



Pragmatical adaptive chaos control from a new double van der Pol system to a new double Duffing system

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ABSTRACT

A new pragmatical adaptive control method for different chaotic systems is proposed. Traditional chaos control is limited to decrease chaos of one chaotic system. This method enlarges the effective scope of chaos control. We can control a chaotic system, e.g. a new chaotic double van der Pol system, to a given chaotic or regular system, e.g. a new chaotic double Duffing system or to a damped simple harmonic system. By a pragmatical theorem of asymptotical stability based on an assumption of equal probability of initial point, an adaptive control law is derived such that it can be proved strictly that the common zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical simulations are given to show the effectiveness of the proposed scheme.

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1. Introduction

Since chaos control was firstly used by Ott et al. [1], it has been studied extensively. Many control methods have been employed to control chaos [2–6]. Simple linear feedback control was proposed [7–9]. Time delay feedback control [10–13], sliding mode control [14–17], backstepping method [18] and adaptive control [19–22] were widely used. However, traditional adaptive chaos control is limited to control the chaotic motion of one chaotic system to regular motion or to fixed point. Proposed pragmatical adaptive control method enlarges the scope of chaos control. We can control a chaotic system to a given simple unchaotic system or to a more complex chaotic system. In current scheme of adaptive control of chaotic motion [23–25], traditional Lyapunov stability theorem and Babalat lemma are used to prove the error vector approaches zero, as time approaches infinity. But the question, why the estimated or given parameters also approach to the uncertain or goal parameters, remains no answer. By a pragmatical theorem of asymptotical stability [29–31] based on an assumption of equal probability of initial points, an adaptive control law is derived such that it can be proved strictly that the common zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical results are given for a chaotic double van der Pol system to be controlled to a chaotic double Duffing system and to a regular damped simple harmonic system.

This paper is organized as follows: In Section 2, a pragmatical adaptive control scheme is given. In Section 3 numerical results of chaos control are given. A chaotic double van der Pol system is controlled to a chaotic double Duffing system and to a regular damped simple harmonic system. Finally, conclusions are given in Section 4.

2. Pragmatical adaptive control scheme

Consider the following chaotic system

$$\dot{x} = f(x, A) + u(t), \quad (1)$$

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where $x = [x_1, x_2, \dots, x_n]^T \in R^n$ denotes a state vector, $A = [A_1, A_2, \dots, A_m] \in R^m$ is a original coefficient vector, and f is a vector function, and $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T \in R^n$ is a control input vector.

The goal system which can be either chaotic or nonchaotic, is

$$\dot{y} = g(y, \hat{B}), \tag{2}$$

where $y = [y_1, y_2, \dots, y_n]^T \in R^n$ denotes a state vector, $\hat{B} = [\hat{B}_1, \hat{B}_2, \dots, \hat{B}_p]^T \in R^p$ is a goal coefficient vector, and g is a vector function. Our goal is to design an adaptive control method and a controller $u(t)$ so that the state vector of the chaotic system (1) asymptotically approaches the state vector of the goal system (2).

The chaos control is accomplished in the sense that the limit of the error vector $e(t) = [e_1, e_2, \dots, e_n]^T$ approaches zero

$$\lim_{t \rightarrow \infty} e = 0, \tag{3}$$

where

$$e = y - x. \tag{4}$$

From Eq. (4) we have

$$\dot{e} = \dot{y} - \dot{x}, \tag{5}$$

$$\dot{e} = g(y, \hat{B}) - f(x, A) - u(t). \tag{6}$$

A Lyapunov function $V(e, \tilde{A}, \tilde{B})$ is chosen as a positive definite function

$$V(e, \tilde{A}, \tilde{B}) = \frac{1}{2} e^T e + \frac{1}{2} \tilde{A}^T \tilde{A} + \frac{1}{2} \tilde{B}^T \tilde{B}, \tag{7}$$

where $\tilde{A} = A - \hat{A}$, $\tilde{B} = B - \hat{B}$, A and B are two column matrices whose elements are the original coefficients of systems (1) and (2) respectively, \hat{A} , \hat{B} are two column matrices whose elements are the goal coefficients of systems (1) and (2), respectively.

Its derivative along any solution of the differential equation system consisting of Eq. (6) and. update parameter differential equations for \hat{A} and \hat{B} is

$$\dot{V}(e) = e^T [g(y, \hat{B}) - f(x, A) - u(t)] + \tilde{A} \dot{\tilde{A}} + \tilde{B} \dot{\tilde{B}}, \tag{8}$$

where $u(t)$, $\dot{\tilde{A}}$, and $\dot{\tilde{B}}$ are chosen so that $\dot{V} = e^T C e$, C is a diagonal negative definite matrix, and \dot{V} is a negative semi-definite function of e and parameter differences \tilde{A} and \tilde{B} . In current scheme of adaptive control of chaotic motion [26–28], traditional Lyapunov stability theorem and Babalat lemma are used to prove the error vector approaches zero, as time approaches infinity. But the question, why the estimated or given parameters also approach to the uncertain or goal parameters, remains no answer. By pragmatial asymptotical stability theorem, the question can be answered strictly.

The stability for many problems in real dynamical systems is actual asymptotical stability, although may not be mathematical asymptotical stability. The mathematical asymptotical stability demands that trajectories from all initial states in the neighborhood of zero solution must approach the origin as $t \rightarrow \infty$. If there are only a small part or even a few of the initial states from which the trajectories do not approach the origin as $t \rightarrow \infty$, the zero solution is not mathematically asymptotically stable. However, when the probability of occurrence of an event is zero, it means the event does not occur actually. If the probability of occurrence of the event that the trajectories from the initial states are that they do not approach zero when $t \rightarrow \infty$, is zero, the stability of zero solution is actual asymptotical stability though it is not mathematical asymptotical stability. In order to analyze the asymptotical stability of the equilibrium point of such systems, the pragmatial asymptotical stability theorem is used.

