



Vendor selection by integrated fuzzy MCDM techniques with independent and interdependent relationships

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ABSTRACT

Vendor selection is an evaluation process that is based on many criteria that uses inaccurate or uncertain data. But while the criteria are often numerous and the relationships between higher-level criteria and lower-level sub-criteria are complex, most conventional decision models cannot help us clarify the interrelationships among the sub-criteria. Our proposed integrated fuzzy multiple criteria decision making (MCDM) method addresses this issue within the context of the vendor selection problem. First, we use triangular fuzzy numbers to express the subjective preferences of evaluators. Second, we use interpretive structural modeling (ISM) to map out the relationships among the sub-criteria. Third, we use the fuzzy analytical hierarchy process (AHP) method to compute the relative weights for each criterion, and we use non-additive fuzzy integral to obtain the fuzzy synthetic performance of each common criterion. Fourth, the best vendor is determined according to the overall aggregating score of each vendor using the fuzzy weights with fuzzy synthetic utilities. Fifth, we use an empirical example to show that our proposed method is preferred to the traditional method, especially when the sub-criteria are interdependent. Finally, our results provide valuable suggestions to vendors on how to improve each sub-criterion so that they can bridge the gap between actual and aspired performance values in the future.

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1. Introduction

In today's highly competitive environment, it is impossible for a company to successfully produce low-cost, high-quality products without satisfactory vendors. The selection of appropriate vendors has long been one of the most important functions of any company's purchasing department. The vendor selection problem is an unstructured, complicated, and multi-criteria decision problem. Over the past two decades, many studies have pointed out that the key is to set effective evaluation criteria for the vendor selection problem (VSP). Earlier works on vendor selection identified 23 criteria (i.e., price, delivery, quality etc.) for evaluating and selecting appropriate vendors and for deciding on the size of the order to be placed with each vendor [13]. In 47 out of 76 articles, vendor selection used more than one criterion (i.e., multi-criteria) [39].

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A vendor selection problem usually involves more than one criterion, and criteria often conflict with each other. In multiple criteria decision making (MCDM), it is usually assumed that the criteria are independent. A considerable number of decision models have been developed based on the MCDM theory, such as preference ranking organization method (PROMETHEE) [2], analytical hierarchy process (AHP) [27,17], discrete choice analysis (DCA) [35], total cost ownership (TCO) [12], and data envelopment analysis (DEA) [40,26]. However, in real life the available information in a MCDM process is usually uncertain, vague, or imprecise, and the criteria are not necessarily independent. To tackle the vagueness in information and the essential fuzziness of human judgment/preference, fuzzy set theory was proposed by Zadeh in 1965 [42], and a decision making method in a fuzzy environment was developed by Bellman and Zadeh [1].

A number of subsequent studies used fuzzy set theory to deal with uncertainty in the vendor selection problem. Holt [19] applied seven decision methods to contractor selection. The design process pointed out the advantages and disadvantages in the vendor selection model by Morlacchi in 1999 [25]. De Boer et al. [11] provided a comprehensive review of the literature concerning supplier selection. In these papers, fuzzy set theory was suggested as a way to improve upon the vendor selection problem. Mikhailov [24] proposed the fuzzy AHP method to determine the weight of each criterion and to score each alternative for each criterion. Kumar et al. [22] presented a fuzzy goal programming approach to solve the vendor selection problem with three objectives. In order to select a suitable partner for strategic alliance, fuzzy set theory can also be used to analyze a multiplicity of complex criteria in an MCDM environment [14]. Moreover, Shyur and Shih [30] developed a hybrid MCDM method for strategic vendor selection by using both the ANP and TOPSIS techniques. In order to solve the measurement of qualitative items, an approach was developed using both quantitative and qualitative data for supplier selection [16]. In sum, fuzzy set theory is useful when the purchase situation is full of uncertainty and imprecision due to the subjectivity of human judgment. Likewise, we will use fuzzy set theory in this paper.

An MCDM problem consists of five basic elements: alternatives, criteria, outcomes, preferences, and information (see Table 1). The multiple criteria decision issue focuses mainly on the identification of the evaluation criteria and on the determination of the preference structure (i.e., weights) [33]. Previous researches on the identification of evaluation criteria in vendor selection have usually focused on products, services, and purchase situations [39,15]. However, there are often too many evaluation criteria in complex problems to determine whether these criteria are dependent on or independent to each other. One solution is to divide a complex system into groups of sub-criteria. We can then use interpretive structural modeling (ISM) to measure the interrelationship among sub-criteria more easily [36–38]. ISM is based on Boolean operations of one-to-one correspondence between a binary matrix and a graphical representation of a directed network. It is used to help identify the structural relationships among criteria in a system [23]. Here we use ISM to help us build a structural relation map to identify the independence or dependence of the sub-criteria of a criterion. We can then combine MCDM techniques with additive and non-additive models to evaluate vendors.

Furthermore, the weights represent general forms used to represent the preference structure of a decision maker. If the importance of a criterion can be properly captured through the weights, the quality of the decision making will be enhanced. Normally, the methods used to demonstrate the importance of criteria often assume additive weights and independence among criteria. But an additive model is not always suitable due to the varying degrees of interactions among the criteria. Also, decision makers may simply regard the criteria as dependent so that inevitably the decision criteria are correlated to each other. On the other hand, the fuzzy integral model does not need to assume independence among criteria, and it can be used in nonlinear situations. This is why we use the Sugeno integral for λ fuzzy measure and use a non-additive ('super-additive') fuzzy integral technique [31,21] to evaluate the synthetic performance of alternatives. These methods have been successfully applied in various circumstances [18,5–8,32,34].

We then use AHP [29] to determine the fuzzy weight of each independent criterion. However, fuzzy numbers must first be defuzzified into BNP numbers before they can be used for comparison. Thus, the defuzzification of the fuzzy weight of a criterion is done by calculating the best nonfuzzy performance (BNP) value of the final weights. The three most common defuzzification methods are mean of maximal, Center of Area (COA), and the α -cut methods [43,41,28]. But the COA method

Table 1
The five primary elements in an MCDM problem

Alternatives	Criteria				
	C_1	...	C_i	...	C_n
	w_1	...	w_i	...	w_n
A_1	f_{11}	...	f_{i1}	...	f_{n1}
\vdots	\vdots		\vdots		\vdots
A_j	f_{1j}	...	f_{ij}	...	f_{nj}
\vdots	\vdots		(Performance scores)		\vdots
A_m	f_{1m}	...	f_{im}	...	f_{nm}
Aspired levels	f_1^*	...	f_i^*	...	f_n^*

Note that A_j : the set of m alternatives we will choose from to make our decision, $j = 1, \dots, m$; C_i : the set of n criteria with which we need to make a good decision, $i = 1, \dots, n$; w_i : the weights a decision maker place on each criterion; f_{ij} : the performance scores of each choice, measured in terms of the criteria. Note that the five elements can evolve over time as situations change. Thus the dynamic change of the "information" represented by the evolution of the elements above will be treated in the decision process.

is simple and does not need to introduce the preferences of any evaluators. So we choose the COA method to transform our fuzzy weights into BNP weights. Finally, the overall score and ranking of each vendor will determine the choice of the best vendor.

We have shown that the MCDM process, particularly in a fuzzy environment, can be used to achieve the goals of practicality, accuracy, and objectivity. Our proposed method establishes an integrated fuzzy MCDM method that incorporates interrelationship and synthetic utility among the sub-criteria of a criterion within the context of the vendor selection problem. This paper is different from previously research in three ways. First, we adopt ISM to build to clarify the interrelations among the sub-criteria of a criterion. Second, we use Sugeno's fuzzy integral with fuzzy measures, a non-additive method, to calculate the synthetic utility of the weights of the interactive sub-criteria Third, the weights of each criteria can be determined using the fuzzy AHP method. And fourth, the resulting fuzzy weights of each criterion can be defuzzified using the COA method. Finally, once we obtain the overall scores for the criteria of each vendor, we can select the best vendor.

As an empirical example, we use our integrated fuzzy MCDM method to evaluate the performance of the vendors for a well-known high-tech manufacturing company in Taiwan. We show that our proposed method is an effective way for selecting an appropriate vendor, especially when there are interdependent sub-criteria in a complex hierarchy of evaluation criteria.

The rest of this paper is organized as follows. In Section 2, problems in a fuzzy environment are discussed in detail. In Section 3, some fundamentals associated with the proposed approach are addressed. In Section 4, an empirical study of the vendor selection problem in Taiwan is presented to show our proposed method. Our results are discussed and compared with those obtained using the simple additive weight (SAW) method. In Section 5, we conclude this paper with some suggestions for future research.

