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An evaluation of the time-varying extended logistic, simple logistic, and Gompertz models for forecasting short product lifecycles

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ABSTRACT

Many successful technology forecasting models have been developed but few researchers have explored a model that can best predict short product lifecycles. This research studies the forecast accuracy of long and short product lifecycle datasets using simple logistic, Gompertz, and the time-varying extended logistic models. The performance of the models was evaluated using the mean absolute deviation and the root mean square error. Time series datasets for 22 electronic products were used to evaluate and compare the performance of the three models. The results show that the time-varying extended logistic model fits short product lifecycle datasets 70% better than the simple logistic and the Gompertz models. The findings also show that the time-varying extended logistic model is better suited to predict market capacity with limited historical data as is typically the case for short lifecycle products.

1. Introduction

With the rapid introduction of new technologies and fast design to satisfy consumer demand, electronic products and services are often replaced within a few years. The product lifecycle for electronic goods, which used to be about 10 years in the 1960s, fell to about 5 years in the 1980s and is now less than two years for consumer electronic products such as cell phones and computers. As product lifecycles become shorter, less data are available for market analysis and technology forecasting. Given the current market situation, smaller datasets must be used to forecast future trends of new electronic products and services. Hasted and Ehlers [1] define a small dataset as the dataset which covers only short time intervals with fewer than 30 data points.

A product lifecycle is typically divided into four stages that include introduction, growth, maturity and decline [2]. During the introduction stage, the product is new to the market with little awareness and as a result there is slow sales growth. The growth stage, on the other hand, is characterized by a period of rapid sales growth resulting from the product being widely accepted by the marketplace. As sales growth declines, the product enters the mature stage, and finally, when the marketplace is saturated with the product or a substitute product is introduced, product sales decline. The product lifecycle is often modeled using growth curves or sigmoidal curves which have an inflection point and approaches a fixed limit [3–9].

Growth curves (the first derivative of the product lifecycle curve) are widely used in technology forecasting [10–16] since technology product growth is often very slow during the introduction stage (e.g., a new product replacing a mature product) which is then followed by rapid exponential growth when barriers to product adoption fall. The growth then approaches a market share limit. The limit reflects the saturation of the marketplace with the product or the replacement of the product with another. The curve also models an inflection or break point where growth ends and decline begins.

Many growth curve models have been developed to forecast the penetration rate of technology based products with the simple logistic curve and the Gompertz curve the most frequently referenced [5,6,9,12]. However, when using these two models to forecast market share, care must be taken to set the upper limit of the curve correctly or the prediction will become inaccurate [9]. The upper limit is the maximum possible value and represents the maximum penetration rate or sales volume. Setting the upper limit to growth can be difficult and ambiguous. If the product will likely be popular and used for decades, then the upper limit is set to 100% of the penetration rate. This means that the product will be completely replaced only after everyone in the market has purchased the product. However, when marketers consider new technology products such as computer games or new model cell phones, the value for the upper limit to market share growth can be difficult to estimate. That is, a computer game can be quickly replaced by another game after only reaching 10% market share.

In order to avoid the problem of estimating the market share capacity for the simple logistic and the Gompertz models, Meyer and Ausubel [14] proposed the extended logistic model. Under this

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model, the capacity (or upper limit) of the curve is not constant but is dynamic over time. Meyer and Ausubel [14] also proposed that technology innovations do not occur evenly through time but instead appear in clusters or "innovation waves". Thus, they formulated an extended logistics model which is a simple logistics model with a carrying capacity $k(t)$ that is itself a logistics function of time. Therefore, the researchers extend the constant capacity (k) of the simple logistic model by embedding the carrying capacity in the constant. This study applies the embedded carrying capacity concept to the study of electronics products using a time-varying extended logistic model.

The emergence of short product lifecycles has been addressed in the supply chain and inventory management literature [7,8,17] and there is general agreement that improved prediction of these lifecycles will benefit the management of supply chains, inventories, and product design. However, these new technology lifecycles are a modern phenomenon and the datasets (which characteristically have fewer data points and shorter time periods) challenge the assumptions and applications of traditional forecasting methods.

Traditional forecasting models, like the simple logistic and Gompertz models, require that the upper limit of the curve be estimated prior to the forecast. Since it is difficult to estimate the demand of a new product or the arrival of a substitute product with limited data, traditional approaches are considered unreliable and inaccurate. Therefore, a time-varying extended logistic model with flexible capacity is proposed where the capacity (or upper limit) of the curve is not constant but is dynamic over time.

The proposition of this research is that the time-varying extended logistic model is better than the simple logistic and the Gompertz models when forecasting both long and short product lifecycles. Six time-series datasets describing market penetration rates and 16 datasets describing cumulative sales volumes were used to evaluate model performance. The electronic consumer goods datasets consist of six sets representing long product lifecycles and 16 sets representing short product lifecycles.

Section 1 of this paper provides an introduction and Section 2 discusses the challenges of forecasting short product lifecycles. Section 3 describes traditional and newly developed technology forecasting models including the simple logistic model, the Gompertz model, and the time-varying extended logistic model. Section 4 describes the analytical process of this study and Section 5 provides an empirical case that compares the performance of the models. The last section provides a summary and conclusion as well as the limitations of the study.

2. Forecasting short product lifecycles

Short product lifecycles have become more common in high technology and fashion-based industries which need to continuously introduce new consumer products to remain competitive [7,17]. New electronic products with more functions, faster speed, and finer quality are continuously being introduced and quickly replace models which may only be one year old. Quell et al. [18] analyzed 37 types of home appliance from 1922 to 1979 and demonstrated that the shortening of product lifecycles is an important issue for product designers and planners. Given the reality of this market condition, the development of new forecasting techniques will improve the competitive response and manufacturing strategy of companies.

In 1969, Bass proposed a diffusion model to forecast the sales volume of new products [3] that used the adoption rates of innovators and imitators. Innovators are buyers that are not influenced by the previous buyers when making purchase decisions while imitators are those who are influenced by earlier buyers. The Bass model has been widely applied by practitioners and modified by researchers to forecast short product lifecycles.

