1 Introduction

The collective dynamics of coupled chaotic system have been a topic of continuous interest in nonlinear dynamics. Many interesting phenomena such as different kinds of synchronization for two interacting chaotic attractors have already been observed in [4],[9]-[11],[13],[14]. Lyapunov exponents can describe the coherence behavior of coupled system with different coupling parameters. For example, when we couple two identical Rossler chaotic systems in the first component (see more details in the following section), with small coupling parameter, the number of positive and zero Lyapunov exponents will both be two. As the coupling parameter increases, when one of the zero exponents becomes negative, the phase synchronization takes place [14]. As the coupling parameter continues to increase, the amplitude synchronization occurs when one of the positive exponents becomes negative and then the collective dynamics become more complicated with desynchronized behavior, as coupling strength is stronger than some threshold. On the other hand, when we couple two identical Lorenz attractors in the first component, as the coupling parameter varies from zero to the value for the onset of phase synchronization, one originally zero exponent becomes positive. This is different from the previous Rossler-Rossler case, although a similarity to Rossler-Rossler case is that amplitude synchronization occurs when one of the positive exponents becomes negative. This distinct behavior tells us the collective dynamics are more complicated for coupled Lorenz-Lorenz case in the parameter interval between zero and the value for the onset of phase synchronization. But the coherence behavior for coupled Rossler-Rossler system is more complicated for the parameter value at which the desynchronization occurs. From the basic message, it is known that the behavior of Lyapunov exponents for a system of coupled Rossler-Rossler system is very different from that of a coupled Lorenz-Lorenz system.

Our interest is that if we couple identical chaotic systems using different coupling scheme, could we have similar variations and bifurcations of Lyapunov exponent? Similarly, will this variation be the same as the one for the difference system comprised of the substraction of one system from the other. The latter is often used to check if the amplitude synchronization takes place. On the other hand, what if we couple a Rossler system with a Lorenz system? Would we still have similar behavior of Lyapunov exponents as in Lorenz case; namely, an additional positive Lyapunov exponent for small coupling parameter, which is originally zero ? It is known that generalized synchronization can occur in such a setting, see [13] and other references mentioned below. If such is the case, will additional positive Lyapunov exponents affect the onset of generalized synchronization?

Primary answers to these questions can be obtained through many numerical computations. We can first use the setting below, as in [7], in which a Rossler (resp. Lorenz) is unidirectionally coupled to a Rossler (resp. Lorenz), or consider a coupled Rossler-Lorenz system and compute the Lyapunov spectrum of the whole system as a function of the coupling parameter. We can also set up a mutually coupled scheme and do the same computations. We hope to gain further understanding after accumulating enough numerical data. This can serve as a preliminary work for the scaling of Lyapunov exponent in coupled chaotic system with different coupling scheme and study connection between Lyapunov exponent and dissipation of the coherence behavior of coupled system.

In Section 2, we will present several definitions of Lyapunov exponent, and discuss the relation among these definitions. In addition, we summarize some theorems about Lyapunov exponent.

In Section 3, we will discuss The subtracted system and the coupled systems as well as their relationships between them. We then investigate the relation between Lyapunov exponents of the whole coupled system along with the ones of the associated difference system. In the second part of this section, some analytic works about variations of Lyapunov exponent with special coupling way will be addressed.

In Section 4, we present some results of numerical experiment of chaotic coupled system with different coupling scheme and provide some numerical illustrations. Section 5 will be devoted to discussion and conclusion.

2 Lyapunov Exponents in Continuous-time Systems

In the first part of this section, we will present the definitions of Lyapunov exponent, and discuss the relations among these definitions. In the second part, we summarize some theorems about Lyapunov exponent.

Before we begin introducing the definitions of Lyapunov exponent formally, we first provide the notion of characteristic exponent of functions, which was first developed by Lyapunov in 1892. The following definition can be found in [Adrianova].

Definition 2.1. *Let $f(t)$ be a complex-valued function defined on the interval $[t_0, \infty)$.*

$$\chi[f] = \limsup_{t \to \infty} \frac{1}{t} \ln |f(t)|$$

is called the **characteristic exponent** *of the function $f(t)$.*