HYDROLOGICAL PROCESSES
Hydrol. Process. **22**, 3741–3748 (2008)
Published online 31 January 2008 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/hyp.6978

Semi-analytical solution for a slug test in partially penetrating wells including the effect of finite-thickness skin

Hund-Der Yeh,[1]* Yen-Ju Chen[1] and Shaw-Yang Yang[2]

[1] *Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan*
[2] *Department of Civil Engineering, Vanung University, Chungli, Taiwan*

Abstract:

This paper presents a new semi-analytical solution for a slug test in a well partially penetrating a confined aquifer, accounting for the skin effect. This solution is developed based on the solution for a constant-flux pumping test and a formula given by Peres and co-workers in 1989. The solution agrees with that of Cooper and co-workers and the KGS model when the well is fully penetrating. The present solution can be applied to simulate the temporal and spatial head distributions in both the skin and formation zones. It can also be used to demonstrate the influences of skin type or skin thickness on the well water level and to estimate the hydraulic parameters of the skin and formation zones using a least-squares approach. The results of this study indicate that the determination of hydraulic conductivity using a conventional slug-test data analysis that neglects the presence of a skin zone will give an incorrect result if the aquifer has a skin zone.

KEY WORDS groundwater; slug test; constant-flux pumping test; skin effect; partially penetrating well; Laplace transform; finite Fourier cosine transform

Received 14 April 2007; Accepted 27 November 2007

INTRODUCTION

A slug test is performed by suddenly removing/(adding) a small amount of water from/(into) the test well and measuring the change of water level at that well simultaneously. The measured test data can then be analysed to determine the aquifer parameters, e.g. hydraulic conductivity and specific storage, representing the hydraulic characteristics around the vicinity of the wellbore. The slug test is commonly used in aquifer-site characterization because of the advantages of low cost, ease of implementation and short test duration, absence of need for post-treatment of a large volume of contaminated water, and relatively minor disturbance to the groundwater flow system.

A skin is considered as a zone of distinct hydraulic conductivity adjacent to the wellbore face caused by a well drilling or completion. A positive skin (also called a low-permeability skin), which usually arises from damage caused by well drilling and incomplete well development, is a zone adjacent to the wellbore with lower hydraulic conductivity than the undisturbed formation. In contrast, a negative skin (also called a high-permeability skin) refers to a zone with higher hydraulic conductivity than the undisturbed formation owing to excessive well completion. Since the slug-test data reflects the hydraulic characteristics of the aquifer near the test well, ignoring the existence of a wellbore skin can lead to significant errors in parameter determination from test-data analysis. In addition, the slug test well commonly partially penetrates the aquifer for most engineering practices. Because the water flux towards a partially penetrating well includes both horizontal and vertical components, the head response in a partially penetrating well differs from that in a fully penetrating well.

A number of mathematical models used to analyse slug test data have been published over the past four decades. For a fully penetrating well, Cooper *et al.* (1967) presented an analytical solution and proposed a corresponding type-curve method considering a finite-diameter well in a confined aquifer. However, the lack of sensitivity in the type-curve match limited the ability to correctly determine the storage coefficient. With regard to the wellbore-skin effect, Faust and Mercer (1984) investigated the effect of a finite-thickness skin on the response of slug tests using a simple analytical solution and numerical modelling. Following the suggestion of Faust and Mercer (1984), Moench and Hsieh (1985) presented a semi-analytical solution for the slug test and utilized a numerical inversion method to generate the type curves. They used type curves to illustrate the influence of a finite-thickness skin on the open well and pressurized slug tests. Yang and Gates (1997) analysed the effect of wellbore skin on slug test results by utilizing both finite-element modelling and field tests. Their results showed that the early- and late-time data, respectively, reflect the groundwater flow within the skin and undisturbed

* Correspondence to: Hund-Der Yeh, Institute of Environmental Engineering, National Chiao Tung University, 75 Po-Ai Street, Hsinchu 30039, Taiwan. E-mail: hdyeh@mail.nctu.edu.tw

formation zones. Yeh and Yang (2006) proposed an analytical solution and numerical results for a slug test performed in a confined aquifer with finite-thickness skin. Later, Yang and Yeh (2007) compared the numerical evaluation of the semi-analytical solution proposed by Moench and Hsieh (1985) with the finite-element model presented by Yang and Gates (1997), simulating a slug test performed in a two-zone confined aquifer.

For a partially penetrating well, Dougherty and Babu (1984) proposed a semi-analytical solution for a slug test in a confined aquifer system, taking into account the skin effect. In their mathematical model, a skin factor, without an exact skin thickness, was used to sum the effect of a disturbed zone around the wellbore. Hyder *et al*. (1994) extended the Dougherty and Babu (1984) solution and developed a mathematical model incorporating the effects of well partial penetration, aquifer anisotropy, and finite-thickness skin in a confined or unconfined aquifer system using a semi-analytical solution.

Peres *et al*. (1989) proposed a different approach for slug test data analysis. They developed a theoretical formula relating the slug test solution and the solution of constant-flux pumping test in a well fully penetrating a confined aquifer system by considering the effect of wellbore storage. The relationship describing the time derivative of the constant-flux solution was equivalent to the slug test solution. Moreover, using Duhamel's theorem, they pointed out that their formula was applicable to various well/aquifer constructions. Based on the formula given by Peres *et al*. (1989) and the solution of a constant-flux pumping test, this paper develops a new slug test solution to describe the head distribution in a confined aquifer system.

The literature regarding the constant-flux pumping test is briefly reviewed below. Novakowski (1989) proposed a semi-analytical solution for a constant-flux pumping test in a well fully penetrating a confined aquifer system, considering skin effects and wellbore storage. Furthermore, he presented a series of type curves, obtained from a numerical inversion of the Laplace-domain solution, to illustrate the influences of skin, wellbore storage, and radial distances on the drawdown distributions. Yeh *et al*. (2003) developed an analytical solution for constant-flux pumping test conducted in a fully penetrating well in a two-zone confined aquifer system. Yang *et al*. (2006) developed an analytical solution for the constant-flux pumping test conducted in a partially penetrating well in a confined aquifer system. Note that the use of the formula given by Peres *et al*. (1989) in obtaining the slug test solution needs a constant-flux solution considering the effects of wellbore storage and the presence of a well partially penetrating a radial two-zone confined aquifer system.