2. The vendor selection problem in a fuzzy MCDM environment

Vendor selection has a significant impact on a company's competitive priorities, such as price, quality, delivery, supporting services, and innovation. The decision making process is complex and usually involves vague information. This is why we study the vendor selection problem in the context of a fuzzy MCDM environment. To achieve a company's purchasing objective, the vendor selection problem may involve m candidate vendors, each denoted by V_j where $j = 1, \dots, m$, from which the best vendor is chosen. A committee is organized with $P + K$ evaluators, where P decision makers are from cross-functional divisions within the company, each denoted by M_p where $p = 1, \dots, P$, and where K experts are outsiders (i.e., consultants), each denoted by E_k where $k = 1, \dots, K$. These P decision-makers come up with and decide on a list of n criteria for vendor selection through brainstorming sessions, from which we will determine the weights of the criteria and discover the interrelations among the sub-criteria of each criterion. The K evaluators are used to evaluate the performance scores of the criteria for each vendor. The integrated fuzzy MCDM model of the vendor selection process is shown in Fig. 1. It is assumed that the top-level criteria are independent of each other (see Appendix 1.1). In this paper, the weights of all the criteria and the performance scores of each vendor are equivalent to linguistic variables, which can be measured by means of several linguistic terms represented by triangular fuzzy numbers (TFNs). The linguistic variables "very good", "good", "fair", "bad", and "very bad" correspond to a fuzzy five-level scale used by the evaluators to score each criterion as "absolutely important", "very strongly important", "essentially important", "weakly important", and "equally important", respectively (see Fig. 2). Table 2 shows the membership function for transforming a five-level linguistic variable scale into triangular fuzzy numbers.

The weight of criterion i given by the evaluator M_p is denoted by $\tilde{w}_i^p = (l_i^p, m_i^p, u_i^p)$, where $i = 1, \dots, n$, and $p = 1, \dots, P$. For criterion i , the fuzzy performance score \tilde{s}_{ij}^k of candidate vendor j in terms of TFNs given by evaluator E_k is denoted by $\tilde{s}_{ij}^k = (LE_{ij}^k, ME_{ij}^k, UE_{ij}^k)$, where $j = 1, \dots, m$, and $k = 1, \dots, K$. For each criterion, a TFN is defined as $(m_i^p - \rho, m_i^p, m_i^p + \rho)$, where m_i^p is the mean of the TFN and ρ is the spread and is a positive number. That is, the real number can be represented as intervals with equal lower and upper bounds. Its membership function is assumed to be normal (i.e., $\sup \mu_{\tilde{s}}(x) = 1$). Note that the value of ρ depends on the characteristics of the criterion. For example, the expression "quality" represents a linguistic variable of a company and it may take on values such as "weakly important". The membership functions can denote the degree of truth that a TFN is equal to a value x within the real interval $[l_i^p, u_i^p]$, and the evaluator M_p can subjectively give his/her range of the linguistic variable "weakly important" = (1,3,5). On the other hand, for the performance scores of vendor j , a TFN is defined as $(ME_{ij}^k - \varepsilon, ME_{ij}^k, ME_{ij}^k + \delta)$, where ME_{ij}^k is the mean of the TFN, ε and δ are real numbers that represent the left and right spreads, where the spreads depend on the subjective perception of the decision-maker. Let's say the company uses the yield rate to measure quality and defines a corresponding TFN (i.e., $\tilde{s}_{ij}^k = (LE_{ij}^k, ME_{ij}^k, UE_{ij}^k)$) for the yield rate on a scale of 1–100, with the evaluator M_p setting the values of ε and δ as 5 and 10, respectively. If the evaluator then gives the company's yield rate a linguistic score of "very good", that score would correspond to a TFN of (85,90,100). In other words, "very good" = (85,90,100).

3. Some fundamentals of the integrated fuzzy MCDM method

In this section, some important fundamentals that are used in the proposed method (see Section 4) are addressed. These fundamentals include the methodology used to clarify the interrelationships among the sub-criteria of a criterion, the

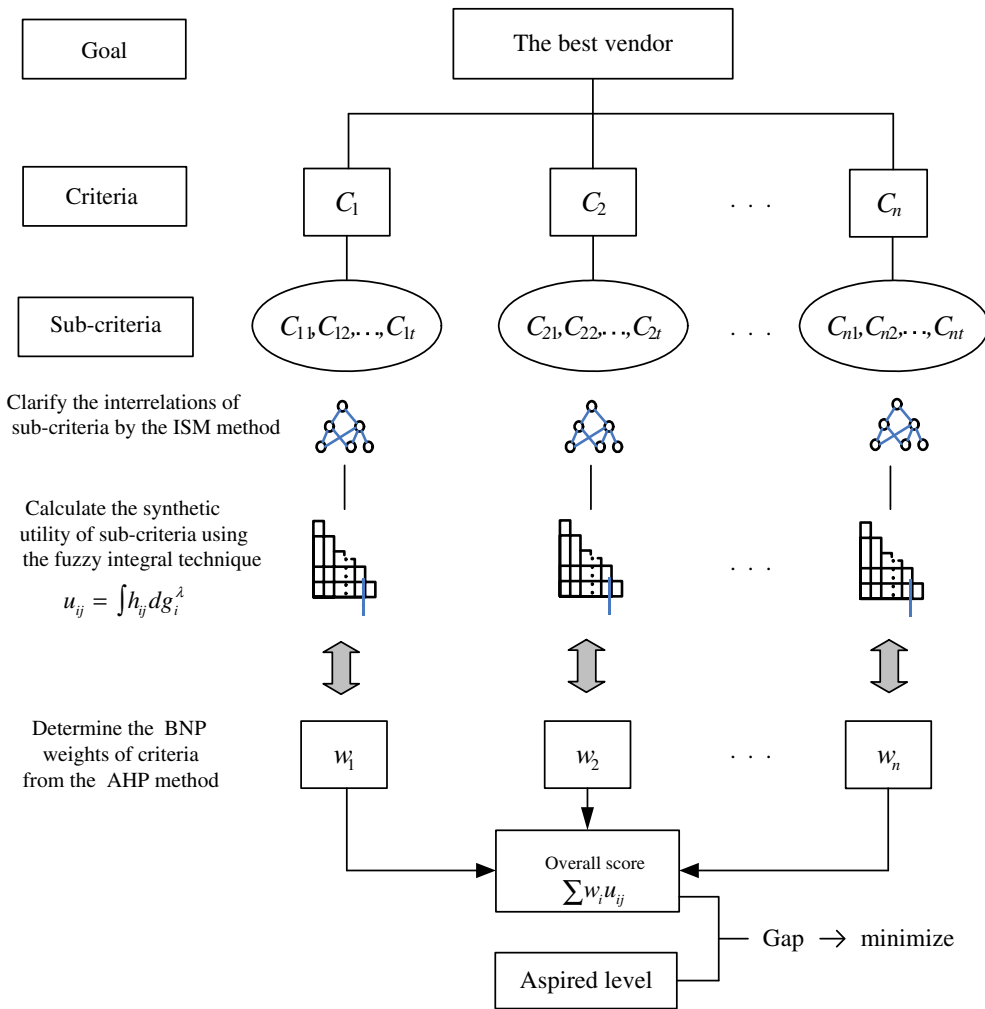


Fig. 1. The integrated evaluation model of vendor selection.

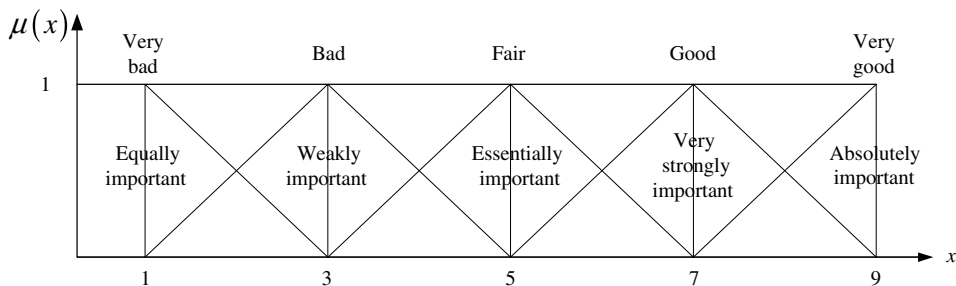


Fig. 2. Membership functions for the linguistic variable defined in this paper (an example).

concept of determining the fuzzy weight for each criterion, and the principle of calculating the synthetic utility with the interactive sub-criteria.