Let X and Y be two manifolds of dimensions m and n ($m < n$), respectively, and φ be a differentiable map from X to Y , then $\varphi(X)$ is subset of Lebesque measure 0 of Y [20]. For an autonomous system

$$\frac{dx}{dt} = f(x_1, \dots, x_n), \tag{9}$$

where $x = [x_1, \dots, x_n]^T$ is a state vector, the function $f = [f_1, \dots, f_n]^T$ is defined on $D \subset R^n$ and $\|x\| \leq H > 0$. Let $x = 0$ be an equilibrium point for the system (9). Then

$$f(0) = 0. \tag{10}$$

Definition. The equilibrium point for the system (9) is pragmatially asymptotically stable provided that with initial points on C which is a subset of Lebesque measure 0 of D , the behaviors of the corresponding trajectories cannot be determined, while with initial points on $D-C$, the corresponding trajectories behave as that agree with traditional asymptotical stability [18,19].

Theorem. Let $V = [x_1, \dots, x_n]^T: D \rightarrow R_+$ be positive definite and analytic on D , such that the derivative of V through Eq. (9), \dot{V} , is negative semi-definite.

Let X be the m -manifold consisted of point set for which $\forall x \neq 0, \dot{V}(x) = 0$ and D is a n -manifold. If $m+1 < n$, then the equilibrium point of the system is pragmatically asymptotically stable.

Proof. Since every point of X can be passed by a trajectory of Eq. (9), which is one-dimensional, the collection of these trajectories, C , is a $(m+1)$ -manifold [22,23].

If $m+1 < n$, then the collection C is a subset of Lebesgue measure 0 of D . By the above definition, the equilibrium point of the system is pragmatically asymptotically stable.

If an initial point is ergodically chosen in D , the probability of that the initial point falls on the collection C is zero. Here, equal probability is assumed for every point chosen as an initial point in the neighborhood of the equilibrium point. Hence, the event that the initial point is chosen from collection C does not occur actually. Therefore, under the equal probability assumption, pragmatical asymptotical stability becomes actual asymptotical stability. When the initial point falls on $D-C$, $\dot{V}(x) < 0$, the corresponding trajectories behave as that agree with traditional asymptotical stability because by the existence and uniqueness of the solution of initial-value problem, these trajectories never meet C .

In Eq. (7) V is a positive definite function of n variables, i.e. p error state variables and $n-p=m$ differences between unknown and estimated parameters, while $\dot{V} = e^T C e$ is a negative semi-definite function of n variables. Since the number of error state variables is always more than one, $p > 1$, $m+1 < n$ is always satisfied, by pragmatical asymptotical stability theorem we have

$$\lim_{t \rightarrow \infty} e = 0 \quad (11)$$

and the estimated parameters approach the uncertain parameters. The pragmatical adaptive control theorem is obtained. Therefore, the equilibrium point of the system is pragmatically asymptotically stable. Under the equal probability assumption, it is actually asymptotically stable for both error state variables and parameter variables. \square

3. Numerical results of the chaos control

A van der Pol [32-34] oscillator driven by a periodic excitation is considered. The equation of motion can be written as

$$\ddot{x} + \varphi \dot{x} + a\dot{x}(x^2 - 1) - b \sin \omega t = 0, \quad (12)$$

where φ, a, b are constant parameters and $b \sin \omega t$ is an external excitation. In Eq. (12), the linear term stands for a conservative harmonic force which determines the intrinsic oscillation frequency. The self-sustaining mechanism which is responsible for the perpetual oscillation rests on the nonlinear term. Energy exchange with the external agent depends on the magnitude of displacement $|x|$ and on the sign of velocity \dot{x} . During a complete cycle of oscillation, the energy is dissipated if displacement $x(t)$ is large than one, and that energy is fed-in if $|x| < 1$. The time-dependent term stands for the external driving force with amplitude b and frequency ω . Eq. (12) can be rewritten as two first order equations

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -\varphi y + a(1 - x^2)y + b \sin \omega t. \end{cases} \quad (13)$$

A new system, the double van der Pol system, studied in this paper consists of two van der Pol systems with mutual coupling terms instead of two external excitations

$$\begin{cases} \frac{dx_1}{dt} = y_1, \\ \frac{dy_1}{dt} = a_1 x_1 + b_1 y_1 + c_1 x_1^2 y_1 + d_1 u_1, \\ \frac{du_1}{dt} = v_1, \\ \frac{dv_1}{dt} = f_1 u_1 + g_1 v_1 + h_1 u_1^2 v_1 + j_1 x_1, \end{cases} \quad (14)$$

where $d_1 u_1, j_1 x_1$ are mutual coupling terms. When $a_1 = -1, b_1 = 0.2, c_1 = -2.4, d_1 = 0.04, j_1 = -0.3, f_1 = -1, g_1 = 2, h_1 = -2$ are original coefficients and initial conditions are $x_1(0) = 3, y_1(0) = 4, u_1(0) = 3$, and $v_1(0) = 4$, chaos of the system are illustrated by phase portraits (Fig. 1).

Case (a) Control a chaotic new double van der Pol system to a new double Duffing system

The goal system is a double Duffing system. The Duffing system is

$$\ddot{x} + a\dot{x} + bx + cx^3 = d \cos \omega t, \quad (15)$$

where a, b, c, d, ω are constant parameters, $d \cos \omega t$ is an external excitation. It can be written as two first order differential equations

$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = -ay - bx - cx^3 + d \cos \omega t. \end{cases} \quad (16)$$

