provide a powerful way to manipulate and control the flow of electromagnetic waves. High transmission of light through sharp bends in PC waveguides has been demonstrated [1]. A 60° photonic band gap (PBG) waveguide bend which has near 100% efficiency at $\lambda = 1.55$ μm was observed [2]. A taper structure which is incorporated into a 60° PC bend for adiabatic mode transformation demonstrated a 90% transmission [3]. By decreasing the size of holes in the immediate vicinity of the bend, the guided mode of the bend can be shifted in frequency to that of a straight waveguide that allows the light of PC waveguides with very low group velocities to be guided around corners [4]. Transmission spectra of a nonresonant PC bend and a cavity-resonant PC bend were measured [5]. The cavity-resonant bend was shown to have a narrowband transmission with a low insertion loss. However, flat dispersion of defect modes results in a very narrow guiding bandwidth. Thus, there have been many bend structures proposed for bandwidth improvement [6]. By making the waveguide become single mode at the PC bend, the bandwidth of transmission can be improved [7]. Significant performance improvements of PC waveguide bends have been numerically and experimentally demonstrated by the topology optimization [8], [9].

The periodicity of the PC waveguide gives rise to mode gaps between different guided modes. Such mode gaps make it possible to create bound states in a waveguide with a constriction and in bends. Bound states in PBG bends closely correspond to cavity modes [10]. In this letter, the cavity mode of a PC waveguide bend and the guided mode of a PC straight waveguide are analyzed. An approach for improving the transmission efficiency of PC waveguide bends through maximization of the overlap integral of fields between the straight waveguide and the waveguide bend is presented.

In the following paragraphs, the mode matching technique for highly efficient transmission in a 60° PC waveguide bend will be presented. By shifting the lattice points around a waveguide bend, the transmission of a 120° PC waveguide bend can also be significantly improved. Finally, the transmission spectrum of this offset PC waveguide bend will be discussed. The numerical simulations are calculated by the two-dimensional (2-D) finite-difference time-domain (FDTD) method. Perfectly matched layers are applied to the four sides of the computational domain to delimit the boundary.

II. MODE MATCHING BY SHIFT OF LATTICE POINTS

The radiation losses due to the scattering at the index discontinuities along the waveguide can be significantly reduced by the mode matching technique [11]. The overlap integral is the inner product of two wave functions over space and matching can be evaluated from the overlap integral of modes. Offset is one kind of mode matching technique used at the waveguide junctions within the bend to minimize the transmission loss. The mechanism is that the relatively low refractive-index region outside the corner operates as a phase-front accelerator of the propagating mode.

We will show that the transmission of a 2-D PC waveguide bend can also be improved by the position offset of lattice points around the bend corner. The wave transmitted through a conventional 60° PC waveguide bend simulated by the FDTD method is shown in Fig. 1(a). The PC structure is a hexagonal array of air holes perforated on the matrix with refractive index $n = 3.44$. The ratio of the radius of air holes over the lattice constant (r/a)

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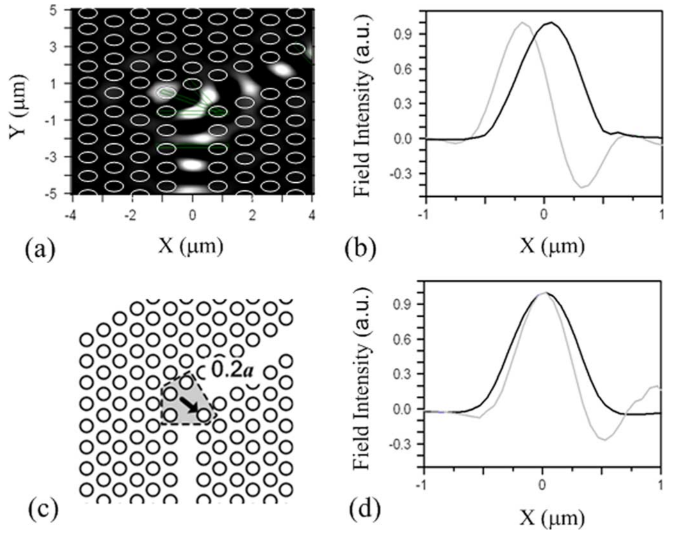

Fig. 1. (a) Wave transmitted through a conventional 60° PC waveguide bend at $f = 0.2350(a/\lambda)$. (b) Field profiles in the straight waveguide (black line) and in the bend without offset (gray line). (c) Lattice points around the bend corner are shifted $0.2a$ along the Γ-K direction indicated by the arrow. (d) Field profiles in the straight waveguide (black line) and in the bend with offset $0.2a$ (gray line).