3.1. Using ISM to clarifying the interrelationships among sub-criteria

In a completely interdependent system, all sub-criteria of the systems are mutually related, directly or indirectly. Thus, any interference with one of the sub-criteria affects all the others. To clarify the interrelationships among the sub-criteria of

Table 2

Linguistic expression for fuzzy scale of relative weights of criteria and performance values of alternatives (an example of one judge)

Intensity of fuzzy scale (example)	Definition of linguistic variables for relative weights of criteria	Definition of linguistic variables for performance values of alternatives
$\bar{1} = (1, 1, 3)$	Equally important	Very bad
$\bar{3} = (1, 3, 5)$	Weakly important	Bad
$\bar{5} = (3, 5, 7)$	Essentially important	Fair
$\bar{7} = (5, 7, 9)$	Very strongly important	Good
$\bar{9} = (7, 9, 9)$	Absolutely important	Very good
$\bar{2}, \bar{4}, \bar{6}, \bar{8}$	Intermediate values between two adjacent judgments Used to represent compromise between the priorities listed above	

a criterion, a reachability matrix is derived from the adjacency matrix by adding the identity matrix and then raising the resulting matrix to successive powers until no new entries are obtained. For the sub-criteria of criterion i , an adjacency matrix (i.e., relation matrix) can be constructed by evaluator M_p . The general form of the adjacency matrix A can be expressed by

$$A = \begin{matrix} & C_{i1} & C_{i2} & \cdots & C_{it} \\ \begin{matrix} C_{i1} \\ C_{i2} \\ \vdots \\ C_{it} \end{matrix} & \begin{pmatrix} 0 & e_{12}^p & \cdots & e_{1t}^p \\ e_{21}^p & 0 & \cdots & e_{2t}^p \\ \vdots & \vdots & \ddots & \vdots \\ e_{t1}^p & e_{t2}^p & \cdots & 0 \end{pmatrix} & , & r = 1, \dots, t; p = 1, \dots, P, \end{matrix} \tag{1}$$

where e_{rr}^p denotes the value of the relation between the r th row and the r 'th column sub-criteria given by evaluator M_p . If the answer given by evaluator M_p for sub-criterion C_{ir} inflecting the sub-criterion C_{ir} is "Yes", then, $e_{rr}^p = 1$; otherwise, the value of $e_{rr}^p = 0$ is given. To obtain the consensus opinion of all evaluators, we use a mode method to calculate the value of the opinions of evaluator M_p for the relationships among sub-criteria in the adjacency matrix: if the majority opinion is "1", the value of the relationship for the sub-criterion is "1", which represents the sub-criteria being related. Likewise, if the majority opinion is "0", the value of the relationship for the sub-criterion is "0", which means the sub-criteria are not related; furthermore, if the majority evaluator answer is "1", this represents the intensities of different dependencies among sub-criteria. Consequently, the mostly frequent value (i.e., 0 or 1) of the comparisons among sub-criteria is called the mode. The mode method gives us the adjacency matrix A . Next, we compute the reachability matrix T by

$$T = (A + I), \tag{2}$$

$$\text{Then, when } T^l = T^{l+1}, l > 1 \text{ stops (stable reachability)}. \tag{3}$$

where I is the identity matrix, l denotes the number of times we multiply T with itself, and T^l denotes the stable reachability matrix. Subsequently, the stable reachability set (R_r) and the priority set (A_r) can be calculated based on Eqs. (4) and (5), respectively. The former includes the element of $C_{ir'}$ for all reachable sub-criteria, whereas the latter includes all sub-criteria of the reachable elements of $C_{ir'}$.

$$R_r = \{C_{ir'} | e_{rr}^p = 1\}, \tag{4}$$

$$A_r = \{C_{ir'} | e_{rr}^p = 1\}. \tag{5}$$

The hierarchy and relationships among sub-criteria can be determined using Eqs. (4) and (5). In addition, the multi-level diagram of the relationships among sub-criteria can be shown as follows:

$$R_r \cap A_r = R_r, \tag{6}$$

For example, let criterion i consist of four sub-criteria C_{i1} , C_{i2} , C_{i3} , and C_{i4} ; the values of the adjacency matrix A between sub-criteria given by evaluator M_p can thus be represented as below. Thus, the adjacency matrix is added to the identity matrix to form a tentative reachability matrix T at $l = 1$, as follows:

$$A = \begin{matrix} & C_{i1} & C_{i2} & C_{i3} & C_{i4} \\ \begin{matrix} C_{i1} \\ C_{i2} \\ C_{i3} \\ C_{i4} \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & , \text{ and } T = A + I = \begin{matrix} & C_{i1} & C_{i2} & C_{i3} & C_{i4} \\ \begin{matrix} C_{i1} \\ C_{i2} \\ C_{i3} \\ C_{i4} \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \end{matrix} \tag{7}$$

Note that the reachability matrix is calculated under the operators of the Boolean multiplication and addition law (i.e., $1 \times 1 = 1, 1 \times 0 = 0 \times 1 = 0, 0 \times 0 = 0, 1 + 1 = 1, 1 + 0 = 0 + 1 = 1$, and $0 + 0 = 0$).

Table 3
The reachability of criteria

C_{ir}	R_r	A_r	$R_r \cap A_r$
1	1,2,4	1,2,3	1,2
2	1,2,4	1,2,3	1,2
3	1,2,3,4	3	3
4	4	1,2,3,4	4

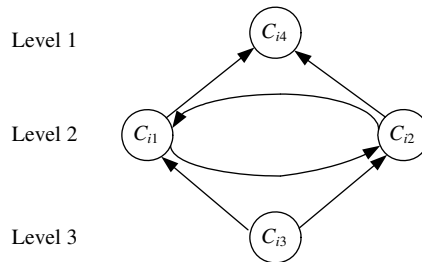


Fig. 3. The directed digraph of multi-level structures.

$$T = (A + I)^2 = \begin{matrix} & C_{i1} & C_{i2} & C_{i3} & C_{i4} \\ \begin{matrix} C_{i1} \\ C_{i2} \\ C_{i3} \\ C_{i4} \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 1^* \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1^* \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix},$$

where the asterisk * indicates the derivative relation which does not emerge in the original relation matrix (i.e., $A + I$). According to Eq. (6), we can determine that the top-level sub-criterion is sub-criterion C_{i4} in Table 3. Then, the row and column corresponding to sub-criterion C_{i4} can be deleted from matrix T . Then, repeating the above steps, the second level can be determined. Based on the reachability matrix and the multi-level diagram, the order result of the original set (i.e., $S = \{C_{i1}, C_{i2}, C_{i3}, C_{i4}\}$) that we divided into the final set (i.e., $S' = \{C_{i1}, \overline{C_{i2}}, C_{i3}, C_{i4}\}$) can be plotted as Fig. 3.

3.2. Fuzzy weights determination

To calculate the fuzzy weights of each criterion, we first define the fuzzy comparison matrix \tilde{A} . The fuzzy comparison matrix is

$$\tilde{A} = [\tilde{a}_{ir}^p]_{(n \times n)p}, \tag{7}$$

where \tilde{A} is an $n \times n$ matrix of pairwise comparisons in which \tilde{a}_{ir}^p is given the value of the pairwise comparison between the criteria in the i th row and the i 'th column by evaluator M_p (obtained from questionnaires). According to Eq. (7), pairwise comparisons of a set of n criteria for their relative importance (weights) can be denoted by $\tilde{w}^k = (\tilde{w}_1^k, \tilde{w}_2^k, \dots, \tilde{w}_n^k)$. When the criteria are independent of each other, the relative weights of vector \tilde{w} can be determined by the fuzzy AHP method as

$$(\tilde{A} - \tilde{\lambda}_{\max} I) \tilde{w} = 0, \tag{8}$$

where \tilde{w} is an $n \times 1$ vector. If this equation has a non-zero solution for \tilde{w} , then $\tilde{\lambda}_{\max}$ (which is a scalar) is an eigenvalue of \tilde{A} , and \tilde{w} is an eigenvector corresponding to $\tilde{\lambda}_{\max}$. I denotes the identity matrix, which is a diagonal matrix with the main diagonal terms equal to 1 and zero elsewhere. Thus, we use the geometric mean method to determine the tentative fuzzy weights of each criterion as follows [3,20,9,4,10]:

$$\tilde{r}_i^p = (\tilde{a}_{i1}^p \otimes \tilde{a}_{i2}^p \otimes \dots \otimes \tilde{a}_{in}^p)^{1/n}, \quad i = 1, 2, \dots, n, \tag{9}$$

$$\tilde{w}_i^p = (l_i^p, m_i^p, u_i^p) = \tilde{r}_i^p \otimes [\tilde{r}_1^p \oplus \tilde{r}_2^p \oplus \dots \oplus \tilde{r}_n^p]^{-1} \tag{10}$$

$$\tilde{w}_i = (l_i, m_i, u_i) = [\tilde{w}_i^1 \oplus \tilde{w}_i^2 \oplus \dots \oplus \tilde{w}_i^p] / P \text{ or } [\tilde{w}_i^1 \otimes \tilde{w}_i^2 \otimes \dots \otimes \tilde{w}_i^p]^{1/P} \tag{11}$$

where \tilde{a}_{in}^p is the fuzzy comparison value of criterion $i1$ to criterion in given by evaluator M_p , and \tilde{r}_i^p is the geometric mean of the fuzzy comparison values of criterion i to criterion n by evaluator M_p .

Furthermore, we use the COA defuzzification method [28] to compute the BNP weights of the criteria. The BNP weight w_i of the fuzzy weight (l_i, m_i, u_i) can be found by computing

$$w_i = l_i + [(m_i - l_i) + (u_i - l_i)]/3. \tag{12}$$

3.3. Synthetic utility with interactive criteria calculation

In this paper we use a fuzzy integral method to obtain the synthetic utilities with interdependence among sub-criteria, where the fuzzy measure (e.g., λ -fuzzy measure) is used to find the grade of importance among sub-criteria. For example, given three sub-criteria A, B and C of a criterion, their synthetic utilities could be expressed as in Fig. 4. In our proposed model, non-additive methods, where the sum between the measure of a set and the measure of its complement is not equal to the measure of the space, are used to evaluate the vendors. Unlike the traditional definition of a measure based on additive properties, the non-additive fuzzy measure and fuzzy integral are applied to evaluate problems with dependent multiple criteria.