The objective of this paper is to develop a new mathematical model for the slug test in a partially penetrating well, taking into consideration the effect of wellbore skin. The Laplace-domain solution of a constant-flux pumping test in a well partially penetrating a radial two-zone confined aquifer system taking into account the effects of wellbore storage is derived first. A series of integral-transform techniques, i.e. Laplace transforms, finite Fourier cosine transforms, and inverse finite Fourier cosine transforms are applied to obtain the constant-flux solution. The slug test solution in the Laplace domain is then obtained by applying the formula given by Peres *et al*. (1989) to the solution of the constant-flux pumping test. A method of numerical Laplace inversion, the routine DINLAP of IMSL (1997) based on the modified Crump algorithm, is employed to generate the time-domain results. This routine has been applied successfully in other groundwater problems (Yang and Yeh, 2002, 2005; Yang *et al*., 2006). Finally, the slug test solution is used to investigate the effects of wellbore skin and the presence of a partially penetrating well on the water level at the test well.

MATHEMATICAL DEVELOPMENT

This section contains two parts: in the first part, a mathematical model is proposed for a constant-flux pumping test in a well partially penetrating the confined aquifer system, taking into account wellbore storage and the skin effect. In the second part, the solution of the constant-flux pumping test is converted into the slug test solution based on the formula given by Peres *et al*. (1989).

Mathematical model of constant-flux pumping test

Figure 1 illustrates the well and aquifer configurations. The assumptions for the constant-flux pumping test model are: (a) the wellbore skin is of finite thickness, homogeneous, and anisotropic throughout the entire thickness of the aquifer; (b) the undisturbed zone is confined, homogeneous, anisotropic, laterally infinite, and vertically finite with a constant thickness; (c) the test well is partially penetrated and has a finite radius; and (d) the initial hydraulic head is constant and uniform throughout the whole aquifer. The governing equations for the skin

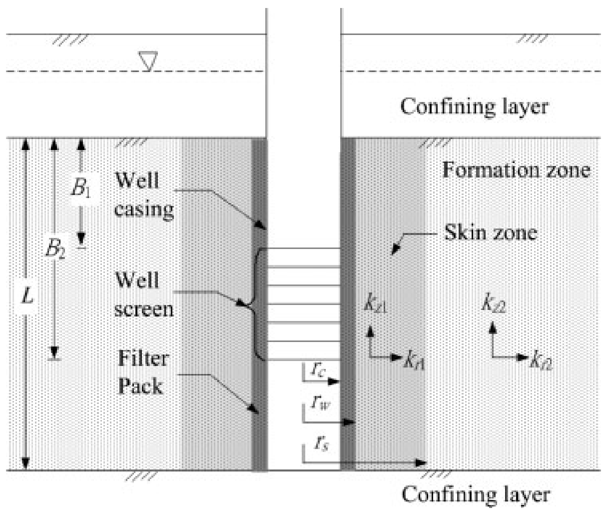

Figure 1. Schematic representation of the partially penetrating well and radial two-zone aquifer system

zone and the formation zone are, respectively (Yang and Yeh, 2005)

$$k_{r1}\left(\frac{\partial^2 h_{c1}}{\partial r^2}+\frac{1}{r}\frac{\partial h_{c1}}{\partial r}\right)+k_{z1}\frac{\partial^2 h_{c1}}{\partial z^2}=S_{s1}\frac{\partial h_{c1}}{\partial t},\ \ r_w\le r\le r_s \tag{1}$$

and

$$k_{r2}\left(\frac{\partial^2 h_{c2}}{\partial r^2}+\frac{1}{r}\frac{\partial h_{c2}}{\partial r}\right)+k_{z2}\frac{\partial^2 h_{c2}}{\partial z^2}=S_{s2}\frac{\partial h_{c2}}{\partial t},\ \ r_s\le r<\infty \tag{2}$$

where h_c is the hydraulic head during a constant-flux pumping test, the subscripts 1 and 2 denote the skin zone and formation zone, respectively, r is the radial distance from the centreline of the pumping well, r_w is the well radius, r_s is the outer radius of the skin zone, z is the vertical direction, t is the time from the start of test, S_S is the specific storage, and k_r and k_z are the radial and vertical components of hydraulic conductivity, respectively.

Equations (1) and (2) are subject to the following boundary conditions. The initial hydraulic heads within the skin and formation zones are both equal to zero, i.e.

$$h_{c1}(r,z,0)=h_{c2}(r,z,0)=0,\ \ r>r_w \tag{3}$$

At an infinite distance from the test well, the hydraulic head is also equal to zero.

$$h_{c2}(\infty,z,t)=0 \tag{4}$$

For a confined aquifer system, impermeable boundaries exist at the bottom and top of the aquifer, thus

$$\frac{\partial h_{c1}(r,0,t)}{\partial z}=\frac{\partial h_{c2}(r,0,t)}{\partial z}=0 \tag{5}$$

and

$$\frac{\partial h_{c1}(r,L,t)}{\partial z}=\frac{\partial h_{c2}(r,L,t)}{\partial z}=0 \tag{6}$$

where L is the thickness of the aquifer. The conservation of mass at the interface between the skin and formation zones can be described by

$$k_{r1}\frac{\partial h_{c1}(r_s,z,t)}{\partial r}=k_{r2}\frac{\partial h_{c2}(r_s,z,t)}{\partial r} \tag{7}$$

Furthermore, the conservation of mass at the test well is approximated as

$$\left(Q-\pi r_c^2\frac{\partial h_{c1}(r_w,z,t)}{\partial t}\right)[U(z-B_1)-U(z-B_2)]=-2\pi r_w k_{r1}(B_2-B_1)\left(\frac{\partial h_{c1}(r,z,t)}{\partial r}\right)_{r=r_w} \tag{8}$$

where Q is a constant discharge pumped from the test well, B_1 and B_2 denote the top and bottom z-coordinates of the screen, respectively, $U\ (z-B_i)$ equals one when $z\ge B_i$ and zero when $0\le z\le B_i$ for $i=1$ or 2. Note that Equation (8) assumes that the flow rate along the well screen is uniform. The second term on the left-hand side (LHS) of Equation (8) reflects the effect of wellbore storage and the term on the right-hand side (RHS) represents the total flux flows across the wellbore screen.