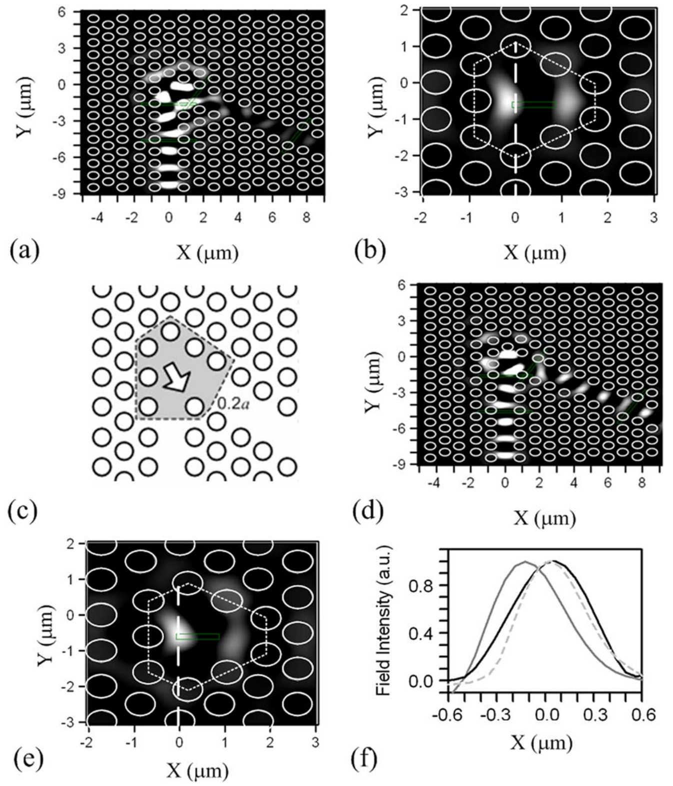

Fig. 2. (a) Wave propagation in a conventional 120° PC waveguide bend at $f = 0.2350$. (b) Field image in a cavity with three missing holes. (c) Lattice points around the bend corner are shifted $0.2a$ along the Γ-K direction indicated by the arrow. (d) Propagation of wave in the 120° PC waveguide bend with offset. (e) Field image in a three-missing- hole cavity with offset. The first annulus of lattice points around the cavity are shifted $0.2a$ along the Γ-K direction. (f) Field profiles in the straight waveguides corresponding to (a) and (d) (black line), in the three-missing-hole cavity without offset corresponding to (b) (gray line), and in the three-missing-hole cavity with offset corresponding to (e) (gray dashed line).

is 0.33. The normalized field profiles of the guided modes in the straight waveguide (black line) and in the bend (gray line) at the normalized frequency $f = 0.2350(a/\lambda)$ are shown in Fig. 1(b). The reference plane of the straight waveguide is a plane cross the waveguide within a unit cell. The reference plane in the bend is a plane along the Γ-K direction. The field profiles are taken at the same time while in the steady state. The intensities of fields are normalized. The overlap integral of these two normalized fields is 0.76. These two fields do not fully match; therefore, the transmission through this PC waveguide bend is only 72.7%.

Bent waveguides can be viewed as one finite and two semi-infinite waveguide sections joined together. Guided modes exist in these two semi-infinite sections and bound states exist in the bend section. The transmission of a PC waveguide bend can be improved by mode matching between the bound states in the waveguide bends and the guided modes in the straight waveguides. A shift of lattice points is used to design and fabricate the smallest PC laser reported to date [12], [13] and can also provide the mode matching in PCs [14]. The distance between the two peaks of fields in Fig. 1(b) is about $0.2a$. If the lattice points around the bend corner are shifted $0.2a$ along the Γ-K direction indicated by the arrow like Fig. 1(c), the normalized field profile in the straight waveguide (black line) and in the bend after adjustment (gray line) are shown in Fig. 1(d). The overlap integral of these two fields is increased to 0.92 and the transmission of this PC waveguide bend is improved to 87.5% due to mode matching. A relatively high refractive-index region exists just outside the bend corner after shifting air holes; therefore, the mechanism of transmission improvement in a PC waveguide bend by shifting the lattice points is different from that of a phase-front accelerator.

The mode matching technique can also be used to design a high efficient 120° PC waveguide bend. The propagation of a wave with $f = 0.2350(a/\lambda)$ in the 120° PC waveguide bend constructed by the material same as that in Fig. 1 is shown in Fig. 2(a). Most of the power of light is reflected backward from the bend corner; therefore, the transmission is only 5.7%. The bent region of the 120° PC waveguide bend is equivalent to a cavity with three missing holes and the cavity field image determined at the resonant frequency through Fourier analysis is shown in Fig. 2(b). It can be seen that the cavity mode does not match to the guided mode in the straight waveguide. The directions for 60° or 120° are the same in a PC structure with a hexagonal array of lattices. Therefore, if the lattice points around the bend corner are shifted $0.2a$ along the Γ-K direction indicated by the arrow like Fig. 2(c), the transmission of this 120° PC waveguide bend with offset is dramatically improved from 5.7% to 87.5%. The propagation of wave in this 120° PC waveguide bend with offset and the field image of the three-missing-hole cavity with offset are shown in Fig. 2(d) and (e), respectively. The field profiles in the straight waveguide (black line), in the three-missing-hole cavity without offset (gray line), and in the three-missing-hole cavity with offset (gray dashed line) are shown in Fig. 2(f). The reference plane of the cavity mode is a plane along the x-direction near the center of cavity. Obviously, it can be seen that the transmission improvement of this PC waveguide bend with offset is caused by mode matching.