Kurawarwala and Matsuo [7] proposed a growth model that forecasts the seasonal sales volume demand of short product lifecycles based on the Bass diffusion model. Thirty-eight monthly data points for five different personal computer products were used to estimate seasonal demand and to compare the fit and forecast performance for three models. The measures used for model comparison were the sum of squared error (SSE), the root mean squared error (RMSE), and the mean absolute deviation (MAD). Zhu and Thonemann [17] used the discrete version of the Bass diffusion model and improved on Kurawarwala and Matsuo [8] model to develop an adaptive forecasting algorithm. The demand data for a PC manufacturer was used to test the forecasting performance of the algorithm. Chen [19] proposed an extended logistic model, which is called the time-varying extended logistic model. This research uses the model from Chen's study of seven home appliance datasets to demonstrate that the extended logistic model improved the forecast of both long and short lifecycle datasets.

Lackman [20] reported that the simple logistic and the Gompertz models are suitable for forecasting high technology products. Morrison [6] also showed that the simple logistic and the Gompertz models can be used to forecast the growth of new products. However, when the author applied the models, the upper limit was set subjectively. Bengisu and Nekhili [9] used the simple logistic and the Gompertz models to predict emerging technologies using publications and patents from science and technology databases and Boretos [21] used the simple logistic model to show that the diffusion of mobile phone technology follows an S-curve.

Meade and Islam [12] compared 17 growth models based on 25 time series datasets describing the telecommunications market. Their literature review shows that the simple logistic model is the most widely used. The authors conclude that basic forecasting models using two or three parameters, such as the simple logistic and Gompertz model, offer the best forecasting performance. Their research used datasets for traditional land-line telephones to compare forecasting models. However, the classic telephone introduced in the 1960s and which remained in use through the 1980s has a long product lifecycle that lasted over 30 years. When there are sufficient data points, the trajectory of the product growth curve is clear and the point of inflection can be calculated. If the point of inflection can be estimated, then the upper limit of the simple logistic and the Gompertz models can also be estimated. The simple logistic model is symmetric about the point of inflection. So if the inflection point is defined, the upper limit is twice the market share that occurs at the inflection point. For the Gompertz model, the point of inflection occurs at 37.79% of the upper limit and the upper limit can also be calculated when the inflection point is found. Bengisu and Nekhili [9] showed that the simple logistic and the Gompertz models are quite valid if the upper limit is correctly identified. However, the data points may not be sufficient (too few) to see the point of inflection and to set the correct upper limit when forecasting short lifecycle products. Therefore, a model with more parameters, for example, the time-varying extended logistic model, is needed to project the trajectory of the growth curve. The time-varying extended logistic model uses a dynamic upper limit that can be estimated from the data.

There is little published research which compares the performance of forecasting models used on short product lifecycle datasets. Thus, this study compares the fit and forecast performance of the simple logistic, the Gompertz, and the time-varying extended logistic models.

3. Technological forecasting models

Many models have been used in forecasting. This section introduces the models which are used in this study and derives the underlying formulas for each.

3.1. Simple logistic curve model

Most biological growth follows an S-shape curve or logistic curve which best models growth and decline over time [14]. Since the adoption of technology and technology-based products is similar to biological growth, the simple logistic model is widely used for technology forecasting. Many new forecasting models were proposed based on the simple logistic model and include innovations such as the Bass diffusion model and extended logistic model [12]. The most important characteristic of simple logistic model is that it is symmetric about the point of inflection. This feature indicates that the process which will happen after the point of inflection is the mirror image of the process that happened before the point.

The model for the simple logistic curve is controlled by three coefficients, a, b, and L is expressed as

$$y_t = \frac{L}{1 + ae^{-bt}} \tag{1}$$

where y_t is the value of interest, L is the maximum value of y_t, a describes the location of the curve, and b controls the shape of the curve. To estimate the parameters for a and b, the equation of the simple logistic model is transformed into a linear function using natural logarithms. The linear model is expressed as

$$Y_t = \ln(y_t/L - y_t) = -\ln(a) + bt \tag{2}$$

where the parameter a and b are then estimated using a simple linear regression. The simple logistic model (Eq. (1)) and the linear model (Eq. (2)) are quoted from Martino's book [22] and the derivations are shown in Appendix 1.

3.2. Gompertz model

The Gompertz model was first used to calculate mortality rates in 1825 and has been widely applied to technology forecasting [22]. Although the Gompertz curve is similar to the simple logistic curve, it is not symmetric about the inflection point which occurs at $t = (\ln(b)/k)$. The Gompertz model reaches the point of inflection early in the growth trend and is expressed as

$$y_t = Le^{-ae^{-bt}} \tag{3}$$

where L is the upper bound which should be set before estimating the parameters a and b. Similar to the methodology of estimating the parameters of the simple logistic model, natural logarithms are used to transform the original Gompertz model to linear equation:

$$Y_t = \ln(\ln(L/y_t)) = \ln(a) - bt \tag{4}$$

and then the parameters are estimated [22]. Eqs. (3) and (4) are quoted from Martino's book [22] and the derivations are shown in Appendix 2.

Although the predictive performance of the simple logistic model and the Gompertz model has been validated by many researchers [13], the models have definite limitations when used to forecast short product lifecycles. The reason is that it is almost impossible to estimate the correct upper limit for a new product when it is first introduced to market place.

Fig. 1 depicts the importance of setting the correct upper limit in the simple logistic and the Gompertz models. As can been seen

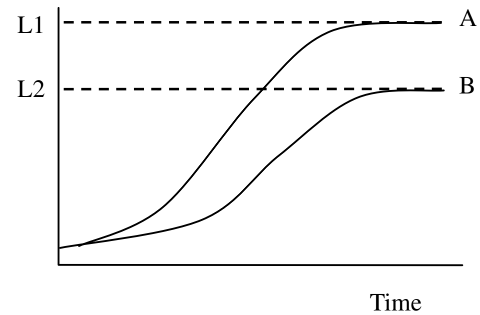


Fig. 1. Curves with different upper limits.

in Fig. 1, curves A and B start at the same point but have different upper limits, $L1$ and $L2$. Since the upper limits are set at different level, the two curves are different, and the prediction results will also be different.

