OPTIMIZING MULTI-RESPONSE PROBLEMS IN THE TAGUCHI METHOD BY FUZZY MULTIPLE ATTRIBUTE DECISION MAKING

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SUMMARY

One of the conventional approaches used in off-line quality control is the Taguchi method. However, most previous Taguchi method applications have only dealt with a single-response problem and the multi-response problem has received only limited attention. The theoretical analysis in this study reveals that Taguchi's quadratic loss function and the indifference curve in the TOPSIS (Technique for order preference by similarity to ideal solution) method have similar features. The Taguchi method deals with a one-dimensional problem and TOPSIS handles multi-dimensional problems. As a result, the relative closeness computed in TOPSIS can be used as a performance measurement index for optimizing multi-response problems in the Taguchi method. Next, an effective procedure is proposed by applying fuzzy set theory to multiple attribute decision making (MADM). The procedure can reduce the uncertainty for determining a weight of each response and it is a universal approach which can simultaneously deal with continuous and discrete data. Finally, the effectiveness of the proposed procedure is verified with an example of analysing a plasma enhanced chemical vapour deposition (PECVD) process experiment. © 1997 by John Wiley & Sons, Ltd.

KEY WORDS: Taguchi method; parameter design; multi-response problem; multiple attribute decision making; TOPSIS method

1. INTRODUCTION

A cost-effective method to improve product quality and operational procedures is with the use of off-line quality control. This area includes those quality control activities used in the product planning, design and production engineering stages (but not during actual production).[1] The Taguchi method, which combines the experimental design techniques with quality loss considerations, is the conventional approach used for off-line quality control. The Taguchi method carefully considers the impact of the various factors influencing performance variation. This method consists of three stages: (a) systems design, (b) parameter design, and (c) tolerance design. A more detailed description of these three design types is provided by Kackar[2] and Phadke.[3] From their investigations, the variability and average of performance are of primary concern.

Product and operational procedures are influenced by design parameters (i.e. factors that are controlled by designers) and noise factors (i.e. factors that cannot be controlled by designers, such as environmental factors). The parameter design of the Taguchi method involves selecting the levels of the design parameters to minimize the effects of the noise factors. That is, the design parameter's settings for a product or a process should be determined so that the product's response has the minimum variation, and its mean is close to the desired target. Experimental design is used in this method to arrange the design parameters and noise factors in the orthogonal arrays. The signal-to-noise (SN) ratio is computed on the basis of quality loss for each experimental combination. Finally, SN ratios are analysed to determine the optimal settings (i.e. control factors and their levels) of the design parameters. The merits and shortcomings of the Taguchi method can be found in References 2, 4 and 5.[2,4,5] However, a customer usually considers more than one quality characteristic in most manufactured products. The Taguchi method can only be used for a single-response case; it cannot be used to optimize a multi-response problem. Engineering judgement has, up until now, been used primarily for the optimization of the multi-response problem in the Taguchi method. Unfortunately, an engineer's judgement will normally increase the uncertainty during the decision making process. Another approach to solve such a problem entails the assigning of a weight for each response. Determining a definite weight for each response in an actual case remains difficult. In addition, a factor which is significant in a single response case is not necessarily significant when considered in a multi-response case. Therefore, a more effective approach is required to solve such a complicated problem.

Fuzzy set theory provides membership functions which represent uncertain and subjective information. Multiple attribute decision making (MADM) refers to a situation in which selections among some courses of action must be made in the presence of

CCC 0748–8017/97/010025–10

Received 5 October 1995
Revised 23 August 1996

multiple, usually conflicting, attributes. In this paper, a systematic procedure is developed via the application of fuzzy set theory to MADM to optimize the multi-response production process. A fuzzy number is first applied to determine the weight for each response. By considering the quality loss of each response, a multi-response performance measurement index is developed on the basis of an MADM method; namely, a technique for order preference by similarity to ideal solution (TOPSIS). The developed index can be used to determine the optimum conditions in the parameter design stage for multi-response problems. The proposed optimization procedure includes a series of steps capable of decreasing the uncertainty in engineering judgement when the Taguchi method is applied. Only the static quality characteristic problem, in which the desired response value is fixed, is discussed in this paper.

The remainder of this paper is organized as follows. A literature review of the multi-response problems in the Taguchi method is given in Section 2. Section 3 introduces MADM problems. Section 4 proposes an optimization procedure for solving the problem of multi-response cases in the Taguchi method. An illustrative example for the implementation of the proposed procedure is provided in Section 5. Concluding remarks are made in Section 6.

2. LITERATURE REVIEW

Derringer and Suich[6] demonstrated how several response variables can be transformed into a desirability function, which can be optimized by univariate techniques. The desirability function approach is simple and permits the user to make subjective judgements on the importance of each response. However, the inexperienced user in assessing a desirability value may lead to inaccurate results. Khuri and Conlon[7] proposed a procedure capable of simultaneously optimizing several response variables that can be represented by polynomial regression models. They used a distance function to measure the deviation from the ideal optimum. By minimizing this function, one can specify suitable operating conditions for the simultaneous optimization of the responses. The notion of using the minimax approach in their method is quite similar to that in the TOPSIS method. However, their method is computationally complicated, thereby making it difficult to explain to practitioners.

