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A Low-Complexity Zero-forcing CFO Compensation Scheme for OFDMA Uplink Systems

Chao-Yuan Hsu and Wen-Rong Wu

***Abstract*—Similar to the conventional orthogonal frequency-division multiplexing (OFDM) system, an OFDM multiple access (OFDMA) system will have a carrier frequency offset (CFO) problem. Since CFOs of all users are different, CFO compensation in the OFDMA uplink system is much more involved. A simple, yet efficient, method is the zero-forcing (ZF) compensation method. However, it involves an inverse of an $N \times N$ CFO-induced ICI matrix, where N is the number of subcarriers. Thus, the complexity can become very high when N is large, a case commonly seen in OFDMA systems. In this work, we propose a low-complexity ZF method to overcome the problem. The main idea is to use Newton's method to solve matrix inversion iteratively. We explore the structure of the CFO-induced ICI matrix and develop a method that can implement Newton's method with fast Fourier transforms (FFTs). As a result, the required computational complexity is significantly reduced from $\mathcal{O}(N^3)$ to $\mathcal{O}(2N \log_2 N)$. Simulations show that, with only three iterations, the proposed method can have similar performance to the direct ZF method.**

***Index Terms*—Orthogonal frequency-division multiple access (OFDMA), carrier frequency offset (CFO), intercarrier interference (ICI), fast Fourier transform (FFT), Newton's method.**

I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) has been shown to be a successful technique to combat a fading channel effect in wireless communications. Since all subcarrier signals overlap orthogonally in the spectrum, it is considered a bandwidth-efficient scheme. An ideal OFDM system has no intercarrier interference (ICI) problem, and it can be easily developed as a frequency-division multiple access (FDMA) scheme. An OFDM-based FDMA system is generally referred to as an OFDMA system, which was proposed in IEEE 802.16e-2005 for broadband wireless access.

In the presence of CFO, the orthogonal property in an OFDM system is destroyed, and ICI is induced, degrading the system performance significantly [1]. Many works have been reported for CFO estimation and compensation. Different from that in OFDM systems, CFO in OFDMA uplink systems causes not only self-interference, but also multiuser interference (MUI) [2], [3]. Various methods have been proposed to solve the problem. One direct method is to estimate the CFO in the base station, and transmit the information back to mobile stations for CFO correction. Another approach is to transmit redundant information in certain subcarriers such that ICI can be cancelled in the receiver end; this approach is known as self-ICI-cancellation [8]−[13].

Yet another viable approach eliminates the need for extra transmission overhead by compensating for ICI in the receiver. CFO compensation methods for OFDMA uplink systems have been reported [14]−[18], [21]. The simplest one is to compensate for ICI with a time-domain phase-rotation for each user [14]. This approach can suppress self-ICI, but it does not take MUI into account. A post-FFT method was proposed [15] to improve the performance of the phase-rotation approach. By combining the parallel interference cancellation (PIC) technique with the method in [15], a more sophisticated scheme was developed [16]. Other PIC-related works can be found in [17], [18]. It can be observed that ICI in a subcarrier mainly comes from neighboring subcarriers. Given this, the ICI matrix is simplified into a banded matrix [19], such that the computational complexity of the zero-forcing (ZF) and minimum-mean-square-error (MMSE) CFO-compensation methods can be reduced. Taking advantage of an interleaved-OFDMA structure, [21] proposes a method that divides the whole system into several smaller subsystems, after which the MMSE method is applied to the subsystems. This method has good performance, and it requires a low computational complexity; however, it is only applicable to an ideal interleaved structure (i.e., uniform subcarrier-spacing for each user). The aforementioned methods were developed for CFO-compensation. CFO estimation methods have also been reported for OFDMA uplink systems [4]−[7].

The ZF method is simple, yet effective for CFO compensation in OFDMA uplink systems. However, because it must invert the ICI matrix, whose dimension equals the number of subcarriers (N), the complexity can become prohibitively high when N is large, as is commonly found in OFDMA systems. In this work, we propose a low-complexity ZF method to cope with the problem [20]. Using Newton's method for iterative matrix inversion and exploring the structure of the ICI matrix, we develop a method that is able to implement

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Newton's method using fast Fourier transforms (FFTs). From simulations, we find that the performance of the proposed method is similar to that of the direct ZF method, while the complexity is reduced from $\mathcal{O}(N^3)$ to $\mathcal{O}(2N\log_2 N)$. The rest of this work is organized as follows: Section 2 describes the signal model of an OFDMA uplink system, Section 3 presents the proposed method in detail and analyzes the required complexity, Section 4 presents the simulation results, and finally, Section 5 presents our conclusions.