3.3. Time-varying extended logistic model

The simple logistic model and the Gompertz model assume that the capacity of technology adoption is fixed and there is an upper bound to growth for these models. However, the adoption of new technology is seldom constant and changes over time. Therefore, researchers have proposed a dynamic carrying capacity and the carrying capacity can be any function [14,23]. As shown by Meyer and Ausubel [14], the original form of simple logistic model is written as

$$\frac{dy_t}{dt} = b \times L \times y_t\left(1 - \frac{y_t}{L}\right) \tag{5}$$

Let $\alpha = b \times L$ and replace the constant L in Eq. (5) with a function $k(t)$, and then Eq. (5) is extended to

$$\frac{dy_t}{dt} = \alpha \times y_t\left(1 - \frac{y_t}{k(t)}\right) \tag{6}$$

where L is the upper limit of the logistic curve and $k(t)$ is the time-varying capacity function similar to the logistic curve.

In Meyer and Ausubel's study, a special $k(t)$ was set to represent a technology which has a bio-logistic growth rate. The setting of $k(t)$ in our research comes from Chen's study [19] with

$$k(t) = 1 - d \times e^{-ct} \tag{7}$$

and c and d are parameters that are estimated. The value of d can be any number and the value of c larger than zero. Our research assumes that the penetration rate capacity will fluctuate with time and may reach 100% but may also be as low as 30% or 50%. The reason for this assumption is that some new products may be introduced to the market and substitute older products. Thus, a product may not always achieve 100% market penetration and may be replaced earlier than expected.

Finally, the time-varying extended logistic model is expressed as

$$y_t = \frac{k(t)}{1 + a \times e^{-bt}} = \frac{1 - d \times e^{-ct}}{1 + a \times e^{-bt}} \tag{8}$$

where $k(t)$ is the capacity that fluctuates with time, and a, b, c, and d are the parameters computed using a nonlinear least squared estimation method provided by a statistic software package like SYSTAT. When this model is tested using sales volume data, the equation is changed to

$$N_t = m * y_t = m * \frac{1 - d \times e^{-ct}}{1 + a \times e^{-bt}} \tag{9}$$

where N_t *is cumulative volume by time t*, and the coefficient m represents the total market sales which is estimated using nonlinear least squares method.

The time-varying extended logistic model is similar to the Bass diffusion model and can be viewed as a special case of this model. The Bass model was developed for predicting sales volume, whereas the time-varying extended logistic model can be modified to predict sales volume as well as other proxies including market penetration rates. Therefore, the time-varying extended logistic model was selected for evaluation over the Bass model.

4. Analytical process

In order to test the forecast accuracy of the simple logistic, Gompertz, and the time-varying extended logistic models, the analytical process is divided into two steps.

Step 1: Model estimation

The first step is used to estimate the models. After reserving the last five data points to test forecast accuracy of the simple logistic, Gompertz, and the time-varying extended logistic models, the remaining data points were used to fit the three models. For the simple logistic and the Gompertz model, Eqs. (2) and (4) are used to estimate coefficients using a simple linear regression. For the time-varying extended logistic model, the coefficients of the models are estimated using nonlinear least squares with SYSTAT statistical software. After the coefficients were computed and the models fitted, the estimated values were calculated.

Step 2: Fit and forecast performance

After the models are constructed, the fit and forecast performance between the three models is conducted. The test consists of checking residuals between actual values and estimated values to measure model performance [7,13]. Two measurements, mean absolute deviation (MAD) and root mean square error (RMSE) are used to calculate residuals.

For the simple logistic and the Gompertz models, the upper limit must be set to obtain accurate results. Setting different upper limit levels of these two models will achieve different prediction results and the fit and predict performance will also be influenced. Thus, several upper limits of the simple and the Gompertz models were set to determine which upper limit would yield the best fit performance.

For forecast performance, the derived models are used to forecast the last five data points of the datasets. In this study, the mean absolute deviation (MAD) and root mean square error (RMSE) are used to measure performance as recommended in the literature [7,13,24]. The mathematical representations are shown below:

$$\mathrm{MAD} = \frac{\sum_{t=1}^{T}|y_t - \hat{y}_t|}{n} \tag{10}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{T}(y_t - \hat{y}_t)^2}{n}} \tag{11}$$

where y_t is the actual value at time t, $\hat{y}_t$ is the estimate at time t, and n is the number of observations. These measurements are based on the residuals, which represent the distance between real data and predictive data. Consequently, if the values of these measures are small, then the fit and prediction performance is acceptable.

5. Empirical results

Twenty-two time-series datasets describing Taiwan penetration rate and cumulative sales volume of electronic products were collected to test the forecast accuracy of the simple logistic model, the Gompertz model and the time-varying extended logistic model. The datasets for market penetration rates were providing by the Directorate General of Telecommunications and Chunghwa Telecom Co. and the Directorate General of Budget [25,26]. The market penetration rate datasets cover six products including color TVs, telephones, washing machines, asymmetric digital subscriber lines (ADSL), mobile Internet subscribers, and broadband networks. The cumulative sales volume datasets were provided by the Taiwan Market Intelligence Center [27]. These datasets cover 16 products including LCD-TV, 19 in. LCD monitors, digital cameras with charge coupled device image sensors (CCD DC), digital cameras with more than five million pixels (DC > 5 million), 802.11g wireless local area networks devices (WLAN 802.11g), cable modems, combo optical disk drives (combo ODD), Barebone computers (Barebone), China personal wireless access systems (China PAS), LCD panels for TV, LCD panels for notebooks, color mobile phones with 65K pixels (color-65k mobile phone), servers, over 30-in. wide LCD-TVs (LCD-TV > 30 in.), Voice over Internet Protocol Integrated Access Devices (VoIP IAD), and Voice over Internet protocol (VoIP) routers.

As shown in Table 1, the estimated sample period, predicted sample period, and sample sizes are presented. The data for color TVs, telephones and washing machines are yearly data points. Since the sample period for these data is greater than 30 years, these products depict a complete product lifecycle (Fig. 2). This research classifies the data for color TVs, telephones and washing machines as long product lifecycles with large datasets for forecasting. The other datasets (ADSL, mobile Internet subscribers, etc.) represent products rapidly brought to market and are categorized as short lifecycle products with limited or small (less than 30 data points) datasets for forecasting.