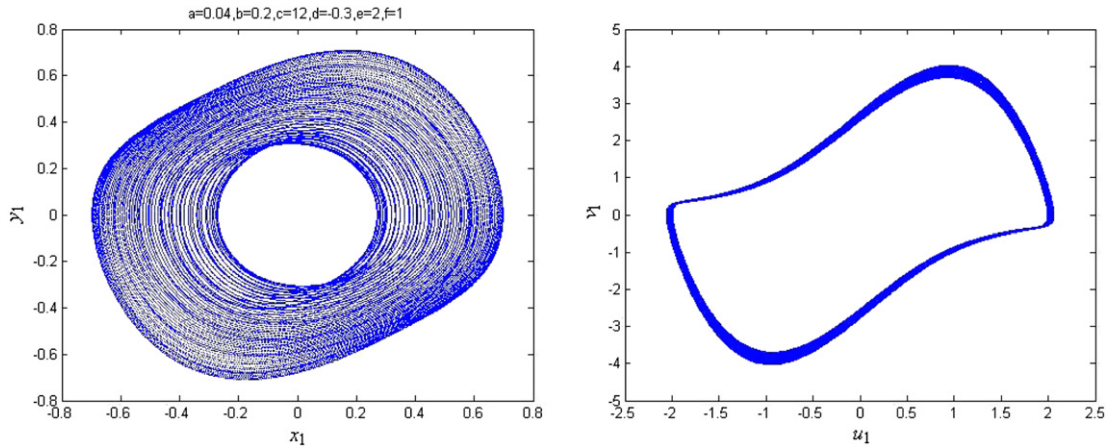


Fig. 1. Phase portraits of the double van der Pol system.

Consider the following double new Duffing system as goal system

$$\begin{cases} \frac{dx_2}{dt} = y_2, \\ \frac{dy_2}{dt} = a_1x_2 + \hat{b}_1y_2 + \hat{c}_2x_2^3 + \hat{d}_1u_2, \\ \frac{du_2}{dt} = v_2, \\ \frac{dv_2}{dt} = f_1u_2 + \hat{g}_1v_2 + \hat{h}_2u^3 + \hat{j}_1x_2. \end{cases} \tag{17}$$

It consists of two Duffing systems in which two external excitations are replaced by two coupling terms. It is an autonomous system with four states where \hat{b}_1 , \hat{c}_2 , \hat{d}_1 , \hat{j}_1 , \hat{g}_1 , and \hat{h}_2 are constant goal coefficients of the system. When $\hat{b}_1 = -0.05$, $\hat{c}_2 = -3$, $\hat{d}_1 = 7$, $\hat{j}_1 = -7$, $\hat{g}_1 = 0.05$, $\hat{h}_2 = -3$, the chaotic behavior is presented in Fig. 2.

In order to lead (x_1, y_1, u_1, v_1) to (x_2, y_2, u_2, v_2) , we add controllers U_1, U_2, U_3 , and U_4 to each equation of Eq. (14), respectively

$$\begin{cases} \frac{dx_1}{dt} = y_1 + U_1, \\ \frac{dy_1}{dt} = a_1x_1 + b_1y_1 + c_1x_1^2y_1 + d_1u_1 + U_2, \\ \frac{du_1}{dt} = v_1 + U_3, \\ \frac{dv_1}{dt} = j_1x_1 + f_1u_1 + g_1v_1 + h_1u_1^2v_1 + U_4. \end{cases} \tag{18}$$

We define error vector $E = [E_1, E_2, E_3, E_4]^T = [x_2, y_2, u_2, v_2]^T - [x_1, y_1, u_1, v_1]^T$. Subtracting Eq. (18) from Eq. (17), we obtain the error dynamics

$$\begin{aligned} \dot{E}_1 &= y_2 - y_1 - U_1, \\ \dot{E}_2 &= a_1x_2 + \hat{b}_1y_2 + \hat{c}_2x_2^3 + \hat{d}_1u_2 - a_1x_1 - b_1y_1 - c_1x_1^2y_1 - d_1u_1 - U_2, \end{aligned}$$

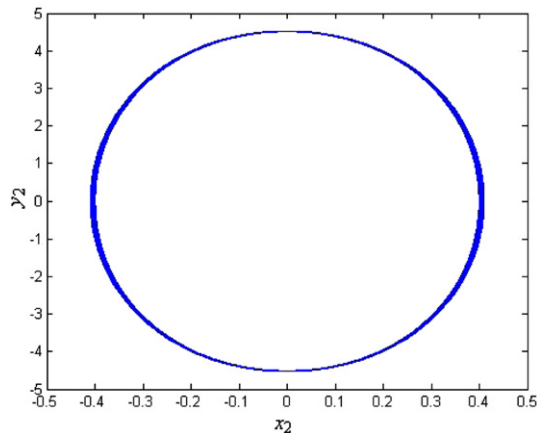


Fig. 2. Phase portraits of the double Duffing system.

$$\begin{aligned} \dot{E}_3 &= v_2 - v_1 - U_3, \\ \dot{E}_4 &= \hat{j}_1 x_2 + f_1 u_2 + \hat{g}_1 v_2 + \hat{h}_2 u_2^3 - j_1 x_1 - f_1 u_1 - g_1 v_1 - h_1 u_1^2 v_1 - U_4, \end{aligned} \tag{19}$$

where $E_1 = x_2 - x_1$, $E_2 = y_2 - y_1$, $E_3 = u_2 - u_1$, $E_4 = v_2 - v_1$.

Choose a Lyapunov function in the form of the positive definite function:

$$V(E_1, E_2, E_3, E_4, \tilde{b}_1, \tilde{c}_1, \tilde{c}_2, \tilde{d}_1, \tilde{j}_1, \tilde{g}_1, \tilde{h}_1, \tilde{h}_2) = \frac{1}{2}(E_1^2 + E_2^2 + E_3^2 + E_4^2 + \tilde{b}_1^2 + \tilde{c}_1^2 + \tilde{c}_2^2 + \tilde{d}_1^2 + \tilde{j}_1^2 + \tilde{g}_1^2 + \tilde{h}_1^2 + \tilde{h}_2^2), \tag{20}$$

where $\tilde{b}_1 = \hat{b}_1 - b_1$, $\tilde{c}_1 = \hat{c}_1 - c_1$, $\tilde{c}_2 = \hat{c}_2 - c_2$, $\tilde{d}_1 = \hat{d}_1 - d_1$, $\tilde{j}_1 = \hat{j}_1 - j_1$, $\tilde{g}_1 = \hat{g}_1 - g_1$, $\tilde{h}_1 = \hat{h}_1 - h_1$, $\tilde{h}_2 = \hat{h}_2 - h_2$ and $\hat{b}_1, \hat{c}_1, \hat{c}_2, \hat{d}_1, \hat{j}_1, \hat{g}_1, \hat{h}_1, \hat{h}_2$ are given parameters, $b_1 = -0.05, c_1 = 0, c_2 = -3, d_1 = 7, j_1 = -7, g_1 = 0.05, h_1 = 0, h_2 = -3$ are goal parameters.