II. Signal Model

In an OFDMA system, the available bandwidth is divided into N equally spaced subbands. Each subband has a bandwidth of $1/(NT_s)$, where T_s is the symbol sampling period. In such a system with Q users, we assume that each user uses $N_s = N/Q$ subcarriers. For user q, the transmit frequency-domain signal at the kth subcarrier is denoted by $\widetilde{x}_k^q$, where $k \in \Upsilon_q$ and Υ_q is the set of the subcarrier indices for user q. It is assumed that $\Upsilon_i \bigcap \Upsilon_j = \emptyset$ for $i \neq j$ and $\bigcup_{q=1}^{Q} \Upsilon_q = \{0, 1, \ldots, N-1\}$. OFDMA adopts the interleaved subcarrier allocation scheme: in other words, $\Upsilon_q = \{q-1, q-1+Q, \ldots, q-1+(\frac{N}{Q}-1)Q\}$. Since the subcarriers assigned to the different users are interleaved throughout the whole bandwidth, this scheme can achieve the maximum frequency diversity. For each user, we assume that the CP (cyclic prefix) length (N_g) is long enough to prevent intersymbol interference and that the channel is time-invariant in an OFDMA symbol period.

Consider a specific OFDMA symbol for user q. The channel output signal after CP removal can be expressed as $\mathbf{y}^q = \mathbf{H}^q\mathbf{x}^q$, where $\mathbf{y}^q$ is the qth user's $N \times 1$ receive time-domain signal vector, and $\mathbf{x}^q$ is the qth user's $N \times 1$ time-domain symbol vector, i.e., $\mathbf{x}^q = (1/\sqrt{N})\mathbf{G}^H\widetilde{\mathbf{x}}^q$. Here $\widetilde{\mathbf{x}}^q$ is the qth user's $N \times 1$ frequency-domain symbol vector, and $\mathbf{G}$ is the $N \times N$ normalized DFT matrix with $\mathbf{G}\mathbf{G}^H = \mathbf{I}_N$, where $\mathbf{I}_N$ is an $N \times N$ identity matrix. $\mathbf{H}^q$ is an $N \times N$ circulant channel matrix with the first $N \times 1$ column vector being $\mathbf{h}^q$, which is the channel response of $\mathbf{x}^q$. Zeros are padded in $\mathbf{h}^q$ since the channel length is assumed to be smaller than N_g. Note that the elements of $\widetilde{\mathbf{x}}^q$ are nonzero only in designated subcarrier positions and $\mathbf{H}^q$ can be decoupled as $\mathbf{G}^H\widetilde{\mathbf{H}}^q\mathbf{G}$, where $\widetilde{\mathbf{H}}^q$ is an $N \times N$ matrix and $\widetilde{\mathbf{H}}^q = diag(\widetilde{\mathbf{h}}^q)$. The notation $diag(\mathbf{g})$ indicates a diagonal matrix with a diagonal vector $\mathbf{g}$, and $\widetilde{\mathbf{h}}^q = \sqrt{N}\mathbf{G}\mathbf{h}^q$. The time-domain OFDMA symbol received from Q users can be expressed as $\mathbf{r} = (1/\sqrt{N})\sum_{q=1}^{Q}\mathbf{E}^q\mathbf{H}^q\mathbf{G}^H\widetilde{\mathbf{x}}^q + \mathbf{v}$, where $\mathbf{E}^q = diag(\mathbf{e}^q)$, $\mathbf{e}^q = [U_q^0, \ldots, U_q^{N-1}]^T$, $U_q^k = exp\{j2\pi\epsilon_q k/N\}$, and ϵ_q is the normalized CFO (with respect to the subcarrier spacing) for user q. Also, $\mathbf{v}$ denotes the $N \times 1$ noise vector. After the FFT operation, we are left with a corresponding frequency-domain signal of $\widetilde{\mathbf{r}} = \sum_{q=1}^{Q}\widetilde{\mathbf{E}}^q\widetilde{\mathbf{y}}^q + \widetilde{\mathbf{v}}$, where $\widetilde{\mathbf{E}}^q = \mathbf{G}\mathbf{E}^q\mathbf{G}^H$, $\widetilde{\mathbf{y}}^q = \widetilde{\mathbf{H}}^q\widetilde{\mathbf{x}}^q$, and $\widetilde{\mathbf{v}} = \sqrt{N}\mathbf{G}\mathbf{v}$. Note that $\widetilde{\mathbf{E}}^q$ is a circulant matrix and its first column is $\widetilde{\mathbf{e}}^q = (1/\sqrt{N})\mathbf{G}\mathbf{e}^q$. Let $\widetilde{\mathbf{x}} = [\widetilde{x}_0, \ldots, \widetilde{x}_{N-1}]^T$ and $\widetilde{\mathbf{h}} = [\widetilde{h}_0, \ldots, \widetilde{h}_{N-1}]^T$ be the composite transmit data and channel frequency-response for all users and $\widetilde{\mathbf{h}}^q = [\widetilde{h}_0^q, \ldots, \widetilde{h}_{N-1}^q]^T$. Then, $\widetilde{x}_k = \widetilde{x}_k^q$ and $\widetilde{h}_k = \widetilde{h}_k^q$ if $k \in \Upsilon_q$. Define a diagonal matrix $\mathbf{S}^q$ such that $\mathbf{S}^q(j,j) = 1$, if $j \in \Upsilon_q$ and $\mathbf{S}^q(j,j) = 0$, otherwise. Thus, we express the received frequency-domain signal as [19], [21]