Applying the Laplace transform and finite Fourier cosine transform, Equations (1) and (2), respectively, can be reduced to

$$\frac{\partial^2 \overline{h}_{c1}}{\partial r^2}+\frac{1}{r}\frac{\partial \overline{h}_{c1}}{\partial r}=\alpha^2\overline{h}_{c1} \tag{9}$$

and

$$\frac{\partial^2 \overline{h}_{c2}}{\partial r^2}+\frac{1}{r}\frac{\partial \overline{h}_{c2}}{\partial r}=\beta^2\overline{h}_{c2} \tag{10}$$

where $\overline{h}_c$ indicates the Fourier–Laplace domain solution of hydraulic head, $\alpha=\sqrt{(S_{s1}p+k_{z1}{w_n}^2)/k_{r1}}$ and $\beta=\sqrt{(S_{s2}p+k_{z2}{w_n}^2)/k_{r2}}$ with $w_n=n\pi/L$ for $n=1,2,\ldots$. The solutions of Equations (9) and (10) subject to the transformed boundary conditions of (4), (7), and (8) can be written, respectively, as

$$\overline{h}_{c1}(r,w_n,p)=\frac{QW_1[CI_0(\alpha r)+DK_0(\alpha r)]}{\pi {r_c}^2p^2\Omega-2\pi r_w pk_{r1}(B_2-B_1)\alpha\psi} \tag{11}$$

and

$$\overline{h}_{c2}(r,w_n,p)=\frac{QW_1EK_0(\beta r)}{\pi {r_c}^2p^2\Omega-2\pi r_w pk_{r1}(B_2-B_1)\alpha\psi} \tag{12}$$

where

$$W_1=\frac{1}{w_n}(\sin w_nB_2-\sin w_nB_1) \tag{13}$$

$$C=K_1(\alpha r_s)K_0(\beta r_s)-\left(\frac{k_{r2}}{k_{r1}}\right)\left(\frac{\beta}{\alpha}\right)K_0(\alpha r_s)K_1(\beta r_s) \tag{14}$$

$$D=I_1(\alpha r_s)K_0(\beta r_s)+\left(\frac{k_{r2}}{k_{r1}}\right)\left(\frac{\beta}{\alpha}\right)I_0(\alpha r_s)K_1(\beta r_s) \tag{15}$$

$$E=K_0(\alpha r_s)I_1(\alpha r_s)+K_1(\alpha r_s)I_0(\alpha r_s) \tag{16}$$

$$\Omega=CI_0(\alpha r_w)+DK_0(\alpha r_w) \tag{17}$$

and

$$\Psi=CI_1(\alpha r_w)-DK_1(\alpha r_w) \tag{18}$$

Furthermore, applying the finite Fourier cosine transforms to Equations (11) and (12), the hydraulic heads of the constant-flux pumping test within the skin and formation zones can then be described, respectively, as

$$\tilde{h}_{c1}(r,z,p)=\frac{1}{L}\left\{\frac{Q(B_2-B_1)[C_0I_0(\alpha_0 r)+D_0K_0(\alpha_0 r)]}{\pi {r_c}^2p^2\Omega_0-2\pi r_w pk_{r1}(B_2-B_1)\alpha_0\Psi_0}\right\}+\frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{QW_1[CI_0(\alpha r)+DK_0(\alpha r)]\cos(w_nz)}{\pi {r_c}^2p^2\Omega-2\pi r_w pk_{r1}(B_2-B_1)\alpha\Psi}\right\} \tag{19}$$

DOI: 10.1002/hyp

and

$$\tilde{h}_{c2}(r,z,p) = \frac{1}{L}\left\{\frac{Q(B_2-B_1)E_0K_0(\beta_0 r)}{\pi {r_c}^2 p^2\Omega_0 - 2\pi r_w p k_{r1}(B_2-B_1)\alpha_0\Psi_0}\right\} + \frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{QW_1\ E\ K_0(\beta\ r)\cos(w_n z)}{\pi {r_c}^2 p^2\Omega - 2\pi r_w p k_{r1}(B_2-B_1)\alpha\Psi}\right\} \quad (20)$$

where the subscript 0 shown in the variables α_0, β_0, C_0, D_0, E_0, Ω_0, Ψ_0 represents n equalling zero for the related variables.

Solution of slug test

Taking wellbore storage and skin effect into consideration for a well fully penetrating a confined aquifer system, Peres *et al.* (1989) provide a formula relating the slug test and constant-flux solutions in the Laplace domain as

$$\tilde{h}(r,z,p) = \frac{\pi {r_c}^2 H_0 p}{Q}\tilde{h}_c(r,z,p) \quad (21)$$

They showed that this relation is valid for any well/aquifer construction when employing Duhamel's theorem. Based on Equation (21), the constant-flux solutions, i.e. Equations (19) and (20), reduce to the Laplace-domain solutions of a slug test for the skin and formation zones, respectively, as:

$$\tilde{h}_1(r,z,p) = \frac{1}{L}\left\{\frac{\pi {r_c}^2 H_0(B_2-B_1)[C_0 I_0(\alpha_0 r) + D_0 K_0(\alpha_0 r)]}{\pi {r_c}^2 p\Omega_0 - 2\pi r_w k_{r1}(B_2-B_1)\alpha_0\Psi_0}\right\} + \frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{\pi {r_c}^2 H_0 W_1[CI_0(\alpha r) + DK_0(\alpha r)]\cos(w_n z)}{\pi {r_c}^2 p\Omega - 2\pi r_w k_{r1}(B_2-B_1)\alpha\Psi}\right\} \quad (22)$$

and

$$\tilde{h}_2(r,z,p) = \frac{1}{L}\left\{\frac{\pi {r_c}^2 H_0(B_2-B_1)E_0K_0(\beta_0 r)}{\pi {r_c}^2 p\Omega_0 - 2\pi r_w k_{r1}(B_2-B_1)\alpha_0\Psi_0}\right\} + \frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{\pi {r_c}^2 H_0 W_1 E\ K_0(\beta r)\cos(w_n z)}{\pi {r_c}^2 p\Omega - 2\pi r_w k_{r1}(B_2-B_1)\alpha\Psi}\right\} \quad (23)$$