Its time derivative along any solution of Eq. (19) and parameter dynamics is

$$\begin{aligned} \dot{V} &= E_1[y_2 - y_1 - U_1] + E_2[a_1 x_2 + \hat{b}_1 y_2 + \hat{c}_2 x_2^3 + \hat{d}_1 u_2 - a_1 x_1 - b_1 y_1 - c_1 x_1^2 y_1 - d_1 u_1 - U_2] + E_3[v_2 - v_1 - U_3] \\ &\quad + E_4[\hat{j}_1 x_2 + f_1 u_2 + \hat{g}_1 v_2 + \hat{h}_2 u_2^3 - j_1 x_1 - f_1 u_1 - g_1 v_1 - h_1 u_1^2 v_1 - U_4] + \tilde{b}_1(-\dot{\tilde{b}}_1) + \tilde{c}_1(-\dot{\tilde{c}}_1) + \tilde{c}_2(-\dot{\tilde{c}}_2) + \tilde{d}_1(-\dot{\tilde{d}}_1) \\ &\quad + \tilde{j}_1(-\dot{\tilde{j}}_1) + \tilde{g}_1(-\dot{\tilde{g}}_1) + \tilde{h}_1(-\dot{\tilde{h}}_1) + \tilde{h}_2(-\dot{\tilde{h}}_2). \end{aligned} \tag{21}$$

Choose

$$\begin{aligned} U_1 &= y_2 - y_1 + E_1, \\ U_2 &= a_1 x_2 + \hat{b}_1 y_2 + \hat{c}_2 x_2^3 + \hat{d}_1 u_2 - a_1 x_1 - b_1 y_1 - c_1 x_1^2 y_1 - d_1 u_1 + E_2 + \tilde{a}_1^2 + \tilde{b}_1^2 + \tilde{c}_1^2 + \tilde{c}_2^2 + \tilde{d}_1^2, \\ U_3 &= v_2 - v_1 + E_3, \\ U_4 &= \hat{j}_1 x_2 + f_1 u_2 + \hat{g}_1 v_2 + \hat{h}_2 u_2^3 - j_1 x_1 - f_1 u_1 - g_1 v_1 - h_1 u_1^2 v_1 + E_4 + \tilde{e}_1^2 + \tilde{f}_1^2 + \tilde{g}_1^2 + \tilde{h}_1^2 + \tilde{h}_2^2, \end{aligned} \tag{22}$$

$$\begin{aligned} -\dot{\tilde{b}}_1 &= \tilde{b}_1 E_2, \\ -\dot{\tilde{c}}_1 &= \tilde{c}_1 E_2, \\ -\dot{\tilde{c}}_2 &= \tilde{c}_2 E_2, \\ -\dot{\tilde{d}}_1 &= \tilde{d}_1 E_2, \\ -\dot{\tilde{j}}_1 &= \tilde{j}_1 E_4, \\ -\dot{\tilde{g}}_1 &= \tilde{g}_1 E_4, \\ -\dot{\tilde{h}}_1 &= \tilde{h}_1 E_4, \\ -\dot{\tilde{h}}_2 &= \tilde{h}_2 E_4. \end{aligned} \tag{23}$$

Eq. (23) is the parameter dynamics. Substituting Eqs. (22) and (23) into Eq. (18), we obtain

$$\dot{V} = E_1^2 - E_2^2 - E_3^2 - E_4^2 < 0$$

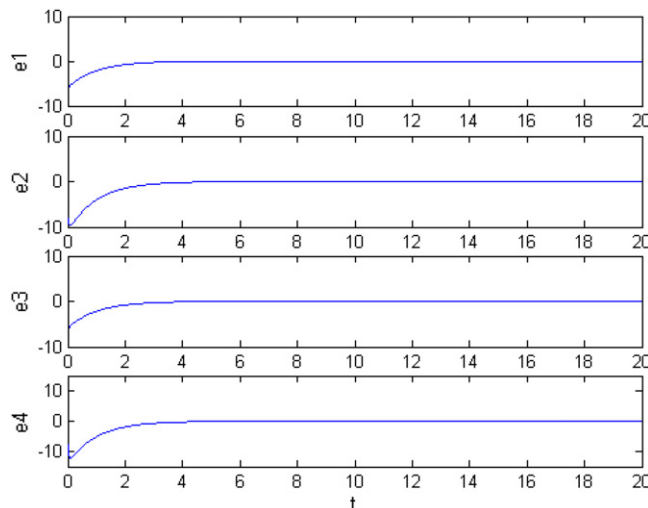


Fig. 3. Time histories of state errors for E_1, E_2, E_3, E_4 for Case (a).

which is negative semi-definite function of E_1, E_2, E_3, E_4 . The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that the common origin of error dynamics (19) and parameter dynamics (20) is asymptotically stable. Now, D is an 8-manifold, $n = 12$ and the number of error state variables $p = 4$. When $E_1 = E_2 = E_3 = E_4 = 0$ and $\tilde{b}_1, \tilde{c}_1, \tilde{c}_2, \tilde{d}_1, \tilde{e}_1, \tilde{g}_1, \tilde{h}_1, \tilde{h}_2$, take arbitrary values, $\dot{V} = 0$, so X is 4-manifold, $m = n - p = 12 - 4 = 8$. $m + 1 < n$ is satisfied. By pragmatistical asymptotical stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The pragmatistical generalized synchronization is obtained. Under the assumption of equal probability, it is actually asymptotically stable. This means that the chaos control for different systems, from a double van der Pol system to a double Duffing system, can be achieved. The simulation results are shown in Figs. 3 and 4.