$$\widetilde{\mathbf{r}} = \widetilde{\mathbf{M}}\widetilde{\mathbf{y}} + \widetilde{\mathbf{v}}, \tag{1}$$

where $\widetilde{\mathbf{y}} = \widetilde{\mathbf{H}}\widetilde{\mathbf{x}}$, $\widetilde{\mathbf{H}} = diag(\widetilde{\mathbf{h}})$, and $\widetilde{\mathbf{M}} = \sum_{q=1}^{Q}\widetilde{\mathbf{E}}^q\mathbf{S}^q$ is the CFO-induced ICI matrix.

III. Proposed CFO Compensation Method

From (1), we see that a straightforward method to compensate for ICI is the ZF method given by $\overline{\mathbf{y}}_{ZF} = \widetilde{\mathbf{M}}^{-1}\widetilde{\mathbf{r}}$ [21]. Although the direct ZF method can completely suppress ICI, it needs to invert an $N \times N$ matrix. When N is large, the required complexity can become very high. Unfortunately, in most real-world applications, N is large. For IEEE 802.16e, N can be as large as 2048. Here, we propose a low-complexity ZF method to solve the problem. The main idea is to use an iterative procedure to avoid the direct matrix inversion. In this work, specifically, we use Newton's method.

Let $\mathbf{W}_k$ be the estimate of $\widetilde{\mathbf{M}}^{-1}$ at the kth iteration. Newton's iteration for matrix inversion [22], [23] can be described as $\mathbf{W}_{k+1} = (2\mathbf{I}_N - \mathbf{W}_k\widetilde{\mathbf{M}})\mathbf{W}_k$ for $k = 0, 1, \ldots, \infty$. Let $\mathbf{R}_k = \mathbf{I}_N - \mathbf{W}_k\widetilde{\mathbf{M}}$ represent the estimation residual. Newton's iteration implies that $\| \mathbf{I}_N - \mathbf{W}_k\widetilde{\mathbf{M}} \| \leq \| \mathbf{I}_N - \mathbf{W}_0\widetilde{\mathbf{M}} \|^{2^k}$ for all k. If $\| \mathbf{I}_N - \mathbf{W}_0\widetilde{\mathbf{M}} \| < 1$, we then have quadratic convergence [24]. From Newton's iteration, we can also see that matrix-to-matrix multiplications are required and that the computational complexity of Newton's iteration is even higher than the direct matrix inversion. Thus, direct application of Newton's method is not feasible.

Now, we develop a method to solve the problem. Using Newton's iteration method, we obtain a sequence of matrices $\{\mathbf{W}_0, \mathbf{W}_1, \ldots, \mathbf{W}_k\}$. Exploring their structures, we can express $\mathbf{W}_k$ as

$$\mathbf{W}_k = \sum_{m=0}^{2^k-1} c_m^k (\mathbf{W}_0\widetilde{\mathbf{M}})^m \mathbf{W}_0, \tag{2}$$

where c_m^k is the coefficient of the mth summation term in (2). Assign c_m^k's as coefficients of a polynomial function of z, i.e., $g_k(z) = c_0^k z^0 + c_1^k z^1 + \ldots + c_{2^k-1}^k z^{2^k-1}$. Then, $g_{k+1}(z)$ can subsequently be derived from $g_k(z)$ as $g_{k+1}(z) = 2g_k(z) - z[g_k(z)]^2$, where $g_0(z) = 1$. Note that (2) is not in the original form of Newton's iteration.