In addition, the fuzzy measure assumes only monotonicity, and boundaries are more general than the conventional Lebesgue measures (which assume additivity). It would be more appropriate to apply a fuzzy measure to determine the grade of importance of multiple criteria in an inherently subjective evaluation process. Here, several properties of the fuzzy measure and the fuzzy integral are presented as follows:

Let A and B be sub-criteria of criterion i , assume that $X = \{x_1, x_2, \dots, x_t\}$, where $r = 1, \dots, t$, is a finite set, and let $P(X)$ denote the power set of X or the set of all subsets of X . A fuzzy measure g over a set X is a function: $g: P(X) \rightarrow [0, 1]$ such that:

$$(1) \quad g_{ir}(\emptyset) = 0, \quad g_{ir}(X) = 1, \tag{13}$$

$$(2) \quad \text{If } A, B \subset P(X) \text{ and } A \subset B, \text{ then } g_{ir}(A) \leq g_{ir}(B). \tag{14}$$

In the fuzzy measure, we adopt a λ -fuzzy measure to gauge the relationship of each sub-criterion. The λ -fuzzy measure [31] is the most widely used fuzzy measure, and it is constrained by a parameter λ , which describes the degree of addition among sub-criteria. Suppose $A, B \subset X$ with $A \cap B = \emptyset$, the λ -fuzzy measure g_λ satisfies the following additional property:

$$g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A)g_\lambda(B), \text{ for } -1 < \lambda < \infty. \tag{15}$$

Based on Eq. (14), the value of λ can be found from $g_\lambda^r(X) = 1$, and in general the fuzzy density denoted as $g_r^\lambda = g_\lambda^r(\{x_r\})$ for sub-criteria can also be obtained by

$$g_r^\lambda(\{x_1, x_2, \dots, x_t\}) = \sum_{r=1}^t g_r^\lambda + \lambda \sum_{r=1}^{t-1} \sum_{r'=r+1}^t g_r^\lambda g_{r'}^\lambda + \dots + \lambda^{t-1} g_1^\lambda g_2^\lambda \dots g_t^\lambda = \frac{1}{\lambda} \left| \prod_{r=1}^t (1 + \lambda g_r^\lambda) - 1 \right|, \tag{16}$$

Based on the above properties one of the following three cases will be discussed.

Case 1 : if $\lambda > 0$, i.e., $g_r^\lambda(A \cup B) > g_r^\lambda(A) + g_r^\lambda(B)$, this implies A and B have a multiplicative effect.

Case 2 : if $\lambda = 0$, i.e., $g_r^\lambda(A \cup B) = g_r^\lambda(A) + g_r^\lambda(B)$, this implies A and B have an additive effect.

Case 3 : if $\lambda < 0$, i.e., $g_r^\lambda(A \cup B) < g_r^\lambda(A) + g_r^\lambda(B)$, this implies A and B have a substitutive effect.

Next, let h be a measurable set function defined on the fuzzy measurable space (X, \mathfrak{N}) and suppose that $h_{ij}(x_1) \geq h_{ij}(x_2) \geq \dots \geq h_{ij}(x_t)$, where $i = 1, \dots, n, j = 1, \dots, m$, then the fuzzy integral of fuzzy measure $g_r^\lambda(\cdot)$ with respect to $h_{ij}(\cdot)$ can be defined as

$$u_{ij} = \int h_{ij} \cdot dg_r^\lambda = h_{ij}(x_t) \cdot g_r^\lambda(H_t) + [h_{ij}(x_{t-1}) - h_{ij}(x_t)] \cdot g_r^\lambda(H_{t-1}) + \dots + [h_{ij}(x_1) - h_{ij}(x_2)] \cdot g_r^\lambda(H_1) \\ = h_{ij}(x_t) \cdot [g_r^\lambda(H_t) - g_r^\lambda(H_{t-1})] + h_{ij}(x_{t-1}) \cdot [g_r^\lambda(H_{t-1}) - g_r^\lambda(H_{t-2})] + \dots + h_{ij}(x_1) \cdot g_r^\lambda(H_1), \tag{17}$$

where $H_1 = \{x_1\}$, $H_2 = \{x_1, x_2\}, \dots, H_t = \{x_1, x_2, \dots, x_t\} = X$. In addition, if $\lambda = 0$ and $g_1^\lambda = g_2^\lambda = \dots = g_t^\lambda$ then $h_{ij}(x_1) \geq h_{ij}(x_2) \geq \dots \geq h_{ij}(x_t)$ is not necessary. The basic concept of Eq. (17) can be expressed as shown in Fig. 5. Based on Eq. (17), the synthetic utility among the sub-criteria of each vendor can be obtained, which we call the unweighted score of the criterion for each vendor. A simple example will be used to show how the value of λ can be obtained as well as how the relationships among criteria, as discussed above, can be determined (see Appendix 3).

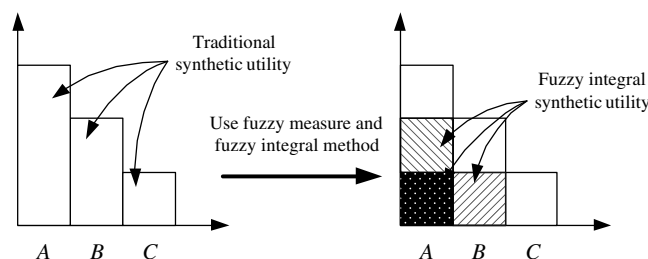


Fig. 4. Comparison between the traditional additive method and the fuzzy integral method for calculating synthetic utility.

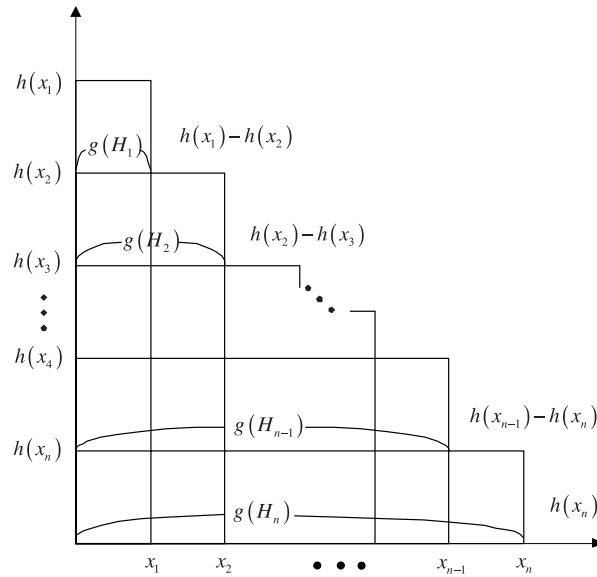


Fig. 5. The basic concept for calculating the synthetic utility using the fuzzy integral method.

4. An example of a vendor selection problem in Taiwan

In this section, we use an empirical example of a vendor selection decision to demonstrate that the integrated fuzzy MCDM technique is more appropriate than the traditional method, especially when sub-criteria are interrelated. This section is divided into four subsections: (1) problem descriptions, (2) data collection via questionnaires, (3) results and analyses, and (4) discussions.

4.1. Problem description

In recent years, high technology industries in Taiwan have grown rapidly, especially the electronic and information-technology (IT) industries. To achieve the strategic initiatives and strengthen the core competencies of a company, it is crucial to select appropriate vendors. In this section, we will use our integrated fuzzy MCDM method on the vendor selection problem of a well-known 3C component manufacturer. Its products mainly include PC enclosures, communications equipment, and consumer electronic products. In 2004, its consolidated revenue was USD 13 billion, and the company has over 100,000 employees around the world. Its customers include such famous companies as Intel, IBM, Dell, HP, Motorola, and Sony. In order to cope with business growth and sustain the company's competitive advantage, the company needs an effective vendor selection model to help it select an appropriate vendor in a new purchasing project. In line with the purchasing request (e.g., 10,000 pieces/week), which was based on scenario writing and brain storming for heat sinks for notebook personal computers in a consumer electronics business division, we will evaluate five candidate vendors ($V_1, V_2, V_3, V_4,$ and V_5). Those vendors who successfully passed the screening processes were eligible for procurement. Moreover, choosing the possible evaluation criteria for the vendor selection involves a decision making team that includes managers from different functional divisions of the case company (i.e., purchasing director, purchasing manager, quality manager, product manager and production manager). The list of criteria and sub-criteria in this vendor selection problem has been chosen based on the professionalism, knowledge, and experience of managers. The major criteria and sub-criteria involved in this vendor selection are listed in Table 4. A purchasing committee with seventeen evaluators containing seven decision-makers (M_1, M_2, \dots, M_7) and ten experts (E_1, E_2, \dots, E_{10}) was organized. Based on the criteria listed in Table 4, the ten experts scored each candidate vendor with respect to the hierarchical evaluation structure.

