LETTER

Blind SNR Estimation with Coherent Function for OFDM Systems

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SUMMARY In OFDM receivers, an accurate signal-to-noise ratio (SNR) estimation is desirable for all sorts of operations involved in high-performance baseband demodulation. In this work, conventional SNR estimation techniques are investigated. Next, a blind SNR estimation scheme for the phase-shift keying (PSK) signals based on the coherence function is proposed. The proposed method is non-data-aided (NDA) and hence bandwidth-efficient. Simulations confirm that the proposed method has the best performance from moderate to high SNRs both in AWGN and dispersive channels; however, the proposed method has worse performance when SNRs are low.
key words: *OFDM, SNR estimation*

1. Introduction

The issues associated with the estimation of signal-to-noise ratio (SNR) are of great importance in many areas of digital communication systems, particularly due to the fact that various signal detection algorithms require knowledge of the SNR for optimal performances. Hence, it is essential to estimate the SNR accurately both in AWGN and dispersive channels. In literature, the works in [1], [2] only estimate the SNR in AWGN channels; the maximum-likelihood (ML) SNR estimation is given in [3]; the minimum-mean-squared-error (MMSE) SNR estimation is given in [4]; the work in [5] estimates the SNR in multipath fading channels by utilizing the pilot subcarriers; and the work in [6] estimates the SNR in multipath fading channels by utilizing the moments of the envelope of the received signal. To achieve the optimum performance in OFDM receivers, a blind SNR estimation scheme that utilizes the coherence function [7] of the phase-shift keying (PSK) signal is proposed. The proposed algorithm is non-data-aided (NDA) which can improve the transmission efficiency of OFDM systems.

2. OFDM System Model

On the transmitter side, N complex data symbols are modulated onto N subcarriers by using the inverse fast Fourier transform (IFFT). The last N_G IFFT samples are copied to form a cyclic prefix (CP) that is inserted at the beginning of each OFDM symbol. By inserting the CP, the guard interval is created so that inter-symbol interference (ISI) can be avoided and the orthogonality among the subcarriers can be sustained. The receiver uses the fast Fourier transform (FFT) to demodulate received data. Detailed analysis of the received frequency-domain data is given in [8] and can be written as a general form as

$$\tilde{X}_{l,k} = X_{l,k}^{dsr} + \nu_k \tag{1}$$

where $X_{l,k}^{dsr} = H_{l,k}X_{l,k}$ is the desired signal, $H_{l,k}$ and $X_{l,k}$ are the channel frequency response (CFR) and the transmitted frequency-domain data at the k-th subcarrier of the l-th symbol, respectively; and ν_k is the AWGN. Finally, the theoretical SNR can be expressed as

$$\rho^{th} = \frac{E\left[\left|X_{l,k}^{dsr}\right|^2\right]}{E[|\nu_k|^2]} = \frac{\sigma_X^2}{2\sigma_\nu^2} \tag{2}$$

where $\sigma_X^2 = E[|X_{l,k}^{dsr}|^2]$ is the desired signal power and σ_ν^2 is the power of the real part/imaginary part of ν_k.

3. Conventional SNR Estimation Techniques

Some conventional techniques in literature are discussed or extended for OFDM systems as follows.

3.1 ML and MMSE Algorithms

The ML SNR estimate $\widehat{\rho}_{ML}$ [3] at the k-th subcarrier is derived from L received OFDM symbols and can be expressed as

$$\widehat{\rho}_{ML} = \frac{\widehat{S}_{ML}}{\widehat{N}_{ML}} \tag{3}$$

$$\widehat{S}_{ML} = \frac{1}{L}\sum_{l=0}^{L-1} \mathrm{Re}\left\{\tilde{X}_{l,k}\widehat{H}_{l,k}^{*}X_{l,k}^{*}\right\} \tag{4}$$

$$\widehat{N}_{ML} = \frac{1}{L}\sum_{l=0}^{L-1}\left|\tilde{X}_{l,k}\right|^2 - \widehat{S}_{ML} \tag{5}$$

where $\widehat{S}_{ML}$ and $\widehat{N}_{ML}$ are ML estimates of the signal and AWGN powers, respectively; $\mathrm{Re}\{\cdot\}$ denotes the real part of its argument; and $\widehat{H}_{l,k}$ is the estimate of $H_{l,k}$.