Also note that our final objective is to obtain the CFO-compensated result $\mathbf{W}_k\widetilde{\mathbf{r}}$, not the matrix inverse $\mathbf{W}_k$ itself. Multiply (2) by $\widetilde{\mathbf{r}}$ and let $\overline{\mathbf{y}}_k = \mathbf{W}_k\widetilde{\mathbf{r}}$ and $\overline{\mathbf{s}}_m = (\mathbf{W}_0\widetilde{\mathbf{M}})^m\mathbf{W}_0\widetilde{\mathbf{r}}$, which gives us

$$\overline{\mathbf{y}}_k = \sum_{m=0}^{2^k-1} c_m^k \overline{\mathbf{s}}_m. \tag{3}$$

From the definition of $\overline{\mathbf{s}}_m$, we obtain the following iterative step: $\overline{\mathbf{s}}_{m+1} = (\mathbf{W}_0\widetilde{\mathbf{M}})\overline{\mathbf{s}}_m$, which allows $\overline{\mathbf{s}}_m$ to be calculated recursively. With this approach, we have transformed the matrix-to-matrix multiplications in (2) into the matrix-to-vector multiplications in (3).

To complete our algorithm, we further let $\mathbf{W}_0$ be diagonal and recall that $\widetilde{\mathbf{M}} = \sum_{q=1}^{Q}\mathbf{G}\mathbf{E}^q\mathbf{G}^H\mathbf{S}^q$. Thus, we can rewrite $\overline{\mathbf{s}}_{m+1}$ as $\overline{\mathbf{s}}_{m+1} = \mathbf{W}_0\mathbf{G}[\sum_{q=1}^{Q}\mathbf{E}^q(\mathbf{G}^H\mathbf{S}^q\overline{\mathbf{s}}_m)]$. Note that

TABLE I
COMPLEXITY COMPARISON OF THE PROPOSED METHOD, THE BANDED ZF METHOD, AND THE DIRECT ZF METHOD.

Complexity	Proposed method	Banded ZF method	Direct ZF method
Real multiplications	$2(2^k-1+Q)N\log_2(N)+2(2^k-1)N\log_2(N/Q)+[8(2^k-1)Q+2(2^k+1)]N+4Q(2S+1)$	$2QN\log 2(N)+(4B^2+14B+6)N-\frac{8}{3}B^3-9B^2-\frac{19}{3}B$	$\frac{4}{3}N^3+5N^2+2QN\log_2(N)-\frac{1}{3}N$
Real divisions	$2Q$	$(2B+2)N-B^2-B$	N^2+N
Real additions	$3(2^k-1+Q)N\log_2(N)+3(2^k-1)N\log_2(N/Q)+[6(2^k-1)Q+2]N+2Q(3S+1)$	$3QN\log 2(N)+(4B^2+11B+3)N-\frac{8}{3}B^3-\frac{15}{2}B^2-\frac{29}{6}B$	$\frac{4}{3}N^3+\frac{7}{2}N^2+3QN\log_2(N)-\frac{11}{6}N$

calculating $\overline{\mathbf{s}}_{m+1}$ only involves vector multiplications, IDFTs, and a DFT. It is well-known that DFT/IDFT can be implemented with FFT/IFFT and the computational complexity can be greatly reduced. Thus, the required computational complexity is reduced from $\mathcal{O}(N^3)$ to $\mathcal{O}((Q+1)N\log_2 N)$.

Utilizing the interleaved-OFDMA structure, the computational complexity can be further reduced. Let $\overline{\mathbf{s}}_m = [\overline{s}_{m,0},\ldots,\overline{s}_{m,N-1}]^T$ and $\mathbf{u}_m^q = \mathbf{S}^q\overline{\mathbf{s}}_m = [u_{m,0}^q,\ldots,u_{m,N-1}^q]^T$. From the definition of $\mathbf{S}^q$, $u_{m,i}^q = \overline{s}_{m,i}$, if $i\in\Upsilon_q$ and $u_{m,i}^q=0$, otherwise. This is to say that $\mathbf{u}_m^q$ corresponds to an upsampled sequence of the desired elements in $\overline{\mathbf{s}}_m$. The nonzero elements in $\mathbf{u}_m^q$, denoted by $\mathbf{d}_m^q = [\overline{s}_{m,q-1},\ldots,\overline{s}_{m,q-1+(N/Q-1)\times Q}]^T$, can be obtained by circularly shifting $\mathbf{u}_m^q$ with $q-1$ elements and downsampling the result with a factor of Q. Let $\overline{\mathbf{d}}_m^q = (1/\sqrt{Q})\mathbf{G}_{N_s}^H\mathbf{d}_m^q$, where $\mathbf{G}_{N_s}$ is an $N_s\times N_s$ normalized DFT matrix, and construct an $N\times 1$ vector by duplicating $\overline{\mathbf{d}}_m^q$ Q times, shown as $\mathbf{a}_m^q = [(\overline{\mathbf{d}}_m^q)^T,\ldots,(\overline{\mathbf{d}}_m^q)^T]^T$. We can then express $\mathbf{G}^H\mathbf{S}^q\overline{\mathbf{s}}_m$ as $\mathbf{G}^H\mathbf{S}^q\overline{\mathbf{s}}_m = \mathbf{C}^q\mathbf{a}_m^q$, where $\mathbf{C}^q = diag([Z_q^0,\ldots,Z_q^{N-1}]^T)$, and $Z_q^k = exp\{j2\pi(q-1)k/N\}$. Note that $\mathbf{C}^q$ results from circlurly shifting $\mathbf{u}_m^q$. As a result, we can implement $\mathbf{G}^H\mathbf{S}^q\overline{\mathbf{s}}_m$ by an IDFT with dimension N/Q instead of N. Using this approach, we can reduce the complexity further with $\overline{\mathbf{s}}_{m+1} = \mathbf{W}_0\mathbf{G}\left(\sum_{q=1}^{Q}\mathbf{E}^q\mathbf{C}^q\mathbf{a}_m^q\right)$. Note that $\mathbf{a}_m^q$ is a column vector and that both $\mathbf{C}^q$ and $\mathbf{E}^q$ are diagonal matrices. As assumed, $\mathbf{W}_0$ is a diagonal matrix. As a result, this approach only involves one FFT of size N, Q IFFTs of size N/Q, and several vector operations. Thus, the computational complexity of the ZF method can be reduced from $\mathcal{O}(N^3)$ to $\mathcal{O}(N\log_2(N^2/Q))$.