4.2. Data collection questionnaires

In order to clarify the interrelationships among the sub-criteria of a criterion in a hierarchical system and to determine the weights of the criteria, the decision-makers were asked to complete a questionnaire (ISM questionnaire, see Appendix 1). Through the expertise and knowledge of the decision-makers, the relationships among the sub-criteria of a criterion can be determined. Meanwhile, the participants were also asked to respond to a questionnaire containing a series of pairwise comparisons using Saaty's nine-point scale (AHP questionnaire, see Appendix 1). In this questionnaire, each question consisted of a pairwise comparison of two elements. For each pairwise comparison, the participants had to determine the level of the

Table 4
Criteria and sub-criteria for the vendor selection

Criterion	Sub-criterion
Quality (C_1)	Quality performance (C_{11}) Quality containment and VDCS feedback (C_{12})
Price & Terms (C_2)	Price (C_{21}) Terms (C_{22}) Responsiveness (C_{23}) Lead time (C_{24})
Supply chain support (C_3)	VMI/VOI hub Set Up cost (C_{25}) Purchase order reactivity (C_{31}) Capacity support & flexibility (C_{32}) Delivery/VMI operation (C_{33})
Technology (C_4)	Technical support (C_{41}) Design involvement (C_{42}) ECN/PCN process (C_{43})

relative importance between the two elements. Furthermore, a questionnaire (Appendix 2) for the evaluation by experts of the synthetic utilities of interactive sub-criteria was also administered. This questionnaire sought the satisfaction level of the experts concerning the candidate vendors using a Likert-type five-point scale. Experts were asked to evaluate different cognitive levels given each grade of candidate vendor according to subjective perceptions.

4.3. Results and analyses

Based on the fundamentals stated in Section 3, the integrated fuzzy MCDM method solves the vendor selection problem in the following steps:

Step 1: Find the interrelation among the sub-criteria of a criterion. Seven evaluators were asked to determine the relationships among the sub-criteria based on the ISM method. Here the seven evaluators assigned values of 0 (no relation) or 1 (relation) on the directed relations of each sub-criteria pair to form a single evaluator's adjacency matrix A . Then the group's adjacency matrix A was determined using the mode method to aggregate the opinion of the seven evaluators. The reachability matrix T was calculated using Eqs. (2) and (3). Based on Eqs. (4) and (5), the stabilized reachability matrix T' could be derived. The hierarchies among the sub-criteria of each criterion could then be plotted as in Fig. 6.

Step 2: Determine the weights of the evaluated criteria. Our next goal was to find the fuzzy weights for the criteria using fuzzy AHP. Based on the fuzzy five-level scale (see Fig. 2), the four evaluation criteria in the vendor selection problem are *quality* (C_1), *price and terms* (C_2), *supply chain support* (C_3), and *technology* (C_4). The seven evaluators also filled out our AHP questionnaire. Their subjective judgments were integrated sequentially to obtain the fuzzy weights of criteria using Eq. (10). The fuzzy weights and the BNP weights for each criterion were also computed using Eq. (12), as shown in Table 5.

Step 3: Find the performance score using linguistic variables. Using the vendor selection satisfaction level questionnaire, the evaluators defined their own subjective range intervals using linguistic variables within a fuzzy scale to determine the performance scores of each vendor. Let \tilde{h}_{ij}^k represents the fuzzy performance score of the i th criterion of the j th vendor by the k th evaluator. We select the fuzzy geometric mean method to aggregate the fuzzy performance score from K evaluators, as shown in Table 6. That is

$$\tilde{h}_{ij} = (\tilde{h}_{ij}^1 \otimes \tilde{h}_{ij}^2 \otimes \dots \otimes \tilde{h}_{ij}^K)^{1/K}. \quad (18)$$

Furthermore, Eq. (12) is used to compute the BNP values of the fuzzy performance score \tilde{h}_{ij} , as shown in Table 7.

4.4. Results and discussions

In our integrated fuzzy MCDM method, we use non-additive multi-criteria evaluation techniques to deal with situations involving interrelations among the sub-criteria of a criterion. We introduced the λ value to represent the properties of substitutive or multiplicative effects between two criteria, where the values of λ ranged from -1 to positive infinite value (∞) (see Section 3.3). Using Eq. (16), the λ value of each criterion can be obtained. A multiplicative effect was found among five sub-criteria of the criterion *price and terms* ($\lambda = 0.1467$), whereas there were substitutive effects among three sub-criteria of the *supply chain support* criterion ($\lambda = -0.1376$) and also among three sub-criteria of the *technology* criterion ($\lambda = -0.1170$). Even though there were synthetic interactive effects, the λ values were still quite low, which could mean that there is still a great deal of room for these vendors to improve their performance. Consequently, the evaluation decision process would be more practical and flexible using the different λ values. The satisfaction values for the five candidate vendors could be gained

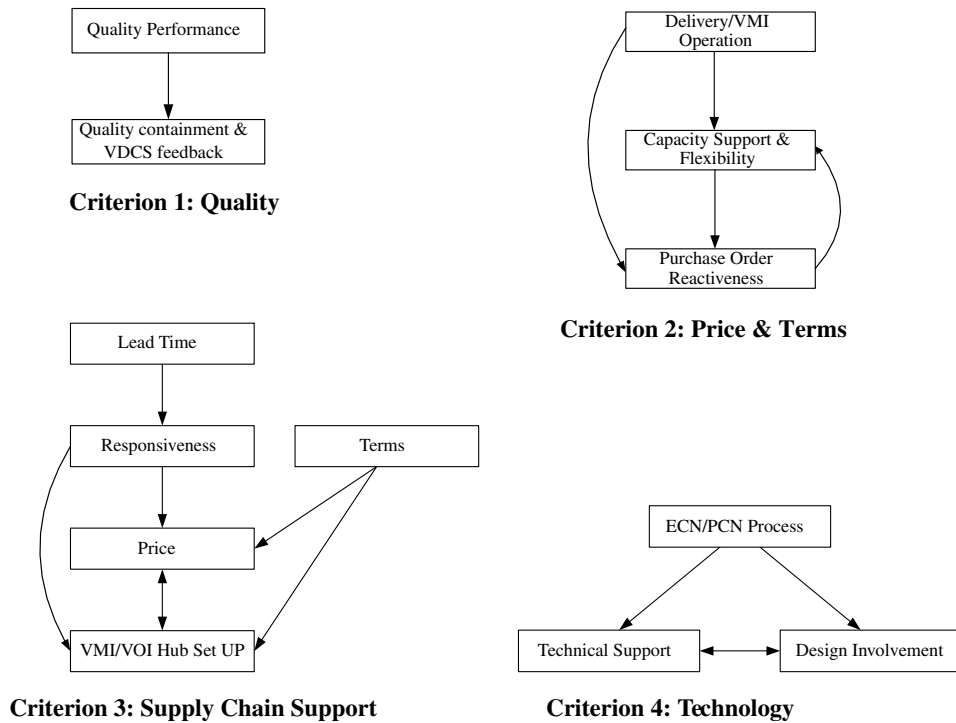


Fig. 6. The multi-level diagram of four criteria derived by the ISM method.

Table 5 Weights of criteria for evaluating the appropriate vendor by fuzzy AHP

Criteria and sub-criteria	Local weights	Overall weights (global)	BNP values	Normalized criteria weights (local)	Normalized sub-criteria weights (global)
Quality	(0.1706, 0.3001, 0.5441)	0.3382	0.3034		
- Quality performance	(0.3954, 0.5284, 0.6940)	(0.0621, 0.1321, 0.2727)	0.1557	0.5206	0.1198
- Quality containment & VDCS feedback	(0.3451, 0.4716, 0.6564)	(0.0543, 0.1179, 0.2579)	0.1434	0.4794	0.1104
Price and terms	(0.2828, 0.5202, 0.9010)		0.5680	0.5095	
- Price	(0.3281, 0.5076, 0.7627)	(0.0516, 0.1269, 0.2997)	0.1594	0.4992	0.1227
- Terms	(0.1802, 0.2783, 0.4306)	(0.0284, 0.0696, 0.1692)	0.0891	0.2788	0.0686
- Responsiveness	(0.0478, 0.0751, 0.1211)	(0.0075, 0.0188, 0.0476)	0.0246	0.0771	0.0189
- Lead time	(0.0563, 0.0838, 0.1349)	(0.0089, 0.0210, 0.0530)	0.0276	0.0864	0.0212
- VMI/VOI hub set-up cost	(0.0360, 0.0551, 0.0930)	(0.0057, 0.0138, 0.0365)	0.0187	0.0584	0.0144
Supply chain support	(0.0659, 0.1128, 0.2086)		0.1291	0.1158	
- Purchase order reactivity	(0.2497, 0.4478, 0.7585)	(0.0393, 0.1120, 0.2981)	0.1498	0.4412	0.1153
- Capacity support & flexibility	(0.1495, 0.2520, 0.4421)	(0.0235, 0.0630, 0.1738)	0.0868	0.2556	0.0668
- Delivery/VMI operation	(0.1813, 0.3001, 0.5222)	(0.0285, 0.0750, 0.2052)	0.1029	0.3032	0.0792
Technology	(0.0413, 0.0669, 0.1304)		0.0795	0.0713	
- Technical support	(0.2053, 0.3601, 0.6347)	(0.0323, 0.0900, 0.2429)	0.1239	0.3632	0.0954
- Design involvement	(0.2174, 0.3823, 0.6588)	(0.0342, 0.0956, 0.2589)	0.1296	0.3797	0.0998
- ECN/PCN process	(0.1527, 0.2576, 0.4447)	(0.0240, 0.0644, 0.1747)	0.0877	0.2571	0.0675

to calculate the fuzzy integral values of each criterion (see Table 8 below). Take V_1 and V_3 , for example, by using the method proposed in this study, the satisfaction values concerning the *quality*, *price and terms*, *supply chain support*, and *technology* criteria for V_1 were 5.140, 7.419, 4.007, and 4.786, respectively. Similarly, the satisfaction values for the same criteria for V_3 were 4.674, 8.863, 1.540, and 2.917, respectively. For the *quality* criterion, the satisfaction values of V_1 and V_3 were the same regardless of whether the simple additive weight method or the non-additive method were used, since the *quality* criterion is based on the assumption of independence. As for the *price and terms* criterion, a higher score was obtained by using our proposed method. In contrast, for the other two criteria, *supply chain support* and *technology*, the scores were lower using our proposed method. It can be seen that if the traditional simple additive weight model is utilized to aggregate the final