In addition, the water level in the test well can be obtained through averaging the head in Equation (22) along the screened interval. That is

$$\tilde{h}_w(p) = \frac{(B_2-B_1)}{L}\left\{\frac{\pi {r_c}^2 H_0[C_0 I_0(\alpha_0 r_w) + D_0 K_0(\alpha_0 r_w)]}{\pi {r_c}^2 p\Omega_0 - 2\pi r_w k_{r1}(B_2-B_1)\alpha_0\Psi_0}\right\} + \frac{2}{(B_2-B_1)L}\sum_{n=1}^{\infty}\left\{\frac{\pi {r_c}^2 H_0 {W_1}^2[CI_0(\alpha r_w) + DK_0(\alpha r_w)]}{\pi {r_c}^2 p\Omega - 2\pi r_w k_{r1}(B_2-B_1)\alpha\Psi}\right\} \quad (24)$$

Since Equations (22)–(24) are rather complicated, the inversions of these solutions to the time domain may not be tractable analytically. The DINLAP routine of IMSL (1997), which is a code of modified Crump algorithm, with accuracy to five decimal places is employed to perform the numerical Laplace inversion.

RESULTS AND DISCUSSION

In this section, the newly derived solution (or model) is compared with the solutions of Cooper *et al.* (1967) and Hyder *et al.* (1994). The influence of a wellbore skin and the presence of a partially penetrating well on slug-test water level responses are investigated using the present solution. Assume that a slug test is operated in a confined and isotropic aquifer system. The aquifer thickness (L) is 20 m with hydraulic conductivity (k_{r2}) 10^{-4} m s^{-1} and specific storage (S_{s2}) 5×10^{-5} m^{-1}. The radii of the well (r_w) and casing (r_c) are 0·0915 m and 0·0508 m, respectively. The initial water level in a test well, H_0, is 1 m and the normalized head is defined as the well water level at any test time divided by the initial well water level.

Comparison with existing solutions

Cooper *et al.*'s solution (1967) was developed to describe the groundwater behaviour during a slug test performed in a finite-diameter well fully penetrating a confined aquifer system. Comparing the normalized head response determined by the present solution with that of Cooper *et al.* (1967), Figure 2 shows good agreement of the normalized head with the result obtained for the case of a slug test is performed in a well fully penetrating a confined aquifer without the presence of skin zone.

Hyder *et al.*'s solution (1994), also called the KGS model, was developed for a partially penetrating well constructed in a confined or unconfined aquifer system and with or without the presence of skin zone. They used a different representation for the mass balance condition at the wellbore; that is

$$-\pi r_c^2 \frac{dh_w(t)}{dt}[U(z-B_1) - U(z-B_2)] = -2\pi r_w k_{r1} (B_2-B_1)\left(\frac{\partial h_{c1}(r,z,t)}{\partial r}\right)_{r=r_w} \quad (25)$$

Notice that the RHS of Equation (25) is dependent on z, yet the LHS is not. The solution of Equation (1) with Equation (25) considered as the boundary condition for a slug test gives an approximate result for the well water level or hydraulic head in the skin zone. Figure 3 shows the simulated results given by the present solution and the KGS model (Hyder *et al*., 1994) for a slug test performed in a well with a wellbore skin. It is assumed that the skin zone has an outer radius of 0·305 m, hydraulic conductivity (k_{r1}) 10^{-3} m s^{-1}, 10^{-4} m s^{-1} or 10^{-5} m s^{-1}, and specific storage (S_{s1}) 5×10^{-5} m^{-1}. The case $k_{r1} = 10^{-4}$ m s^{-1} implies that the wellbore skin is absent. Figure 3a shows that the normalized head responses in a fully penetrating well simulated by the present solution coincide with those of the KGS model (Hyder *et al*., 1994) for positive skin, negative skin, and no skin cases. In contrast, the normalized head for a well partially penetrating a confined aquifer with screen length 6 m and 3 m are plotted in Figures 3b and 3c, respectively. The screens are located on the top of the confined aquifer. Figure 3c represents an extreme case, with penetration ratio only 0·15. In both figures, the normalized head simulated by the present solution is almost identical with those of the KGS model (Hyder *et al*., 1994), except that the negative-skin case produced by the present solution shown in Figure 3c gives slightly lower values at intermediate times compared with those of the KGS model (Hyder *et al*., 1994). These differences in normalized heads are attributed to the fact that the KGS model (Hyder *et al*., 1994) used the depth-averaging hydraulic head along the screened interval to represent the well water level and assumed the flux through the screen to be equal to the change of well water level with respect to time as the boundary condition of the screen part. In contrast, in the proposed mathematical model the hydraulic head was derived first and then the depth average of the hydraulic head was taken as the well water level. The differences in normalized heads simulated by the present solution and the KGS model (Hyder *et al*., 1994) may increase with the skin-zone hydraulic conductivity.