The final task is to determine $\mathbf{W}_0$. A good initial value can reduce the number of iterations significantly. Letting $\mathbf{W}_0 = diag([w_0,w_1,\ldots,w_{N-1}]^T)$, we propose a minimum-Frobenius-norm criterion to obtain optimum initial values. The criterion is shown as $\mathbf{W}_{opt,0} = \arg\min_{\mathbf{w}_0}\|\mathbf{I}_N-\mathbf{W}_0\widetilde{\mathbf{M}}\|_F^2$, where $\|\mathbf{R}\|_F$ denotes the Frobenius norm of $\mathbf{R}$. The optimum initial values can be obtained by setting $\partial\{\|\mathbf{I}_N-\mathbf{W}_0\widetilde{\mathbf{M}}\|_F^2\}/\partial w_k^* = 0$. Thus, we can express the optimal initial value $w_{opt,k}$ as $w_{opt,k} = \widetilde{m}_{k,k}^*/\sum_{j=0}^{N-1}|\widetilde{m}_{k,j}|^2$, where $\widetilde{m}_{i,j} = \widetilde{\mathbf{M}}(i,j)$. For further complexity reduction, we approximate $w_{opt,k}$ as $w_{opt,k}\approx\widetilde{m}_{k,k}^*/\sum_{j=<k-S:k+S,N>}|\widetilde{m}_{k,j}|^2$, where S is the number of ICI terms considered ($0\le S\le N/2-1$), and $<i:j,N>$ denotes a sequence of $\{i-N\lfloor i/N\rfloor, i+1-N\lfloor(i+1)/N\rfloor,\ldots,j-N\lfloor j/N\rfloor\}$ (i and j are integers and $i\le j$). The approximation is based on the fact that the ICI in a subcarrier mainly comes from neighboring subcarriers.

For the direct ZF method, we apply Gaussian elimination [25] to implement the matrix inversion. Finally, for signal detection, we apply a one-tap frequency domain channel equalizer to each subcarrier. The result can be expressed as $\overline{\mathbf{x}} = \widetilde{\mathbf{H}}^{-1}\overline{\mathbf{y}}$, where $\overline{\mathbf{x}}$ is the estimate of $\widetilde{\mathbf{x}}$ while $\overline{\mathbf{y}}$ is the CFO-compensated $\widetilde{\mathbf{y}}$. The complexity comparison for the proposed and existing methods is shown in Table I. For convenience, the methods in [15] and [19] are referred to as the CLJL and banded ZF methods, respectively. In Table I, B is the ICI matrix bandwidth for the banded ZF method.

IV. SIMULATIONS

In this section, we present simulation results to evaluate the performance of the proposed method. Here, we use an interleaved-OFDMA uplink system with $N=2048$, $Q=16$, and $N_g = 128$. The modulation scheme is 16-QAM. The channel length, L, is set to 127 for all users, and the power delay profile for the qth user is described by the exponential function $\sigma_{q,l}^2 = e^{-\alpha_q l}/\sum_{m=0}^{L-1}e^{-\alpha_q m}$, where l is the tap index and α_q is a parameter of the function. Here, we let $\{\alpha_1,\alpha_2,\ldots,\alpha_Q\} = \{0,0.2,0.4,\ldots,3\}$. Each channel tap fades independently, and it has a Rayleigh distribution. The averaged bit-error-rate (BER) is adopted as the performance index, and CFOs for all users are set to {0.1, -0.2, -0.05, 0.2, -0.3, 0, -0.1, 0.4, -0.3, 0.05, 0, -0.1, 0.05, -0.1, 0.3, 0.15}. It is found that the performance of the proposed method with $S=2$ is almost the same as that with $S=1023$. Thus, in the following simulations, we only consider the condition $S=2$.