Only limited attention has been given to multi-response problems in the Taguchi method. Logothetis and Haigh[8] applied the multiple regression technique and the linear programming approach to optimize a five-response process by the Taguchi method. However, sufficient details were not provided in their work to establish their procedure. Moreover, if the t-values of the regression coefficients are insignificant or the value of R^2 (the coefficient of determination) is low, their method's application could be limited. Their method increases the computational process complexity, thereby making it difficult for use on the shop floor. Vining and Myers[9] applied the dual response approach to achieve some of the goals of the Taguchi philosophy, specifically to obtain a target condition on the mean while minimizing the variance. They focused on the single-response problem. Castillo and Montgomery[10] further showed that the generalized reduced gradient (GRG) algorithm can lead to better solutions than those obtained with the dual response approach. They also demonstrated that the GRG algorithm can be applied to a multiple response problem. However, the above three methods may be difficult for those users having limited statistical training.

Phadke[3] used the Taguchi method to study the surface defects and wafer thickness in the polysilicon deposition process for a VLSI circuit manufacturer. Based on the judgement of relevant experience and engineering knowledge, trade-offs were made in Phadke's investigation to select the optimum factor levels for a problem with multiple quality characteristics. By human judgement, the validity of the experimental results cannot be easily assured. Contradictory results could be reached by different engineers addressing the problem. Therefore, the uncertainty in the optimum factor levels is increased. Phadke's approach can usually only be used by an experienced engineer.

Hung[11] transformed various types of quality characteristics (smaller-the-better, larger-the-better and nominal-the-best) into the nominal-the-best characteristics with a target of 0 and gave a weight to each quality characteristic for computing the SN ratio. However, his method could not handle a problem involving continuous and discrete data. When the weight of a particular quality characteristic is increased, the optimum conditions will not move toward the same direction of that quality characteristic; therefore, this result is unsatisfactory.

Shiau[12] assigned a weight to each SN ratio of the quality characteristic and summed the weighted SN ratios for computing the performance measurement of a multi-response problem. For example, there are two quality characteristics with SN ratios: $SN_1 = -10\log L_1$ and $SN_2 = -10\log L_2$, where L_1 and L_2 represent the quality losses of these two characteristics. As a result, the weighted SN ratio for this two-response problem will be $SN_0 = w_1(SN_1) + w_2(SN_2)$, where w_i is the weight of the ith response. If $SN_0 = -10\log L$, where L can be viewed as the total quality loss, we then have $L = L_1^{w_1} \cdot L_2^{w_2}$. This equation is difficult to explain from the perspective of the Taguchi method's quality loss.

Tai, Chen and Wu[13] claimed that quadratic modelling was invalid for non-symmetric loss functions. In their investigation, empirical loss functions were developed for a multi-response problem involving six variables and nine responses for the surface mount process. Multiple responses can be converted into a single response on the basis of the quality loss of each response. However, these empirical loss functions can only be used in a particular process. When their method is applied, the empirical loss functions need to be determined in advance. Consequently, the complexity of the problem is increased if this does not occur.

Pignatiello[14] presented a quadratic loss function for multiple-response quality engineering problems. The expected loss function was expressed in terms of a variance component and a squared deviation-from-target component. To minimize the expected loss function, a predictive regression model (univariate response) can be established by using controllable variables. The repeated procedure was applied to minimize the expected loss by following the descent direction and establishing a new local experimentation region. One disadvantage of his method is that the cost matrix is difficult to determine, thereby making it nearly impossible to estimate the predictive regression model precisely. Another limitation is that additional experimental observations are required in conparison to the traditional Taguchi method. Several different strategies were also discussed in Pignatiello's work. However, these strategies were either impractical or infeasible for determining the optimal factor/level combination.

Tong, Su and Wang[15] proposed a procedure to determine the multi-response signal-to-noise (MRSN) ratio through the integration of the quality loss for all responses with the application of Taguchi's SN ratios. In their method, it was still difficult to determine the weight ratio for responses. In addition, the quality loss at each trial was divided by the maximum quality loss in the total of the trials. In this case, it is likely that the optimal factor/level combination could be dominated by the 'maximum quality loss'. This fact is not desired. Accordingly, a more effective procedure is proposed in this work to optimize multi-response problems in the Taguchi method.

3. MULTIPLE ATTRIBUTE DECISION MAKING

3.1. *Multiple attribute decision making*

Multiple attribute decision making (MADM) involves the selection among some alternatives each having multiple, usually conflicting, attributes. From a practical viewpoint, the number of alternatives is predetermined in the MADM problems. The term 'attributes' is referred to as a 'goal' or a 'criterion'. MADM problems have common characteristics. For instance, multiple attributes usually conflict with each other. Each attribute has a different measurement unit. The relative importance of each attribute is usually given by a set of weights. Many MADM methods are available, with each one having its own characteristics and applicability. Hwang and Yoon[16] classified MADM problems on the basis of the type of information from the decision maker and the salient features of the information. According to their taxonomy, TOPSIS, in which the information given is a cardinal preference of the attributes, is the most suitable technique for this study. A detailed description of this method is provided later in this paper.