Effect of wellbore-skin type

Assume that a slug test is operated in a confined aquifer with a skin zone adjacent to the wellbore. The skin zone has an outer radius (r_s) of 0·305 m and specific storage (S_{s1}) of 5×10^{-5} m^{-1}. Four different radial hydraulic conductivities of the skin zone (k_{r1}) are considered: $k_{r1} = 10^{-3}$ m s^{-1} for a negative skin case, $k_{r1} = 10^{-4}$ m s^{-1} for a no skin case, and $k_{r1} = 10^{-5}$ m s^{-1} or 10^{-6} m s^{-1} for a positive skin case. The partially penetrating screen is located at $B_1 = 0$ m and $B_2 = 6{\cdot}0$ m. Figure 4a shows the influence of wellbore-skin type on the slug test data for fully and partially penetrating wells. The normalized head difference shown in Figure 4b represents the differences in normalized head between the cases discussed in Figure 4a and the no skin case for a fully penetrating well ($B_1 = 0$ m, $B_2 = 20{\cdot}0$ m and $k_{r1} = 10^{-4}$ m s^{-1}).

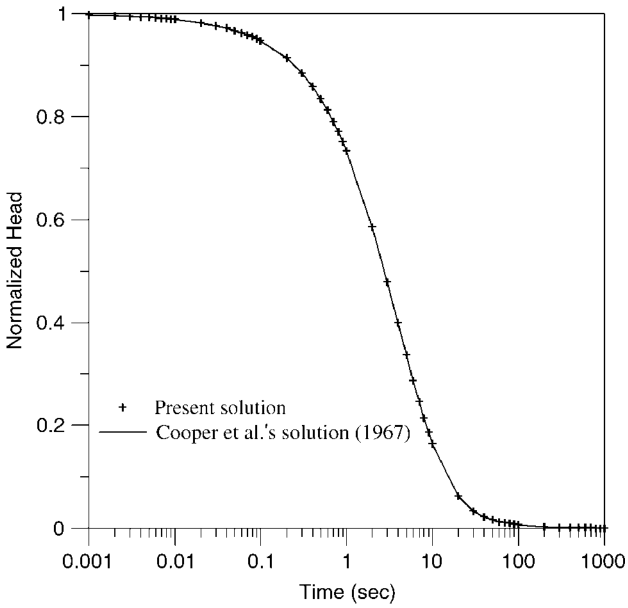

Figure 2. Plots of normalized head versus time determined by the present solution and Cooper *et al*.'s solution (1967) for a slug test performed in a fully penetrating well

Under a fully or partially penetrating well condition, Figure 4a shows that the recovery rate of water level at the test well for the positive skin case is remarkably slower than that for the no skin case, while the recovery rate for a negative skin case is moderately swifter than that for the no skin case. The slow recovery of the well water level for the positive skin case is attributed to the low hydraulic conductivity of the skin zone. Accordingly, it is noteworthy that ignoring the presence of a positive skin will result in underestimation of hydraulic conductivity determined from slug-test data analysis. On the other hand, the influence of a negative skin on the recovery rate of well water level is not so obvious; however, overestimation of hydraulic conductivity will occur if one disregards it. In short, the normalized head of a partially penetrating well is obviously slower than that of a fully penetrating well under the same aquifer condition. Thus, disregarding the presence of a partially penetrating well in the slug test data analysis will also cause underestimation of the hydraulic conductivity. Figure 4b shows that the largest normalized head difference is about 0·9, occurring at 20 s for the positive-skin case $k_{r1} = 10^{-6}$ m s^{-1} with a partially penetrating well. Significant underestimation of hydraulic conductivity will occur if one ignores the presence of positive skin, and treats the partially penetrating well as a fully penetrating well when analyzing slug test data.

Effect of wellbore skin thickness

The thickness of the skin zone may range from a few millimetres to several metres (Novakowski, 1989). Assume that a slug test performed in a confined aquifer system which has a skin zone outer radius (r_s) of 0·150 m, 0·305 m or 0·610 m. Fully and partially penetrating wells are both considered in the investigation.

