Spin Hall effect in a Josephson contact

A. G. Mal'shukov[1,2,3] and C. S. Chu[2,3]
[1]*Institute of Spectroscopy, Russian Academy of Sciences, 142190 Troitsk, Moscow oblast, Russia*
[2]*Department of Electrophysics, National Chiao Tung University, Hsinchu 30010, Taiwan*
[3]*Physics Division, National Center for Theoretical Sciences, Hsinchu 30043, Taiwan*
(Received 30 May 2008; published 9 September 2008)

The spin Hall effect on the Josephson tunneling through a two-dimensional normal contact with a spin-orbit split conduction band has been studied in the diffusive regime and at the zero electric bias. Linearized Usadel equations for triplet components of the pairing function in the presence of intrinsic spin-orbit interaction have been derived. These equations have been employed for analysis of the spin Hall effect induced by the supercurrent. We predict that a nondissipative out-of-plane spin Hall polarization accumulates at lateral edges and an in-plane polarization is induced throughout the entire normal region. At the same time, in contrast to the spin Hall effect in normal systems, the spin current is absent in the considered case of the stationary Josephson effect.

DOI: 10.1103/PhysRevB.78.104503 PACS number(s): 72.25.Dc, 71.70.Ej, 73.40.Lq, 74.50.+r

I. INTRODUCTION

Various spintronic applications have attracted much recent interest in the spin-orbit interaction (SOI) effects on electron transport in normal metals and semiconductors. This interaction gives rise to fundamental transport phenomena, such as the spin Hall effect (SHE) (for a review, see Ref. 1), and electric spin orientation.[1,2] The former shows up in a spin-polarization flux flowing perpendicular to the electric current. This flux, in its turn, gives rise to the spin polarization at the sample boundaries. The spin polarization can also be directly induced by the electric current in the sample bulk, that is, the electric spin orientation effect. These effects demonstrate delicate coupling of spin and charge degrees of freedom in electron transport. On the other hand, we have the interplay between SOI and superconductivity. Indeed, various systems including superconductor-ferromagnet superconductor junctions[3] (F stands for ferromagnet), superconductor-normal-superconductor (SNS)[4,5] and superconductor-normal (SN) (Refs. 6) contacts, or bulk superconductors,[6,7] have been considered. In Refs. 4 and 5 the effect of the SOI onto the Josephson current has been studied in the case when the normal (N) part of a SNS junction is a normal metal with a noticeable SOI. We will, however, focus on a different problem. Namely, we are going to consider the spin Hall current and spin accumulation associated with the supercurrent across the junction. As was pointed out in Refs. 6 and 7, SOI causes the admixture of triplet components in the pairing function. This sort of singlet-triplet coupling finds similarity in the spin-charge coupling in normal systems. Thus, one might expect that phenomena closely related to SHE could manifest themselves also in superconducting structures, such as SNS junctions. But, after a moments' thought, it becomes clear that this analogy cannot go too far. For example, at least in the case of zero voltage across the junction, the spin Hall current cannot be a linear response to the supercurrent. It is because, on the one hand, these two currents are, by nature, of opposite parities in time; but, on the other hand, the parities must be the same in the case of stationary nondissipative superconducting transport. Yet, it is still of great interest to look for another signature of SHE: the accumulation of magnetization in response to the supercurrent in superconducting structures. It should be noted that, despite a formal similarity, such a magnetization is fundamentally distinct from that induced by the normal SHE since it is determined by a coherent many-particle quantum state and, hence, is not subject to dissipative processes of spin diffusion and relaxation that take place in normal systems.