3.2. *Fuzzy multiple attribute decision making*

The classical MADM methods cannot effectively cope with uncertain (or imprecise) information. The use of the fuzzy set theory is a perfect means to resolve such a difficulty. Fuzzy MADM methods are designed to solve MADM problems with fuzzy data. A good source of existing fuzzy decision making studies can be found in the work of Zimmermann.[17] The existing fuzzy MADM approaches have two major drawbacks. First, cumbersome computations are required, thereby limiting fuzzy MADM's applicability to real world problems. Secondly, most approaches require that the problem's data be presented in a fuzzy format, even though they are crisp in nature. Converting crisp data into a fuzzy format will increase computational efforts. Accordingly, Chen and Hwang[18] proposed an approach to overcome these difficulties. Their approach is composed of two major phases. The first phase converts fuzzy data into crisp scores. When the problem encountered contains only crisp data, classical MADM methods can be used to determine the ranking order of alternatives in the second phase. The procedure described in Section 4 is proposed on the basis of Chen and Hwang's approach to solving fuzzy MADM problems.

3.3. *TOPSIS*

TOPSIS considers that the chosen alternative should have the shortest distance from the ideal solution and the longest distance from the negative-ideal solution. Such an approach is both comprehensible and functional. This approach stipulates only that the attributes must be numerical and comparable. For example, let a MADM problem be expressed in matrix format as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & & x_{mn} \end{pmatrix} \end{array} \quad (1)$$

where A_i ($i = 1, 2, \ldots, m$) are possible alternatives; x_j ($j = 1, 2, \ldots, n$) are attributes with which alternative performances are measured; x_{ij} is the performance of alternative A_i with respect to attribute x_j. The procedure of TOPSIS can be described in the following six steps.[16]

Step 1. Calculate the normalized decision matrix, $R = [r_{ij}]_{m \times n}$:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \; i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n \tag{2}$$

Step 2. Calculate the weighted normalized decision matrix, $V = [v_{ij}]_{m \times n}$:

$$v_{ij} = w_j r_{ij}, \; i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n \tag{3}$$

where w_j is the weight of the jth attribute and

$$\sum_{j=1}^{n} w_j = 1.$$

Step 3. Determine the ideal and negative-ideal solutions:

(a) The ideal solution:

$$\begin{aligned} A^* &= \{(\max v_{ij} | j \in J), \\ &\quad (\min v_{ij} | j \in J') | i = 1, 2, \ldots, m\} \\ &= \{V_1^*, V_2^*, \ldots, V_j^*, \ldots, V_n^*\}, \end{aligned} \tag{4}$$

(b) The negative-ideal solution:

$$\begin{aligned} A^- &= \{(\min v_{ij} | j \in J), \\ &\quad (\mathrm{Max}\ v_{ij} | j \in J') | i = 1, 2, \ldots, m\} \\ &= \{V_1^-, V_2^-, \ldots, V_j^-, \ldots, V_n^-\}, \end{aligned} \tag{5}$$

where $J = \{j = 1, 2, \ldots, n | j$ associated with benefit criteria$\}$; $J' = \{j = 1, 2, \ldots, n | j$ associated with cost criteria$\}$.

Step 4. Calculate the separation measures: The separation of each alternative from the ideal one is given as

$$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - V_j^*)^2}, \; i = 1, 2, \ldots, m \tag{6}$$

The separation of each alternative from the negative-ideal solution is given as

$$S_i^- = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^-)^2}, \; i = 1, 2, \ldots, m \tag{7}$$

Step 5. Calculate the relative closeness to the ideal solution: The relative closeness of A_i with respect to A^* is defined as

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, \; i = 1, 2, \ldots, m \tag{8}$$

Step 6. Rank the preference order: The alternative with the largest relative closeness is the best choice.

3.4. *A comparison of Taguchi's loss function and the indifference curve in TOPSIS*

Taguchi's quadratic loss function is presented in Figure 1, where y is the quality characteristic of a product and T is the target value for y. Notably, at $y = T$, the loss is zero. The loss increases slowly when y is near T; however, the loss increases rapidly as y goes further away from T. In Figure 1, the loss increases by AB when y is increased 1 unit nearing T. The loss increases by CD when y is increased 1 unit further away from T. Obviously, CD $>$ AB. When the TOPSIS method is applied, some typical indifference curves for a two attributes problem are drawn and are presented in Figure 2. In Figure 2, when $C_i^* \geq 0{\cdot}5$ or is near 1 (close to the ideal solution A^*), the marginal rate of substitution decreases with an increase in V_1. On the other hand, when $C_i^* < 0{\cdot}5$ or goes further from 1 (away from the ideal solution A^*), the marginal rate of substitution increases with an increase in V_1. From equation (8), we have $cS_i^* - (1 - c)S_i^- = 0$, where $0 < c < 1$. This equation implies that the indifference curves observed in TOPSIS can be viewed as hyperbolae. Taguchi's quadratic loss function and the indifference curve in TOPSIS have similar features. The latter can be viewed as an extension of the former. Notably, Taguchi deals with one-dimensional problems, in contrast, TOPSIS handles multi-dimensional problems. As a result, the relative closeness computed in TOPSIS can be used as a performance measurement index for optimizing multi-response problems in the Taguchi method.