The MMSE SNR estimate [4] can be written as

$$\widehat{\rho}_{MMSE} = \frac{\widehat{S}_{MMSE}}{\widehat{N}_{MMSE}} \tag{6}$$

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$$\widehat{S}_{MMSE} = \frac{1}{L}\sum_{l=0}^{L-1}\left|\tilde{X}_{l,k}\right|^2 - \widehat{N}_{MMSE} \tag{7}$$

$$\widehat{N}_{MMSE} = \frac{1}{L}\sum_{l=0}^{L-1}\left|\tilde{X}_{l,k} - \widehat{H}_{l,k}X_{l,k}\right|^2 \tag{8}$$

where $\widehat{S}_{MMSE}$ and $\widehat{N}_{MMSE}$ are MMSE estimates of the signal and AWGN powers, respectively.

As can be seen, the ML and MMSE estimators need the estimate of the CFR and the knowledge of the transmitted data; therefore, it is data-aided. This also means that the performances of these SNR estimators are dependent on the channel estimation algorithms. Moreover, when $\widehat{H}_{l,k}$ is derived from the often used least-square (LS) channel estimation, both (5) and (8) will lead to zero, and the SNR estimation is invalidated.

3.2 Error Vector Magnitude (EVM) Algorithm

The EVM [2] is referred to the magnitude of the error vector and can be expressed as

$$\widehat{\rho}_{EVM} = \frac{\widehat{S}_{EVM}}{\widehat{N}_{EVM}} \tag{9}$$

$$\widehat{S}_{EVM} = \left(\frac{1}{N}\sum_{k=0}^{K-1}\widehat{X}_{l,k,I/Q}\right)^2 \tag{10}$$

$$\widehat{N}_{EVM} = \frac{1}{N}\sum_{k=0}^{K-1}\left(\widehat{X}_{l,k,I/Q}\right)^2 - \left(\frac{1}{N}\sum_{k=0}^{K-1}\widehat{X}_{l,k,I/Q}\right)^2 \tag{11}$$

where $\widehat{S}_{EVM}$ and $\widehat{N}_{EVM}$ are EVM estimates of the signal and AWGN powers, respectively; and $\widehat{X}_{l,k,I/Q}$ is the estimate of the I/Q component of the transmitted data.

3.3 Boumard's Algorithm

In [5], Boumard utilizes the characteristic of the pilot subcarriers; therefore, it is data-aided, and the SNR at the k-th subcarrier can be expressed as follows:

$$\widehat{\rho}_{BA} = \frac{\widehat{S}_{BA}}{\widehat{N}_{BA}} \tag{12}$$

$$\widehat{S}_{BA} = \frac{1}{L}\sum_{l=0}^{L-1}\tilde{X}_{l,k}\tilde{X}^*_{l+1,k} \tag{13}$$

$$\widehat{N}_{BA} = \frac{1}{L}\sum_{l=0}^{L-1}\left|\tilde{X}_{l,k}\right|^2 - \widehat{S}_{BA} \tag{14}$$

where $\widehat{S}_{BA}$ and $\widehat{N}_{BA}$ are Boumard's estimates of the signal and AWGN powers, respectively.

3.4 Second- and Fourth-Order Moments (M_2M_4) Algorithm

After extensive survey of SNR estimation techniques, we found that, in [6], the SNR estimation of a real-valued single-carrier signal is useful due to its simplicity. It was derived by utilizing the 2nd and 4th moments of the envelope of received frequency-domain data in slow-fading channels. Besides [1], [6] and [9] are also based on high-order moments. We generalize and extend the technique in [6] to the complex-valued signals for the OFDM systems. Since the derivation is similar to [6], it is omitted here. The key parameters and results are written as follows. First, one can define the following 2nd and 4th moments of $\left|\tilde{X}_{l,k}\right|$ at the k-th subcarrier based on time sample averages:

$$\widehat{M} \triangleq \frac{1}{L}\sum_{l}\left|\tilde{X}_{l,k}\right|^2, \tag{15}$$

$$\widehat{Q} \triangleq \frac{1}{L}\sum_{l}\left|\tilde{X}_{l,k}\right|^4, \tag{16}$$

and

$$\widehat{R} \triangleq \frac{1}{L}\sum_{l}\left|\tilde{X}_{l,k}\right|^2\left|\tilde{X}_{l+1,k}\right|^2. \tag{17}$$

Then, the M_2M_4 SNR estimate can be written as

$$\widehat{\rho}_{MA} = \frac{\widehat{S}_{MA}}{\widehat{N}_{MA}} \tag{18}$$

$$\widehat{S}_{MA} = \left(\widehat{M}^2 + \widehat{R} - \widehat{Q}\right)^{1/2} \tag{19}$$

$$\widehat{N}_{MA} = \widehat{M} - \widehat{S}_{MA} \tag{20}$$

where $\widehat{S}_{MA}$ and $\widehat{N}_{MA}$ are M_2M_4 estimates of the signal and AWGN powers, respectively.