Case (b) Control a chaotic double van der Pol system to a exponentially damped simple harmonic system
 Consider the following exponentially damped simple harmonic system

$$\begin{cases} \frac{dx_3}{dt} = -\hat{\lambda}_1 x_3, \\ \frac{dy_3}{dt} = -\hat{b}_1 y_3, \\ \frac{du_3}{dt} = v_3, \\ \frac{dv_3}{dt} = -\hat{f}_1 u_3. \end{cases} \tag{24}$$

In the first equation of Eq. (18), $k_1 = 1$.

$$\begin{cases} \frac{dx_1}{dt} = k_1 y_1 + U_1, \\ \frac{dy_1}{dt} = a_1 x_1 + b_1 y_1 + c_1 x_1^2 y_1 + d_1 u_1 + U_2, \\ \frac{du_1}{dt} = v_1 + U_3, \\ \frac{dv_1}{dt} = j_1 x_1 + f_1 u_1 + g_1 v_1 + h_1 u_1^2 v_1 + U_4, \end{cases} \tag{25}$$

where $k_1 = 1, a_1 = -1, b_1 = 0.2, c_1 = -2.4, d_1 = 0.04, j_1 = -0.3, f_1 = -1, g_1 = 2, h_1 = -2, \lambda_1 = 0$.

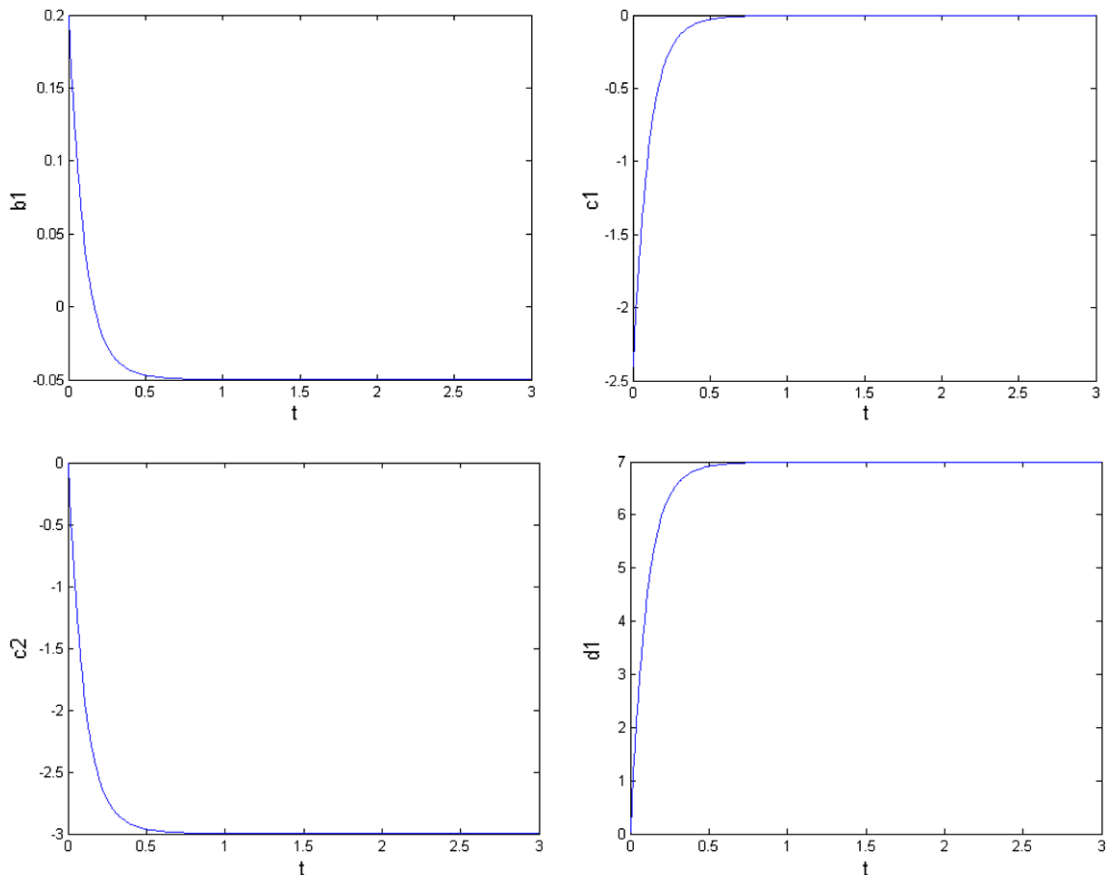


Fig. 4. Time histories of coefficients $a_1, b_1, c_1, c_2, d_1, j_1, f_1, g_1, h_1, h_2$ for Case (a).