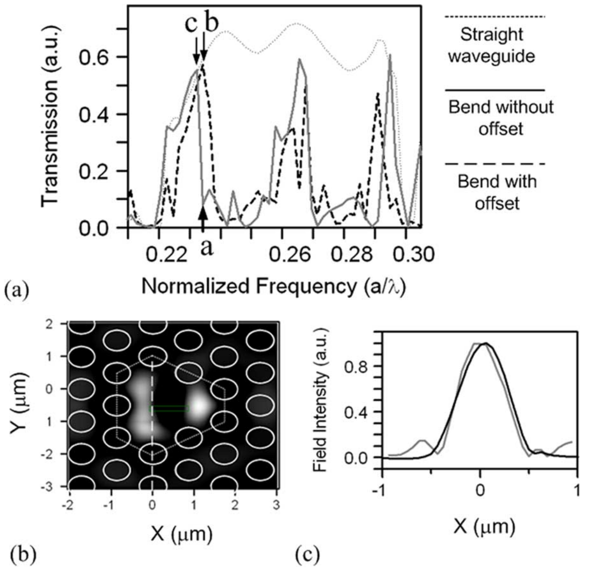

Fig. 3. (a) Transmission spectra of the straight PC waveguide, the 120° PC waveguide bend without offset, and the 120° PC waveguide bend with offset $0.2a$. Point $\mathbf{a}$ is the low transmission point of the 120° PC bend without offset at $f = 0.2350(a/\lambda)$. Point $\mathbf{b}$ is the high transmission point of the 120° PC waveguide bend with offset $0.2a$ at $f = 0.2350(a/\lambda)$. Point $\mathbf{c}$ is a high transmission point of the 120° PC bend without offset at $f = 0.2315(a/\lambda)$. (b) Field image in the three-missing-hole cavity without offset at $f = 0.2315(a/\lambda)$. (c) Field profiles in the straight waveguide (black line) and in the three-missing-hole cavity without offset (gray line) at $f = 0.2315(a/\lambda)$.

III. TRANSMISSION SPECTRA

The transmission spectra of the straight PC waveguide, the 120° PC waveguide bend without offset, and the 120° PC waveguide bend with $0.2a$ offset calculated by the fast Fourier transform (FFT) are shown in Fig. 3(a). We can see that the transmission of the straight waveguide is high within the PBG range from 0.216 to $0.307(a/\lambda)$. On the other hand, the PC waveguide bends show a narrowband transmission. Point $\mathbf{a}$ in Fig. 3(a) is the low transmission point of the 120° PC bend without offset at $f = 0.2350(a/\lambda)$ and its transmission is only 5.7%, corresponding to Fig. 2(a). Point $\mathbf{b}$ in the figure is the high transmission point of the 120° PC waveguide bend with offset $0.2a$ at $f = 0.2350(a/\lambda)$ and its value is improved to 87.5%, corresponding to Fig. 2(d). Point $\mathbf{c}$ is a high transmission point of the 120° PC bend without offset at $f = 0.2315(a/\lambda)$. The frequency shift of the spectra for a PC waveguide bend due to this lattice shifting is observed. The field image in the cavity without offset at $f = 0.2315(a/\lambda)$ is shown in Fig. 3(b). Why the transmission at point $\mathbf{c}$ is high can be seen from Fig. 3(c). The field profiles in the straight waveguide (black line) and in the three-missing-hole cavity without offset (gray line) at $f = 0.2315(a/\lambda)$ are well matched. These cases show that the transmission of a sharp PC waveguide bend can be significantly improved if the modes of the bound state in the waveguide bend and the guided mode in the straight waveguide can be matched.

IV. CONCLUSION

We have shown that a high efficiency PC waveguide bend can be designed by mode-matching technique. By shifting lattice points around the bend corner, the bound state in the waveguide bend and the guided mode in the straight waveguide can be matched. In this letter, the transmissions of the 60° and 120° PC waveguide bend with mode matching are improved from 72.7% to 87.5% and from 5.7% to 87.5%, respectively. The bound state in a waveguide bend is similar to a cavity mode; therefore, the PC waveguide bend performs a narrowband transmission. The frequency shift of the spectra for a PC waveguide bend due to this lattice shifting is observed.

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