Fig. 2 shows the penetration rate for the six products and Fig. 3 shows the cumulative sales volume for the 16 short lifecycle

Table 1
Estimated and predicted sample period and sample size

Proxy	Product	Estimated sample period		Predicted sample period		Sample size
		From	To	From	To	
Penetration rate	Color TV	1974	1999	2000	2004	31
	Telephone	1970	1999	2000	2004	35
	Washing Machine	1974	1999	2000	2004	31
	ADSL	2000Q2	2005Q2	2005Q3	2006Q3	26
	Mobile Internet	2001Q4	2005Q2	2005Q3	2006Q3	20
	Broadband network	2000Q2	2005Q2	2005Q3	2006Q3	26
Cumulative sales volume	LCD-TV	2003Q1	2006Q1	2006Q2	2007Q2	18
	19 in. LCD monitor	2003Q1	2006Q1	2006Q2	2007Q2	18
	CCD DC	2003Q1	2006Q1	2006Q2	2007Q2	18
	DC > 5 million	2003Q1	2006Q1	2006Q2	2007Q2	18
	WLAN (802.11g)	2003Q1	2006Q1	2006Q2	2007Q2	18
	Cable modem	2003Q1	2006Q1	2006Q2	2007Q2	18
	Combo ODD	2003Q1	2006Q1	2006Q2	2007Q2	18
	Barebones	2003Q1	2006Q1	2006Q2	2007Q2	18
	China PAS	2003Q1	2006Q1	2006Q2	2007Q2	18
	LCD panel for TV	2003Q1	2006Q1	2006Q2	2007Q2	18
	LCD panel for notebook	2003Q1	2006Q1	2006Q2	2007Q2	18
	Color-65k mobile phone	2003Q1	2006Q1	2006Q2	2007Q2	18
	Server	2003Q1	2006Q1	2006Q2	2007Q2	18
	LCD-TV > 30 in.	2004Q1	2006Q2	2006Q3	2007Q2	14
	VoIP IAD	2004Q1	2006Q2	2006Q3	2007Q2	14
	VoIP router	2004Q1	2006Q2	2006Q3	2007Q2	14

Source: Directorate General of Budget and Directorate General of Telecommunications and Chunghwa Telecom Co. 25,26, and Market Intelligence Center Taiwan 27.

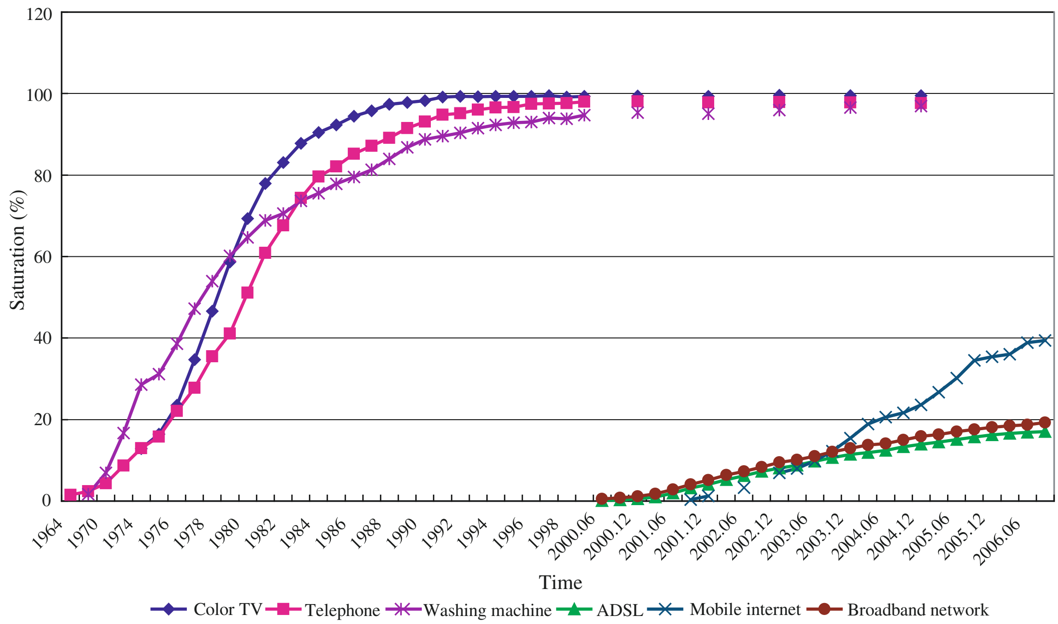
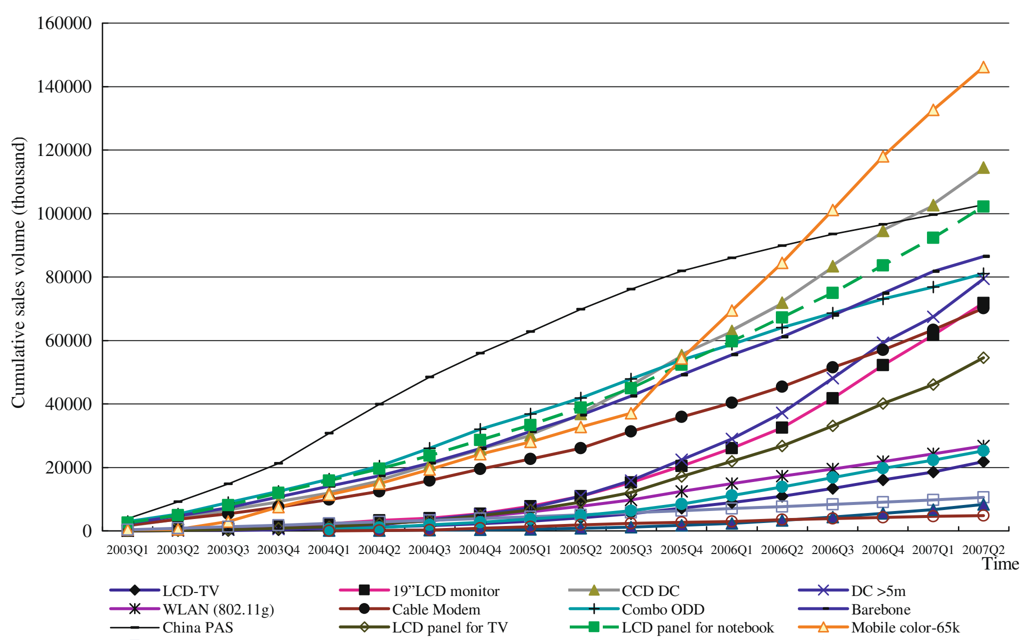


Fig. 2. Market growth for saturation datasets.