4. PROPOSED OPTIMIZATION PROCEDURE

The most frequent issues encountered in multi-response problems are (a) the conflict among responses, (b) a different measurement unit for each response, and (c) a difficulty in assigning a set of weights to the present information regarding the relative importance of each response. To solve these issues, a systematic optimization procedure is proposed in this section of the paper. The proposed

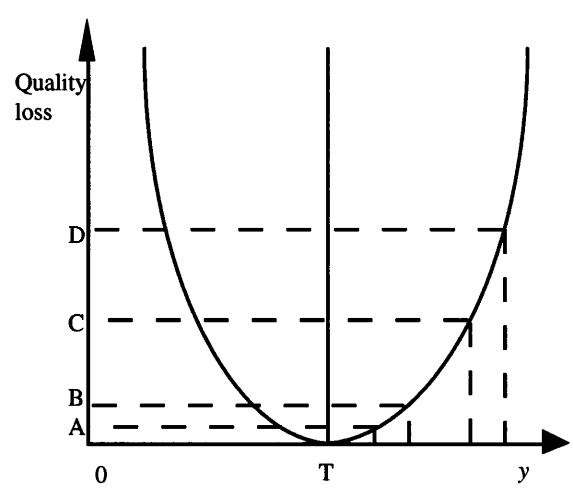

Figure 1. Taguchi's quadratic loss function

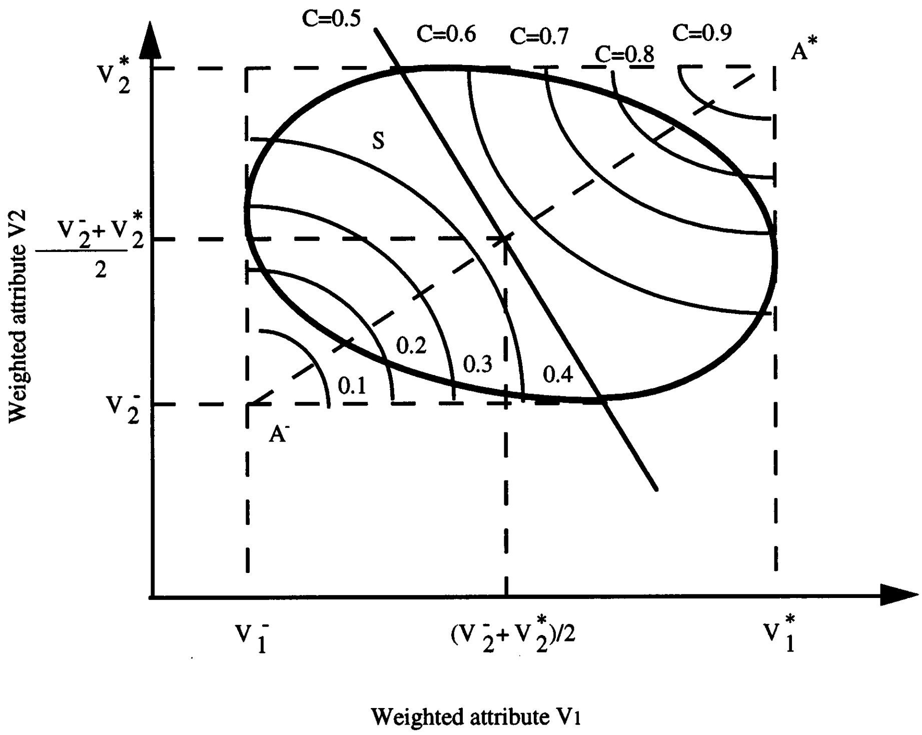

Figure 2. Typical indifference curves observed in TOPSIS[10]

procedure assumes that decision data are fuzzy. That is, the relative importance of each response is fuzzified in order to incorporate unquantifiable and/or imperfect information into a decision. In order to reduce the computational complexity and to satisfy the perspective of Taguchi's quality loss, TOPSIS is applied to find a performance measurement index for each trial. This index is calculated from the relative closeness as obtained from equation (8). This index is referred to here as a 'TOPSIS' value. The larger the TOPSIS value the better the product quality is implied. As a result, the traditional Taguchi method can be applied on the basis of TOPSIS values. The proposed optimization procedure is described as follows in six detailed steps:

Step 1. *Transform the relative importance of each response into a fuzzy number*

(a) Express the relative importance of each response by the linguistic term, as determined from the experience of an engineer.
(b) Establish a formal scale system which can be used to convert linguistic terms into their corresponding fuzzy numbers.
(c) Find a conversion scale which matches all of the linguistic terms. If more than one scale is found, the scale with the least number of terms (the simplest scale) is to be used for conversions.

Step 2. *Assign crisp scores to the selected conversion scale* (*fuzzy number*)

(a) Apply a fuzzy scoring method to convert fuzzy numbers into crisp scores.
(b) Normalize these crisp scores in order to obtain a set of weights to represent the relative importance of each response such that

$$\sum_{j=1}^{n} w_j = 1$$

where w_j is the weight of the jth response ($j = 1, 2, \ldots, n$).