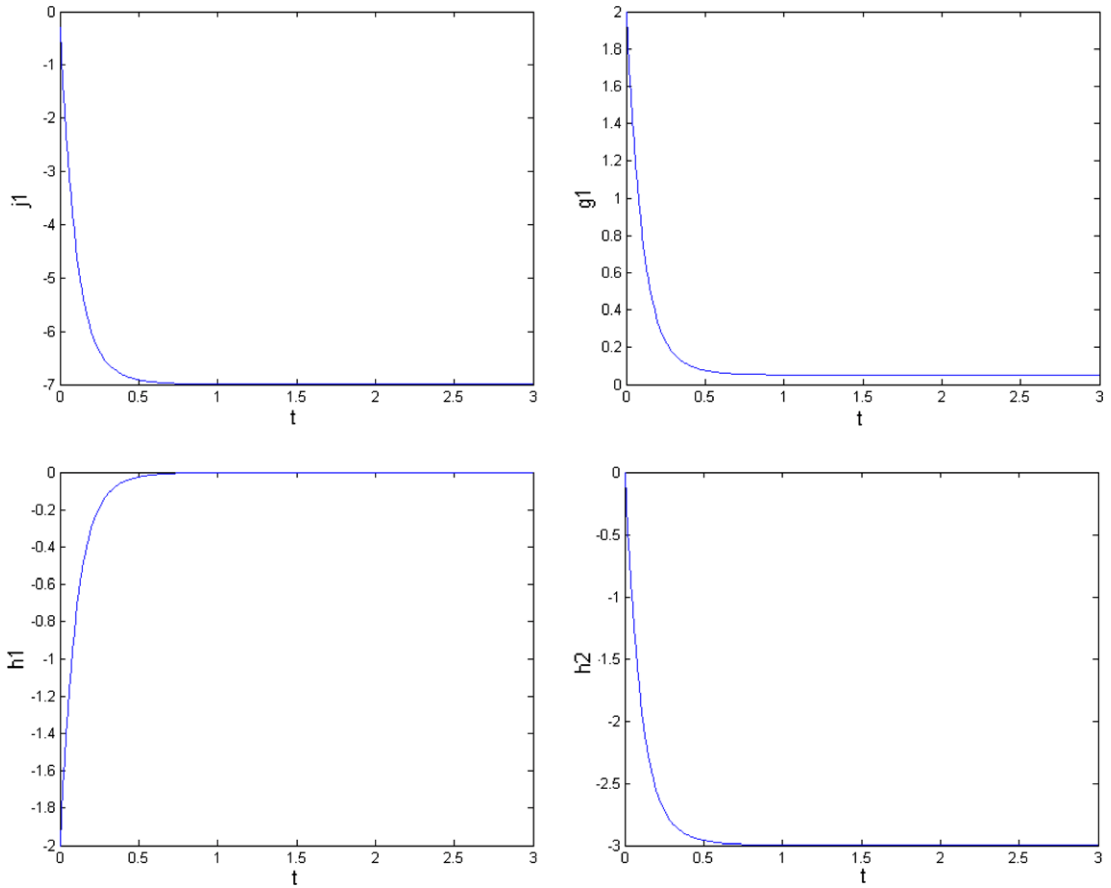


Fig. 4 (continued)

We define error vector $E = [E_1, E_2, E_3, E_4]^T = [x_3, y_3, u_3, v_3]^T - [x_1, y_1, u_1, v_1]^T$. Subtracting Eq. (25) from Eq. (24), we obtain the error dynamics

$$\dot{E}_1 = -\lambda_1 x_3 - k_1 y_1 - U_1,$$

$$\dot{E}_2 = -\hat{b}_1 y_3 - a_1 x_1 - b_1 y_1 - c_1 x_1^2 y_1 - d_1 u_1 - U_2,$$

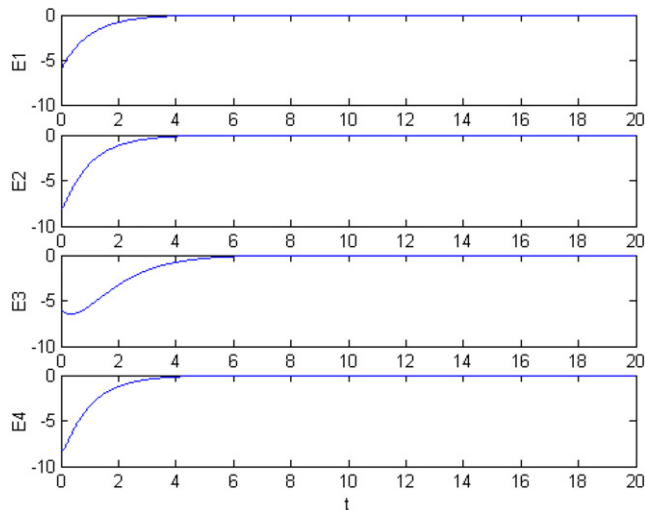


Fig. 5. Time histories of state errors for E_1, E_2, E_3, E_4 for Case (b).

$$\begin{aligned} \dot{E}_3 &= v_3 - v_1 - U_3, \\ \dot{E}_4 &= -\hat{f}_1 u_3 - j_1 x_1 - f_1 u_1 - g_1 v_1 - h_1 u_1^2 v_1 - U_4, \end{aligned} \tag{26}$$

where $E_1 = x_2 - x_1$, $E_2 = y_2 - y_1$, $E_3 = u_2 - u_1$, $E_4 = v_2 - v_1$.

Choose a Lyapunov function in the form of the positive definite function

$$V(E_1, E_2, E_3, E_4, \tilde{k}_1, \tilde{a}_1, \tilde{b}_1, \tilde{c}_1, \tilde{d}_1, \tilde{j}_1, \tilde{f}_1, \tilde{g}_1, \tilde{\lambda}_1) = \frac{1}{2} (E_1^2 + E_2^2 + E_3^2 + E_4^2 + \tilde{k}_1^2 + \tilde{a}_1^2 + \tilde{b}_1^2 + \tilde{c}_1^2 + \tilde{d}_1^2 + \tilde{j}_1^2 + \tilde{f}_1^2 + \tilde{g}_1^2 + \tilde{h}_1^2 + \tilde{\lambda}_1^2), \tag{27}$$

where $\tilde{k}_1 = \hat{k}_1 - k_1$, $\tilde{a}_1 = \hat{a}_1 - a_1$, $\tilde{b}_1 = \hat{b}_1 - b_1$, $\tilde{c}_1 = \hat{c}_1 - c_1$, $\tilde{d}_1 = \hat{d}_1 - d_1$, $\tilde{j}_1 = \hat{j}_1 - j_1$, $\tilde{f}_1 = \hat{f}_1 - f_1$, $\tilde{g}_1 = \hat{g}_1 - g_1$, $\tilde{h}_1 = \hat{h}_1 - h_1$, $\tilde{\lambda}_1 = \hat{\lambda}_1 - \lambda_1$, and $\hat{k}_1, \hat{a}_1, \hat{b}_1, \hat{c}_1, \hat{d}_1, \hat{j}_1, \hat{f}_1, \hat{g}_1, \hat{h}_1, \hat{\lambda}_1$, are goal parameters, $\hat{k}_1 = 0$, $\hat{a}_1 = 0$, $\hat{b}_1 = 2$, $\hat{c}_1 = 0$, $\hat{d}_1 = 0$, $\hat{j}_1 = 0$, $\hat{f}_1 = 2.3$, $\hat{g}_1 = 0$, $\hat{h}_1 = 0$, $\hat{\lambda}_1 = 3$.