We will consider SHE and the electric spin orientation in the case of the Josephson tunneling through a two-dimensional (2D) N contact (see Fig. 1). The SOI in the normal contact may be caused by impurities or it can be of an intrinsic origin, due to the crystal field in noncentrosymmetric lattices. We will consider the intrinsic SOI represented by the Hamiltonian $H_{so}=\boldsymbol{\sigma}\cdot\mathbf{h}_{\boldsymbol{k}}$, where $\boldsymbol{\sigma}$ is the Pauli spin vector. The spin-orbit field $\mathbf{h}_{\boldsymbol{k}}$, which is an odd function of the electron wave vector $\boldsymbol{k}$, can be given, for example, by Rashba[8] or Dresselhaus[9] SOI, as well as by their combination. In this case the vector $\mathbf{h}_{\boldsymbol{k}}$ lies in the plane of the 2D system. The electron transport through the contact will be treated within the diffusion approximation so that the length of the junction L, the condensate wave-function penetration depth into N region L_c, and the spin precession length $L_{so}=v_F/2h$ with v_F as the Fermi velocity and h as the angular-averaged spin-orbit field are all assumed to be much larger than the electron mean-free path l. For our Josephson setting,

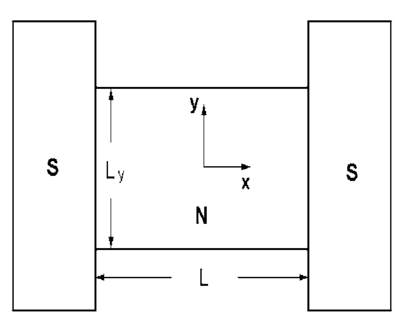

FIG. 1. Josephson contact with S and N denoting superconducting and normal regions, respectively.

the bias voltage across the junction is zero and the supercurrent is determined by the phase difference between the two S electrodes. Our analysis involves a standard semiclassical treatment of Gor'kov's equations in the diffusion approximation (for a review, see Ref. 10). Our goal is to derive Usadel-type equations and to calculate the spin density induced by the SHE.

II. USADEL EQUATIONS

In the considered case of the thermal equilibrium, all observables of interest can be expressed via retarded and advanced Green's functions. The corresponding Gor'kov's equations in the Nambu representation have the form

$$\left(i\frac{\partial}{\partial t} - \check{H} - \check{\Sigma}^{r/a}\right)\check{G}^{r/a}(X,X') = \delta(X-X'), \qquad (1)$$

where r and a denote retarded and advanced functions, respectively, $X=(t,\mathbf{r})$ and

$$\check{H} = \frac{\tau_3}{2m^*}\hat{k}^2 - \tau_3\mu + \boldsymbol{\sigma}\cdot\mathbf{h}_{\hat{\mathbf{k}}}, \qquad (2)$$

with the momentum operator $\hat{\mathbf{k}}=-i\partial/\partial\boldsymbol{r}$, and the chemical potential μ. After averaging the initial Green's functions over random positions of short-range impurities, the self-energy in Eq. (1) takes the form[11]

$$\check{\Sigma}^{r/a}(X,X') = \frac{\tau_3}{2\tau\pi N_F}\check{G}^{r/a}(t,t',\boldsymbol{r},\boldsymbol{r})\tau_3\delta(\boldsymbol{r}-\boldsymbol{r}'), \qquad (3)$$

where τ is the elastic-scattering time. Unperturbed Green's functions are easily obtained from Eq. (1). In the momentum representation and after the time Fourier transform, they can be written as

$$\check{G}^{0r/a}(\omega,\boldsymbol{k}) = (\omega - \tau_3 E_k - \boldsymbol{\sigma}\cdot\mathbf{h}_{\mathbf{k}} \pm i\Gamma)^{-1}, \qquad (4)$$

where $E_k=(k^2/2m^*)-\mu$. Below we will perform calculations for retarded functions and drop the labels r and a.

Proximity to superconducting contacts results in appearance of anomalous (proportional to τ_1 and τ_2) Green's functions. These functions are inhomogeneous in space. In order to calculate them, we will follow a well-known procedure in the framework of the semiclassical approximation.[12] First, we perform the Fourier transform with respect to $X-X'$ introducing, accordingly, the frequency and wave-vector variables ω and $\boldsymbol{k}$. The center-of-mass variables will remain intact and will be denoted as $\boldsymbol{r}$. Since the problem is stationary, the corresponding center-of-time variable is absent. Taking into account that variations of G on the scale of the Fermi wavelength are small, Eq. (1) should be expanded in terms of gradients $\partial/\partial\boldsymbol{r}$. The next step is to simplify the self-energy part of Eq. (1), keeping only the terms linear in the anomalous part. Such a linearization can be done if the transparency of the SN contact is small or the leads are close to the superconducting critical temperature. By combining Eq. (1) and its conjugate one and by making use of the fact that $\mathbf{h}_{\boldsymbol{k}}$ is an odd function of $\boldsymbol{k}$ for the anomalous part G_{12}, we obtain the equation