DOI: 10.1002/hyp

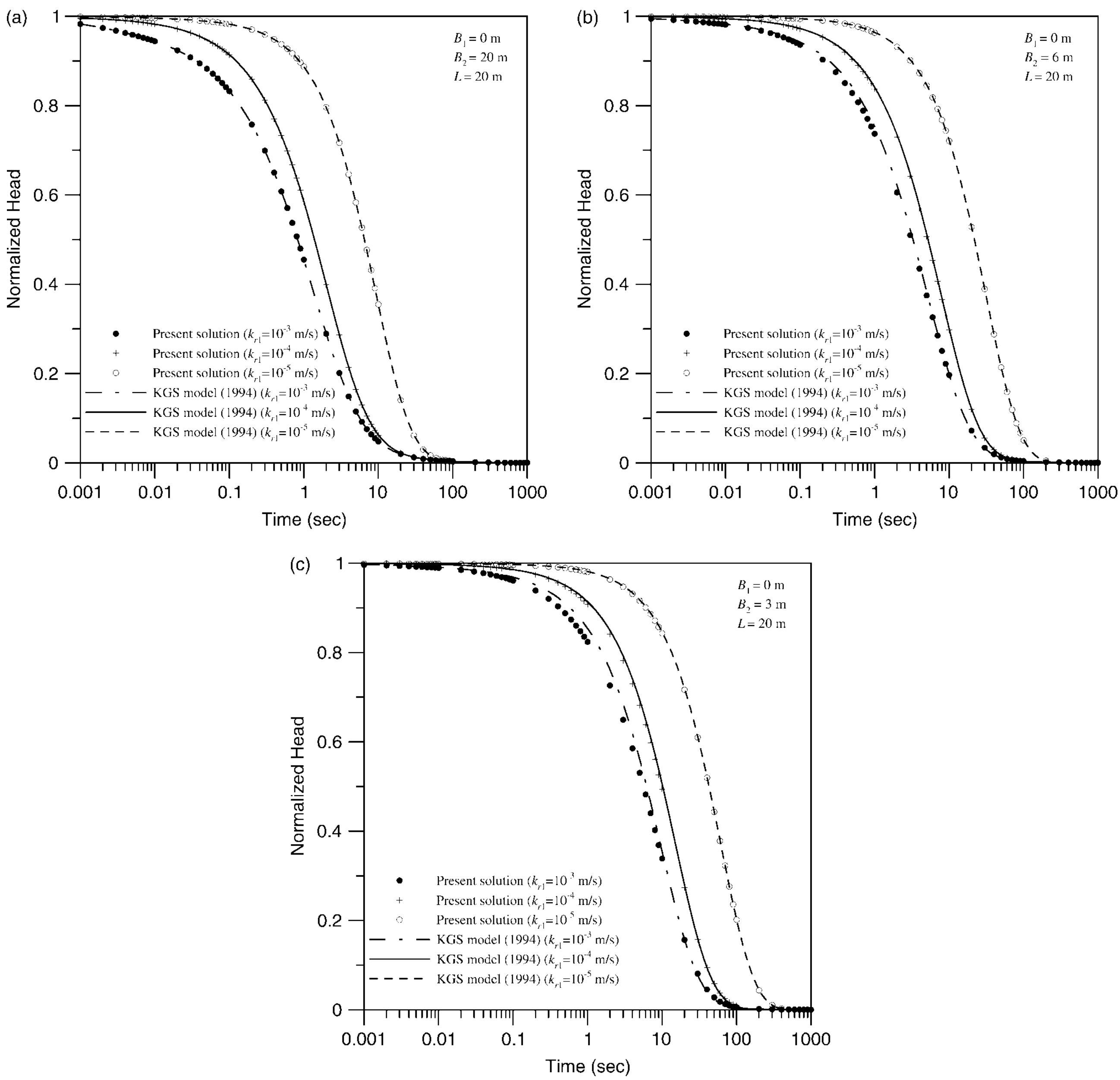

Figure 3. Plots of normalized head versus time determined by the present solution and the KGS model (Hyder *et al.*, 1994) for (a) a fully penetrating well, (b) a partially penetrating well with $B_1 = 0$ m and $B_2 = 6$ m and aquifer thickness $L = 20$ m, and (c) a partially penetrating well with $B_1 = 0$ m and $B_2 = 3$ m and aquifer thickness $L = 20$ m

The screen of the partially penetrating well is assumed to be perforated from $B_1 = 0$ m to $B_2 = 6{\cdot}0$ m. The influence of skin thickness on well water level is presented in Figure 5. Figure 5a illustrates the normalized head for a positive skin case with $k_{r1} = 10^{-5}$ m s^{-1}, while Figure 5b displays the negative skin case with $k_{r1} = 10^{-3}$ m s^{-1}.

Figure 5a displays curves of normalized head versus time for both fully and partially penetrating well cases. The figure shows that the normalized head decreases with increasing skin thickness for the positive skin case, therefore, the normalized head takes more time to recover in a thicker positive skin zone. Figure 5b shows the recovery rate of well water level increasing with skin thickness for the negative skin case. On the other hand, the recovery rate of well water level in a partially penetrating well is obviously slower than that in a fully penetrating well under the same aquifer conditions. In addition, the normalized head for the positive skin case shown in Figure 5a is stabilized more slowly than that for the negative skin case shown in Figure 5b, as mentioned in the previous section.

CONCLUSIONS

A new semi-analytical solution for a slug test in a partially penetrating well taking account of the wellbore skin effect has been developed via the formula of Peres *et al.* (1989). The present solution for the simulated normalized head is in good agreement with the solutions

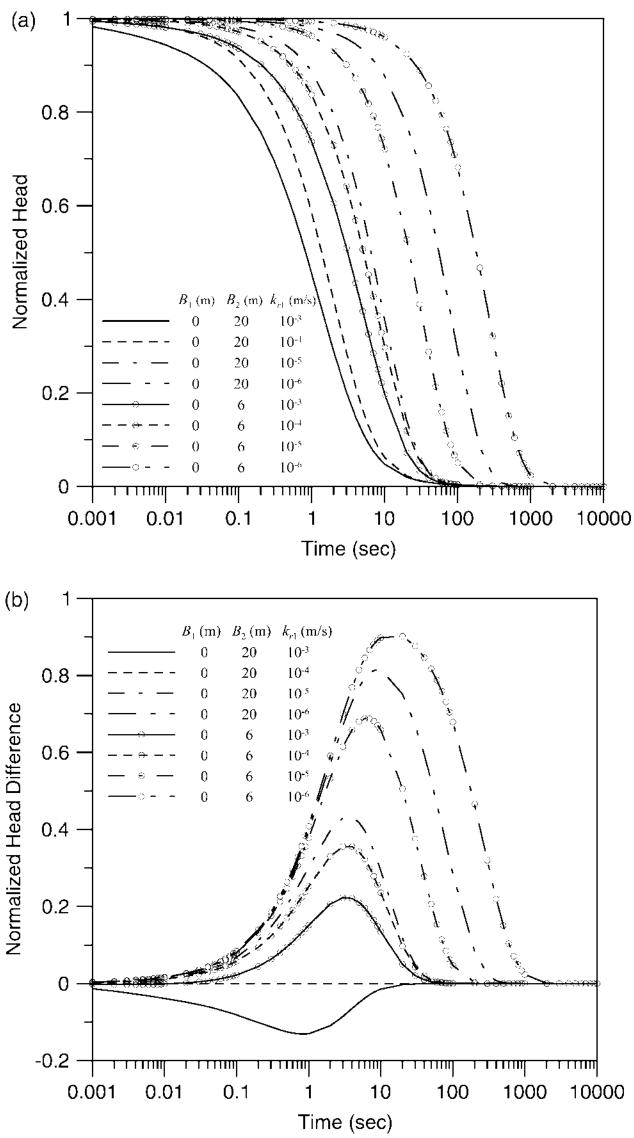

Figure 4. Plots for a fully penetrating well ($B_2 = 20$ m) or partially penetrating well ($B_2 = 6$ m) with skin hydraulic conductivities $K_{r1} = 10^{-3}$, 10^{-4}, 10^{-5} or 10^{-6} m s^{-1} and formation hydraulic conductivity $K_{r2} = 10^{-4}$ m s^{-1}. (a) Normalized head versus time. (b) Normalized head difference versus time when compared with the results obtained for the case of no skin and a fully penetrating well ($K_{r1} = 10^{-4}$ m s^{-1}, $B_1 = 0$ m and $B_2 = 20$ m)