Its time derivative along any solution of Eq. (19) and parameter dynamics is

$$\begin{aligned} \dot{V} &= E_1 [-\hat{\lambda}_1 x_2 - k_1 y_1 - U_1] + E_2 [-\hat{b}_1 y_3 - a_1 x_1 - b_1 y_1 - c_1 x_1^2 y_1 - d_1 u_1 - U_2] + E_3 [v_2 - v_1 - U_3] \\ &\quad + E_4 [-\hat{f}_1 u_3 - j_1 x_1 - f_1 u_1 - g_1 v_1 - h_1 u_1^2 v_1 - U_4] + \tilde{k}_1 (-\dot{\hat{k}}_1) + \tilde{a}_1 (-\dot{\hat{a}}_1) + \tilde{b}_1 (-\dot{\hat{b}}_1) + \tilde{c}_1 (-\dot{\hat{c}}_1) + \tilde{d}_1 (-\dot{\hat{d}}_1) \\ &\quad + \tilde{j}_1 (-\dot{\hat{j}}_1) + \tilde{f}_1 (-\dot{\hat{f}}_1) + \tilde{g}_1 (-\dot{\hat{g}}_1) + \tilde{h}_1 (-\dot{\hat{h}}_1) + \tilde{\lambda}_1 (-\dot{\hat{\lambda}}_1). \end{aligned} \tag{28}$$

Choose

$$\begin{aligned} U_1 &= -\hat{\lambda}_1 x_2 - k_1 y_1 + E_1, \\ U_2 &= -\hat{b}_1 y_3 - a_1 x_1 - b_1 y_1 - c_1 x_1^2 y_1 - d_1 u_1 + \tilde{a}_1^2 + \tilde{b}_1^2 + \tilde{c}_1^2 + \tilde{d}_1^2 + E_2, \\ U_3 &= E_3, \\ U_4 &= -\hat{f}_1 u_3 - j_1 x_1 - f_1 u_1 - g_1 v_1 - h_1 u_1^2 v_1 + \tilde{j}_1^2 + \tilde{f}_1^2 + \tilde{g}_1^2 + \tilde{h}_1^2 + E_4, \end{aligned} \tag{29}$$

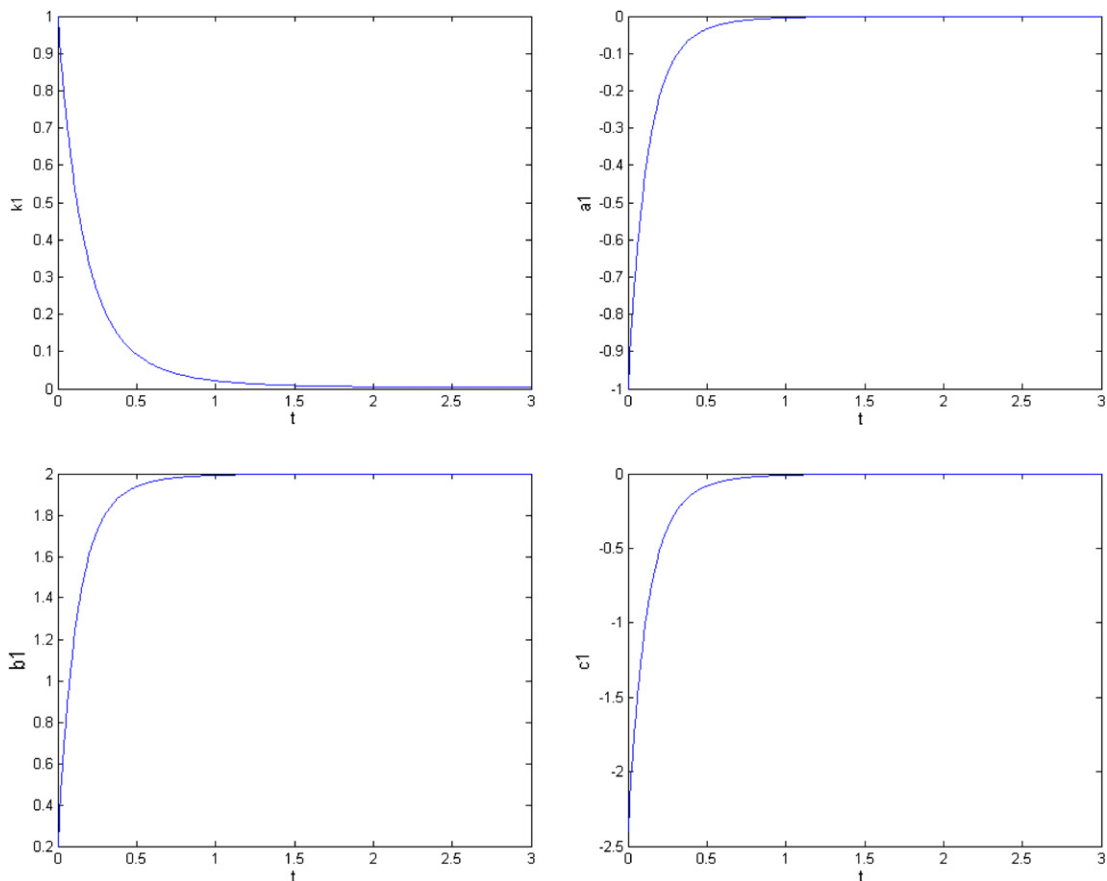


Fig. 6. Time histories of coefficients $k_1, a_1, b_1, c_1, d_1, j_1, f_1, g_1, h_1, \lambda_1$ for Case (b).