For example, the linguistic terms suggested by Chen and Hwang[18] are summarized in Table I. This scale system is both comprehensible and feasible for practical applications. Table I is capable of converting linguistic terms into fuzzy numbers. By applying Chen and Hwang's fuzzy ranking method (using left and right scores), the crisp scores of fuzzy numbers in Table I are computed in Table II. The relative importance of three responses is assumed here to be very high, medium and low. Scales 3, 6, 7 and 8 in Table I contain these three terms. The simplest scale—scale 3—is chosen as our conversion scale. The corresponding crisp scores for these three responses can be found in Table II. They are: 0·909 (very high), 0·500 (medium) and 0·283 (low). These three scores can be normalized by a simple calculation. For instance, for the first response (very high), the normalized weight can be 0·537 = 0·909/(0·909 + 0·500 + 0·283). The normalized weights for the other two responses are 0·296 and 0·167.

Step 3. *Compute the quality loss*

In this step, the quality loss for each response is computed. Notably, the quality loss computation for

Table I. Linguistic terms used in the study

Scale No. of terms used	1 two	2 three	3 five	4 five	5 six	6 seven	7 nine	8 eleven
1. extremely high								yes
2. very high			yes		yes	yes	yes	yes
3. high–very high							yes	yes
4. high	yes	yes	yes	yes	yes	yes	yes	yes
5. fairly high				yes	yes		yes	
6. mol high						yes		yes
7. medium	yes	yes	yes	yes		yes	yes	yes
8. mol low						yes		yes
9. fair low				yes	yes		yes	
10. low		yes	yes	yes	yes	yes	yes	yes
11. low–very low							yes	yes
12. very low			yes		yes	yes	yes	yes
13. none								yes

Table II. Crisp scores of fuzzy numbers

Scale No. of terms used	1 two	2 three	3 five	4 five	5 six	6 seven	7 nine	8 eleven
1. extremely high								0·954
2. very high			0·909		0·917	0·909	0·917	0·864
3. high–very high							0·875	0·701
4. high	0·750	0·833	0·717	0·885	0·750	0·773	0·750	0·667
5. fairly high				0·700	0·584		0·630	
6. mol high						0·637		0·590
7. medium	0·583	0·500	0·500	0·500		0·500	0·500	0·500
8. mol low						0·363		0·410
9. fair low				0·300	0·416		0·370	
10. low		0·166	0·283	0·115	0·250	0·227	0·250	0·333
11. very–very low							0·125	0·299
12. very low			0·091		0·083	0·091	0·083	0·136
13. none								0·046

the ‘nominal-the-best’ response is based on the loss after adjusting the mean on target. According to the Taguchi method, the following three formulae are given:

$$L_{ij} = k_1 \frac{1}{r} \sum_{k=1}^{r} y_{ijk}^2 \tag{9}$$

for the smaller-the-better response

$$L_{ij} = k_2 \frac{1}{r} \sum_{k=1}^{r} \frac{1}{y_{ijk}^2} \tag{10}$$

for the larger-the-better response

$$L_{ij} = k_3 \left(\frac{s_{ij}}{\bar{y}_{ij}}\right)^2 \tag{11}$$

for the nominal-the-best response

where L_{ij} is the quality loss for the jth response at the ith trial, y_{ijk} is the observed data for the jth response at the ith trial, kth repetition, r is the number of replications for each response,

$$\bar{y}_{ij} = \frac{1}{r} \sum_{k=1}^{r} y_{ijk}$$

$$s_{ij}^2 = \frac{1}{r-1} \sum_{k=1}^{r} (y_{ijk} - \bar{y}_{ij})^2$$

k_1, k_2, k_3 are quality loss coefficients, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, r$.

Step 4. Determine the TOPSIS value for each trial

(a) Let

$$r_{ij} = \frac{L_{ij}}{\sqrt{\sum_{i=1}^{m} L_{ij}^2}} \tag{12}$$

and

$$v_{ij} = w_j r_{ij}, \; i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n, \tag{13}$$

where w_j is the weight of the jth response obtained from step 2.

(b) Apply equations (4)–(8) to compute the relative closeness of each trial (C_i^*).

(c) The TOPSIS value in the ith trial is set to C_i^*.

Step 5. *Determine the optimal factor/level combination*

(a) Estimate the factor effects based on the TOPSIS value.
(b) Determine the optimal control factors and their levels.

Step 6. *Conduct the confirmation experiment*

A confirmation experiment should be performed to verify that the optimum condition derived by the experiment actually yields an improvement. If the predicted and observed SN ratios for each response are close to each other, one can conclude that the additive model on which the experiment was based is a good approximation. As a result, the recommended optimum condition can be adopted for the process under study. If the predicted and observed SN ratios for one of the responses do not match, one may suspect that the additive model is inadequate and that the interactions are important. In the latter case, another experiment may be necessary to achieve the required objective.

5. IMPLEMENTATION

A case study is presented in this section which verifies the effectiveness of the proposed optimization procedure. This case study involves the improvement of a plasma enhanced chemical vapour deposition (PECVD) process in the fabrication of ICs. This case study was conducted by the Industrial Technology Research Institute located in Taiwan. The three-inch wafers were mounted on holders (boats). Each boat can carry five wafers. The deposition process entails depositing a uniform layer of silicon nitride (SiN*x*) with a specified thickness as one step in the IC fabrication process. In the past, the uniformity of the output was unstable. The reason for this low uniformity was unknown to the process engineers. Therefore, these engineers were unaware of how to adjust the multiple settings of the process parameters when the quality of the wafer was not meeting the requirements. In this study, an experiment was performed to determine the effects of process parameters on the silicon nitride deposition process in order to raise the quality to meet requirements. Optimal settings could hopefully be found in this experiment such that a high uniformity (i.e. low variability) for the response can be achieved.