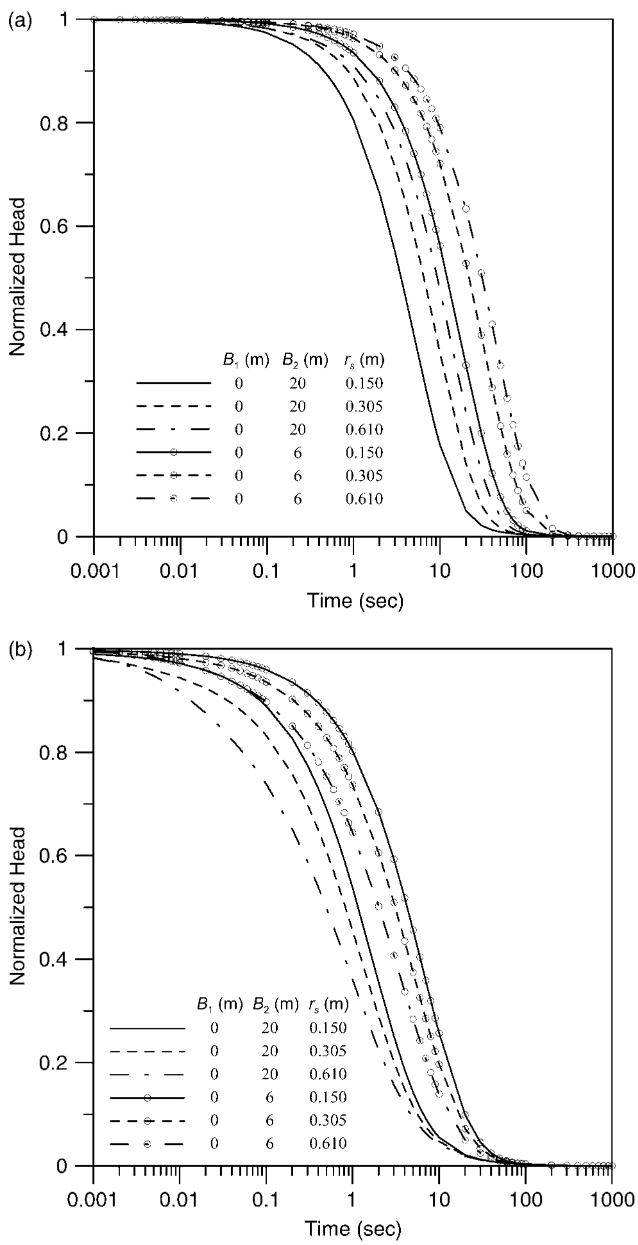

Figure 5. Plots of normalized head versus time in a fully or partially penetrating well ($B_1 = 0$ m and $B_2 = 6$ m) with $K_{r2} = 10^{-4}$ m s^{-1} and (a) a positive skin case with $K_{r1} = 10^{-5}$ m s^{-1}, (b) a negative skin case with $K_{r1} = 10^{-3}$ m s^{-1}. The outer radius of the skin zone is 0·150, 0·305 or 0·610 m

of Cooper *et al.* (1967) and the KGS model (Hyder *et al.*, 1994) in the case of a well fully penetrating a confined aquifer system. For the case of a partially penetrating well, a difference between the present solution and the KGS model (Hyder *et al.*, 1994) for the normalized head is due to different assumptions made for the boundary condition along the wellbore screen.

The new solution is used to examine the effect of wellbore skin on the well water level in a slug test. The recovery of water level during a slug test on an aquifer with a positive skin is slower than that of an aquifer without the skin; in contrast, the recovery of water level in an aquifer with a negative skin is more rapid than in an aquifer without the skin. The hydraulic conductivity of the formation determined from the recovery slug-test data analysis will be underestimated if one ignores the presence of a positive skin; while it will be overestimated if one disregards the presence of a negative skin. In addition, when the skin thickness increases, the water level recovery rate decreases for the positive skin case; on the other hand, it increases for the negative skin case. Obviously, the presence of a partially penetrating well also has an effect on the water level recovery rate in the well.

ACKNOWLEDGEMENTS

This study is partially supported by the Taiwan National Science Council under the grant NSC 94-2211-E -009-012.

REFERENCES

Cooper HH Jr, Bredehoeft JD, Papadopulos IS. 1967. Response of a finite–diameter well to an instantaneous charge of water. *Water Resources Research* **3**(1): 263–269.

Dougherty DE, Babu DK. 1984. Flow to a partially penetrating well in a double-porosity reservoir. *Water Resources Research* **20**(8): 1116–1122.

Faust CR, Mercer JW. 1984. Evaluation of slug tests in wells containing a finite-thickness skin. *Water Resources Research* **20**(4): 504–506.

Hyder Z, Butler JJ Jr., McElwee CD, Liu W. 1994. Slug tests in partially penetrating wells. *Water Resources Research* **30**(11): 2945–2957.

IMSL. 1997. *Math/Library*. Volumes 1and 2, Houston, Texas: Visual Numerics, Inc.

Moench AF, Hsieh PA. 1985. Analysis of slug test data in a well with finite thickness skin. US Geological Survey: Menlo Park, CA 94025; 17–29.

Novakowski KS. 1989. A composite analytical model for analysis of pumping tests affected by well bore storage and finite thickness skin. *Water Resources Research* **25**(9): 1937–1946.

Peres AMM, Onur M, Reynolds AC. 1989. A new analysis procedure for determining aquifer properties from slug test data. *Water Resources Research* **25**(7): 1591–1602.

Yang YJ, Gates TM. 1997. Wellbore skin effect in slug-test data analysis for low-permeability geologic materials. *Ground Water* **35**(6): 931–937.