The performances of five methods, namely, the conventional, CLJL, direct ZF, banded ZF, and proposed methods, are compared in our simulations. The conventional method is that described in [14]. Figure 1 shows the simulation results. From this figure, we find that the conventional and CLJL methods both have a serious error floor phenomenon. This is because compensation of the qth user's CFO may cause other users' CFOs to become enlarged, contributing to an increased MUI. The performance of the proposed method with three iterations can approach that of the direct ZF method. The complexity of the banded ZF method depends greatly on its band dimension, denoted as B. For a fair comparison, we let B be 16 for the banded ZF method. In this case, the complexities of the banded ZF method and the proposed method ($k=3$) are roughly equal. Figure 1 shows that the proposed method performs much better than the banded ZF method ($B=16$).

To see the impact of the CFO magnitude, we consider a scenario in which the Qth user's CFO is increased from 0 to 0.5. The CFOs of the other $(Q-1)$ users remain the same as

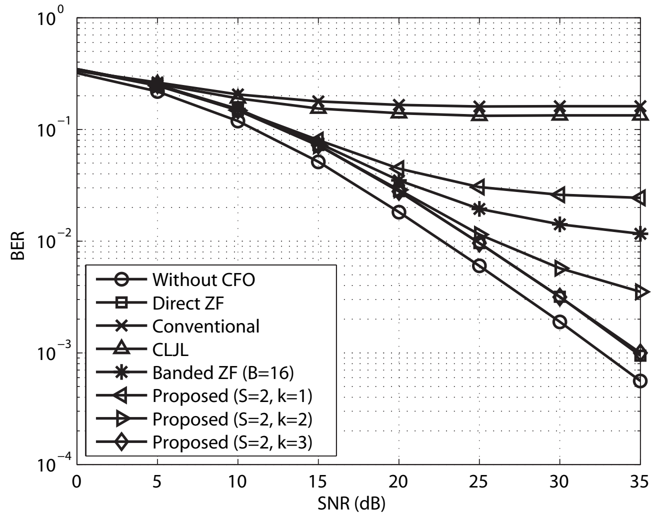

Fig. 1. BER performance comparison for the conventional, CLJL, proposed, and direct ZF methods (16-QAM modulation, and CFOs = {0.1, -0.2, -0.05, 0.2, -0.3, 0, -0.1, 0.4, -0.3, 0.05, 0, -0.1, 0.05, -0.1, 0.3, 0.15}).

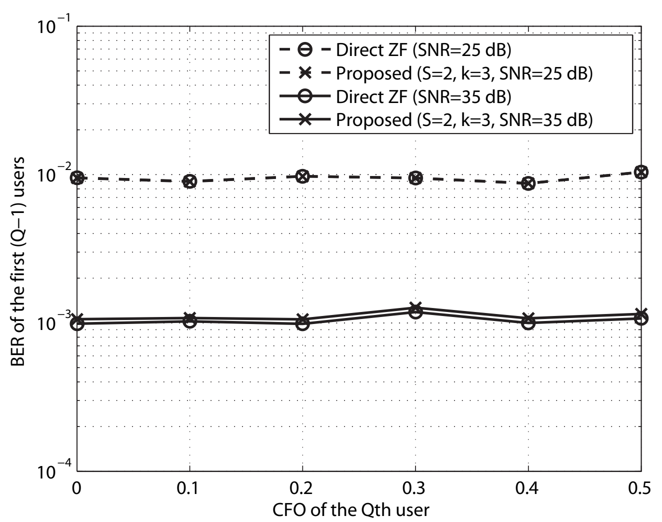

Fig. 2. BER performance comparison for the proposed and direct ZF methods (16-QAM modulation, CFOs of the first (Q-1) users = {0.1, -0.2, -0.05, 0.2, -0.3, 0, -0.1, 0.4, -0.3, 0.05, 0, -0.1, 0.05, -0.1, 0.3}, and the CFO of the Qth user increases from 0 to 0.5).

those in Fig. 1. Fig. 2 shows the averaged BER of the first $(Q-1)$ users. From this figure, we can see that the proposed method $(k = 3)$ is largely unaffected by increasing the CFO of the Qth user. Also, the proposed method has the same performance as the direct ZF method. Since OFDMA is a multiuser system, the near-far phenomenon may occur. To see the impact of the phenomenon, we consider a scenario in which the powers of the first $(Q - 1)$ users are equal, while that of the Qth user is varied. The power of the Qth user over that of one of the remaining users is defined as the near-far power ratio κ, which ranges from -15 dB to 15 dB. Here, CFOs are set as shown in Fig. 1. Figure 3 shows the averaged BER of the first $(Q-1)$ users. From this figure, we see that the near-far effect does affect BER, slightly. For $\kappa \leq 5$ dB, there is almost no performance degradation. We also find that the proposed and direct ZF methods have similar performance regardless of the value of κ.