$$\left(2\omega - \boldsymbol{v}\cdot\hat{\mathbf{q}} + \frac{i}{\tau}\right)G_{12} - \{\mathbf{h}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma}, G_{12}\} - \frac{1}{2}[\delta\mathbf{h}_{\boldsymbol{k},\hat{\mathbf{q}}}\cdot\boldsymbol{\sigma}, G_{12}] = I_{sc}, \qquad (5)$$

where $\delta\mathbf{h}_{k,\hat{\mathbf{q}}}=(\hat{\mathbf{q}}\cdot\nabla_k)\mathbf{h}_{\boldsymbol{k}}$ with $\hat{\mathbf{q}}=-i\partial/\partial\mathbf{r}$ and

$$I_{sc} = -\frac{1}{2\tau\pi N_F}(G^0_{11}g_{12} + g_{12}G^0_{22}). \qquad (6)$$

The subscripts of G^0 denote the matrix elements in the Nambu space and $g_{12}=\Sigma_{\boldsymbol{k}}G_{12}$. The 2×2 matrix $G_{12,\alpha\beta}$ can be transformed to the conventional pairing function $F_{\alpha\bar{\beta}} \equiv G_{12,\alpha\beta}$, where $\bar{\beta}$ denotes the spin projection opposite to β. Further, it is convenient to decompose F into triplet F_1,F_{-1}, and F_0 and singlet F_s components as

$$F_0 = \frac{F_{12}+F_{21}}{\sqrt{2}}, \quad F_s = \frac{F_{12}-F_{21}}{\sqrt{2}}$$

$$F_1 = F_{11}, \quad F_{-1} = F_{22}. \qquad (7)$$

After this transformation, it is easy to see that the last term in the left-hand side of Eq. (5) is responsible for a coupling between the singlet and triplet components of the pairing function. Besides, the singlet-triplet coupling also originates from the spin-dependent parts of G^0_{11} and G^0_{22} in Eq. (6). Due to such coupling, the triplet component of F is generated within the junction between two singlet S electrodes.

The Usadel diffusion equation can be obtained from Eq. (5) by iterating it with respect to small $\omega\tau$, $(\boldsymbol{v}\cdot\hat{\mathbf{q}})\tau$, and $\mathbf{h}_{\boldsymbol{k}}\tau$ up to the second order in the last two parameters. By this way, components of F_m are expressed in terms of $I_{\rm sc}$ and the last term in the left-hand side of Eq. (5). Further summation over $\boldsymbol{k}$ leads to the closed diffusion equation for $f_m=\Sigma_{\boldsymbol{k}}F_m$.

The derivation of the diffusion equation and the following analysis will be restricted to a limiting case of the strong SOI so that $L_{\rm so}\ll L_c$. From the theory of SNS contacts,[10] it follows that $L_c=\min(L,L_T)$ where $L_T=\sqrt{D/k_BT}$ is the thermal diffusion length. It is assumed that the energy gap in superconducting contacts $|\Delta|\gg k_BT$ and D/L^2. If the N region is represented by a narrow gap semiconductor quantum well, $L_{\rm so}$ may vary from less than a micron to several microns. For example, the Dresselhaus interaction in a GaAs/AlGaAs quantum well provides the spin splitting $2h=0.1$ meV at $n=5\times 10^{11}$ cm^{-2} (Ref. 13) that gives $L_{\rm so}\approx 2$ μm. This length strongly decreases, if a strong Rashba SOI takes place in addition to the Dresselhaus interaction. Hence, at low enough temperatures and the junction lengths of several μm, the requirement $L_{\rm so}\ll L_c$ can be realized in practice. Taking the leading terms we arrive at the following diffusion equation for the triplet pairing function $f_m=\Sigma_{\boldsymbol{k}}F_m$ where $(m=0,1,-1)$:

$$2i\omega f = \tau\left\langle\left(-i\boldsymbol{v}\cdot\frac{\partial}{\partial\boldsymbol{r}} + 2\mathbf{J}\cdot\mathbf{h}_{\boldsymbol{k}}\right)^2\right\rangle f + \mathrm{M}f_s, \qquad (8)$$

where $\mathbf{J}$ is the vector of 3×3 angular moment operators and $\langle\ldots\rangle$ denotes the angular averaging over the Fermi surface. The triplet-singlet coupling is given by

$$\mathrm{M}_0 = 0, \quad \mathrm{M}_{\pm 1} = \frac{4\tau^2}{\sqrt{2}}\langle \mathrm{h}_{\boldsymbol{k}}^{\mp}(\mathbf{h}_{\boldsymbol{k}} \times \delta\mathbf{h}_{\boldsymbol{k},\hat{\mathbf{q}}})\rangle, \tag{9}$$

with $\mathrm{h}_{\boldsymbol{k}}^{\mp}=\mathrm{h}_{\boldsymbol{k}}^{x}\mp i\mathrm{h}_{\boldsymbol{k}}^{y}$. The singlet f_s satisfies the similar equation, where the second term in angular brackets is absent and the mixing with triplets is represented by the Hermitian conjugate to M. Since triplets, in their turn, are expressed through f_s, such a mixing gives rise to a correction term in the closed equation for f_s. From Eqs. (8) and (9) it is easy to evaluate this correction as $\sim \omega h^2\tau^2/(k_F^2 L_{\rm so}^2) f_s \ll 2\omega f_s$. Therefore, the effect of SOI on f_s and, hence, on the Josephson current is weak in the semiclassical case, which is in agreement with Ref. 5. We also neglect a depairing effect on f_s due to exchange Zeeman field associated with the finite spin Hall polarization. This effect is weak as evaluated in the discussion in Sec. IV. Hence, the singlet function f_s is given by the well-known unperturbed solution in the SNS contact.

Without the last term in the right-hand side in Eq. (8), it [Eq. (8)] formally coincides with the spin-diffusion equation for two-dimensional electron gas (2DEG) in a zero electric field.[14] The spin-diffusion equation in the presence of the electric field has been derived in Ref. 15 for the case of the Rashba SOI and for a general SOI in Ref. 16. After a linear transformation to spin-density variables,[14] Equation (8) will coincide with these more general equations, if, apart from a constant factor, f_s is formally identified with the electric-field potential. Hence, a coupling of the spin to the electric field in normal spin transport appears to be very similar to the singlet-triplet coupling in Eq. (8). We note, however, a principal distinction from the normal transport. Equation (8) is written not for an observable spin density but for the anomalous Green's function, which plays the role of the Cooper pair wave function. Hence, the observables, such as spin densities, cannot be directly represented by a solution of the diffusion equation but must be calculated from bilinear combinations of f_m as will be done below.

III. SPIN DENSITY

Let us consider an example of the Rashba SOI. In this case $h_{\boldsymbol{k}}^x=\alpha k_y$ and $h_{\boldsymbol{k}}^y=-\alpha k_x$. For a homogeneous in y direction case, all functions depend only on x and we get $f_0=0$ and $f_1=f_{-1}$ with f_1 satisfying the equation

$$D\frac{\partial^2}{\partial x^2}f_1 - \Gamma_{\rm so}f_1 = i\frac{\alpha\tau\Gamma_{\rm so}}{\sqrt{2}}\frac{\partial}{\partial x}f_s, \tag{10}$$

where $\Gamma_{\rm so}=2\tau\alpha^2k_F^2$ is the D'yakonov-Perel' spin-relaxation time.[17] The small left hand side of Eq. (8) has been neglected in Eq. (10). Neglecting the third and higher derivatives of f_s, the solution of Eq. (10) can be written as

$$f_1 = -i\frac{\alpha\tau}{\sqrt{2}}\frac{\partial}{\partial x}f_s + \psi(x), \tag{11}$$

where, in order to satisfy appropriate boundary conditions[18] at $x=\pm L/2$, $\psi(x)$ is taken as a linear combination of $\exp(\pm kx)$ with $k=\sqrt{D/\Gamma_{\rm so}}=1/L_{\rm so}$. Since $L \gg L_{so}$, this function is important only close to the boundaries. We, however, are interested in the bulk solution given by the first term in Eq. (11).