Yang SY, Yeh HD. 2002. Solution for flow rates across the wellbore in a two-zone confined aquifer. *Journal of Hydraulic Engineering, ASCE* **128**(2): 175–183.

Yang SY, Yeh HD. 2005. Laplace-domain solutions for radial two-zone flow equations under the conditions of constant-head and partially penetrating well. *Journal of Hydraulic Engineering, ASCE* **131**(3): 209–216.

Yang SY, Yeh HD, Chiu PY. 2006. A closed form solution for constant flux pumping in a well under partial penetration condition. *Water Resources Research* **42**(5): W05502, DOI:10·1029/2004WR003889.

Yang SY, Yeh HD. 2007. On the solutions of modeling slug test performed in a two-zone confined aquifer. *Hydrogeology Journal* **15**(2): 297–305. DOI:10·1007/s10040-006-0100-x.

Yeh HD, Yang SY, Peng HY. 2003. A new closed-form solution for a radial two-layer drawdown equation for groundwater under constant-flux pumping in a finite-radius well. *Advanced Water Resources* **26**: 747–757.

Yeh HD, Yang SY. 2006. A novel analytical solution for a slug test conducted in a well with a finite-thickness skin. *Advanced Water Resources* **29**(10): 1479–1489.

NOTATION

$B_1 =$ distance from the top of aquifer to the top of test-well screen

$B_2 =$ distance from the top of aquifer to the bottom of test-well screen

$C = K_1(\alpha r_s)K_0(\beta r_s) - \left(\frac{k_{r2}}{k_{r1}}\right)\left(\frac{\beta}{\alpha}\right)K_0(\alpha r_s)K_1(\beta r_s)$

$C_0 = K_1(\alpha_0 r_s)K_0(\beta_0 r_s) - \left(\frac{k_{r2}}{k_{r1}}\right)\left(\frac{\beta_0}{\alpha_0}\right)K_0(\alpha_0 r_s)K_1(\beta_0 r_s)$

$D = I_1(\alpha r_s)K_0(\beta r_s) + \left(\frac{k_{r2}}{k_{r1}}\right)\left(\frac{\beta}{\alpha}\right)I_0(\alpha r_s)K_1(\beta r_s)$

$D_0 = I_1(\alpha_0 r_s)K_0(\beta_0 r_s) + \left(\frac{k_{r2}}{k_{r1}}\right)\left(\frac{\beta_0}{\alpha_0}\right)I_0(\alpha_0 r_s)K_1(\beta_0 r_s)$

$E = K_0(\alpha r_s)I_1(\alpha r_s) + K_1(\alpha r_s)I_0(\alpha r_s)$

$E_0 = K_0(\alpha_0 r_s)I_1(\alpha_0 r_s) + K_1(\alpha_0 r_s)I_0(\alpha_0 r_s)$

$H_0 =$ initial water level in a test well

$h_{c1} =$ hydraulic head of constant-flux pumping test within a skin zone

$h_{c2} =$ hydraulic head of constant-flux pumping test within a formation zone

$\tilde{h}_1 =$ hydraulic head of slug test within a skin zone in the Laplace domain

$\tilde{h}_2 =$ hydraulic head of slug test within a formation zone in the Laplace domain

$\tilde{h}_{c1} =$ hydraulic head of constant-flux pumping test within a skin zone in the Laplace domain

$\tilde{h}_{c2} =$ hydraulic head of constant-flux pumping test within a formation zone in the Laplace domain

$\tilde{h}_w =$ well water level of slug test in the Laplace domain

$\overline{h}_{c1} =$ hydraulic head of constant-flux pumping test within a skin zone in the Fourier–Laplace domain

$\overline{h}_{c2} =$ hydraulic head of constant-flux pumping test within a formation zone in the Fourier–Laplace domain

$I_0 =$ Modified Bessel functions of the first kind of order zero

$I_1 =$ Modified Bessel functions of the first kind of order one

$K_0 =$ Modified Bessel functions of the second kind of order zero

$K_1 =$ Modified Bessel functions of the second kind of order one

$k_{r1} =$ radial component of hydraulic conductivity in a skin zone

$k_{r2} =$ radial component of hydraulic conductivity in a formation zone

$k_{z1} =$ vertical component of hydraulic conductivity in a skin zone

$k_{z2} =$ vertical component of hydraulic conductivity in a formation zone

$L =$ thickness of aquifer

$p =$ Laplace variable

$Q =$ Constant-flow rate through the wellbore

$r =$ radial distance from the centreline of well

$r_c =$ radius of casing

$r_s =$ outer radius of a skin zone

$r_w =$ effective radius of test well

$S_{s1} =$ specific storage within a skin zone

$S_{s2} =$ specific storage within a formation zone

$t =$ time since the start of test

$U =$ unit step function

$W_1 = \frac{1}{w_n}(\sin w_n B_2 - \sin w_n B_1)$

$w_n = \frac{n\pi}{L}, n = 1, 2, \ldots$

$z =$ vertical direction

$\alpha = \sqrt{\left(\frac{S_{s1}}{k_{r1}}\right)p + \left(\frac{k_{z1}}{k_{r1}}\right)w_n^2}, n = 1, 2, \ldots$

$\alpha_0 = \sqrt{\left(\frac{S_{s1}}{k_{r1}}\right)p}$

$\beta = \sqrt{\left(\frac{S_{s2}}{k_{r2}}\right)p + \left(\frac{k_{z2}}{k_{r2}}\right)w_n^2}, n = 1, 2, \ldots$

$\beta_0 = \sqrt{\left(\frac{S_{s2}}{k_{r2}}\right)p}$

$\Omega = CI_0(\alpha r_w) + DK_0(\alpha r_w)$

$\Omega_0 = C_0 I_0(\alpha_0 r_w) + D_0 K_0(\alpha_0 r_w)$

$\Psi = CI_1(\alpha r_w) - DK_1(\alpha r_w)$

$\Psi_0 = C_0 I_1(\alpha_0 r_w) - D_0 K_1(\alpha_0 r_w)$