Table 6
Fuzzy performance score of candidate vendors for sub-criteria

Candidate vendor (V_j)	Sub-criteria				
	C_{11}	C_{12}	C_{21}	C_{22}	C_{23}
V_1	(2.94, 4.12, 5.81)	(4.37, 6.11, 7.71)	(4.25, 5.81, 7.24)	(3.87, 5.25, 6.98)	(3.66, 5.35, 6.66)
V_2	(3.88, 5.51, 7.08)	(4.82, 6.02, 7.66)	(4.11, 5.77, 7.29)	(5.25, 6.91, 8.30)	(3.97, 5.69, 7.17)
V_3	(4.02, 5.84, 7.22)	(2.22, 3.42, 5.06)	(5.07, 6.72, 7.82)	(3.99, 5.54, 6.84)	(3.21, 4.81, 6.17)
V_4	(1.90, 2.93, 4.58)	(2.39, 3.94, 5.69)	(1.86, 3.33, 5.00)	(1.30, 2.46, 4.14)	(2.45, 3.66, 5.53)
V_5	(3.86, 5.68, 7.09)	(2.29, 3.81, 5.49)	(1.34, 2.04, 3.56)	(2.90, 4.28, 5.63)	(2.88, 4.19, 5.58)
	C_{24}	C_{25}	C_{31}	C_{32}	C_{33}
V_1	(4.47, 6.14, 7.53)	(5.14, 6.83, 8.33)	(2.67, 4.44, 6.02)	(5.75, 6.96, 8.55)	(3.74, 5.40, 6.92)
V_2	(6.57, 7.91, 8.86)	(4.56, 6.18, 7.71)	(5.23, 6.78, 8.14)	(5.18, 6.72, 8.18)	(4.33, 6.09, 7.66)
V_3	(2.46, 3.76, 5.34)	(2.16, 3.70, 5.23)	(2.52, 4.12, 5.94)	(3.14, 4.52, 6.25)	(2.76, 4.42, 5.88)
V_4	(2.04, 3.43, 5.02)	(3.89, 5.19, 7.05)	(2.77, 4.09, 5.81)	(2.71, 4.24, 5.91)	(1.95, 3.31, 4.50)
V_5	(4.43, 5.64, 7.49)	(2.75, 4.20, 5.47)	(2.69, 4.29, 5.78)	(3.62, 5.38, 6.98)	(2.61, 4.08, 5.71)
	C_{41}	C_{42}	C_{43}		
V_1	(1.86, 3.18, 4.85)	(4.88, 6.42, 7.83)	(3.22, 5.18, 6.85)		
V_2	(4.33, 5.93, 7.44)	(6.36, 7.85, 8.91)	(4.73, 6.36, 7.79)		
V_3	(4.02, 5.53, 6.83)	(2.52, 3.98, 5.68)	(2.84, 4.14, 5.88)		
V_4	(2.39, 3.59, 5.46)	(2.24, 3.93, 5.43)	(3.49, 5.07, 6.50)		
V_5	(2.43, 4.07, 5.55)	(1.84, 2.71, 4.36)	(2.24, 3.37, 4.88)		

Table 7
Defuzzied performance score of candidate vendors for sub-criteria

Candidate vendor (V_j)	BNP values of sub-criteria												
	C_{11}	C_{12}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{31}	C_{32}	C_{33}	C_{41}	C_{42}	C_{43}
V_1	4.289	6.065	5.768	5.367	5.221	6.051	6.767	4.376	7.089	5.355	3.295	6.378	5.083
V_2	5.491	6.167	5.721	6.819	5.609	7.782	6.150	6.718	6.692	6.026	5.897	7.705	6.293
V_3	5.692	3.568	6.534	5.459	4.730	3.854	3.693	4.196	4.635	4.353	5.460	4.060	4.288
V_4	3.136	4.006	3.395	2.633	3.880	3.494	5.377	4.224	4.287	3.253	3.812	3.868	5.022
V_5	5.541	3.863	2.315	4.272	4.215	5.853	4.138	4.254	5.326	4.131	4.016	2.969	3.499

Table 8
Fuzzy integral results of each dimension for candidate vendors

Criteria (C_i)	AHP weights	λ value	Candidate vendor (V_j)				
			V_1	V_2	V_3	V_4	V_5
Quality	0.3034	–	5.140 (5.140)	5.815 (5.815)	4.674 (4.674)	3.553 (3.553)	4.737 (4.737)
Price and terms	0.5095	0.1467	7.419 (5.691)	8.733 (6.214)	8.863 (5.692)	5.766 (3.342)	6.621 (3.414)
Supply chain support	0.1158	–0.1376	4.007 (5.366)	3.619 (6.502)	1.540 (4.356)	1.847 (3.946)	2.521 (4.491)
Technology	0.0713	–0.1170	4.786 (4.925)	4.190 (6.685)	2.917 (4.627)	2.437 (4.144)	2.283 (3.486)
Overall scores	–	–	6.988 (5.432)	7.842 (6.160)	7.117 (5.152)	5.062 (3.533)	5.961 (3.945)

Note that the 1st criterion represents the criterion without synthetic effect; therefore, it will be not necessary to use the fuzzy integral method. The 2nd–4th criteria represent criteria with synthetic effect, so the fuzzy integral method is needed. “()” represents the weighted synthetic scores of candidate vendors calculated using the traditional AHP method.

overall scores, it will underestimate when the criteria have multiplicative effects (i.e., *price and terms*) and overestimate when the criteria have substitutive effects (i.e., *supply chain support* and *technology*). These results are consistent across vendors. This observation implies that criteria with synthetic interactive effects are more reasonable than those obtained by the traditional additive evaluation process.

Moreover, from Table 9, the ranks of the overall scores of the five candidate vendors were found to be $V_2 \succ V_3 \succ V_1 \succ V_5 \succ V_4$, where $A \succ B$ means that A is preferred to B . Obviously, the best vendor is V_2 . The results can also be compared with the traditional AHP method. The overall scores can be obtained and the ranking is $V_2 \succ V_1 \succ V_3 \succ V_5 \succ V_4$. Although the same best vendor was found using different methods, the ordinal ranking of the vendors were different. In other words, different methods still resulted in different ranking of the vendors. Furthermore, we employed the concept of the ideal point to represent the results of the analysis in Fig. 7. These aspired/desired/ideal points (10 scores) represent

Table 9
Results obtained using the traditional AHP method and the proposed method

Candidate vendor (V_j)	Traditional AHP method		Proposed method	
	BNP	Ranking	BNP	Ranking
V_1	5.432	2	6.988	3
V_2	6.160 ^a	1	7.842 ^a	1
V_3	5.152	3	7.117	2
V_4	3.533	5	5.062	5
V_5	3.945	4	5.961	4

^a Represents the appropriate vendor.

