The Visual Computer

Short Communication

A note on computing the saddle values in isosurface polygonization

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The ambiguity problem of the marching cubes method can be solved by constructing isosurfaces that are topologically consistent with the trilinear interpolant. This can be achieved by computing the saddle values of the interpolant. In this note, we analyze the trilinear interpolant and derive an efficient method of computing the saddle values.

Key words: Volume visualization – Marching cubes – Topology consistency – Saddle value

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1 Introduction

The marching cubes method (Lorensen 1987) is a popular technique for visualizing a set of volume data. This method uses 15 base cases to construct polygons for each cube to approximate the isosurfaces $F(x, y, z) = t$ for various values of t. The advantages of the polygon-based approach make it easy to adopt the technique on hardware platforms and to execute it efficiently. Unfortunately, the method has an ambiguity problem that causes anomalies in the rendered images (Durst 1988).

Several improved approaches have been proposed (Montani et al. 1994; Nielson et al. 1991; Pasco 1988; Wilhelms and Gelder 1990). These methods try to resolve ambiguous situations by preventing possible "holes" in the isosurfaces. However, these approaches could introduce the topological inconsistency problem (Natarajan 1994), which confuses our perception of the observed objects, especially when supersampling the original data is necessary.

In 1994, Natarajan developed a technique that computes topologically consistent isosurfaces by examining the *saddle values* of a trilinear interpolant. This technique also improves the method proposed by Pasco in 1988, which investigated saddle values for the bilinear interpolant only.

In this note, we analyze the trilinear interpolant. We show that the cost for computing the body saddle value can be reduced.

2 An alternative method for computing the saddle value

A more efficient method for computing saddle values is obtained by factorization of the interpolant. We now discuss the 2D case.

A 2D bilinear interpolant $B(x, y)$ defined in a square enclosed by four vertices p_{00}, p_{01}, p_{10} and p_{11} (Fig. 1) is

$$B(x, y) = axy + bx + cy + d, \tag{1}$$

where $a = v_{00} + v_{11} - v_{10} - v_{01}$, $b = v_{10} - v_{00}$, $c = v_{01} - v_{00}$, and $d = v_{00}$. The bilinear interpolant $B(x, y)$ can be factorized into

$$B(x, y) = a(x + c/a)(y + b/a) + d - bc/a.$$

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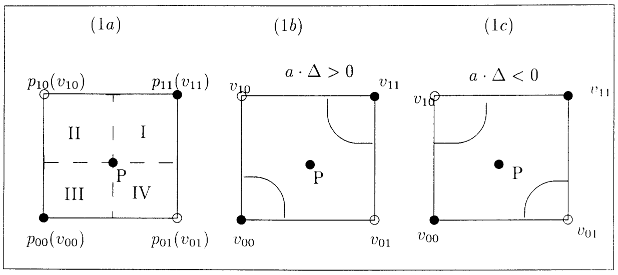

Fig. 1a–c. *P* is the saddle point that must be inside the square. The two asymptotes partition the square into four quadrants that are ordered as shown in **a**. If $a \cdot \Delta > 0$, the hyperbola lies in quadrants 1 and 3 (see **b**). Otherwise the hyperbola lies in quadrants 2 and 4 (see **c**)

Whenever a threshold t is given, function $B = t$ can be written as

$$B - t = 0 = a\left(x + \frac{c}{a}\right)\left(y + \frac{b}{a}\right) - \Delta, \tag{2}$$

where $\Delta = t - d + bc/a$.
Geometrically, Eq. 2 is a hyperbola centered at point $P = (-c/a,\ -b/a)$. P must lie in the square enclosed by p_{00}, p_{01}, p_{11}, and p_{10}. This fact can be proved by arguing that $0 \leq -c/a,\ -b/a \leq 1$ as follows. Note that $-c/a = (v_{00} - v_{01})/((v_{00} - v_{01}) + (v_{11} - v_{10}))$. Since $v_{00} - v_{01}$ and $v_{11} - v_{10}$ are both positive or both negative in the ambiguous situations, $-c/a$ must be greater than 0 and less than 1. Similar discussions leads to $0 \leq -b/a \leq 1$. Since P lies inside the square, the pair of asymptotes of the hyperbola partition the square into four quadrants as shown in Fig. 1a.
If a and Δ are both positive or both negative, then the hyperbola lies in the first and third quadrants. In this case, v_{10} and v_{01} are connected (Fig. 1b). Similarly, if one of a and Δ is positive and the other is negative, then the hyperbola lies in the second and fourth quadrants. v_{00} and v_{11} are connected (Fig. 1c). In fact, P and Δ are the *face saddle point* and the *face saddle value*, respectively.
The same discussions can be extended to a trilinear interpolant. A trilinear interpolant F interpolating the values in a cube can be expressed as follows:

$$F(x, y, z) = axyz + bxy + cyz + dxz + ex + fy + gz + h, \tag{3}$$

where coefficients $a, b, \ldots, h$ are computed directly from $v_{000}, v_{001}, \ldots, v_{111}$ (Lin and Ching 1996). If we fix x and factorize F, then we have

$$F(x, y, z) = (ax + c)\left(y + \frac{dx + g}{ax + c}\right) \times \left(z + \frac{bx + f}{ax + c}\right) + SA, \tag{4}$$

where

$$SA = ex + h - (dx + g)(bx + f)/(ax + c). \tag{5}$$

For a fixed value of x, Eq. 4 is a bilinear interpolant as discussed in the 2D case. The saddle point (x_s, y_s, z_s) of this bilinear function can be represented parametrically as

$$SP = \left(x, -\frac{dx + g}{ax + c},\ -\frac{bx + f}{ax + c}\right), \tag{6}$$

and the saddle value is SA.
If the x-coordinate of the body saddle point of F is known, the body saddle value SA can be obtained directly by evaluating Eq. 5. That is to say, the simplest way to get the body saddle value is first to compute

$$x_s = \frac{1}{a}\left(-c \pm \sqrt[2]{\frac{(af - bc)(cd - ag)}{ae - bd}}\right)$$

and then substitute x_s into Eq. 5.

3 Conclusions

The connectivity of the vertices of a 3D cube is determined by six face saddle values and one body saddle value.
Our analysis did not lead to a more efficient way to compute the face saddle values. Two multiplications and one addition operation are needed to obtain a face saddle value. To compute the body saddle value, Natarajan needs 26 multiplications, 18 additions, and 1 unit computation of square root arithmetic operations. In total, there are 38 multiplications and 24 additions, plus 1 square root arithmetic to determine the connectivity of

vertices. Our analysis in this note could not eliminate the square root operation. However, we can reduce the number of multiplications from 26 to 13, and the number of addition operations from 18 to 10. In total, 25 multiplications and 18 additions are required.

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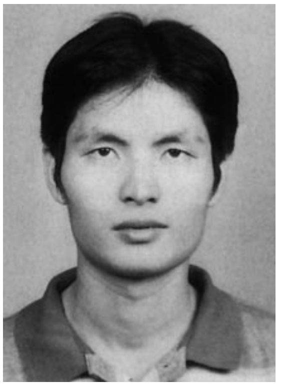

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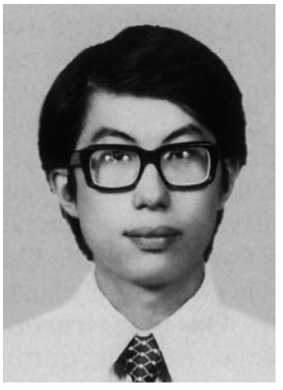

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