Note: the unit of wireless internet is ten thousand.
Source: ASIP, Market Intelligence Center, Taiwan

Fig. 3. Market growth for cumulative datasets.

products. Fig. 2 shows that color TVs, telephones, and washing machines have entered the mature stage of the product lifecycle. Therefore, a clear upper limit for these products can be set. On the other hand, the curves for ADSL, mobile Internet subscribers, and other short lifecycle products are still evolving, making it difficult to define the stage of product lifecycle or to predict when these products will stop growing.

For the long product lifecycle datasets, the upper limit is set at 100%. For the short product lifecycle datasets, different upper limits are set to achieve the best estimates. The possible upper limit for the short lifecycle is set at three different levels to include optimistic, a conservative, and a pessimistic settings. An optimistic upper limit means that the product is new to the market and has potential to grow. A pessimistic setting means that the product almost reached the upper limit to market growth. Between the optimistic and pessimistic limits is the conservative setting. The conservative setting models a product that has been in the market for a while and has reached about one-third or one-half of the upper limits to growth.

The penetration rate datasets use upper limits of 100%, 50%, and 30%. However, since the current penetration rate of the mobile Internet is 40%, the pessimistic setting is change to 50% and conservative upper limit is changed to 60%. For the cumulative sales volume datasets, the upper limit is set based on the multiple of the most recent observation as recommended by Meade and Islam [13]. The optimistic, conservative, and pessimistic upper limits are 5 times, 3 times, and 1.5 times the most recent observation which is based on the cumulative sales volume of the second quarter in 2007. In fact, 5 times the most recent observation means that the proportion of the current cumulative sales volume to maximum sales volume (upper limit) is 20%. Thus, the current cumulative sales volume only reaches 20% of the upper limit and there is still 80% of the maximum sales volume remaining to sell. So the setting of 5 times the most recent observation is an optimistic setting. A pessimistic setting of the cumulative sales volume dataset is set at 1.5 times the most recent observation. Using the cable modem dataset as an example, the most recent cumulative sales volume is 70,106,900 units and the pessimistic upper limit is 105,160,350 units. This means that the current cumulative sales volume has already reached two-third of the upper limit and has entered the mature stage of the product lifecycle.

Tables 2 and 3 provide the fit and forecast performance for the penetration dataset. The evaluation rule is that the smaller the va-

Table 2
Fitting performance measures for the time-varying extended logistic, Gompertz, and the simple logistic models – penetration rate datasets

Model	Index	Color TV	Phone	Washing machine	ADSL			Mobile Internet			Broadband network		
					L = 100%	L = 50%	L = 30%	L = 100%	L = 60%	L = 50%[a]	L = 100%	L = 50%	L = 30%
Extended logistic	MAD	**0.0053**	**0.0063**	**0.0098**		**0.0036**			**0.0057**			**0.0027**	
	RMSE	**0.0071**	**0.0083**	**0.0118**		**0.0043**			**0.0073**			**0.0032**	
Gompertz	MAD	0.0297	0.0290	0.0192	0.0154	0.0140	0.0102	0.0134	0.0077	0.0059	0.0146	0.0114	0.0075
	RMSE	0.0502	0.0440	0.0290	0.0228	0.0170	0.0119	0.0156	0.0094	0.0076	0.0176	0.0132	0.0084
Simple logistic	MAD	0.0361	0.0323	0.0276	0.0329	0.0274	0.0212	0.0337	0.0256	0.0217	0.0243	0.0075	0.0153
	RMSE	0.0551	0.0453	0.0384	0.0483	0.0372	0.0264	0.0439	0.0304	0.0248	0.0328	0.0084	0.0180

Note: L – upper limit.
Boldface number means the best performance among three models.
[a] The current saturation rate of mobile Internet is over 30%.

Table 3
Forecasting performance measures for the time-varying extended logistic, Gompertz, and the simple logistic models – penetration rate datasets

Models	Index	Color TV	Phone	Washing machine	ADSL			Mobile Internet			Broadband network		
					L = 100%	L = 50%	L = 30%	L = 100%	L = 60%	L = 50%[a]	L = 100%	L = 50%	L = 30%
Extended logistic	MAD	**0.0025**	**0.0049**	**0.0021**	**0.0140**			**0.0117**			**0.0036**		
	RMSE	**0.0026**	**0.0057**	**0.0034**	**0.0147**			**0.0152**			**0.0041**		
Gompertz model	MAD	0.0042	0.0125	0.0095	0.1037	0.0671	0.0345	0.0627	0.0132	0.0146	0.0842	0.0531	0.0237
	RMSE	0.0043	0.0130	0.0100	0.1060	0.0688	0.0352	0.0699	0.0154	0.0166	0.0867	0.0545	0.0243
Simple logistic model	MAD	0.0045	0.0171	0.0140	0.2846	0.1688	0.0833	0.2397	0.0991	0.0503	0.1895	0.1185	0.0556
	RMSE	0.0046	0.0174	0.0143	0.2933	0.1716	0.0839	0.2503	0.1021	0.0518	0.1958	0.1210	0.0562

Note: L – upper limit.
Boldface number means the best performance among three models.
[a] The current saturation rate of mobile Internet is over 30%.

Table 4
Fitting performance measures for the time-varying extended logistic, Gompertz, and the simple logistic models – cumulative shipment volume data

Model	Saturation specification	Index	LCD-TV	19 in. LCD monitor	CCD DC	DC > 5 million	WLAN 802.11g	Cable modem	Combo ODD	Barebones	China PAS	Panel for TV	LCD-TV > 30 in.	VoIP IAD
Extended logistic		MAD	**51**	**167**	**451**	**161**	**586**	**207**	**435**	**165**	**398**	**147**	**11**	**82**
		RMSE	**58**	**212**	**593**	**188**	**772**	**282**	**538**	**184**	**483**	**182**	**15**	**100**
Gompertz	5*2007Q2	MAD	192	364	1314	205	3219	1022	2204	1412	12,717	430	54	199
	volume	RMSE	268	434	1542	274	4634	1293	2802	1740	14,616	579	82	222
	3*2007Q2	MAD	141	412	998	297	2026	770	1696	1036	12,298	305	70	198
	volume	RMSE	174	495	1120	440	2637	927	2095	1208	14,041	412	104	224
	1.5*2007Q2	MAD	134	13,488	989	582	2026	354	678	553	2762	401	99	314
	volume	RMSE	172	15,179	1226	954	2510	443	740	655	3357	575	143	386
Simple logistic	5*2007Q2	MAD	900	1958	3853	2461	14,679	2484	4570	3543	14,105	2445	65	660
	volume	RMSE	1767	3925	5931	5439	27,124	3656	6438	5080	16,670	4943	119	1021
	3*2007Q2	MAD	823	1803	3487	2238	13,007	2259	4076	3170	9084	2231	54	588
	volume	RMSE	1569	3521	5160	4842	23,147	3213	5542	4390	11,961	4371	98	875
	1.5*2007Q2	MAD	656	1456	2615	1744	9414	1705	2820	2270	6280	1773	29	423
	volume	RMSE	1148	2635	3440	3562	14,974	2187	3478	2817	7605	3182	49	551