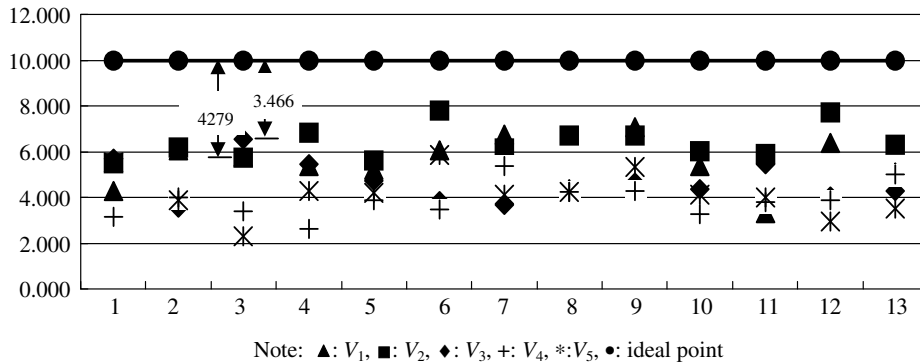


Fig. 7. The performance scores of the appropriate vendor and the ideal point in each sub-criterion.

points at which all the criteria of each vendor would be optimized, thus emphasizing the gaps between the appropriate vendor (V_2) and the ideal points. In Fig. 7, it can be seen that V_2 has the highest score in most sub-criteria, except in the sub-criteria *quality performance* (C_{11}), *price* (C_{21}), *VMI/VOI hub set up* (C_{25}), and *capacity support and flexibility* (C_{32}). For instance, for V_2 and V_3 , the gaps between individual scores and the ideal point of the *price* sub-criterion are 4.279 and 3.466, respectively. Similarly, it can also be shown from Table 8 that the satisfaction values of V_2 in the *quality* and *price and terms* criteria are 5.815 and 8.733, respectively. Although the values are larger than one-half of the aspired point, the gaps to the aspired points are 4.185 and 1.267, respectively. Similarly, the satisfaction values for the *supply chain support* and *technology* criteria are 3.619 and 4.190, respectively. These values are less than the aspired point of the two criteria and the gaps for these two criteria are more than the gaps for the *quality* and *price and terms* criteria. Based on the above results, some suggestions can be made to stimulate the creativity and improve the performance of the appropriate vendor (V_2) for some poor sub-criteria via different strategies (e.g., continuous improvement, price negotiation tactics, supply chain integration planning, etc.) to achieve the aspired/desired values; the ideal vendor can also be achieved through R&D and innovation/creativity. This information is useful for new vendors in this purchase project. Furthermore, with respect to the ordinal ranking of the weights of sub-criteria, the global weight values of *price* and *quality performance* are 0.1227 and 0.1198, respectively (see Table 5 below). Thus, the degree of importance of the two sub-criteria is higher than those of the other sub-criteria. These results show that the evaluators are most concerned about price and quality when selecting the appropriate vendor, which is consistent with the results found in a real purchase project.

5. Conclusion

Vendor selection is a very complicated multiple criteria problem. The information available for use in multiple criteria decision making is usually uncertain, vague, or imprecise, and the criteria are not necessarily independent. In addition, if a criterion were to contain additional sub-criteria, there would be a stronger possibility of correlation among sub-criteria. However, traditional MCDM methods are based on the assumption of independence among sub-criteria, and the analytical framework for vendor selection processes has failed to consider the violation of this assumption.

In this paper, we demonstrate an integrated fuzzy MCDM technique, which is more appropriate for selecting the best vendor. Here we introduce fuzzy numbers to express linguistic variables that express the subjective judgment of evaluators. In addition, we employ the ISM method to clarify the interrelationships of intertwined sub-criteria in the complex structural hierarchy of a vendor selection problem. The final fuzzy weights of each criterion can also be obtained by applying the fuzzy geometric mean method. Furthermore, we validate the use of non-additive fuzzy integral in the evaluation process when

criteria are interdependent. Our proposed method may avoid overestimation when the criteria have substitutive effects or underestimation when the criteria have multiplicative effects. Our results show that the fuzzy integral method is better and more reasonable than the traditional method. We further find that the difference in preference order will depend on the different λ values with respect to the effect of criteria. This will provide useful information regarding substitutive or multiplicative effects among the criteria under consideration. By the concept of ideal point, we can provide the available information and strategies to stimulate the creativity of and improvement the appropriate vendor that they might achieve the aspired/ desired values. Therefore, we demonstrate that the non-additive multiple criteria evaluation techniques are more appropriate than the traditional method and provide practitioners with a valuable tool for use in a fuzzy MCDM environment to solve vendor selection problems.

Appendix A. ISM questionnaire

This questionnaire is about vendor selection and the evaluation of relative sub-criteria and criteria. Because of your expertise and knowledge, your opinions will set up the relationship among criteria and sub-criteria. I can then establish an analysis model of vendor selection.

Instructions for this questionnaire:

Example criterion 3: Supply chain support. Under this criterion, there are three sub-criteria – purchase order reactivity, capacity support and flexibility, and delivery/VMI operation – which are shown below.

Row 1 means that if you think that purchase order reactivity does not exist, and the sub-criterion capacity support and flexibility does have an influence then you should give it a score of “1”. Similarly, if purchase order reactivity does not exist, and there is no influence of the sub-criterion delivery/VMI operation, then you should give it a score of “0”.

“Supply chain support” criteria

	Purchase order reactivity	Capacity support and flexibility	Delivery/VMI operation
Purchase order reactivity	0	1	0
Capacity support and flexibility	0	0	1
Delivery/VMI operation	1	1	0

According to defined evaluation criteria, we utilize a simple relation matrix to verify the independence between criteria. Seven decision makers were invited to evaluate the relations of criteria, including quality (Q), price and terms (P), support chain support (S), and technology (T). In addition, a threshold value (let $\eta = 0.5$) was given by decision makers through discussions to decide whether the criteria were independent. Furthermore, we used an arithmetic mean method to calculate the mean values of the opinions of the seven decision makers concerning the relationships between criteria in the relation matrix (R). Thus, the mean relation matrix (\bar{R}) could be obtained. Take the four criteria evaluated by the first decision maker, for example, the values given for the relationships between criteria to form the R matrix were as follows:

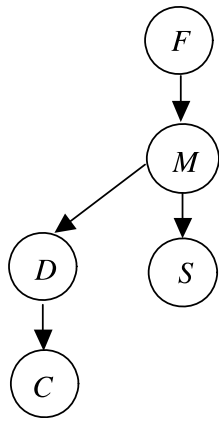
$$R = \begin{matrix} & Q & P & S & T \\ \begin{matrix} Q \\ P \\ S \\ T \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 1 \\ & 0 & 0 & 1 \\ & & 0 & 0 \\ & & & 0 \end{pmatrix} \end{matrix}$$

Next, using an arithmetic mean method, the evaluations of the relations between the four criteria made by the seven decision-makers were summarized as the values of the relation matrix (R'). Finally, the mean relation matrix $\bar{R} = [\bar{r}_{ij}]_{n \times n}$ could also be obtained as follows:

$$R' = \begin{matrix} & Q & P & S & T \\ \begin{matrix} Q \\ P \\ S \\ T \end{matrix} & \begin{pmatrix} 0 & 3 & 1 & 2 \\ & 0 & 2 & 2 \\ & & 0 & 1 \\ & & & 0 \end{pmatrix} \end{matrix}, \text{ and } \bar{R} = \begin{matrix} & Q & P & S & T \\ \begin{matrix} Q \\ P \\ S \\ T \end{matrix} & \begin{pmatrix} 0 & 0.43 & 0.14 & 0.29 \\ & 0 & 0.29 & 0.29 \\ & & 0 & 0.14 \\ & & & 0 \end{pmatrix} \end{matrix}$$

If the value of the mean relation matrix \bar{R} is more than the threshold value ($\bar{r}_{ij} \geq \eta$), the pairwise comparison criteria can be considered independent, whereas if not, they are dependent criteria. Clearly, all values of the mean relation matrix \bar{R} are smaller than 0.5. Thus, the four criteria should be regarded as independent.

We also use a simple example to show the steps of the ISM. Assume the family members consist of father (F), mother (M), son (S), daughter (D), and cat (C) and the relationships can be represented as the relation matrix in Fig. A.1. Based on the concept of the ISM method, the relation matrix is added to the identity matrix to form the T matrix as follows:



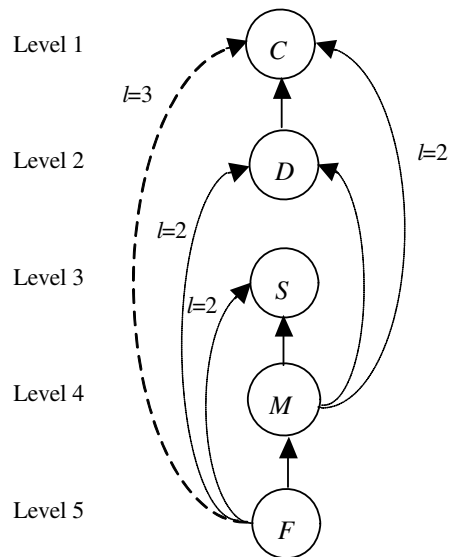
$$T = \begin{matrix} & \begin{matrix} F & M & S & D & C \end{matrix} \\ \begin{matrix} F \\ M \\ S \\ D \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}, \text{ and } T = A + I = \begin{matrix} & \begin{matrix} F & M & S & D & C \end{matrix} \\ \begin{matrix} F \\ M \\ S \\ D \\ C \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Next, the tentative reachability matrix is obtained by powering the matrix T to satisfy Eq. (3). Last, by calculating the limiting power of the matrix T , when $T^l = T^{l+1}$, $l > 1$ stop, the limiting reachability matrix obtained is as follows:

$$\begin{matrix} & \begin{matrix} F & M & S & D & C \end{matrix} \\ \begin{matrix} F \\ M \\ S \\ D \\ C \end{matrix} & \begin{bmatrix} 1 & 1 & 1^* & 1^* & 0 \\ 0 & 1 & 1 & 1 & 1^* \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}, \text{ and } \begin{matrix} & \begin{matrix} F & M & S & D & C \end{matrix} \\ \begin{matrix} F \\ M \\ S \\ D \\ C \end{matrix} & \begin{bmatrix} 1 & 1 & 1^* & 1^* & 1^{**} \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}.$$

(When $l = 2$) (When $l = 3$)

Because matrix $T^3 = T^4$, matrix T^3 is stop. the limiting reachability matrix (when $l = 2$), the asterisk * indicates the derivative relation which does not emerge in the original relation matrix. For instance, the intersection of row 1 and column 3 represents the effects that father (F) has on mother (M) and has on son (S). Similarly, when $l = 3$, the asterisk ** means the effects that father (F) has on mother (M) and has on daughter (D) and has on cat (C). In order to determine the levels of the elements in a hierarchical structure, the reachability set and the priority set are derived based on Eqs. (4) and (5). The first level can be derived according to Eq. (6) and is the father. The other levels can also be determined with the same procedures (Table A.1). Note that the reachability matrix generates the relationships between the father and the cat.