3.5 Discussion of Conventional Techniques

Summarizing the introduced techniques, first, it is observed that the ML and MMSE estimators need the channel estimates and the knowledge of the transmitted data in addition to the received data. Second, the EVM estimator needs the estimates of the transmitted data. Third, the Boumard's estimation needs the received data and the pilot subcarriers. Finally, the M_2M_4 estimation is NDA and only needs the received data.

4. Proposed SNR Estimation

The coherence function [7] is often used to determine the degree to which one observed phase-shift keying (PSK) signal is related to another observed signal. If we have uncorrelated noise and the coherent signal, the amount of the powers due to noise and coherent signal can be determined. The concept of coherence function can be applied to the OFDM systems, so that the SNR, at the k-th subcarrier, can be written as

$$\widehat{\rho}_k = \frac{\left|\widehat{\gamma}_k\right|^2}{1 - \left|\widehat{\gamma}_k\right|^2} \tag{21}$$

where

$$\left|\widehat{\gamma}_k\right|^2 \triangleq \frac{\left|E\left[\tilde{X}_{l,k}\tilde{X}^*_{l+1,k}\right]\right|^2}{E\left[\left|\tilde{X}_{l,k}\right|^2\right]E\left[\left|\tilde{X}_{l+1,k}\right|^2\right]},\ k \in P,\ 0 \le \left|\widehat{\gamma}\right|^2 \le 1 \quad (22)$$

is the magnitude-squared-coherence (MSC) at the k-th subcarrier of the l-th symbol, and P is the pilot set. The MSC is the cross-power spectrum of two consecutive symbols normalized by their auto-power spectra, which can be viewed as the correlation between like frequency components of the received frequency-domain data. Note that $\widehat{\rho}_k$ can be averaged over subcarriers to improve its accuracy.

Originally, the MSC in [7] was realized by the commonly-used technique of time sample average, which is based on the assumption of signal ergodicity. Unfortunately, it is only applicable for data-aided (DA) systems. In OFDM systems, since there is only one data sample at each pilot subcarrier within an OFDM symbol, and the number of pilot subcarriers is a fraction of N, say $N/32$, the MSC might suffer from large estimation error and long acquisition time. Therefore, to blindly estimate the MSC without known data (or pilots) and derive the SNR estimate accurately in reasonable amounts of symbols, we propose to average the MSC over all N subcarriers within an OFDM symbol as follows

$$\overline{\left|\widehat{\gamma}\right|}^2 = \frac{\left(\frac{1}{N}\sum_k \left|\tilde{X}_{l,k}\tilde{X}^*_{l+1,k}\right|\right)^2}{\left(\frac{1}{N}\sum_k \left|\tilde{X}_{l,k}\right|^2\right)\left(\frac{1}{N}\sum_k \left|\tilde{X}_{l+1,k}\right|^2\right)} \quad (23)$$

where the bar on top of $\left|\widehat{\gamma}\right|$ denotes the sample average over N subcarriers. Finally, the SNR can be obtained similar to (21) by

$$\widehat{\rho}' = \frac{\overline{\left|\widehat{\gamma}\right|}^2}{1 - \overline{\left|\widehat{\gamma}\right|}^2}. \quad (24)$$

It should be noted that the proposed method has the same property as the M_2M_4 estimation, i.e., NDA and requiring only the received data for the SNR estimation.

5. Simulations and Discussions

Monte Carlo simulations are conducted to evaluate the performance by means of the estimation's normalized mean-squared error (NMSE)

$$\text{NMSE} \triangleq E\left[\left(\frac{\widehat{\rho} - \rho^{th}}{\rho^{th}}\right)^2\right] \quad (25)$$

where $\widehat{\rho}$ is the SNR estimate. The following parameters of an OFDM system are assumed: there are $N = 256$ subcarriers; the guard interval is $N_G = N/8 = 32$ samples; the simulated modulation scheme is quadrature phase-shift keying (QPSK); the signal bandwidth is 2.5 MHz; the radio frequency is 2.4 GHz; the subcarrier spacing is 8.68 kHz; the OFDM symbol duration is $115.2\,\mu s$; and there are 8 pilot subcarriers. In each simulation run, 1,000 symbols are tested. All the results are obtained by averaging over 2,000 independent channel realizations. To verify the performance of the proposed technique, we compare the proposed method with the following techniques: EVM, Boumard's, original MSC (21), and M_2M_4 in the AWGN and multipath fading channels.