Our next step is to calculate the spin-polarization density associated with triplet components of the pairing function. This polarization is given by

$$S^i(\boldsymbol{r}) = \frac{i}{2}\sum_{\boldsymbol{k}}\int\frac{d\omega}{2\pi}n_F(\omega)\times\mathrm{Tr}\{\sigma^i[G^r_{\boldsymbol{k}11}(\omega,\boldsymbol{r}) - G^a_{\boldsymbol{k}11}(\omega,\boldsymbol{r})]\}, \tag{12}$$

where n_F is the equilibrium Fermi distribution function. It is easy to see that the nonzero value of Eq. (12) is provided by triplet components of anomalous Green's functions, which contribute to G_{11} with a correction term $\propto f^2$. Up to the leading second order with respect to f_s and keeping only the linear terms of the triplet f_m $(m=1,-1,0)$, for the retarded function we obtain from Eqs. (1)–(4),

$$\sum_{\boldsymbol{k}}\mathrm{Tr}[\sigma^i G^{r/a}_{\boldsymbol{k}11}] = \frac{\mp 1}{\pi N_F}\left[\frac{i\delta^{iz}}{2}(f_0^{r/a}f_s^{+r/a} - f_s^{r/a}f_0^{+r/a}) + \frac{1}{\sqrt{2}}(f_i^{r/a}f_s^{+r/a} + f_s^{r/a}f_i^{+r/a})\right], \tag{13}$$

where $f_y=(f_1+f_{-1})/2$ and $f_x=-i(f_1-f_{-1})/2$. The conjugate functions $f^+(\omega)=-f^*(-\omega)$.

In the case of Rashba SOI $f_x=f_0=0$ and $f_y=f_1$. The latter is given by Eq. (11). Then, from Eq. (13) it immediately follows that only the y projection of the spin density is finite. Using the relations $f_s^a(\omega)=f_s^r(-\omega)$ and $f_m^a(\omega)=-f_m^r(-\omega)$ $(m=1,-1,0)$, we arrive to the spin polarization,

$$S^y(x) = eN_F\alpha\tau\frac{J}{\sigma_{\rm dc}}, \tag{14}$$

where $\sigma_{\rm dc}$ is the dc conductivity of the normal metal and J is the Josephson current density,

$$J = \frac{eD}{4\pi^2N_F}\int d\omega n_F(\omega)\left[\left(f_s^r\frac{\partial f_s^{+r}}{\partial x} - \frac{\partial f_s^r}{\partial x}f_s^{+r}\right) - (r \rightleftarrows a)\right]. \tag{15}$$

The spin polarization in Eq. (14) coincides with polarization induced in normal metals by the electric field E,[2] if the Josephson current is substituted for the normal dissipative dc current $J_{\rm dc}=\sigma_{\rm dc}E$. Similar effect has been predicted by Edelstein[6] for bulk superconductors and at NS boundary, providing the supercurrent flows along the SN interface.

Let us now check, if the analogy with the electric spin orientation extends to the spin Hall effect. Hence, our goal is to calculate J_y^z, which is the y projection of a spin flux polarized in the z direction. The corresponding spin current operator can be written as $J_y^z=\{\sigma_z,\mathrm{v}_y\}/2$ where the velocity $v_y=k_y/m^*+\partial(\boldsymbol{\sigma}\cdot\boldsymbol{h_k})/\partial k_y$. Since it has been assumed that $h_z=0$, one gets $J_y^z=\sigma_z k_y/m^*$. The spin Hall current $J_{\rm sH}$, in its turn, can be derived from Eq. (12) with σ^i substituted for J_y^z. Keeping the same leading terms as in calculation of the spin density, we arrive at $J_{\rm sH}=0$. This result does not depend on whether $\boldsymbol{h_k}$ is given by the Rashba or Dresselhaus interactions. That is very distinct from the normal spin Hall effect, where in the diffusive regime the spin Hall conductance is zero for the Rashba SOI, but finite for the cubic Dresselhaus interaction.[19] In general, as it was discussed above, the zero value of J_{sH} in superconducting transport follows from the time inversion symmetry.