Note: Boldface number means the best performance among three models.

lue for MAD and RMSE, the better the fit and prediction performance. As shown in Tables 2 and 3, the time-varying extended logistic model has the best fit and prediction for both long and short lifecycle products. Tables 4 and 5 provide the fit and forecast performance for the cumulative sales volume dataset. Table 4 shows that time-varying extended logistic model has the best fit performance for all products. Table 5 shows that the time-varying extended logistic model has the best forecast performance for the majority of the products.

Table 6 summarizes the comparative results of the time-varying extended logistic model, the simple logistic and the Gompertz models. The fit and forecast performance are compared and ranked using the root mean square error (RMSE) which is widely used for measuring the performance [7,13,23]. As can be seen in Table 6, the time-varying extended logistic model has the best predictive performance for 13 products among the 18 products for which the model converged. The model has the second best predictive performance for 4 products, and the worst predictive performance for LCD-TV > 30 in. data. The Gompertz model predicts best for 4 product datasets and has the second best forecast performance for three models. The simple logistic model only predicts well for the LCD-TV > 30 in. data. In summary, the time-varying extended logistic model is 70% better in prediction than the other models.

In order to test whether the root mean square error of the time-varying extended logistic ($RMSE_{ei}$) is smaller than error of the simple logistic ($RMSE_{si}$) and Gompertz ($RMSE_{gi}$) models, we first calculate the statistics $RMSE_{ei} - RMSE_{si}$ and $RMSE_{ei} - RMSE_{gi}$. Then these statistics are used to test the null hypotheses that $RMSE_{ei} = RMSE_{si}$ ($H0_a$) and $RMSE_{ei} = RMSE_{gi}$ ($H0_b$) using one-tail sign test. The reason why the one-tail sign test is chosen is because the distribution of RMSE is unknown and the sample size is small, so a nonparametric test is used. A sign test only needs a count of the number of sample value exceeding a defined constant which is equal to zero in this case [28].

Table 7 presents the *P*-values for the fit and forecast performance between the time-varying extended logistic model, the simple logistic model and the Gompertz model. As shown in Table 7, all *P*-values of sign test are smaller than 0.05, which means there are statistically significant differences among the three models at the 95% level. Further, the time-varying extended logistic model outperforms than the simple logistic and Gompertz models in both fit and forecast performance.

Table 6
Fitting and Forecasting performance ranks of the extended logistic, Gompertz, and the simple logistic models

Product	Fitting			Forecasting		
	Extended logistic	Gompertz	Simple logistic	Extended logistic	Gompertz	Simple logistic
Color TV	1	2	3	1	2	3
Phone	1	2	3	1	2	3
Washing machine	1	2	3	1	2	3
ADSL	1	2	3	1	2	3
Mobile Internet	1	2	3	1	2	3
Broadband network	1	3	2	1	2	3
LCD-TV	1	2	3	1	2	3
19 in. LCD monitor	1	2	3	2	1	3
CCD DC	1	2	3	1	2	3
DC > 5 million	1	2	3	2	1	3
WLAN (802.11g)	1	2	3	1	2	3
Cable modem	1	2	3	1	2	3
Combo ODD	1	2	3	1	2	3
Barebones	1	2	3	1	2	3
China PAS	1	2	3	1	2	3
LCD panel for TV	1	2	3	2	1	3
LCD-TV > 30 in.	1	3	2	3	2	1
VoIP IAD	1	2	3	2	1	3

Note: 1 means the model with the lowest RMSE and best performance. For the Gompertz and the simple logistic models, the capacity with the lowest RMSE is compared.

The simple logistic and the Gompertz models are limited by the shape of the growth curve. For example, the simple logistic curve is symmetric about the point of inflection, so when the datasets do not have these characteristics, the simple logistic does not predict well. The Gompertz curve is an asymmetric S-curve and the Gompertz model reaches the inflection point before the market penetration has reached half the upper limit [13]. Thus, the Gompertz model may be more suitable for certain types of short lifecycle products than the simple logistic model. As shown in Tables 3 and 5, the wrong capacity will lead to an error in prediction. If industrial policy or enterprise decisions are made based on a model

Table 5
Forecasting performance measures for the time-varying extended logistic, Gompertz, and the simple logistic models – cumulative shipment volume data

Model	Saturation specification	Index	LCD-TV	19 in. LCD monitor	CCD DC	DC > 5 million	WLAN 802.11g	Cable modem	Combo ODD	Barebones	China PAS	Panel for TV	LCD-TV > 30 in.	VoIP IAD
Extended logistic		MAD	**265**	6072	**923**	10,068	12,030	**2253**	**1878**	**2040**	**3115**	5217	497	1342
		RMSE	**301**	7758	**1157**	12,434	**12,505**	**2911**	**2029**	**2153**	**3305**	5846	734	1740
Gompertz	5*2007Q2	MAD	1820	**2372**	11,176	**1813**	63,530	7910	23,077	14,504	16,194	5076	583	1548
	volume	RMSE	1913	**2706**	12,062	**2303**	69,259	8238	24,689	15,519	20,024	5433	697	2034
	3*2007Q2	MAD	280	6805	3223	5194	32,902	3396	15,549	7971	11,017	**604**	887	**302**
	volume	RMSE	360	7814	3452	5582	35,553	3419	16,541	8421	13,492	**703**	1077	**403**
	1.5*2007Q2	MAD	2707	13,488	9608	14,066	**11,484**	4205	2651	3031	14,329	6885	1364	2069
	volume	RMSE	3116	15,179	10,662	15,342	12,692	5052	2764	3530	14,703	7865	1668	2426
Simple logistic	5*2007Q2	MAD	26,450	64,846	70,320	104,201	365,213	36,609	61,892	53,560	43,670	78,644	2433	11,727
	volume	RMSE	29,106	72,596	76,969	116,523	396,302	39,564	66,579	58,014	51,122	86,272	3142	14,568
	3*2007Q2	MAD	17,740	44,268	50,134	68,273	241,342	26,680	44,945	38,728	78,007	51,581	1643	8250
	volume	RMSE	18,836	47,682	53,567	73,370	254,235	28,212	47,516	41,109	81,173	54,536	2060	10,020
	1.5*2007Q2	MAD	6734	16,090	16,953	25,192	86,925	9222	16,668	13,437	29,775	19,611	**295**	2516
	volume	RMSE	6786	16,182	17,113	25,473	87,717	9271	17,002	13,582	30,183	19,761	**356**	2881

Note: Boldface number means the best performance among three models.