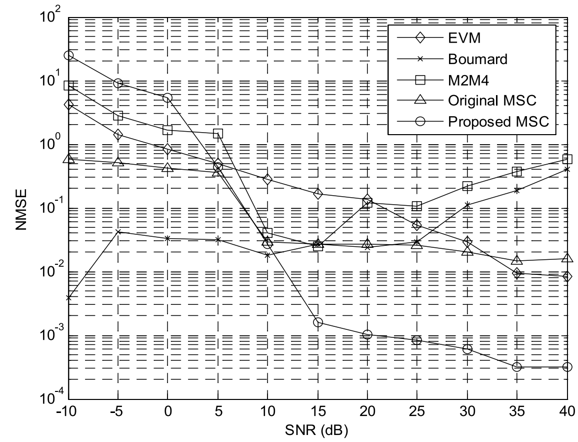

Fig. 1 SNR estimations in the AWGN channel.

5.1 Performance in the AWGN Channel

The performance in the AWGN channel is shown in Fig. 1. As can be seen, the Boumard's has the best performance when the SNR ranges from very low (< 0 dB) to low (0–10 dB) values; and the proposed method has the best performance when the SNR ranges from moderate (10–25 dB) to high (> 25 dB) values. In addition, the performance of the proposed method consistently improves when the SNR increases.

5.2 Performance in Multipath Channels

To verify the performance of the proposed technique in multipath channels, the channel is assumed to have N_G paths. The channel taps are randomly generated by independent zero-mean, complex Gaussian variables with $\sum_\tau \sigma^2_{h_\tau} = 1$, where $\sigma^2_{h_\tau} = E[|h(\tau)|^2]$ is the power of the τ - th channel tap. The performances of the proposed method in time-variant and — invariant multipath channels are shown in Fig. 2. For clarity, we only show the performances of conventional methods in time-invariant multipath channels. The normalized Doppler frequency in time-variant channels is 0.035 which corresponds to a maximum Doppler frequency of 303.8 Hz, i.e., a mobile speed of 117 km/hr.

The theoretical SNR in time-invariant channels is the true SNR. In contrast, in time-variant channels, besides AWGN, the received signal also suffers from inter-carrier interference (ICI). Therefore, with the result in [10], the theoretical SNR can be shown to be

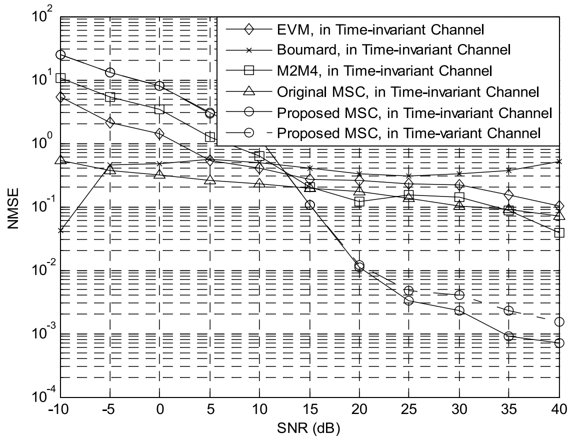

Fig. 2 SNR estimations in the multipath fading channel.

$$\rho^{th} = \frac{N + 2\sum_{\Delta_n=1}^{N-1} (N - \Delta_n)\, J_0(\beta\Delta_n)}{N(N-1) - 2\sum_{\Delta_n=1}^{N-1} (N - \Delta_n)\, J_0(\beta\Delta_n) + 2\sigma_\nu^2} \tag{26}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind; $\beta \triangleq 2\pi f_d T/N$, f_d represents the maximum Doppler shift, T is the symbol duration; and Δ_n is the time difference due to time selectivity of channels.

As can be seen, the performance of the proposed method in multipath channels is similar to that in AWGN channels and is only slightly lower than it. In addition, the performance of the proposed method in time-variant channels is also slightly lower than that in time-invariant channels.

5.3 Discussions

In summary, the proposed method has the best performance from moderate to high SNRs that we are interested in. Therefore, we can say that the proposed method has the best performance in typical SNR conditions. In addition, the proposed method is less sensitive to multipath fading channels; hence, the proposed method can be used for practical environments.

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