Besides J_{sH}, in normal systems the dc current together with SOI gives rise to the accumulation of the z component of spin at the lateral edges of the sample.[16,20–22] In the case of the Josephson junction, the z projection of the spin density is given by Eq. (12) and the first term in Eq. (13). Hence, it is proportional to the f_0 component of the pairing function which, in its turn, can be found from Eq. (8), which near lateral edges of the sample, takes the form

$$0 = D\frac{\partial^2 f_0}{\partial y^2} + 4i\langle v_y \mathbf{h}_{\boldsymbol{k}}\rangle\left(\mathbf{J}_{01}\frac{\partial f_1}{\partial y} + \mathbf{J}_{0,-1}\frac{\partial f_{-1}}{\partial y}\right) - 2\Gamma_{\mathrm{so}} f_0,$$

$$0 = D\frac{\partial^2 f_i}{\partial y^2} + 4i\langle v_y \mathbf{h}_{\boldsymbol{k}}\rangle \mathbf{J}_{i0}\frac{\partial f_0}{\partial y} - \Gamma_{\mathrm{so}}(f_i - f_i^b). \qquad (16)$$

where the subscript $i=\pm 1$ and $f_i^b = -\mathrm{M}_i f_s/\Gamma_{\mathrm{so}}$. For simplicity we assumed that $\langle h_{\boldsymbol{k}}^x h_{\boldsymbol{k}}^y\rangle = 0$, which does not take place if SOI is, for example, the sum of Rashba and Dresselhaus interactions. The boundary conditions to this equation depend on the type of the boundary as discussed in Refs. 16, 21, and 22. Let us consider hard wall boundaries of 2DEG at $y = \pm L_y/2$. In this case one can borrow the boundary conditions for Eq. (8) from Refs. 16 and 21. In normal systems these conditions correspond to the vanishing spin current at $y=\pm L_y/2$. In our case similar equations can be written for triplet "currents" $\boldsymbol{j}=\Sigma_k \boldsymbol{v} F$. We thus have $j^y|_{y=\pm L_y/2}=0$ where the zero triplet component is given by

$$j_0^y = -D\frac{\partial f_0}{\partial y} - 2i\tau\langle\{v_y[\mathbf{h}_{\boldsymbol{k}} \times (\sqrt{2}\boldsymbol{f} + \tau\delta\mathbf{h}_{\boldsymbol{k},\hat{\mathbf{q}}} f_s)]\}\rangle, \qquad (17)$$

where $\boldsymbol{f}=(f_x,f_y)$. The first term in this equation is the diffusive current, the first term in the brackets is determined by the spin precession in the effective spin-orbit field, and the last term is associated with the singlet-triplet coupling. The following analysis of Eq. (16) with the above boundary conditions is the same as for SHE in normal systems. Indeed, in the case of Rashba SOI, taking $f_x=0$ and $f_y=f_1=f_{-1}\equiv f_{\pm 1}^b$ in the form of the first term of Eq. (11), one can easily see that the second term in Eq. (17) vanishes. Hence, the solution of Eq. (16) is simply $f_0=0$ and $f_i=f_i^b$. This, according to Eqs. (12) and (13), leads to $S_z=0$. For other type of boundaries the resulting z polarization may be finite.[22] It is also finite in the case of the hard wall boundary and cubic Dresselhaus SOI $h_{\boldsymbol{k}}^x=\gamma k_x(k_y^2-\kappa^2)$ and $h_{\boldsymbol{k}}^y=-\gamma k_y(k_x^2-\kappa^2)$. It is easy to see that for this interaction the second term in Eq. (17) does not turn to zero. Calculating the spin-charge coupling in Eq. (9) and $f_{\pm 1}^b$, the solution of Eq. (8) can be expressed in the form $f_0 = \chi(y)(\partial f_s/\partial x)$ where χ is a real function of y. Then, Eqs. (12), (13), and (15) give

$$S^z = -eN_F\chi(y)(J/\sigma_{\mathrm{dc}}). \qquad (18)$$

The numerical plots for the function χ, in their turn, can be taken from Fig. 1 of Ref. 16.