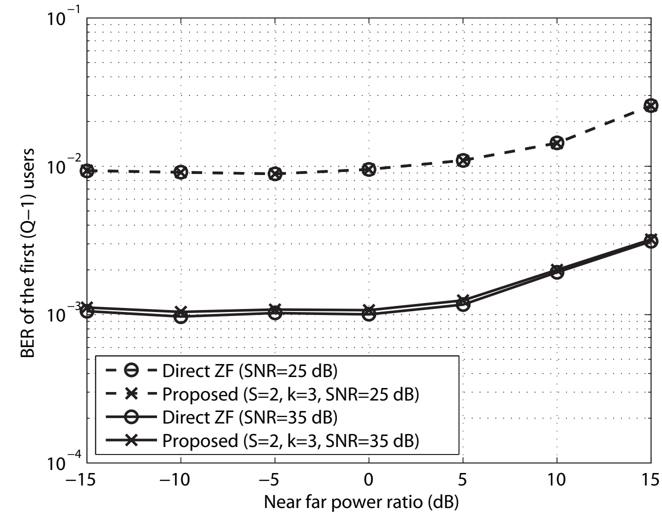

Fig. 3. BER performance comparison for the proposed method $(k = 3)$ and the direct ZF method in the near-far scenario (16-QAM modulation, and CFOs = {0.1, -0.2, -0.05, 0.2, -0.3, 0, -0.1, 0.4, -0.3, 0.05, 0, -0.1, 0.05, -0.1, 0.3, 0.15}).

TABLE II
COMPLEXITY COMPARISON OF THE DIRECT ZF METHOD, THE BANDED ZF METHOD, AND THE PROPOSED METHOD WHEN $N = 2048$ AND $Q = 16$.

Methods	Real multiplications	Real divisions	Real additions
Direct ZF	11474937856	4196352	11469003776
Banded ZF ($B = 16$)	3275760	69360	3532168
Proposed ($S = 2$, $k = 3$)	3109184 (0.000271, 0.949150)	32 (0.000008, 0.000461)	3236064 (0.000282, 0.916170)

Table II shows the computational complexities of the direct ZF, banded ZF, and proposed algorithms. In the table, the two numbers inside each set of parentheses (in the forth row) are the ratios of the number of operations (indicated by each column) required for the proposed method to those of the direct ZF and banded ZF methods, respectively. From this table, we can see that the real multiplications/additions/divisions required for the proposed method are 0.000271/0.000282/0.000008 times those for the direct ZF method. It is apparent that the proposed method requires a much lower complexity. Although the banded ZF method can have low complexity, its performance is not satisfactory. At a similar complexity, the proposed method outperforms the banded ZF method.

V. Conclusions

In this work, we propose a low-complexity CFO-compensation method for an interleaved-OFDMA uplink system. The proposed method is an efficient implementation of the ZF method. Using Newton's iteration for matrix inversion and exploring the structure inherent in the CFO-induced ICI matrix, we develop a method that can be implemented with FFTs. As a result, the complexity can be reduced to $\mathcal{O}(2N\log_2 N)$. Since the FFT/IFFT module is already available in OFDMA transceivers, implementation of the proposed method only requires limited extra circuits, facilitating its real-world application.

REFERENCES

[1] T. Pollet, M. Van Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise," *IEEE Trans. Commun.*, vol. 43, no. 7, pp. 191–193, Feb./Mar./Apr. 1995.

[2] L. Wei and C. Schlegel, "Synchronization requirements for multi-user OFDM on satellite mobile and two-path Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 43, no. 2–4, pp. 887–895, Feb./Apr. 1995.

[3] M. S. El-Tanany, and Y. Wu, and L. Hazy, "OFDM uplink for interactive broadband wireless: analysis and simulation in the presence of carrier, clock and timing errors," *IEEE Trans. Broadcast.*, vol. 47, pp. 3–19, Mar. 2001.

[4] Z. Cao, U. Tureli, and Y.-D. Yao, "Deterministic multiuser carrier-frequency offset estimation for interleaved OFDMA uplink," *IEEE Trans. Commun.*, vol. 52, no. 9, pp. 1585–1594, Sept. 2004.

[5] M.-O. Pun, M. Morelli, and C.-C. J. Kuo, "Maximum-likelihood synchronization and channel estimation for OFDMA uplink transmissions," *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 726–736, Apr. 2006.