The two responses (in order of importance) are: (a) RI: refractive index, in which the target value is 2, and (b) DT: deposition thickness, in which the target value is 1000 Å. The priority of the RI response is higher than that of the DT response. Following a discussion with the IC process engineers, the relative importances of these two responses are assumed to be 'high' and 'medium', respectively. In the conducted experiment, eight controllable factors were selected for optimization. These factors and their alternative levels are listed in Table III. The standard array L_{18} was selected for the experiment. The data for eighteen experiments are summarized in Table IV.

5.1. *The conventional Taguchi approach*

The difficulties encountered in optimizing multi-response problems are illustrated in the conventional analysis based on the Taguchi method. The factor effects on SN ratios are illustrated in Figure 3. According to the Taguchi method, the larger the SN ratio, the better the quality. Therefore, the tentative optimum setting can be separately made in the following:

$$\text{RI response: } A_1B_3C_2D_1E_3F_1G_1H_3 \tag{14}$$

$$\text{DT response: } A_1B_1C_3D_2E_2F_2G_2H_3 \tag{15}$$

Based on this observation, these two responses can be optimized by setting factor A at level 1 and setting factor H at level 3. However, determining the optimal settings for factors B, C, D, E, F and G can be complicated. For instance, factor B set at level 3 creates an advantage for the RI response, but a disadvantage for the DT response. In contrast, factor B set at level 1 creates an advantage for the DT response, but a disadvantage for the RI response. This observation illustrates that different levels of the same factor can be optimum for different responses. As a result, the decision is not clear.

5.2. *The proposed optimization procedure*

When the proposed procedure was applied in this case study, the relative importances of responses were first transformed into fuzzy numbers. From Table I, scales 1, 2, 3, 4, 6, 7 and 8 contain linguistic terms 'high' and 'medium'. Scale 1 with the least number of terms is selected as the conversion scale. From Table II, the crisp scores for the two responses are: 0·750 (high) and 0·583 (medium). These two scores were then normalized and the normalized weights were 0·562 and 0·438 which were obtained for the RI response and the DT response, respectively. Therefore, the TOPSIS value for each trial could be determined by using equations (11), (12), (13) and equations (4)–(8). The computational results are summarized in the last column of Table IV. The main effects on the TOPSIS values are summarized in Table V and their corresponding factor effects are plotted in Figure 4. The controllable factors on a TOPSIS value in order of their significance are: F, E, H, B, C, D, G and A. The larger the TOPSIS value would imply the better the quality; consequently, the tentative optimal condition can be set as $A_1B_2C_3D_2E_2F_2G_2H_3$. The predicted SN ratios under the optimum condition and the corresponding two-standard-deviation confidence

Table III. Control factors and their levels

Factor		Level 1	Level 2	Level 3
A.	Cleaning method	No	Yes	—
B.	The chamber temperature	100°C	200°C	300°C
C.	Number of runs after the chamber has been cleaned	1st	2nd	3rd
D.	The flow rate of SiH_4	6%	7%	8%
E.	The flow rate of N_2	30%	35%	40%
F.	The chamber pressure	160 mtorr	190 mtorr	220 mtorr
G.	R. F. power	30 watt	35 watt	40 watt
H.	Deposition time	11·5 min	12·5 min	13·5 min

*Starting levels are identified by underscore

Table IV. Data summary by experiment

Expt. no.	Factors A	B	C	D	E	F	G	H	Deposition thickness (DT) 1	2	3	4	5	Refractive index (RI) 1	2	3	4	5	Average DT	Average RI	TOPSIS value
1	1	1	1	1	1	1	1	1	694	839	728	688	704	2·118	1·919	1·985	2·085	2·056	730·6	2·033	0·8290
2	1	1	2	2	2	2	2	2	918	867	861	874	851	2·205	2·240	2·234	2·165	2·275	874·2	2·224	0·9718
3	1	1	3	3	3	3	3	3	936	954	930	1058	958	2·677	2·643	2·714	2·456	2·565	967·2	2·611	0·8423
4	1	2	1	1	2	2	3	3	765	828	842	768	801	2·096	1·997	1·949	2·046	2·000	800·8	2·018	0·9263
5	1	2	2	2	3	3	1	1	709	743	753	752	989	2·032	2·007	1·943	2·003	1·845	789·2	1·966	0·7686
6	1	2	3	3	1	1	2	2	795	785	846	722	833	1·860	1·838	1·842	1·999	1·858	796·2	1·879	0·8668
7	1	3	1	2	1	3	2	3	711	816	1085	787	1150	2·012	1·909	1·797	1·930	1·819	909·8	1·893	0·5820
8	1	3	2	3	2	1	3	1	580	644	602	607	811	1·834	1·760	1·760	1·782	1·744	648·8	1·776	0·8251
9	1	3	3	1	3	2	1	2	590	812	627	595	609	1·719	1·707	1·676	1·704	1·675	646·6	1·696	0·8302
10	2	1	1	3	3	2	2	1	917	1142	1126	916	966	2·097	1·911	1·889	2·014	1·960	1013·4	1·974	0·7851
11	2	1	2	1	1	3	3	2	1389	1405	1219	2063	1392	1·927	1·860	1·945	1·539	1·867	1293·6	1·828	0·6350
12	2	1	3	2	2	1	1	3	865	914	993	838	893	1·963	1·881	1·812	1·923	1·899	900·6	1·896	0·9086
13	2	2	1	2	3	1	3	2	827	884	884	851	1066	1·903	1·829	1·788	1·863	1·767	902·4	1·830	0·8706
14	2	2	2	3	1	2	1	3	787	805	780	776	976	2·103	2·020	2·011	2·107	1·968	824·8	2·042	0·8733
15	2	2	3	1	2	3	2	1	739	779	745	724	976	2·182	2·080	2·071	2·179	1·968	792·6	2·096	0·7598
16	2	3	1	3	2	3	1	2	724	721	690	1023	915	2·274	2·166	2·215	2·103	2·203	814·6	2·192	0·7284
17	2	3	2	1	3	1	2	3	771	806	785	869	859	1·942	1·905	1·909	1·916	1·900	818·0	1·914	0·9800
18	2	3	3	2	1	2	3	1	712	781	749	692	760	2·077	1·961	1·985	2·101	1·980	738·8	2·021	0·9017