IV. DISCUSSION

For a numerical evaluation of the spin polarization, we write an expression for the critical Josephson current through a long SNS junction in the form[23] $J_c = a\sigma_{\mathrm{dc}}\epsilon_T/eL$ where $\epsilon_T = D/L^2$ is the Thouless energy and a is a dimensionless parameter. The latter increases up to ~ 10 at $k_BT < \epsilon_T$ and decreases at higher temperatures. In its turn, the spin polarization in Eq. (14) can be written in the form $S^y = N_F\Delta_0/2$, where Δ_0 is an effective spin splitting, which has been measured in Ref. 24 by the Faraday rotation method. The z component of the spin polarization is of the same order of magnitude as S^y because Eqs. (16) and (17) acquire a dimensionless form when y is measured in units of L_{so} and $f_0, f_{\pm 1}$ are rescaled as f_0/f_1^b, $f_{\pm 1}/f_1^b$. Substituting the above expression for the Josephson current into Eq. (14) and taking into account that $D\tau = l^2/2$, we get

$$\Delta_0 = \alpha a \frac{l^2}{L^3}. \qquad (19)$$

Taking the Rashba parameter of an asymmetric InAs-based quantum well[25] $\alpha = 5\times 10^{-12}$ eV m, for $L=1$ μm, $l = 0.1$ μm, and $a\sim 5$ at $\Delta/\epsilon_T=100$, $k_BT/\epsilon_T=5$ (see Ref. 23), we arrive at $\Delta_0=0.25$ μeV. For comparison, we note that the Faraday rotation method[24] allows to measure even smaller values of Δ_0. It should be noted that the above number is evaluated on the verge of the diffusion approximation applicability since L_{so} is only three times larger than l. Also L_{so} is not sufficiently small compared to L and L_T so that the approximation of the strong SOI is not very accurate. An obvious trend, however, is that the spin density increases with higher l, stronger SOI, and larger a. Therefore, a theory valid in the case of short ballistic junctions carrying high Josephson currents is necessary to study the regime where SHE is able to induce high spin densities. It should be taken into account that at the higher magnetization, the depairing effect due to an exchange interaction ignored in Eqs. (5) and (8) might become important. The exchange Zeeman energy produced by polarized electrons can be evaluated as $E_{ex} \sim e^2 S/(k_F\epsilon_0)$. Taking $\Delta_0=0.25$ μeV and $k_F a_B^* \sim 1$, where the effective Bohr radius is $a_B^* = \hbar^2\epsilon_0/m^* e^2$, with $\epsilon_0 \sim 10$, $m^*/m = 0.23$ we obtain $E_{\mathrm{ex}} \sim 4\cdot 10^{-2}$ μeV. This energy is much less than ϵ_T and, hence, the Cooper pair energy 2ω in the Usadel Eq. (8). Therefore, for the chosen parameters this depairing effect can be ignored.

In conclusion, the stationary spin Hall effect induced by a supercurrent across an SNS junction has been studied in the diffusive regime for a relatively strong SOI in the 2D junction. We found out that the spin Hall current is forbidden by the time inversion symmetry. On the other hand, the out-of-plane magnetization accumulates at lateral edges in a very close analogy to SHE in normal systems. Also, similar to the electric spin orientation, the spin polarization parallel to 2DEG is finite throughout the entire N region. On the other hand, such a close analogy takes place only in the considered above limiting case of a long junction. We also expect a behavior quite distinct from SHE in normal systems in the case of the Josephson effect driven by a dc electric bias.

ACKNOWLEDGMENTS

This work was supported by Russian RFBR with Grant No. 060216699, by Taiwan NSC with Grant No. 96-2811-M-009-038–MY3 and NCTS with Grant No. 93-2119-M-007-002, and by MOE-ATU Grant.