Note: — — — represents the inflects of $l=2$
 - - - - - represents the inflects of $l=3$

Fig. A1. Hierarchical structure of the elements.

A.1. AHP questionnaire

This research is about appropriate vendor evaluation. The purpose of this questionnaire is to ask your opinions concerning vendor selection. With your opinions, we can build a vendor selection evaluation model.

A.1.1. Part I: description of and instructions for this questionnaire

In this research, a vendor is defined and the level of relative importance of a criterion is chosen from five different levels, namely “absolutely important”, “very strongly important”, “essentially important”, “weakly important”, and “equally important” on a fuzzy five-level scale, through which its range is defined (range lies between 1 and 9). The scale of “1–9” is defined (see Fig. 2 in Section 2). For instance, if you think the subjective perception value of decision maker might be $\tilde{3} = (\underline{2}, \underline{3}, \underline{4})$. Please indicate your values on a TFN scale as follows:

$$\tilde{1} = (\square, \square, \square), \quad \tilde{2} = (\square, \square, \square), \quad \tilde{3} = (\square, \square, \square), \quad \tilde{4} = (\square, \square, \square), \quad \tilde{5} = (\square, \square, \square), \quad \tilde{6} = (\square, \square, \square),$$

$$\tilde{7} = (\square, \square, \square), \quad \tilde{8} = (\square, \square, \square), \quad \tilde{9} = (\square, \square, \square).$$

According to your experience of vendor selection, please answer this questionnaire based on your own opinions. The descriptions of each level are shown below:

Intensity of fuzzy scale (example)	Definition of linguistic variables for relative weights of criteria
1	Equally important
3	Weakly important
5	Essentially important
7	Very strongly important
9	Absolutely important
2,4,6,8	Intermediate values between two adjacent judgments. Used to represent compromises between the priorities listed above.

Note that 1–9 represents $\tilde{1}$ – $\tilde{9}$, respectively.

A.1.2. Part II: questionnaire

According to the suggestions of experts and scholars, this research is investigating a draft measurement for four vendor selection criteria, namely quality, price and terms, supply chain support, and technology. According to your subjective perceptions, please indicate relative importance levels in terms of pairs of the criteria listed below.

Pairwise of criterion	Relative importance (“9” is maximum, “1” is minimum)														Pairwise of criterion			
	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	1:1	1:2	1:3	1:4	1:5	1:6		1:7	1:8	1:9
Quality																		Price and Terms
Quality																		Supply chain support
Quality																		Technology
Price and terms																		Supply chain support
Price and terms																		Technology
Supply chain support																		Technology

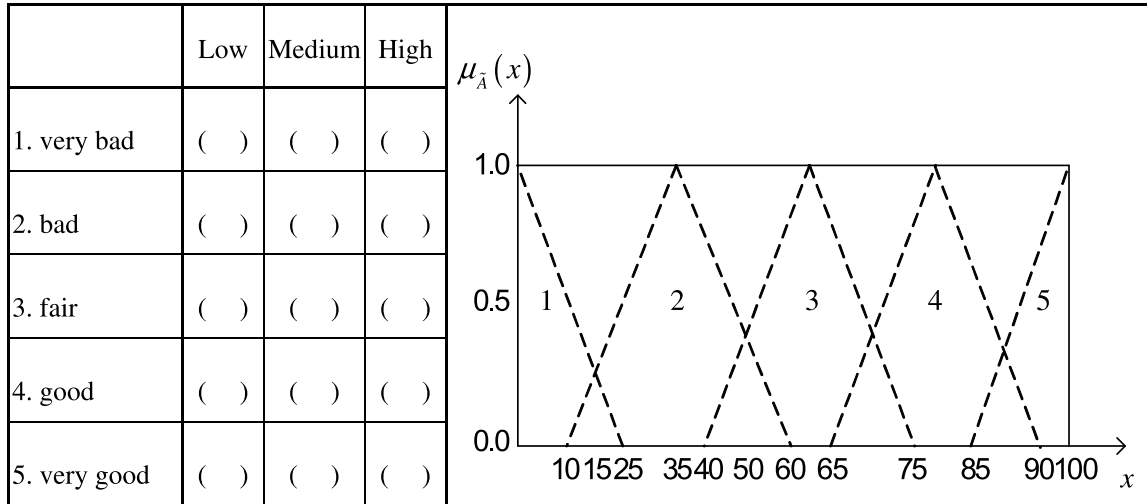
Table A1
The reachability set and priority set

Elements	R_r	A_r	$R_r \cap A_r$
1	1,2,3,4,5	1	1
2	2,3,4,5	1,2	2
3	3	1,2,3	3
4	4,5	1,2,4	4
5	5	1,2,4,5	5

Appendix B. Vendor selection satisfaction level questionnaire

This questionnaire is about your satisfaction level toward vendor selection. Please answer this questionnaire according to your subjective perceptions of vendor selection.

This questionnaire adopts a Likert-type five-point scale. It has five different levels – “very bad”, “bad”, “fair”, “good”, and “very good” – on a fuzzy five-level scale, through which its range is defined (range lies between 0 and 100). Give a score between 1 and 100 to indicate three different scales (i.e., low, medium, high). For example, you might think the satisfaction level a linguistic score of “very good”, that score would or correspond to a TFN of (80,90,100) respectively. Please answer this questionnaire according to your perceptions.



Example criterion 1: quality. Under this criterion, there are two sub-criteria: quality performance, and quality containment and VDCS feedback.

Sub-criteria	Performance	Vendor 1(V ₁)	Vendor 2(V ₂)	Vendor 3(V ₃)	Vendor 4(V ₄)	Vendor 5(V ₅)
C ₁₁ : quality performance	Very good Good Fair Bad Very bad					
C ₁₂ : quality containment and VDCS feedback	Very good Good Fair Bad Very bad					

Appendix C

In this article, we utilize non-additive Choquet integrals to aggregate fuzzy performance scores with weights. Here we give an example to compare the results with those obtained using the traditional assumption of independence among the criteria under consideration.

Considering the case of an employer who would like to promote a new manager for a company, the evaluation committee sets three criteria, management skill (C₁), professional knowledge (C₂) and experience (C₃). Three persons, P₁, P₂ and P₃, are interviewed, and the scores from the evaluators are summed up as shown in the following table:

Employer	Management skill (C ₁)	Professional knowledge (C ₂)	Experience (C ₃)
P ₁	90	80	50
P ₂	50	60	90
P ₃	70	75	70

In addition, the committee sets the weights of each criterion as

$$g_i(\{C_1\}) = g_i(\{C_2\}) = 0.45; \quad g_i(\{C_3\}) = 0.3;$$

$$g_i(\{C_1, C_2\}) = 0.5; \quad g_i(\{C_2, C_3\}) = g_i(\{C_1, C_3\}) = 0.9.$$

where $g_i(\ast)$ indicates the values of the fuzzy measure for the criteria.

Using the fuzzy integral with the above fuzzy measure and the traditional method (e.g., simple weight method, SWM), the evaluation results are as shown in the following table:

Employer	Synthetic score	
	Fuzzy integral method	Simple weight method
P_1	69.50 ^a	76.25 ^b
P_2	68.00	63.75
P_3	72.25	71.88

^a Interactive case among criteria:

$$g_i(\{C_1\}) = 0.45; g_i(\{C_1, C_2\}) = 0.5; g_i(\{C_1, C_2, C_3\}) = 1.$$

Synthetic score = $\int h dg = (90 - 80) \times 0.45 + (80 - 50) \times 0.5 + 50 \times 1 = 69.50$.

^b Additive case among criteria: find the weights of each criterion through normalization. That is

$$g_i(\{C_1\}) = g_i(\{C_2\}) = 0.375; g_i(\{C_3\}) = 0.25.$$

Synthetic score = $90 \times 0.375 + 80 \times 0.375 + 50 \times 0.25 = 76.25$.

From the above results, we find that if $g_i(\{x_1\}) + g_i(\{x_2\}) = 0.9 < 1$, then $\lambda > 0$. This implies that the relations among criteria have a multiplicative effect. In other words, this can increase the overall score of an alternative if the criteria are enhanced simultaneously. Based on the above results, we can see the difference between the two methods to identify the best alternative by ranking using the synthetic scores. Thus, the fuzzy integral method is more suitable than a traditional method (e.g., SAW) when there are non-independent effects (either substitutive or multiplicative) among the considered criteria.

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