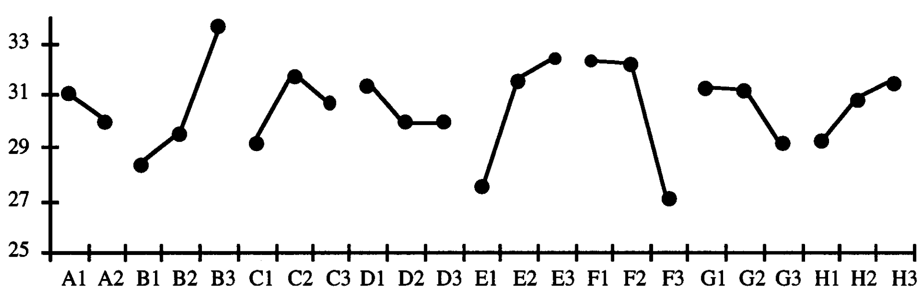

Figure 3. (a) Factor effects on SN ratios (RI response)

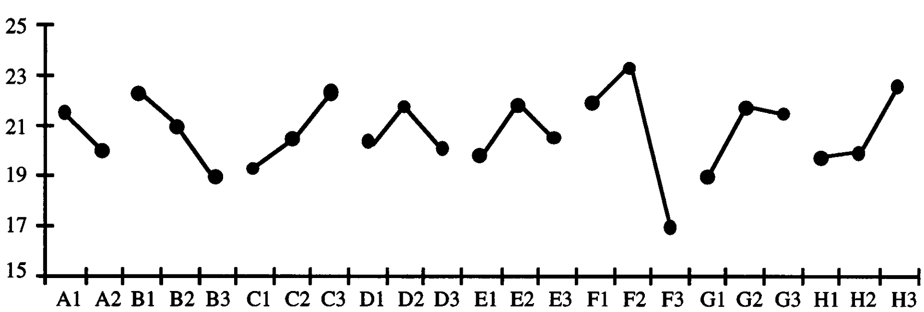

Figure 3. (b) Factor effects on SN ratios (DT response)

limits for the prediction errors are computed and presented in Table VI.

A confirmation experiment will verify the optimal condition. The results under the optimum condition $A_1B_2C_3D_2E_2F_2G_2H_3$ and under the starting condition $A_2B_1C_2D_2E_2F_2G_2H_2$ are tabulated in Table VI. According to the data in Table VI, an improvement in refractive index is 5·47 dB (the variance dropped to 32 per cent) and the deposition thickness is 9·89 dB (the variance dropped to 10 per cent). The factor

Table V. Main effects on TOPSIS values

Factors	Level 1	Level 2	Level 3	Max–Min
A	0·8269	0·7603	—	0·0666
B	0·7286	0·8442	0·8079	0·1156
C	0·7869	0·7423	0·8516	0·1093
D	0·7267	0·8339	0·8202	0·1072
E	0·6813	0·8533	0·8461	0·172
F	0·8800	0·8814	0·6194	0·262
G	0·8230	0·8243	0·7335	0·0908
H	0·8116	0·7172	0·8521	0·1349

effects on the averages of the RI response and the DT response are plotted in Figures 5(a) and 5(b), respectively. Factor D has only a slight effect on the TOPSIS value and the average of the DT response, but a more significant effect on the average of the RI response. Factor G has a slight effect on the TOPSIS value and the average of the RI response, but a more significant effect on the average of the DT response. Factors D and G can be chosen as adjustment factors for RI response and DT response, respectively. For instance, if the averages of these two responses under the optimum condition are not satisfactory, the flow rate of SiH_4 can be increased and the R. F. power decreased. This occurrence subsequently causes the RI response to be close to its target value (2) and the DT response close to its target value (1000 Å).