[1] H.-A. Engel, E. I. Rashba, and B. I. Halperin, in *Handbook of Magnetism and Advanced Magnetic Materials*, edited by H. Kronmüller and S. Parkin (Wiley, Chichester, UK, 2007).
[2] V. M. Edelstein, Solid State Commun. **73**, 233 (1990); F. T. Vas'ko and N. A. Prima, Sov. Phys. Solid State **21**, 994 (1979); L. S. Levitov, Yu. V. Nazarov, and G. Eliashberg, Zh. Eksp. Teor. Fiz. **88**, 229 (1985) [Sov. Phys. JETP **61**, 133 (1985)]; A. G. Aronov and Yu. B. Lyanda-Geller, Pis'ma Zh. Eksp. Teor. Fiz. **50**, 398 (1989) [JETP Lett. **50**, 431 (1989)].
[3] E. V. Bezuglyi, A. S. Rozhavsky, I. D. Vagner, and P. Wyder, Phys. Rev. B **66**, 052508 (2002); E. A. Demler, G. B. Arnold, and M. R. Beasley, *ibid.* **55**, 15174 (1997); S. Oh, Y.-H. Kim, D. Youm, and M. R. Beasley, *ibid.* **63**, 052501 (2000); F. S. Bergeret, A. F. Volkov, and K. B. Efetov, *ibid.* **75**, 184510 (2007).
[4] L. Dell'Anna, A. Zazunov, R. Egger, and T. Martin, Phys. Rev. B **75**, 085305 (2007); I. V. Krive, L. Y. Gorelik, R. I. Shekhter, and M. Jonson, Fiz. Nizk. Temp. **30**, 535 (2004); [Low Temp. Phys. **30**, 398 (2004)]; Z. H. Yang, Y. H. Yang, J. Wang, and K. S. Chan, J. Appl. Phys. **103**, 103905 (2008).
[5] O. V. Dimitrova and M. V. Feigel'man, JETP **102**, 652 (2006).
[6] V. M. Edelstein, Phys. Rev. B **67**, 020505(R) (2003); Phys. Rev. Lett. **75**, 2004 (1995).
[7] L. P. Gor'kov and E. I. Rashba, Phys. Rev. Lett. **87**, 037004 (2001).
[8] Yu. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984).
[9] G. Dresselhaus, Phys. Rev. **100**, 580 (1955).
[10] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005); A. A. Golubov, M. Yu. Kupriyanov, and E. Ilichev, *ibid.* **76**, 411 (2004).
[11] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1975).
[12] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, New York, 1981).
[13] P. Pfeffer and W. Zawadzki, Phys. Rev. B **52**, R14332 (1995); W. Zawadzki and P. Pfeffer, Semicond. Sci. Technol. **19**, R1 (2004).
[14] A. G. Mal'shukov and K. A. Chao, Phys. Rev. B **61**, R2413 (2000).
[15] E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. **93**, 226602 (2004); A. A. Burkov, A. S. Nunez, and A. H. MacDonald, Phys. Rev. B **70**, 155308 (2004).
[16] A. G. Mal'shukov, L. Y. Wang, C. S. Chu, and K. A. Chao, Phys. Rev. Lett. **95**, 146601 (2005).
[17] M. I. D'yakonov and V. I. Perel', Zh. Eksp. Teor. Fiz. **60**, 1954 (1971) [Sov. Phys. JETP **33**, 1053 (1971)].
[18] M. Yu. Kupriyanov and V. F. Lukichev, Zh. Eksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].
[19] A. G. Mal'shukov and K. A. Chao, Phys. Rev. B **71**, 121308(R) (2005).
[20] G. Usaj and C. A. Balseiro, Europhys. Lett. **72**, 631 (2005); R. Raimondi, C. Gorini, P. Schwab, and M. Dzierzawa, Phys. Rev. B **74**, 035340 (2006).
[21] O. Bleibaum, Phys. Rev. B **74**, 113309 (2006).
[22] V. M. Galitski, A. A. Burkov, and S. Das Sarma, Phys. Rev. B **74**, 115331 (2006); İ. Adagideli and G. E. W. Bauer, Phys. Rev. Lett. **95**, 256602 (2005); A. Brataas, A. G. Mal'shukov, and Ya. Tserkovnyak, New J. Phys. **9**, 345 (2007); A. G. Mal'shukov, L. Y. Wang, and C. S. Chu, Phys. Rev. B **75**, 085315 (2007).
[23] P. Dubos, H. Courtois, B. Pannetier, F. K. Wilhelm, A. D. Zaikin, and G. Schön, Phys. Rev. B **63**, 064502 (2001).
[24] Y. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Nature (London) **427**, 50 (2004).
[25] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997).