[6] J. J. van de Beek, P. O. Börjesson, M.-L. Boucheret, D. Landström, J. M. Arenas, P. Ödling, C. Östberg, M. Wahlqvist, and S. K. Wilson, "A time and frequency synchronization scheme for multiuser OFDM," *IEEE J. Select. Areas Commun.*, vol. 17, no. 11, pp. 1900–1914, Nov. 1999.

[7] M. Moreli, "Timing and frequency synchronization for the uplink of an OFDMA system," *IEEE Trans. Commun.*, vol. 52, no. 2, pp. 296–306, Feb. 2004.

[8] Y. Zhao and S. Häggman, "Intercarrier interference self-cancellation scheme for OFDM mobile communication systems," *IEEE Trans. Commun.*, vol. 49, no. 7, pp. 1185–1191, July 2001.

[9] J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM," *IEEE Trans. Commun.*, vol. 47, no. 3, pp. 365–369, Mar. 1999.

[10] K. Sathananthan, R. M. A. P. Rajatheva, and S. B. Slimane, "Cancellation technique to reduce intercarrier interference in OFDM," *IEEE Electron. Lett.*, vol. 36, no. 25, pp. 2078–2079, Dec. 2000.

[11] K. Sathananthan and C. Tellambura, "Partial transmit sequence and selected mapping schemes to reduce ICI in OFDM systems," *IEEE Commun. Lett.*, vol. 6, no. 8, pp. 313–315, Aug. 2002.

[12] S.-H. Tsai, Y.-P. Lin, and C.-C. J. Kuo, "An approximately MAI-free multiaccess OFDM system in carrier frequency offset environment," *IEEE Trans. Signal Processing*, vol. 53, no. 11, pp. 4339–4353, Nov. 2005.

[13] H. Yoo, and D. Hong, "Edge sidelobe suppressor scheme for OFDMA uplink systems," *IEEE Commun. Lett.*, vol. 7, no. 11, pp. 534–536, Nov. 2003.

[14] A. M. Tonello and S. Pupolin, "Performance of single user detectors in multitone multiple access asynchronous communications," in *Proc. IEEE 55th VTC*, vol. 1, May 2002, pp. 199–203.

[15] J. Choi, C. Lee, H. W. Jung, and Y. H. Lee, "Carrier frequency offset compensation for uplink of OFDM-FDMA systems," *IEEE Commun. Lett.*, vol. 4, pp. 414–416, Dec. 2000.

[16] D. Huang and K. B. Letaief, "An interference-cancellation scheme for carrier frequency offsets correction in OFDMA systems," *IEEE Trans. Commun.*, vol. 53, pp. 1155–1165, July 2005.

[17] M. Huang, X. Chen, S. Zhou, and J. Wang, "Iterative ICI cancellation algorithm for uplink OFDMA system with carrier-frequency offset," in *Proc. IEEE 62nd VTC-2005-Fall*, vol. 3, Sept. 2005, pp. 1613–1617.

[18] R. Fantacci, D. Marabissi, and S. Papini, "Multiuser interference cancellation receivers for OFDMA uplink communications with carrier frequency offset," in *Proc. IEEE GLOBECOM '04*, vol. 5, Nov. 2004, pp. 2808–2812.

[19] Z. Cao, U. Tureli, Y.-D. Yao, and P. Honan, "Frequency synchronization for generalized OFDMA uplink," in *Proc. IEEE GLOBECOM '04*, vol. 2, Nov. 2004, pp. 1071–1075.

[20] C.-Y. Hsu and W.-R. Wu, "Low-complexity CFO compensation for uplink OFDMA systems," in *Proc. IEEE PIMRC'06*, Helsinki, Finland, Sept. 2006.

[21] Z. Cao, U. Tureli, and Y.-D. Yao, "Analysis of two receiver schemes for interleaved OFDMA uplink," in *Proc. Conference Record of the Thirty-Sixth Asilomar Conference on Signals, Systems and Computers*, vol. 2, Nov. 2002, pp. 1818–1821.

[22] G. Schulz, "Iterative Berechnung der reziproken Matrix," *Z. Angew. Math. Mech.*, v. 13, pp. 57–59, 1933.

[23] A. S. Householder, *The Theory of Matrix in Numerical Analysis.* New York: Dover Publications, 1964.

[24] V. Y. Pan, Y. Rami, and X. Wang, "Structured matrices and Newton's iteration: unified approach," *Linear Algebra Appl.*, vol. 343-344, pp. 233–265, Mar. 2002.

[25] S. J. Leon, *Linear Algebra With Applications,* 5th ed. Upper Saddle River, NJ: Prentice-Hall, 1998.