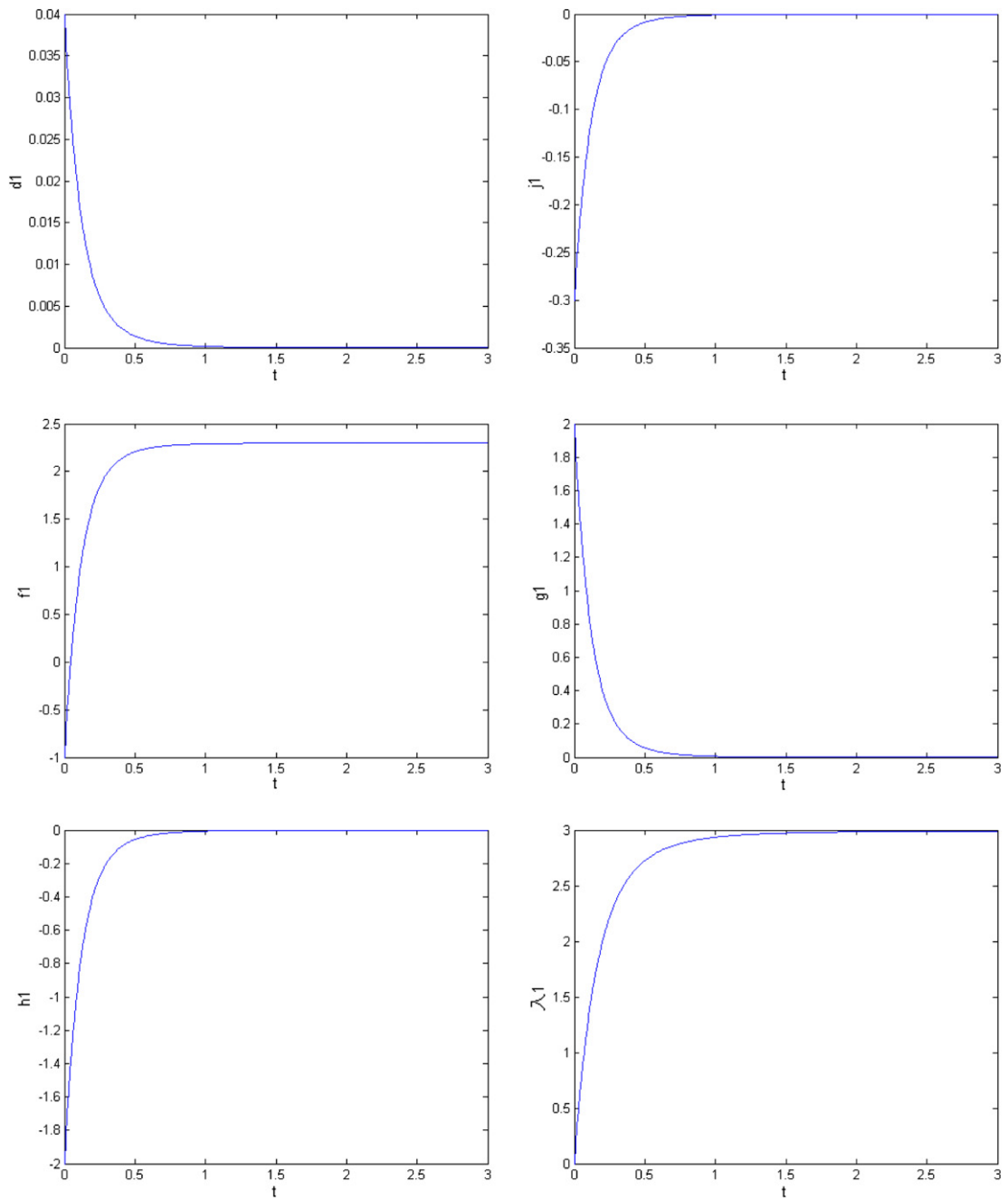


Fig. 6 (continued)

$$\begin{aligned}
 -\dot{k}_1 &= \tilde{k}_1 E_1, \\
 -\dot{a}_1 &= \tilde{a}_1 E_2, \\
 -\dot{b}_1 &= \tilde{b}_1 E_2, \\
 -\dot{c}_1 &= \tilde{c}_1 E_2, \\
 -\dot{d}_1 &= \tilde{d}_1 E_2, \\
 -\dot{j}_1 &= \tilde{j}_1 E_4, \\
 -\dot{g}_1 &= \tilde{g}_1 E_4, \\
 -\dot{h}_1 &= \tilde{h}_1 E_4, \\
 -\dot{\lambda}_1 &= \tilde{\lambda}_1 E_1.
 \end{aligned}$$

(30)

Eq. (30) is the parameter dynamics. Substituting Eqs. (29) and (30) into Eq. (28), we obtain

$$\dot{V} = E_1^2 - E_2^2 - E_3^2 - E_4^2 < 0$$

which is negative semi-definite function of E_1, E_2, E_3, E_4 . The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that the common origin of error dynamics (26) and parameter dynamics (27) is asymptotically stable. Now, D is an 8-manifold, $n = 12$ and the number of error state variables $p = 4$. When $E_1 = E_2 = E_3 = E_4 = 0$ and $\tilde{k}_1, \tilde{a}_1, \tilde{b}_1, \tilde{c}_1, \tilde{d}_1, \tilde{j}_1, \tilde{f}_1, \tilde{g}_1, \tilde{h}_1, \tilde{\lambda}_1$, take arbitrary values, $\dot{V} = 0$, so X is 4-manifold, $m = n - p = 12 - 4 = 8$. $m + 1 < n$ is satisfied. By pragmatistical asymptotical stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The pragmatistical generalized synchronization is obtained. Under the assumption of equal probability, it is actually asymptotically stable. This means that the chaos control for different systems, from a double van der Pol system to an exponentially damped-simple harmonic system, can be achieved. The simulation results are shown in Figs. 5 and 6.

4. Conclusions

To control chaotic systems to different systems is study by new pragmatistical adaptive control method. The pragmatistical asymptotical stability theorem fills the vacancy between the actual asymptotical stability and mathematical asymptotical stability. The conditions of the Lyapunov function for pragmatistical asymptotical stability are lower than that for traditional asymptotical stability. By using this theorem, with the same conditions for Lyapunov function, $V > 0, \dot{V} \leq 0$, as that in current scheme of adaptive chaos control, we not only obtain the adaptive control of chaotic systems but also prove that the estimated parameters approach the uncertain values. Traditional chaos control is limited to decrease chaos of one chaotic system. This method enlarges the effective scope of chaos control. We can control a chaotic system to a given chaotic system or to a given regular system.

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