Table 7
The *P*-value of sign test

	Fitting performance	Forecasting performance
$H0_a$: $RMSE_{ei} = RMSE_{si}$ $H1_a$: $RMSE_{ei} < RMSE_{si}$	0^{***}	0.0001^{***}
$H0_b$: $RMSE_{ei} = RMSE_{gi}$ $H1_b$: $RMSE_{ei} < RMSE_{gi}$	0^{***}	0.0482^{***}

Note: The equation for *P*-value of one-tail sign test can be expressed as $P\text{-valueofsigntest} = \sum_{k=0}^{S^+}\binom{n'}{k}(0.5)^n$, where S^+ = the number of $RMSE_{ei} > RMSE_{si}$ (or $RMSE_{gi}$); n' = the number of $(n - S^0)$; n = sample size; S^0 = the number of $RMSE_{ei} = RMSE_{si}$ (or $RMSE_{gi}$); $RMSE_{ei}$ = the RMSE value of time-varying extended logistic model; $RMSE_{si}$ = the RMSE value of the simple logistic model; $RMSE_{gi}$ = the RMSE value of the Gompertz model.
*** 95% Significant level.

using the wrong upper limit, a serious forecast error can be made. Since the time-varying extended logistic model uses more parameters to capture the trend of products, the fit and forecast performance are improved.

This research used 22 product datasets to test the performance of the simple logistic, the Gompertz and the time-varying extended logistic model. However, the datasets for LCD panel for notebooks, color-65k mobile phones, servers, and VoIP routers, would not converge when using the time-varying extended logistic model to estimate the coefficients. A similar situation was reported by Meade and Islam [13]. Their research used 25 telecommunications market datasets to compare the performance of 17 growth curve models. For their study, half of the datasets would not converge when estimating the coefficients of models. Our study showed four products would not converge among 22 products yielding a proportion less than 20%. Therefore, our convergence results are consistent with earlier research.

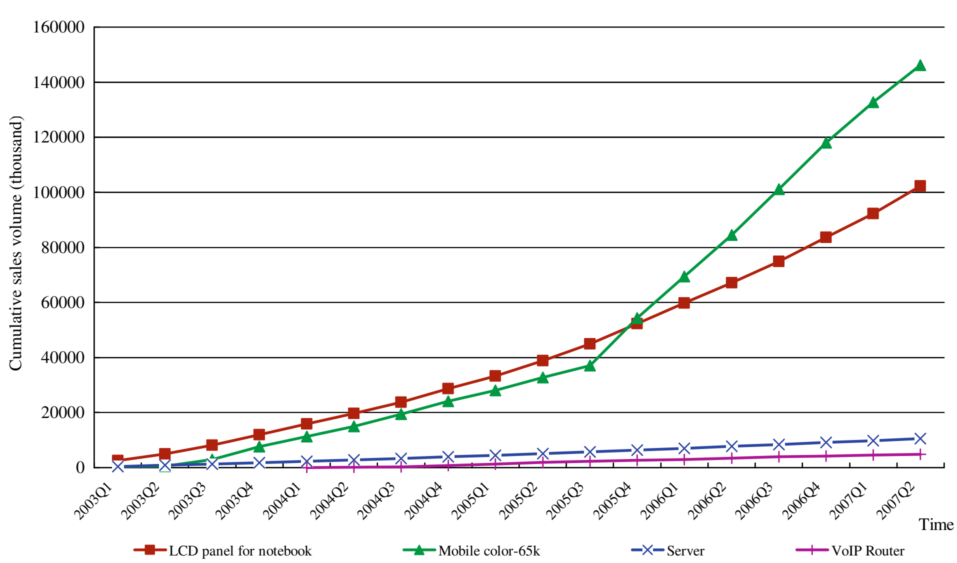
When using the simple logistic and Gompertz models, the upper limit (L) must be set and then a linear transformation method is applied to calculate parameters using Eqs. (2) and (4). Since only two parameters are estimated, it is easy for the models to converge. However, since the upper limit of the time-varying extended logistic model is dynamic with time, more parameters are needed to capture the trace. Therefore, the linear transformation method used in the simple logistic and Gompertz models cannot be used to estimate the parameters and a nonlinear estimation method must be used. For the cumulative sales volume dataset, five parameters are estimated using 14–18 data points which causes an increase in nonconvergence for the extended logistics model.

The four products with data that would not converge provide some insight. These datasets are linear and the curve for the color-65k mobile phone has an obvious jump (Fig. 4). Meade and Islam's research [12] used telephone data from Sweden to compare the simple logistic, extended logistic, and the local logistic models. They concluded that the extended logistic model had the worst performance. Although the setting of the extended logistic model is different with this research, Meade and Islam's study serves as a useful example. The growth curve of the Swedish telephone dataset is linear. Therefore, the time-varying extended logistic model should not have been used. If forecasters wish to apply the time-varying extended logistic model, then they should confirm that the data has an S-shape prior to the forecast.

6. Discussion and conclusion

This study compares the fit and prediction performance of the simple logistic, Gompertz, and the time-varying extended logistic models for 22 electronic products. Since the simple logistic and Gompertz curves require the correct upper limit settings for accurate market growth rate predictions, these two models may not be suitable for short product lifecycles with limited data. Therefore, to solve this problem, the time-varying extended logistic model was tested. Since the time-varying extended logistic model estimates the time-varying capacity from the data, it tends to perform better for both long and short lifecycle products if the data are not linear. The results show that the time-varying extended logistic model outperforms the simple logistic and the Gompertz models in most of product datasets where the data has the beginnings of an S-shape.