6. CONCLUSIONS

Multi-response problems using the Taguchi method can be solved by assuming that the weight for each response is known and that the weight is presented by crisp numbers. However, it is difficult for the weight to be directly assigned by an engineer in most cases. Moreover, fuzzy set theory can be used to incorporate data which cannot be precisely assessed. In this study, a procedure involving the introduction of fuzzy data into a MADM problem has been proposed to achieve the optimization of multi-response problems in the Taguchi method. The procedure includes the following steps: (a) transformation of the relative importance of each response, (b) assignment of crisp scores for the selected conversion scale, (c) computation of the quality loss, (d) determination of the TOPSIS value, (e) determination of the optimal factor/level combi-

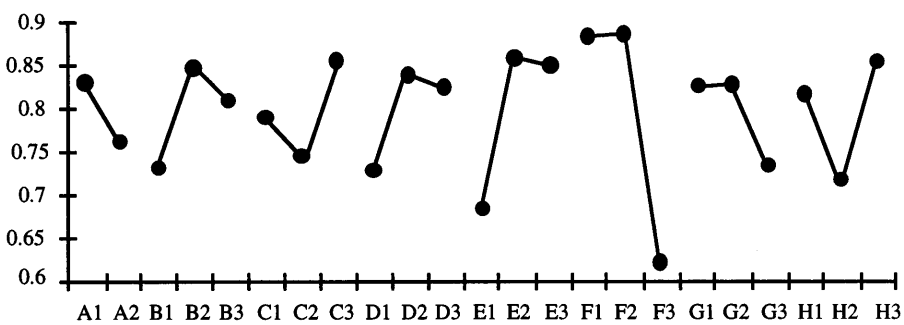

Figure 4. Factor effects on TOPSIS values

Table VI. Results of confirmation experiment

		Starting condition	Optimum condition (prediction)	Optimum condition (confirmation)	Improvement
Refractive index	SN	32·09	32·29± 5·94	37·56	5·47dB
	Average	2·0216		1·9074	
	Variance	0·00198		0·000638	
Deposition thickness	SN	22·58	27·99± 6·57	32·47	9·89dB
	Average	1043·267		1039	
	Variance	6000·8		610·5	

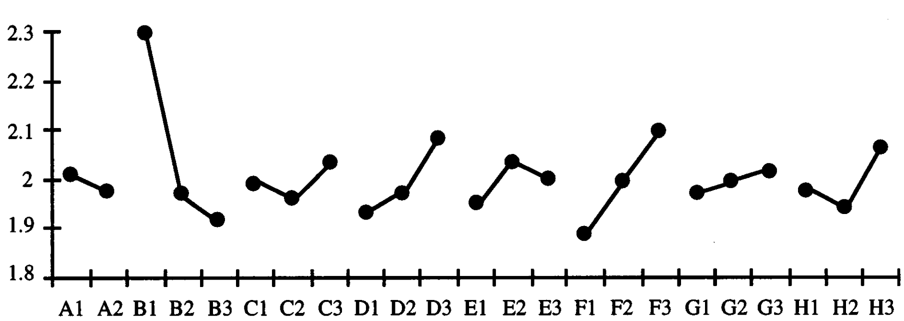

Figure 5. (a) Factor effects on the average of the RI response

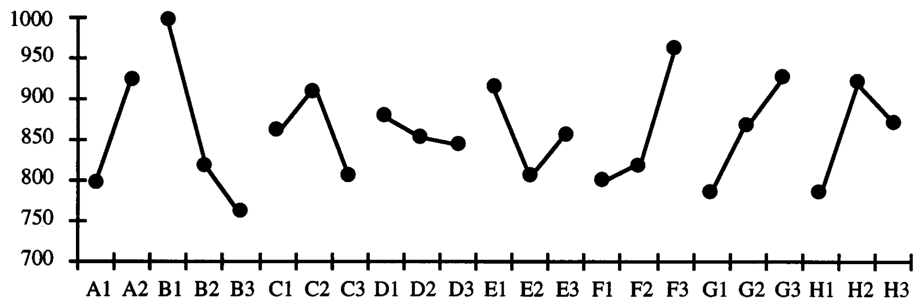

Figure 5. (b) Factor effects on the average of the DT response

nation, and (f) performance analysis of a confirmation experiment. Theoretical analysis in this study reveals that Taguchi's quadratic loss function and the indifference curve in the TOPSIS method have similar features and are compatible. The Taguchi method deals with a one-dimensional problem and TOPSIS handles a multi-dimensional problem. As a result, the relative closeness computed in TOPSIS can be used as a performance measurement index for optimizing multi-response problems in the Taguchi method. In the opinion of the authors, four significant contributions are achieved in the proposed procedure. First, the relative importance of each response can be expressed easily by the linguistic term. Secondly, only one performance measurement (TOPSIS value) is required for the multiple responses at each experimental trial. Thirdly, the procedure is a universal approach which can be used in any type of multi-response problems. Fourthly, the proposed method can simultaneously deal with a multi-response problem involving both continuous and discrete data types. Additionally, an experiment on the plasma enhanced chemical vapour deposition process in the IC manufacturing field has been performed to substantiate the authors' proposed optimization procedure.

ACKNOWLEDGEMENTS

The authors would like to thank the National Science Council of the R. O. C. for financial support of this manuscript under Contract No. NSC-84–2121-M-009–022. The Industrial Technology Research Institute (Taiwan) is also appreciated for assistance in accumulating the experimental data in this work.

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