When forecasting the future growth and market for products, forecasters need to study the shape and the characteristics of



Note: the unit of wireless internet is ten thousand.
Source: ASIP, Market Intelligence Center, Taiwan

Fig. 4. Market growth for LCD panel for notebooks, color-65k mobile phones, servers, and VoIP routers.

the growth curve before selecting a suitable model. Although the time-varying extended logistic model has better performance for forecasting short lifecycle products, care must be taken when using this model. The extended logistic model may only be suitable for data that grows as an S-curve and may not be suitable for linear data or for curves with many anomalous data points. A possible solution for these types of datasets may be to apply smoothing techniques or data re-interpretation techniques. Smoothing and data re-interpretation techniques were first proposed by Tukey [29] and are commonly used in exploratory data analysis. Further research can be conducted using Tukey smoothing and data re-interpretation to see if the extended logistic model can be forced to converge and therefore find broader applications for a wider range of short product lifecycle datasets.

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Appendix 1

$$y_T = \frac{L}{1 + ae^{-bt}} \tag{1}$$

$$y_t + y_t ae^{-bt} = L$$

$$y_t ae^{-bt} = L - y_t$$

$$ae^{-bt} = \frac{L - y_t}{y_t}$$

$$\frac{1}{a}e^{bt} = \frac{y_t}{L - y_t}$$

$$\ln\left(\frac{1}{a}e^{bt}\right) = \ln\left(\frac{y_t}{L - y_t}\right)$$

$$-\ln a + bt = \ln\left(\frac{y_t}{L - y_t}\right) = Y_t$$

$$\therefore Y_t = \ln(y_t/L - y_t) = -\ln a + bt \tag{2}$$

Appendix 2

$$y_t = Le^{-ae^{-bt}} \tag{3}$$

$$\frac{y_t}{L} = e^{-ae^{-bt}}$$

$$\ln\left(\frac{y_t}{L}\right) = -ae^{-bt}$$

$$\ln\frac{L}{y_t} = ae^{-bt}$$

$$Y_t = \ln\left(\ln\frac{L}{y_t}\right) = \ln(ae^{-bt})$$

$$\therefore Y_t = \ln(\ln(L/y_t))\ln a - bt \tag{4}$$

Appendix 3

Cumulative sales volume dataset

Sample period	Products															
	LCD-TV	19 in. LCD monitor	CCD DC	DC > 5 million	WLAN (802.11g)	Cable modem	Combo ODD	Barebone	China PAS	LCD panel for TV	LCD panel for notebook	Mobile color-65k	Server	LCD-TV > 30 in.	VoIP IAD	VoIP router
2003Q1	26.900	49.921	1540.638	33.759	56.588	1752.000	2946.440	2300.000	3815.000	25.340	2559.158	390.000	410.000			
2003Q2	63.100	389.660	3706.334	132.559	215.973	3614.000	5230.362	4720.000	9096.000	97.940	4956.234	490.000	850.000			
2003Q3	164.100	763.660	6537.814	302.049	500.027	5379.000	8882.726	7229.000	14799.000	359.369	8059.782	2893.000	1295.000			
2003Q4	407.500	1245.868	9196.234	602.352	958.930	7552.000	12461.777	10677.000	21275.000	795.749	11955.402	7557.000	1792.000			
2004Q1	787.500	2173.868	12021.892	1159.524	1572.550	9832.000	16351.273	14115.000	30759.000	1307.749	15826.702	11353.000	2295.000	84.000	425.000	30.800
2004Q2	1194.500	3273.868	15788.372	1905.414	2244.595	12385.000	20438.203	17378.000	39869.000	2280.749	19619.702	14937.000	2809.000	146.000	1055.200	68.800
2004Q3	1603.900	4025.868	20911.238	3480.309	3154.315	15865.000	26021.378	21389.000	48429.000	3207.749	23735.702	19398.000	3334.000	225.000	1869.500	267.600
2004Q4	2178.600	5417.368	25733.319	5189.401	4499.246	19520.000	32006.914	25943.000	55984.000	4821.049	28662.202	24106.000	3900.000	338.000	2739.500	769.300
2005Q1	2993.600	7735.268	30158.223	7221.865	5928.487	22604.000	36858.493	31263.000	62744.000	6606.549	33278.502	28003.000	4487.300	509.000	3698.500	1309.300
2005Q2	4059.600	10957.768	36820.687	10907.315	7716.311	26086.000	41830.072	36523.000	69737.900	9046.649	38795.202	32758.000	5083.605	781.000	4773.500	1923.300
2005Q3	5432.200	15307.768	45611.350	15795.081	9722.111	31326.000	47856.446	42451.000	76124.574	12112.899	44874.202	37099.000	5699.927	1219.000	6337.500	2345.300
2005Q4	7101.652	20404.988	55382.907	22373.650	12482.311	35912.000	53790.890	49106.000	81922.139	17078.509	52274.002	54268.000	6353.141	1712.000	8489.500	2661.300
2006Q1	8883.652	26032.988	63007.848	28958.121	14918.811	40315.000	58765.654	55533.000	85990.326	21967.809	59708.202	69451.000	7014.901	2403.000	11166.500	2967.300
2006Q2	10896.652	32548.245	71902.554	37172.826	17202.311	45434.900	64048.872	61134.000	89875.326	26745.219	67162.002	84473.000	7681.721	3284.000	13853.950	3471.300
2006Q3	13379.652	41731.245	83420.201	48149.297	19519.511	51546.900	68619.391	67911.000	93475.326	33127.219	74899.002	101103.000	8369.621	4330.000	16776.350	3880.300
2006Q4	16097.652	52201.245	94502.554	59281.062	21876.301	56997.900	73063.269	74853.000	96525.326	40059.219	83663.352	117994.000	9076.997	5505.000	19727.650	4178.300
2007Q1	18563.652	61755.245	102643.731	67488.821	24246.615	63382.900	76889.575	81808.000	99545.326	46143.219	92258.352	132689.000	9789.320	6699.000	22328.650	4575.300
2007Q2	21794.652	71821.525	114524.165	79399.864	26800.515	70106.900	81083.108	86487.000	102795.326	54545.239	102214.352	146140.000	10506.950	8355.000	25181.450	4836.300

Unit: thousand.
Source: Market Intelligence Center Taiwan 27.

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